UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 29 May 2013 13.30 to 15.30

EXPERIMENTAL AND THEORETICAL PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A candidate Higgs boson decays into two photons with energies 68 GeV and 73 GeV, separated by an angle of 128°. The energies are known to a precision of 10%; the angle has no uncertainty. Estimate the mass of the Higgs boson and its uncertainty. [4]

(b) Draw a Standard-Model Feynman diagram for the decay $B_s^0 \to J/\psi\,\phi$ and suggest the most likely decay modes for the J/ψ $[c\bar{c}]$ and ϕ $[s\bar{s}]$ states. The B_s^0 meson is a $\bar{b}s$ state. [4]

(c) An isotope of plutonium, ^{239}Pu, is an α-emitter with a half-life of 24,120 years. What is the initial activity of 1 kg of ^{239}Pu? [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) excited states of nuclei;

(b) parity violation;

(c) electroweak unification.

3 *Attempt* **either** *this question* **or** *question 4.*
Explain how the hadron wavefunction,

$$\psi_{\text{hadron}} = \psi_{\text{space}}\, \psi_{\text{spin}}\, \psi_{\text{flavour}}\, \psi_{\text{colour}},$$

leads to the prediction of an octet of spin-$\frac{1}{2}$ states and a decuplet of spin-$\frac{3}{2}$ states for the lowest-mass baryons formed from u-, d-, and s-quarks. [7]

Briefly explain the origin of the baryon mass formula

$$M_{qqq} = m_1 + m_2 + m_3 + A' \left[\frac{\mathbf{S_1} \cdot \mathbf{S_2}}{m_1 m_2} + \frac{\mathbf{S_1} \cdot \mathbf{S_3}}{m_1 m_3} + \frac{\mathbf{S_2} \cdot \mathbf{S_3}}{m_2 m_3} \right]$$

where A' is a constant. [2]

Show that the masses of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ uus bound states can be written as

$$\begin{aligned} M_{uus} &= 2m_u + m_s + A' \left[\frac{1}{4m_u^2} - \frac{1}{m_u m_s} \right] \quad \text{for} \quad J^P = \frac{1}{2}^+ \\ M_{uus} &= 2m_u + m_s + A' \left[\frac{1}{4m_u^2} + \frac{1}{2m_u m_s} \right] \quad \text{for} \quad J^P = \frac{3}{2}^+ . \end{aligned}$$

[8]

The Σ^+ ($J^P = \frac{1}{2}^+$) and Σ^{*+} ($J^P = \frac{3}{2}^+$) baryons are bound states of uus quarks. Calculate the predicted masses of the Σ^+ and Σ^{*+} from the baryon mass formula and compare with the measured values of $M(\Sigma^+) = 1.19\ \text{GeV}/c^2$ and $M(\Sigma^{*+}) = 1.38\ \text{GeV}/c^2$. [3]

The Σ^+ and Σ^{*+} decay predominantly via the following decay modes

$$\begin{aligned} \Sigma^+ &\rightarrow p[uud] + \pi^0[(u\bar{u} - d\bar{d})/\sqrt{2}] \\ \Sigma^{*+} &\rightarrow \Lambda[uds] + \pi^+[u\bar{d}] \end{aligned}$$

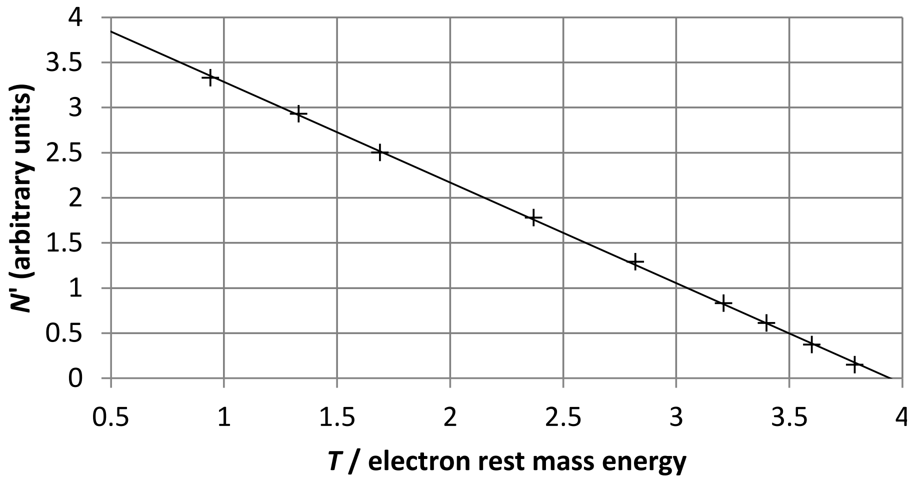
where the quark contents of the hadrons are shown in square brackets. Draw Feynman diagrams for these decays and explain why the total widths, $\Gamma(\Sigma^+) \approx 8 \times 10^{-12}$ MeV and $\Gamma(\Sigma^{*+}) \approx 36$ MeV, are significantly different. [5]

[*Mass of the u-quark,* $m_u \approx 362\,\text{MeV}/c^2$*; mass of the s-quark,* $m_s \approx 537\,\text{MeV}/c^2$*;* $A' = 0.026\,\text{GeV}^3/c^6$.]

4 *Attempt* **either** *this question* **or** *question 3.*

Outline the Fermi theory of β-decay, making clear any assumptions made, and show how it gives rise to (i) Fermi and Gamow–Teller transitions and (ii) allowed and forbidden transitions. [10]

In the β-decay process ${}^{91}_{39}\mathrm{Y} \rightarrow {}^{91}_{40}\mathrm{Zr} + e^- + \bar{\nu}_e$, the number of electrons emitted in the momentum range p to $p + \mathrm{d}p$ is denoted by $N(p)\mathrm{d}p$. The quantity $N'(p)$ is defined as $N'(p) \equiv \sqrt{N(p)/(p^2 F(p))}$, where $F(p)$ is the Fermi function that takes into account the effect of the nuclear field on the electron. In the figure below, N' is plotted as a function of the electron kinetic energy T.



Show why there is a linear dependence in this plot. Use the plot to find the difference between the ground-state energies of yttrium-91 and zirconium-91, expressing your answer in MeV. [8]

The Shell Model of the nucleus predicts that, following a number of closed shells containing 28 nucleons of any one type, the sequence of states is

$$2p_{3/2},\ 1f_{5/2},\ 2p_{1/2},\ 1g_{9/2},\ 2d_{5/2},\ 1g_{7/2}, \cdots .$$

Deduce the most likely type of transition to be involved in the β-decay of yttrium-91. [7]

END OF PAPER