UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Saturday 30 May 2015 1.30 pm to 3.30 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)
MPHIL IN NUCLEAR ENERGY

PARTICLE & NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *printed sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) The spin-parity (J^P) and excitation energies of the five lowest-energy states of $^{174}_{72}$Hf are

J^P	0^+	2^+	4^+	6^+	8^+
E/keV	0	91	297	608	1009

Show that these states are consistent with being rotational excitations, obtain a value for the moment of inertia of the $^{174}_{72}$Hf nucleus and compare your result to the expectation if $^{174}_{72}$Hf is assumed to be a rigid spherical rotator. [4]

[*A solid sphere of mass m and radius R has moment of inertia* $I = \frac{2}{5}mR^2$.]

(b) The light spin-half quarks (d, u and s) form eighteen $L = 0$ ground state baryons with a multiplet structure of eight $J^P = \frac{1}{2}^+$ and ten $J^P = \frac{3}{2}^+$ states. If quarks were spin-zero particles, and all other considerations are unchanged, by considering the required symmetry of the flavour part of the wavefunction explain why only a single $L = 0$ baryon would be observed. [4]

(c) Neutrinos can undergo charged-current interactions with electrons,

$$\nu_\ell + e^- \to \nu_e + \ell^- ,$$

where $\ell = \{e, \mu, \tau\}$. Calculate the minimum laboratory frame neutrino energy for this reaction to be kinematically possible for the three different neutrino flavour states. [4]

[*In natural units, the masses of the charged leptons are:* $m_e = 0.511\,MeV$, $m_\mu = 106\,MeV$ *and* $m_\tau = 1777\,MeV$. *Neglect the masses of the neutrinos and assume the initial-state electron is at rest.*]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following:

(a) the evidence for quarks and colour; [13]

(b) nuclear fission and fusion;

(c) the deuteron and its implications for the form of the strong nuclear force.

3 *Attempt* **either** *this question* **or** *question four.*

The Higgs boson is a spin-zero scalar particle with mass $m_{\mathrm{H}} \approx 125$ GeV. It has been proposed that the properties of the Higgs can be studied in a muon collider through the resonant reaction $\mu^+\mu^- \to \mathrm{H} \to \mathrm{b\bar{b}}$, with a cross section described by the Breit-Wigner formula

$$\sigma(E) = \left(\frac{\pi g}{p^2}\right)\left(\frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \frac{1}{4}\Gamma^2}\right).$$

Define the meaning of the symbols in this formula and explain the origin of the $1/p^2$ dependence. [6]

Write down the values of g for $\mu^+\mu^- \to \mathrm{Z} \to \tau^+\tau^-$ and $\mu^+\mu^- \to \mathrm{H} \to \tau^+\tau^-$, explaining the physical significance. [2]

Show that the full-width at half-maximum height of the resonance is Γ. [2]

The coupling of the Higgs boson to the fundamental fermions is proportional to the fermion mass, m_f. The main decay modes of the Higgs boson are: $\mathrm{H} \to \mathrm{b\bar{b}}$, $\mathrm{H} \to \mathrm{c\bar{c}}$, $\mathrm{H} \to \tau^+\tau^-$, $\mathrm{H} \to \mathrm{WW}$, $\mathrm{H} \to \mathrm{gg}$, $\mathrm{H} \to \mathrm{ZZ}$ and $\mathrm{H} \to \gamma\gamma$, where g represents a gluon. Draw the lowest-order Feynman diagram for $\mu^+\mu^- \to \mathrm{H} \to \mathrm{b\bar{b}}$ and the lowest-order Feynman diagrams for the Higgs decays to $\mathrm{b\bar{b}}$, gg and $\gamma\gamma$. [4]

In the Standard Model, the sum of the branching ratios of the Higgs to $\tau^+\tau^-$, $\mathrm{c\bar{c}}$ and $\mathrm{b\bar{b}}$ is 67 % and the partial decay width of the Higgs boson to $\tau^+\tau^-$ is

$$\Gamma(\mathrm{H} \to \tau^+\tau^-) = \frac{m_\tau^2 m_{\mathrm{H}}}{8\pi v^2},$$

where v is the vacuum expectation of the Higgs field. Stating any assumptions, answer the following questions:

(a) find the ratio of $\Gamma(\mathrm{H} \to \tau^+\tau^-) : \Gamma(\mathrm{H} \to \mathrm{c\bar{c}}) : \Gamma(\mathrm{H} \to \mathrm{b\bar{b}})$;

(b) calculate the cross-section for $\mu^+\mu^- \to \mathrm{H} \to \mathrm{b\bar{b}}$ at the peak of the resonance, expressing your answer in both S.I. and natural units;

(c) explain why a $\mu^+\mu^-$ collider is being considered rather than resonant production in e^+e^- collisions;

(d) explain why the branching ratio for $\mathrm{H} \to \mathrm{WW}$ is not zero, despite the fact that $m_{\mathrm{H}} < 2m_{\mathrm{W}}$. [11]

[*Assume that the masses of the fermions are m_e = 0.5 MeV, m_μ = 106 MeV, m_τ = 1.8 GeV, m_c =1.5 GeV and m_b=5 GeV. Note that $\hbar c$ = 197 MeV fm and 1 barn = 10^{-28} m^2.*]

4 *Attempt* **either** *this question* **or** *question three.*

The diagram on the next page shows some of the low-lying energy levels of the isobars ${}^{17}_{7}$N, ${}^{17}_{8}$O, ${}^{17}_{9}$F and ${}^{17}_{10}$Ne, with spins and parities indicated. The energies of the levels (in MeV), relative to the respective ground states are indicated. The figures in brackets show the respective ground state *nuclear* energies relative to the ground state of ${}^{17}_{8}$O.

i) Without detailed derivations, give a brief account of nuclear beta decay. You should include a description of the selection rules, Sargent's law and its origin and the requirements in terms of the *nuclear masses* that determine whether β decays are energetically allowed. [7]

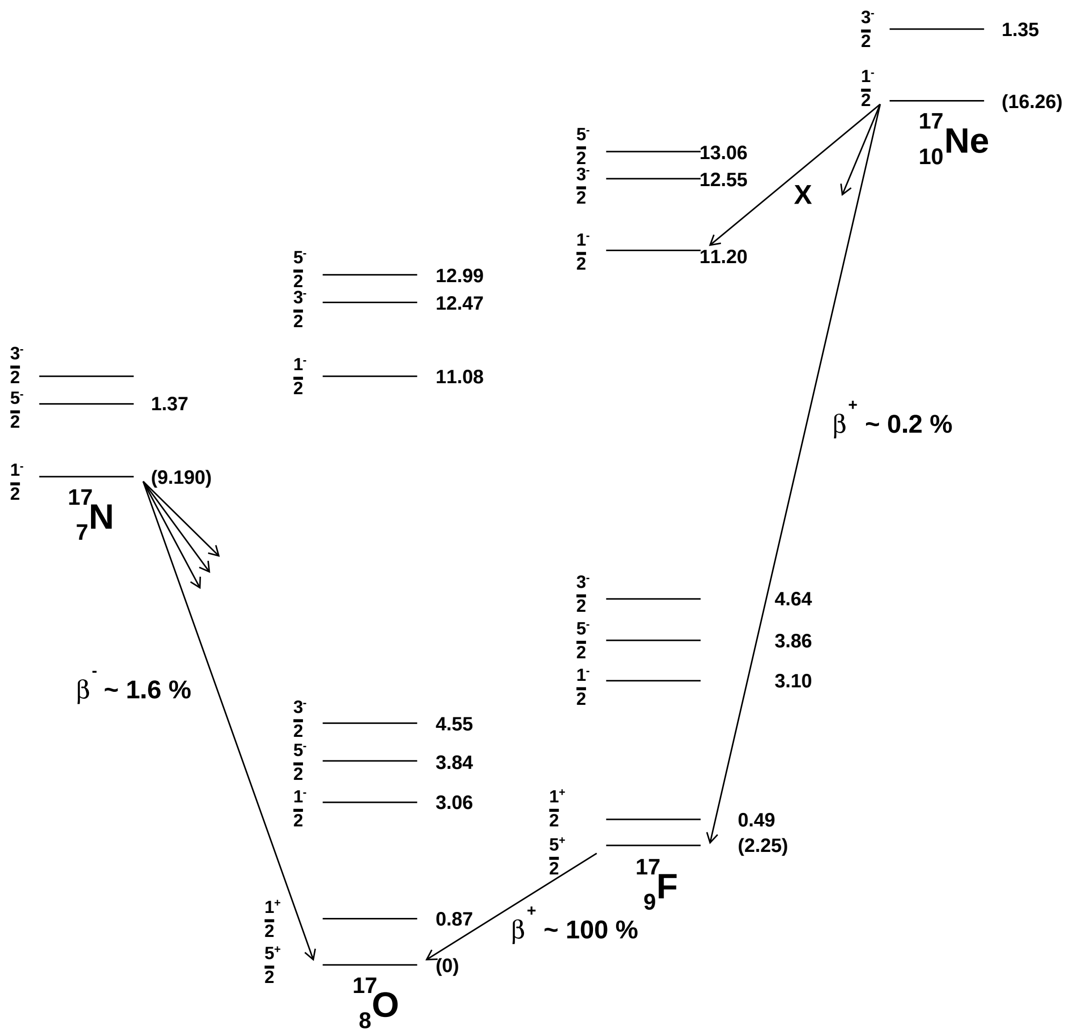
ii) Treating the nucleus as a sphere of uniform charge density, use the mass difference between the ground states of ^{17}O and ^{17}F to provide an estimate of the charge radius of the $A = 17$ isobars. Take the masses of the proton and neutron to be $m_\mathrm{p} = 938.272\,\mathrm{MeV}/c^2$ and $m_\mathrm{n} = 939.566\,\mathrm{MeV}/c^2$. [6]

The lifetimes of the ground states of ^{17}N, ^{17}F and ^{17}Ne are, respectively, 6.1 s, 95 s and 0.15 s. with respective branching fractions to the *ground states* of the daughter nuclei of 1.6 %, 100 % and 0.2 , as indicated on the diagram.

iii) Classify the three labelled ground-state β transitions shown in the diagram and calculate their partial decay widths. [4]

iv) From the ratios of the partial decay widths, $\Gamma(\mathrm{N} \to \mathrm{O})/\Gamma(\mathrm{Ne} \to \mathrm{F})$ and $\Gamma(\mathrm{N} \to \mathrm{O})/\Gamma(\mathrm{F} \to \mathrm{O})$, for the labelled β transitions, obtain estimates for the corresponding ratio of the squared matrix elements and comment on the results. [4]

v) Consider the β^+ transition, marked X in the diagram, from the ground state of ${}^{17}_{10}$Ne to its corresponding analogue state in ${}^{17}_{9}$F. Classify this decay and estimate its branching fraction, clearly explaining your reasoning. [4]



The low-lying states of $^{17}_{7}$N, $^{17}_{8}$O, $^{17}_{9}$F and $^{17}_{10}$Ne. The energy levels (in MeV) are given with respect to the respective ground state. The figures in brackets give the ground state nuclear energy levels relative to the ground state of $^{17}_{8}$O. The two lowest-lying states of both O and F have $J^P = \frac{5}{2}^+$ and $\frac{1}{2}^+$, all other states shown have negative parity.

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