PNP 2015

1) a) $^{174}_{72}Hf$

J^P	0^+	2^+	4^+	6^+	8^+
E/keV	0	91	297	608	1009

Show that these states are consistent with being rotational excitations - $E \propto J(J+1)$ . .

expected ratios $4^+/2^+ = \frac{20}{6}$ $\frac{42}{20}$ $\frac{72}{42}$

~~ca~~ observed 3.26 2.05 1.66 ~ consistent

nuclear mirror symmetry restricts sequence of rotational states to even angular momentum quantum numbers

moment of inertia

$E = \frac{\hbar^2 J(J+1)}{2I}$ $\Rightarrow$ $I = 2.27 \times 10^{-54}\ kg\,m^2$

rigid spherical rotor - $I = \frac{2}{5} mR^2$, $R = R_0 A^{1/3}$

$A = 174$, $R_0 = 1.2 fm$

$I = 5.22 \times 10^{-54}\ kg\,m^2$

or $I = 6.1 \times 10^{-54}\ kg\,m^2$ for $R_0 = 1.3\ fm$

~ 3x larger than inferred value

b) If quarks were spin-0 particles, explain why only a single L=0 baryon would be observed

baryon is now a spin-0 boson

- need ψ_{baryon} symmetric

$\psi_{spatial}$ symmetric for L=0, ψ_{colour} always antisymmetric

$\psi_{baryon} = \psi_{spatial}\psi_{spin}\psi_{flavour}\psi_{colour}$

need $\psi_{spin}\psi_{flavour}$ antisymmetric

for 3x S=0 quarks, ψ_{spin} is symmetric - need $\psi_{flavour}$ antisymmetric under exchange of any 2 quarks - uds only

c) $\nu_l + e^- \to \nu_e + l^-$

find min lab frame neutrino energy, assuming initial electron at rest, for 3 neutrino flavours

$$s = (E_\nu + m_e)^2 - E_\nu^2 = m_l^2 \quad - \text{min}$$

$$2E_\nu m_e + m_e^2 = m_l^2$$

$l = e$: $E_\nu = 0$

$l = \mu$: $E_\nu = \dfrac{m_\mu^2 - m_e^2}{2m_e} = 10.99 \text{ GeV}$

$l = \tau$: $E_\nu : \dfrac{m_\tau^2 - m_e^2}{2m_e} = 3090 \text{ GeV}$

3) $\sigma(E) = \left(\dfrac{\pi g}{p^2}\right) \dfrac{\Gamma_i \Gamma_f}{(E-E_0)^2 + \frac{1}{4}\Gamma^2}$

g - takes into account spins of initial particles
- ratio of no of spin states for resonant state to no of spin states of initial state - probability that initial particles collide in correct spin state to form a resonance

p - centre of mass momentum

Γ_i - partial width for reaction $x + X \to Z^*$

Γ_f - partial width for reaction $Z^* \to y + Y$

Γ - total width for reaction $x + X \to y + Y$

where overall reaction is $x + X \to Z^* \to y + Y$

E = energy, E_0 = energy of resonance

$1/p^2$ dependence - state with $E = E_0$, mean lifetime τ formed at $t=0$ has $\psi(t) = \psi(0) e^{-iE_0 t} e^{-t/2\tau}$

frequencies present: $f(\omega) = f(E) = \int_0^\infty \psi(0) e^{-t(i(E-E_0) + 1/2\tau)} \, dt$

$$F(E) = \frac{\psi(0)}{i(E_0 - E) + 1/2\tau} = \frac{i\psi(0)}{E - E_0 + i/2\tau} \quad , \quad \Gamma = 1/\tau$$

probability of state with energy E is $|F(E)|^2$

$$= \frac{|\psi(0)|^2}{(E-E_0)^2 + \Gamma^2/4}$$

$\frac{1}{p^2}$ dependence comes from density of final states

g values for $\mu^+\mu^- \to Z \to \tau^+\tau^-$, $\mu^+\mu^- \to H \to \tau^+\tau^-$

$$g = \frac{2.1 + 1}{(2.\frac{1}{2}+1)(2.\frac{1}{2}+1)} = \frac{3}{4} \qquad g = \frac{0+1}{(2.\frac{1}{2}+1)^2} = \frac{1}{4}$$

Show that FWHM at resonance is Γ

$E = E_0$ at resonance $\Rightarrow$ $\sigma_{res} = \frac{4\pi g \Gamma_i \Gamma_f}{p^2 \Gamma^2}$

half max $\sigma = \frac{2\pi g \Gamma_i \Gamma_f}{p^2 \Gamma^2} = \frac{\pi g}{p^2} \frac{\Gamma_i \Gamma_f}{(E-E_0)^2 + \Gamma^2/4}$

$$2\left[(E-E_0)^2 + \Gamma^2/4\right] = \Gamma^2$$

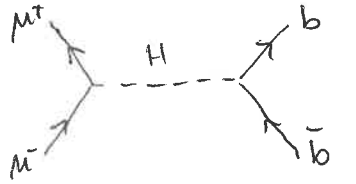
$$\frac{1}{2}\Gamma^2 = 2(E-E_0)^2$$

$$\frac{1}{2}\Gamma = \pm(E - E_0) \quad , \quad E = E_0 \pm \frac{1}{2}\Gamma$$

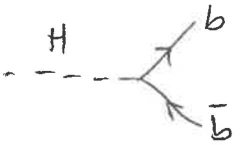
width at half max = difference between solutions

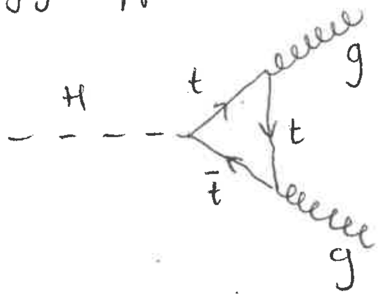
$$= E_0 + \frac{1}{2}\Gamma - (E_0 - \frac{1}{2}\Gamma) = \Gamma$$

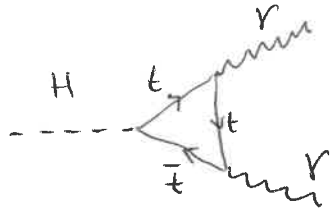
$\mu^+\mu^- \to H \to b\bar{b}$



Higgs decay to $b\bar{b}$, gg, $\gamma\gamma$







decays to massless particles · must occur indirectly, via massive particles eg top quark

Sum of branching ratios for $H \to \tau^+\tau^-$, $c\bar{c}$, $b\bar{b}$ is 67%

partial decay width to $\tau^+\tau^-$ · $\Gamma(H \to \tau^+\tau^-) = \dfrac{m_\tau^2 M_H}{8\pi v^2}$

a) ratio $\Gamma(H \to \tau^+\tau^-) : \Gamma(H \to c\bar{c}) : \Gamma(H \to b\bar{b})$

coupling $\propto$ mass $\Rightarrow$ $\Gamma \propto m^2$

need factor of 3 for $c\bar{c}$, $b\bar{b}$ decays (colour)

· ratio $= 1.8^2 : 3(1.5)^2 : 3(5^2) = 3.24 : 6.75 : 75$

b) cross section for $\mu^+\mu^- \to H \to b\bar{b}$ at resonance

$$\sigma = \frac{\pi g}{p^2} \frac{4\Gamma_\mu \Gamma_b}{\Gamma^2}$$

$g = \frac{1}{4}$, $p = \frac{1}{2} M_H$

$\Gamma_\mu = \left(\frac{m_\mu}{m_\tau}\right)^2 \Gamma_\tau$, $\Gamma_b = 3\left(\frac{m_b}{m_\tau}\right)^2 \Gamma_\tau$

$\Gamma_\tau \left(1 + 3\left(\frac{m_c}{m_\tau}\right)^2 + 3\left(\frac{m_b}{m_\tau}\right)^2\right) = 0.67\Gamma \Rightarrow \Gamma = 39.15\,\Gamma_\tau$

$$\sigma = \pi \left(\frac{2}{M_H}\right)^2 \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_b}{m_\tau}\right)^2 3 \cdot \frac{1}{39.15^2} = \cancel{1.649 \times 10^{-6}\ (\mathrm{GeV})^{-2}}$$

$= \cancel{1.649 \times 10^{-12}\ (\mathrm{MeV})^{-2}}$

$= \cancel{6.40 \times 10^{-8}\ \mathrm{fm}^2}$

$= \cancel{0.64\ \mathrm{nb}}$

$\sigma = 4.21 \times 10^{-14}\ (\mathrm{MeV})^{-2}$

$= 1.63 \times 10^{-9}\ \mathrm{fm}^2$

$= 0.016\ \mathrm{nb}$