PNP 2012

1) c) Show that 3 $J^P = 2^+$ quanta can couple to give total $J^P = 0^+, 2^+, 3^+, 4^+, 6^+$

M_J	$M_{J_1}, M_{J_2}, M_{J_3}$	J
6	2 2 2	6
5	2 2 1	6
4	2 2 0, 2 1 1	6, 4
3	2 1 0, 2 2 -1, 1 1 1	6, 4, 3
2	2 0 0, 2 2 -2, 2 1 -1, 1 1 0	6, 4, 3, 2
1	1 0 0, 2 -2 1, 1 -1 1, 2 -1 0	6, 4, 3, 2
0	0 0 0, 0 2 -2, 0 1 -1, 1 1 -2, -1 -1 2	6, 4, 3, 2, 0

$J^P = 0^+, 2^+, 3^+, 4^+, 6^+$ (parity $= (+1)^3 = +1$)

a) range of interaction between hadrons via pion exchange

$M_\pi = 140\,\text{MeV}$

range $r = \frac{1}{M_\pi} = 7.14\times10^{-3}\,(\text{MeV})^{-1} = 7.14\times10^{-6}\,(\text{GeV})^{-1}$

$r = 7.14\times10^{-6} \times 6.6\times10^{-25} \times c = 1.41\times10^{-15}\,m$

range $= 1.41\,\text{fm}$

b) high energy electron collides with atomic electron
incident electron energy at threshold for e^+e^- production

$e^-e^- \to e^-e^-e^+e^-$

min $s = (4m_e)^2$

lab frame $(E_e + m_e)^2 - p^2 = 16m_e^2$

$2m_e^2 + 2E_e m_e = 16m_e^2$

$E_e = 7m_e = 3.6\,\text{MeV}$

Outline how Feynman diagrams are used to calculate particle scattering and decay processes

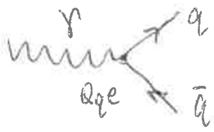
Used to calculate matrix elements M which determine scattering rate $\Gamma \propto |M|^2$ from Fermi's Golden Rule
need overall matrix element given by sum over all possible Feynman diagrams - lowest order diagrams give greatest contribution to M as each internal line introduces a propagator $\propto \frac{1}{q^2 - m^2}$

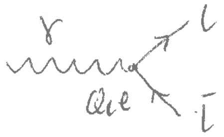
Coupling strength at each vertex, c_i

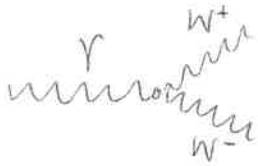
$M = \prod_i c_i$ for each diagram

$c_i = Qe \,/\, V_{ckm} g_w \,/\, g_z \,/\, \sqrt{\alpha_s}$ for EM / weak / strong interactions

Vertices arising in Feynman diagrams
EM - mediated by photons (γ), which don't carry electric charge
↳ no self interactions
EM vertex never changes type or flavour of particle
Coupling constant Qe, only coupling to charged particles
Allowed vertices

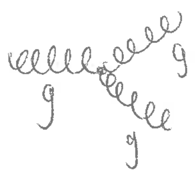




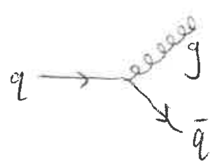


+ antiparticles

QCD - mediated by gluons, which carry colour charge - can have self interactions:



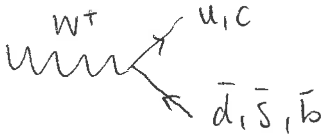
coupling ($= \sqrt{\alpha_s}$) only to particles carrying colour charge (quarks and gluons
Allowed vertices

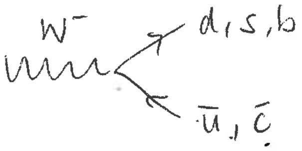


never changes flavour

Weak - mediated by $W^\pm$, Z bosons

$W^\pm$ - charged current - coupling to quarks, leptons, p, Z with strength g_W for leptons, $V_{CKM}\, g_W$ for quarks





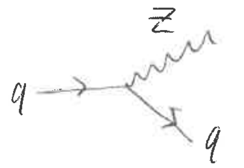
Charged current vertex always changes quark flavour

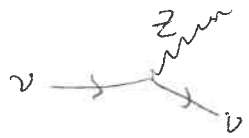
$W^\pm$ preferentially couples to quarks in same generation but quark mixing allows coupling across generations (much weaker)

- Cabibbo suppressed interactions

Z boson - neutral current - never changes particle type or flavour

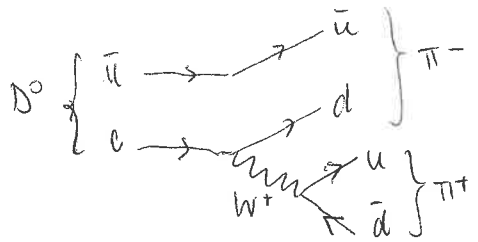
Z couplings are a mixture of EM and weak couplings





a) $D^0 \to \pi^+\pi^-$

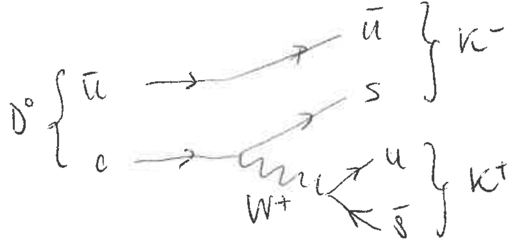
$c\bar{u} \to u\bar{d}, d\bar{u}$



Cabibbo suppressed

$D^0 \to K^+K^-$

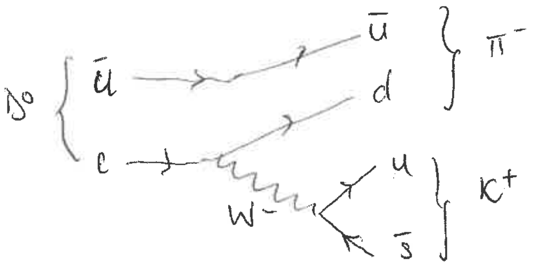
$c\bar{u} \to u\bar{s}, s\bar{u}$



Cabibbo suppressed

$D^0 \to K^+\pi^-$

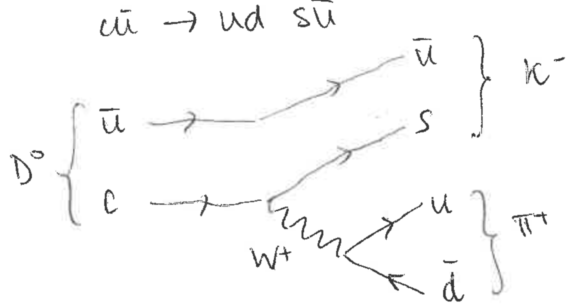
$c\bar{u} \to u\bar{s}\ d\bar{u}$



doubly Cabibbo suppressed

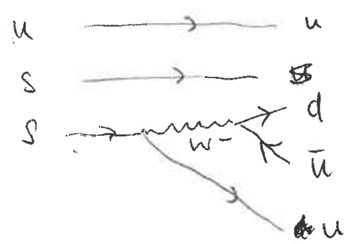
$D^0 \to \pi^+K^-$

$c\bar{u} \to u\bar{d}\ s\bar{u}$



b) $\Xi^0 \to \Lambda^0 \pi^0$

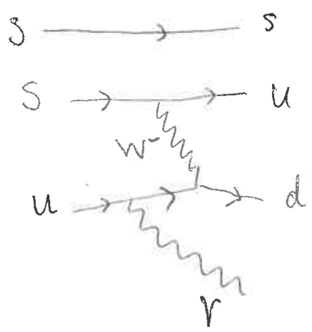
uss $\to$ uds, $u\bar{u}/d\bar{d}$



$\Xi^0 \to \Lambda^0 \bar{K}^0$

kinematically forbidden

$\Xi^0 \to \Lambda^0 \gamma$

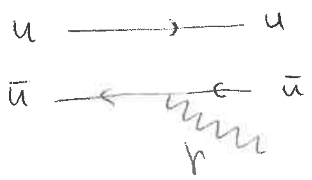


extra vertex - suppressed compared to $\Lambda^0 \pi^0$ decay

c) $\eta'^0 \to \rho^0 \gamma$

$\eta'^0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$



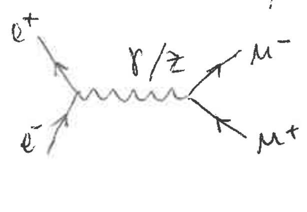
$\eta'^0 \to \pi^0 \gamma$

$J^P = 0 \to 0$ - forbidden - $0 \to 0$ not allowed for any γ decay

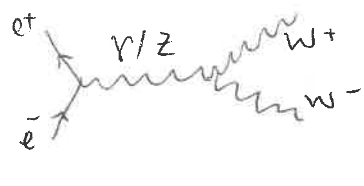
$\eta'^0 \to \pi^0 \rho^0$

$J^P = 0^- \to 0^- \oplus 1^-$ - could be weak decay (parity violated)

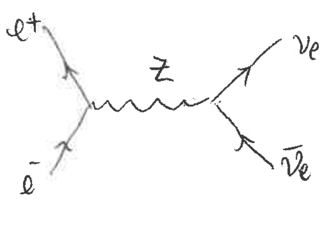
d) $e^+ e^- \to \mu^+ \mu^-$



$e^+ e^- \to W^+ W^-$



$e^+ e^- \to \nu_e \bar{\nu}_e$



$\mu^+\mu^-$ dominates at low energy

at $E_{cm} \sim M_Z$, $\nu_e \bar{\nu}_e$ production allowed

at $E_{cm} \sim 2M_W$, W^+W^- production allowed

at higher energies reaction strengths are comparable

4) Breit-Wigner for $x + X \to Z^* \to y + Y$

a) Γ_x = partial width for ~~d~~ reaction $x + X \to Z^*$
- rate of formation of Z^*

Γ_y = partial width for reaction $Z^* \to y + Y$
- rate of decay of Z^*

Γ = total width for reaction $x + X \to y + Y$ via resonant state

$\Gamma = \sum_i \Gamma_i$ - sum over all decay channels

b) derive form of denominator

QM description of decaying states:

State with energy $E = E_0$, mean lifetime τ, formed at $t=0$

$\psi(t) = \psi(0) e^{-iE_0 t} e^{-t/2\tau}$

frequencies present in wavefunction:

$$f(\omega) = F(E) = \int_0^\infty \psi(t) e^{-iEt}\, dt = \int_0^\infty \psi(0) e^{-t(i(E_0 - E) + 1/2\tau)}\, dt$$

$$F(E) = \frac{i\psi(0)}{(E_0 - E) - i/2\tau}, \quad \Gamma = 1/\tau$$

probability of finding state with energy E is $|f(E)|^2$

$$= \frac{|\psi(0)|^2}{(E - E_0)^2 + \Gamma^2/4}$$

c) Justify form for g:

takes into account spins of initial particles - ratio of no of spin states for resonant state to total no of spin states for $x + X$ system

- probability that x and X collide in correct spin state to form Z^*

Assume all particles are non-relativistic

target X initially at rest

x has kinetic energy T_x in lab frame

$$V_{cmF} = \frac{p_x}{m_x + m_X}$$

$$T_x = \frac{1}{2} m_x v_x^2 = \frac{p_x^2}{2m_x} \Rightarrow p_x = \sqrt{2 m_x T_x}$$

$$V_{cmF} = \frac{\sqrt{2m_x T_x}}{m_x + m_X}$$

in CM frame, $v_x = \dfrac{\sqrt{2m_x T_x}}{m_x} - \dfrac{\sqrt{2m_x T_x}}{m_x + m_X}$

$$v_X = \frac{-\sqrt{2m_x T_x}}{m_x + m_X}$$

$$T_* = \frac{1}{2} m_x \left[\sqrt{2m_x T_x}\left(\frac{1}{m_x} - \frac{1}{m_X + m_x}\right)\right]^2 + \frac{1}{2} m_X \frac{2m_x T_x}{(m_x + m_X)^2}$$

$$= \frac{m_x^2 m_X^2 T_x}{m_x^2 (m_x + m_X)^2} + \frac{m_X m_x T_x}{(m_x + m_X)^2} = \frac{m_X T_x}{m_x + m_X} = \frac{\mu T_x}{m_x}$$

expression for p_*

$p_* = m_X v_X$ in CM frame

$$p_* = \frac{m_X \sqrt{2m_x T_x}}{m_x + m_X} = \frac{m_X}{m_x + m_X}\sqrt{\frac{2m_x^2 T_*}{\mu}} = \sqrt{2\mu T_*}$$

resonance $\alpha + {}^{12}C \to {}^{16}O + \gamma$

α particle has lab KE $10.10\,MeV$ at resonance

CM momentum p_* at resonance

$$\mu = \frac{m_\alpha m_c}{m_\alpha + m_c} = 2795.80 \frac{MeV}{c^2}$$

$$T_x = 10.10\,MeV$$

$$T_* = \frac{\mu T_\alpha}{m_\alpha} = 7.57\,MeV$$

$$p_* = \sqrt{2\mu T_*} = 205.8 \frac{MeV}{c}$$

Find lab KE at which resonance excited in $p + {}^{15}N$ collisions

mass difference of initial states is $M_p + M_N - (M_\alpha + M_C) = 5.46\,MeV$

new $T^* = 7.57 - 5.46\,MeV = 2.11\,MeV$

lab KE $T_p = \frac{M_p T^*}{\mu}$

$\mu = \frac{M_p M_N}{M_p + M_N} = 879\,MeV \Rightarrow T_p = 2.25\,MeV$

ratio $\frac{\sigma(\alpha + {}^{12}C \to {}^{16}O + \gamma)}{\sigma(p + {}^{15}N \to {}^{16}O + \gamma)} = \frac{g_1}{g_2}\left(\frac{P_{*2}}{P_{*1}}\right)^2 \frac{\Gamma_\alpha}{\Gamma_p}$

$\frac{g_1}{g_2} = \frac{(2J_p+1)(2J_N+1)}{(2J_\alpha+1)(2J_C+1)} = 4$ - $J_\alpha = J_C = 0,\ J_p = J_N = \frac{1}{2}$

$P_{*1} = 205.8\,\frac{MeV}{c}$, $P_{*2} = \sqrt{2\mu T_{*2}} = 60.9\,\frac{MeV}{c}$

$\Gamma = 0.20\,MeV$, $\Gamma_\alpha = 0.15\,MeV$, $\Gamma_0 = 0.10\,eV$

$\Gamma_p = \Gamma - \Gamma_\alpha - \Gamma_0 = 0.05\,MeV$

ratio $= 4\left(\frac{60.9}{205.8}\right)^2 \frac{0.15}{0.05} = 1.05$