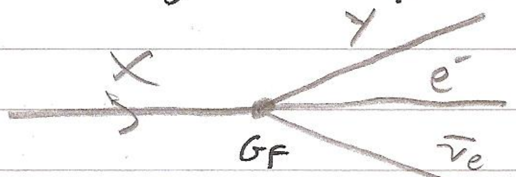
June 2005 Paper 3 Section A (PNP)

A1. B. Brief notes:

a) Fermi theory of β decay

- 4 fermion contact interaction
- No propagator in matrix element, instead effects included in coupling constant G_F
- Matrix element has the form $G_F \langle \text{initial} | \text{final} \rangle$
- β decay has form



thus matrix element is $G_F \langle X | Y e^- \bar{\nu}_e \rangle$

- In integral form $M_{if} = \int \psi_X^* \, \psi_Y \, \psi_{e^-} \, \psi_{\bar{\nu}_e} \, d^3r$
- We consider the electron and neutrino as free particles that weakly interact with the nucleus at the energies we are considering.
- Thus $\psi_{e^-} = e^{-i p_{e^-} \cdot r}$ and $\psi_{\bar{\nu}_e} = e^{-i p_{\bar{\nu}_e} \cdot r}$
- Element large when $\psi_{e^-} \psi_{\bar{\nu}_e} \sim 1$ i.e. $p_{e^-} = -p_{\bar{\nu}_e}$ thus orbital angular momentum of system unchanged $\Rightarrow$ parity of X unchanged. – This is an allowed transition.
- If X, Y are mirror nuclei, the ~~energy~~ wavefunction of the decaying neutron in X is similar to the wavefunction of the decay product proton in Y thus $\int \psi_X^* \psi_Y \, d^3r \simeq 1$ – this is a superallowed transition.

- The $e^-, \bar{\nu}_e$ may carry away orbital angular momentum $\ell > 0$. In this case the term $\psi_{e^-}\psi_{\bar{\nu}_e} = e^{-i(p_e+p_{\bar{\nu}_e})\cdot r} = 1 - i(p_e+p_{\bar{\nu}_e})\cdot r + \frac{1}{2}[(p_e+p_{\bar{\nu}_e})\cdot r]^2 + \dots$
- If an allowed transition is not ~~all~~ possible due to selection rules (i.e the parities of ~~the~~ X, Y are different) then it may occur by a forbidden transition. (each term of $p \cdot r$ is an extra power of $r\cos\theta$ $\Rightarrow$ parity change in matrix element).

- Nucleus J-J coupled, $e^-, \bar{\nu}_e$ are not $\Rightarrow$ must consider spins of $e^-, \bar{\nu}_e$ system
- Can be spin 0 singlet or spin 1 triplet
- Thus an allowed transition may only change J by ± 1 between X and Y.

b) nuclear mass.

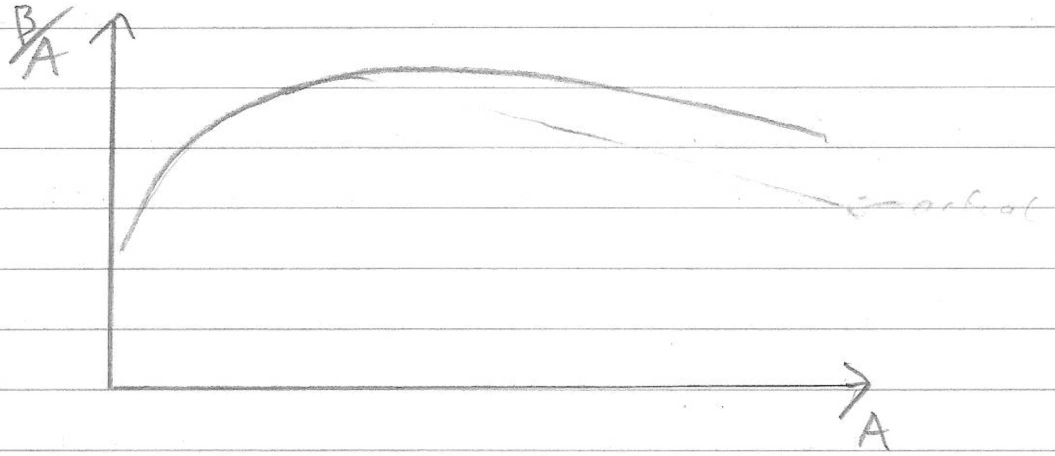
- Stable nuclei are a minimum in ~~ene~~ mass (and hence energy) across isobars.

- For a nucleus with A nucleons and Z protons the mass is given by $m(A,Z) = Zm_p + (A-Z)m_N - B$ where B is the binding energy.

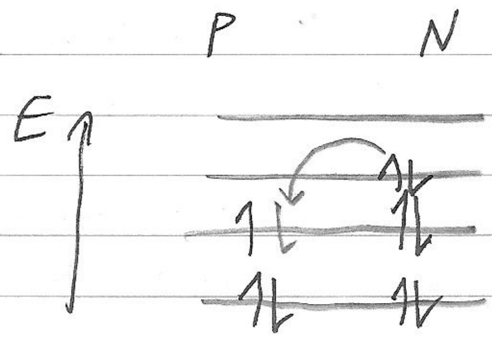
- As a theoretical model of this binding energy, the nucleus can be approximated as a spherical drop of ~~liquid.~~ charged liquid.

- Thus there will be an energy term associated with the volume of the liquid $= a_v A$ as we assume volume is proportional to number of nucleons.

- There will be a term for the energy penalty of the free surface $= -a_s A^{\frac{2}{3}}$
- There will also be a coulomb term $\propto \frac{q^2}{r}$ $= -a_c \frac{Z^2}{A^{\frac{1}{3}}}$
- Experimental data can be used to find a_v, a_s, a_c
- This gives the rough graph of binding energy per nucleon considering $N \simeq Z$



- But this does not explain why $N \simeq Z$ or why ~~magic~~ even numbers of N, Z give ~~exceptionally~~ more stable nuclei.
- Considering the nucleus as confining its constituent ~~pro~~ nucleons, ~~only discrete~~ in a fermi gas, only discrete energy levels are allowed.
- Thus if $N \neq Z$, there will be a large energy difference compared to if $N \simeq Z$ as shown. Thus there tend to be equal numbers of protons and neutrons.



- Protons and ~~nect~~ neutrons have spin $\frac{1}{2}$ so forming pairs of spin up and down reduces the free energy associated with unpaired spin.
This explains why even number of protons and neutrons are favoured.

c) The Nuclear shell model

- Especially stable nuclei at certain 'magic numbers' of Z, N.

- There are greater a numbers of isotopes and isotones at these magic numbers.

- The nuclear shell model treats nucleons as independently moving inside a Saxon-Woods potential.

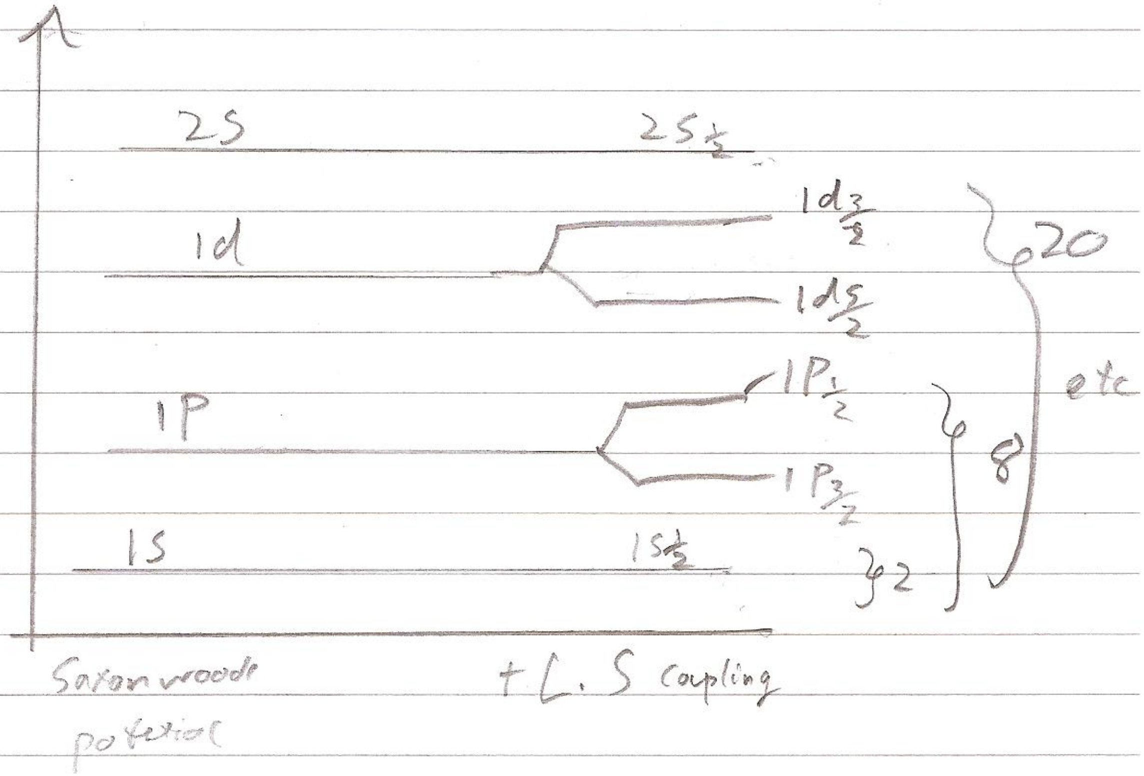
- The nucleons can be treated as independently moving because if they were not there would be many collisions resulting in exchange of energy.

- Assuming all nearby states are occupied, there cannot be any exchange of energy $\Rightarrow$ no collisions.

- To get the magic numbers, a spin-orbit interaction needs to be included in the potential.

- This splits the energy levels for all $l \neq 0$ states into two.

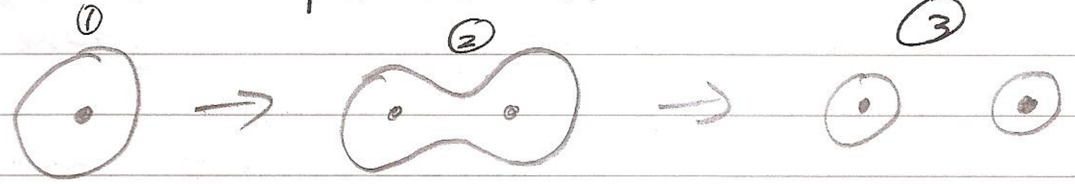
- In this way, the magic numbers are recovered.



d. nuclear fission

- Fission is the event where a large parent nucleus splits into two daughter nuclei

- Expect this to occur only if the binding energies of the two daughter nuclei are combined are less than the parent nucleus'.

- 

The semi empirical mass formula gives the binding energy in terms of surface area and coulomb repulsion of the nucleus.

- In ② the surface area is increased greatly but the coulomb repulsion has not decreased ⇒ potential barrier to fission.

- We can therefore consider this a tunneling problem of the daughter nucleus out of the parent.

- Probability of fission $p \propto e^{-2G}$ where the Gamow factor $G \propto m^{\frac{1}{2}} E_f$ where m is the mass of the escaping particle and E_f the height of the fission barrier.

- This is why α decay is much more likely than fission: the mass of the α particle is small compared to most daughter nuclei.

- In most cases spontaneous fission is very slow. To occur, the energy barrier must be overcome by exciting the nucleus, usually with neutrons which do not feel the Coulomb repulsion.

- Large momentum $\Rightarrow$ small λ $\Rightarrow$ small Breit Wigner cross sections.

- To acheive a sustainable chain reaction, need to slow neutrons to allow capture.

- In commercial ~~ne~~ fission reactors, elastic scattring from graphite is used to slow neutrons.

A2. Light Baryons made from 3 of any combination of u, d, s ~~guard~~ quarks

- 3 cases: Baryon composed of 3 of same quark
- 〃 〃 2 〃
- All quarks have different flavours.

① All quarks have same flavour: i.e uuu, ddd, sss.
- flavour symmetric under exchange
- spin symmetric 〃 〃

⇒ must be in $S=\frac{3}{2}$ state.

∴ 3 in $\frac{3}{2}$ state.

② 2 quarks have same flavour.
- for similar quarks, flavour and spin symmetric.
- other quark can be spin up or down as it is non identical

⇒ 6 for both $\frac{1}{2}$ and $\frac{3}{2}$ states.

③ 3 quarks with different flavours.
- ~~Spin wavefunction independent for each quark~~
- flavour antisymmetric / symmetric
- spin antisymmetric / symmetric

for $S=\frac{3}{2}$ must be symmetric
for $S=\frac{1}{2}$ could be symmetric or antisymmetric

⇒ 1 for $\frac{3}{2}$, 2 for $\frac{1}{2}$.

Total: 10 for $\frac{3}{2}$, 8 for $\frac{1}{2}$.

- $q_1 q_2 q_3$ baryon state can ~~have~~ be in an octet (spin $\frac{1}{2}$) or decuplet (spin $\frac{3}{2}$) state.
- Only difference is the spin interaction between the ~~two~~ constituent quarks.

- Magnetic moment of a particle $\mu = \frac{e}{2m} \underline{S}$
- Thus interaction between two particles of form $\mu_1 \cdot \mu_2 \propto \frac{S_1 \cdot S_2}{m_1 m_2}$
- ~~This~~ When summed over all pairs of constituent particles gives the 'perturbation' to the mass as given.

For the uus bound state:

$S_1 \cdot S_2 = \frac{1}{2}(S_{tot}^2 - S_1^2 - S_2^2)$

~~Spin 1/2:~~ ~~$S_u \cdot S_u = \frac{1}{2}(1 - \frac{1}{4} - \frac{1}{4}) = \frac{1}{4}$~~

~~$S_u \cdot S_s = \frac{1}{2}(0 - \frac{1}{4} - \frac{1}{4}) = -\frac{1}{4}$~~

~~$S_u \cdot S_s = \frac{1}{2}($~~

S_u & S_s

Combined spin 1 or 0 — spin $\frac{1}{2}$

~~$= \frac{1}{2}(\frac{1}{4}$~~

~~$S_u \cdot S_u = \frac{1}{2}(0 - \frac{1}{2}(\frac{1}{2}+1) + \frac{1}{2}(\frac{1}{2}-1)$~~

~~$S_u \cdot S_u = \frac{1}{2} S_{tot}^2 - \frac{3}{4}$~~

~~Split up so that S_u constitutes the entire spin wavefunction of the two u quarks~~

I HATE SPIN STATISTICS!

for Spin $\frac{3}{2}$: Combined spin wave function $U \sim S_u \otimes S_u$ has spin 1, S_s has spin $\frac{1}{2}$

$$S_u \cdot S_u = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$ as must be parallel to have combined spin 1.

Can be found the more correct way!

$$S_u \cdot S_u = \frac{1}{2}(S_{tot}^2 - S_u^2 - S_u^2) \qquad S_{tot} = S_u \otimes S_u = U$$
$$= \frac{1}{2}\left(1(1+1) - \frac{1}{2}(\tfrac{1}{2}+1) \cdot 2\right)$$
$$= \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

$\therefore$ have $A'\left(\frac{1}{4m_u^2}\right)$

Instead of 2 $S_u \cdot S_s$ terms, have to consider combined spin wavefunction $U \cdot S_s$.

$$U \cdot S_s = \frac{1}{2}(J^{P\,2} - U^2 - S_s^2)$$
$$= \frac{1}{2}\left(\tfrac{3}{2}(\tfrac{3}{2}+1) - 2 - \tfrac{3}{4}\right)$$
$$= \frac{1}{2}$$

$\therefore$ have $A'\left(\frac{1}{2m_u m_s}\right)$

probably did it stupid way.

for $J^P = \frac{1}{2}^+$

still have $A'\left(\frac{1}{4m_u^2}\right)$

but $V \cdot S_s = \frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}\right) - 2 - \frac{3}{4}\right)$

$= -1$

$\therefore$ have $A'\left(\frac{-1}{m_u m_s}\right)$

$GeV^3 \to MeV^3$

$m(\Sigma^+) = 2m_u + m_s + 0.026 \times 10^9 \left(\ldots \right)$

$= 1.18\ GeV$

very close to experimental value of 1.19 GeV.

$m(\Sigma^{+*}) = \cancel{1.58\ GeV.}\ 1.38\ GeV$

~~This is significantly different from the experimental~~ Calculator malfunction!

very close to value of 1.38 GeV