PNP 2017

1) a) threshold energies for reactions

$\bar{\nu}_e + p \rightarrow e^+ + n$ and $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$

if proton is at rest

$$\sqrt{s} = m_e + m_n$$

$$(E_\nu + m_p)^2 - E_\nu^2 = (m_e + m_n)^2$$

$$m_p^2 + 2m_pE_\nu = (m_e + m_n)^2$$

$$E_\nu = \frac{(m_e + m_n)^2 - m_p^2}{2m_p} = 1.81 \text{ MeV}$$

for muon reaction replace m_e with m_μ

$$E_\nu = 113 \text{ MeV}$$

b) Nuclear shell model prediction for magnetic dipole moments of ${}^{15}_{7}N$ and ${}^{17}_{8}O$

$g_p = 5.586$, $g_n = -3.826$

$\mu = g_j j \mu_N$

${}^{15}_{7}N$ - unpaired proton with $l=1$, $j=\frac{1}{2}$, $s=\frac{1}{2}$

use $g_l = 1$, $g_s = g_p$

$$g_j = g_l \frac{l(l+1) + j(j+1) - s(s+1)}{2j(j+1)} + g_p \frac{s(s+1) + j(j+1) - l(l+1)}{2j(j+1)}$$

$$= \frac{2 + 3/4 - 3/4}{3/2} + 5.586 \frac{3/4 + 3/4 - 2}{3/2} = \frac{4}{3} - \frac{1}{3} \cdot 5.586$$

$$\mu = \frac{1}{2} \cdot \frac{1}{3}(4 - 5.586)\mu_N = -0.264\,\mu_N$$

${}^{17}_{8}O$ - unpaired neutron with $l=2$, $j=\frac{5}{2}$, $s=\frac{1}{2}$

use $g_l = 0$, $g_s = g_n$

$$g_j = -3.826 \frac{3/4 + 35/4 - 6}{35/2} = \frac{1}{5} \cdot -3.826$$

$$\mu = \frac{5}{2} \cdot \frac{1}{5} \cdot -3.826\mu_N = 1.91\,\mu_N$$

c) couplings g_L and g_R of Z bosons to fermions are

$g_{L,R} \propto (I_3)_{L,R} - Q\sin^2\theta_W$, $\sin^2\theta_W = 0.23$

$B(Z \to e^+e^-) = 3.36\%$

Find $B(Z \to \bar{\nu}_e \nu_e)$

LH e^+e^- $\quad I_3 = -\frac{1}{2}$

RH e^+e^- $\quad I_3 = 0$

$Q = -1$

$B_{ee} \propto |g_L|^2 + |g_R|^2 = (0.23 - \frac{1}{2})^2 + 0.23^2 = 0.1258$

ν $\quad I_3 = +\frac{1}{2}$, LH only

$Q = 0$

$B_{\nu\nu} \propto 1/4$

$$\frac{B_{\nu\nu}}{3.36} = \frac{1/4}{0.1258} \Rightarrow B_{\nu\nu} = 6.68\% \text{ for } Z \to \nu_e \bar{\nu}_e$$

3) How do half lives of α decays of even-even nuclei depend on energy release, Q_α and atomic number Z of parent nucleus?

$$\ln\lambda \sim -\frac{Z'}{Q_\alpha^{1/2}} + \text{const.} \quad , \quad Z' = Z-2$$

$$\lambda = \frac{1}{\tau} \Rightarrow \ln\tau \sim \frac{Z-2}{Q_\alpha^{1/2}}$$

decay rates of ^{228}Ra and ^{228}Ac (α decay)

^{228}Ra : $Q_\alpha = 227.9828 - 223.97692 - 4.00151)\, Muc^2 = 4.37 \times 10^{-3}\, muc^2$
$= 6.57 \times 10^{-13}$ J

^{228}Ac : $Q_\alpha = 7.55 \times 10^{-13}$ J

expect $\sim$ same $\tau_{1/2}$

need Q_α in MeV $\Rightarrow \tau_{1/2} \sim \exp\left[\frac{88}{\sqrt{4.375}}\right] \sim 10^{10}$ yr

half lives for α decay much greater than β decay half lives
↳ α decays of ^{228}Ra and ^{228}Ac not seen

~~α decay of ^{232}Th~~

Allowed J^P of nucleus Y in $X \to Y + \alpha$

X = even-even $- J^P = 0^+$

final parity $P = (-1)^{l_\alpha}(-1)^{l_Y} = +1$ to conserve parity (strong interaction)

need l_α, l_Y both odd or both even

both odd $- \ P_Y = -1$, & $J_Y = 1, 3, 5 \ldots$

both even $- \ P_Y = +1$, $J_Y = 0, 2, 4 \ldots$

$J^P = 0^+, 1^-, 2^+, 3^-, \ldots$

α decay of ^{232}Th

decay to ground state - energy released $E_0 = \Delta m$

$E_0 = m(Th) - m_\alpha - m(Ra) = 4.37 \times 10^{-3}\ m_u c^2$

$E_0 = \alpha$ KE + recoil energy

$\xleftarrow[V_{Ra}]{}$ Ra $\quad$ $\alpha \xrightarrow[V_\alpha]{}$

momentum: $m_\alpha V_\alpha = m_{Ra} V_{Ra}$

KE: $\frac{1}{2} m_\alpha V_\alpha^2 + \frac{1}{2} m_{Ra} V_{Ra}^2 = E_0$

$$E_0 = \frac{1}{2} m_\alpha V_\alpha^2 \Big(1 + \underbrace{\frac{m_\alpha}{m_{Ra}}}_{= 0.0176}\Big) = 4.37 \times 10^{-3}\ m_u c^2$$

$$\frac{1}{2} m_\alpha V_\alpha^2 = \frac{4.37 \times 10^{-3}\ m_u c^2}{1.0176} = 6.4545 \times 10^{-13}\ J = 4034\ keV$$

= KE of α particle

α particle has highest KE when Th decays to Ra ground state - highest observed KE = 4011.2 keV - consistent with α KE for decay to ground state

Ratio of excitation energy for 2 excited states of ^{228}Ra

total energy released $E_0 = 4105$ keV

$$E_0 = \frac{1}{2} M_\alpha v_\alpha^2 \left(1 + \frac{M_\alpha}{M_{Ra}}\right) + E_{ex}$$

for $\frac{1}{2} M_\alpha v_\alpha^2 = 3948.5$ keV, $E_{ex} = 87.0$ keV

for $\frac{1}{2} M_\alpha v_\alpha^2 = 3810.0$ keV, $E_{ex} = 228$ keV

* ratio of excited states

$$\frac{4011 - 3810}{4011 - 3948.5} \sim 3.2$$ - consistent with being rotational excitations

compatible with ratio $J(J+1)$ for 4^+, 2^+ ~~ex~~ excited states

level of agreement of β-decay half lives with Sargent's rules (^{228}Ra, ^{228}Ac)

Q values : $= M_X - M_Y - m_e - m_\nu = 6 \times 10^{-4}$ (^{228}Ra decay)

2.83×10^{-3} (^{228}Ac decay)

Sargent's rule : $\tau_{1/2} \propto E_0^5$ $\Rightarrow$ ratio of $E_0 \sim 4.28 \times 10^{-4}$

ratio of half lives $\sim 10^4$

ground state of ^{228}Ac has $J^P = 3^+$, then in order of increasing energy, excited states have $J^P = 1^-, 1^+, 1^-, 1^+$

^{228}Ra - β-decay to ground and excited states of ^{228}Ac

(^{228}Ra is even-even $\Rightarrow J^P = 0^+$)

$J^P = 0^+ \to 1^+$ - no parity change

$l_{e\nu}$ even - $l_{e\nu} = 0$, $S_{e\nu} = 1$ $\Rightarrow$ GT allowed

$J^P = 0^+ \to 1^-$ - parity change - $l_{e\nu}$ odd

$l_{e\nu} = 1$, $S_{e\nu} = 0$ or 1 - F / GT 1st forbidden decay to ground state - $J^P = 0^+ \Rightarrow 3^+$

no parity change - l_{ev} even

$l_{ev} = 2$, $S_{ev} = 1$ - GT 2nd forbidden

allowed transitions ($0^+ \to 1^+$) should have the highest relative intensities (40%, 30%)

2nd forbidden decay to ground state - not seen

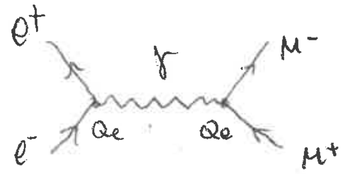
1st forbidden decays ($0^+ \to 1^-$) have ~~a~~ lower intensities than $0^+ \to 1^+$ - 20%, 10%

^{228}Ra 0^+

	Energy	Intensity
1^+	39.1keV/12.7keV	30% / 40%
1^-	39.5keV/25.6keV	10% / 20%
1^+	39.1keV/12.7keV	30% / 40%
1^-	39.5keV/25.6keV	10% / 20%
3^+		0%

4) $e^+e^- \to \mu^+\mu^-$

$\sigma_{\mu\mu} = \dfrac{4\pi\alpha^2}{3s}$

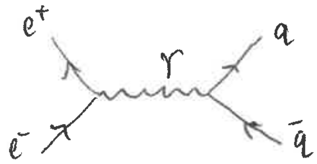


coupling $\propto Q_e$ at each vertex $= \sqrt{\alpha}$

multiply couplings in Feynman diagram to find matrix element
$M \propto \alpha$

$\sigma \propto |M|^2 \propto \alpha^2$

$e^+e^- \to$ hadrons



ratio $\sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-)$

hadronic decays for $2m_b < \sqrt{s} < 2m_t$

$u\bar{u}$, $c\bar{c}$, $d\bar{d}$, $s\bar{s}$, $b\bar{b}$

coupling $\propto \frac{2}{3}\alpha$ for up type quarks

$-\frac{1}{3}\alpha$ for down type quarks

Γ_{uu}, $\Gamma_{cc} \propto \frac{4}{9}\alpha^2$, Γ_{dd}, Γ_{ss}, $\Gamma_{bb} \propto \frac{1}{9}\alpha^2$

factor of 3 for colour - total hadron cross section

$\sigma \propto 3.\frac{4}{9}\alpha^2 \times 2 + 3.\frac{1}{9}\alpha^2 \times 3 = \left(\frac{8}{3} + 1\right)\alpha^2$

for $\mu^+\mu^-$ decay, $\sigma \propto \alpha^2 \Rightarrow$ ratio $\frac{11}{3}$

Measured max cross section $= 1.75 - 0.8$ nb $= 0.95$ nb

at $\sqrt{s} = 10.585$ GeV

calculate using ~~[illegible]~~ $\longrightarrow$

$\sigma = \dfrac{4\pi\alpha^2}{3s}$

Spin parity of resonance produced directly in e^+e^- collisions

$J^P = 1^-$

Estimate mass and total width of $\Upsilon(4S)$ meson $(b\bar{b})$

peak at 10.585 GeV $\therefore m(\Upsilon(4S)) \doteq 10.585$ GeV

FWHM $= 10.592 - 10.571 = 0.021$ GeV

- total width $\Gamma = 21$ MeV

Γ_{ee} for decay $\Upsilon(4S) \to e^+e^-$

$$\sigma_{res} = \frac{\pi g}{p^2}\frac{\Gamma_i \Gamma_F}{\Gamma^2/4} = \frac{4\pi g}{p^2}\frac{\Gamma_{ee}\Gamma_F}{\Gamma^2}$$

$$p = \tfrac{1}{2} m_\Upsilon \quad , \quad g = \frac{2.1+1}{(2.\frac{1}{2}+1)^2} = \frac{3}{4}$$

$$\sigma_{res} = \frac{12\pi}{m_\Upsilon^2}\frac{\Gamma_{ee}\Gamma_F}{\Gamma^2}$$

$\sigma_{res} = 0.95\,\text{nb} = 0.95\times10^{-7}\,\text{fm}^2 = 2.436\times10^{-12}\,(\text{MeV})^{-2}$

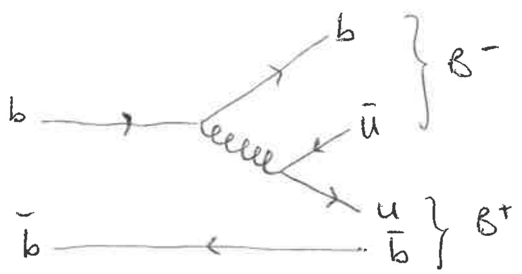
Note $\Gamma_F = \Gamma_{hadrons} \sim \Gamma$

$$\Gamma_{ee} = \frac{\sigma_{res} M_\Upsilon^2 \Gamma}{12\pi} = 1.92\times10^{-4}\ \text{MeV}$$

multiply by correction for σ_{res}

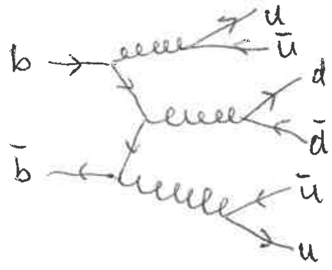
$b\bar{b}$ decays - $\Upsilon(nS)$

mass of $\Upsilon(4S)$ > 2 x mass of B mesons $\Rightarrow$ can decay via strong force to 2 x B

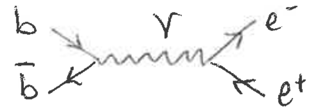


$\Upsilon(4S)$ decay

other $b\bar{b}$ resonances ($n<4$) must decay via Zweig-suppressed diagram to pions - B mesons too massive



- $n<4$ decays
  - decay to B mesons kinematically forbidden



decay to e^+e^- - same diagram for any of $\Upsilon(nS)$ resonances

Γ_{ee} similar for any of the resonances
but Zweig suppressed diagram is higher order than diagram for $\Upsilon(4S) \to$ hadrons diagram (6 vertices compared to 2) so cross sections / Γ much smaller for $\Upsilon(nS) \to$ hadrons ($n<4$) and total width is much smaller than for $n=4$

$m(u\bar{b}) = m_u + m_b + A \frac{S_u \cdot S_b}{m_u m_b}$, spin 0 $\Rightarrow S_1 \cdot S_2 = -\frac{3}{4}$

$m(u\bar{b}) = 5280 \text{ MeV} = 0.31 \text{ GeV} + 5 \text{ GeV} - \frac{3}{4} \frac{A}{5 \times 0.31 (\text{GeV})^2}$

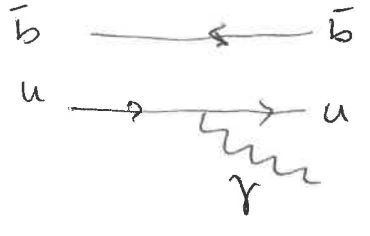
$A = 0.062 (\text{GeV})^3$

spin - 1 counterparts - $S_1 \cdot S_2 = \frac{1}{4}$

$m(u\bar{b}) = 0.31 \text{ GeV} + 5 \text{ GeV} + \frac{1}{4} \frac{0.062}{5 \times 0.31} \text{ GeV} = 5.35 \text{ GeV}$ (spin 1)

mass difference $(B^* - B) = 70 \text{ MeV} < m_\pi$ $\therefore$ can't decay via strong force

EM decay



B^* mesons decay to B mesons by EM interaction