UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 28 May 2014 9.00 am to 11.00 am

PHYSICS (3)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (3)

ADVANCED QUANTUM PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

ADVANCED QUANTUM PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A particle of mass m moves in a potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Determine the probability of finding the particle outside the classical turning points when it is in the ground state described by $\psi_0 = A\exp\left(-\frac{1}{2}m\omega x^2/\hbar\right)$. [4]

[*You may wish to use* $(2/\sqrt{\pi})\int_a^b \exp(-y^2)dy = \mathrm{erf}(b) - \mathrm{erf}(a)$*, where* $\mathrm{erf}(0) = 0$, $\mathrm{erf}(1) = 0.84$ *and* $\mathrm{erf}(\infty) = 1$.]

(b) A two-level system, described by $|\psi_1\rangle$ and $|\psi_2\rangle$ with energies E_1 and E_2, is perturbed by a sudden impulse described by the Hamiltonian $\widehat{H} = \widehat{U}\delta(t)$, where the operator $\widehat{U}$ has only off-diagonal matrix elements in this basis. If the system is initially in state $|\psi_1\rangle$, find, to lowest order, the probability that a transition occurs. [4]

(c) The carbon atom ground state has a p^2 configuration. What are the allowed levels in the LS-coupling scheme and which has the lowest energy according to Hund's rules? [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the postulates of Quantum Mechanics;

(b) the central field approximation and the Hartree method;

(c) the properties and operation of lasers.

3 *Attempt* **either** *this question* **or** *question 4.*

Show how the raising and lowering operators for spin $\frac{1}{2}$ particles can be constructed from the Pauli spin matrices

$$\widehat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \widehat{\sigma}_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \text{ and } \widehat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

[4]

Two oppositely-charged spin $\frac{1}{2}$ particles (spins $\widehat{s}_1 = (\hbar/2)\,\widehat{\sigma}_1$ and $\widehat{s}_2 = (\hbar/2)\,\widehat{\sigma}_2$) are coupled in a system with a spin-spin interaction energy ΔE. The system is placed in a uniform magnetic field $\boldsymbol{B}$ which is aligned in the z-direction. The Hamiltonian for the system is

$$\widehat{H} = \frac{\Delta E}{4}\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 - (\widehat{\mu}_1 + \widehat{\mu}_2) \cdot \boldsymbol{B}$$

where $\widehat{\mu}_j = (g_j\mu_0/\hbar)\widehat{s}_j$ is the magnetic moment of the j-th particle, with $\mu_0 = q\hbar/(2m)$; symbols q and m denote the charge and mass of the particle, respectively.

Using the spin basis states $|\uparrow_1\uparrow_2\rangle$, $|\downarrow_1\downarrow_2\rangle$, $|\uparrow_1\downarrow_2\rangle$ and $|\downarrow_1\uparrow_2\rangle$, show that the Hamiltonian can be written as

$$\widehat{H} = \begin{pmatrix} H_{11} & 0 & 0 & 0 \\ 0 & H_{22} & \frac{\Delta E}{2} & 0 \\ 0 & \frac{\Delta E}{2} & H_{33} & 0 \\ 0 & 0 & 0 & H_{44} \end{pmatrix},$$

with

$$\begin{aligned} H_{11} &= \frac{\Delta E}{4} - \frac{\mu_0 B}{2}(g_1 + g_2), \\ H_{22} &= -\frac{\Delta E}{4} - \frac{\mu_0 B}{2}(g_1 - g_2), \\ H_{33} &= -\frac{\Delta E}{4} + \frac{\mu_0 B}{2}(g_1 - g_2), \\ H_{44} &= \frac{\Delta E}{4} + \frac{\mu_0 B}{2}(g_1 + g_2). \end{aligned}$$

[9]

Determine the eigenvalues of the system. [4]

What are the energy levels and degeneracies for the system in the region of (a) zero magnetic field and (b) a high magnetic field? [4]

Sketch the energies as a function of B. [4]

4 *Attempt* **either** *this question* **or** *question 3.*

Discuss the applicability and use of the approximation

$$\varDelta E_i = \langle i|\widehat{H}_1|i\rangle + \sum_{k\neq i} \frac{|\langle i|\widehat{H}_1|k\rangle|^2}{E_i - E_k} + \cdots ,$$

which gives the energy shift $\varDelta E_i$ of eigenstate $|i\rangle$, with initial energy E_i, of a quantum system resulting from the addition of a Hamiltonian $\widehat{H}_1$ to the existing one. [8]

A rigid linear molecule, in angular momentum eigenstate $|J, m\rangle$ with rotational energy $BJ(J + 1)$ and electric dipole moment p, is subjected to an electric field E. J is the quantum number for rotational angular momentum and m the quantum number for the component in the direction of the electric field. Explain why there can be no correction of first order in E to the rotational energy. [4]

Show that, for $J \neq 0$, the second-order correction to the energy of the state $|J, m\rangle$ is

$$\frac{(pE)^2 \left[J(J+1) - 3m^2\right]}{2BJ(J+1)(2J-1)(2J+3)}.$$

What is the second-order correction for the $J = 0$ state? [8]

For the linear molecule OCS the rotational transition $J = 0 \rightarrow J = 1$ occurs at 12.16 GHz. In a static electric field of 10^5 V m^{-1}, the transition splits into two components separated by 3.21 MHz. What is the dipole moment of OCS? [5]

[*You may use the result*

$$\langle J, m|\cos\theta|J' - 1, m'\rangle = \langle J' - 1, m'|\cos\theta|J, m\rangle = \left(\frac{J^2 - m^2}{4J^2 - 1}\right)^{\frac{1}{2}} \delta_{JJ'}\delta_{mm'},$$

where θ is the azimuthal angle.]

END OF PAPER