UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Saturday 2 June 2012 13.30 to 15.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) As the Sun rises over a still lake, an angle of elevation is reached where the reflected image of the Sun is completely linearly polarised in a plane parallel to the lake's surface. Explain the effect and calculate the angle, given that the refractive index of water is 1.33. [4]

(b) Two Hertzian dipoles are driven by alternating current sources of the same amplitude. The first dipole is 1 cm long and operates at a wavelength of 20 cm, while the second is 0.5 cm long and operates at a wavelength of 25 cm. What is the ratio of powers emitted by the two dipoles? [4]

(c) Estimate the magnitude of the aberration (i.e. the apparent angular shift) of a star due to the Earth's orbital motion around the Sun, when the star's light is incident normal to the Earth's trajectory. [4]

[*A simple geometrical argument is sufficient. The Sun–Earth distance is* $1.5 \times 10^{11} m$.]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) radiation from an accelerating point charge;

(b) temporal coherence and the power spectrum, giving two examples;

(c) the choice of gauge for the electromagnetic potentials.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain briefly how the polarisation state of a light beam can be described by a Jones vector. [1]

Describe the concept of birefringence and the effect of a quarter-wave plate on plane-polarised light incident normal to its surface, as a function of the angle between the direction of polarisation and one of the plate's transverse principal axes Ox and Oy. Calculate the thickness d of the plate for light at angular frequency ω_0, explaining any quantities you introduce. [6]

Consider now a frequency ω that differs slightly from ω_0. Show that the Jones matrix Q for the quarter-wave plate, with its fast axis along Ox, can be written as $Q = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i}e^{\mathrm{i}\epsilon} \end{pmatrix}$, where $\epsilon \ll 1$, and write down an expression for ϵ. Say why the Jones matrix H for a similar half-wave plate is $H = \begin{pmatrix} 1 & 0 \\ 0 & -e^{2\mathrm{i}\epsilon} \end{pmatrix}$. [3]

In an attempt to make a quarter-wave plate that works over the whole visible spectrum, a combination of Q and H is used. Light plane-polarised along the x-axis is passed through a half-wave plate that has been rotated through an angle $\alpha = 15°$ from that axis. It then passes into a quarter-wave plate that has been rotated through a *further* angle $\beta = 60°$. Calculate the Jones vector of the resulting wave and show that it is proportional to $\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix}$ if $\epsilon = 0$. What type of polarisation is this? [6]

Show that the Jones vector remains proportional to $\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix}$ to first order in ϵ, by calculating the ratio of the two components (work to 3 significant figures). [4]

In order to make a quarter-wave plate that can be switched on and off rapidly, consider using (a) an electric field (Kerr effect) and (b) a magnetic field (Faraday effect) applied to some appropriate material. Explain the effect of each type of polariser on linearly-polarised light and say why only one of these would be suitable. [5]

[*The matrix that rotates the axes through an angle θ is*

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$
]

(TURN OVER

4 *Attempt* **either** *this question* **or** *question 3.*

Define the terms 4-current, 4-potential and 4-gradient, and use them to show that the conservation of charge holds in all frames. [5]

The transformation of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields between two frames S and S′ in standard configuration (i.e. whose origins and axes coincide at $t = t' = 0$, and where the S′ frame moves at velocity $\boldsymbol{u}$ in the positive x-direction) can be written as:

$$E'_x = E_x\,, \qquad E'_{y,z} = \gamma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})_{y,z}\,,$$

$$B'_x = B_x\,, \qquad B'_{y,z} = \gamma(\boldsymbol{B} - \boldsymbol{u} \times \boldsymbol{E}/c^2)_{y,z}\,.$$

An observer at rest in frame S′ sees a stationary line charge, distributed along the x'-axis with uniform charge per unit length λ'.

(a) Calculate the electric and magnetic fields seen by this observer at a distance r' from the line of charge and derive, with the aid of the transformation laws quoted above, the corresponding fields seen by an observer who is at rest in S. [7]

(b) Find the charge density λ in S, and also the current that flows in this frame. Show that your results are consistent with the fields derived in (a). [5]

(c) In S, another line charge, which is stationary and has charge density $-\lambda$, is now superposed on the original line charge. Use the results derived in (b) to write down the total $\boldsymbol{E}$ and $\boldsymbol{B}$ fields and the charge density and current in S′, and show that these too are consistent. [5]

(d) Explain how the situation in (c) is related to an ordinary wire. [3]

END OF PAPER