UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 28 May 2015 9.00 am to 11.00 am

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)
OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) What is the Jones matrix for a linear polarizer in the xy-plane with its polarizing direction at an angle θ to the x-axis? An optical system consists of a sequence of optical elements: (i) a fixed linear polarizer; (ii) a linear polarizer rotating in its own plane with angular frequency ω; (iii) a linear polarizer rotating in its own plane with angular frequency 2ω and in the opposite sense to the first; (iv) a second fixed linear polarizer. At $t = 0$ all the polarizing directions are parallel to the x-axis. If an unpolarized beam of intensity I is incident on the system, show that the output intensity is $\frac{1}{2} I \cos^2 \omega t \cos^2 2\omega t \cos^2 3\omega t$. [4]

(b) A superconductor can be modelled semiclassically as a plasma of charged particles of mass m, charge q and number density n, whose motion is undamped. Show that an applied static magnetic field cannot penetrate such a material for distances much larger than the London penetration depth $\lambda_{\mathrm{L}} = c/\omega_{\mathrm{p}}$ where $\omega_{\mathrm{p}} = \sqrt{nq^2/\epsilon_0 m}$. [4]

(c) Show that the *phase* of an oscillation is a Lorentz invariant and therefore that, for a travelling wave of angular frequency ω and wavevector $\boldsymbol{k}$, the quantity $(\omega/c, \boldsymbol{k})$ is a four-vector. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the Faraday effect;

(b) the physical mechanisms leading to the broadening of atomic spectral lines;

(c) Thomson and Rayleigh scattering.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain what are meant by the terms *radiation resistance*, *power gain* and *effective area* when applied to radio antennas. [5]

Outline briefly the basic arguments which show that, for a receiving antenna connected to a matched load and operating at wavelength λ, the effective area for absorption A_{eff} and the power gain G are related by

$$A_{\text{eff}}(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi) \quad . \qquad [10]$$

A radar set operating at frequency ν has an antenna (used for both transmission and detection) which emits a narrow beam of radio waves with angular Poynting flux given (using conventional polar co-ordinates) by

$$N(\theta, \phi) = C \exp\left(-\frac{(\theta - \pi/2)^2}{2\alpha^2}\right) \exp\left(-\frac{\phi^2}{2\alpha^2}\right) \quad ,$$

where $\alpha \ll \pi/2$ and C is a constant. The total emitted power is P. The receiver input impedance R is matched to the antenna's radiation resistance. If the detection limit is set by V_{m}, the minimum r.m.s. voltage which can be detected across R, find an expression for the maximum range D at which the radar set can detect a perfectly conducting sphere of radius a ($\ll c/\nu$) placed on the x-axis. [10]

[*The impedances of both the antenna and receiver can be assumed to be purely real.*

The Johnson noise mean-square voltage fluctuations in the frequency range $\nu \to \nu + \mathrm{d}\nu$ *for a resistor* R *at temperature* T *is* $\langle V^2 \rangle = 4k_{\text{B}}TR\mathrm{d}\nu$. *The total power emitted by an oscillating electric dipole of r.m.s. magnitude* p *is* $\dfrac{8\pi^3\nu^4p^2}{3\epsilon_0c^3}$, *and the polarizability of a conducting sphere of radius* a *is* $4\pi\epsilon_0a^3$.]

4 *Attempt* **either** *this question* **or** *question 3.*

The transformation equations for the components of the electric and magnetic fields between the laboratory frame S and a frame S′ moving at speed v along the x-axis are:

$$
\begin{aligned}
E'_x &= E_x & \qquad B'_x &= B_x \\
E'_y &= \gamma\,(E_y - vB_z) & \qquad B'_y &= \gamma\left(B_y + \frac{v}{c^2}E_z\right) \\
E'_z &= \gamma\,(E_z + vB_y) & \qquad B'_z &= \gamma\left(B_z - \frac{v}{c^2}E_y\right)
\end{aligned}
$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

Show that if an electromagnetic wave is (arbitrarily) polarized when viewed in frame S, its polarization state is the same when viewed in frame S′. [9]

The upper surface of a thick slab of dielectric with isotropic refractive index n lies in the xy-plane and is moving at speed v along the x-axis. A beam of unpolarized light travelling along the z-axis is incident on the moving slab from above. Explain why the reflected beam is in general partially polarized. [8]

For what value of v is the reflected beam completely linearly polarized? In which direction is the polarization? [8]

END OF PAPER