UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 26 May 2016 9.00 am to 11.00 am

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)
MPHIL IN NUCLEAR ENERGY

PARTICLE & NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *printed sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) In the nuclear shell model, the lowest-lying nucleon energy levels are, in order of increasing energy:

$1s_{\frac{1}{2}}$, $1p_{\frac{3}{2}}$, $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, ...

Predict the spin-parity (J^P) of the ground states of the following nuclides: ${}^{17}_{8}\mathrm{O}$, ${}^{10}_{5}\mathrm{B}$, ${}^{15}_{7}\mathrm{N}$, ${}^{10}_{4}\mathrm{Be}$. [4]

(b) The masses of light mesons can be modelled as the sum of their constituent quarks' masses and a term corresponding to the colour magnetic spin-spin interaction. In this model, show that the mass of the η meson can be expressed as a linear combination of that of the ρ^0, ϕ, π^0, K^0 and K^{*0} in the following way:

$$m(\eta) = -\frac{1}{3}m(\pi^0) - 2m(\rho^0) + 4m(\mathrm{K}^{*0}) + \frac{4}{3}m(\mathrm{K}^0) - 2m(\phi)\ . \qquad [4]$$

[*You can use the information below and use the approximation that the down quark and the up quark have equal mass.*

particle	$q\bar{q}$	J^P
π^0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	0^-
ρ^0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	1^-
η	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	0^-
ϕ	$s\bar{s}$	1^-
K^0	$d\bar{s}$	0^-
K^{*0}	$d\bar{s}$	1^-

]

(c) Neutrinos produced in the thermonuclear reactions in the sun can be detected through elastic scattering with electrons in matter. Using energy and momentum conservation, show that the kinetic energy of the scattered electron satisfies

$$T_e = \frac{2m_e E_\nu^2 \cos^2\theta}{(E_\nu + m_e)^2 - E_\nu^2 \cos^2\theta},$$

where θ is the angle between the scattered electron and the incoming neutrino in the lab frame. Solar neutrinos from the reaction ${}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$ have an energy of 0.86 MeV. Determine the maximum kinetic energy that can be transferred to the scattered electron in an elastic collision. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following:

(a) muonic atoms; [13]

(b) the Fermi gas model of the nucleus;

(c) the incompleteness of the Standard Model and theories that go beyond the Standard Model.

(TURN OVER

3 *Attempt* **either** *this question* **or** *question four.*

(a) For the following decays, state which is the dominant force and draw the relevant Feynman diagram:

i) $J/\psi \to \mu^+\mu^-$ [2]

ii) $W^+ \to e^+\nu_e$ [2]

iii) $\Delta^{++} \to p\pi^+$ [2]

iv) $\tau^- \to \nu_\tau\pi^-$ [2]

(b) Derive the momentum spectrum of the electron in nuclear beta decay and show that the total decay rate is proportional to the fifth power of the released energy. [6]

(c) Draw the Feynman diagram of the dominant decay of the muon. The decay rate of the muon, according to Fermi theory, is given by

$$\Gamma_\mu = \frac{G_F^2}{192\pi^3} m_\mu^5.$$

Explain why Fermi theory can be applied to muon decay, and calculate the muon lifetime in seconds. [4]

(d) Use Fermi theory also to calculate the lifetime of the τ lepton, the c quark and the b quark. [7]

[*In natural units, the masses of the charged leptons are:* $m_e = 0.511\,\mathrm{MeV}$, $m_\mu = 106\,\mathrm{MeV}$ and $m_\tau = 1777\,\mathrm{MeV}$.
For the masses of the heavy quarks, you can use $m_b = 5.0\,\mathrm{GeV}$ and $m_c = 1.5\,\mathrm{GeV}$.
For the elements of the CMK matrix, $|V_{ud}| = |V_{cs}| = 0.97$, $|V_{us}| = |V_{cd}| = 0.22$, $|V_{cb}| = 0.040$.
Furthermore, you can use $G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $\hbar = 6.582 \times 10^{-22}\,\mathrm{MeV\,s}$.]

4 *Attempt* **either** *this question* **or** *question three.*

(a) The deuteron has spin-parity $J^P = 1^+$. Assuming that the deuteron is dominated by the orbital angular momentum state $\ell = 0$, and noting that no excited states exist, what can be concluded about the nature of the $\mathrm{n-p}$ force and about the existence of nn and pp bound states? [4]

(b) Based on the masses given below, calculate the binding energy B of the deuteron. [1]

(c) The magnetic moment of the deuteron is $0.857\ \mu_{\mathrm{N}}$ and it has a non-zero electric quadrupole moment. What does this imply for the structure of the deuteron? [4]

(d) The deuteron can be modelled by a spherically symmetric square-well potential with depth V_0 and radius b, and can be described by the radial wave equation:

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + V(r)u(r) = Eu(r),$$

where $u(r) = rR(r)$.

Solve the wave equation for this system and find the equation that relates V_0 to b and the binding energy B. [8]

From scattering experiments it is known that $b = 2.1$ fm. Determine the value of V_0 using the approximation that $B \ll V_0$. [4]

(e) A deuteron can be formed by the neutron capture of a proton, in the reaction $\mathrm{np} \rightarrow \mathrm{d}\gamma$. If the neutron and proton are assumed to be at rest, the energy of the photon will be slightly smaller than the binding energy of the deuteron. Explain why, and calculate the difference $B - E_\gamma$. [4]

[*In natural units,* $m_{\mathrm{p}} = 938.272\,\mathrm{MeV}$, $m_{\mathrm{n}} = 939.565\,\mathrm{MeV}$, $m_{\mathrm{d}} = 1875.613\,\mathrm{MeV}$, $\mu_{\mathrm{p}} = 2.793\mu_{\mathrm{N}}$, $\mu_{\mathrm{n}} = -1.913\mu_{\mathrm{N}}$, $\hbar c = 197.3\,\mathrm{MeV\,fm}$.]

END OF PAPER