PNP 2018

1) a) $e^+e^- \to Z^0 \to e^+e^-$

max cross section $2.0\,nb$ at $\sqrt{s} = 91.2\,GeV$

estimate branching ratio for decay $Z_0 \to e^+e^-$

$$g = \frac{2J_Z+1}{(2J_e+1)^2} = \frac{3}{4}$$

at resonance, $\sigma_{max} = \frac{\pi g}{p^2} \frac{(\Gamma_{ee})^2}{\Gamma^2/4} = \frac{4\pi g}{p^2} B_{ee}^2$

$p = \frac{1}{2}\sqrt{s} \Rightarrow \sigma_{max} = \frac{12\pi}{s} B_{ee}^2$

$$B_{ee} = \sqrt{\frac{\sigma_{max}\, s}{12\pi}} = 3.4\%$$

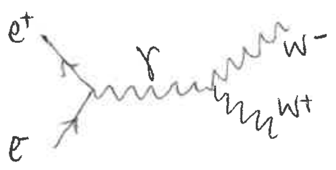
b) possible multipoles for γ decay of excited nuclear state of spin-parity $\frac{5}{2}^+$ to lower energy state with $J^P = \frac{5}{2}^+$

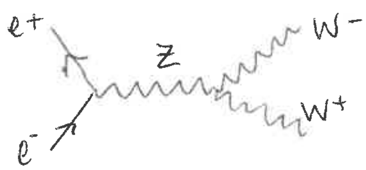
no parity change   EL transitions have L even
ML transitions have L odd

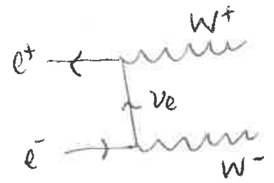
multipoles M_1, E2, M3, E4, M5

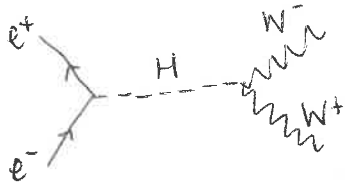
M1, E2 likely to make greater contribution

c) 4 leading order Feynman diagrams for $e^+e^- \to W^+W^-$









negligible contribution to cross section from Higgs diagram
coupling $\propto$ mass, $e^\pm$ have very small mass $\Rightarrow$ very small coupling to Higgs

B2 $M(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A} - \delta(A)$

$$\delta(A) = \begin{cases} a_P A^{-3/4} & \text{even-even} \\ -a_P A^{-3/4} & \text{odd-odd} \\ 0 & \text{even-odd} \end{cases}$$

Spontaneous fission $(A,Z) \to (A_1, Z_1) + (A_2, Z_2)$

$A_1 = yA,\ Z_1 = yZ,\quad A_2 = (1-y)A,\ Z_2 = (1-y)Z$

Show that energy release for this process is a maximum for symmetric fission $(y = \frac{1}{2})$

ignoring pairing terms, estimate $\frac{Z^2}{A}$ value above which fission should be energetically possible

energy release $E_0 = M(A,Z) - M(A_1,Z_1) - M(A_2,Z_2)$

$$E_0 = m_p(Z - yZ - (1-y)Z) - (A-Z)m_n(1-y-(1-y)) - a_V A(1-y-(1-y))$$
$$+ a_S A^{2/3}(1 - y^{2/3} - (1-y)^{2/3}) + \frac{a_C Z^2}{A^{1/3}}(1 - y^{5/3} - (1-y)^{5/3})$$
$$+ a_A \frac{(A-2Z)^2}{A}(1-y-(1-y))$$

$$= a_S A^{2/3}(1 - y^{2/3} - (1-y)^{2/3}) + \frac{a_C Z^2}{A^{1/3}}(1 - y^{5/3} - (1-y)^{5/3})$$

$$\frac{\partial E_0}{\partial y} = \frac{2}{3} a_S A^{2/3}(-y^{-1/3} + (1-y)^{-1/3}) + \frac{a_C Z^2}{A^{1/3}} \frac{5}{3}(-y^{2/3} + (1-y)^{2/3})$$

$= 0$ for max energy release

for $y = 1-y$ $(y = \frac{1}{2})$, $\frac{\partial E_0}{\partial y} = 0$

- max energy released for $y = \frac{1}{2}$ - symmetric fission

threshold for fission to be energetically possible - $E_0 = 0$

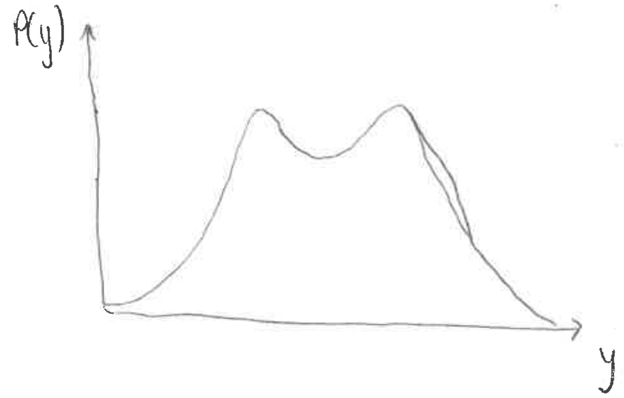
$$a_S A^{2/3}(1 - 2(\tfrac{1}{2})^{2/3}) + \frac{a_C Z^2}{A^{1/3}}(1 - 2(\tfrac{1}{2})^{5/3}) = 0$$

$$-0.260\, a_S + 0.370\, a_C \frac{Z^2}{A} = 0$$

$$\frac{Z^2}{A} = 17.6$$

in practise $\frac{Z^2}{A}$ must be much larger - nuclei have to deform before splitting, passing through intermediate state where surface energy increases but Coulomb energy hasn't yet been reduced much
- tunnelling process, must overcome potential barrier
Coulomb energy only decreases significantly after fission
only favourable for nuclei with large $\frac{Z^2}{A}$

distribution of y values for fission fragments



Z, N near magic numbers
Z/N is the same as parent

Estimate excitation energies of $^{236}U^*$ and $^{239}U^*$ formed when ^{235}U and ^{238}U capture a ~~neutron~~ neutron of negligible KE

$$\Delta m = a_v + a_s\left(A^{2/3} - (A+1)^{2/3}\right) + a_c Z^2\left(\frac{1}{A^{1/3}} - \frac{1}{(A+1)^{1/3}}\right) + a_A\left[\frac{(A-2Z)^2}{A} - \frac{(A+1-2Z)^2}{A+1}\right]$$

$$+ a_p (A+1)^{-3/4} \quad \text{for } A = 235$$

$$\Delta m = -1.94 + 1.4 - 9.15 + 0.556 + 15.8 = 6.66 \text{ MeV}$$

for $A = 238$, pairing term has opposite sign

$$\Delta m = 15.8 - 1.94 + 1.37 - 9.51 - 0.551 + 5.17 = 5.18 \text{ MeV}$$

difference in excitation energies

$E_{ex} = 6.66$ MeV (^{235}U)
$\quad\quad 5.18$ MeV (^{238}U)

due to opposite sign of pairing term

Observed excitation energies

^{235}U - 6.5 MeV

^{238}U - 4.8 MeV

activation energies

^{235}U - 6.2 MeV

^{238}U - 6.6 MeV

Explain why thermal neutrons can induce rapid fission of ^{235}U but not ^{238}U

^{235}U - excitation energy > activation energy - neutron induced fission occurs down to zero neutron energy with large cross section

^{238}U - excitation energy < activation energy - neutron induced fission only possible above neutron threshold energy ~ 2 MeV

Nuclear reactor design - prompt neutrons from fission have high energy (~ 2MeV) $\Rightarrow$ small fission cross section

To exploit large ^{235}U fission cross section, neutrons must be moderated down to thermal energy before being absorbed - collisions with rods of ^{12}C

B3 Explain why the existence of spin $\frac{3}{2}$ baryons with flavour content uuu, ddd, sss provides evidence for the existence of the colour degree of freedom

Baryons are fermions - need overall wavefunction ψ_{baryon} antisymm under exchange of any 2 quarks.

if $\psi_{baryon} = \psi_{spatial} \psi_{spin} \psi_{flavour} \psi_{colour}$:

$\psi_{spatial}$ is symm. for $l=0$ baryons

ψ colour is always antisymm

If $J = \frac{3}{2}$, all quarks have spin up and ψ_{spin} is symm

$\psi_{flavour}$ is symm if all quarks have same flavour

$\therefore$ without colour degree of freedom, overall ψ_{baryon} would be symmetric for baryons with $J = \frac{3}{2}$ and quark content uuu, ddd, sss $\Rightarrow$ require a colour component which is antisymmetric under quark exchange

uu quark pair in a uus baryon

for overall antisymmetric ψ_{baryon}, require product $\psi_{spin} \psi_{flavour}$ to be symmetric

for the uu part, $\psi_{flavour}$ is symm $\Rightarrow$ require ψ_{spin} symm under uu exchange $\Rightarrow$ total spin 1 for uu part

mass term $A' \dfrac{S_1 \cdot S_2}{m_1 m_2}$ - interaction of 2 quark magnetic dipole moments

mass of uus baryon

spin $\frac{3}{2}$ - all quarks have spin up

$S_i \cdot S_j = \frac{1}{2}(S^2 - S_i^2 - S_j^2) = \frac{1}{2}[S(S+1) - S_i(S_i+1) - S_j(S_j+1)]$

$S = 1$ for any pair of quarks $\Rightarrow$ $S_i \cdot S_j = \frac{1}{4}$ for any quark pair spin $\frac{3}{2}$: $M_{uus} = 2m_u + m_s + A'\left[\frac{S_{u_1}\cdot S_{u_2}}{m_u^2} + \frac{S_{u_1}\cdot S_s + S_{u_2}\cdot S_s}{m_u m_s}\right]$

$= 2m_u + m_s + \frac{A'}{4}\left[\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right]$

spin $\frac{1}{2}$: uu part has $S = 1$

$M_{uus} = 2m_u + m_s + A'\left[\frac{S_{u_1}\cdot S_{u_2}}{m_u^2} + \frac{S_{u_1}\cdot S_s + S_{u_2}\cdot S_s}{m_u m_s}\right]$

where $S_{u_1}\cdot S_{u_2} = \frac{1}{2}\left[S(S+1) - S_{u_1}(S_{u_1}+1) - S_{u_2}(S_{u_2}+1)\right]$

$= \frac{1}{4}$

$S^2 = S_{u_1}^2 + S_{u_2}^2 + S_s^2 + 2\left[S_{u_1}\cdot S_{u_2} + S_{u_1}\cdot S_s + S_{u_2}\cdot S_s\right]$

$\frac{3}{4} = \frac{3}{4}\cdot 3 + \frac{1}{2} + 2\left[S_{u_1}\cdot S_s + S_{u_2}\cdot S_s\right]$

$-2 = 2\left[S_{u_1}\cdot S_s + S_{u_2}\cdot S_s\right]$

$M_{uus} = 2m_u + m_s + A'\left[\frac{1/4}{m_u^2} + - \frac{1}{m_u m_s}\right]$

$= 2m_u + m_s + \frac{A'}{4}\left[\frac{1}{m_u^2} - \frac{4}{m_u m_s}\right]$

ratio of mass differences $M(\Sigma^*) - M(\Sigma)$ and $M(\Xi^*) - M(\Xi)$

with $m_u = m_d$, quark model predicts difference

$\frac{A'}{4}\left(\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right) - \frac{A'}{4}\left(\frac{1}{m_u^2} - \frac{4}{m_u m_s}\right) = \frac{3A'}{2m_u m_s}$

same value predicted for $M(\Sigma^*) - M(\Sigma)$ and $M(\Xi^*) - M(\Xi)$

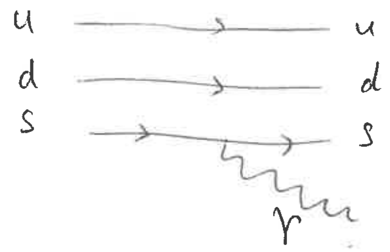
data - $M(\Sigma^*) - M(\Sigma) = 190, 192, 194$

$M(\Xi^*) - M(\Xi) = 213, 217$

ratio ~ 1.12 - agreement to within $\sim 10\%$

Σ^0 decays as $\Sigma^0 \to \Lambda^0 + \gamma$

Feynman diagram



Energy of emitted photon

in Σ^0 rest frame, initial energy m_Σ

$$E_\Lambda + E_\gamma = m_\Sigma$$

$$E_\Lambda^2 = (m_\Sigma - E_\gamma)^2 = m_\Sigma^2 + E_\gamma^2 - 2m_\Sigma E_\gamma$$

$$E_\Lambda^2 = m_\Lambda^2 + p_\Lambda^2 = m_\Lambda^2 + E_\gamma^2$$ as photon and Λ^0 have equal and opposite momenta

$$m_\Lambda^2 + E_\gamma^2 = m_\Sigma^2 + E_\gamma^2 - 2m_\Sigma E_\gamma$$

$$E_\gamma = \frac{m_\Sigma^2 - m_\Lambda^2}{2m_\Sigma} = \frac{1193^2 - 1116^2}{2.1193} = 74.5 \text{ MeV}$$

spin $\frac{3}{2}$ baryons more likely to decay via strong force

proton has nothing lighter to decay to $\Rightarrow$ must decay via weak force

neutron can't decay via strong force as pion mass greater than difference between proton and neutron masses

$\Lambda, \Sigma, \Xi, \Omega$ would have to decay to kaons if they decayed via strong force

but mass difference $< m_{kaon}$ $\therefore$ can't decay via strong force

Δ, Σ^*, Ξ^* can all decay via strong force

$\Delta \to n + \pi$, $\Sigma^* \to \Sigma + \pi$, $\Xi^* \to \Xi\pi$

involve $g \to q\bar{q}$

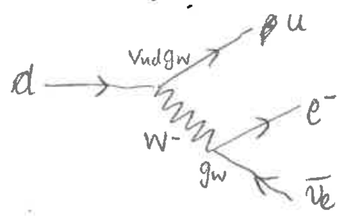
B4 precise determination of $|V_{ud}|$ from measurements of $0^+ \to 0^+$ nuclear β decays

Classify : $0^+ \to 0^+ e\nu$

no parity change $\Rightarrow$ $l_{e\nu}$ even

$l_{e\nu} = 0$ allowed - allowed Fermi decay

involves interaction



couplings g_W, $V_{ud}\, g_W$

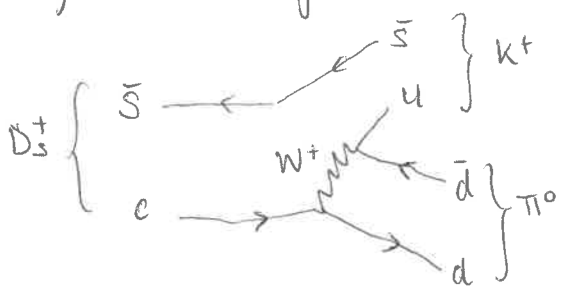
Matrix element $\mathcal{M} \propto V_{ud}\, g_W^2$

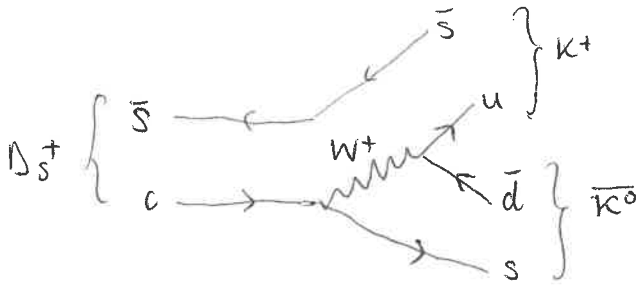
decay rate $\Gamma \propto |\mathcal{M}|^2 \propto |V_{ud}|^2 g_W^4$

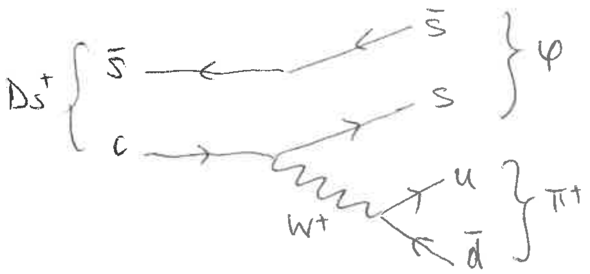
D_s^+ decays to $K^+\pi^0$, $K^+\overline{K^0}$, $\varphi\pi^+$

branching ratios 6.3×10^{-4}, 2.9×10^{-2}, 4.5×10^{-2}

Feynman diagrams







$K^+\pi^0$: $\Gamma \propto |V_{ud}V_{cd}|^2 g_W^4$

$K^+\overline{K^0}$: $\Gamma \propto |V_{ud}V_{cs}|^2 g_W^4$

$\varphi\pi^+$: $\Gamma \propto |V_{cs}V_{ud}|^2 g_W^4$

expect ratio

$|V_{ud}V_{cd}|^2 : |V_{cs}V_{ud}|^2 : |V_{cs}V_{ud}|^2$

$= 0.053 : 1 : 1$

observed ratio

$0.022 : 1 : 1.55$

D_s^+, π, K have $J^P = 0^-$

φ has $J^P = 1^-$

decays of the form $0^- \to 0^- + 0^-$ must have $l=0$ in final state $\Rightarrow$ total parity afterwards $P = (-1)^0 = +1$

↳ parity violated in D_s^+ decays to K, π mesons are weak

For a decay $0^- \to 1^- + 0^-$ must have $l=1$ in final state

so parity conserved ($=-1$ before and after) - not necessarily weak decay for $D_s^+ \to \varphi\pi^+$

estimate branching ratio for $W^- \to e^-\bar{\nu}_e$

possible decay products

$e^-\bar{\nu}_e$, $\mu^-\bar{\nu}_\mu$, $\tau^-\bar{\nu}_\tau$, $u\bar{d}$, $u\bar{s}$, $u\bar{b}$, $c\bar{s}$, $c\bar{d}$, $c\bar{b}$

$\Gamma = \Gamma_{e\bar{\nu}_e}\left(3 + 3\left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right) + 3\left(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2\right)\right)$

$= 9\,\Gamma_{e^-\bar{\nu}_e}$

$B_{ee} = \frac{1}{9}$

fraction of W^- decays that contain a b quark in final state

$f = 3\left(|V_{ub}|^2 + |V_{cb}|^2\right) \cdot \frac{1}{9} = 0.057\%$

muon mass 106 MeV, lifetime $\tau_\mu = 2.2\,\mu s$

decays as $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

assuming Sargent's rule applies to muon and top quark decays find top quark lifetime

$M_t = 175$ GeV

analogous decay to muon decay is $t \to b e \nu$

$$BR(t \to be\nu) = \frac{\Gamma(t \to be\nu)}{\Gamma_t} = \tau_t \cdot \Gamma(t \to be\nu)$$

branching ratio $= \frac{1}{9}$ $\Rightarrow$ $\tau_t = \dfrac{1}{9\Gamma(t \to be\nu)}$

$\Gamma \propto E_0^5$: $$\frac{\Gamma(t \to be\nu)}{\Gamma(\mu \to e\nu\nu)} = |V_{tb}|^2 \frac{M_t^5}{M_\mu^5}$$

$$\Gamma(\mu \to e\nu\nu) = \frac{1}{\tau_\mu}$$

$$\tau_\mu \Gamma(t \to be\nu) = |V_{tb}|^2 \left(\frac{M_t}{M_\mu}\right)^5$$

$$\frac{1}{9}\tau_\mu = \tau_t |V_{tb}|^2 \left(\frac{M_t}{M_\mu}\right)^5$$

$$\tau_t = \frac{1}{9}\tau_\mu \left(\frac{M_\mu}{M_t}\right)^5 \frac{1}{|V_{tb}|^2} = \frac{1}{9}\, 2.2\times10^{-6} \left(\frac{0.106}{175}\right)^5 \frac{1}{0.999^2} = 2\times10^{-23}\text{ s}$$

implications - top quarks decay before they can hadronise

- lifetime too short