UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 25 May 2016 1.30 pm to 3.30 pm

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)
OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) A charged particle with kinetic energy T passes through a slab of material with refractive index n. Electromagnetic radiation is observed in a direction at angle θ to the particle's direction of motion. What is the particle's mass? [4]

(b) Compare the coherence length and coherence width of sunlight at the surface of the Earth. [4]

(c) At time $t = 0$ an electron is in a circular orbit of radius a_0 around a proton. Show that, according to classical physical arguments alone and assuming that the fraction loss of energy per orbit is small, the radius r of the orbit will decrease with t according to:

$$r^3 = a_0^3 - 4r_0^2 ct$$

where $r_0 = \dfrac{e^2}{4\pi\epsilon_0 mc^2}$ is the classical radius of the electron. [4]

$\left[\textit{The instantaneous power } P \textit{ radiated by an electron with acceleration } \boldsymbol{\alpha} \textit{ is given by the Larmor formula } P = \dfrac{\mu_0 e^2}{6\pi c}|\boldsymbol{\alpha}|^2.\right]$

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) birefringence;

(b) synchrotron radiation;

(c) photonic crystals.

3 *Attempt* **either** *this question* **or** *question 4.*

Define the *magnetic vector potential* $\boldsymbol{A}$ and explain what is meant by a *gauge transformation*. [3]

Show that in free space in the presence of a steady free current density $\boldsymbol{J}$ and with suitable choice of gauge:

$$\boldsymbol{\nabla}^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}, \quad [4]$$

and hence show that:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int_{\text{all space}} \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} \, \mathrm{d}^3\boldsymbol{r}'. \quad [3]$$

If $\boldsymbol{J}$ varies explicitly with time, explain how this expression should be modified to take into account the effects of *retardation*. [2]

A long straight wire lies along the z-axis. At $t = 0$ a current I is suddenly established uniformly along the wire, which remains always electrically uncharged. Show that at time t at a distance r ($< ct$) from the wire:

$$\boldsymbol{A}(r, t) = \widehat{\boldsymbol{z}} \, \frac{\mu_0 I}{2\pi} \ln\left(\frac{ct}{r} + \sqrt{\left(\frac{ct}{r}\right)^2 - 1} \right). \quad [7]$$

Hence calculate the electric and magnetic fields $\boldsymbol{E}(r, t)$ and $\boldsymbol{H}(r, t)$, and the Poynting vector $\boldsymbol{N}(r, t)$ for some fixed value of r. [4]

Sketch graphs of, and comment on, your results. [2]

[*You may use the result* $\displaystyle\int \frac{\mathrm{d}z}{\sqrt{z^2 + a^2}} = \ln\left(z + \sqrt{z^2 + a^2}\right) + \text{constant}$.

In cylindrical co-ordinates (r, θ, z), $\nabla \times \boldsymbol{A} = \begin{vmatrix} \widehat{\boldsymbol{r}}/r & \widehat{\boldsymbol{\theta}} & \widehat{\boldsymbol{z}}/r \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{vmatrix}$.]

(TURN OVER

4 *Attempt* **either** *this question* **or** *question 3.*

The power $\langle P \rangle$ radiated by an electric dipole p is given by

$$\langle P \rangle = \frac{\mu_0 \langle \ddot{p}^2 \rangle}{6\pi c}.$$

Show that Rayleigh scattering from spherical dielectric particles of size much smaller than the wavelength of the incident radiation is strongly dependent on the frequency of the radiation. [4]

If the incident radiation is propagating along Ox and is linearly polarized along Oz, describe, without detailed calculation, the angular profile of the scattered radiation intensity and its polarization. [2]

Radiation propagating along Ox is scattered by a needle-shaped particle at the origin O. The particle has length much less than the radiation wavelength, and diameter much smaller still, and is formed from a material with a very large dielectric constant. The long axis of the particle is oriented along the direction defined by (θ, ϕ) in standard spherical polar co-ordinates. For a given incident intensity I_0, what are the polarization states and relative intensities of the radiation scattered along the Oy-direction when the incident radiation is:

(a) linearly polarized along Oz;

(b) linearly polarized along Oy. [6]

Use your results for cases (a) and (b) to deduce the polarization states of the radiation scattered along the Oy-direction when the incident radiation is

(c) right hand circularly polarized;

(d) unpolarized? [4]

In case (a) above, the particle is now replaced by a diffuse cloud of similar particles with random orientation. The inter-particle spacings are large compared with the coherence length of the incoming radiation so that the particles scatter mutually incoherently. Neglecting multiple scattering, calculate the relative intensities of radiation measured by a distant observer along Oy through a linear polarizer with its axis first parallel to Ox, and then parallel to Oz, and comment on your results. [7]

How would this be modified if the randomly oriented particles were densely packed to form a continuous dielectric medium? [2]

END OF PAPER