Problem 19

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Entropy for a fair 3-sided die:

$$-\sum_{i=1}^{3} p_i lg(p_i) = -\frac{1}{3} lg(\frac{1}{3}) * 3 = -lg(\frac{1}{3}) = lg(3) \approx 1.585$$

Entropy for a weighted 6-sided die:

$$-\sum_{i=1}^{6} p_i lg(p_i) = \frac{1}{6} lg(6) + \frac{1}{9} lg(9) * 3 + \frac{1}{4} lg(4) * 2 = \frac{1}{6} lg(6) + \frac{1}{3} lg(9) + 1 \approx 2.487$$

An algorithm for simulating the fair 3-sided die with the weighted 6-sided die is below(1).

Algorithm 1 algorithm

```
while true: do
    roll die
    if roll == 2 or roll == 3 or roll == 4: then
        print 'A'
    else
        roll again
        if roll == 5 or roll == 6: then
            print 'B'
        else
            print 'C'
        end if
    end if
end while
```

This algorithm is correct because we can treat the two rolls as independent events. The probability of rollina an A is $3 * \frac{1}{9} = \frac{1}{3}$. The probability of rolling either B or C will each have a chance of occuring $\frac{2}{3}$ of the time (i.e. Prob(B or C) = Prob($\bar{\text{A}}$). The probability of B is then $\frac{2}{3} * (\frac{1}{4} + \frac{1}{4}) = \frac{1}{3}$. The probability of C can be calculated in a similar fashion, $\frac{2}{3} * (\frac{1}{6} + 3 * \frac{1}{9}) = \frac{1}{3}$.

The expected number of dice rolls can be computed through the expected value. $E[X] = \sum_{i=1}^{n} p_i v_i = \frac{1}{3} * 1 + \frac{2}{3} * 2 = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$. The entropy of the the algorithm per output is then: $\frac{5}{3}(\frac{1}{6}lg(6) + \frac{1}{3}lg(9) + 1) \approx 4.146$. The entropy is then compared to the scenario with our fair 3-sided die to compute the entropy loss. $\frac{\frac{5}{3}(\frac{1}{6}lg(6)+\frac{1}{3}lg(9)+1)-lg(3)}{\frac{5}{3}(\frac{1}{6}lg(6)+\frac{1}{3}(9)+1)} \approx \frac{4.146-1.585}{4.146} \approx 0.6177$.