UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 31 May 2019 1.30 pm to 3.30 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

PARTICLE AND NUCLEAR PHYSICS

*Candidates offering this paper should attempt a total of* ***five*** *questions:* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

*The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains* ***five*** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 Consider the following decay channels of the $\Omega^-$ particle

$$\Omega^- \to \Lambda^0 K^-$$
$$\Omega^- \to \Lambda^0 \pi^-.$$

Draw the lowest-order Feynman diagram for each decay and explain which one is the less probable decay. [4]

[*The quark contents for the particles involved are $\Omega^-$ $[sss]$, $\Lambda^0$ $[uds]$, $\pi^-$ $[\bar{u}d]$ and $K^-$ $[\bar{u}s]$.*]

2 Consider the neutron $[udd]$ and the $\Lambda^0$ $[uds]$ $\beta$ decays

$$n^0 \to p^+ e^- \bar{\nu}_e$$
$$\Lambda^0 \to p^+ e^- \bar{\nu}_e.$$

Draw the lowest-order Feynman diagram for each decay, and compute the Cabibbo angle $\theta_C$ using the ansatz that the density of final states is proportional to $(M - m_p)^5$, where $M$ is the mass of the decaying particle and $m_p$ that of the proton. [4]

[$m_{p^+} = 938.3\,\text{MeV}$, $m_{n^0} = 939.6\,\text{MeV}$, $m_{\Lambda^0} = 1115.7\,\text{MeV}$, $\tau_{n^0} = 880\,\text{s}$, $\tau_{\Lambda^0} = 2.63 \cdot 10^{-10}\,\text{s}$, $\mathcal{B}(n^0 \to p^+ e^- \bar{\nu}_e) = 1$, $\mathcal{B}(\Lambda^0 \to p^+ e^- \bar{\nu}_e) = 8.32 \cdot 10^{-4}$.]

3 For each of the following $\beta$ decays, indicate whether Fermi or Gamow-Teller matrix elements are involved, and specify whether they are super-allowed, allowed or forbidden and to which degree ($n^{\text{th}}$ forbidden with integer $n$). [4]

$^{129}_{53}\text{I}(\tfrac{7}{2}^+) \to {}^{129}_{54}\text{Xe}(\tfrac{3}{2}^+)$

$^{73}_{36}\text{Kr}(\tfrac{3}{2}^-) \to {}^{73}_{35}\text{Br}(\tfrac{1}{2}^-)$

$^{158}_{70}\text{Yb}(0^+) \to {}^{158}_{69}\text{Tm}(2^-)$

$^{22}_{12}\text{Mg}(0^+) \to {}^{22}_{11}\text{Na}(0^+)$ with $\log(f\tau_{1/2}) = 3.48$

SECTION B

*Attempt **two** questions from this section*

4 Explain the principle behind ${}^{14}_{6}$C radio-dating for organic matter. [4]

What is the average lifetime of a ${}^{14}_{6}$C nucleus? [1]

Using the semi-empirical mass formula to find the most bound $A = 14$ isobar state(s), deduce the decay modes of the ${}^{14}_{6}$C. Are the decay products stable? [8]

What is the natural activity of a gram of carbon? [3]

The carbon of an (organic) artefact found in an Egyptian tomb has an activity of 0.13 Bq per gram. How old is this artefact? [3]

*The ${}^{14}_{6}$C half-life is 5730 years. The atmospheric concentration of ${}^{14}_{6}$C is about* $1.3 \cdot 10^{-12}$ *times the concentration of ${}^{12}_{6}$C, the most abundant carbon isotope.*

*The semi-empirical mass formula for nuclear masses is*

$$M(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A} - \delta(A),$$

*where* $m_p = 938.3\,\text{MeV}$, $m_n = 939.6\,\text{MeV}$ *and in which nuclear masses are best described with* $a_V = 15.8\,\text{MeV}$, $a_S = 18.0\,\text{MeV}$, $a_C = 0.72\,\text{MeV}$, $a_A = 23.5\,\text{MeV}$. *The pairing term $\delta(A)$ is not relevant here.*

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5 The $\rho$ meson always decays into two $\pi$ mesons via the strong interaction.

Write down the equations corresponding to the conservation of the total angular momentum and the conservation of parity. Are they compatible? [3]

Which exchange symmetry must a two-pion state fulfil? Why is the decay $\rho \to \pi^0\pi^0$ not observed? [4]

The isospin of the pion $I(\pi)$ is 1. Using the notation $|I, q\rangle$, where $I$ is the isopin and $q$ the electric charge, the three possible charge states of the pion are $|\pi^+\rangle = |1, 1\rangle$, $|\pi^0\rangle = |1, 0\rangle$ and $|\pi^-\rangle = |1, -1\rangle$. Write down all the eigenstates of the total isospin of a two-pion system. [4]

Deduce the isospin of the $\rho$ meson and the decay modes for each of the charge states. [4]

Which charge states of the $\rho$ meson are eigenstates of the charge-conjugation operator? Give their eigenvalues. [4]

[*The spin and parity of the $\rho$ meson and of the pion are $J^P(\rho) = 1^-$ and $J^P(\pi) = 0^-$.* ]

6 Consider the decay of a particle $A$ of mass $M$ into $n$ daughter particles of masses $m_1, \ldots, m_n$

$$A \to a_1 + \cdots + a_n.$$

Write down the relevant conservation laws in the centre-of-mass frame. [2]

Compute in the centre-of-mass frame the energies $E_1$ and $E_2$ of the daughter particles in the case of a two-body decay, $A \to a_1 + a_2$. [6]

In the case of a three-body decay, $A \to a_1 + a_2 + a_3$, compute the minimal energy $E_1^{\mathrm{min}}$ for the daughter particle $a_1$ in the centre-of-mass frame. [3]

Compute also the maximal energy $E_1^{\mathrm{max}}$ for the daughter particle $a_1$ in the centre-of-mass frame. (It may be useful to distinguish the cases when (i) at least one of $a_2$ and $a_3$ has mass and (ii) when both $a_2$ and $a_3$ are massless.) [8]

END OF PAPER