UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 31 May 2017 1.30 pm to 3.30 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

PARTICLE AND NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **six** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

Natural units of $\hbar = c = \mu_0 = \epsilon_0 = 1$ are used throughout this paper.

1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) Compute the threshold energies for the reactions $\bar{\nu}_e + p \to e^+ + n$ and $\bar{\nu}_\mu + p \to \mu^+ + n$ for antineutrinos incident on a stationary proton. What are the implications of these threshold energy values for the detection of flavour oscillations of antineutrinos produced by nuclear reactors? [4]

[*The neutron and proton masses are* $m_n = 939.6\,\text{MeV}$, $m_p = 938.3\,\text{MeV}$. *The electron and muon masses are* $m_e = 0.511\,\text{MeV}$ *and* $m_\mu = 105.7\,\text{MeV}$. *Antineutrinos can be assumed to be massless.*]

(b) In the shell model of the nucleus, the nucleon energy levels are ordered in increasing energy as $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, … . What is the shell model prediction for the magnetic dipole moment of the nuclides ${}^{15}_{7}\text{N}$ and ${}^{17}_{8}\text{O}$, in units of the nuclear magneton, μ_N? [4]

[*The g-factors for the magnetic dipole moments of the proton and neutron are* $g_p = +5.586$ *and* $g_n = -3.826$. *The Landé g-factor formula is*

$$g_j = g_\ell \frac{\ell(\ell+1) + j(j+1) - s(s+1)}{2j(j+1)} + g_s \frac{s(s+1) + j(j+1) - \ell(\ell+1)}{2j(j+1)} .$$
]

(c) In the Standard Model, the couplings g_L and g_R of the Z boson to fermions are $g_{L,R} \propto (I_3)_{L,R} - Q \sin^2\theta_W$, where I_3 denotes weak isospin, and Q is the electric charge of the fermion in units of e. The decay $Z \to e^+e^-$ has a branching ratio of 3.36%. Assuming $\sin^2\theta_W = 0.23$, compute the branching ratio for the decay $Z \to \bar{\nu}_e \nu_e$. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) experimental determinations of the nuclear charge radius and charge density : explain how electron scattering measurements can be used to obtain information on the charge radius and density of the nucleus, and summarise the results obtained. Briefly mention other techniques which can be used to obtain such information;

(b) QCD : summarise the key features of Quantum Chromodynamics (QCD) as a theory of the strong interactions, and discuss the experimental evidence in its favour;

(c) the top quark, W boson and Higgs boson : give examples of processes occurring in high energy particle collisions which lead to the production of top quarks, W bosons or the Higgs boson. Discuss the decays of these particles, and explain how, taken together, measurements of their masses, m_{t}, m_{W} and m_{H}, can be used to test the electroweak sector of the Standard Model.

(TURN OVER

3 *Attempt* **either** *this question* **or** *question 4.*

The first four steps in the radioactive decay chain which begins with ^{232}Th are

$$\begin{array}{ccccccccc} ^{232}\mathrm{Th} & \xrightarrow{\alpha} & ^{228}\mathrm{Ra} & \xrightarrow{\beta^-} & ^{228}\mathrm{Ac} & \xrightarrow{\beta^-} & ^{228}\mathrm{Th} & \xrightarrow{\alpha} & ^{224}\mathrm{Ra}\,, \\ & (1.4\times10^{10}\,\mathrm{y}) & & (5.75\,\mathrm{y}) & & (6.1\,\mathrm{h}) & & (1.9\,\mathrm{y}) & \end{array}$$

where the half-life of each α-decay or β^--decay is given in brackets.

How, approximately, do the half-lives of α-decays of even-even nuclei depend on the energy release, Q_α, and on the atomic number, Z, of the parent nucleus? Using the ground state nuclear masses given at the end of this question, obtain order of magnitude estimates of the decay rates for the α-decays of ^{228}Ra and ^{228}Ac, and explain why these α-decays are not observed in practice. [5]

Deduce the permitted spin-parities (J^P) of the nucleus Y produced in the α-decay $X \to Y + \alpha$ of an even–even nucleus X. [4]

In the α-decay of ^{232}Th, the α-particle spectrum contains components with kinetic energies 4011.2 keV, 3948.5 keV, and 3810.0 keV. Taking the recoil of the daughter nucleus into account, verify that the highest of these kinetic energies is approximately that expected for α-decay to the ground state of ^{228}Ra. [4]

Obtain the ratio of excitation energies for two of the excited states of ^{228}Ra, and comment on the likely origin and spin-parity values of these excited states. [3]

Quantify the level of agreement of the measured β-decay half-lives of ^{228}Ra and ^{228}Ac with Sargent's Rule. [m_e = 0.00055 amu.] [3]

The ground state of ^{228}Ac has spin-parity 3^+. In order of increasing energy, the lowest lying excited states of ^{228}Ac have spin-parities 1^-, 1^+, 1^- and 1^+. The electron spectrum produced in the β-decay of ^{228}Ra consists of four components, with end-point energies and relative intensities of 39.5 keV (10%), 39.1 keV (40%), 25.6 keV (20%), and 12.7 keV (30%). Classify the β-decays of ^{228}Ra to the ground state and excited states of ^{228}Ac, and comment on the extent to which the β-decays observed are consistent with these classifications. [6]

The table below gives the ground state nuclear masses, m_N, of selected nuclides, in units of 1 amu = 931.494095 MeV *:*

nuclide	m_N/amu	*nuclide*	m_N/amu
$^{232}_{90}$Th	231.98868	$^{224}_{86}$Rn	223.97692
$^{228}_{88}$Ra	227.98280	$^{224}_{87}$Fr	223.97567
$^{228}_{89}$Ac	227.98220	$^{224}_{88}$Ra	223.97194
$^{228}_{90}$Th	227.97937	$^{4}_{2}$He	4.00151

4 *Attempt* **either** *this question* **or** *question 3.*

The leading order QED prediction for the cross section of the non-resonant process $e^+e^- \to \mu^+\mu^-$ is $\sigma_{\mu\mu} = 4\pi\alpha^2/3s$, where $\alpha \approx 1/137$ is the fine structure constant and $\sqrt{s}$ is the centre of mass energy. Draw the leading-order Feynman diagram for this process, and explain how the factor α^2/s arises from the Feynman rules. [3]

Draw the quark-level QED Feynman diagram for the non-resonant production of hadrons in e^+e^- collisions. Obtain the leading order QED prediction for the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ of non-resonant cross sections, for values of $\sqrt{s}$ in the range $2m_b < \sqrt{s} < 2m_t$, where m_b and m_t are the masses of the bottom and top quarks, respectively. [3]

The plot overleaf shows measurements of $\sigma_{\text{had}} = \sigma(e^+e^- \to \text{hadrons})$ in the region of the $\Upsilon(4S)$ bottomonium ($b\bar{b}$) resonance. To correct for inefficiencies of the detector, the cross section measurements must be multiplied by a scale factor, which can be assumed to be independent of $\sqrt{s}$. The $\Upsilon(4S)$ resonance contribution can be assumed to be described by the Breit-Wigner cross section which, in standard notation, is given by

$$\sigma(E) = \frac{\pi g}{p^2} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \Gamma^2/4} \,.$$

The branching ratio for decays of the $\Upsilon(4S)$ to hadrons is known to be close to 100%.

Estimate the scale factor which must be applied to the cross section measurements to bring the measured continuum (non-resonant) cross section into agreement with the leading order QED prediction. [$1\,\text{b} = 10^{-28}\,\text{m}^2$, $\hbar c = 197.5\,\text{MeV fm}$, $1\,\text{fm} = 10^{-15}\,\text{m}$.] [3]

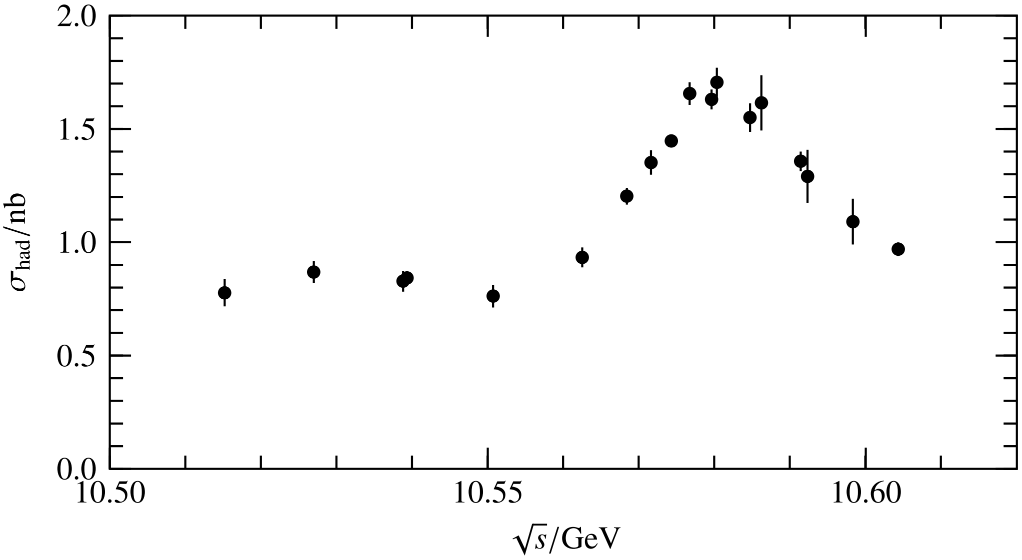
What must be the spin-parity, J^P, of a resonance produced directly in e^+e^- collisions? [1]

Estimate the mass and total width of the $\Upsilon(4S)$ meson, and estimate the partial width, Γ_{ee}, for the decay $\Upsilon(4S) \to e^+e^-$. [5]

The lightest mesons containing a b quark, the B^+ ($\bar{b}u$) and B^0 ($\bar{b}d$) mesons, have masses of about 5280 MeV. The $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances have masses 9460 MeV, 10023 MeV and 10355 MeV, respectively. Draw Feynman diagrams for the decays $\Upsilon \to e^+e^-$ and $\Upsilon \to$ hadrons for the various $\Upsilon(nS)$ resonances ($n \leq 4$), and explain why the $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances have values of Γ_{ee} similar to that of $\Upsilon(4S)$, but have total widths which are smaller by at least two orders of magnitude. [4]

In the quark model, the mass of a $q_1\bar{q}_2$ meson includes a contribution from a spin-spin interaction of the form $A\widehat{S}_1 \cdot \widehat{S}_2/(m_1 m_2)$, where A is a positive constant. Assuming quark masses $m_u = m_d = 0.31\,\text{GeV}$ and $m_b = 5.0\,\text{GeV}$, use the measured B meson (B^+ and B^0) masses to estimate the value of the constant A, and obtain a prediction for the masses of their $\bar{b}u$ and $\bar{b}d$ spin-one counterparts, the B^* mesons. If your prediction is correct, how must the B^* mesons decay? [$m_\pi \approx 140\,\text{MeV}$.] [6]

(TURN OVER for continuation of question 4


2.0
1.5
1.0
0.5
0.0
$\sigma_{\mathrm{had}}/\mathrm{nb}$
10.50
10.55
10.60
$\sqrt{s}/\mathrm{GeV}$

END OF PAPER