NST II ETP(4) MAY 2010

2010 Marking Scheme

B Question 1

• → 1 Mark

a) At band center we can assume (parabolic band)

$$E(k) = E_0 + \frac{\hbar^2 k^2}{m^*} \quad \bullet$$

From this we get

$$\frac{d^2E(k)}{dk^2} = \frac{\hbar^2}{m^*} \Rightarrow m^* = \frac{\hbar^2}{d^2E(k)/dk^2} \Rightarrow m^* \simeq 0.5 m_e \quad \bullet$$

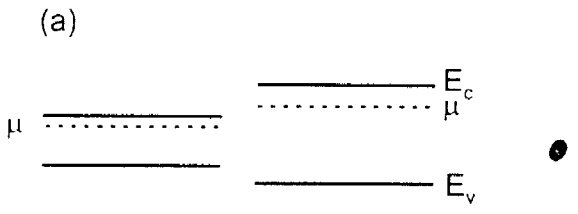
Cyclotron resonance frequency ν

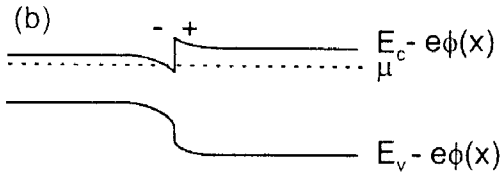
$$\nu = \frac{\omega}{2\pi} = \frac{eB}{m^*} \simeq 3.52 \cdot 10^{11} Hz \quad \bullet$$

(4 MARKS)

$B = 1$ T and $\frac{d^2E(k)}{dk^2} \approx 2.44 \cdot 10^{-38}$ m^{4}kgs^{-2}.

b) First part, either graphically or in text form.





• either sketch or explain

half mark each point

- Two semiconductors with different Fermi energies and different band gaps are brought into contact.
- Chemical potentials have to equalize.
- Valence and conductance band of wide-band gap material is shifted by the difference in chemical potentials.
- With right doping level discontinuity in contact region drops below Fermi energy.

Potential well with infinite walls:

$$E_n = \frac{(\hbar \pi n)^2}{2mL^2} \quad \bullet$$

and thus

$$\Delta E = \frac{3 * (\hbar\pi)^2}{2mL^2} \approx 2.7 \cdot 10^{-20} J \approx 168\text{meV}$$

(4 MARKS)

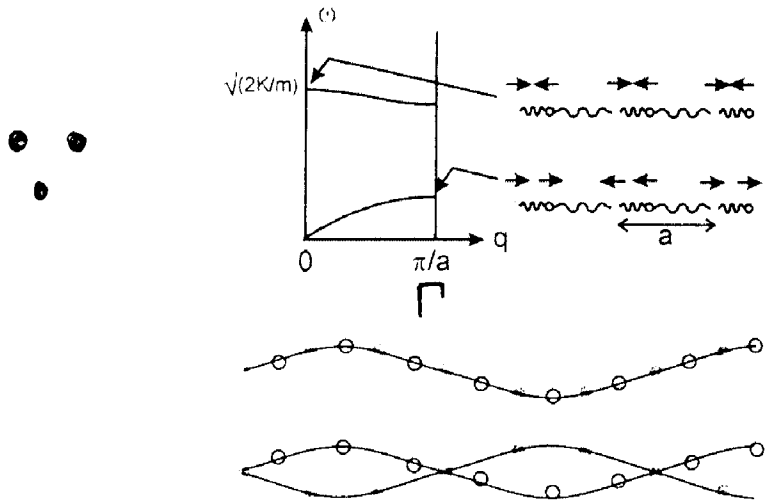
c) For small ka linearize sin

$$\omega^2 \approx \frac{4C}{M}\left(\frac{1}{2}ka\right)^2$$

$$v = \frac{d\omega}{dk} = \sqrt{\frac{C}{M}}a$$

for small k.

Acoustical and optical phonons. And dispersion relation:



Maximum wave vector is π/a with a the lattice constant.
(4 MARKS)

Question 2

Bookwork

a) (13 MARKS in total)a) Spin energies in field

$$\epsilon_{\mathbf{k}\uparrow} = \epsilon_{\mathbf{k}} + \mu_B H$$

$$\epsilon_{\mathbf{k}\downarrow} = \epsilon_{\mathbf{k}} - \mu_B H$$

To calculate the susceptibility we need the magnetization $M = \mu_b(n_\uparrow - n_\downarrow)$, where

$$n_\uparrow - n_\downarrow = \mu_B H g(\mu)$$

and thus (using that the spin splitting is small) we get the susceptibility

$$\frac{M}{H} = \chi = \mu_B^2 g(\mu)$$

b) Bookwork

- The specific heat in a metal is $C = \gamma T + \alpha T^3$.
- First term is the electronic contribution second term lattice contribution.
- γ, the Sommerfeld parameter,
- γ is proportional to density of states at the Fermi energy E_F.
- Since $E_F = \hbar^2 k_f^2 / 2m$ and $\gamma \propto 1/E_F$ we have that $\gamma \propto m$, where m is the effective mass.
- For a heavy fermion superconductor there is an energy gap at E_F below the critical temperature which proves that the electronic C disappears.

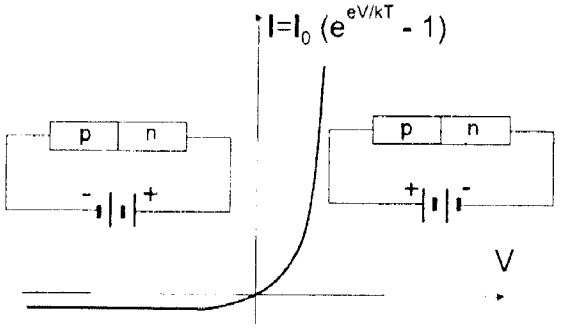
c) Bookwork + a little bit of general knowledge

- Solar cells are $p - n$ junctions
- Incident photons create electron-hole pairs
- When carriers reach junction they are separated by built-in field
- This creates a dipole in the solar cell
- A small electrical bias is created
- There are direct and indirect semiconductors, ideally direct semiconductors should be used
- Armorphous Si is most commonly used because of simplified fabrication and 2.5 times higher absorption than crystalline Si

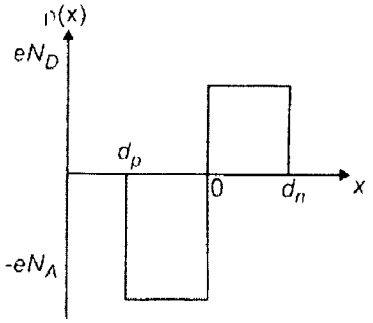
B Question 3

Bookwork

IV characteristic with formula and reverse and forward bias sketched



and comments on diffusion currents, potential barrier and hole generation current. Sketch



Diffusion currents. In equilibrium with no external voltage bias, there is no net current flowing across the junction. But of course to maintain chemical equilibrium, there must be microscopic current flows across the barrier in both directions, that cancel on average.

Potential barrier. The depletion regime of the junction is a high-resistance in comparison to the n- or p-type doped semiconductors. Any potential applied across the device is dropped almost entirely across the depletion layer. The overall potential seen by a (positively charged) hole is therefore ϕ_b - V , where ϕ_b is the barrier height at equilibrium.

Hole generation current. On the n-type side of the depletion regime, the majority carriers are electrons, but detailed balance means that there will always be some small density of minority holes. Any minority carrier diffusing into the depletion regime will be swept down the potential into the p-type regime. This generates a current (from right to left, of holes, and therefore negative) $-J_h^{gen}$ It is not much dependent on the external bias V , because of the large inbuilt potential drop in the depletion regime.

all bookwork upto here!

Poisson equation just integrated twice since in region $0 < x < d_n$

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\epsilon\epsilon_0} \simeq -\frac{eN_D}{\epsilon\epsilon_0}$$

integrate to get electric field in n- or p-doped zone

$$E(x) = \frac{dV(x)}{dx} = -\frac{eN_D}{\epsilon\epsilon_0}(d_n - x)$$

again to get $V(x)$

$$V(x) = V_n(\infty) - \frac{eN_D}{2\epsilon\epsilon_0}(d_n - x)^2$$

$$V(x) = V_p(\infty) - \frac{eN_A}{2\epsilon\epsilon_0}(d_p - x)^2$$

From the continuity at $x = 0$ we get V_D as was asked

$$\frac{e}{2\epsilon\epsilon_0}(N_D d_n^2 + N_A d_p^2) = V_n(\infty) - V_p(\infty) = V_D$$

There has to be charge neutrality

$$N_D d_n = N_A d_p$$

Combining the last two equations leads to

$$d_n = \left(\frac{2\epsilon\epsilon_0 V_D}{e}\frac{N_A/N_D}{N_A + N_D}\right)^{1/2}$$

$$d_p = \left(\frac{2\epsilon\epsilon_0 V_D}{e} \frac{N_D/N_A}{N_A + N_D} \right)^{1/2}$$

With the numbers we get

$$d_n = 2.47 \cdot 10^{-6}\text{m}$$

$$d_p = 0.87 \cdot 10^{-6}\text{m}$$

This we can estimate the junction capacitance to

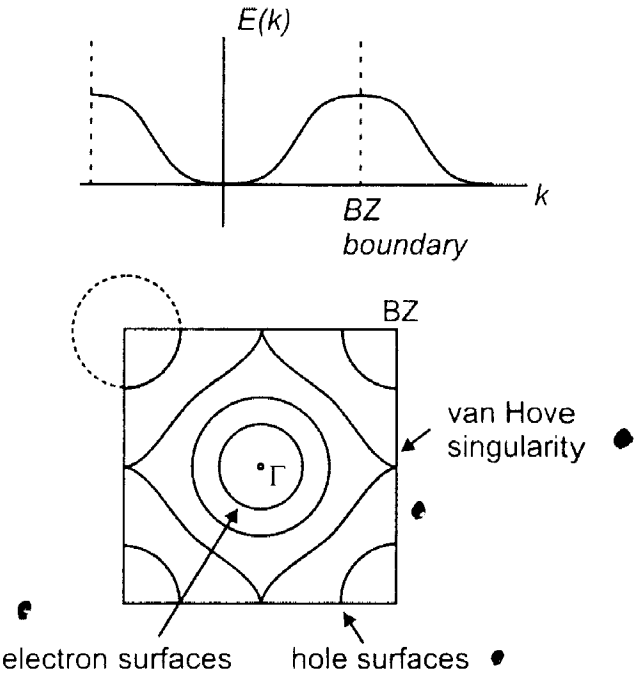
$$C = \frac{\epsilon\epsilon_r A}{d_n + d_p} \approx 3.18 \cdot 10^{-14}\ \text{F}$$

And the fastest response time is given by the $1/RC \approx 3.14$ GHz. However, d_n and d_p will depend on the applied voltage since then instead of V_D we have $V_D - U$ where U is the outer voltage. So we have $C \propto 1/\left(\sqrt{V_D - U}\right)$ which means that f_c gets lower for $U > 0$ and larger for $U < 0$.

B Question 4

Book work

Fermi surface perpendicular to Brillouin zone (BZ) boundary:
$E(k)$ is periodic. It follows that $E(k+g) = E(k)$, where g is the reciprocal lattice vector.
At the BZ boundary, dispersion $E(k)$ is flat (due to hybridization with states near $k-g$).
For the group velocity we have $v = \frac{1}{\hbar}\nabla_k E(k)$. v has zero projection perpendicular to BZ boundary, thus v lies in the BZ boundary plane.
Iso-surfaces of E like at the Fermi energy E_F are perpendicular to $\nabla_k E(k)$, which is parallel to BZ boundary. From this we can conclude that E_F iso-surface is perpendicular to BZ boundary.
Sketch:



Calculate hole density with Onsager. First calculate area of Fermi surface A_k:

$$A_k = \frac{2\pi e}{\hbar}\frac{1}{\Delta(1/B)} \approx 173 \cdot 10^{18}\ \mathrm{m}^{-2} \simeq 1.73\ \text{Å}^{-2}$$

using $\Delta(1/B) \approx 18,100$ T from question. Then calculate area of BZ:

$$\left(\frac{2\pi}{a}\right)^2 = \left(\frac{2\pi}{3.86\ \text{Å}}\right)^2 = 2.65\ \text{Å}^{-2}$$

this corresponds to 2 holes per site. The hole concentration per site is thus:

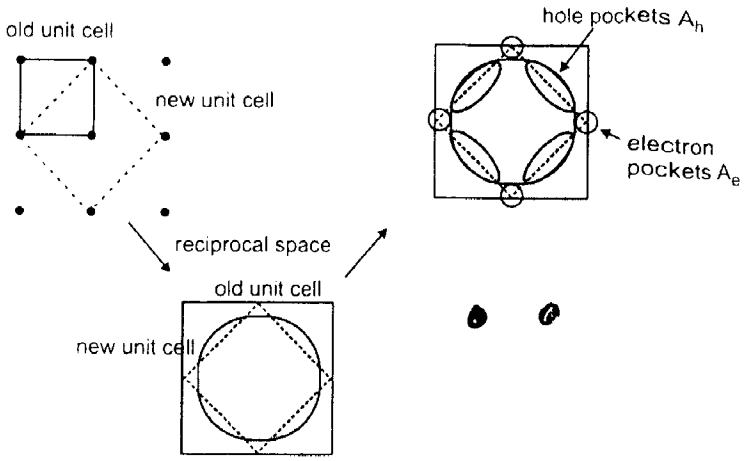
$$p = 2 \cdot \frac{1.73}{2.65} = 1.306 \text{ per site}$$

YBCO has $p = 1.1$ per site. So we can estimate the expected oscillations from the Tl2201 frequency

$$F_{YBCO} \approx F_{Tl2201} \cdot \frac{1.1}{1.3} \gg 540\ \text{T}$$

from this we can conclude that this is not a simple cylindrical Fermi surface. Additional periodic potential could lead to doubling of unit cell, as suggested in the question. This means that the Fermi surface now intersects BZ at right angles. This leads to the formation of pockets for electrons and holes.

Sketch



The actual sizes of the pockets depends on the size of the hybridization gap, but the hole density dictates the difference in area. Total number of holes in sample:

$$2N_h = (2A_h - A_e + A_{NBZ})\frac{A_{sample}}{(2\pi)^2}$$

where we have 2 holes per site. Now we get with $A_{sample} = a^2 N$ and $A_{NBZ} = \frac{1}{2}\left(\frac{2\pi}{a}\right)^2$

$$1.1 = \frac{N_h}{N} = \frac{2A_h - A_e + A_{NBZ}}{2A_{NBZ}} = \frac{A_h}{A_{NBZ}} - \frac{A_e}{A_{NBZ}} + 1$$

which leads to

$$\frac{A_h}{A_{NBZ}} - \frac{A_e}{A_{NBZ}} = 1.1 - 1 = 0.1$$

How A

For "T1220l" we had with $A = 1.3$ a frequency of $F \approx 18,100$ T. The new frequency is then just

$$F_h = 18,100\,\mathrm{T}\frac{0.1}{2}\frac{1}{1.3} \approx 700\,\mathrm{T}$$

(One can also use Onsager to calculate again).

Conclusion: Actual oscillations are probably due to electron pocket or more complicated band structure.