UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Tuesday 31 May 2022 9.00 am to 11.00

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

ELECTRODYNAMICS AND OPTICS

Candidates offering this paper should attempt a total of ***five*** *questions: all* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains ***seven*** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

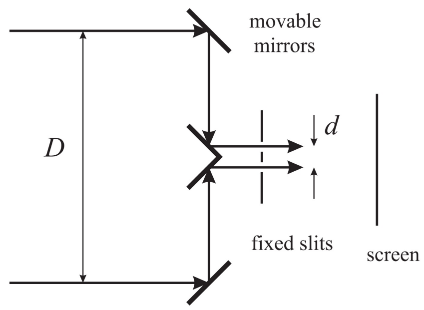
SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 A shower of high energy electrons passes through a material with refractive index $n = 1.4$. A detector embedded in the material has its axis parallel to the electrons' direction of motion and can detect radiation that falls on it at an angle of less than 20° with respect to its axis. Explain why the electrons emit radiation and calculate the range of electron speeds that is detected. [4]

2 A transmitting antenna with power gain $G = 10^5$ points towards a receiving antenna with an effective area $A = 1.8$ m^2. The two antennas are separated by a distance $d = 1$ km. The transmitting antenna is connected to a matched source that produces a spectrum equivalent to the Johnson noise of a resistor at the physical temperature of $T_t = 10^4$ K. Calculate the effective temperature of the internal resistance of the receiving antenna.
[*Hint: The Nyquist formula for the mean-square voltage fluctuation of a resistor R at temperature T in the frequency range* $\nu \to \nu + d\nu$ *is* $\langle V^2 \rangle = 4k_B T R d\nu$.] [4]

3 Michelson used his Stellar Interferometer (shown below) for the first time to measure the diameter of the red supergiant Betelgeuse, which has an angular diameter $\alpha = 22.6 \times 10^{-8}$ rad and a peak emission at $\lambda = 570$ nm. Show quantitatively that the simple Young's slit set-up, with a typical distance d between the slits in the millimeter range, is not sensitive enough to measure the star's diameter. In his improved set-up Michelson introduced a system of movable mirrors, with variable spacing D. Explain how this improves the sensitivity of the measurement and calculate the value of D at which we expect to measure vanishing interference fringes on the screen. [4]



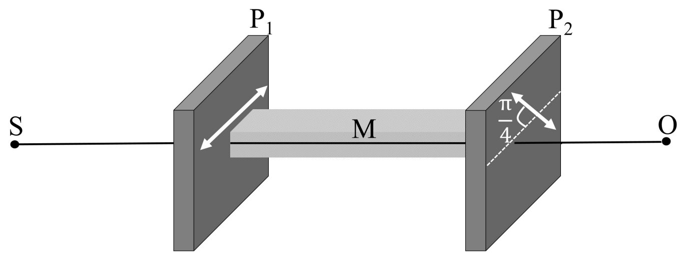
SECTION B

Attempt two questions from this section

4 (a) Explain what is meant by optical birefringence and obtain an expression for the specific rotatory power of a birefringent material of thickness d in terms of its refractive indices. [3]

(b) Materials like flint glass become birefringent when exposed to a weak magnetic field B_0. Without detailed calculations, explain why this is. [3]

Consider the set-up below. S is a monochromatic unpolarised light source, O is the observer, while P_1 and P_2 are two identical polarisers with the transmitting axis of P_2 rotated by $\frac{\pi}{4}$ with respect to that of P_1, as shown in the figure. M is a 1 m long block of flint glass with Verdet coefficient, defined as the specific rotatory power per unit of magnetic field, $\Upsilon = 9$ rad$\mathrm{T}^{-1}\mathrm{m}^{-1}$. A magnetic field B_0 is applied along the long axis of the block.



(c) Calculate the Jones matrix of the composite device. [5]

(d) Give an expression of the transmitted intensity in terms of Υ. Hence, find the minimum value and direction of the magnetic field at which the transmission is maximal. [4]

(e) If the direction of the magnetic field is kept fixed, how does the transmitted intensity change if: [4]

1. The position of the light source S and the observer O are exchanged?
2. The two polarisers P_1 and P_2 are exchanged?

5 The components of the magnetic and electric field emitted by a Hertzian dipole are:

$$B_\phi = \frac{\mu_0 \sin\theta}{4\pi}\left\{\frac{[\dot{p}]}{r^2} + \frac{[\ddot{p}]}{rc}\right\}$$

$$E_\theta = \frac{\sin\theta}{4\pi\varepsilon_0}\left\{\frac{[p]}{r^3} + \frac{[\dot{p}]}{r^2c} + \frac{[\ddot{p}]}{rc^2}\right\}$$

$$E_r = \frac{2\cos\theta}{4\pi\varepsilon_0}\left\{\frac{[p]}{r^3} + \frac{[\dot{p}]}{r^2c}\right\}.$$

(a) Show that the instantaneous total radiated power of a point Hertzian dipole with moment $\mathbf{p} = p\,\hat{z}$ is

$$P = \frac{\mu_0}{6\pi c}[\ddot{p}]^2,$$

specifying what is meant by the notation $[\ddot{p}]$. [4]

A non magnetic dielectric sphere of radius R and uniform dielectric constant ε is permanently polarised and at time $t = 0$ its surface charge is distributed as $\sigma(\theta) = \sigma_0 \cos\theta$, where θ is the polar angle with respect to the $\hat{z}$ axis. The sphere is then set into rotation about the $\hat{x}$ axis with angular frequency ω.

(b) Show that at $t = 0$ the sphere is equivalent to a dipole of moment $\mathbf{p} = \frac{4}{3}\pi\sigma_0 R^3$ $\hat{z}$. Hence, show that the average radiated power $\bar{P}$ when the sphere is set into rotation is proportional to ω^4, and find the proportionality constant. [7]

(c) Assuming no other source of energy loss, find how the sphere's angular frequency varies with time. [4]

The same sphere initially unpolarised is traversed by a low frequency linearly polarised monochromatic radiation with wavelength $\lambda \gg R$.

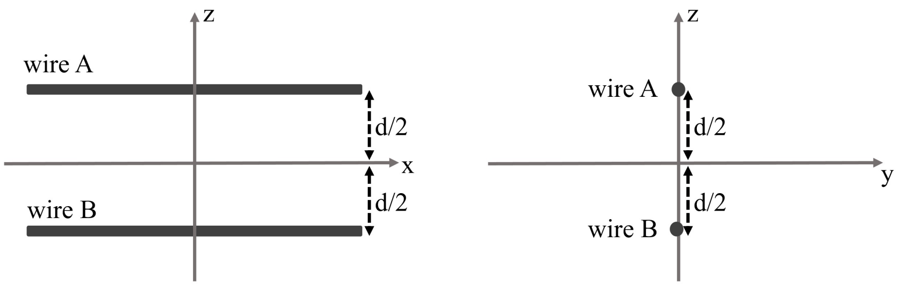
(d) Knowing that an electric field $\mathbf{E}$ induces a surface charge distribution $\sigma(\theta) = \sigma_0 \cos\theta$ with $\sigma_0 = 3\varepsilon_0\frac{\varepsilon-1}{\varepsilon+2}E$, calculate the cross section of scattering. How does it compare to the size of the sphere? [4]

6 Consider two inertial frames of reference O and O', with O' moving with velocity v relative to O along the positive $\hat{x}$ direction.

(a) Show how the Lorentz transformations for the electric and magnetic fields written below can be obtained from the transformation properties of the field-strength tensor. [3]

$$
\begin{aligned}
E'_x &= E_x & B'_x &= B_x \\
E'_y &= \gamma\left(E_y - vB_z\right) & B'_y &= \gamma\left(B_y + \frac{v}{c^2}E_z\right) \\
E'_z &= \gamma\left(E_z + vB_y\right) & B'_z &= \gamma\left(B_z - \frac{v}{c^2}E_y\right) .
\end{aligned}
$$

Consider two long filaments of a dielectric material separated by a distance d, as shown in the figure below. The filaments carry an excess charge, distributed uniformly with density per unit length η in their rest frame. The cross section is considered negligible.



(b) The components of the electric field generated by filament A and B in their respective rest frames are:

$$
E^0_{A,y} = \frac{\eta}{2\pi\varepsilon_0}\frac{y}{(z-\frac{d}{2})^2 + y^2}; \quad E^0_{A,z} = \frac{\eta}{2\pi\varepsilon_0}\frac{z-\frac{d}{2}}{(z-\frac{d}{2})^2 + y^2}
$$

$$
E^0_{B,y} = \frac{\eta}{2\pi\varepsilon_0}\frac{y}{(z+\frac{d}{2})^2 + y^2}; \quad E^0_{B,z} = \frac{\eta}{2\pi\varepsilon_0}\frac{z+\frac{d}{2}}{(z+\frac{d}{2})^2 + y^2} .
$$

Give an expression for the x, y and z components of the electric and magnetic fields in terms of $E^0_{A,y}$, $E^0_{A,z}$, $E^0_{B,y}$ and $E^0_{B,z}$ in the laboratory frame and in the frame of reference moving with velocity v along the positive $\hat{x}$ direction, in the two cases described below: [8]

1. Case 1: Both filaments A and B are set into motion with velocity v along $\hat{x}$.
2. Case 2: Only filament A is set into motion with velocity v along $\hat{x}$.

(c) Calculate the force experienced by filament B for the two cases described above, in the laboratory frame O. [4]

(d) Describe the phenomenon of spin-orbit coupling and give an example of a physical system in which this effect becomes relevant. [2]

(e) Describe how spin-orbit coupling affects the behaviour of the excess charge density accumulated on the filaments in both Case 1 and Case 2. [2]

END OF PAPER