UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 28 May 2014 1.30 pm to 3.30 pm

PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)
MPHIL IN NUCLEAR ENERGY

PARTICLE & NUCLEAR PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *printed sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A charged particle moving at a speed βc through a medium with refractive index n can emit Čerenkov photons at an angle α to its direction of motion if βc exceeds the speed of light in that medium. (c is the speed of light in vacuo.) Write down a formula relating the Čerenkov angle α to β and n. [*In order to derive or check your answer you may find it helpful to draw a sketch showing the relationship between the advancing particle and its Čerenkov wave-front.*] [4]

(b) Given that the ω^0 meson (mass 783 MeV/c^2) and the π^0 meson (mass 135 MeV/c^2) both have a non-zero overlap with the $u\bar{u}$ wavefunction, and given that the ω^0 has spin-parity quantum numbers $J^P = 1^-$ whereas the π^0 has $J^P = 0^-$, discuss whether the ω^0 can decay to a state containing $\pi^0\pi^0$ and nothing else. [4]

(c) Estimate the size of the ratio of cross sections:
$\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ for electrons and positrons colliding at centre of mass energies approximately half way between the threshold for charm quark pair production and the threshold for bottom quark pair production. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **one** of the following: [13]

(a) the Semi-Empirical Mass Formula;

(b) determination of nuclear size and shape;

(c) nuclear alpha decays.

3 *Attempt* **either** *this question* **or** *question 4.*

Relativistic kinematics:

Suppose that a massive particle C of mass m_C decays (isotropically in its rest frame) into a massless particle t and a massive particle B of mass m_B. Suppose further that B then decays (isotropically in its rest frame) into a massless particle s and a massive particle A of mass m_A. You may assume that $m_C > m_B > m_A > 0$.

(a) Determine:

(i) the magnitude of the three-momentum of particle C in the rest frame of particle B in terms of any relevant masses, and

(ii) the magnitude of the three-momentum of particle A in the rest frame of particle B in terms of any relevant masses. [5]

(b) Hence, or otherwise, determine the largest and smallest values which can be taken by the following invariant mass-squared quantities:

(i) $m_{\mathrm{st}}^2 \equiv (p_\mathrm{s} + p_\mathrm{t})^\mu (p_\mathrm{s} + p_\mathrm{t})_\mu \equiv (E_\mathrm{s} + E_\mathrm{t})^2 - (\boldsymbol{p}_\mathrm{s} + \boldsymbol{p}_\mathrm{t})^2$;

(ii) $m_{\mathrm{At}}^2 \equiv (p_\mathrm{A} + p_\mathrm{t})^\mu (p_\mathrm{A} + p_\mathrm{t})_\mu \equiv (E_\mathrm{A} + E_\mathrm{t})^2 - (\boldsymbol{p}_\mathrm{A} + \boldsymbol{p}_\mathrm{t})^2$. [10]

Hadronic interactions:

The particles X^0 and Y^- can be produced in the strong interaction processes $K^- + p \to \pi^0 + X^0$ and $K^- + p \to K^+ + Y^-$ respectively.

(c) Draw a Feynman diagram for each of these processes and thereby deduce the quark content of the X^0 and the Y^-. [4]

The X^0 and Y^- decay by the mechanisms $X^0 \to \Lambda + \gamma$ and $Y^- \to \Lambda + \pi^-$.

(d) What can be deduced about the lifetimes, spins, and/or parities of the X^0 and Y^- based on the information provided? [6]

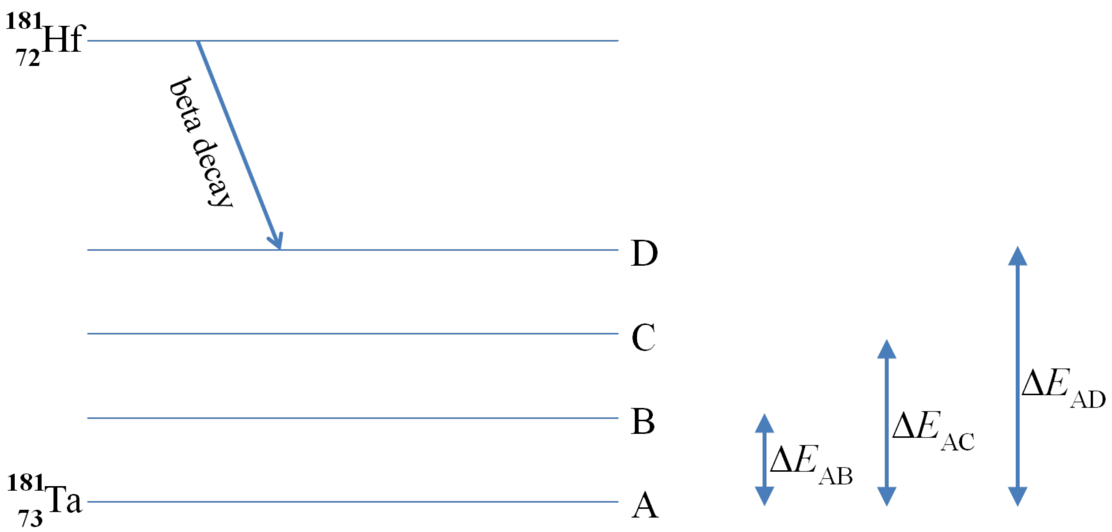
[*The quark content of the* K^+ *is* $u\bar{s}$. *The* X^0 *has mass* $1193\,\mathrm{MeV}/c^2$. *The* Y^- *has mass* $1321\,\mathrm{MeV}/c^2$. *The* Λ *has mass* $1116\,\mathrm{MeV}/c^2$ *and* $J^P = \frac{1}{2}^+$. *The charged pion has mass* $140\,\mathrm{MeV}/c^2$ *and* $J^P = 0^-$. *The photon has* $J^P = 1^-$.]

4 *Attempt* **either** *this question* **or** *question 3.*

This page consists of data, which you will need to answer the questions appearing on the next (facing) page.

Nuclear Gamma Decays:

The diagram below shows the lowest four nuclear energy levels of $^{181}_{73}$Ta, named A to D. The displacements of the energy levels of the excited states with respect to the ground state are labelled by the quantities ΔE_{AB}, ΔE_{AC} and ΔE_{AD}. Assume that transitions between $^{181}_{73}$Ta states, if they are permitted, are rapid, and occur only through gamma radiation. Also shown is the ground state of $^{181}_{72}$Hf. This undergoes beta-decay to one of the $^{181}_{73}$Ta nuclear energy levels, as indicated. The energy level spacings are not drawn to scale.



A sample of $^{181}_{72}$Hf is placed inside a gamma ray spectrometer in order to examine the structure of $^{181}_{73}$Ta nuclear energy levels. The following gamma ray energies, intensities and multipolarities are seen by the spectrometer, together with no other spectral lines of comparable or greater intensities:

Spectral line	γ energy (keV)	Intensity (arbitrary units)	Multipolarity
1	133.02 ± 0.02	16.5	E2
2	136.25 ± 0.02	2.4	M1+E2
3	345.85 ± 0.2	2.4	E2
4	482.0 ± 0.2	14.1	M1+E2
5	615.5 ± 0.5	0.04	M3

(a) From the energy and intensity data in the table above, together with the preceding explanatory text, determine the values that ΔE_{AB}, ΔE_{AC} and ΔE_{AD} could take. You should find that there are four possible solutions. For each solution draw an energy level diagram clearly showing which energy levels are connected by each of the five spectral lines. [7]

(b) Describe what the symbols $E1$, $E2$, $E3$, $M1$, $M2$ and $M3$ mean, and indicate (i) the angular momentum and parity quantum numbers J^P associated with the gamma radiation for each, and (ii) the expected hierarchy among the decay rates for decays mediated by these six modes. [5]

(c) Giving examples, explain, in general terms, how the presence and absence of spectral lines with measured multipolarities and energies at certain intensities can be used to constrain the spins and parities of excited states of a nucleus with known energy levels. [5]

(d) Assume that the ground state A has angular momentum and parity quantum numbers $J^P = \frac{7}{2}^+$. Of the four energy level diagrams sketched in (a) use the one in which spectral lines 2 and 4 are both decays to the ground state. Making use of any of the spectrometer data (including multipolarities) provided earlier, deduce what you can about the allowed values of the spin and parity quantum numbers J^P for the remaining levels, B, C and D. [8]

END OF PAPER