

Q.1 (a) $\sum m_j$: good quantum number m_J [1]

Max m_J defies max J [1]

Max m_J due to addition principle
 $= 5/2 + 3/2 + 1/2 + (-1/2) = 8/4$

$\Rightarrow \underline{\text{Max } J = 4}$ [2]

(b) Two possible approaches

(i) Due to centrifugal barrier $V^{\text{eff}}(r) = \frac{L(L+1)\hbar^2}{2mr^2}$ [2]

only $l=0$ partial wave reaches
 scatterer in low energy limit. [2]

OR ii) Range of scattering vector $0 < \Delta k < 2|k|$ ^{k_i incident wavevector} [1]
 if $\Delta k a \ll 1$ phase of all scatterers [1]
 within potential are the same (0)
 \Rightarrow all scattering in phase irrespective of
 direction of outgoing wave \rightarrow isotropic
 requires $ka \ll 1$ - low energy [2]

(c)

T	L	m_L
T	↓	1
T		0
T		-1

Max $J = 1$ [1]
 Max L (given S) = 1 [1]
 $> 1/2$ full $J = L + S = 2$ [1]
 $3 P_2$ [1]

Q2 (a) Variational method - OM.

i) Theorem $\frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \geq E_0$ [1]

(ii) Proof [2]

(iii) How to apply - variational parameter [1]

(iv) Excited states - not bound, a guess, for E_1, E_2 except for symmetry, [1]or fixed basis set for $|\psi_0\rangle, |\psi_1\rangle$ [1]

(v) Use to prove existence of bound state [1]

b) Landau levels

(i) 2D electron gas + \perp magnetic field [1]

(ii) Different gauges - different description of states. [1]

(iii) Massive degeneracy in levels of energy $(n + 1/2)\hbar\omega_c$ $\omega_c = \frac{eB}{m_e}$ [1]

(iv)

(v) is 3D normal dispersion along field [1]

(vi) Further splitting due to spin magnetic energy [1]

(vii) Number of states per level = $\frac{eDA}{h}$ [27](c) ~~Spectrum of~~ Helium atom. Ground state of neutral Helium atomi) 2 electron atom - angular momentum states no longer degenerate. \sim Each electron is hydrogen-like state. [1]

ii) Identical particle symmetry

 $S = 1$ sym requires antisym spatial [1] $S = 0$ antisym requires sym spatial

(iii) Spatial state - antisymmetry requires electrons in different single particle states - some states forbidden [1]

Q2 cont

- iv) Antisym state lower in energy [1]
 - exchange interaction
- v) Further splitting of J states [1]
 - (for $S=1$ state only)
- vi) Example of lower states [2]

OR

Diagrammatically - All relevant

$S=0$ [1]

$S=1$ [1]

1S_0 3P_0 3P_1 3P_2 3S_1 $^3P_{2,1,0}$ $^3D_{3,2,1}$ etc

$(1r)(3d)$ $(1r)(3p)$ $(1s)(3d)$
 $(1r)(2p)$ [1]

$(1r)(3d)$ $(1r)(3p)$ $(1r)(3d)$

$(1s)(2s)$ -

- $(1s)(2p)$
 - [exchange splitting] [1]
 $(1s)(2s)$

$(1r)^2$ - [1]

No more
 other

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we λ is designated order (4).

Q3 (i)

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}_1, \quad |\phi_n\rangle = |\psi_n\rangle + \lambda \sum_{j \neq n} c_j |\psi_j\rangle$$

$$E_n^{\approx} = E_n + \lambda E_n'$$

$$\hat{H} |\phi_n\rangle = E_n |\phi_n\rangle$$

$$(\hat{H}_0 + \lambda \hat{H}_1) (|\psi_n\rangle + \lambda \sum_{j \neq n} c_j |\psi_j\rangle) = (E_n + \lambda E_n') (|\psi_n\rangle + \lambda \sum_{j \neq n} c_j |\psi_j\rangle)$$

$$\hat{H}_0 |\psi_n\rangle + \lambda \sum_{j \neq n} c_j \hat{H}_1 |\psi_j\rangle = E_n |\psi_n\rangle + E_n' \sum_{j \neq n} c_j |\psi_j\rangle$$

$$\delta_{jj'} = \langle \psi_j | \psi_{j'} \rangle$$

$$\langle \psi_{j'} | \hat{H}_1 | \psi_n \rangle + c_j E_j = E_n c_j, \quad j' \neq n$$

$$c_j = \frac{\langle \psi_{j'} | \hat{H}_1 | \psi_n \rangle}{E_n - E_j}$$

$$|\phi_n\rangle = |\psi_n\rangle + \sum_{j \neq n} \frac{\langle \psi_j | \hat{H}_1 | \psi_n \rangle}{E_n - E_j} |\psi_j\rangle$$

$$\hat{H}_1 = \hat{V} = -eqx$$

$$|\phi_0\rangle = |\psi_0\rangle + \frac{\langle \psi_1 | -eqx | \psi_0 \rangle}{-E_1} |\psi_1\rangle$$

$$\langle \psi_1 | x | \psi_0 \rangle = \langle \psi_1 | \psi_1 \rangle = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\therefore |\phi_0\rangle = |\psi_0\rangle + \frac{eq}{\hbar\omega} \sqrt{\frac{\hbar}{2m\omega}} |\psi_1\rangle$$

$$O.p.v = \langle \phi_0 | qx | \phi_0 \rangle = \langle \psi_0 | qx | \psi_0 \rangle + \frac{eq^2 \hbar}{2m\omega} \langle \psi_0 | x | \psi_1 \rangle + \langle \psi_1 | x | \psi_0 \rangle$$

$$= \frac{eq^2 \hbar}{2m\omega} (\langle \psi_0 | x | \psi_1 \rangle + \langle \psi_1 | x | \psi_0 \rangle)$$

$$= \frac{2eq^2 \hbar}{2m\omega} = 2eq^2 \frac{\hbar}{2k} = \frac{eq^2}{k}$$

$$\frac{1}{2} kx^2 - eqx = \frac{1}{2} k \left(x - \frac{eq}{k} \right)^2 - \frac{e^2 q^2}{2k}$$

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⑤

Q3 (ii)

For S.H.O $\langle \phi_n | x | \psi_0 \rangle = 0$

except for $n=1$

\therefore Only $n=1$ term in perturbation theory
is non-zero so expression is
exact to 1st order even though have
only included $j=1$ term. [2]

However larger E - get higher order
terms ie scattering from $\psi_1 \rightarrow \psi_2$ etc etc
and these are not included in our expression
[2]

\rightarrow We do not see this as for small E
do not need to worry about normalisation
but clearly expression will eventually break
down

Normalisation of perturbed state also a problem.
When expression is correctly normalised the
 $\langle p \rangle_{\text{max}}$ for your state $|\psi_0\rangle, |\psi_1\rangle$ [2]

ie the $|\phi\rangle = \frac{1}{\sqrt{2}}(|\psi_0\rangle + |\psi_1\rangle)$

and $p_{\text{max}} = q \cdot \langle \phi | x | \phi \rangle$
 $= q \langle \psi_0 | x | \psi_1 \rangle = q \sqrt{\frac{\hbar}{2m\omega}}$

On axes, ground state wavefunction is Gaussian

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(6)

Q4.
$$\begin{aligned} & [H_0 + H' \cos(\omega t)] (c_1(t) \exp(i\omega_1 t) |\psi_1\rangle \\ & \quad + c_2(t) \exp(i\omega_2 t) |\psi_2\rangle) \\ & = i\hbar \frac{\partial}{\partial t} (c_1(t) \exp(i\omega_1 t) |\psi_1\rangle \\ & \quad + c_2(t) \exp(i\omega_2 t) |\psi_2\rangle) \end{aligned} \quad [2]$$

$$\begin{aligned} & H_0 |\psi_1\rangle c_1(t) \exp(i\omega_1 t) = E_1 |\psi_1\rangle c_1(t) \exp(i\omega_1 t) \\ & i\hbar \frac{\partial \exp(i\omega_1 t)}{\partial t} [c_1(t) |\psi_1\rangle] = \hbar\omega_1 \exp(i\omega_1 t) |\psi_1\rangle c_1(t) \end{aligned}$$

Term cancel, similar for $|\psi_2\rangle$ [1]

$$\Rightarrow H' \cos(\omega t) [c_1(t) \exp(i\omega_1 t) |\psi_1\rangle + c_2(t) \exp(-i\omega_2 t) |\psi_2\rangle] \quad [2]$$

$$\begin{aligned} & = i\hbar \frac{\partial c_1(t)}{\partial t} \exp(-i\omega_1 t) |\psi_1\rangle \\ & \quad + i\hbar \frac{\partial c_2(t)}{\partial t} \exp(-i\omega_2 t) |\psi_2\rangle \end{aligned}$$

$$\begin{aligned} \langle \psi_2 | & \rightarrow \hbar\omega'_2 \left(\exp(i\omega t) + \exp(-i\omega t) \right) \frac{c_1(t)}{2} \exp(-i\omega_1 t) \\ \langle \psi_2 | H' | \psi_1 \rangle \cdot (\hbar\omega')^* & = i\hbar \frac{\partial c_2}{\partial t} \exp(-i\omega_2 t) \quad [1] \end{aligned}$$

$$\begin{aligned} \rightarrow \frac{\omega'}{2} c_1(t) & [\exp(i(\omega + \omega_0)t) + \exp(i(\omega_0 - \omega)t)] \\ & = i \frac{\partial c_2(t)}{\partial t} \quad [1] \end{aligned}$$

$$\begin{aligned} \langle \psi_1 | & \rightarrow \hbar\omega'_1 \frac{1}{2} (\exp(i\omega t) + \exp(-i\omega t)) c_2(t) \exp(-i\omega_2 t) \\ & = i\hbar \frac{\partial c_1}{\partial t} \exp(-i\omega_1 t) \quad [1] \end{aligned}$$

$$\begin{aligned} \rightarrow \frac{\omega'}{2} c_2(t) & [\exp(i(\omega - \omega_0)t) + \exp(-i(\omega + \omega_0)t)] \\ & = i \frac{\partial c_1}{\partial t} \quad [1] \end{aligned}$$

Bookwork

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cur

$$\rightarrow w \sim w_0 \quad w - w_0 \text{ very small}$$

$$w + w_0 \gg w - w_0$$

Notation by
quantum in
lecture

Terms in $(w + w_0)$ rapidly oscillating [1]
~~do not~~ tend to produce destructive
interference

say $c_1 \sim 1 \quad c_2 \sim 0 \quad \text{at } t=0$

$$c_2(t) \sim \frac{1}{i(w + w_0)} \int_0^t \exp[i(w + w_0)t'] dt' \frac{(w')^*}{2}$$

oscillating [1]

whereas $w - w_0$ term can produce
steady increase in $c_2(t) \sim \frac{1}{i} \frac{(w')^*}{2} t$ [1]

spin $\frac{1}{2}$ nuclear energy levels $\pm \frac{1}{2} \gamma \hbar B$

$$w_0 = \gamma B \quad \begin{array}{c} \uparrow \quad |\psi_1\rangle \\ \downarrow \quad |\psi_2\rangle \end{array}$$

*
(w')
(w')

For $|w - w_0|$

$$\rightarrow \frac{(w')^*}{2} c_1(t) = i \frac{\partial c_2(t)}{\partial t}$$

$$\frac{(w')}{2} c_2(t) = i \frac{\partial c_1(t)}{\partial t}$$

$$\rightarrow \frac{|w'|^2}{4} c_1(t) = - \frac{\partial^2 c_1}{\partial t^2} \quad [2]$$

Jan

$$c_1(t) = c_1(0) \cos\left(\frac{|w'|}{2} t\right)$$

$$c_2(t) = \frac{(w')^*}{i|w'|} \sin\left(\frac{|w'|}{2} t\right)$$

See in lecture

with error

by

Spn is a direr = $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\frac{(w')^x}{i(w')} = 1 \quad (\text{rev}) \quad [1]$$

$$\frac{(\psi')^*}{\psi} = -i \rightarrow \text{Finden y direkt}$$

choice of B_+ but $|w'| = \gamma B \frac{1}{2} h$ [3]
 $\gamma B h \epsilon = \pi$

s_{p-1} is given by $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ [1]

$$\frac{(\omega')^2}{i/v'} = -1 \rightarrow \text{Field in } -x \text{ direction}$$

$t, 0$ etc as before [1]

$$-2 \quad \text{spu} \quad \rightarrow \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Field anywhere is a y place (1)

mit $\frac{|w'|}{2} t = \frac{\pi}{2}$. $\gamma^{\text{Dht}} = 2\pi$
[1]

$$\nexists w \approx w_0 \quad \text{or} \quad w \neq w_0$$

Then $|c_2(t)|$ never becomes 1. [1]

x , y rotation not possible [2]

$$b_m \text{ not } -2 \quad [1]$$

1. Learning
 2. Technology
 3. Business
 4. Marketing
 5. Finance
 6. Operations
 7. Human Resources
 8. Information Systems
 9. Legal
 10. Environmental
 11. Social
 12. Political
 13. Economic
 14. Cultural
 15. Religious
 16. Philosophical
 17. Artistic
 18. Scientific
 19. Medical
 20. Engineering
 21. Architecture
 22. Design
 23. Education
 24. Healthcare
 25. Transportation
 26. Energy
 27. Environment
 28. Climate
 29. Weather
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