UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Tuesday 28 May 2013 13.30 to 15.30

EXPERIMENTAL AND THEORETICAL PHYSICS (1)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (1)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

THERMAL AND STATISTICAL PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) Find the root-mean-squared voltage across a 10^{-10} F capacitor in parallel with a resistor, in thermal equilibrium at room temperature. [4]

(b) Sketch the temperature dependence of the internal energy and of the heat capacity for a two-level system with energy-level spacing $\hbar\Omega$. [4]

(c) Estimate the Bose–Einstein condensation temperature of ^{4}He from the Fermi temperature of ^{3}He, $T_F = 5.0$ K, at the same number density. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) equipartition of energy and the fluctuation–dissipation theorem;

(b) black-body radiation and the temperature dependence in the Stefan–Boltzmann law;

(c) equations of state of real gases.

3 *Attempt* **either** *this question* **or** *question 4.*

State the grand partition functions for non-interacting fermions and bosons occupying a single quantum state at energy ϵ, in contact with a reservoir at chemical potential μ. Derive the resulting average occupation number of that quantum state for both fermions and bosons. [5]

A point defect in a solid may be occupied by 0, 1 (spin up or down) or 2 electrons, and the solid provides a reservoir of electrons at chemical potential μ. The energy for occupation by a single electron is ϵ, and that for 2 electrons is $2\epsilon + U$, where U is the Coulomb repulsion energy between the electrons. Show that the average electron occupancy of the defect can be written as

$$\bar{n} = \frac{2e^{(\mu-\epsilon)\beta} + 2e^{(2\mu-2\epsilon-U)\beta}}{1 + 2e^{(\mu-\epsilon)\beta} + e^{(2\mu-2\epsilon-U)\beta}} \, ,$$

where $\beta = 1/k_{\mathrm{B}}T$. [4]

In the limiting case $U = 0$, sketch $\bar{n}$ as a function of μ. [3]

For finite U sketch $\bar{n}$ as a function of μ in the low-temperature limit $U\beta \gg 1$. [*You may find it helpful to consider the relative sizes of the exponential terms for different regimes of* μ.] [6]

Show that the variance of the particle number can be expressed as

$$\sigma^2 \equiv \left\langle n^2 \right\rangle - \bar{n}^2 = k_{\mathrm{B}}T \left(\frac{\partial \bar{n}}{\partial \mu}\right)_T \, .$$ [4]

In the low-temperature limit $U\beta \gg 1$, find σ^2 for $\mu = \epsilon$ and sketch σ^2 for the whole range of μ. [3]

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4 *Attempt* **either** *this question* **or** *question 3.*

A metal that undergoes a ferromagnetic phase transition at a temperature T_c can be described within the Landau theory of phase transitions by a free-energy density

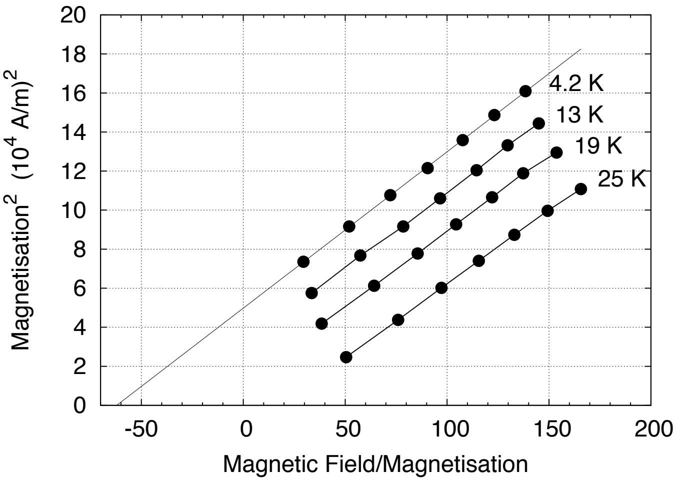
$$F(M) = \frac{a(T)}{2}M^2 + \frac{b}{4}M^4 - \mu_0 HM \,,$$

where M is the component of the magnetisation along the z-direction, H is the magnitude of the applied magnetic field, which points along the z-direction, $a(T) = \alpha(T - T_c)$, and α and b are positive phenomenological parameters.

Sketch $F(M)$ for (i) $H = 0$ and $T > T_c$, (ii) $H = 0$ and $T < T_c$, and (iii) $H > 0$ and $T > T_c$. In each case, indicate the average magnetisation and the associated free energy. [6]

Explain why for $H = 0$ no odd powers of M contribute to the free energy and find an expression for the equation of state, which links H to M. Show that for $H \to 0$ and $T > T_c$ the magnetic susceptibility $\chi \equiv (\partial M/\partial H)_T = \mu_0/a(T)$. [5]

Measurements of the isotherms $M(H)$ of a low-temperature magnet are plotted in the modified form M^2 vs. H/M in the figure below. Assuming that Landau theory applies, extract $a(T)$ and b and determine the ferromagnetic transition temperature T_c. [6]



Show that, for $H = 0$ and $T < T_c$, the minimum of the free energy follows $F = -a(T)^2/4b$. Hence find an expression for the heat-capacity jump at T_c and determine its expected size. [5]

In a related material, which also becomes ferromagnetic at low temperature, it is found that $b < 0$. How should one modify the Landau expansion for $F(M)$ in this case? What does $b < 0$ imply for the nature of a ferromagnetic phase transition? [3]

END OF PAPER