answers

02 December 2010
5:34 PM

1

(a) bookwork based on $\cos\theta_c = \frac{1}{\beta n}$ [1]

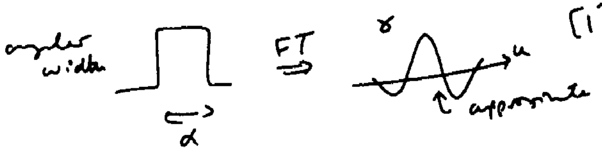
and $E = \gamma m c^2$ [1]

with $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$n = \frac{1}{\cos\theta_c \sqrt{1 - \left(\frac{mc^2}{E}\right)^2}} = 1\cdot 38 \sim 1\cdot 4$$ [1] [1]

(b) spatial coherence

visibility $\gamma(u = kd) = FT\{I(\theta)\}$ [1]



[1] zeros at $u = \frac{2\pi}{\alpha} = kd = \frac{2\pi d}{\lambda}$ [1]

so $d \sim \frac{\lambda}{\alpha} = \frac{500\text{nm}}{(4.10^8/2.10^{15})} \sim 2.5\text{m}$ [1]

(c) phase: $\Delta\phi = \pi/2$ so $J = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ [1]

$e^{\pm i\Delta\phi}$ [1]

rot^n by $45°$ $= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ [1]

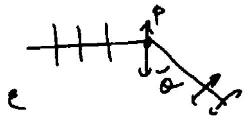
$$J = \frac{i}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$ [1]

2. brief notes

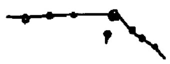
(a) Scattering by particles

induced dipole p

reradiates $\langle P \rangle = \dfrac{\mu_0 \langle \ddot{p}^2 \rangle}{6\pi c}$



$N_1 \sim \sin^2\theta$



$N_2 \sim$ const

pol^n degree : $\dfrac{N_2 - N_1}{N_2 + N_1} \sim \dfrac{1-\sin^2\theta}{1+\sin^2\theta}$

gives pol^n from scattered light $\therefore$ sky

(b) Power gain : directionality

$$G(\theta,\phi) = \frac{N(\theta,\phi)}{\int N \, dS}$$

draw for ED, EQ

Effective area : $A_{eff} = \dfrac{P_{load}}{N_{incident}}$

'absorption' cross section

can be $>$ geometrical area

interference of scattered reemission

$A_{eff} = \dfrac{\lambda^2}{4\pi} G$ always

(c) $J = (c\rho, \underline{J})$ transforms as 4-vector

so charge $\leftrightarrow$ currents

$\Box^2 A = \mu_0 J$

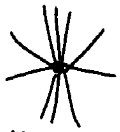
neutral current carrying wire $\Longrightarrow$ charges

in another frame

neutral current carrying wire $\Longrightarrow$ charges in stationary frame

$\nwarrow$ B $\qquad\qquad$ $\mathcal{E}$ $\nearrow$

$\longleftrightarrow$ same forces

field lines of moving charge  $\rightarrow v$

uniform moving charge does not radiate

3.

· Bookwork,

$$\frac{[p]}{r^3} \qquad \frac{[\dot{p}]}{r^2 c} \sim \frac{p_0}{r^2}\frac{\omega}{c}$$ [1]

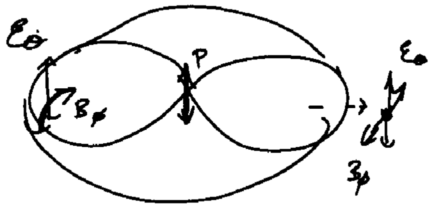
$\nwarrow$ near field

$$\frac{[\ddot{p}]}{rc^2} \sim \frac{p_0}{r}\left(\frac{\omega}{c}\right)^2 \sim \frac{p_0}{r^3}\left(\frac{2\pi r}{\lambda}\right)^2$$ [1]

$\nwarrow$ dominates far field at large $r \gg \lambda$ [1]

$\frac{1}{r}$ term expected for radiation

$$E_\theta = \frac{\sin\theta}{4\pi\varepsilon_0}\frac{[\ddot{p}]}{rc^2}$$



$$B_\phi = \frac{\mu_0 \sin\theta}{4\pi}\frac{[\ddot{p}]}{rc} = \frac{E_\theta}{c}$$

- in-phase
- orthogonal
- $E_\theta = cB_\phi$ [4]
- linearly-polarised

Bookwork:

[1]

$$N = \mathcal{E} \wedge \mathcal{H}$$

$$= E_\theta \frac{B_\phi}{\mu_0}$$

[1]

$$= \frac{\mu_0}{16\pi^2 c}\sin^2\theta\,\frac{[\ddot{p}]^2}{r^2}$$

$$p = qd\,e^{-i\omega t}$$

[1]

$p = qd\,e^{-i\omega t}$

$\ddot{p} = -\omega^2 qd\,e^{-i\omega t}$ [1]

$\langle [\ddot{p}]^2 \rangle = \frac{1}{2}\,\omega^4\,(qd)^2$ [1]

$$\langle N \rangle = \frac{\mu_0 \omega^4 q^2 d^2}{32\pi^2 c}\,\frac{\sin^2\theta}{r^2}$$

$$\langle P \rangle = \int \langle N \rangle . dS \qquad [1]$$

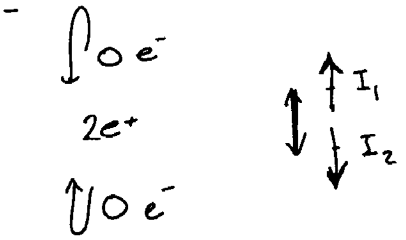
$$= \underline{\qquad} \int_0^\pi \sin^2\theta \,\frac{1}{r^2}\, 2\pi r^2 \sin\theta\, d\theta$$

$$= \underline{\qquad}\ 2\pi \int_0^\pi \sin^3\theta\, d\theta$$

$$\int \sin\theta(1-\cos^2\theta)\,d\theta = \left[-\cos\theta\right]_0^\pi + \left[\tfrac{1}{3}\cos^3\theta\right]_0^\pi = 4/3$$

$$= \frac{\mu_0 \omega^4 q^2 d^2}{12\pi c} \qquad [1]$$

<u>Similar to notes:</u>

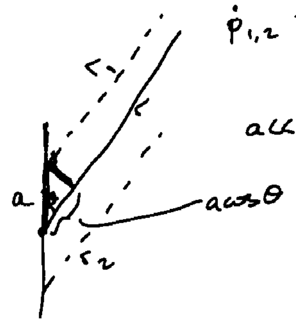


$$B_\phi = \frac{\mu_0 \sin\theta}{4\pi r c}\left\{ [\ddot{p}_1(r_1)] + [\ddot{p}_2(r_2)] \right\} \qquad [1]$$

$\dot{I}(t - r_1/c) \quad - \quad \dot{I}(t - r_2/c)$

opp dir^n

$\dot{p}_{1,2} = \pm I d$



acc r: $\quad r_1 \sim r - a\cos\theta$ [2]

$\qquad\quad r_2 \sim r + a\cos\theta$

$$\dot{I}(t - \tfrac{r_1}{c}) \simeq \dot{I}(t - \tfrac{r}{c}) - a\cos\theta\, \frac{\partial \dot{I}(t - r/c)}{\partial r}$$

[1]

so $\quad \dot{I}(t - \tfrac{r_1}{c}) - \dot{I}(t - \tfrac{r_2}{c}) = 2a\cos\theta\, \dfrac{\partial \dot{I}(t - r/c)}{\partial r}$

[illegible] [1]

but $\frac{\partial}{\partial r}[X] = -\frac{1}{c}[\dot{X}]$ [1]

$\frac{2a\cos\theta}{c}[\ddot{I}]$

$B_\phi = \frac{\mu_0 \sin\theta\cos\theta}{2\pi r c^2} a [\dddot{p}]$ [1]

$\sim \omega[\ddot{p}]$

$P = \frac{cB_\phi^2}{\mu_0}$

so power radiated $= \frac{\mu_0 \omega^6}{8\pi^2} \frac{q^2 d^2 a^2}{c^3} 2\pi \underbrace{\int_0^\pi \sin^2\theta\cos^2\theta \sin\theta \, d\theta}$ [1]

$\int \sin\theta(\cos^2\theta - \cos^4\theta)\,d\theta$

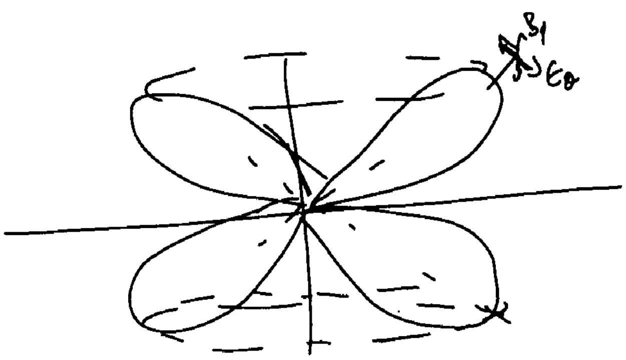
$= \left[-\frac{\cos^3\theta}{3}\right]_0^\pi + \left[\frac{\cos^5\theta}{5}\right]_0^\pi$

$= \frac{4}{15}$ [1]

$= \frac{\mu_0 \omega^6 q^2 d^2 a^2}{15\pi c^3}$ [1]

ratio to dipole : $\frac{EQ}{ED} = \frac{12\,\omega^2 a^2}{15 c^2} = \frac{12}{15}\left(\frac{2\pi a}{\lambda}\right)^2$

$= 8.8 \times 10^{-7}$ [1]



lobes @ $\pm 45°$ [2]

Bookwork:

- 4-vector is a 4D quantity with cpts which transform between inertial frames according to [2] the Lorentz transformation.

eg.

$\underline{R} = (ct, \underline{r})$ ← three space-like cpts [1]

↑ one time-like

Lorentz Invariant is inner product of two 4-vectors

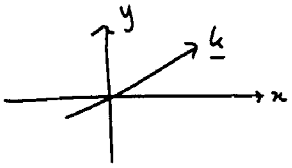
↑ doesn't depend on inertial frame [2]

$$[Lw]^{+} \cdot [Lv] = w^{+} \cdot \underbrace{L^{+}L}\, v = w^{+} \cdot v$$

$$A \cdot B = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3$$

[in notes]

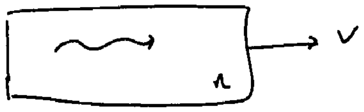
- $\underline{K} = \left(\frac{\omega}{c}, \underline{k}\right)$ [1]



consider $\underline{k} = (k\cos\theta, k\sin\theta, 0)$ [1]

$$L = \begin{matrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{matrix}$$ [1]

$$\frac{\omega'}{c} = \gamma\left(\frac{\omega}{c} - \beta k\cos\theta\right)$$
$$k'\cos\theta' = \gamma\left(k\cos\theta - \beta\frac{\omega}{c}\right)$$ [1]
$$k'\sin\theta' = k\sin\theta$$

- 

$\frac{\omega}{ck} = \frac{1}{n}$ in S (stationary) [1]

$$\frac{\omega'}{ck'} = \frac{1}{n'}$$

for $\parallel x$, $\theta = 0$ so eqns become [seen in examples]

$$\frac{\omega'}{c} = \gamma\left(\frac{\omega}{c} - \beta k\right) = \gamma k\left(\frac{1}{n} - \beta\right)$$ [1]

$$k' = \gamma\left(k - \beta\frac{\omega}{c}\right) = \gamma k\left(1 - \beta/n\right)$$ [1]

divide

$$\frac{\omega'}{k'} = \frac{1}{n'} = \frac{\frac{1}{n} - \beta}{}$$

$1 - \beta/n$

$$\text{so} \quad n' = \frac{n-\beta}{1-n\beta}$$ [1]

$$\simeq (n-\beta)(1+n\beta+\ldots)$$

$$= n - \beta + n^2\beta + \ldots$$

$$= n + \beta(n^2-1) + \ldots$$

If photon in same direction as v, n' increases [1]

since $k' = k \underbrace{\gamma(1-\beta/n)}_{<1}$ so $\lambda' > \lambda$ red-shifted [1]

- if $/\!/ y$, $\theta = \pi/2$ so eq^{n}s become [new] [1]

$$\left.\begin{aligned} \frac{\omega'}{c} &= \gamma\frac{\omega}{c} \\ k'\cos\theta' &= -\gamma\beta\frac{\omega}{c} \end{aligned}\right\} \text{divide} \quad n'\cos\theta' = -\beta$$

$$k'\sin\theta' = k$$

[1]

$$\left.\begin{aligned} \tan\theta' &= -\frac{n}{\gamma\beta} \\ \cos\theta' &= -\beta/n' \end{aligned}\right\} \sin\theta' = \frac{n}{\gamma\beta}\cdot\frac{\beta}{n'} = \frac{n}{\gamma n'}$$ [1]

so $\theta' > 90°$ slightly

$$\frac{\sin^2\theta'}{\cos^2\theta'} = \frac{n^2}{\gamma^2\beta^2} = \frac{1-\beta^2/n'^2}{\beta^2/n'^2} = \frac{n'^2-\beta^2}{\beta^2}$$

$$n'^2 = \frac{n^2}{\gamma^2} + \beta^2$$

$$\text{so} \quad n' = \frac{n}{\gamma}\sqrt{1+\left(\frac{\gamma\beta}{n}\right)^2}$$ [1]

- Simple calc^{n}: [notes] [1]

$$\phi = \underline{K}\cdot\underline{R} = \left(\frac{\omega}{c}, \underline{k}\right)\cdot(ct, r)$$ Lorentz invariant [1

so phase cannot change at output port. [1]

- birefringence: Δn in different orthogonal axes [1]

- birefringence: Δn in diff'nt orthogonal axes [1]

normally due to crystal structure: $n_o \neq n_e$ (1)

[note] [new] normally <u>same</u> propagation direction, diff'nt polarisation directions [1]

here polarisation not mentioned