UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Monday 28 May 2018 1:30 pm to 3:30 pm

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

QUANTUM CONDENSED MATTER PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1** *and* **two** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

draft: 17th May 2018

QUANTUM CONDENSED MATTER PHYSICS

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

1 *Attempt* **all** *parts of this question.*

(a) Consider a linear chain consisting of atoms all of the same mass m and alternating elastic coupling constants K and K'. Write the equations of motion and sketch the dispersion of the acoustic and optical phonon branch. What is the $q \to 0$ limit of the optical phonon frequency to leading order when $K \gg K'$? [4]

(b) A MOSFET metal/insulator/p-type semiconductor structure is set up so that it just begins to form an n-type inversion layer. Sketch the valence and conduction energy bands, clearly indicating the inversion layer and depletion region. Do the chemical potentials in the metal and in the semiconductor have to be the same? Explain your answer. [4]

(c) Using the expressions for the Hall coefficient R_{H} and conductivity σ in terms of carrier density n, carrier charge q, and mobility μ (as appropriate), find a relation between the Hall field E_{H} and longitudinal field E parallel to the current, in an n-doped semiconductor under a magnetic field B perpendicular to E. What value of B (in tesla) is needed for the Hall field to reach 1.5% of the longitudinal field, if the semiconductor has mobility $\mu = 900\ \mathrm{cm^2V^{-1}s^{-1}}$? [4]

SECTION B
Attempt **two** *questions from this section*

B2 Sketch the bending of the conduction and valence bands, and of the electron and hole concentrations, across an unbiased p-n junction. Indicate clearly the majority and minority carriers on either side of the junction, and sketch the behaviour of the carrier density across the junction. [5]

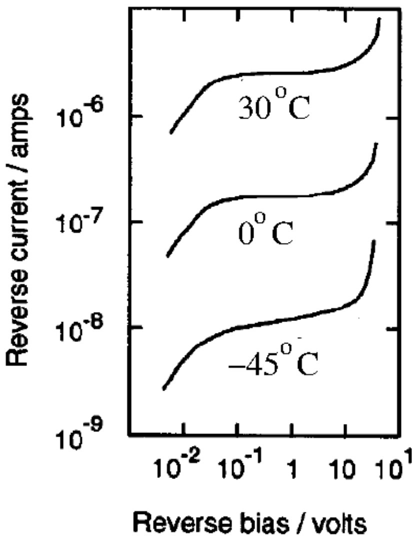
Discuss briefly how such junctions are used in light emitting diodes. [3]

Discuss the origin of the two terms in the diode equation (or diode IV characteristic) in reverse bias,

$$I = I_{\text{sat}} \left[1 - \exp\left(\frac{-eV}{k_B T} \right) \right] ,$$

where e is the hole charge, V is the applied voltage difference (bias voltage), k_B is the Boltzmann constant, and T is the temperature. What determines the value of I_{sat}? [5]

The IV characteristic measured on a germanium p-n diode under reverse bias is shown in the figure below. Explain how you can derive the band gap of a material from such curves, and estimate its value for germanium using the data in the figure. [6]



B3 Describe briefly the concept of linear combination of atomic orbitals (LCAO) applied to the case of a diatomic molecule with only one orbital per atom (as discussed in lectures). You should include explicitly the eigenvalue equation. [5]

Consider modelling a three dimensional transition metal with two energy bands: one corresponding to states $|\phi_{\boldsymbol{k}}\rangle$ with a broad, approximately parabolic dispersion $E_1(\boldsymbol{k}) = \langle\phi_{\boldsymbol{k}}|\widehat{H}|\phi_{\boldsymbol{k}}\rangle = \hbar^2 k^2/2m$; and the other corresponding to states $|\chi_{\boldsymbol{k}}\rangle$ with negligible dispersion $E_2(\boldsymbol{k}) = \langle\chi_{\boldsymbol{k}}|\widehat{H}|\chi_{\boldsymbol{k}}\rangle = E_d > 0$. Here $\widehat{H}$ is the electronic Hamiltonian, m is the mass of a free electron, and $\boldsymbol{k}$ is the electronic state wavevector. Interactions between the two bands, $\langle\phi_{\boldsymbol{k}}|\widehat{H}|\chi_{\boldsymbol{k}}\rangle = V$, can be assumed to be independent of $\boldsymbol{k}$. Construct approximate eigenstates $|\psi_{\boldsymbol{k}}\rangle$ of $\widehat{H}$ as linear superpositions of $|\phi_{\boldsymbol{k}}\rangle$ and $|\chi_{\boldsymbol{k}}\rangle$, and show that the dispersion $E(\boldsymbol{k})$ of the hybridised eigenstates can be written as

$$E(\boldsymbol{k}) = \frac{E_1(\boldsymbol{k}) + E_d}{2} \pm \left[\left(\frac{E_1(\boldsymbol{k}) - E_d}{2}\right)^2 + |V|^2\right]^{1/2} .$$ [5]

Sketch the band structure for $V = 0$ and for $0 < |V| \ll E_d$, and compare the asymptotic behaviour in the two cases for $k \to 0$ and $k \to \infty$. [*Ignore potential intersections with the Brillouin zone boundary.*] [5]

Assume that, at some low temperature, a metal described by the band structure above has equal occupancy of both bands for $V = 0$. Obtain the increase in the magnitude of the Fermi wave vector k_{F} when V is non-zero. Show that the ratio of the Fermi velocity v_{F} of the hybridised band to that of the unhybridised (broad) band $v_{\mathrm{F}}^{(1)}$ can be written as

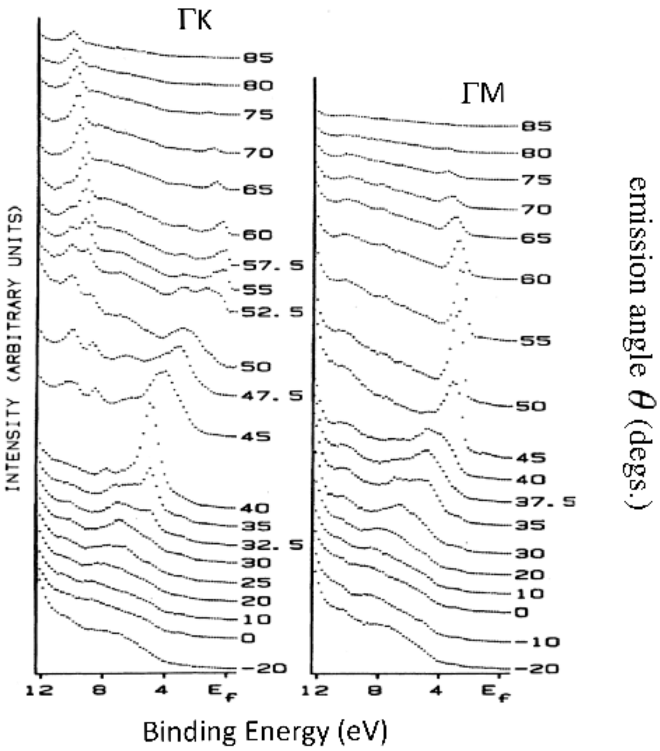
$$v_{\mathrm{F}}/v_{\mathrm{F}}^{(1)} = \frac{2^{1/3}}{(2^{2/3}-1)^2}\frac{|V|^2}{E_d^2} ,$$

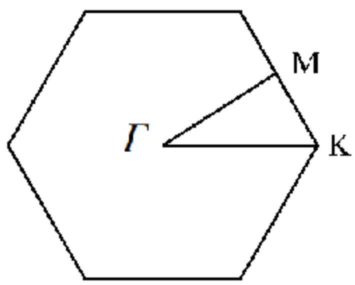
in the limit $|V| \ll E_d$. [4]

B4 Describe how angle-resolved photoemission measurements work and how they can be used to obtain the electron spectral function of a material. You should include: a labelled diagram of the relevant energies; the kinetic energy of the photoemitted electron E_{kin}; the incident photon energy $\hbar\omega$; the work function of the material ϕ; the Fermi energy E_{F}; and the initial and final photoemitted electron energies E_{i} and E_{f}. [9]

Give examples of two other experimental measurements that provide evidence for the electronic band structure in crystalline solids, and discuss briefly how they work. [4]

The figure below shows the intensity of electrons photoemitted from a sheet of graphite (whose band structure has dispersion only in the plane parallel to the sheet). The emission angles were chosen towards the K and M points in the Brillouin zone, as illustrated in the figure. The spectra were obtained using incident photons of energy 19.5 eV. Energies are measured with respect to the work function, which lies 3.8 eV below the ground state vacuum.





Sketch qualitatively the variation of the energy of the highest valence bands as a function of $k_{f\|}$ for the direction Γ K and Γ M in the Brillouin zone. [2]

Using this information, estimate the value of θ required for the highest valence band to reach the K point at the Brillouin zone boundary. [*The lattice parameter of graphite is* $a = 0.246$ nm *and the value of* $k_{f\|}$ *at the Brillouin zone boundary corners is* $4\pi/3a$.] [4]

END OF PAPER