Electrodynamics and Optics 2013

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Use at your own risk! These are just a few thoughts of how one might attempt these questions. There will almost certainly be mistakes either in the algebra or in the transcription, if you find one, feel free to get in touch (cm672)

1a
Use the Airy disc:

$1.22\ \frac{\lambda}{D} = 0.61\ radians$

So the dish must be pointed in approximately the right direction but there's a 35^o leeway.

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b

- **Spatial coherence - the degree to which a signal is coherent when analysed at different points of the wavefront at the same time.**
- **Temporal coherence - the degree to which a signal is coherent when analysed at different points in time at the same point in the wavefront.**

Temporal coherence length: $l_c \simeq \frac{\lambda^2}{\Delta\lambda}$ **visible light** $\lambda \approx 500nm$

$\Delta\lambda \approx 300nm \qquad so \qquad l_c \approx 830nm$

Spatial coherence width: $w_c \approx \frac{\lambda}{\alpha} \quad = \quad 2.58mm$

C

Consider the electric field in the capacitor $E = \frac{\sigma}{\epsilon}$ when stationary. In another frame, charge is still the same and whilst length contraction occurs in the same direction as the motion, the area (perpendicular to the motion) is the same in any frame. So as the charge density is the same in each frame, the Electric field (in the direction of motion) remains unchanged.

Field in solenoid: $B = \frac{\mu_o NI}{l} = \frac{\mu_o N}{l}\frac{\mathrm{d}q}{\mathrm{d}t}$

Length contraction: $l' = \frac{l}{\gamma}$ **and time dilation:** $\frac{\mathrm{d}q}{\mathrm{d}t} = \gamma\frac{\mathrm{d}q}{\mathrm{d}t'}$

So in any frame these two effects cancel to leave the B-field parallel to the axis of the solenoid unchanged in any frame.

3

Consider two perpendicular components of the electric field.

If they are in phase or anti-phase the resulting electric field will be linearly polarised and remain in that direction.

If they are $\frac{\pi}{2}$ out of phase the resulting electric field will rotate around the propagation direction hence: circular polarisation. It will be left-handed or right-handed depending on the convention and which way the electric field rotates.

Jone's vectors represent the polarisation state of EM radiation.

For example: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **- linearly polarised in x-direction**

$\begin{pmatrix} 1 \\ i \end{pmatrix}$ **- circularly polarised (left-handed)**

in general a Jones vector can be represented as $\begin{pmatrix} a \\ be^{i\delta} \end{pmatrix}$ **where a and b are the relative magnitudes and** δ**, the relative phase.**

Jone's matrices represent the effect of optical components on the polarisation state. For instance, linearly polarised light passing through components A then B then C.

$$\underline{\underline{J_C}} \cdot \underline{\underline{J_B}} \cdot \underline{\underline{J_A}} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

A linear polariser will take unpolarised light and transmit linearly polarised light. If this light is passed into a quarter-wave plate with the polarisation direction at 45^o **to the fast axis, circularly polarised light will be produced.**

Linear polariser at angle θ **to the x-axis:** $\begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix}$

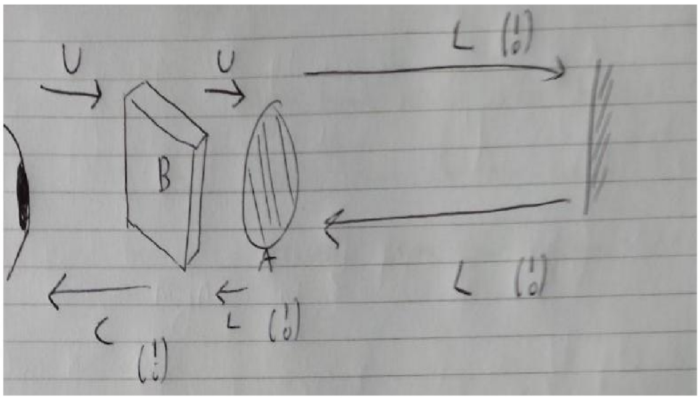
Quarter-wave plate Jone's matrix: $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Resulting polarisation $= \begin{pmatrix} cos(\theta) \\ isin(\theta) \end{pmatrix}$

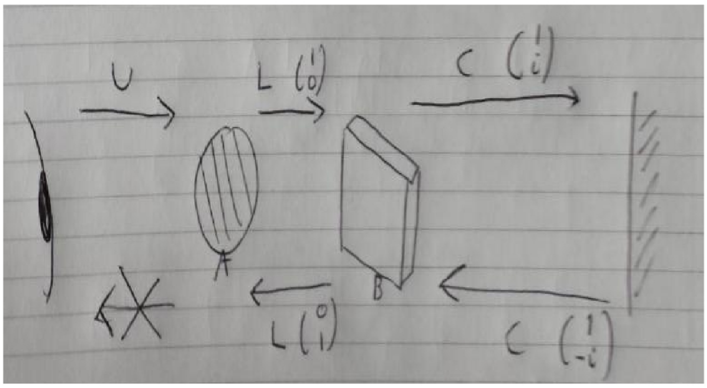
(For $\theta = 45^o$**)** $= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ **<u>Left-handed circular polarisation</u>**

The quarter-wave plate will not change the polarisation state (or rather lack of it) of the unpolarised light, so the resulting light will be linearly polarised. An observer (the eye) will not notice the difference between different polarisation states so the images will be identical regardless of which way round the components go. The images will be darkened because only half of the unpolarised light will be transmitted by the linear polariser (Malus' law).

a) Unpolarised light from the eye remains unpolarised as it passes through the quarter-wave plate, becoming linearly polarised then refelcted, passing back through the linear polariser then circularly polarised by the quarter-wave plate.



b) Unpolarised light, initially linearly polarised then circularly polarised by the quarter-wave plate. The mirror reverses the handedness of the circular polarisation. When passed back through the quarter-wave plate it results in linear polarisation orthogonal to the linear polariser. No light is therefore transmitted, leading to the dark image.



4

A Hertzian dipole is a theoretical construction comprising 2 separated opposite charges where the separation distance oscillates. It can be used in electrodynamics to model antennae with similar field patterns and to perform calculations of gain, effective area, radiation resistance etc.

The square brackets indicate we are to evaluate the property at the retarded time i.e $t - \frac{r}{c}$ **. It takes time for the for changes of field, travelling at c, to propagate a distance r.**

Terms

- $\propto \frac{1}{r^3}$ **dipole terms dominate close to dipole.**
- $\propto \frac{1}{r^2}$ **induction terms that never dominate.**
- $\propto \frac{1}{r}$ **radiation terms that dominate when** $r >> \lambda(>> d)$

Taking only radiation terms:

$\underline{B} = \frac{\mu_o}{4\pi} \frac{[\ddot{m}] sin(\theta)}{c^2 r} \underline{\hat{\theta}}$ $\qquad$ $\underline{E} = \frac{1}{4\pi\epsilon_o} \frac{[\ddot{m}] sin(\theta)}{c^3 r} \underline{\hat{\phi}}$

so Poynting vector $\implies$ $\underline{N} = \frac{1}{\mu_o} \underline{E} \times \underline{B}$

$$\underline{N} = \frac{1}{16\pi^2 \epsilon_o c^5 r^2} [\ddot{m}]^2 sin^2(\theta) \underline{\hat{r}}$$

Total power radiated, integrate over unit sphere:

$$\int\int \underline{N} \cdot \mathrm{d}\underline{\Omega} = \int_0^{2\pi} \int_0^{\pi} \frac{1}{16\pi^2 \epsilon_o c^5 r^2} [\ddot{m}]^2 sin^2(\theta) \cdot r^2 sin(\theta) \mathrm{d}\theta \ \mathrm{d}\phi$$

$$= \frac{[\ddot{m}]^2}{8\pi\epsilon_o c^5} \int_0^{\pi} sin^3(\theta) \mathrm{d}\theta = \boxed{\frac{[\ddot{m}]^2}{6\pi\epsilon_o c^5}}$$

Radiation resistance is the effective resistance an antenna presents in an electronic circuit. Radiation radiates (obviously!) energy away much like a resistor.

$P = \frac{[\ddot{m}]^2}{6\pi c^5 \epsilon_o}$ **so if** $m = m_o e^{i\omega t}$ **and** $m_o = I_o A$

then $< P >= \frac{I_o^2 A^2 \omega^2}{12\pi c^5 \epsilon_o}$

and $P = I^2R$ **so** $R = \frac{A^2\omega^2}{12\pi c^5\epsilon_o}$

Faraday induction: $\xi = \frac{-\mathrm{d}\Phi_B}{\mathrm{d}t}$

$\Phi_B = \pi b^2 B sin(\omega t) \quad so \quad \xi = -\pi b^2 B\omega cos(\omega t)$

$\xi = IR \quad so \quad I = \frac{-\pi b^2 B\omega cos(\omega t)}{R}$

$< I >^2 = \frac{\pi^2 b^4 B^2 \omega^2}{2R^2}$

$P = \frac{\pi^2 b^4 B^2 \omega^2}{2R}$ **Energy comes from kinetic energy of spinning satellite.**

The frequency will be 500Hz (wavelength 600km). The magnetic field is of the order of micro-Tesla (10^{-6}). This yields a power of 0.49W.

The atmosphere is opaque to long wavelengths (below the plasma frequency) and the power is also very low. So it would be impossible to detect this signal on the earth.

Without the atmosphere between them, detecting the signal from another satellite seems more feasible. However, the power is so low and there are many astronomical sources of similar wavelength so it would be difficult to detect. One would have to use a high-gain antenna and look for a narrow-band signal. (Most astronomical signals are fairly broad-band)

END OF PAPER