Problem 18

Ryan Burmeister

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$\sum_{i=1}^{n} p_i lg(\frac{1}{p_i})$ where n events are possible and each event has probability p_i. If assume a biased coing scenario with the probability of the coin landing on heads as p, then the probability of the coin landing on tails is $1-p$. If we expand the right side of the Shannon entropy equation with the biased coin scenario, you get $-plg(p) + (1-p)lg(\frac{1}{1-p})$. In the unbiased coin scenario ($p = .5$), the shannon entropy is 1. To calculate the number of flips necessary for a biased coin to obtain the same shannon entropy as an unbiased coin, one can solve for p. In the case where $p = .75$, the shannon entropy is ≈ 0.811278. Therefore, at least two flips are necessary to obtain one bit of information. In the more general sense, the least value of n for which the following equation holds true give the number of flips required for a biased coin to give the same information as an unbiased one, $-plg(p) + (1-p)lg(\frac{1}{1-p}) \geq \frac{1}{n}$.