UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 30 May 2018 13.30 pm to 15.30 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

PARTICLE AND NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are question* **A1** *from Section A and* **two** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Metric graph paper	Approved calculator allowed
Rough workpad	
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

SECTION A

Answers should be concise; relevant formulae may be assumed without proof.

A1 *Attempt* **all** *parts of this question.*

(a) In standard notation, the Breit-Wigner formula for the cross section for the production and subsequent decay of a resonant state R is

$$\sigma(a + b \to R \to X) = \frac{\pi g}{p_i^2} \frac{\Gamma_{ab}\Gamma_X}{(E - E_0)^2 + \Gamma^2/4} \,,$$

where $g = (2J_R + 1)/(2J_a + 1)(2J_b + 1)$. The process $\mathrm{e^+e^-} \to \mathrm{Z^0} \to \mathrm{e^+e^-}$ is observed to have a maximum cross section of 2.0 nb occuring at a centre of mass energy $\sqrt{s} = 91.2\,\mathrm{GeV}$. Estimate the branching ratio for the decay $\mathrm{Z^0} \to \mathrm{e^+e^-}$. [4]
[$1\,\mathrm{b} = 10^{-28}\,\mathrm{m}^2$, $\hbar c = 197.5\,\mathrm{MeV}$.]

(b) List all possible multipoles for the γ-decay of an excited nuclear state of spin-parity $(5/2)^+$ to a lower energy state which also has spin-parity $(5/2)^+$. Comment on the likely relative contribution of each multipole to the overall decay rate. [4]

(c) At leading order, the process $\mathrm{e^+e^-} \to \mathrm{W^+W^-}$ is described by four Feynman diagrams, one of which involves the Higgs boson. Draw these diagrams, and explain why the Higgs diagram gives a negligible contribution to the cross section. [4]

SECTION B
Attempt **two** *questions from this section*

B2 In standard notation, the Semi-Empirical Mass Formula (SEMF) for nuclear masses is

$$M(A,Z) = Zm_{\rm p} + (A-Z)m_{\rm n} - a_{\rm V}A + a_{\rm S}A^{2/3} + a_{\rm C}\frac{Z^2}{A^{1/3}} + a_{\rm A}\frac{(A-2Z)^2}{A} - \delta(A)\,,$$

where the pairing term $\delta(A)$ is given by:

$$\delta(A) = \begin{cases} +a_{\rm P}A^{-3/4} & \text{for even-even nuclei}\,, \\ -a_{\rm P}A^{-3/4} & \text{for odd-odd nuclei}\,, \\ 0 & \text{for even-odd nuclei}\,. \end{cases}$$

Nuclear masses are well described with $a_{\rm V} = 15.8\,\text{MeV}$, $a_{\rm S} = 18.0\,\text{MeV}$, $a_{\rm C} = 0.72\,\text{MeV}$, $a_{\rm A} = 23.5\,\text{MeV}$ and $a_{\rm P} = 33.5\,\text{MeV}$.

Consider a spontaneous fission process of the form $(A,Z) \to (A_1,Z_1) + (A_2,Z_2)$ with $A_1 = yA$, $Z_1 = yZ$, and $0 < y < 1$. Using the SEMF, show that the energy release for this process is expected to be a maximum for the case of symmetric fission. Ignoring any pairing terms, estimate the value of Z^2/A above which fission is energetically possible. [5]

Explain why, in practice, spontaneous fission is only observed for nuclides with values of Z^2/A much larger than this estimate. Sketch the distribution of y values which is typically observed for the fission fragments. [4]

Using the SEMF, estimate the excitation energies of the $^{236}\text{U}^*$ and $^{239}\text{U}^*$ nuclear states formed when ^{235}U and ^{238}U nuclei, respectively, capture a neutron of negligible kinetic energy. Identify the term in the SEMF which is primarily responsible for the difference in the predicted excitation energies for these two cases. [*Uranium has atomic number* $Z = 92$.] [6]

The observed excitation energies following low energy (thermal) neutron capture by ^{235}U and ^{238}U are approximately 6.5 MeV and 4.8 MeV, respectively. The fission activation energies (the potential barrier height which must be overcome for fission to occur) for ^{235}U and ^{238}U are approximately 6.2 MeV and 6.6 MeV, respectively. Explain why thermal neutrons can induce rapid fission of ^{235}U but not of ^{238}U. Discuss the implications of the energy dependence of the cross sections for neutron induced fission for the design of nuclear reactors which use uranium as a fuel. [4]

B3 The table at the bottom of the page lists the ground state ($\ell = 0$) baryons composed of u, d and s quarks, together with their spins and masses.

Explain why the existence of spin $3/2$ baryons with flavour content uuu, ddd or sss provides evidence for the existence of the colour degree of freedom. Explain also why the uu quark pair in a uus baryon must be in a state with total spin $S_{\text{uu}} = 1$. [4]

In the quark model of hadrons, the baryon mass receives a contribution $A'(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)/(m_i m_j)$ from each pair of quarks within the baryon, where $\boldsymbol{S}_i$ is the spin operator for the i'th quark, m_i is its mass, and A' is a constant. What is the physical origin of such terms? [1]

By squaring total spin operators of the form $\boldsymbol{S}_1 + \boldsymbol{S}_2$ and $\boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3$, or otherwise, show that the quark model predicts the mass of the uus spin $1/2$ baryon to be

$$M_{\text{uus}} = 2m_{\text{u}} + m_{\text{s}} + \frac{A'}{4}\left(\frac{1}{m_{\text{u}}^2} - \frac{4}{m_{\text{u}} m_{\text{s}}}\right),$$

and obtain the corresponding expression for the mass of the spin $3/2$ uus baryon. Hence obtain the quark model prediction for the ratio of the mass differences $M(\Sigma^*) - M(\Sigma)$ and $M(\Xi^*) - M(\Xi)$, and compare with observations in the table below. [6]

The Σ^0 baryon decays as $\Sigma^0 \to \Lambda^0 + \gamma$ with a branching ratio of 100%. Draw a leading order Feynman diagram for this decay, and compute the energy of the emitted photon in the Σ^0 rest frame. [4]

State which of the baryons in the table below decay dominantly via the strong interaction, justifying your answer. Draw an example of a Feynman diagram for such a decay. [*The pion mass is about* 140 MeV*; the mass of the lightest meson which contains a strange quark is about* 500 MeV.] [4]

Baryon name	Quark content	Spin	Mass / MeV
n, p	udd, uud	1/2	940, 938
Λ^0	uds	1/2	1116
$\Sigma^-, \Sigma^0, \Sigma^+$	dds, uds, uus	1/2	1197, 1193, 1189
Ξ^-, Ξ^0	dss, uss	1/2	1322, 1315
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	3/2	1232, 1232, 1232, 1232
$\Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}$	dds, uds, uus	3/2	1387, 1385, 1383
Ξ^{*-}, Ξ^{*0}	dss, uss	3/2	1535, 1532
Ω^-	sss	3/2	1673

B4 The magnitudes $|V_{qq'}|$ of the elements of the CKM matrix are approximately

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.974 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}.$$

The most precise determination of $|V_{ud}|$ is obtained from measurements of $0^+ \to 0^+$ nuclear β-decays. Classify β-decays of this type, and state why and how their decay rate depends on V_{ud}. [4]

The D_s^+ meson is observed to decay to $K^+\pi^0$, $K^+\overline{K^0}$ and $\phi\pi^+$ with branching ratios of approximately 6.3×10^{-4}, 2.9×10^{-2} and 4.5×10^{-2}, respectively. Draw leading order Feynman diagrams for each of these decays, and quantify the degree to which the differences in branching ratio can be understood in terms of the CKM matrix elements involved in each case. [*The* D_s^+*,* K^+ *and* $\overline{K^0}$ *mesons have quark content* $c\bar{s}$*,* $u\bar{s}$ *and* $s\bar{d}$*, respectively.*] [5]

The D_s^+, π, and K mesons have spin-parity 0^-; the ϕ meson has spin-parity 1^-. Explain why this information alone is sufficient to establish that the decays of D_s^+ to $K^+\pi^0$ and $K^+\overline{K^0}$ are weak, while its decay to $\phi\pi^+$ is not necessarily weak. [3]

Estimate the branching ratio for the decay $W^- \to e^-\bar{\nu}_e$, and estimate the fraction of W^- decays that contain a b quark in the final state. [3]

The muon has a mass of about 106 MeV, a lifetime of about 2.2 μs, and decays as $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ with a branching ratio of 100%. The top quark has a mass of about 175 GeV. Assuming Sargent's Rule applies to muon and top quark decays, obtain an order of magnitude estimate of the top quark lifetime, and comment on the implications of the estimated value. [4]

END OF PAPER