2020 Paper 6 : Particles and Nuclear.

Section A.

① $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

$\approx 3 \text{ colours} \times \dfrac{\sum_1^{N\text{quarks}} e_q^2}{1^2}$ quark charges / muon charge.

• 5 GeV udsc

$R = 3 \times (e_u^2 + e_d^2 + e_s^2 + e_c^2)$

$= 3 \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right)$

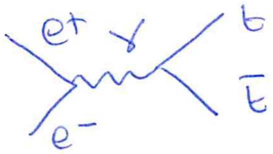
$= \frac{10}{3} = \underline{\underline{3.33}}$ 1

• 15 GeV udscb

$R = \frac{10}{3} + 3 \times e_b^2 = \frac{10}{3} + 3 \times \frac{1}{9} = \frac{11}{3} \approx \underline{\underline{3.7}}$ 1

• 360 GeV udscbt 1

$R = \frac{11}{3} + 3 \times e_t^2 = \frac{11}{3} + 3 \times \frac{4}{9} = \frac{15}{3} = \underline{\underline{5}}$

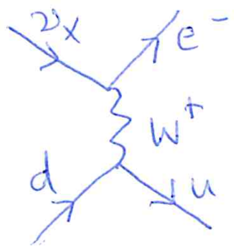


Assumes only photon propogator. 1

Above m_Z (90GeV) get $e^+e^- \to Z^0 \to t\bar{t}$

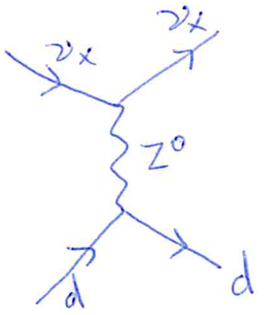
Above $2m_W$ (2×80GeV) get $e^+e^- \to W^+W^-$

(2) a) $\nu_x + d \to u + e^-$



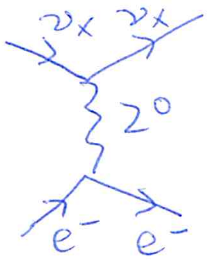
Charged current <u>only</u>; $\nu_x = \nu_e$. 1

b) $\nu_x + d \to d + \nu_x$ 1



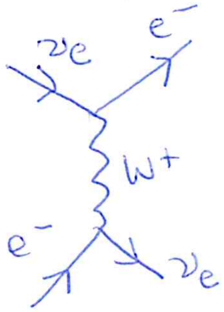
Neutral current only; all ν_x.

c) $\nu_x + e^- \to \nu_x + e^-$



Neutral current; all ν_x 1

and.



Charged current; $\nu_x = \nu_e$. 1

(3)

$$n + {}^{115}_{49}In \xrightarrow{{}^{116}_{49}In^*}$$

$J \quad \tfrac{1}{2} \quad \tfrac{9}{2} \quad 5$

$E = 1.46\,eV$

$\sigma_{TOT} = 5\times10^4$ barns.

$m_n = 1.6\times10^{-27}$ kg.

$1b = 10^{-28}\,m^2$

Total Cross-section $\quad \sigma_{TOT} = \dfrac{g\pi\lambda\!\!\!^-{}^2\Gamma_n\Gamma}{\Gamma^2/4} = \dfrac{4g\pi\lambda\!\!\!^-{}^2\Gamma_n}{\Gamma}$

$$g = \frac{(2J_{2^*}+1)}{(2J_n+1)(2J_{In}+1)} = \frac{(2\times5+1)}{(2\times\frac{1}{2}+1)(2\times\frac{9}{2}+1)} = \underline{\underline{\frac{11}{20}}}$$

Elastic ~~Neutron~~ ~~Capture~~ cross-section $\quad \sigma_n = \dfrac{4g\pi\lambda\!\!\!^-{}^2\Gamma_n^2}{\Gamma^2}$

$$\frac{\sigma_n}{\sigma_{TOT}} = \frac{\Gamma_n}{\Gamma}$$

$$\underline{\underline{\sigma_n = \frac{\Gamma_n}{\Gamma}\sigma_{TOT}}}$$

$$\lambda\!\!\!^- = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mE}} = \frac{1.05\times10^{-34}}{(2\times1.6\times10^{-27}\times1.46\times1.6\times10^{-19})^{1/2}} = \underline{\underline{3.8\times10^{-12}\,m.}}$$

Now $\dfrac{\Gamma_n}{\Gamma} = \dfrac{\sigma_{TOT}}{4g\pi\lambda\!\!\!^-{}^2}$

$$\therefore \sigma_n = \frac{\sigma_{TOT}{}^2}{4g\pi\lambda\!\!\!^-{}^2} = \frac{(5\times10^4\times10^{-28})^2}{4\times\frac{11}{20}\times\pi\times(3.8\times10^{-12}\,m)^2} = \underline{\underline{0.05\,b}}$$

$$\therefore \text{Probability} = \frac{0.05}{5\times10^4} \sim \underline{\underline{10^{-6}}}$$

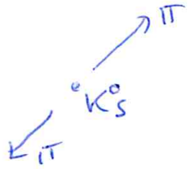
(3) $\Gamma(K_S^0 \to \pi^+\pi^-) = \dfrac{p_\pi}{8\pi m_K^2} |M_{fi}|^2$

$m_K = 498\ \text{MeV}/c^2$

$m_\pi = 140\ \text{meV}/c^2$

$|M_{fi}|^2 = 1.545 \times 10^{-7}\ \text{MeV}/c^3$

In K_S^0 com frame



$E_\pi = m_K/2 = 249\ \text{meV}$

$p_\pi = (E_\pi^2 - m_\pi^2)^{1/2} = (249^2 - 140^2)^{1/2} = 205.9\ \text{meV}/c$ 1

$\Gamma(K_S^0 \to \pi^+\pi^-) = \dfrac{205.9}{8\pi\, 498^2} \times 1.545 \times 10^{-7} = 5.1 \times 10^{-12}\ \text{meV}$. 1

$\dfrac{\Gamma(K_S^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to X)} = 0.69$

X ← anything

$\therefore\ \tau_{K_S^0} = \dfrac{1}{\Gamma(K_S^0 - X} = \dfrac{0.69}{5.1 \times 10^{-12}} = 1.35 \times 10^{11}\ \text{meV}^{-1}$. 1

$1\ \text{GeV}^{-1} = 6.6 \times 10^{-25}\ \text{s} \Rightarrow 1\ \text{meV}^{-1} = 6.6 \times 10^{-22}\ \text{s}$.

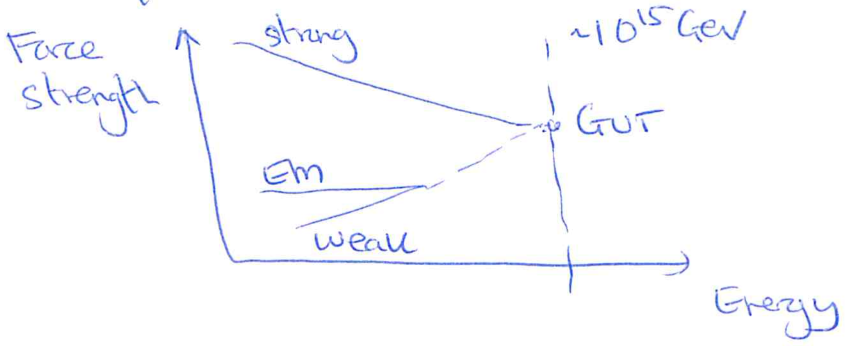
$\therefore\ \tau_{K_S^0} = 1.35 \times 10^{11} \times 6.6 \times 10^{-22} = 0.89 \times 10^{-10}\ \text{s}$ 1

or $\underline{\underline{89\ \text{ps}}}$

Q4

BOOKWORK START

3 a) Grand Unified Theories – aim to unite strong with electroweak, and ultimately, gravity interactions. Underpins many ideas of theories beyond the Standard Model.



Suggests unification at $\sim 10^{15}$ GeV

Gravity significant at Planck Mass $\sim 10^{19}$ GeV

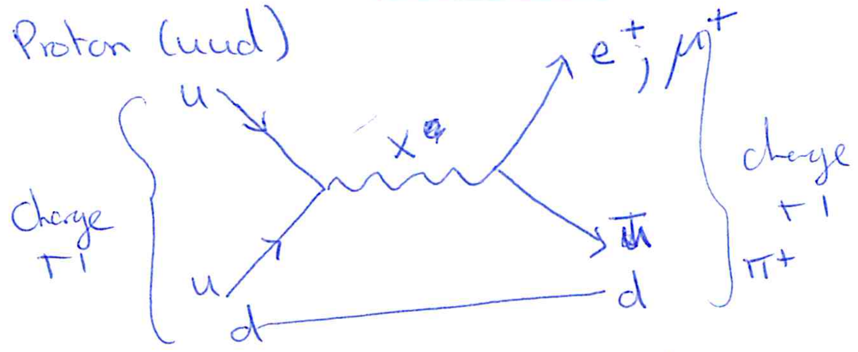
Baryon number violation – baryon number $= \frac{1}{3}(N_q - N_{\bar{q}})$

where $N_{q}^{(-)}$ ~~are~~ is the number of valence quarks and antiquarks in a composite particle.

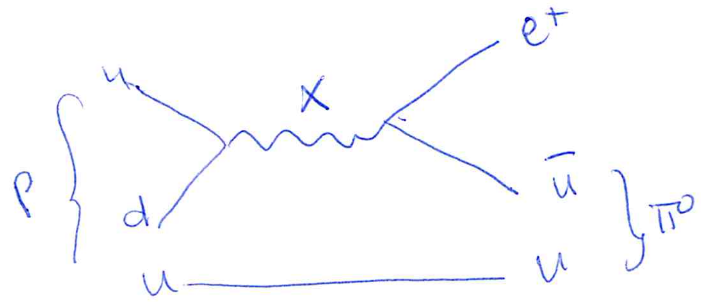
GUT models allow for the changing of a baryon into leptons and antiquarks, thus violating both baryon and lepton number 3

BOOKWORK END

b) Proton (uud)

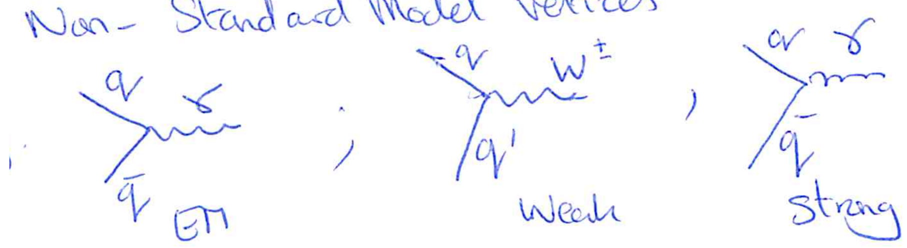


$p \rightarrow e^+ \pi^+$



$p \rightarrow e^+ \pi^0$

Non-Standard Model vertices



SM Vertices

4

Q5

c) $\Gamma \sim$ coupling $\times$ propogator $\times$ phase space

Sargents rule $\Gamma \propto E^5$

$$M \sim \frac{\alpha_G}{(m_x^2 - q^2)}$$

$$\therefore \Gamma \sim \frac{\alpha_G^2}{(m_x^2 - q^2)^2} m_p^5$$

Process dominated by propogator $m_x^2 >> q^2$.

$$\therefore \Gamma \sim \frac{\alpha_G^3 m_p^5}{m_x^4}$$

4

d) $$\tau = \frac{1}{\Gamma} = \frac{m_x^4}{\alpha_G^2 M_p^5} = \frac{(2 \times 10^{14})^4}{(0.024)^2 \times (0.938)^5} \ \text{GeV}^{-1}$$

3

$$= 5.2 \times 10^{35} \text{ s}$$

e) Experiment: Superkamiokande. BOOKWORK

Need lots of protons → heavy water

$p \to e^+ \pi^0$
$\quad \hookrightarrow \gamma\gamma$

e+ and γ interact electromagnetically → light.
Cherenkov light detected with PMTs surrounding water.
Shield from cosmic rays & backgrounds → underground and use inner volume only.
Calibrate using lasers or radioactive material.

5

Q5

BOOKWORK START

6 a) Evidence in favour of Shell Model

– Total angular momentum and parities of ground states

Magic even-even nuclei $J^P = 0^+$

even-odd " J^P of odd nucleon

odd-odd " $J = |j_1 + j_2| \ldots |j_1 - j_2|$ j_i = odd nucleons

jj coupling dominant

$P = (-1)^{l_1} \times (-1)^{l_2}$

– magnetic moments near closed shells

even-odd nuclei μ of odd nucleon

Schmidt lines

Not so great as fails since not all nucleons paired off.

– Magic numbers 2, 8, 20 etc $\Sigma(2j+1)$

Spin-orbit coupling – interaction between nucleon spin and nuclear force $\Delta E \propto -\underline{S} \cdot \underline{L}$

– Neutron binding energies

Binding energy of last nucleon high c.f. predicted value of SEMF.

– Spontaneous neutron emitters e.g. $^{17}_{8}O$, $^{36}_{87}Kr$ N = magic number +1

BOOKWORK END

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b) $^{14}_{7}N$ odd-odd

7 neutrons, last in $1p_{1/2}$

7 protons, " " $1p_{1/2}$

$P = (-1)^{l_1}(-1)^{l_2} = (-1)^1(-1)^1 = +$

$J = |j_1 + j_2| \ldots |j_1 - j_2| = 0, 1$

$\Big\}$ 0^+ or 1^+

$^{30}_{14}Si$ even-even $\rightarrow$ $J^P = 0^+$

$^{39}_{16}S$ even-odd

23 neutrons, last in $1f_{7/2}$ $\rightarrow$ $J^P = 7/2^-$ 6

$^{17}_{9}F$ even-odd

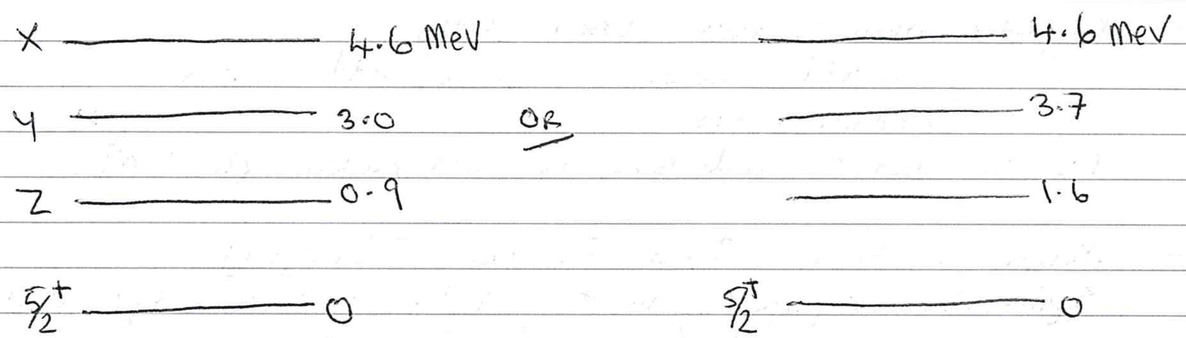
9 protons, last in $1d_{5/2}$ $\rightarrow$ $J^P = \frac{5}{2}^+$

$d \rightarrow l = 2$

b) $^{17}_{9}F$ E1 2.1, 3.7, 4.6 MeV
M1 1.6 MeV
E2 0.9, 1.6 MeV
faint 3.0 MeV

Ground state $5/2^+$

Possible combinations of energy levels for given γ-radiations emitted:



Scheme A

E1 (4.6): γ has $J^P = 1^-$; final state $5/2^+$, parity changed
$\therefore$ Initial state $X = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$

E2 (0.9): γ has $J=2$, final state $5/2^+$, parity unchanged

$\therefore$ Initial state $Z = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+, \frac{9}{2}^+$

However, no M1 at 0.9 MeV with $J_\gamma = 1$
$\therefore$ not $\frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$

$\therefore$ $Z = \frac{1}{2}^+$ or $\frac{9}{2}^+$

E1 (3.7): γ has $J=1$, final state has $\frac{1}{2}^+$ or $\frac{9}{2}^+$, parity changed

$\therefore$ Initial state $X = \frac{3}{2}^-$ or $\frac{7}{2}^-$

M1 (1.6): γ has $J=1$, ~~Final~~ Initial state $\frac{3}{2}^-$ or $\frac{7}{2}^-$, parity unchanged

$\therefore$ Final state $Y = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ or $\frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-$

However no E1 (3.0) J=1 final state $= \frac{5}{2}^+$, parity changed

$\therefore$ Initial state not $\frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$

$$\therefore \; Y = \frac{1}{2}^- \text{ or } \frac{9}{2}^-$$

Thus EITHER OR

EITHER		OR	
$\frac{3}{2}^-$	4.6 MeV	$\frac{7}{2}^-$	4.6 MeV
$\frac{1}{2}^-$	3.0	$\frac{9}{2}^-$	3.0
$\frac{1}{2}^+$	0.9	$\frac{9}{2}^+$	0.9
$\frac{5}{2}^+$	0	$\frac{5}{2}^+$	0

3.0 MeV faint radiation is likely to be M2 in either case $\Delta J = 2$

1	10^{-3}	10^{-6}	$\frac{M2}{E1} \sim 10^{-6}$
E1	E2 M1	E3 M2	

Scheme B

E1 (4.6): γ has $J=1$, final state $\frac{5}{2}^+$, parity changed

$$X = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$$

M1 (1.6): γ has $J=1$, final state $\frac{5}{2}^+$, parity unchanged

$$Z = \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$$

E1 (3.7) $\quad Y = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$

E2 (0·9) : $X = \frac{3}{2}^-$ $Y = \cancel{\frac{1}{2}^-}, \cancel{\frac{3}{2}^-}, \cancel{\frac{5}{2}^-}, \frac{7}{2}^-$ No M1 (0·9)

$X = \frac{5}{2}^-$ $Y = \frac{1}{2}^-, \cancel{\frac{3}{2}^-}, \cancel{\frac{5}{2}^-}, \cancel{\frac{7}{2}^-}, \frac{9}{2}^-$

$X = \frac{7}{2}^-$ $Y = \frac{3}{2}^-, \cancel{\frac{5}{2}^-}, \cancel{\frac{7}{2}^-}, \cancel{\frac{9}{2}^-}, \frac{11}{2}^-$

$\therefore$ $X = \frac{3}{2}^-$ and $Y = \frac{7}{2}^-$

or $X = \frac{7}{2}^-$ and $Y = \frac{3}{2}^-$

E1 (2.1) : $Y = \frac{3}{2}^-$ $Z = \cancel{\frac{1}{2}^+}, \frac{3}{2}^+, \frac{5}{2}^+$

$Y = \frac{7}{2}^-$ $Z = \frac{5}{2}^+, \frac{7}{2}^+, \cancel{\frac{9}{2}^+}$

No E1 (3.0) $\rightarrow$ $Z \neq \frac{5}{2}^+$

Thus,

EITHER		OR	
$\frac{3}{2}^-$	4.6	$\frac{7}{2}^-$	4.6
$\frac{7}{2}^-$	3.7	$\frac{3}{2}^-$	3.7
$\frac{7}{2}^+$	1.6	$\frac{3}{2}^+$	1.6
$\frac{5}{2}^+$	0	$\frac{5}{2}^+$	0

3.0 MeV radiation likely to be M2 in either case.

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