UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 31 May 2018 1.30 pm to 3.30 pm

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)
OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1** *and* **two** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

draft: 17th May 2018

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) In superconducting aluminium a small externally applied magnetic field decays exponentially with a decay length of $\lambda_L = 22\,\text{nm}$. Calculate the density of conduction electrons assuming that the effective mass is the bare electron mass. [4]

(b) A cube of non-magnetic uniaxial crystal of side $L = 1\,\text{cm}$ is illuminated with a ray of unpolarised light under normal incidence. We observe two parallel rays exiting the crystal separated by $\Delta = 0.1\,\text{mm}$. Calculate the angles α_o and α_e between the $\boldsymbol{E}$ and $\boldsymbol{D}$ fields of the ordinary and the extraordinary beams and explain your reasoning. [5]

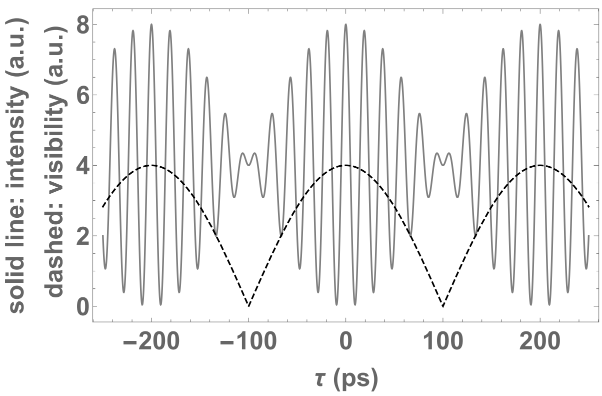
(c) Electrons with a kinetic energy of 1GeV produce a cone of Cerenkov radiation with an opening angle $\alpha = 97.5°$ in a water-filled detector. Calculate the index of refraction n of water to three significant digits. [3]

SECTION B
Attempt **two** *questions from this section*

B2 Draw an annotated sketch of a Fourier transform spectrometer based on a Michelson interferometer and describe its working principle. How does its absolute resolution $\delta\lambda$ scale with the range of movement L of the movable mirror? [5]

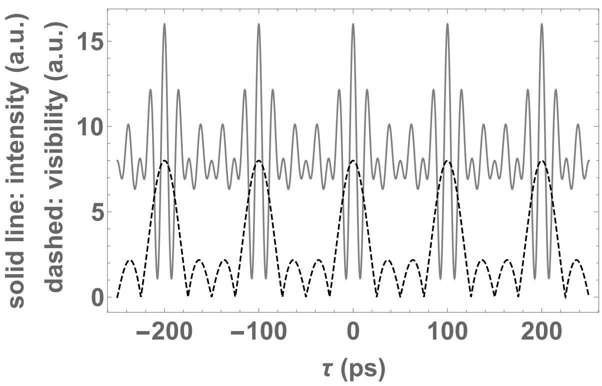
Assume that a Michelson interferometer uses a non-polarizing beam splitter and that its two arms affect the polarization of the light differently. What is the resulting visibility for a perfectly coherent, monochromatic, incoming wave if the two arms result in (i) two linearly polarized beams with an angle θ between their polarization directions, (ii) a linear and a circular polarized beam. [3]

A Fourier transform spectrometer is used to analyse the sum of two monochromatic waves. Calculate the difference in frequencies from the below interferogram, where τ denotes the time delay introduced by the two arms. [3]



Assume that each of the monochromatic components is pressure broadened with $\tau_1 = 1$ ns. What is the resulting spectral line-shape? Draw an annotated sketch of the expected interferogram and discuss the changes you would expect compared to the above case. [3]

The same spectrometer is now used to analyse light containing N monochromatic components with frequencies $\omega_n = \omega_0 + n \cdot \Delta\omega$ ($n \in 0, 1, 2, \ldots, N-1$). Extract N and $\Delta\omega$ from the following interferogram and explain your reasoning. [5]



B3 Calculate the strength of the oscillating current $I(t)$ in a Hertzian dipole in terms of the maximum electric dipole moment p_0, the fixed distance between the point charges d, and the oscillation frequency ω. [2]

For a Hertzian dipole aligned along the z-axis, three of the six components of the magnetic and electric field vanish in spherical coordinates. Which ones and why? [2]

Starting from a general expression for the retarded vector potential $A(\boldsymbol{r}, t)$, derive the vector potential of the Hertzian dipole in the *far field*. Show that it only depends on the distance r from the dipole and the oscillating dipole moment and calculate the dependency. [3]

State and sketch the radial distribution of the emitted power in the far field of a single Hertzian dipole that is aligned along the z-axis (i) in the (x, z) plane and (ii) the (x, y) plane. If this dipole is used as a receiver for radiation coming from the x direction, what is the polarization response of the antenna? [3]

Describe what is meant by the terms power gain $G(\theta, \phi)$ and effective area $A_{\text{eff}}(\theta, \phi)$ of an antenna and outline the basic arguments why for any antenna the effective area and the power gain must be proportional to each other. [3]

A Hertzian dipole aligned along the z-axis and placed at the origin is driven at a frequency $\omega = 2\pi \times 100\,\text{MHz}$ such that the radiated power is $P_0 = 1\,\text{W}$. A second Hertzian dipole with the same orientation is placed at the point $(x, y, z) = (100\,\text{m}, 0, 0)$ and is connected to a matched load. Calculate the power dissipated in the matched load. [4]

Assume now that the transmitting antenna is emitting short pulses of the above radiation. Between these transmissions, the antenna is connected to a matched load and is used to receive signals reflected from the second antenna. For the same geometry as above, calculate the power dissipated in the first antenna during a reflected pulse. How does this depend on the distance between the antennas? [2]

[*You may assume without proof that power gain and effective area are related by:*

$$A_{\mathit{eff}}(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi)$$
]

B4 Explain the terms 4-vector and Lorentz invariant and state the experimental evidence that the electric charge of a proton is Lorentz invariant. [4]

Consider an object with charge density ρ_0 in its rest frame and calculate the charge density in a frame moving at velocity v relative to the object. Demonstrate that $J = (c\rho, \boldsymbol{J})$, where $\boldsymbol{J}$ is the familiar 3D electrical current, is a 4-vector. [2]

Assume two parallel, current carrying wires at distance d with currents $I_1 = I_2$. The wires are identical and carry no net charge. Calculate the direction and strength of the force per unit length using conventional magnetic forces. [2]

We will now aim to reproduce the above result by considering only electrostatic forces in appropriate coordinate systems. Calculate the direction and strength of the resulting force *per unit length* in the lab frame by separately considering the forces on the positive and negative charge densities ρ_2^+, ρ_2^-, respectively, in one of the wires. While the positive charges are static, the negative charges move with drift velocity v. Show that this relativistic result matches that of the above calculation. [7]

[*The Lorentz transformation for the transverse force on an object is given by*

$$\boldsymbol{F}'_\perp = \frac{\boldsymbol{F}_\perp}{\gamma(1 - \boldsymbol{u}\cdot\boldsymbol{v}/c^2)},$$

where $\boldsymbol{v}$ *is the velocity of* S' *relative to* S *and* $\boldsymbol{u}$ *is the velocity of the object in* S.]

Using again only electrostatic forces in appropriate coordinate systems, explain why the case $I_2 = -I_1$ leads to a force in the opposite direction. [4]

END OF PAPER