UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 1 June 2017 9.00 am to 11.00 am

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)
OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

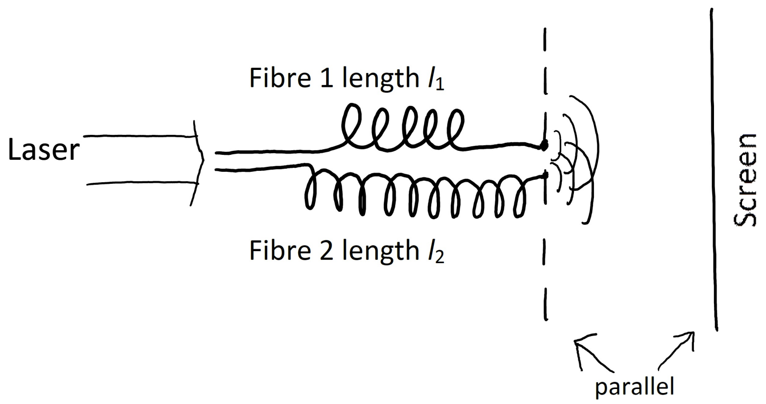
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

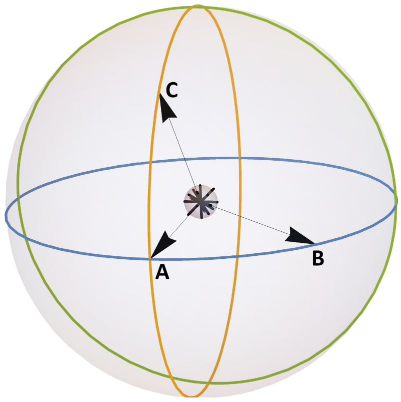
1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) Illuminating a flat glass surface with unpolarised light leads to the observation that, for a particular angle of illumination θ, the reflected light is linearly polarised. Explain this effect and calculate θ for a glass with a refractive index of $n = 1.5$. [4]

(b) Light from an approximately single-frequency laser is split into two beams of equal amplitude and is guided through two separate single-mode fibres with lengths $l_1 = 10$ m and $l_2 = 20$ m. The output ends of the fibres are placed a small distance from each other and the resulting interference pattern is observed on a screen as shown in the diagram below. The laser light coming from the left hits the two fibre-ends with the same phase. Sketch and describe the interference signal for the cases of (i) an ideal and (ii) a realistic single-frequency laser. Assuming a Lorentzian line shape, what is the linewidth of the laser if the fringe visibility is $V = 0.5$? Assume that the phase velocity of the fibre is given by $n_{\text{eff}} = 2$. [4]



(c) You are observing the total intensity of radiation from a Hertzian dipole of unknown orientation from a distance R = 100 m in the direction labelled A on the diagram below. You measure an intensity of I = 0.1 W/m^2. Repeating the measurement after rotating the detector position by 45°around the dipole in two orthogonal directions, B and C, gives an identical result. What is the orientation of the dipole and the total radiated power? [4]



In this diagram, black arrows denote the three detector positions while the inner sphere represents the dipole and the short lines illustrate possible dipole orientations.

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) why the sky appears blue, covering the relevant scattering mechanism and stating and explaining the wavelength dependence;

(b) building a steerable antenna without moving elements;

(c) Čerenkov radiation, covering both its creation and its angular emission characteristics.

3 *Attempt* **either** *this question* **or** *question 4.*

What is the most general form of a Jones vector describing a pure polarisation, and how many degrees of freedom does it have? [2]

Consider a setup of three consecutive linear polarisers along the z-axis that is illuminated with right-hand circular polarised light. The first polariser transmits light that is linearly polarised along the x direction, and the third transmits only y-polarised light. The middle polariser can be rotated in the x, y-plane. Calculate and plot the transmitted intensity fraction as a function of the rotation angle θ of the middle polariser, where $\theta = 0$ corresponds to the polariser transmitting the x-polarisation. [4]

Explain how a waveplate works and derive the Jones matrix for a quarter-wave plate. [4]

Calculate and plot the transmission of the setup above if the middle polariser is replaced by a rotatable half-wave plate, whose fast axis corresponds with the x-axis for $\theta = 0$. [4]

Repeat the above analysis for a quarter-wave plate instead of the half-wave plate. [2]

Is it possible to create any arbitrary elliptical polarisation state starting from left-hand polarised light using only half-wave and quarter-wave plates? Justify your answer by either an explicit construction or by arguing its impossibility. [4]

In many applications one wants to be able to switch the polarisation rapidly by applying either an electric field (Kerr effect) or a magnetic field (Faraday effect) to some appropriate material. Explain the effects in both cases and argue why only one of these would be suitable for creating a switchable quarter-wave plate. [5]

[*The 2D rotation matrix through an angle θ is given by:*

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$
]

4 *Attempt* **either** *this question* **or** *question 3.*

Describe the concept of gauge freedom in the definition of the magnetic vector potential. [2]

Consider the vector potential of an infinitely long, homogeneous solenoid, whose symmetry axis is the z-axis. In cylindrical coordinates, two of the three components of the vector potential vanish in a particular well-known gauge. Which gauge is this and which component is non-zero? [2]

How does the magnitude of the non-zero component scale with the distance r from the axis of the solenoid? Consider both $r < R$ and $r > R$, where R is the radius of the solenoid. [3]

Consider an infinitely long dielectric cylinder of radius R and $\epsilon = \mu = 1$ with surface charge density σ rotating around its symmetry axis at angular frequency ω. The symmetry axis of the cylinder is the z-axis. Calculate and sketch the resulting electric field. [3]

Now assume that the rotating cylinder considered above is uniformly charged with volume charge density ρ. Calculate and sketch the resulting magnetic field inside the cylinder. [*Consider the solid cylinder as a set of charged hollow cylinders.*] [4]

Consider an electron initially at rest at a distance $d > R$ from the axis of an infinitely long solenoid with radius R and n windings per unit length. What is the final velocity of the electron if an initial current I in the solenoid is almost instantly reduced to zero. [4]

Describe a possible experiment to quantitatively measure the vector potential outside of the above solenoid for a constant current I, and express the measured quantity as a function of I. Comment on the effect of gauge freedom on your approach. [4]

Describe the magnetic field created by a rotating, uniformly charged sphere of radius R for distances $r \gg R$. How does the average strength of the field scale with r in this limit? [3]

END OF PAPER