UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 02 June 2021 11 am to 1 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

PARTICLE AND NUCLEAR PHYSICS

*Candidates offering this paper should attempt a total of* ***five*** *questions:* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

*Natural units of* $\hbar = c = \mu_0 = \epsilon_0 = 1$ *are used throughout this paper.*

*The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains* ***eight*** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| 2 × 20 Page Answer Book | Mathematical Formulae handbook |
| Metric graph paper | Approved calculator allowed |
| Rough workpad | |
| Yellow master coversheet | |

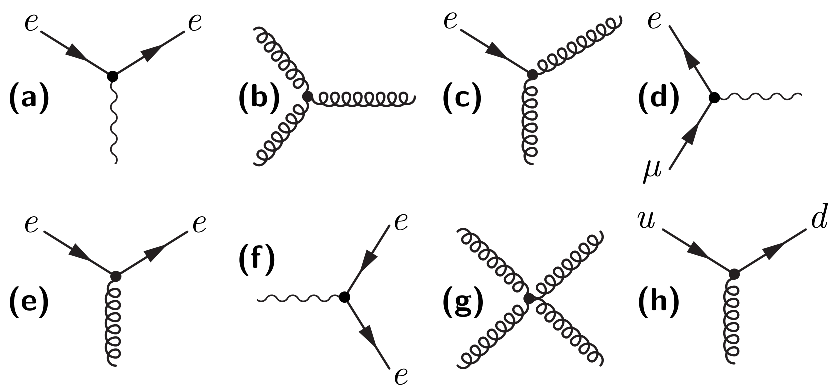
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 The PEP-II storage ring at Stanford collided electrons with an energy of 9 GeV head-on with positrons. Calculate the minimum energy required for the positrons in order to produce an $\Upsilon(4S)$ resonance with a mass of 10 580 MeV. Determine the velocity of the produced $\Upsilon(4S)$ resonance in the laboratory frame in natural units ($\beta$). Neglect the electron and positron masses in your calculations in this question. [4]

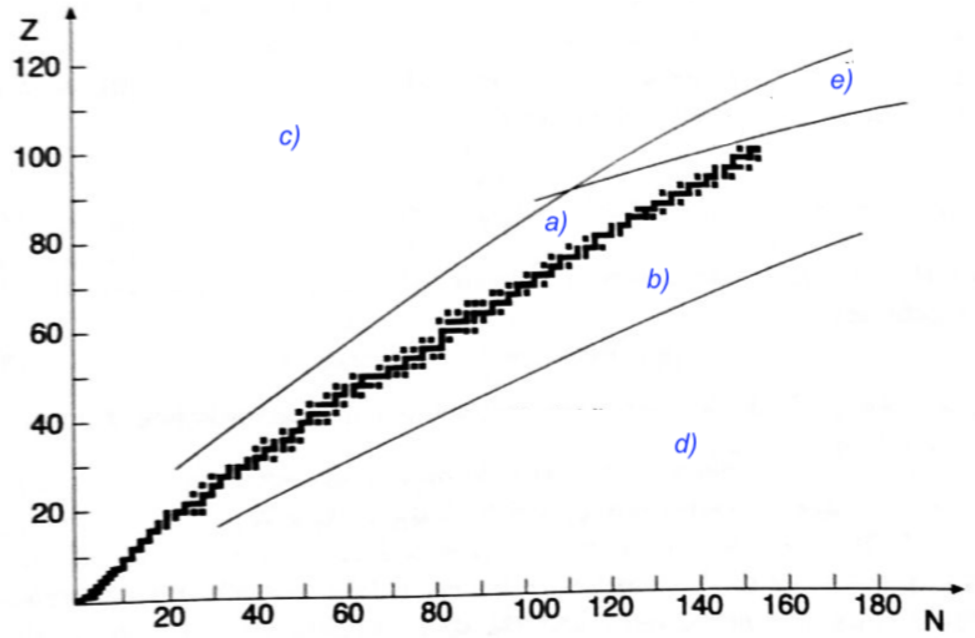
2 Consider the interaction vertices sketched below.




- Indicate the ones that are valid in QED.
- Indicate the ones that are valid in QCD.

Describe briefly (one sentence) one method to measure the strong coupling constant $\alpha_S$ at an electron-positron collider (e.g., LEP) and draw the related Feynman diagram(s), correctly labelling the particles involved. [4]

3 Consider the chart of $\beta$-stable nuclides shown below. Name and describe the nuclear transition using the $(Z, N) \rightarrow ...$ notation in each of the regions a), b), and e), where $Z$ and $N$ are the number of protons and neutrons, respectively. Give *quark-level* Feynman diagrams for the processes in a) and b), correctly labelling the particles involved. What will happen with nuclides in regions c) and d)? [4]



SECTION B

*Attempt two questions from this section*

4 (a) Explain the meaning of the terms parity, parity violation and their role in the Standard Model of particle physics. [4]

(b) Briefly outline the C. S. Wu experiment and how it demonstrated parity violation in the decay of $^{60}_{27}\mathrm{Co}$ nuclei. [5]

(c) The $K^+$ meson can decay to the final states $\pi^+\pi^0$ and $\pi^+\pi^+\pi^-$. Draw a lowest-order Feynman diagram for each of these decay modes. Show whether they conserve or violate parity. [4]

(d) The $\overline{B^0_d}$ meson can decay to the final states $D^+\pi^-$ and $\pi^+\pi^-$. Draw a lowest-order Feynman diagram for each of these decay modes. [2]

(e) Estimate the ratio of branching fractions

$$\frac{B(\overline{B^0_d} \to D^+\pi^-)}{B(\overline{B^0_d} \to \pi^+\pi^-)}$$

where you may ignore the difference between the masses of the $D^+$ and the $\pi^+$. Compare your estimate to the measured value of $\sim 500 \pm 10$. Comment on your results. [4]

*All mesons in the question have spin-parity* $0^+$ *and their quark contents are* $\pi^+\,[u\bar{d}]$; $\pi^-\,[\bar{u}d]$; $\pi^0\,[\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})]$; $K^+\,[u\bar{s}]$; $D^+\,[c\bar{d}]$; *and* $\overline{B^0_d}\,[b\bar{d}]$.

*The Cabibbo-Kobayashi-Maskawa quark mixing matrix can be written as*

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} & \sin^3\theta_{\mathrm{C}} \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} & \sin^2\theta_{\mathrm{C}} \\ \sin^3\theta_{\mathrm{C}} & -\sin^2\theta_{\mathrm{C}} & 1 \end{pmatrix},$$

*where* $\theta_{\mathrm{C}}$ *is the Cabibbo angle and* $\sin\theta_{\mathrm{C}} = 0.225$.

5 Consider the decays $\pi^- \to \mu^- + \bar{\nu}_\mu$ and $\pi^- \to e^- + \bar{\nu}_e$ in the rest frame of the $\pi^-$. In the following, we shall calculate the ratio $R$ of the decay rates of the two processes, i.e.,

$$R \equiv \frac{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)}{\Gamma(\pi^- \to e^- + \bar{\nu}_e)} \, .$$

For this, we will use Fermi's Golden Rule, in simplified form given by:

$$\Gamma = 2\pi \cdot |\mathcal{M}|^2 \cdot \rho(E_0) \, ,$$

where $\mathcal{M}$ is the matrix element for the $\pi^- \to \ell^- \bar{\nu}_\ell$ transition with $\ell = e, \mu$, and $\rho(E_0)$ is the density of final states as a function of the initial state energy $E_0$. In all following calculations, neglect the mass of the neutrino.

(a) Show that the magnitude of the three-momentum of the charged lepton $\ell$ is given by

$$|\mathbf{p}_\ell| = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} \, .$$

[*Start from four-momentum conservation and remember that we are in the rest frame of the $\pi$.*]

[3]

(b) The density of states is given by

$$\rho(E_0) = \frac{\mathrm{d}n}{\mathrm{d}E_0} = \frac{\mathrm{d}n}{\mathrm{d}|\mathbf{p}_\ell|} \cdot \frac{\mathrm{d}|\mathbf{p}_\ell|}{\mathrm{d}E_0} \, .$$

Show that the Jacobian is given by

$$\frac{\mathrm{d}|\mathbf{p}_\ell|}{\mathrm{d}E_0} = \frac{m_\pi^2 + m_\ell^2}{2m_\pi^2} \, .$$

[*Start from (a), and identify the variable in that expression that corresponds to $E_0$.*]

[2]

(c) Show that the naïve expectation for $R$ considering *only* the phase space available for the decay products is given by

$$\frac{\rho_\mu}{\rho_e} = \left[\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_e^2}\right]^2 \cdot \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 + m_e^2} \, ,$$

where $\rho_e$ and $\rho_\mu$ represent the densities of states for the decays $\pi^- \to e^- + \bar{\nu}_e$ and $\pi^- \to \mu^- + \bar{\nu}_\mu$, respectively. Calculate $R$ using these assumptions.

[*The masses of the involved particles are:*
$m_{\pi^-} = 140$ MeV, $m_{\mu^-} = 106$ MeV, $m_{e^-} = 0.5$ MeV.]

[3]

(TURN OVER

(d) Having calculated $\rho_\mu/\rho_e$, we focus on $|\mathcal{M}|^2$ in the following.
Draw the dominant quark-level Feynman diagram for the $\pi^- \to \ell^- + \bar{\nu}_\ell$ decay processes. [1]

(e) Sketch the allowed spin and momentum configuration(s) for the final-state particles for the $\pi^- \to \mu^- + \bar{\nu}_\mu$ decay in the $\pi^-$ rest frame. [2]

(f) Show that the lepton velocity $\beta_\ell$ is given by:

$$\beta_\ell = \frac{m_\pi^2 - m_\ell^2}{m_\pi^2 + m_\ell^2}$$

Calculate $\beta_e$, $\beta_\mu$.

[*Start from four-momentum conservation and use that* $E = \gamma m$ *and* $|\mathbf{p}| = \beta\gamma m$ *for massive particles.*]

[3]

(g) The ratio $R$ is experimentally measured to be around 8000, i.e. the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$ absolutely dominates over $\pi^- \to e^- + \bar{\nu}_e$, despite the smaller phase space available. In a few sentences, give an explanation why using the concept of helicity. [3]

(h) Sketch and describe an experimental setup to generate a focused beam of $\bar{\nu}_\mu$. You can use a proton accelerator, a target, and any combination of energy-momentum filters, magnets, and collimators. Keep in mind that the beam should be as pure in $\bar{\nu}_\mu$ as reasonably achievable. [2]

6 (a) Name and give a brief physical interpretation of one or two sentences for each of the eight terms in the semi-empirical mass formula, as reproduced on the following page, for the masses of atoms. [4]

(b) Sketch the individual contributions to the binding energy per nucleon $E_{\mathrm{bind}}/A$ according to the semi-empirical mass formula as a function of $A$. For this, ignore the term $\delta(A)/A^{1/2}$ and represent all other contributions to the nuclear binding energy in *one* single plot. Label the individual contributions and the axes. Indicate the typical value for $E_{\mathrm{bind}}/A$ on the $y$-axis. [4]

(c) The potassium isotope $^{40}_{19}\mathrm{K}$ can undergo $\beta$-decays into $^{40}_{18}\mathrm{Ar}$ and $^{40}_{20}\mathrm{Ca}$, both of which are $\beta$-stable. Explain using the semi-empirical mass formula why *both* decays $^{40}_{19}\mathrm{K} \to {}^{40}_{18}\mathrm{Ar}$ and $^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}$ are (energetically) possible. [2]

(d) Give the two possible processes for the decay $^{40}_{19}\mathrm{K} \to {}^{40}_{18}\mathrm{Ar}$. Which of the two is energetically favoured and will hence dominate? Give the main contribution to the difference in the energy balance between the two processes. [2]

(e) The isotope $^{40}_{19}\mathrm{K}$ contributes about 20% to the total radiation dose of humans and other organisms. An average human body of 70 kg contains 0.0164 g of $^{40}_{19}\mathrm{K}$. Calculate the activity in Bq. $^{40}_{19}\mathrm{K}$ has a half-life of $t_{1/2} = 1.3 \times 10^9$ years. [4]

(f) Due to its long lifetime, the isotope $^{40}_{19}\mathrm{K}$ allows radio-dating of minerals. This is achieved by comparing the ratio of $^{40}_{19}\mathrm{K}$ to $^{40}_{18}\mathrm{Ar}$, under the assumption that all of the $^{40}_{18}\mathrm{Ar}$ trapped in a mineral originates from $^{40}_{19}\mathrm{K}$ decays, and considering the branching ratio of 11% for the $^{40}_{19}\mathrm{K} \to {}^{40}_{18}\mathrm{Ar}$ decay. The ratio of $^{40}_{19}\mathrm{K} : {}^{40}_{18}\mathrm{Ar}$ inside a bore core from 10 km depth is measured to be $1 : 0.3$. Calculate the age of the rock probed by the bore core. Does this support the claim that the rock dates back to the formation of the Earth's crust $2.5 \times 10^9$ years ago? [3]



(TURN OVER

*In standard notation, the* ***semi-empirical mass formula*** *(also known as the Weizsäcker formula) for the masses of atoms is*

$$M(A,Z) = Zm_p + Zm_e + (A-Z)m_n \tag{1}$$

$$+ \underbrace{a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + \frac{\delta(A)}{A^{1/2}}}_{\textit{Nuclear binding energy } E_{bind}} \tag{2}$$

*A possible parametrisation for the terms representing the* ***nuclear binding energy*** *is given by:*

$$\begin{aligned} a_1 &= -15.7 \text{ MeV} \\ a_2 &= 17.2 \text{ MeV} \\ a_3 &= 0.71 \text{ MeV} \\ a_4 &= 23.3 \text{ MeV} \\ \delta(A) &= \begin{cases} -11.2 \text{ MeV} & \textit{for even-even nuclei} \\ 0 \text{ MeV} & \textit{for even-odd nuclei} \\ +11.2 \text{ MeV} & \textit{for odd-odd nuclei} \end{cases} \end{aligned}$$



END OF PAPER