UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Monday 07 June 2021 11:00am to 13:00pm

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

Quantum Condensed Matter Physics

*Candidates offering this paper should attempt a total of **five** questions: **three** questions from Section A and **two** questions from Section B.*

*The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 For a monoatomic crystal with a simple-cubic lattice, show that the number of electronic states per atom in the first Brilloin zone is $2N$ per band, where N is the number of atoms in the crystal. How does this help to explain which materials are metals or insulators? [4]

2 Explain why metals are generally highly reflective at optical frequencies and transparent in the ultraviolet. [4]

3 Sketch and comment on the phonon modes and dispersion relation for an ideal diatomic chain in which both atoms have the same mass and one of the spring constants is much larger than the other, indicating the relevant boundary of the Brillouin zone. [4]

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SECTION B
Attempt two questions from this section

4 This question explores the tight-binding model for electrons in periodic lattice, leading to their band structure.

(a) State Bloch's theorem for a particle in a periodic potential. [3]

(b) For a one-dimensional chain of atoms in a fixed periodic potential, assume the only significant wavefunction overlap is for nearest neighbours a distance a apart. Use the tight binding method and Bloch wavefunctions to show that the band energy states, E, vary with wave-vector k as

$$E(k) = E_0 + 2t\,\cos(ka)\ ,$$

and define the parameters t and E_0. [5]

(c) Extend this calculation to a solid consisting of a simple cubic lattice of atoms and show that the width of the band of allowed energy states is $12|t|$. [3]

(d) Show that, for small values of k, the constant energy surfaces in part (c) are spheres in $\boldsymbol{k}$-space, and derive an expression for the effective mass. [3]

(e) Assume t is given by

$$t = -2E_1 \exp\left(-a/a_1\right)\ ,$$

where E_1 is a constant with units of energy, and a_1 is a constant with units of length. Write down an expression for the variation of the effective mass with atomic separation a, for small values of k, and sketch the form of this variation. [3]

(f) The solid above, with $a = a_1$, is compressed uniformly such that there is a fractional reduction of 0.01 in the value of a. Calculate the fractional change in the width of the band. [2]

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5 This question explores the application of kinetic theory for electrons and phonons to estimate electronic and thermal conductivity of metals.

(a) Show that the electronic contribution to the heat capacity of a metal is of the form:

$$C = ANk_{\mathrm{B}}^2 T/E_F \ , \qquad [4]$$

where E_F is the Fermi energy, N is the number of conducting electrons, and A is a constant that need not be evaluated.

(b) Using this result and simple kinetic theory arguments obtain the Wiedermann-Franz law, which relates the electrical conductivity σ of a metal to its thermal conductivity K. [6]

(c) Under what circumstances do you expect the Wiedermann-Franz law to hold? [3]

(d) For a typical monovalent metal, what is the relationship between the size of the Fermi wave vector and the wavevector of the most energetic phonons in the crystal. How is this relevant to the scattering processes that limit the electrical and thermal conductivity at room temperature and below? [6]

[*The constant A can be taken as $\pi^2/2$.*]

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6 A semiconductor p-n junction has a doping profile such that across the entire junction the doping is linearly graded. The total doping concentration, $n(x)$, in the interval is given by

$$n(x) = n_{\mathrm{D}}(x) - n_{\mathrm{A}}(x) = \alpha x \ ,$$

where x is the perpendicular distance from the plane of the junction interface, and α is a constant representing the compensated doping gradient. $n_{\mathrm{D}}(x)$ and $n_{\mathrm{A}}(x)$ represent the donor and acceptor doping concentrations, respectively, as a function of x.

(a) Assuming all the donors and acceptors are ionised in the depletion region, show that Poisson's equation for the electrostatic potential can be written as

$$\frac{\partial^2 V}{\partial x^2} = -\frac{e\alpha x}{\varepsilon_0 \varepsilon} \ ,$$

where e is the electronic charge. Outside the depletion region, $\partial^2 V/\partial x^2 = 0$. [2]

(b) A bias V_{ext} is applied to the p-type side of the junction, relative to the other side. Sketch the variation in the energies of the bottom of the conduction band, the top of the valence band and the chemical potential, as a function of x across the depletion region, for $V_{\mathrm{ext}} = 0$ and for $V_{\mathrm{ext}} < 0$. At $V_{\mathrm{ext}} = 0$, there is a built-in band offset eV_{b} across the junction. What is the origin of this offset? [4]

(c) Describe the various currents flowing across the junction at $V_{\mathrm{ext}} = 0$, and how they change when the external bias is non-zero. [3]

(d) By solving Poisson's equation, derive an expression for the electric field as a function of position within the depletion region. At what value of x will the magnitude of the electric field be a maximum? [3]

(e) Show that the width of the depletion region at $V_{\mathrm{ext}} = 0$ is given by

$$w = \left(\frac{12 \varepsilon_0 \varepsilon V_{\mathrm{b}}}{e\,\alpha} \right)^{1/3} \ .$$ [3]

(f) If Q is the stored charge per unit area of interface, calculate the differential capacitance per unit area of the junction, $C = \mathrm{d}Q/\mathrm{d}V_{\mathrm{ext}}$, and comment on its dependence on V_{ext}. [4]

END OF PAPER

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