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Lent/Easter Terms 2022

Part II Particle and Nuclear Physics
Examples Sheet 3

Weak Interactions

20. Tau Decay, Tripos A-style question.
Draw a Feynman diagram for each of the principal decay modes of the τ^-. Show that the τ lepton decay branching fractions should be approximately in the ratios

$$(\tau^- \to e^- \nu_\tau \bar{\nu}_e) \ : \ (\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu) \ : \ (\tau^- \to \text{hadrons}) = 1 : 1 : 3$$

The actual ratios are approximately 1.02 : 1 : 3.5. Suggest a possible explanation.

Estimate the mean lifetime of the τ lepton, given that the branching fraction for $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ is 18%. You may assume the decay rate for $X \to e^- \bar{\nu}_e \nu_X$ is given by Sargent's Rule

$$\Gamma = \frac{G_F^2 E_0^5}{60\pi^3}$$

.

[You may use $m_\tau = 1.777$ GeV/c^2, $m_\mu = 0.106$ GeV/c^2 and take the mean μ lifetime to be 2.197×10^{-6} s.]

21. Threshold Energy, Tripos A-style question.
In a beam of antineutrinos, it is proposed to search for $\bar{\nu}_\tau$ via their interactions on nucleons in a stationary target to produce τ-leptons.

    (a) Draw the Feynman diagram for the simplest such production process.
    (b) Calculate the minimum energy of the $\bar{\nu}_\tau$ which would permit τ-lepton production.
    (c) What is the energy of the produced τ-lepton when the $\bar{\nu}_\tau$ has this threshold energy ?
    (d) How far will the τ-lepton travel on average before decaying, given that its mean lifetime is 290 fs ?

[The masses of τ^+, proton and neutron are 1.777 GeV/c^2, 0.938 GeV/c^2 and 0.940 GeV/c^2 respectively.]

22. Omega Decay, Tripos A-style question.
The Ω^- baryon (sss), produced in the event shown in Q.4 is seen to decay weakly through the decay chain $\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$ and $\Lambda^0 \to \mathrm{p}\pi^-$. Draw the Feynman diagrams for the decays of the Ω^-, the Ξ^0 and the Λ^0.

Draw a Feynman diagram for the strong decay $\Omega^- \to \Xi^-\overline{\mathrm{K}^0}$ and explain why the decay is not observed. With the aid of a Feynman diagram explain why the weak decay $\Omega^- \to \Lambda^0\pi^-$ is strongly suppressed.

[The strange hadrons have quark compositions and masses $\Omega^-(sss)$ 1.67 GeV, $\Xi^0(uss)$ 1.31 GeV, $\Xi^-(dss)$ 1.32 GeV, $\overline{\mathrm{K}^0}(s\bar{d})$ 498 MeV, $\Lambda^0(uds)$ 1.12 GeV]

Electroweak Unification

23. Z Mass Peak, Tripos B-style question.

(a) In lectures we deduced that the couplings of the Z boson to fermions should be of the form:

$$g_Z[I_3 - Q\sin^2\theta_\mathrm{W}] \quad where \quad g_Z = \frac{e}{\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}} \, .$$

In this expression, Q is the electric charge (in units of e), I_3 is the weak isospin of the fermion species and helicity state being considered and the weak mixing angle is given by $\sin^2\theta_\mathrm{W} \approx 0.23$. The decay rate into $f\bar{f}$ (assumed massless compared with the Z) is proportional to $(g_\mathrm{L}^2 + g_\mathrm{R}^2)$ where g_L and g_R are the couplings to left-handed and right-handed fermions respectively. Compare the Z decay rates to pairs of charged lepton pairs of each species, neutrino-antineutrino pairs, and to u-like and d-like quark-antiquark pairs. Hence predict the branching fractions for Z decay to $\tau^+\tau^-$, to neutrinos and to hadrons.

(b) In the OPAL experiment at LEP the cross-section for $e^+e^- \to \tau^+\tau^-$ was measured at various centre-of-mass energies. Some of the results are shown below. Plot these data

E_cm/GeV	$\sigma(e^+e^- \to \tau^+\tau^-)$/nb
88.481	0.2769 ± 0.0235
89.442	0.4892 ± 0.0091
90.223	0.8331 ± 0.0368
91.283	1.4988 ± 0.0213
91.969	1.1892 ± 0.0235
92.971	0.7089 ± 0.0105
93.717	0.4989 ± 0.0276

and make estimates of the Z boson mass, m_Z, the total width of the Z boson, Γ_Z, and the partial decay width to $\tau^+\tau^-$, Γ_τ, (assuming lepton universality of the Neutral Current). Compare the branching fraction for $\mathrm{Z} \to \tau^+\tau^-$ with your predictions from (a), and comment.

Why is the measured resonance curve asymmetric? Indicate what other effects need to be taken into account when accurately determining m_Z, Γ_Z and Γ_τ

(c) Estimate the total decay width, Γ_Z, and the lifetime of the Z boson using the resonant cross-section ratio,

$$\frac{\sigma(e^+e^- \to Z \to \text{hadrons})}{\sigma(e^+e^- \to Z \to \mu^+\mu^-)} = 20.7,$$

and the measured values of the Z partial decay widths, $\Gamma(Z \to \mu^+\mu^-) = 83.3$ MeV and $\Gamma(Z \to \nu_\mu\bar{\nu_\mu}) = 166.5$ MeV. Make clear any assumptions you make.

24. <u>W Width, Tripos A-style question.</u>
The number of neutrino species can be estimated from the total width of the W boson. Using the Standard Model prediction of the partial width for $\mathrm{W}^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}$ decays,

$$\Gamma(\mathrm{W}^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}) = \frac{\mathrm{G_F}}{\sqrt{2}}\frac{\mathrm{M_W^3}}{6\pi},$$

the mass of the W boson, $M_\mathrm{W} = 80.385 \pm 0.015$ GeV/c^2 and the total width, $\Gamma_\mathrm{W} = 2.085 \pm 0.042$ GeV, estimate the number of light neutrino species. Make clear your assumptions.
[$G_\mathrm{F} = 1.2 \times 10^{-5}$ GeV^{-2}. *]*

25. <u>ν Scattering, Tripos A-style question.</u>
Draw *all* possible lowest order Feynman diagrams for the following neutrino scattering or annihilation processes:

(a) $\nu_\mathrm{e}\,\mathrm{e}^- \to \nu_\mathrm{e}\,\mathrm{e}^-$
(b) $\bar{\nu}_\mathrm{e}\,\mathrm{e}^- \to \bar{\nu}_\mathrm{e}\,\mathrm{e}^-$
(c) $\nu_\mu\, e^- \to \nu_\mu\, e^-$
(d) $\bar{\nu}_\mu\,\mathrm{e}^- \to \bar{\nu}_\mu\,\mathrm{e}^-$
(e) $\nu_\mathrm{e}\,\mathrm{n} \to \mathrm{e}^-\,\mathrm{p}$

26. <u>Weak Force & Conservation, Tripos A-style question.</u>
Consider each of the groups of processes given below. In each group, with the aid of Feynman diagrams using the Standard Model vertices, determine which processes are allowed and which are forbidden. By considering the strength of the forces involved, rank the processes in each group in order of expected rate.

(a) $\pi^0 \to \gamma\gamma$	$\pi^0 \to \pi^-\mathrm{e}^+\nu_\mathrm{e}$	and	$\pi^0 \to \nu\bar{\nu}$;
(b) $e^+e^- \to \tau^+\tau^-$	$\bar{\nu}_\mu + \tau^- \to \tau^- + \bar{\nu}_\mu$	and	$\nu_\tau + \mathrm{p} \to \tau^+ + \mathrm{n}$;
(c) $\mathrm{B}^0(\bar{b}d) \to \mathrm{D}^-(\bar{c}d)\pi^+$	$\mathrm{B}^0 \to \pi^+\pi^-$	and	$\mathrm{B}^0 \to J/\psi\mathrm{K}^0$
(d) $\mathrm{D}^0(c\bar{u}) \to \mathrm{K}^-\pi^+$	$\mathrm{D}^0 \to \pi^+\pi^-$	and	$\mathrm{D}^0 \to \mathrm{K}^+\pi^-$.

Neutrino Oscillations

27. <u>ν Oscillations, Tripos B-style question.</u>

(a) Show that if there are two neutrino mass eigenstates ν_2 and ν_3 with masses m_2 and m_3 and energies E_2 and E_3, mixed so that

$$\begin{aligned}\nu_\mu &= \nu_2\cos\theta + \nu_3\sin\theta \\ \nu_\tau &= -\nu_2\sin\theta + \nu_3\cos\theta\end{aligned}$$

then the number of muon neutrinos observed at a distance L from the muon source is

$$|\nu_\mu(L)|^2 = |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\,\sin^2\left\{\left(\frac{E_3-E_2}{2\hbar}\right)\frac{L}{c}\right\}\right].$$

(b) If m_2 and m_3 are very much less than the neutrino momentum, p, show that

$$|\nu_\mu(L)|^2 \approx |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\,\sin^2\left\{A\left(\frac{(m_2^2 - m_3^2)L}{p}\right)\right\}\right]$$

where A is a constant.

(c) In 2005 the MINOS experiment started to study neutrino oscillations by pointing a beam of 1-5 GeV/c muon neutrinos from Fermilab, Illinois, at the 5400 ton MINOS far dectector in the SOUDAN mine in Minnesota, 730 km away. The experiment aimed to make a precise measurement of $m_3^2 - m_2^2$.

Sketch the expected energy spectrum of muon neutrinos at the MINOS far detector if $\sin^2(2\theta) = 0.90$ and $m_3^2 - m_2^2 = 2.5 \times 10^{-3}$ $(\mathrm{eV/c^2})^2$. Assume that the energy spectum of neutrinos produced by the beam at Fermilab is of uniform intensity in the range 1-5 GeV and zero elsewhere (i.e. a top-hat function).

(d) If muon neutrinos oscillate into tau neutrinos, will any τ leptons (produced by charged current interactions) be observed in the MINOS far detector ?

Hint: You may find the result of qu.21 useful.

[$A = 1.27$ s^{-1} if m_2 and m_3 are measured in eV/c^2, p in GeV/c and L in km. The mass of the τ^- is 1.777 GeV/c^2.]

Numerical answers

20. 0.3 ps

21. (b) 3.47 GeV; (c) 2.88 GeV; (d) 110 μm; (e) $\sim$ 20%

23. (a) 0.034 ($e^{\pm}$, $\mu^{\pm}$, $\tau^{\pm}$); 0.068 (for each neutrino flavour); 0.118 (each up-type quark); 0.152 (each down-type quark); 0.692 (all hadrons) ; (c) 2.47 GeV, 2.66×10^{-25} s

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Suggested Tripos Questions

Weak interaction: 2018 B4, 2017 1(c), 2011 3

Electroweak unification: 2018 A1(b), 2015 3, 2013 1(a)

Neutrino Oscillation: 2009 (3) A4