NATURAL SCIENCES TRIPOS Part II

Friday 29 May 2009 9.00 am to 12.00 noon

EXPERIMENTAL AND THEORETICAL PHYSICS (3)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (3)

Candidates offering the whole *of this paper should attempt a total of* six *questions, three from Section A* and *three from Section B. The questions to be attempted are* A1, A2 *and* one *other question from Section A and* B1, B2 *and* one *other question from Section B.*

Candidates offering half *of this paper should attempt a total of* three *questions,* either *three from Section A* or *three from Section B. The questions to be attempted are* A1, A2 *and* one *other question from Section A or* B1, B2 *and* one *other question from Section B. These candidates will leave after* 90 minutes.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains 7 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.

Answers to each *section should be tied up separately, with the numbers of the questions attempted written clearly on the cover sheet.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough workpad
Blue coversheets (2)
Treasury tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

PARTICLE AND NUCLEAR PHYSICS

A1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) For a centre-of-mass energy $\sqrt{s} = 20\,\text{GeV}$ calculate the predicted value of

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

[4]

[*You may neglect the contribution from the Z boson.* $m_\text{b} \sim 5\,GeV$ *and* $m_\text{t} \sim 175\,GeV$.]

(b) For each of the following decays draw the dominant lowest order Feynman diagram:

i) $\Delta^+(\text{uud}) \to \text{n}(\text{udd}) + \pi^+(\text{u}\overline{\text{d}})$;

ii) $\text{K}^+(\text{u}\overline{\text{s}}) \to \mu^+\nu_\mu$;

iii) $\phi(\text{s}\overline{\text{s}}) \to e^+ + e^-$, note that $J^P_\phi = 1^-$.

Which process would you expect to have the greatest decay rate? [4]

(c) The radius of a nucleus with A nucleons is given by $R \approx r_0 A^{\frac{1}{3}}$. Assuming that the nuclear force is short ranged and that the only process occurring is neutron absorption, estimate the probability that an energetic neutron will pass through 5 mm of lead. [4]

[*The density of lead* ^{208}Pb *is* $11350\,kg\,m^{-3}$; $r_0 = 1.3 \times 10^{-15}\,m$ *and take the mass of a nucleon to be* $1.67 \times 10^{-27}\,kg$.]

A2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) alpha decay;

(b) electroweak unification;

(c) QCD.

A3 *Attempt* **either** *this question* or *question A4.*

The semi-empirical mass formula (SEMF) for a nucleus of Z protons and $(A-Z)$ neutrons is

$$M(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{\frac{2}{3}} + a_C Z^2 A^{-\frac{1}{3}} + a_A \frac{(A-2Z)^2}{A} + \delta,$$

where $m_n - m_p = 1.3\,\text{MeV}$, $a_V = 15.8\,\text{MeV}$, $a_S = 18.0\,\text{MeV}$, $a_C = 0.7\,\text{MeV}$, $a_A = 23.5\,\text{MeV}$ and $\delta = \pm 33.5 A^{-\frac{3}{4}}\,\text{MeV}$ or 0. Explain the origin of each of the terms in the SEMF, stating clearly how the different values of δ relate to A and Z. [6]

The principal decays of the low lying states of ${}^{108}_{47}\text{Ag}$ are shown below, with the energies of the observed gamma-rays given in keV. Use the SEMF to show that for the β^- and β^+ decays of ${}^{108}_{47}\text{Ag}$,

$$M(A, Z\pm 1) - M(A,Z) \approx \pm(m_p - m_n) \pm 2a_C Z A^{-\frac{1}{3}} + \frac{4a_A}{A}(1 \pm 2Z \mp A) - 2.0\,\text{MeV},$$

and hence estimate the energy released in decays of the ground state of ${}^{108}_{47}\text{Ag}$ to ${}^{108}_{48}\text{Cd}$ and ${}^{108}_{46}\text{Pd}$. Hence explain how the SEMF accounts for the fact that certain odd-odd nuclei such as ${}^{108}_{47}\text{Ag}$ undergo both β^- and β^+ decays. [8]

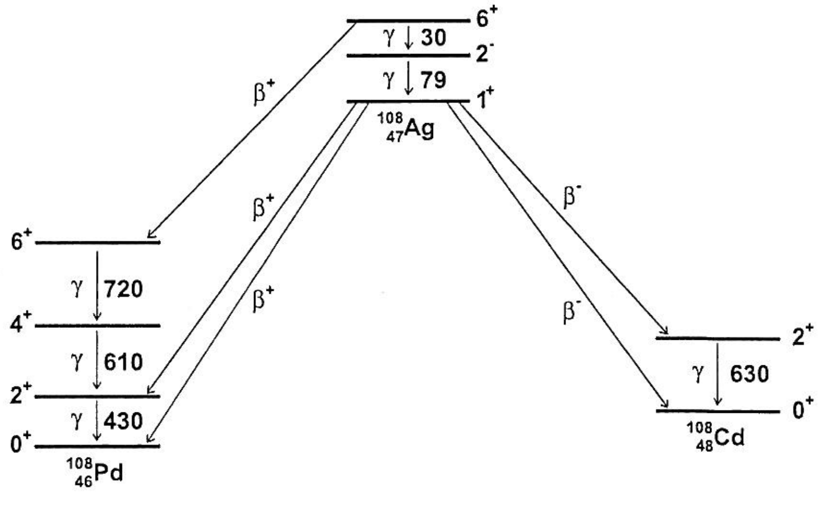
By considering the relevant γ and β decay selection rules:

(a) classify the β decays of the 6^+ and 1^+ states of ${}^{108}_{47}\text{Ag}$ as Gamow-Teller or Fermi, allowed or forbidden; [3]

(b) classify the various γ decays; [2]

(c) explain why no β^+ decay to the 4^+ state of ${}^{108}_{46}\text{Pd}$ is observed; [3]

(d) suggest why the 6^+ state of ${}^{108}_{47}\text{Ag}$ is metastable and has a very long lifetime compared to that of the ground state. [3]



$[m_p = 938.3\,MeV,\ m_n = 939.6\,MeV,\ and\ m_e = 0.5\,MeV.]$

(TURN OVER

A4 *Attempt* **either** *this question* **or** *question A3.*

Fission reactors produce $\overline{\nu}_e$ from $n \rightarrow p + e^- + \overline{\nu}_e$. The $\overline{\nu}_e$ can be detected from their $\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$ and $\overline{\nu}_e + p \rightarrow e^+ + n$ interactions. Draw the lowest order Feynman diagrams for each of these three processes. [5]

Taking $m_p = 938.3\,\text{MeV}$, $m_n = 939.6\,\text{MeV}$, and $m_e = 0.5\,\text{MeV}$, calculate the minimum $\overline{\nu}_e$ energy in the laboratory frame (where the struck proton is at rest) for which the process $\overline{\nu}_e + p \rightarrow e^+ + n$ is kinematically allowed. [5]

In the two-flavour treatment of neutrino oscillations, the $\overline{\nu}_e$ survival probability is given by

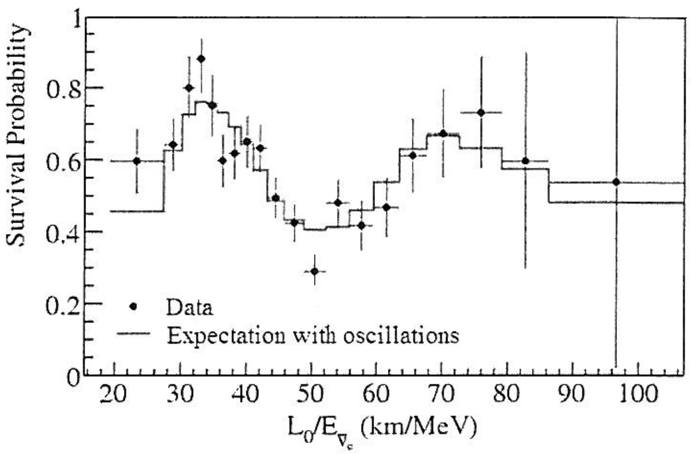
$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2[\text{eV}^2]L[\text{m}]}{E_\nu[\text{MeV}]}\right).$$

Explain the meaning of the symbols in this formula. [3]

The KamLAND detector, which has a mass of $5 \times 10^5\,\text{kg}$, detects $\overline{\nu}_e$ from several fission reactors located at distances between 130 and 240 km from the experiment. The flux of $\overline{\nu}_e$ at the detector is approximately $10^5\,\text{cm}^{-2}\,\text{s}^{-1}$. At the mean $\overline{\nu}_e$ energy of 4 MeV, $\sigma(\overline{\nu}_e + p \rightarrow e^+ + n) = 2 \times 10^{-42}\,\text{cm}^2$. Estimate the expected number of reactor $\overline{\nu}_e$ interactions in the KamLAND detector in two years. [6]

The Figure below shows the KamLAND data for the ratio of the number of $\overline{\nu}_e$ interactions to the expected number in the absence of neutrino oscillations, plotted as a function of $L_0/E_{\overline{\nu}_e}$, where $E_{\overline{\nu}_e}$ is the measured neutrino energy and L_0 is the effective mean distance to the reactors. Explain, in as much detail as possible, the main features of this plot and obtain a value for Δm^2. [4]

In the plot, the line labelled "Expectation with oscillations" accounts for the fact that the reactors are at different distances from the detector. Explaining your reasoning, estimate the value of $\sin^2 2\theta$. [2]



SECTION B

ASTROPHYSICAL FLUID DYNAMICS

B1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) A cool extended gas cloud of mass M collapses under gravity while remaining close to hydrostatic equilibrium at all times. Find the total energy radiated by the time the average temperature has reached T^*. State clearly your assumptions. [4]

(b) Two cylindrical jets of incompressible fluid of radius a and velocities $(0, 0, \pm v_z)$ collide head-on at the origin, and spread out radially to form a sheet in the $z = 0$ plane. Show that the thickness d of this sheet at a large distance r from the origin is a^2/r. [You may assume that the flow is everywhere laminar and dissipationless.] [4]

(c) Show that the vertical density structure, $\rho(z)$ for a self-gravitating slab of polytropic gas, $p = K\rho^2$, in hydrostatic equilibrium, satisfies the differential equation

$$\frac{d^2\rho}{dz^2} + \frac{2\pi G}{K}\rho = 0.$$

[4]

B2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) convection;

(b) the virial theorem;

(c) Alfvén waves.

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B3 *Attempt* **either** *this question* **or** *question B4.*

Describe briefly the circumstances under which shocks are likely to form in astrophysical systems. [3]

Derive the Rankine-Hugoniot equations

$$\begin{aligned} \rho_2 u_2 &= \rho_1 u_1 \\ p_2 + \rho_2 u_2^2 &= p_1 + \rho_1 u_1^2 \\ \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} + \frac{u_2^2}{2} &= \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} + \frac{u_1^2}{2} \end{aligned}$$

for a plane adiabatic shock, and briefly state their physical meaning. [8]

For a strong shock front (i.e., one for which the upstream pressure is negligible), and assuming normal incidence, show that the downstream velocity u_2 is related to the upstream velocity u_1 by

$$u_2 = \frac{\gamma - 1}{\gamma + 1} u_1 .$$

[5]

Hydrogen free-falling onto a white dwarf passes through a stationary shock close to the surface at radius R_0. Show that the shock temperature is given by

$$k_B T = \frac{3}{16}\frac{G M_* m_p}{R_0},$$

where M_* is the mass of the star, and m_p is the mass of the proton. [7]

What is $k_B T$ if $R_0 = 6000\,\mathrm{km}$ and $M_* = 1\,M_\odot$? [2]

[$1\,M_\odot = 2 \times 10^{30}\,\mathrm{kg}$; $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$; $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$.]

B4 *Attempt* **either** *this question* **or** *question B3.*

An accretion disc around a star is in circular motion with angular velocity $\Omega(R)$, with a slow superposed radial velocity, $u_R(R)$. Show that the equation of mass conservation is

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\Sigma u_R\right) = 0,$$

where $\Sigma(R)$ is the surface density of the material. [4]

Combining this equation with the Navier-Stokes equation for the fluid yields

$$\frac{\partial}{\partial t}\left(R^2\Sigma\Omega\right) + \frac{1}{R}\frac{\partial}{\partial R}\left(R^3\Sigma\Omega u_R\right) = \mathcal{G} \qquad (*),$$

where $\mathcal{G} = 1/(2\pi R)\,(dG/dR)$ and $G(R)$ is the viscous torque acting on a ring of material at radius R.

Give a physical interpretation of the terms on the left hand side of equation (*). [3]

Also demonstrate that

$$G(R) = 2\pi\nu\Sigma R^3\frac{\partial \Omega}{\partial R},$$

where ν is the kinematical viscosity. [5]

In a thin steady accretion disc, the constant mass accretion rate is $\dot{m}$. Show that the surface density Σ is related to Ω through the equation

$$\Sigma = \frac{1}{2\pi\nu S}\left(\dot{m}\Omega - \frac{C}{R^2}\right),$$

where $S(R) = -R\,(\partial\Omega/\partial R)$ is the shear. [10]

Assuming that S vanishes at $R = R_0$, which is close to the accreting star, evaluate C in terms of $\Omega(R_0)$, and interpret C in physical terms. [3]

END OF PAPER