UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 29 May 2015 9.00 am to 11.00 am

PHYSICS (5)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (5)

ASTROPHYSICAL FLUID DYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

ASTROPHYSICAL FLUID DYNAMICS

1 *Answer* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A black hole of mass 2×10^{37}kg is located in a the central region of a galaxy where it is steadily accreting gas, composed of atomic hydrogen only. Assuming that at a distance of 3×10^{16} m from the black hole the gas number density is 10^6 m^{-3} and that the gas at this location has the free-fall velocity (from infinity), derive the accretion rate onto the black hole. [4]

(b) A sound wave crosses a discontinuity from a medium with density ρ_1 and temperature T_1 to a medium with density ρ_2 and temperature T_2. The two media are in pressure equilibrium. Show that the ratio of the velocity perturbations of the reflected wave to the incident wave is

$$r = \frac{\rho_1\sqrt{T_1} - \rho_2\sqrt{T_2}}{\rho_1\sqrt{T_1} + \rho_2\sqrt{T_2}}$$

(assume that both fluids have the same adiabatic index γ and that both fluids can be approximated as perfect gases of the same mean molecular weight). [4]

(c) Demonstrate the Virial Theorem $2T + \Omega = 0$ for a cloud of particles, where T is the total kinetic energy and Ω is the total gravitational potential energy. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) magneto-hydrodynamic waves;

(b) the Bernoulli equation and its applications;

(c) mass-radius scaling relations for stars.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain the physical meaning of the three Rankine-Hugoniot relations:

$$\rho_1 u_1 = \rho_2 u_2;$$

$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2;$$

$$\frac{1}{2}u_1^2 + \mathcal{E}_1 + \frac{p_1}{\rho_1} = \frac{1}{2}u_2^2 + \mathcal{E}_2 + \frac{p_2}{\rho_2} \quad , \qquad (*)$$

where ρ is density, u is velocity measured in the frame of the shock, p is pressure, $\mathcal{E}$ is the internal energy per unit mass, and the subscripts 1,2 refer to upstream (pre-shock) and downstream (post-shock) conditions respectively. [3]

Show that, for an adiabatic shock in a perfect gas with adiabatic index γ, equation (*) can be rewritten as

$$\frac{c_{s,1}^2}{\gamma - 1} + \frac{1}{2}u_1^2 = \frac{c_{s,2}^2}{\gamma - 1} + \frac{1}{2}u_2^2 \quad ,$$

where $c_{s,1}$, $c_{s,2}$ are the speeds of sound in the up-stream and down-stream gas. Hence or otherwise show that

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1 + 2/M_1^2} \quad ,$$

where M_1 is the upstream Mach number. [7]

Discuss the limit of the density contrast for a strong adiabatic shock. [3]

Show that for an isothermal shock in a perfect gas the density contrast is $\rho_2/\rho_1 = M_1^2 = (u_1/c_{s,1})^2$ where $c_s = \sqrt{p/\rho}$ is now the isothermal sound speed. Discuss the implications of this relation for the physics of isothermal shocks in comparison to the adiabatic case. [6]

Suppose that the pre-shocked medium is made of molecular hydrogen and that the shock dissociates all molecules. Derive, in the isothermal shock case, the relation between density contrast ρ_2/ρ_1 and the velocities u_1 and u_2 of the pre-shocked and post-shocked gas, respectively, relative to the shock front. (Assume that both media behave as a perfect gas.) [6]

4 *Attempt* **either** *this question* **or** *question 3.*

(a) Show that, for a self gravitating cloud of a perfect gas with uniform density ρ and uniform temperature T, the maximum stable mass (Jeans mass) is proportional to $T^{3/2}\rho^{-1/2}$. [6]

(b) Briefly explain why in a gas thermal stability requires $\left(\partial\dot{Q}/\partial T\right)_P > 0$, where $\dot{Q}$ is the net cooling rate. [2]

A hydrogen gas cloud is photoionized by a constant ultraviolet flux, which produces a constant heat input Γ to the gas. The cooling of the gas is given by the radiation resulting from recombination, and has the form $\dot{Q}_{\rm rad} = \frac{3}{2}n^2\alpha_B k_{\rm B}T$, where $n = n_{\rm e} = n_{\rm p}$ is the gas number density, T is the gas temperature (assuming $T_{\rm e} = T_{\rm p} = T$), and α_B is the recombination coefficient. Assuming that the medium behaves like a perfect gas, determine whether the system is thermally stable. [4]

(c) A young star generates a spherically symmetric wind that reaches the escape velocity $v_{\rm esc}$ at a distance $R_{\rm esc}$ from the star. The gas in the outflow behaves adiabatically and the gas motion is dominated by the stellar gravitational potential. Determine the gas velocity as a function of radius. [6]

(d) Assume in the previous part that $R_{\rm esc} = 3 \times 10^{11}$ m, $M_* = 4 \times 10^{30}$ kg and the gas number density is $n(R_{\rm esc}) = 10^8$ m^{-3} (assume the wind to be composed of atomic hydrogen). Derive a formula for the mass outflow rate of the wind. [3]

(e) Assume that the wind of the previous part behaves as a perfect gas and expands adiabatically. Derive the variation of pressure and of temperature as a function of radius. [4]

END OF PAPER