UNIVERSITY OF CAMBRIDGE

Answers: ADVANCED QUANTUM PHYSICS 2014

1 (a) Classical limits when

$$\begin{aligned} E_0 &= \frac{1}{2}\hbar\omega = \frac{1}{2}m\omega^2 x^2 \\ x &= \pm\sqrt{\frac{\hbar}{m\omega}} \end{aligned}$$

Probability, P, that particle is outside the classical limits is [1]

$$P = \frac{\int_{\sqrt{\frac{\hbar}{m\omega}}}^{\infty} \exp\{-\frac{m\omega}{\hbar}x^2\}dx}{\int_0^{\infty} \exp\{-\frac{m\omega}{\hbar}x^2\}dx}$$

Substitute $y = \sqrt{\frac{m\omega}{\hbar}}x$, $dy = \sqrt{\frac{m\omega}{\hbar}}dx$, and $(x = 0, y = 0)$, $(x = \infty, y = \infty)$ and $x = \sqrt{\frac{\hbar}{m\omega}}, y = 1$ to obtain [1]

$$\begin{aligned} P &= \frac{\int_1^{\infty} \exp\{-y^2\}dy}{\int_0^{\infty} \exp\{-y^2\}dy} \\ &= 1 - 0.843 \\ &= 0.16 \end{aligned}$$

[2]

(b) From first-order perturbation theory

$$c_{i\to f} = -\frac{\imath}{\hbar}\int_0^t \langle\psi_1|\widehat{H}(t')|\psi_2\rangle \exp\{\imath\frac{(E_1 - E_2)}{\hbar}t'\}dt'$$

[1]

Represent 2 states by matrix $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$.

Then,

$$\begin{aligned} c_{i\to f} &= -\frac{\imath}{\hbar}\int_0^t \begin{pmatrix} \psi_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & U \\ U & 0 \end{pmatrix}\delta(t')\begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}\exp\{\imath\frac{(E_1 - E_2)}{\hbar}t'\}dt' \\ &= -\frac{\imath}{\hbar}\int_0^t \begin{pmatrix} \psi_1 & 0 \end{pmatrix}\begin{pmatrix} U\psi_2\delta(t') \\ 0 \end{pmatrix}\exp\{\imath\frac{(E_1 - E_2)}{\hbar}t'\}dt' \\ &= -\frac{\imath}{\hbar}U\psi_1^*\psi_2\int_0^t \delta(t')\exp\{\imath\frac{(E_1 - E_2)}{\hbar}t'\}dt' \\ &= -\frac{\imath}{\hbar}U\psi_1^*\psi_2\exp\{\imath\frac{(E_1 - E_2)}{\hbar}t\} \end{aligned}$$

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Therefore, probability $P_{i\to f}$ is given by [2]

$$\begin{aligned} P_{i\to f} &= |c_{i\to f}|^2 \\ &= \left(-\frac{\imath}{\hbar}U\psi_1^*\psi_2\exp\{\imath\frac{(E_1-E_2)}{\hbar}t\}\right)\left(\frac{\imath}{\hbar}U\psi_1\psi_2^*\exp\{-\imath\frac{(E_1-E_2)}{\hbar}t\}\right) \\ &= \left(\frac{U}{\hbar}\right)^2 \end{aligned}$$

[1]

(c) 2 electrons in p^2 configuration.

$s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \Rightarrow S = 0, 1$

$l_1 = 1, l_2 = 1 \Rightarrow L = 0, 1, 2$

Using notation ${}^{2S+1}L_J$, all possible states are ${}^1S_0$, ${}^1P_1$, ${}^1D_2$, ${}^3S_1$, ${}^3P_{0,1,2}$, ${}^3D_{1,2,3}$. However, ${}^1P_1$, ${}^3S_1$, ${}^3D_{1,2,3}$ are not allowed due to identicle particle symmetry.

Therefore allowed states are ${}^1S_0$, ${}^1D_2$, ${}^3P_{0,1,2}$.

Hund's rules states that the level with the highest S and then the lowest J, for a sub-shell less than half full, has the lowest energy. Therefore, the ground state is ${}^3P_0$.

2 (a) *Bookwork:* **Postulates of Quantum Mechanics**

- The state of a QM system is specified by the wavefunction $\Psi(\boldsymbol{r},t)$. The wavefunction must be single-valued, continuous and finite. [1]
- The probability of finding a particle in the volume element $d^3\boldsymbol{r}$ at time t is $|\Psi(\boldsymbol{r},t)|^2 d^3\boldsymbol{r}$. For a single particle, the net probability of finding it at some point in space must be unity leading to the normlization condition $\int_{-\infty}^{+\infty}|\Psi(\boldsymbol{r},t)|^2 d^3\boldsymbol{r} = 1$. [2]
- To every observable in classical mechanics there corresponds a linear, Hermitian operator in QM. If the system is in an eignstate of the linear, Hermitian operator $\widehat{A}$ with eigenvalue a, then any measuerment of the quantity A will yield a. [2]
- If a system is in a state described by a normalised wavefunction Ψ, then the average value of the observable corresponding to the operator $\widehat{A}$ is $\langle A\rangle = \int_{-\infty}^{+\infty}\Psi^*\widehat{A}\Psi d^3\boldsymbol{r}$. A linear, Hermitian operator leads to real expectation values. [2]
- An arbitrary state, describing the same system, can be expanded in the complete set of eigenfunctions of the operator $\widehat{A}$ ($\widehat{A}\Psi_i = a_i\Psi_i$) as $\chi = \sum_i^n c_i\Psi_i$, where n may go to infinity. In this case, the probability of obtaining the result a_i from the measurement of $\widehat{A}$ is $P(a_i) = |\langle\Psi_i|\chi\rangle|^2 = |c_i|^2$ and the expectation value is $\langle A\rangle = \sum_i a_i|c_i|^2$. [2]

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- A measurement of the state χ, which leads to the eigenvalue a_i, causes the wavefunction to "collapse" into the corresponding eigenstate Ψ_i. [2]
- The wavefunction or state function os a system evolves in time according to the time-dependent SE $i\hbar\frac{\partial\Psi}{\partial t} = \widehat{H}\Psi(\boldsymbol{r},t)$, where $\widehat{H}$ is the Hamiltonian. If χ is an eigenstate of $\widehat{H}$, it follows that $\Psi(\boldsymbol{r},t) = \Psi(\boldsymbol{r},0)\exp -iEt/\hbar$. [2]

(b) *Bookwork:* **Central field approximation and the Hartree method**

- Hamiltonian for a multi-electron atom can be written as

$$\widehat{H} = \sum_i \left[-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_i}\right] + \sum_{i<j}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_{ij}}$$

  where the last term represents the Coulomb interaction between constituent electrons. [2]
- The central-field approximation is based upon the observation that the electron interaction term contains a large central (spherically symmetric) component arising from the "core electrons". A closed shell has an electron density distributions that is isotropic. [1]
- Develop a perturbative scheme using $\widehat{H} = \widehat{H}_0 + \widehat{H}_1$, where

$$\widehat{H}_0 = \sum_i \left[-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_i} + U_i(r_i)\right]$$

  and

$$\widehat{H}_1 = \sum_{i<j}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_{ij}} - \sum_i U_i(r_i).$$

  The one-electron potential $U_i(r_i)$, which is assumed central incorporates the "average" effect of the other electrons. $\widehat{H}_0$ is seperable into a sum of terms for each electron, so that the overall wavefunction can be factorized into components for each electron.
  Basic idea is to solve the SE for $\widehat{H}_0$ and then to treat $\widehat{H}_1$ as a perturbation. [2]
- The potentials $U_i(r_i)$ can be estimated using the "self-consistent field method"; a variational approach due to Hartree.
- If electrons are considered independent, the wavefunction can be factorized

$$\Psi(\{\boldsymbol{r}_i\}) = \Psi_{i1}(\boldsymbol{r}_1)\Psi_{i2}(\boldsymbol{r}_2)\cdots\Psi_{iN}(\boldsymbol{r}_N)$$

  with $i_k \equiv (n\ell m_\ell m_s)_k$. [1]
- Using this wavefunction as a trial state, the variational energy is given by

$$E = \langle\Psi|\widehat{H}|\Psi\rangle = \sum_i\int d^3r\psi_i^*\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}\right)\psi_i + \frac{1}{4\pi\epsilon_0}\sum_{i<j}\int d^3r\int d^3r'\psi_i^*(\boldsymbol{r})\psi_j^*(\boldsymbol{r}')\frac{}{|\boldsymbol{r}}$$

[2]

•Variational principle - vary $E[\psi_i]$ wrt complex wavefunction ψ_i (normalised). Normalization can be imposed using Lagrange multipliers. Following the variation, get the Hartree equations:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}\right)\psi_i + \frac{1}{4\pi\epsilon_0}\sum_{j\neq i}\int d^3r'|\psi_j(\boldsymbol{r}')|^2\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}\psi_i(\boldsymbol{r}) = \epsilon_i\psi_i(\boldsymbol{r}).$$

The 3rd term represents the electrostatic potential experienced by the individual electrons. [2]

•The set of possible trial functions that minimizes the energy are determined by the effective potential

$$U_i(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\sum_{j\neq i}\int d^3r'|\psi_j(\boldsymbol{r}')|^2\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

[1]

•One electron energies are given by (multiply Hartree equations by $\psi_i^*(\boldsymbol{r})$ and integrate):

$$\epsilon_i = \int d^3r\psi_i^*\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}\right)\psi_i + \frac{1}{4\pi\epsilon_0}\sum_{j\neq i}\int d^3r'd^3r|\psi_j(\boldsymbol{r}')|^2\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}|\psi_i(\boldsymbol{r})|^2.$$

and the variational energy by

$$E = \sum_i \epsilon_i - \frac{1}{4\pi\epsilon_0}\sum_{i<j}\int d^3r'd^3r|\psi_j(\boldsymbol{r}')|^2\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}|\psi_i(\boldsymbol{r})|^2.$$

[2]

(c) *Bookwork:* **Properties and operation of lasers**

Laser - light amplification by stimulated emission of radiation.

Properties:

•Monochromaticity (one atomic transition, spectral line width,Q of the cavity)

•Coherence, including discussion of spatial (irregularities in optical phase) and temporal coherence (time duration over which beam is well-defined). Coherence length for various light sources.

•Brightness (high power per unit area, energy in narrow spectrum of atomic transition, example)

•Ultra-short pulse generation (time duration of pulses, mode-locking, examples)

[5]

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Operating principles (diagram,pumping energy, positive optical feedback, resonant cavity, mirrors, losses: useful(output coupling) and useless(absorption, scattering, refelctivity)) [4]

Gain mechanism (population inversion, rate equations and laser threshold condition) [4]

3 *Bookwork:*
The spin raising and lowering operators are $\widehat{\sigma}_{\pm} = \widehat{\sigma}_x \pm \imath\widehat{\sigma}_y$. Therefore,

$$\begin{aligned}\widehat{\sigma}_+ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \imath \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \\ \widehat{\sigma}_- &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \imath \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}\end{aligned}$$

[4]

$$\begin{aligned}\widehat{\sigma}_x &= \frac{1}{2}\left(\widehat{\sigma}_+ + \widehat{\sigma}_-\right) \\ \widehat{\sigma}_y &= \frac{1}{2\imath}\left(\widehat{\sigma}_+ - \widehat{\sigma}_-\right)\end{aligned}$$

and

$$\begin{aligned}\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 &= \widehat{\sigma}_{1x}\widehat{\sigma}_{2x} + \widehat{\sigma}_{1y}\widehat{\sigma}_{2y} + \widehat{\sigma}_{1z}\widehat{\sigma}_{2z} \\ &= \frac{1}{4}\left(\widehat{\sigma}_{1+} + \widehat{\sigma}_{1-}\right)\left(\widehat{\sigma}_{2+} + \widehat{\sigma}_{2-}\right) - \frac{1}{4}\left(\widehat{\sigma}_{1+} - \widehat{\sigma}_{1-}\right)\left(\widehat{\sigma}_{2+} - \widehat{\sigma}_{2-}\right) + \widehat{\sigma}_{1z}\widehat{\sigma}_{2z} \\ &= \frac{1}{2}\left(\widehat{\sigma}_{1+}\widehat{\sigma}_{2-} + \widehat{\sigma}_{1-}\widehat{\sigma}_{2+}\right) + \widehat{\sigma}_{1z}\widehat{\sigma}_{2z}\end{aligned}$$

In matrix form

$$\begin{aligned}\widehat{\sigma}_+ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \imath \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \\ \widehat{\sigma}_- &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \imath \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}\end{aligned}$$

Hence,

$$\begin{aligned}\widehat{\sigma}_+|\uparrow\rangle &= 0 = \widehat{\sigma}_-|\downarrow\rangle \\ \widehat{\sigma}_-|\uparrow\rangle &= 2|\downarrow\rangle \\ \widehat{\sigma}_+|\downarrow\rangle &= 2|\uparrow\rangle \\ \widehat{\sigma}_z|\uparrow\rangle &= |\uparrow\rangle \\ \widehat{\sigma}_z|\downarrow\rangle &= -|\downarrow\rangle\end{aligned}$$

and

$$\begin{aligned}
\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 |\uparrow_1\uparrow_2\rangle &= |\uparrow_1\uparrow_2\rangle \\
\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 |\downarrow_1\uparrow_2\rangle &= 2|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle \\
\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 |\uparrow_1\downarrow_2\rangle &= 2|\downarrow_1\uparrow_2\rangle - |\uparrow_1\downarrow_2\rangle \\
\widehat{\sigma}_1 \cdot \widehat{\sigma}_2 |\downarrow_1\downarrow_2\rangle &= |\downarrow_1\downarrow_2\rangle
\end{aligned}$$

Matrix elements of Hamiltonian are then given by

$$\begin{aligned}
\widehat{H}|\uparrow_1\uparrow_2\rangle &= \frac{\Delta E}{4}|\uparrow_1\uparrow_2\rangle - \left(\frac{g_1\mu_0}{\hbar}\frac{\hbar}{2}|\uparrow_1\uparrow_2\rangle + \frac{g_2\mu_0}{\hbar}\frac{\hbar}{2}|\uparrow_1\uparrow_2\rangle\right) B \\
&= \left[\frac{\Delta E}{4} - (g_1+g_2)\frac{\mu_0 B}{2}\right]|\uparrow_1\uparrow_2\rangle \\
&= H_{11}|\uparrow_1\uparrow_2\rangle
\end{aligned}$$

$$\begin{aligned}
\widehat{H}|\downarrow_1\downarrow_2\rangle &= \frac{\Delta E}{4}|\downarrow_1\downarrow_2\rangle + \left(\frac{g_1\mu_0}{\hbar}\frac{\hbar}{2}|\downarrow_1\downarrow_2\rangle + \frac{g_2\mu_0}{\hbar}\frac{\hbar}{2}|\downarrow_1\downarrow_2\rangle\right) B \\
&= \left[\frac{\Delta E}{4} + (g_1+g_2)\frac{\mu_0 B}{2}\right]|\downarrow_1\downarrow_2\rangle \\
&= H_{44}|\downarrow_1\downarrow_2\rangle
\end{aligned}$$

$$\begin{aligned}
\widehat{H}|\downarrow_1\uparrow_2\rangle &= \frac{\Delta E}{4}\left(2|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle\right) - \left(-\frac{g_1\mu_0}{\hbar}\frac{\hbar}{2}|\downarrow_1\uparrow_2\rangle + \frac{g_2\mu_0}{\hbar}\frac{\hbar}{2}|\downarrow_1\uparrow_2\rangle\right) B \\
&= \frac{\Delta E}{2}|\uparrow_1\downarrow_2\rangle + \left[(g_1-g_2)\frac{\mu_0 B}{2} - \frac{\Delta E}{4}\right]|\downarrow_1\uparrow_2\rangle \\
&= H_{23}|\uparrow_1\downarrow_2\rangle + H_{22}|\downarrow_1\uparrow_2\rangle
\end{aligned}$$

$$\begin{aligned}
\widehat{H}|\uparrow_1\downarrow_2\rangle &= \frac{\Delta E}{4}\left(2|\downarrow_1\uparrow_2\rangle - |\uparrow_1\downarrow_2\rangle\right) - \left(\frac{g_1\mu_0}{\hbar}\frac{\hbar}{2}|\uparrow_1\downarrow_2\rangle - \frac{g_2\mu_0}{\hbar}\frac{\hbar}{2}|\uparrow_1\downarrow_2\rangle\right) B \\
&= \frac{\Delta E}{2}|\downarrow_1\uparrow_2\rangle + \left[-(g_1-g_2)\frac{\mu_0 B}{2} - \frac{\Delta E}{4}\right]|\uparrow_1\downarrow_2\rangle \\
&= H_{32}|\downarrow_1\uparrow_2\rangle + H_{33}|\uparrow_1\downarrow_2\rangle
\end{aligned}$$

[9]

Eigenvalues: we already have two diagonal elements, so that

$$\begin{aligned}
E_1 &= \left[\frac{\Delta E}{4} - (g_1+g_2)\frac{\mu_0 B}{2}\right] \\
E_2 &= \left[\frac{\Delta E}{4} + (g_1+g_2)\frac{\mu_0 B}{2}\right]
\end{aligned}$$

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We obtain the characteristic equation for the other two eigenvalues from the 2×2 matrix. We have

$$\left[(g_1 - g_2)\frac{\mu_0 B}{2} - \frac{\Delta E}{4} - E\right]\left[-(g_1 - g_2)\frac{\mu_0 B}{2} - \frac{\Delta E}{4} - E\right] - \left[\frac{\Delta E}{2}\right]^2 = 0$$

$$E^2 + \frac{\Delta E}{2}E + \left[\frac{\Delta E}{4}\right]^2 - \frac{1}{4}\left[(g_1 - g_2)\mu_o B\right]^2 = 0$$

$$\begin{aligned} E_\pm &= -\frac{\Delta E}{4} \pm \frac{1}{2}\sqrt{\left(\frac{\Delta E}{2}\right)^2 + ((g_1 - g_2)\mu_o B)^2 + \frac{3}{4}(\Delta E)^2} \\ &= -\frac{\Delta E}{4}\left(1 \mp 2\sqrt{1 + x^2}\right) \end{aligned}$$

where

$$x = \frac{\mu_0 B(g_1 - g_2)}{\Delta E}$$

[4]

Energy levels for a

•zero $\boldsymbol{B}$ field:

$$\begin{aligned} E_1 &= E_4 = \frac{\Delta E}{4} \\ E_+ &= \frac{\Delta E}{4} \\ E_- &= -\frac{3}{4}\Delta E \end{aligned}$$

Degeneracy 2

•high $\boldsymbol{B}$ field:

$$\begin{aligned} E_1 &= -\frac{\mu_0 B}{2}(g_1 + g_2) \\ E_+ &= \frac{\mu_0 B}{2}(g_1 - g_2) \\ E_- &= -\frac{\mu_0 B}{2}(g_1 - g_2) \\ E_4 &= \frac{\mu_0 B}{2}(g_1 + g_2) \end{aligned}$$

Degeneracy 4

[4]

Sketch of energy levels. Use some convenient numbers $\Delta E = 1.0$, $g_1 = 2.0$, $g_2 = 1.0$ and $\mu_0 = 1.0$. [4]

4 *TODO: Bookwork* [8]

Molecule, state $|J,m\rangle$ with rotational energy $BJ(J+1)$, dipole moment p in field E.

$$H_1 = -pE\cos\theta$$

$\langle i|H_1|i\rangle = 0$ from parity, since H_1 odd and $|i\rangle$ has parity $(-1)^J$. [4]

Second order corrections to state $|J,m\rangle$ come from

$$|\langle J,m|\cos\theta|J-1,m|^2 = \left(\frac{J^2-m^2}{4J^2-1}\right)$$

with

$$E_i - E_k = B\left[J(J+1) - (J-1)J\right] = 2BJ$$

and

$$|\langle J,m|\cos\theta|J+1,m|^2 = \left(\frac{(J+1)^2-m^2}{4(J+1)^2-1}\right)$$

with

$$E_i - E_k = B\left[J(J+1) - (J+1)(J+2)\right] = -2B(J+1)$$

Therefore,

$$\Delta E_i = \frac{(pE)^2}{2B}\left[\frac{J^2-m^2}{(4J^2-1)J} - \frac{(J+1)^2-m^2}{(4(J+1)^2-1)(J+1)}\right]$$

$$\begin{aligned}[...] &= \frac{J^2-m^2}{(4J^2-1)J} - \frac{(J+1)^2-m^2}{(4(J+1)^2-1)(J+1)} \\ &= .... \\ &= \frac{J(J+1)-3m^2}{J(2J-1)(J+1)(2J+3)}\end{aligned}$$

Hence,

$$\Delta E_i = \frac{(pE)^2}{2B}\left[\frac{J(J+1)-3m^2}{J(2J-1)(J+1)(2J+3)}\right]$$

When $J=0$, only the $|\langle 0,0|\cos\theta|1,0\rangle|^2 = \frac{1-0}{4-1}$ term contributes. Therefore,

$$\begin{aligned}\Delta E_i(0\to 0) &= (pE)^2\left[\frac{|\langle 0,0|\cos\theta|1,0\rangle|^2}{0-2B}\right] \\ &= -\frac{(pE)^2}{6B} \qquad (1)\end{aligned}$$

[8]

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For the OCS molecule, the unperturbed transition $J = 0 \rightarrow J = 1$ occurs at 12.16 GHz.

$$\Delta E = 12.16 \text{ GHz} = B \times 1 \times 2 - B \times 0 \times 1 = 2B$$

In the static field of 10^5 Vm^{-1}, tranistion splits due to transitions from $|1, \pm 1\rangle \rightarrow |0, 0\rangle$ or $|1, 0\rangle \rightarrow |0, 0\rangle$. The energy split is given by

$$\Delta E_{|1,\pm 1\rangle} - \Delta E_{|1,0\rangle} = \frac{3}{10} \frac{(pE)^2}{2B}$$

Hence, dipole moment

$$\begin{aligned} p &= \frac{1}{E} \sqrt{\frac{10h^2}{3} (12.16 \text{ GHz} \times 3.21 \text{ MHz}} \\ &= 2.39 \times 10^{-30} \text{ Cm} \\ &= 14.95e \text{ pm} \end{aligned}$$

[5]

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