UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 3rd June 2022 09:00 to 11:00

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

PARTICLE AND NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of ***five*** *questions: all* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

As in the presentation in lectures, natural units of $\hbar = c = \mu_0 = \epsilon_0 = 1$ are used throughout this paper.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains 11 sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 Describe the three main kinds of β-decay, and the kinematic conditions under which each occurs. At fixed mass number, A, how many isotopes are typically stable with respect to β-decay and why? [4]

2 Describe the principles underpinning radio-carbon dating. [4]

3 The Standard Model has four gauge bosons: g, W, Z and γ. The Feynman rules of this theory include fundamental vertices which couple together **only** certain groups of these gauge bosons. A fundamental vertex of The Standard Model coupling together N gauge bosons (i.e., one having N legs) is called an 'N-gauge-boson vertex'. [4]

(a) How many two-gauge-boson vertices exist in The Standard Model? If the number is non-zero, draw a picture of each vertex, clearly indicating the type of boson on each leg.

(b) Repeat (a) for three-gauge-boson vertices.

(c) Repeat (a) for four-gauge-boson vertices.

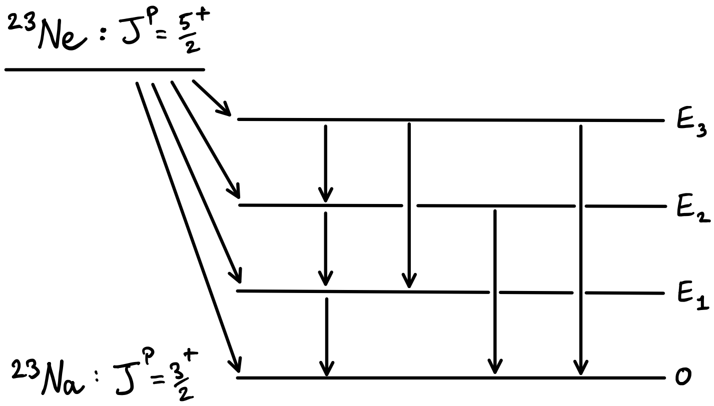
(d) Repeat (a) for five-gauge-boson vertices.

SECTION B
*Attempt **two** questions from this Section.*

4 ^{23}Ne is unstable. Over time it converts to ^{23}Na. Among the products emitted during its decay are photons and these are observed at energies of $\gamma_1 = 0.440$ MeV, $\gamma_2 = 0.906$ MeV, $\gamma_3 = 1.636$ MeV, $\gamma_4 = 2.076$ MeV, $\gamma_5 = 2.542$ MeV and $\gamma_6 = 2.982$ MeV.

(a) By what nuclear process(es) does ^{23}Ne decay to ^{23}Na ? [2]

(b) How many energy-level schemes of the form shown below are compatible with the gamma ray energy spectrum reported? [*You should find more than one.*] Draw an energy-level diagram for each scheme that you find, making sure to label on it: (i) which γ_i corresponds to which transition, and (ii) the values taken by the excitation energies E_1, E_2 and E_3. [3]



Observation of a large sample of ^{23}Ne undergoing complete decay reveals that the numbers of emitted gamma rays having energies $\gamma_1, \gamma_2, \ldots, \gamma_6$ are (in that order) in the relative ratios $N_1 : N_2 : \ldots : N_6$ with $N_1 = 1.27 \times 10^6$, $N_2 = 1.00$, $N_3 = 3.85 \times 10^4$, $N_4 = 3.77 \times 10^3$, $N_5 = 1.03 \times 10^3$ and $N_6 = 1.47 \times 10^3$.

(c) Which energy-level scheme found in part (b) is the most probable given the gamma ray emission-ratio information just given? Justify your choice. [2]

(d) For the most-probable scheme identified in (c) above, and denoting the branching ratios of ^{23}Ne to the 0$^{\text{th}}$, 1$^{\text{st}}$, 2$^{\text{nd}}$ and 3$^{\text{rd}}$ excited states of ^{23}Na by the symbols Γ_0, Γ_1, Γ_2 and Γ_3 respectively:

(i) write down a simple equation relating Γ_0, Γ_1, Γ_2 and Γ_3; [2]

(ii) write down additional formulae relating Γ_1, Γ_2 and Γ_3 to the symbols $N_1, \ldots, N_6$ and Γ_0 so that you may: [8]

(iii) solve all the equations above simultaneously to determine the numerical values of Γ_1, Γ_2 and Γ_3 under the assumption that $\Gamma_0 = 67\%$. [2]

5 (a) Explain how the hadron wavefunction

$$\psi_{\text{hadron}} = \psi_{\text{space}}\psi_{\text{spin}}\psi_{\text{flavour}}\psi_{\text{colour}}$$

leads to the prediction of an octet of spin-$\frac{1}{2}$ states and a decuplet of spin-$\frac{3}{2}$ states for the lowest-mass baryons formed from up, down and strange quarks. [6]

(b) Briefly explain the origin of the baryon mass formula

$$M_{qqq} = m_1 + m_2 + m_3 + A'\left[\frac{\boldsymbol{S}_1 \cdot \boldsymbol{S}_2}{m_1 m_2} + \frac{\boldsymbol{S}_2 \cdot \boldsymbol{S}_3}{m_2 m_3} + \frac{\boldsymbol{S}_3 \cdot \boldsymbol{S}_1}{m_3 m_1}\right],$$

where A' is a constant. [2]

(c) Derive the specific form the above mass formula takes for the case of a spin-$\frac{1}{2}$ baryon composed of one up and two strange quarks. Repeat this exercise to determine how the mass of a spin-$\frac{3}{2}$ bound state with the same quark content depends on the same quantities. [*In both cases show your working and leave each answer in terms of m_u, m_s and A' only.*] [8]

(d) The Ξ^0 ($J^P = \frac{1}{2}^+$) and Ξ^{*0} ($J^P = \frac{3}{2}^+$) baryons are bound states of *uss* quarks. Using your formulae from part (c), calculate numerical values for mass predictions for the Ξ^0 and Ξ^{*0}. [*The quark masses may be taken to be m_u = 362 MeV and m_s = 537 MeV. A' = 0.026 GeV^3.*] [1]

(e) The experimentally measured values of the masses are $M(\Xi^0) = 1314.86 \pm 0.20$ MeV and $M(\Xi^{*0}) = 1531.79 \pm 0.34$ MeV. What level of agreement between predicted and measured masses might one reasonably expect the baryon mass formula to provide, and is the level of agreement you saw in (d) better or worse than this? [2]

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6 Some parts of this question expect you to extract information from a set of tables reproduced from the latest edition of ‘The Review of Particle Physics’. These tables (which can be found after the statement of the question) may include both information which is irrelevant to you, and terms or symbols which are not fully explained. The question is therefore testing your ability to extract information from unfamiliar sources regularly used in particle physics research.

(a) Explain why the positively charged pion’s decay mode to $\mu^+\nu_\mu$ dominates over its decay to $e^+\nu_e$. [3]

(b) Looking at the decay modes of the π^0, suggest possible reasons for the difference between the branching fractions stated for the $e^+e^+e^-e^-$ and e^+e^- final states. [4]

(c) Draw the lowest order Feynman diagrams for the processes (i) $\pi^0 \to \pi^+e^-\bar{\nu}_e$ and (ii) $\pi^+ \to e^+\nu_e\pi^0$. Why do the tables list a branching ratio for only one of these two processes? What branching ratio is reported for it? What physical reasons might lead to it being that big or that small? [5]

(d) Make a prediction for the π^- branching fraction into (i) $\mu^-\bar{\nu}_\mu\gamma$ and (ii) $e^-\nu_e e^-e^+$. [2]

(e) Suppose that a Ξ^0 baryon is produced in a vacuum at the centre of an infinitely large box at time $t_0 = 0$. What are the most probable contents of the box if inspected at a later time $t_1 = 10^{-12}$ s? Describe the two main routes or ‘histories’ through which the contents of the box might be expected to evolve over time. Cover the period from t_0 until the contents of the box reach steady state, making clear on what timescales different events might be expected to happen. How probable are the two end states you have considered? [5]

N BARYONS
$(S = 0, I = 1/2)$

p, $N^+ = uud$; n, $N^0 = udd$

p

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.00727646663 \pm 0.00000000009$ u (S = 2.9)
Mass $m = 938.272081 \pm 0.000006$ MeV [a]
$|m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10}$, CL = 90% [b]
$|\frac{q_{\overline{p}}}{m_{\overline{p}}}|/(\frac{q_p}{m_p}) = 1.00000000000 \pm 0.00000000007$
$|q_p + q_{\overline{p}}|/e < 7 \times 10^{-10}$, CL = 90% [b]
$|q_p + q_e|/e < 1 \times 10^{-21}$ [c]
Magnetic moment $\mu = 2.7928473446 \pm 0.0000000008\ \mu_N$
$(\mu_p + \mu_{\overline{p}}) / \mu_p = (0.002 \pm 0.004) \times 10^{-6}$
Electric dipole moment $d < 0.021 \times 10^{-23}$ e cm
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm^3 (S = 1.2)
Charge radius, μp Lamb shift $= 0.84087 \pm 0.00039$ fm [d]
Charge radius $= 0.8409 \pm 0.0004$ fm [d]
Magnetic radius $= 0.851 \pm 0.026$ fm [e]
Mean life $\tau > 3.6 \times 10^{29}$ years, CL = 90% [f] ($p \rightarrow$ invisible mode)
Mean life $\tau > 10^{31}$ to 10^{33} years [f] (mode dependent)

n

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.0086649159 \pm 0.0000000005$ u
Mass $m = 939.565413 \pm 0.000006$ MeV [a]
$(m_n - m_{\overline{n}})/ m_n = (9 \pm 6) \times 10^{-5}$
$m_n - m_p = 1.2933321 \pm 0.0000005$ MeV
$= 0.00138844919(45)$ u
Mean life $\tau = 879.4 \pm 0.6$ s (S = 1.6)
$c\tau = 2.6362 \times 10^8$ km
Magnetic moment $\mu = -1.9130427 \pm 0.0000005\ \mu_N$
Electric dipole moment $d < 0.18 \times 10^{-25}$ e cm, CL = 90%
Mean-square charge radius $\langle r_n^2 \rangle = -0.1161 \pm 0.0022$ fm^2 (S = 1.3)
Magnetic radius $\sqrt{\langle r_M^2 \rangle} = 0.864^{+0.009}_{-0.008}$ fm
Electric polarizability $\alpha = (11.8 \pm 1.1) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (3.7 \pm 1.2) \times 10^{-4}$ fm^3
Charge $q = (-0.2 \pm 0.8) \times 10^{-21}$ e
Mean $n\overline{n}$-oscillation time $> 8.6 \times 10^7$ s, CL = 90% (free n)
Mean $n\overline{n}$-oscillation time $> 4.7 \times 10^8$ s, CL = 90% [g] (bound n)
Mean nn'-oscillation time > 448 s, CL = 90% [h]

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n DECAY MODES			Fraction (Γ_i/Γ)	Confidence level
$p e^- \overline{\nu}_e$			100 %	
$p e^- \overline{\nu}_e \gamma$		[l]	(9.2±0.7) × 10^{-3}	
hydrogen-atom $\overline{\nu}_e$			< 2.7 × 10^{-3}	95%
Charge conservation (Q) violating mode				
$p \nu_e \overline{\nu}_e$	Q		< 8 × 10^{-27}	68%

LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For $I = 1$ (π, b, ρ, a): $u\overline{d}$, $(u\overline{u}-d\overline{d})/\sqrt{2}$, $d\overline{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\overline{u} + d\overline{d}) + c_2(s\overline{s})$

π^0

$I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9768 \pm 0.0005$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.43 \pm 0.13) \times 10^{-17}$ s (S = 1.2)
$c\tau = 25.3$ nm

π^0 DECAY MODES			Fraction (Γ_i/Γ)	Scale factor/ Confidence level
2γ			(98.823±0.034) %	S=1.5
$e^+ e^- \gamma$			(1.174±0.035) %	S=1.5
γ positronium			(1.82 ±0.29) × 10^{-9}	
$e^+ e^+ e^- e^-$			(3.34 ±0.16) × 10^{-5}	
$e^+ e^-$			(6.46 ±0.33) × 10^{-8}	
4γ			< 2 × 10^{-8}	CL=90%
$\nu \overline{\nu}$		[e]	< 2.7 × 10^{-7}	CL=90%
$\nu_e \overline{\nu}_e$			< 1.7 × 10^{-6}	CL=90%
$\nu_\mu \overline{\nu}_\mu$			< 1.6 × 10^{-6}	CL=90%
$\nu_\tau \overline{\nu}_\tau$			< 2.1 × 10^{-6}	CL=90%
$\gamma \nu \overline{\nu}$			< 1.9 × 10^{-7}	CL=90%
Charge conjugation (C) or Lepton Family number (LF) violating mo				
3γ	C		< 3.1 × 10^{-8}	CL=90%
$\mu^+ e^-$	LF		< 3.8 × 10^{-10}	CL=90%
$\mu^- e^+$	LF		< 3.4 × 10^{-9}	CL=90%
$\mu^+ e^- + \mu^- e^+$	LF		< 3.6 × 10^{-10}	CL=90%

$\pi^{\pm}$

$I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57039 \pm 0.00018$ MeV (S = 1.8)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

$\pi^{\pm} \rightarrow \ell^{\pm}\nu\gamma$ form factors [a]

$F_V = 0.0254 \pm 0.0017$
$F_A = 0.0119 \pm 0.0001$
F_V slope parameter $a = 0.10 \pm 0.06$
$R = 0.059^{+0.009}_{-0.008}$

π^+ DECAY MODES			Fraction (Γ_i/Γ)	Confidence level	(M
$\mu^+\nu_\mu$		[b]	(99.98770 ± 0.00004) %		
$\mu^+\nu_\mu\gamma$		[c]	$(2.00 \pm 0.25) \times 10^{-4}$		
$e^+\nu_e$		[b]	$(1.230 \pm 0.004) \times 10^{-4}$		
$e^+\nu_e\gamma$		[c]	$(7.39 \pm 0.05) \times 10^{-7}$		
$e^+\nu_e\pi^0$			$(1.036 \pm 0.006) \times 10^{-8}$		
$e^+\nu_e e^+ e^-$			$(3.2 \pm 0.5) \times 10^{-9}$		
$\mu^+\nu_\mu\nu\overline{\nu}$			$< 9 \times 10^{-6}$	90%	
$e^+\nu_e\nu\overline{\nu}$			$< 1.6 \times 10^{-7}$	90%	
Lepton Family number (*LF*) or Lepton number (*L*) violating modes					
$\mu^+\overline{\nu}_e$	*L*	[d]	$< 1.5 \times 10^{-3}$	90%	
$\mu^+\nu_e$	*LF*	[d]	$< 8.0 \times 10^{-3}$	90%	
$\mu^- e^+ e^+ \nu$	*LF*		$< 1.6 \times 10^{-6}$	90%	

Ξ BARYONS
($S = -2, I = 1/2$)

$\Xi^0 = uss$, $\Xi^- = dss$

Ξ^0

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

P is not yet measured; + is the quark model prediction.

Mass $m = 1314.86 \pm 0.20$ MeV
$m_{\Xi^-} - m_{\Xi^0} = 6.85 \pm 0.21$ MeV
Mean life $\tau = (2.90 \pm 0.09) \times 10^{-10}$ s
$c\tau = 8.71$ cm
Magnetic moment $\mu = -1.250 \pm 0.014\ \mu_N$

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Ξ^0 DECAY MODES			Fraction (Γ_i/Γ)		Confidence level
$\Lambda\pi^0$			(99.524 ± 0.012) %		
$\Lambda\gamma$			(1.17 ± 0.07)	$\times 10^{-3}$	
$\Lambda e^+ e^-$			(7.6 ± 0.6)	$\times 10^{-6}$	
$\Sigma^0\gamma$			(3.33 ± 0.10)	$\times 10^{-3}$	
$\Sigma^+ e^- \overline{\nu}_e$			(2.52 ± 0.08)	$\times 10^{-4}$	
$\Sigma^+ \mu^- \overline{\nu}_\mu$			(2.33 ± 0.35)	$\times 10^{-6}$	
$\Delta S = \Delta Q$ (SQ) violating modes or $\Delta S = 2$ forbidden ($S2$) modes					
$\Sigma^- e^+ \nu_e$	SQ	$<$	9	$\times 10^{-4}$	90%
$\Sigma^- \mu^+ \nu_\mu$	SQ	$<$	9	$\times 10^{-4}$	90%
$p\pi^-$	$S2$	$<$	8	$\times 10^{-6}$	90%
$p e^- \overline{\nu}_e$	$S2$	$<$	1.3	$\times 10^{-3}$	
$p \mu^- \overline{\nu}_\mu$	$S2$	$<$	1.3	$\times 10^{-3}$	

Λ BARYONS
$(S = -1, I = 0)$

$\Lambda^0 = uds$

Λ

$I(J^P) = 0(\frac{1}{2}^+)$

Mass $m = 1115.683 \pm 0.006$ MeV
$(m_\Lambda - m_{\overline{\Lambda}}) / m_\Lambda = (-0.1 \pm 1.1) \times 10^{-5}$ (S = 1.6)
Mean life $\tau = (2.632 \pm 0.020) \times 10^{-10}$ s (S = 1.6)
$(\tau_\Lambda - \tau_{\overline{\Lambda}}) / \tau_\Lambda = -0.001 \pm 0.009$
$c\tau = 7.89$ cm
Magnetic moment $\mu = -0.613 \pm 0.004\ \mu_N$
Electric dipole moment $d < 1.5 \times 10^{-16}\ e$cm, CL = 95%

Λ DECAY MODES		Fraction (Γ_i/Γ)	Confidence level
$p\pi^-$		(63.9 ± 0.5) %	
$n\pi^0$		(35.8 ± 0.5) %	
$n\gamma$		$(1.75 \pm 0.15) \times 10^{-3}$	
$p\pi^-\gamma$	[o]	$(8.4 \pm 1.4) \times 10^{-4}$	
$pe^-\overline{\nu}_e$		$(8.32 \pm 0.14) \times 10^{-4}$	
$p\mu^-\overline{\nu}_\mu$		$(1.57 \pm 0.35) \times 10^{-4}$	
Lepton (L) and/or Baryon (B) number violating decay modes			
π^+e^-	L,B	$< 6 \times 10^{-7}$	90%
$\pi^+\mu^-$	L,B	$< 6 \times 10^{-7}$	90%
π^-e^+	L,B	$< 4 \times 10^{-7}$	90%
$\pi^-\mu^+$	L,B	$< 6 \times 10^{-7}$	90%
K^+e^-	L,B	$< 2 \times 10^{-6}$	90%
$K^+\mu^-$	L,B	$< 3 \times 10^{-6}$	90%
K^-e^+	L,B	$< 2 \times 10^{-6}$	90%
$K^-\mu^+$	L,B	$< 3 \times 10^{-6}$	90%
$K^0_S\nu$	L,B	$< 2 \times 10^{-5}$	90%
$\overline{p}\pi^+$	B	$< 9 \times 10^{-7}$	90%

END OF PAPER