Pt II Astrophysics **Lent, 2022**

Astrophysical Fluid Dynamics

Example sheet 3

1. Show that for a strong shock (where the upstream Mach number M_1 is large), the downstream Mach number M_2 satisfies

$$M_2^2 \simeq \frac{\gamma - 1}{2\gamma}.$$

   Hence obtain an equation for the sound-speed ratio c_2/c_1.

   A shock from a supernova travelling through the surrounding interstellar medium is observed to be travelling with speed 3000 km/s. What is the temperature immediately behind the shock? [You may assume, if you wish, the surrounding interstellar medium to have temperature 100K and density 10^7 particles m^{-3}.] (*From 1997 Examination*)

2. A stellar wind is maintained at a temperature of $T = 2 \times 10^6$K by magnetic heating; calculate the radius at which it achieves the isothermal sound speed if the star from which it blows has mass M. You may assume the gas is made of fully ionized hydrogen. Evaluate your answer when M is the mass of the Sun $M_\odot = 2 \cdot 10^{30}$kg.

3. Isothermal gas of pressure $\rho_0 c_\mathrm{I}^2$ and density ρ_0 at large distances from a star is steadily accreted by a star of mass M at the origin.

   Calculate the accretion rate assuming that the gas remains isothermal. At what radius does the infalling gas achieve the sound velocity?

   If $c_\mathrm{I} = 1$km/s and $n_\infty = 10^9$ hydrogen molecules/ m^3 and the mass of the hydrogen atom is $1.67 \cdot 10^{-27}$kg evaluate this radius in terms of the solar radius $R_\odot = 7 \cdot 10^5$km assuming that the stellar mass is equal to the mass of the Sun ($M_\odot = 2 \times 10^{30}$ kg).

   Calculate the accretion rate in kg/s. How long will it take in seconds and in years to double the mass of the star whose initial mass is $M_\odot$? How does this time depend on that initial mass?

4. Clusters of ionising stars, total luminosity L, sweep out cavities in the interstellar medium whose undisturbed density is ρ_0. Use the similarity solution method to determine the evolution of cavity radius r with time t – i.e. determine a, b, c, where $r \,\alpha\, L^a\, \rho_0^b\, t^c$.

   These bubbles stall when their expansion velocities become of order the sound speed in the interstellar medium. Show that the area occupied by a stalled bubble is proportional to L and hence comment on how the porosity of the interstellar medium [defined as the fractional area of the galaxy as seen from above the disk occupied by stalled bubbles] depends on how ionising stars (with given total luminosity) are organised into clusters.

   If the disc of a galaxy can be approximated by a uniform density gas slab with a sharp edge at height z, comment on the different behaviours of clusters of small and large L.

5. A star is described by a polytropic equation of state, index n. For what values of n is such a star stable against convection? In the case that the star is convectively unstable, describe its structure and how energy is transported from its core to a distant point outside the star. [Assume that the star consists of fully ionized, monatomic hydrogen.]

6. A self-gravitating slab of gas is heated by cosmic rays (constant heating rate per unit mass) and cooled by optically thin thermal bremsstrahlung for which the cooling rate per unit mass is proportional to $\rho T^{0.5}$. Discuss the stability of the slab to (a) thermal and (b) convective instabilities.

7. Calculate the ratio of the free fall time to the sound wave crossing time for a uniform gaseous sphere containing one Jeans mass.

   Show that if such a sphere contracts homogeneously (i.e. maintaining uniform density) and isothermally, the number of Jeans masses it contains rises as $\Re^{3/2}$ where $\Re$ is the collapse factor (i.e. the ratio of the initial value of the radius to its current value).

8. A thin disc of gas, rotating around an object mass M, is supported in the vertical (z) direction by a balance between pressure forces and the z-component of the central object's mass. Write down the equation of hydrostatic equilibrium in the z direction. If the density distribution is of the form $\rho(z) \propto (z_m^2 - z^2)^2$ deduce the pressure distribution and polytropic index, n, for the disc. Is the disc stable against convection if composed of a) monatomic b) diatomic gas?

9. A jet propagates in the z direction through a hydrostatic slab of isothermal gas, with temperature T_s $\left(\text{i.e. one with density distribution } \rho = \rho_0 \operatorname{sech}^2\left(\frac{z}{z_s}\right)\right)$. If the jet is also isothermal, with temperature T_j, write down the density distribution in the jet, $\rho_j(z)$, explaining carefully what boundary condition applies at the jet/slab interface.

   Show that the minimum cross-sectional area of the jet, $A_{\rm min}$, is given by

$$A_{min} = \frac{\dot{M}}{\rho_{0j} c_j} \exp\left[\frac{\mu}{2R_* T_j}\left(c_j^2 - \left(\frac{\dot{M}}{\rho_{0j} A_0}\right)^2\right)\right]$$

   where $\dot{M}$ is the mass flux in the jet, c_j is the sound speed in the jet and ρ_{0j}, A_0 are the density and cross-sectional area at the base of the jet ($z = 0$).

   Write down the height, $z_{\rm min}$, of this minimum and the jet velocity at this point. Show furthermore that the cross-sectional area as a function of z, $A(z)$, is:

$$A(z) = \frac{A_0 \cosh^2\left(\frac{z}{z_s}\right)}{\left[1 + 2\left(\frac{A_0 \rho_{0j}}{\dot{M}}\right)^2 \frac{R_* T_j}{\mu} \ln\left[\cosh^2\left(\frac{z}{z_s}\right)\right]\right]^{1/2}}$$

   and sketch the shape of the jet. [Ignore the effects of gravity on the jet structure].