UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 29 May 2013 9.00 am to 11.00 am

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A circular satellite dish has a radius of 0.5 m and is used to receive TV signals at a frequency of 600 MHz from a geostationary satellite. Suggest how well the normal to the dish must be aligned with the line of sight to the satellite. Estimate the ratio of the voltages at the terminals of a simple dipole antenna and the satellite dish, when both are pointed at the satellite. [4]

(b) Explain the terms spatial and temporal coherence. Estimate the spatial and temporal coherence lengths of visible light from the planet Jupiter (angular diameter 40 arc seconds) when viewed from the Earth's surface. [4]

(c) The components of the electromagnetic fields **E** and **B** that are parallel to the direction of motion of a high-speed frame moving uniformly through the laboratory are the same whether measured in the moving frame or the laboratory frame. Explain how this arises in the case of (i) the electric field in a charged parallel-plate capacitor observed when stationary and when moving perpendicular to the plates, and (ii) the magnetic field in a long solenoid carrying a current observed when stationary and when moving parallel to its long axis. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the electric field of a uniformly moving point charge;

(b) the processes causing polarisation of naturally occurring light in the environment observed on the Earth;

(c) synchrotron radiation.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain what are meant by the terms *linear polarisation* and *circular polarisation* in the context of electromagnetic radiation, and how *Jones matrices* and *Jones vectors* may be used to analyse the polarisation response of an optical system. [6]

A circularly-polarising filter converts unpolarised light into circularly-polarised light. Explain carefully how you could make such a filter using a linear polariser and a quarter-wave plate. Demonstrate that it produces circularly polarised light from unpolarised light travelling in the forward direction. [6]

Such a circular polariser is used both with light travelling through it in the forward direction and also with light travelling through it in the reverse direction. What happens for unpolarised light travelling in the reverse direction through the filter? [3]

A circularly-polarising filter for use with a photographic camera has sides designated A and B. In normal use, light from the scene being photographed passes through the filter in the direction from A to B, with side A facing the scene, and side B facing the camera lens. Such a filter is detached from the camera and viewed by an observer who places it directly in front of one eye. Explain why the observer sees the scene looking darker than when viewed without the filter, both with side B nearest to his eye, and with side A nearest to his eye. [2]

The observer also notes the following:

(a) With side B nearest to his eye, he observes his reflection in a mirror, and notes that he can see his eye through the reflected image of the filter.

(b) With side A nearest to his eye, he observes his reflection in the mirror, and notes that he can no longer see his eye, the image of the filter being dark.

Explain each of these observations carefully with the aid of diagrams indicating the polarisation states in each part of the system. [8]

[*The Jones vectors for left-handed and right-handed circular polarisations of light travelling in the z-direction are* $\begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix}$ *and* $\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix}$*, respectively.*]

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4 *Attempt* **either** *this question* **or** *question 3.*

What is meant by the term *Hertzian dipole*? Give one example of its use in electrodynamics. [4]

In spherical-polar coordinates (r, θ, ϕ) centred on, and aligned to, a Hertzian magnetic dipole of moment m, the electromagnetic fields arising from the dipole may be written as

$$\boldsymbol{B} = \left(\frac{\mu_0}{4\pi}\frac{2[m]\cos\theta}{r^3} + \frac{\mu_0}{4\pi}\frac{2[\dot{m}]\cos\theta}{cr^2}\right)\hat{\boldsymbol{r}} + \left(\frac{\mu_0}{4\pi}\frac{[m]\sin\theta}{r^3} + \frac{\mu_0}{4\pi}\frac{[\dot{m}]\sin\theta}{cr^2} + \frac{\mu_0}{4\pi}\frac{[\ddot{m}]\sin\theta}{c^2 r}\right)\hat{\boldsymbol{\theta}}$$

$$\boldsymbol{E} = -\left(\frac{1}{4\pi\epsilon_0}\frac{[\dot{m}]\sin\theta}{c^2 r^2} + \frac{1}{4\pi\epsilon_0}\frac{[\ddot{m}]\sin\theta}{c^3 r}\right)\hat{\boldsymbol{\phi}}$$

where $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ are unit vectors in the coordinate directions. Explain what each term represents and the meaning of the square brackets [ ]. [4]

Show that the total power radiated by such a dipole is given by

$$P = \frac{[\ddot{m}]^2}{6\pi\epsilon_0 c^5}\ .$$ [4]

Explain what is meant by the term *radiation resistance* in this context, and find an expression for the radiation resistance of a Hertzian magnetic dipole of area A in which there is a sinusoidally oscillating current of angular frequency ω. [4]

A rapidly spinning satellite in orbit around the Earth may be modelled as a single loop of wire of radius b rotating at angular frequency ω about a diameter in the Earth's magnetic field of flux density B. (You may assume that the direction of the field is perpendicular to the spin axis.) Find an expression for the total power radiated if the resistance of the wire is 10^{-4} Ohms. Where does the radiated energy come from? [4]

If the satellite is spinning at a rate of 500 revolutions per second, and has a radius of 1 m, suggest reasons why this radiation could not be detected in practice on the surface of the Earth. Discuss whether it could be detected by another satellite 100 km away. [5]

END OF PAPER