UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 3 June 2011 1.30 pm to 3.00 pm

EXPERIMENTAL AND THEORETICAL PHYSICS (4B)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4B)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **4** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUANTUM CONDENSED MATTER PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) A diatomic linear chain comprises alternating atoms with masses m_1 and m_2 spaced by a distance a. Sketch the displacement patterns for the two longitudinal phonon modes with wavevector $\pi/(2a)$. What is the ratio of their frequencies? [4]

(b) Indium oxide doped with tin is widely used as a metallic conductor that is transparent across the visible spectrum. Provide an estimate for the upper limit to the number density of free electrons. [Hint: visible light covers the wavelength range 400–700 nm.] [4]

(c) Electrons in a certain metal show a lifetime between elastic scattering events of 10^{-12} s at low temperature. What value of magnetic field is needed in order to observe magnetic quantisation of electron energy levels? [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) Fermi liquids;

(b) the field-effect transistor;

(c) coordination number for materials with covalent, ionic, van der Waals and metallic bonding.

3 *Attempt* **either** *this question* **or** *question 4.*

State Bloch's theorem for a particle in a periodic potential. [2]

For a one-dimensional chain of carbon atoms spaced a distance a apart, consider only the interaction between the p_z orbitals on adjacent sites. Use the tight-binding method and Bloch wavefunctions to show that the allowed band states have energies, E, that depend on wavevector, k, as

$$E(k) = E_0 - 2t\cos(ka),$$

where E_0 is a constant and t is the transfer integral. [6]

For a chain comprising N carbon atoms formed as a ring molecule with cyclic boundary conditions show that there are N independent values of k. [2]

For the case $N = 6$ (benzene) show the allowed values of k on a sketch of $E(k)$ for $-\pi/a \leq k \leq \pi/a$. Each carbon atom in benzene contributes one electron to these orbitals. By placing two electrons (spin up and spin down) in the lower energy states, show that the energy difference between the highest filled and the lowest empty states is $2t$. [4]

Using the same procedure, determine the value of this energy difference in the case where $N = 8$. [4]

For many carbon chains and rings of this type, bond lengths are found to alternate (dimerisation). Explain why this may lower the energy of occupied band states. Discuss why benzene ($N = 6$) does not dimerise but that the molecule with $N = 8$ does. [4]

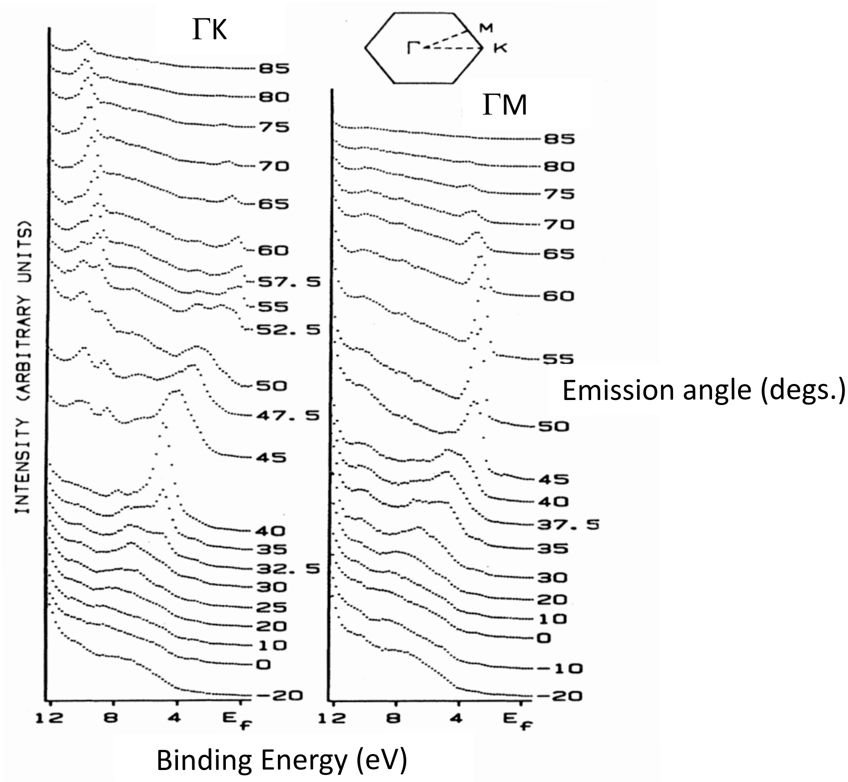
Carotene contains a linear chain of 22 carbon atoms. By treating this as a cyclic molecule estimate the energy difference between the highest filled and the lowest empty states (take a value for t of 3 eV). Discuss why the observed value of about 2.3 eV is considerably higher. [3]

4 *Attempt* **either** *this question* **or** *question 3.*

Describe the photoelectric effect. State the relationship between the kinetic energy of a photo-emitted electron from the surface of a material, E_{kin}, the incident photon energy and the workfunction of the material, using a labelled diagram for these energies. Explain why measurement of the energy dependence of the photoemission intensity provides information about the density of valence-band states. [5]

Describe two other experimental measurements that provide evidence for the electronic bandstructure in crystalline solids. [6]

Explain why the angle with respect to the normal to the surface, θ, of the photo-emitted electron depends only on E_{kin} and the value of the k-momentum vector of the electron projected in the plane of the surface, $k_{\parallel}$. Show that $\hbar k_{\parallel} = \sqrt{2mE_{\mathrm{kin}}}\sin\theta$, where m is the electron mass. [4]



The plots show the intensity of photoemitted electrons measured at different emission angles with respect to the normal to the surface, for a sheet of graphite (whose bandstructure for valence band states shows dispersion only in the plane of the sheet), with emission angles towards the K and M points in the Brillouin zone, as illustrated also in the figure. The spectra were obtained using incident UV photons of energy 21.2 eV, and energies are set with respect to the workfunction that lies 4.7 eV below the vacuum level.

Estimate the value of θ required for the highest valence band to reach the K point at the Brillouin-zone boundary, showing that this matches the experimental value on the plot above of 55° (noting that the lattice parameter of graphite, a, is 0.246 nm and that the value of $k_{\|}$ at the Brillouin-zone boundary corners is $4\pi/(3a)$). [4]

Sketch qualitatively the variation of the energy of the highest valence band versus $k_{\|}$ for the directions ΓK and ΓM. [3]

Graphite is considered to be a semi-metal. Does the valence-band structure you have sketched support this description? [3]

END OF PAPER