NATURAL SCIENCES TRIPOS Part II

Friday 30th May 2008 13.30 to 16.30

EXPERIMENTAL AND THEORETICAL PHYSICS (3)

Candidates offering the **whole** *of this paper should attempt a total of* **six** *questions, three from Section A* **and** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A and* **B1**, **B2** *and* **one** *other question from Section B.*

Candidates offering **half** *of this paper should attempt a total of* **three** *questions,* **either** *three from Section A* **or** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A or* **B1**, **B2** *and* **one** *other question from Section B.*

Answers to **each** *question should be tied up separately, with the number of the question written clearly on the cover sheet.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin. This paper contains 6 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work paper
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

PARTICLE AND NUCLEAR PHYSICS

A1 *Attempt this question.*

Give concise answers to all three parts of the question. Relevant formulae may be assumed without proof.

(a) By considering the minimum centre-of-mass energy, $\sqrt{s}$, of the final state in the reaction $\nu_\tau + n \to p + \tau^-$, calculate the minimum *laboratory* frame energy of the tau neutrino if this reaction is energetically allowed. Assume that the target neutron is at rest in the laboratory frame. [4]

[*Assume* $m_n = m_p = 0.9\,\text{GeV}$ *and* $m_\tau = 1.8\,\text{GeV}$.]

(b) In Natural Units the cross-section for the QED process $e^+e^- \to \mu^+\mu^-$ is

$$\sigma = \frac{4\pi\alpha^2}{3s}.$$

At a centre-of-mass energy of $\sqrt{s} = 20\,\text{GeV}$, the QED contribution to the cross-section is measured to be $2.40 \times 10^{-38}\,\text{m}^2$. Obtain a value for α and comment on your result. [4]

[$\hbar c = 0.197\,\text{GeV fm}$.]

(c) Treating the nucleus as a charged liquid drop, the binding energy per nucleon can be expressed as

$$\frac{B(A,Z)}{A} = a_V - a_S A^{-\frac{1}{3}} - a_c Z^2 A^{-\frac{4}{3}}.$$

What is the minimum value of the mass number, A, for which the fission reaction $(A,Z) \to (A/2, Z/2) + (A/2, Z/2)$ is energetically allowed if $A = 2Z$? Comment on your answer. [4]

[a_S = *16.8* MeV, *and* a_c = *0.72* MeV.]

A2 *Attempt this question.*

Write brief notes on **two** of the following: [13]

(a) Fermi theory of nuclear beta decay;

(b) charmonium;

(c) the quark model of hadrons and the derivation of the proton wavefunction. Your answer should include a discussion of the relevant symmetries;

(d) the measurement of the mass and width of the Z boson at LEP.

A3 *Attempt* **either** *this question* **or** *question A4.*
Outline the experimental evidence for magic numbers in nuclear physics. [5]
In the shell model of the nucleus the single-particle energy levels are:

$$1s_{\frac{1}{2}}, 1p_{\frac{3}{2}}, 1p_{\frac{1}{2}}, 1d_{\frac{5}{2}}, 2s_{\frac{1}{2}}, 1d_{\frac{3}{2}}, 1f_{\frac{7}{2}}, 2p_{\frac{3}{2}}, ...$$

Describe the main features of the nuclear shell model and explain how it accounts for the first four magic numbers: 2, 8, 20 and 28. [7]

Show that the shell model predicts that the spin-parity of the ground state of ${}^{17}_{9}\mathrm{F}$ is $\frac{5}{2}^{+}$ and give corresponding predictions for ${}^{15}_{7}\mathrm{Ne}$ and ${}^{33}_{16}\mathrm{S}$. [3]

In a study of the first three excited states of ${}^{17}_{9}\mathrm{F}$ the following gamma transitions were observed:

E1 : 2.6 MeV, 4.2 MeV, 4.7 MeV;
M1 : 1.6 MeV;
E2 : 0.5 MeV, 1.6 MeV.

In addition a weaker transition of energy 3.1 MeV was seen. The 0.5 MeV gamma-ray corresponds to a transition between the first excited state and the ground state. Assuming that the first excited state is a single particle excitation of the nucleus, suggest the likely spin-parity assignment for this excited state and discuss whether this is consistent with the observed gamma transition. [3]

Clearly explaining your reasoning, draw a possible decay scheme for ${}^{17}_{9}\mathrm{F}$ showing the energy levels of the first three excited states, the spin-parity assignments and the gamma-ray transitions given above. Discuss the likely nature of the 3.1 MeV gamma transition. [7]

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A4 *Attempt* **either** *this question* **or** *question A3.*

Discuss what is meant by a Feynman diagram in particle physics. [3]

What factors determine whether a given particle decay mode is allowed and, if it is allowed, what factors determine the particle decay rate? [4]

For each of the following processes, *either* draw the lowest order Feynman diagram, *or* explain why the decay is forbidden:

(a) $\mu^- \to e^-\overline{\nu}_e\nu_\mu$;

(b) $\mu^- \to e^-\overline{\nu}_\mu\nu_e$;

(c) $\mu^- \to e^-e^+e^-$;

(d) $\Omega^- \to \Xi^0\pi^-$;

(e) $\Omega^- \to \Lambda\pi^-$;

(f) $\eta \to \pi^0$;

(g) $\eta \to \gamma\gamma$;

(h) $\eta \to \mu^+\mu^-$;

(i) $\eta \to \pi^+\pi^-$;

(j) $\eta \to \pi^+\pi^-\pi^0$;

(k) $\eta \to \pi^+\pi^-\gamma$. [11]

Draw the lowest order Feynman diagrams for each of the following decays. For each pair of decays explain which is likely to have the largest branching ratio, clearly stating *all* relevant factors:

(a) $D^+ \to \pi^+\overline{K}^0$ and $D^+ \to \pi^+\pi^0$;

(b) $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^-\pi^+$;

(c) $\pi^- \to \mu^-\overline{\nu}_\mu$ and $K^- \to \mu^-\overline{\nu}_\mu$. [7]

The photon has $J^P = 1^-$. *The masses, quark content, and spin-parities of the hadrons above are:*

Particle	Mass/MeV	quark content	J^P
π^0	135	$(u\overline{u} - d\overline{d})$	0^-
π^+, π^-	140	$u\overline{d}$, $d\overline{u}$	0^-
K^+, $\overline{K}^0$, K^-	495	$u\overline{s}$, $s\overline{d}$, $s\overline{u}$	0^-
η	550	$(u\overline{u} + d\overline{d} - 2s\overline{s})$	0^-
Λ	1115	uds	$\frac{1}{2}^+$
Ξ^0	1315	uss	$\frac{1}{2}^+$
Ω^-	1670	sss	$\frac{3}{2}^+$
D^+	1870	$c\overline{d}$	0^-

SECTION B

ASTROPHYSICS

B1 *Attempt this question.*

Give concise answers to all three parts of the question. Relevant formulae may be assumed without proof.

(a) What is the Eddington luminosity limit? Why is it important for studies of high-energy astrophysical objects? [3]

(b) The Einstein-de Sitter world model has dynamics $a = (t/t_0)^{2/3}$, where $t_0 = 2/3H_0$ and H_0 is Hubble's constant. Estimate the epoch at which the cosmic microwave background radiation had temperature 4,000 K. [4]

(c) A distant star is observed as it passes close to the edge of the Sun during a solar total eclipse. The angular displacement of the image due to the Sun's gravity alone is approximately 1.75arcsec. Estimate the angular displacement due to the gravity of the Moon, given that the Sun is approximately 400 times further from the Earth than the Moon. [5]
[$G = 6.67 \times 10^{-11}\mathrm{m^3s^{-2}kg^{-1}}$; $M_\odot = 1.99 \times 10^{30}\mathrm{kg}$; $R_\odot = 6.96 \times 10^8\mathrm{m}$.]

B2 *Attempt this question.*

Write brief notes on **two** of the following: [13]

(a) dark energy. You should mention both the observational evidence and the problems of interpretation;

(b) the neutrino astrophysics of the Sun and its significance for astrophysics and fundamental particle physics;

(c) primordial nucleosynthesis and its significance for cosmology;

(d) superluminal motions in astrophysical jets.

B3 *Attempt* **either** *this question* **or** *question B4.*

A bounded self-gravitating sphere of gas is in hydrostatic equilibrium, such that the inward force of gravity is everywhere balanced by the outward pressure gradients in the gas.

Show that a lower limit to the central pressure is given by

$$p_c = \frac{GM^2}{8\pi R^4} + p_{\mathrm{ext}} ,$$

where M and R are the mass and radius of the star and p_{ext} is the external pressure. [6]

What is the virial theorem for stars? Use the theorem to work out a lower limit to the central temperature of a $1M_\odot$ star, explaining your assumptions. [6]

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Compare and contrast the physics, internal structures and lifetimes of $1M_\odot$ and $20M_\odot$ main sequence stars, discussing their formation, evolution while on the main sequence, and the endpoints of stellar evolution. [13]
[$G = 6.67 \times 10^{-11}\mathrm{m^3s^{-2}kg^{-1}}$; $k_\mathrm{B} = 1.38 \times 10^{-23}\mathrm{JK^{-1}}$; $M_\odot = 1.99 \times 10^{30}\mathrm{kg}$; $R_\odot = 6.96 \times 10^8\mathrm{m}$; $m_\mathrm{H} = 1.67 \times 10^{-27}\mathrm{kg}$.]

B4 *Attempt* **either** *this question* **or** *question B3.*
The Schwarzschild metric has the form

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{rc^2}\right)\mathrm{d}t^2 - \frac{1}{c^2}\left[\frac{\mathrm{d}r^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right] .$$

Explain clearly the physical meaning of all the quantities appearing in this expression. [4]

Without detailed mathematics, outline the procedure for deriving the energy equation for a point mass m in Schwarzschild space-time

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(r\dot{\phi}^2)\left(1 - \frac{2GM}{rc^2}\right) - \frac{GMm}{r} = \frac{1}{2}mc^2(k^2 - 1) .$$

[4]

Derive the corresponding equation for Galilean space-time and explain carefully the differences between the two equations. [4]

Rewrite the energy equation above in terms of the dimensionless specific angular momentum $\eta = h^2/(r_\mathrm{g}^2c^2)$, where $h = r^2\dot{\phi}$ and $r_\mathrm{g} = 2GM/c^2$ is the Schwarzschild radius. [2]

Use your result to:

(i) show that particles with specific angular momentum $\eta \leq 4$ can fall into the singularity at $r = 0$; [5]

(ii) explain why there is a last circular stable orbit about a black hole, and find its radius. [6]

END OF PAPER

Should be $\frac{1}{2}m(r\dot{\phi})^2$