

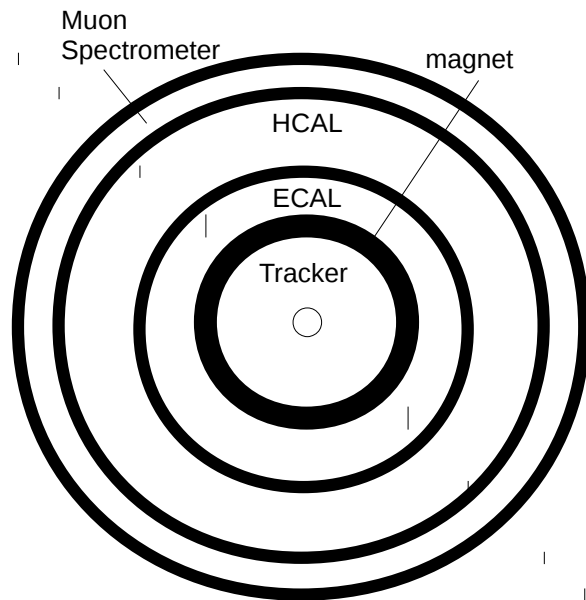
Part II Particle and Nuclear Physics

Examples Sheet 2

Colliders and Detectors

10. Detector Signatures, Tripos A-style question.

For each e^+e^- process below, sketch the signature in a typical cylindrical detector e.g.



- (a) $e^+e^- \rightarrow \mu^+\mu^-e^+e^-$
- (b) $e^+e^- \rightarrow \mu^+\mu^-\gamma$
- (c) $e^+e^- \rightarrow \nu\bar{\nu}\gamma$
- (d) $e^+e^- \rightarrow \tau^+\tau^-$, where the taus decay as $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$ and $\tau^+ \rightarrow \pi^+\pi^0\bar{\nu}_\tau$
- (e) $e^+e^- \rightarrow \pi^+n\bar{p}\pi^0K^+K^-$

Draw the lowest order Feynman diagram for (a)-(d).

11. Detector Resolution, Tripos A-style question.

- (a) In an experiment, the momentum measurement accuracy of the tracking detector is 1% for 1 GeV muons. What is the momentum accuracy for 20 GeV muons in the same apparatus?

- (b) The energy resolution for 1 GeV electrons in the electromagnetic calorimeter is 0.5%. What is the energy resolution for 10 GeV electrons?

Feynman Diagrams and QED

12. QED Feynman Diagrams, Tripos A-style question.

Draw the lowest order Feynman diagram(s) for each of the following processes:

- (a) $\gamma \rightarrow e^+e^-$ (in matter)
- (b) $e^- + e^- \rightarrow e^- + e^-$
- (c) $e^+ + e^- \rightarrow e^+ + e^-$
- (d) $e^+ + e^- \rightarrow \mu^+ + \mu^-$
- (e) $e^+ + e^- \rightarrow \gamma + \gamma$
- (f) $\gamma + \gamma \rightarrow \gamma + \gamma$ (“Delbruck scattering” – forbidden classically)

13. π^0 Decay, Tripos A-style question.

- (a) The π^0 ($J^P = 0^-$) decays predominantly to $\gamma\gamma$ but is also seen to decay to $e^+e^-\gamma$ (“Dalitz decay”), to $e^+e^-e^+e^-$ and to e^+e^- with branching fractions of 1.2%, 3.2×10^{-5} and 2×10^{-7} respectively. Draw the leading order Feynman diagrams for each of these decays. Based on the coupling constants involved (ignoring propagator effects etc.), give *rough* estimates of the branching fractions for each decay.
- (b) The ρ^0 ($J^P = 1^-$) decays to e^+e^- with a branching fraction of 4×10^{-5} . Draw the Feynman diagram for this decay and comment on the difference between the $\pi^0 \rightarrow e^+e^-$ and $\rho^0 \rightarrow e^+e^-$ partial widths.

[The π^0 and ρ^0 lifetimes are $8.4 \times 10^{-17}s$ and $4.4 \times 10^{-24}s$ respectively.]

14. Drell-Yan, Tripos A-style question.

The Drell-Yan process, which is the production of charged lepton pairs in hadron-hadron interactions ($\pi N \rightarrow \mu^+\mu^- + \text{anything, for example}$), proceeds via quark-antiquark annihilation into a single virtual photon. Draw a typical Feynman diagram for this process. Show that the Drell-Yan cross-sections in π^+p , π^+n , π^-p and π^-n interactions would be expected to be in the ratio 1 : 2 : 8 : 4.

What would you expect to be the Drell-Yan cross-sections for pp and $\bar{p}p$ collisions compared with the Drell-Yan cross-section for π^+p interactions ?

QCD and the Quark Model

15. Spin and Parity, Tripos A-style question.

When π^- mesons are stopped in deuterium they form “pionic atoms” (π^-d) which usually undergo transitions to an atomic s-state ($\ell = 0$), whereupon the capture reaction $\pi^-d \rightarrow nn$

occurs and destroys them. (The fact that capture normally occurs in an s-state is established from studies of the X-rays emitted in the transitions before capture). Given that the deuteron has spin-parity $J^P = 1^+$ and the pion has spin 0, show that these observations imply that the pion has negative intrinsic parity.

16. Hadron Masses, Tripos A-style question.

- (a) Verify the quark model predictions given in the lectures for the following meson masses:

Meson	Calculated (MeV)	Observed (MeV)
π	140	138
K	484	496
η	559	549
ρ	780	776
ω	780	783
K*	896	892
ϕ	1032	1020

[Assume $m_u=m_d=310$ MeV, $m_s=483$ MeV and that the spin-spin interaction coefficient $A= 0.0615$ GeV³ in this case.]

What would the model predict for the mass of the η' meson (measured mass 958 MeV)?

- (b) What must be the total spin of any pair of quarks in the baryons in the $J^P=\frac{3}{2}^+$ decuplet? Hence predict the masses of the decuplet baryons and compare your predictions with the measured values.

[Assume in this case $m_u=m_d=360$ MeV, $m_s=540$ MeV and the spin-spin interaction coefficient $A= 0.026$ GeV³.]

17. p and n Moments, Tripos B-style question.

Derive the magnetic moments of the proton and neutron in the quark model as follows:

- (a) Assuming all the quarks are in $\ell=0$ states what must be the total spin of the two u quarks in the proton? (Give reasons for your answer).
(b) Hence show that the wave function for a proton in the $s_z=+\frac{1}{2}$ state can be written as

$$\frac{1}{\sqrt{6}}(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow)$$

and derive a similar expression for the neutron.

- (c) Assuming u and d quarks have equal mass, write their magnetic moments in terms of this quark mass.
(d) Hence predict the ratio of the proton and neutron magnetic moments. Compare with the observed values: $\mu_p = 2.79\mu_N$, $\mu_n = -1.91\mu_N$. What value for the quark mass is needed to give these values? Is this sensible?

- (e) Consider now the magnetic moments of the Σ^+ (uus) and Σ^- (dds) baryons, which are also members of the spin- $\frac{1}{2}$ octet. Show that

$$\mu_{\Sigma^+} - \mu_{\Sigma^-} = \frac{4}{5}[\mu_p - \mu_n] \ .$$

Test using measured values: $\mu_{\Sigma^+} = 2.458 \pm 0.010\mu_N$, $\mu_{\Sigma^-} = -1.160 \pm 0.025\mu_N$.

18. ρ^0 Decay, Tripos B-style question.

Consider the decay of the ρ^0 meson ($J^P = 1^-$) in the following decay modes:

- (a) $\rho^0 \rightarrow \pi^0\gamma$
- (b) $\rho^0 \rightarrow \pi^+\pi^-$
- (c) $\rho^0 \rightarrow \pi^0\pi^0$
- (d) $\rho^0 \rightarrow e^+e^-$

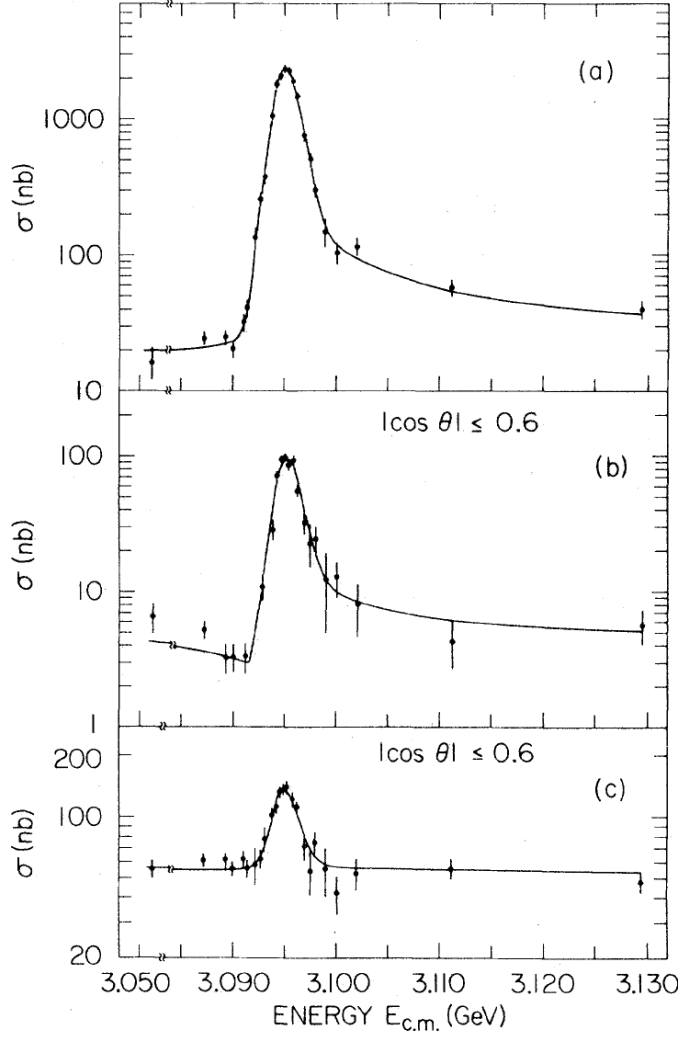
In each case, draw an appropriate Feynman diagram and determine whether the process is allowed or forbidden. By considering the strength of the forces involved, list the decay modes in order of expected rate.

The ratio of partial widths $\Gamma(\rho^0 \rightarrow \pi^0\gamma)/\Gamma(\omega^0 \rightarrow \pi^0\gamma)$ is approximately 0.1 while the ratio $\Gamma(\rho^0 \rightarrow e^+e^-)/\Gamma(\omega^0 \rightarrow e^+e^-)$ is approximately 10. Suggest an explanation for these observations.

[The quark wavefunctions for the mesons are $\frac{1}{\sqrt{2}}[u\bar{u} - d\bar{d}]$ for both π^0 and ρ^0 , and $\frac{1}{\sqrt{2}}[u\bar{u} + d\bar{d}]$ for ω^0 .]

19. J/ψ Meson, Tripos B-style question.

The figure shown below (from Boyarski *et al.*, Phys. Rev. Lett. **34** (1975) 1357) shows the original e^+e^- annihilation cross-section measurements from the Mark II Collaboration which contributed to the discovery of the J/ψ meson. The measurements were made during a fine scan of the e^\pm beam energies at the SPEAR storage ring at SLAC which consisted of oppositely circulating e^+ and e^- beams of equal energy. Figure (a) shows the cross-section for the process $e^+e^- \rightarrow \text{hadrons}$, (b) shows the cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ and (c) shows the cross-section for $e^+e^- \rightarrow e^+e^-$. The latter two were measured in a limited acceptance $|\cos\theta| < 0.6$, where θ is the polar angle of the produced leptons.



The observed width (a few MeV) of the J/ψ resonance peak is predominantly caused by the energy spread inherent in the e^+ , e^- beams at each measured point. The *relative* centre-of-mass energy between measurement points is however known very precisely (to about 1 part in 10^4). The actual J/ψ width is much smaller than the observed width, but can be extracted from the data as follows:

- (a) The Breit-Wigner formula for the scattering of two particles of spin s_1 and s_2 in the region of a resonance of spin J is:

$$\sigma(E) = \frac{\lambda^2}{4\pi} \frac{(2J+1)}{(2s_1+1)(2s_2+1)} \frac{\Gamma_i \Gamma_f}{[(E - E_0)^2 + \Gamma^2/4]}$$

where λ is the de Broglie wavelength of the incoming particles in the centre of mass frame, E is the centre of mass energy, E_0 is the resonance energy, Γ is the total width of the

resonance and Γ_i (Γ_f) is the partial width for decay into the initial (final) state. Show that, for the production of the J/ψ resonance in e^+e^- collisions, the integrated elastic cross-section under the resonance peak is given by

$$\sigma' \equiv \int \sigma_{el}(E)dE \approx \frac{3}{8}\lambda^2 B^2 \Gamma$$

where B is the branching fraction for the decay $J/\psi \rightarrow e^+e^-$.

- (b) Assume that, at each scan point, the beam energy spread produces a spread of centre of mass energies E' distributed about the average centre of mass energy E according to a probability distribution $f(E' - E)$. Show that the measured area under the resonance peak, $\int \sigma_{meas}(E)dE$, is equal to the true area under the peak, $\int \sigma(E)dE$.
- (c) Given that the differential cross-section $d\sigma/d\Omega$ for the process $e^+e^- \rightarrow J/\psi \rightarrow e^+e^-$ is proportional to $1 + \cos^2 \theta$ where θ is the angle between the final e^- and the beam direction, calculate the fraction of J/ψ decays contained within the acceptance region $|\cos \theta| < 0.6$ imposed for the e^+e^- and $\mu^-\mu^+$ channels.
- (d) Use the data in the figure to estimate the quantities σ' and B defined above¹. You should obtain $\sigma' \sim 500 \text{ nb MeV}$ and $B \sim 0.065$ for the leptonic decays, after correcting for the limited acceptance. Hence estimate Γ and Γ_{ee} for the J/ψ .
- (e) The corresponding widths for the ϕ meson are $\Gamma=4.4 \text{ MeV}$ and $\Gamma_{ee}=1.37 \text{ keV}$. Discuss why the J/ψ and ϕ mesons have similar leptonic widths Γ_{ee} but very different total widths Γ .

¹Note that the measured cross-sections contain a significant non-resonant contribution which must be subtracted. Note also that the scales of the graphs are logarithmic.

Numerical answers

11. (a) 20%; (b) 0.16%.
13. (a) 1, 0.028, 5×10^{-5} , 5×10^{-5} (just counting powers of α); (b) $1.56 \mu\text{eV}$, 6 keV
14. $\sigma(p\bar{p}) = 17\sigma(\pi^+p)$
16. (a) $m(\eta') = 349 \text{ MeV}/c^2$; (b) $S=1$, $m(\Delta) = 1.230 \text{ GeV}/c^2$, $m(\Sigma^*) = 1.383 \text{ GeV}/c^2$, $m(\Xi) = 1.535 \text{ GeV}/c^2$, $m(\Omega^-) = 1.687 \text{ GeV}/c^2$
17. (a) $S=1$; (d) $\mu_p/\mu_n = -1.5$, Mass of quark = $330 \text{ MeV}/c^2$
19. (c) 50.4%; (d) using $\sigma' \approx 500 \text{ nb MeV}$, $B \approx 0.065$ should obtain $\Gamma \approx 70 \text{ keV}$, $\Gamma_{ee} \approx 4.6 \text{ keV}$

Suggested Tripos Questions

Feynman Diagrams: 2016 3(a), 2010 3, 2008 A4 2008 (3) A4, 2009 (3) A1(b), 2010 (3) A1(c)

QCD: 2018 B3 last part, 2014 3, 2013 1(b)

Hadron physics and quark model: 2018 B3, 2017 4, 2016 1(b)