PNP 2016

1) a) predict spin-parity of ${}^{10}_{5}B$, ${}^{15}_{7}N$, ${}^{10}_{4}Be$, ${}^{17}_{8}O$

${}^{17}_{8}O$ - unpaired neutron in $1d_{5/2}$

$l = 2,\ j = \frac{5}{2} \Rightarrow J^P = \frac{5}{2}^+$

${}^{10}_{4}Be$ - even-even nucleus - $J^P = 0^+$.

${}^{15}_{7}N$ - unpaired proton in $1p_{1/2}$

$L = 1,\ j = 1/2 \Rightarrow J^P = \frac{1}{2}^-$

${}^{10}_{5}B$ - unpaired proton and neutron, both in $1p_{3/2}$

$j_p = \frac{3}{2}^- = j_n \Rightarrow$ total $J = 0, 1, 2, 3$

$J^P = 0^+, 1^+, 2^+, 3^+$

b) Show that $m(\eta) = -\frac{1}{3}m(\pi^0) - 2m(\rho^0) + 4m(K^{*0}) + \frac{4}{3}m(K^0) - 2m(\varphi)$

$m(\pi^0) = 2M_u - \frac{3A}{4M_u^2}$ $\qquad m(\rho^0) = 2M_u + \frac{A}{4M_u^2}$

$m(\varphi) = 2M_s + \frac{A}{4M_s^2}$ $\qquad m(K^0) = M_u + M_s - \frac{3A}{4M_uM_s}$

$m(K^{*0}) = M_d + m_s + \frac{A}{4M_uM_s}$ $\qquad m(\eta) = \frac{2}{3}\left(2M_s - \frac{3A}{4M_s^2}\right) + \frac{1}{3}\left(2M_u - \frac{3A}{4M_u^2}\right)$

RHS: $-\frac{1}{3}\left(2M_u - \frac{3A}{4M_u^2}\right) - 2\left(2M_u + \frac{A}{4M_u^2}\right) + 4\left(M_u + M_s + \frac{A}{4M_uM_s}\right)$

$+ \frac{4}{3}\left(M_u + M_s - \frac{3A}{4M_uM_s}\right) - 2\left(2M_s + \frac{A}{4M_s^2}\right)$

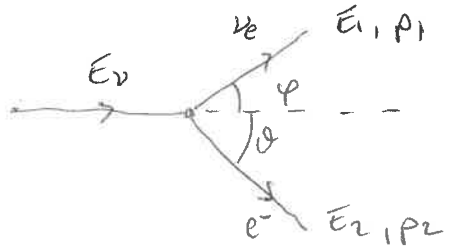
$= M_u\left(-\frac{2}{3} - 4 + 4 + \frac{4}{3}\right) + M_s\left(4 + \frac{4}{3} - 4\right) + \frac{A}{4M_u^2}(1 - 2)$

$+ \frac{A}{4M_uM_s}(4 - 4) - \frac{A}{2M_s^2}$

$= \frac{2}{3}M_u + \frac{4}{3}M_s - \frac{A}{4M_u^2} - \frac{A}{2M_s^2}$

$= \frac{2}{3}\left(2M_s - \frac{3A}{4M_s^2}\right) + \frac{1}{3}\left(2M_u - \frac{3A}{4M_u^2}\right) = m(\eta)$

c) elastic scattering of neutrinos off electrons (stationary electrons)



show that electron kinetic energy satisfies

$$T_e = \frac{2m_e E_\nu^2 \cos^2\vartheta}{(E_\nu + m_e)^2 - E_\nu^2\cos^2\vartheta}$$

energy conservation $\quad E_\nu + m_e = E_1 + E_2$

momentum $\quad E_\nu = p_2\cos\vartheta + p_1\cos\varphi$

$$0 = p_2\sin\vartheta + p_1\sin\varphi$$

$$p_1^2\sin^2\varphi + p_1^2\cos^2\varphi = p_1^2 = p_2^2\sin^2\vartheta + (E_\nu - p_2\cos\vartheta)^2$$

$$E_1^2 = p_1^2 = (E_\nu + m_e - E_2)^2$$

$$(E_\nu + m_e - E_2)^2 = p_2^2\sin^2\vartheta + (E_\nu - p_2\cos\vartheta)^2$$

$$E_2 = T_e + m_e \quad , \quad p_2^2 = E_2^2 - m_e^2$$

$$(E_\nu + m_e - T_e - m_e)^2 = (E_2^2 - m_e^2)\sin^2\vartheta + (E_\nu - \sqrt{E_2^2 - m_e^2}\cos\vartheta)^2$$

$$(E_\nu - T_e)^2 - (T_e^2 + 2T_e m_e)\sin^2\vartheta = E_\nu^2 + (E_2^2 - m_e^2)\cos^2\vartheta - 2E_\nu\sqrt{E_2^2 - m_e^2}\cos\vartheta$$

$$\cancel{E_\nu^2} + \cancel{T_e^2} - 2E_\nu T_e - (\cancel{T_e^2} + 2T_e m_e) + (T_e^2 + 2\cancel{T_e m_e})\cos^2\vartheta = \cancel{E_\nu^2} + (T_e^2 + \cancel{2m_e T_e})\cos^2\vartheta - 2E_\nu\sqrt{T_e^2 + 2m_e T_e}\cos\vartheta$$

$$2E_\nu\sqrt{T_e^2 + 2m_e T_e}\cos\vartheta = 2E_\nu T_e + 2T_e m_e$$

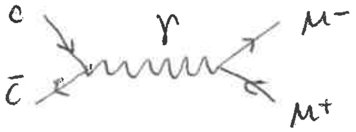
$$E_\nu^2(T_e^2 + 2m_e T_e)\cos^2\vartheta = E_\nu^2 T_e^2 + T_e^2 m_e^2 + 2T_e^2 m_e E_\nu$$

$$T_e(E_\nu^2 + m_e^2 + 2m_e E_\nu - E_\nu^2\cos^2\vartheta) = 2m_e E_\nu^2\cos^2\vartheta$$

$$T_e = \frac{2m_e E_\nu^2\cos^2\vartheta}{(E_\nu + m_e)^2 - E_\nu^2\cos^2\vartheta}$$

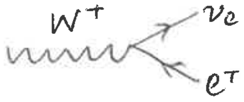
3) Feynman diagrams

a) i) $J/\psi \to \mu^+\mu^-$



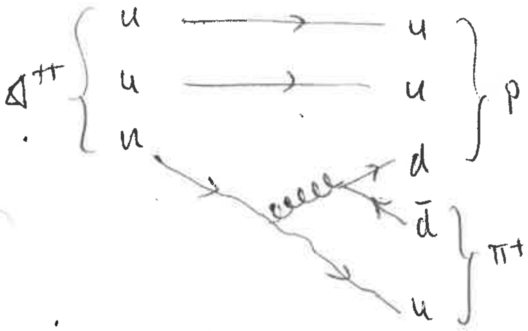
EM

ii) $W^+ \to e^+ \nu_e$



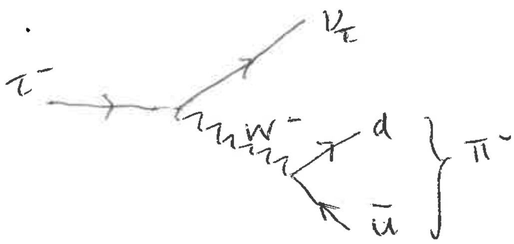
weak

iii) $\Delta^{++} \to p\pi^+$



strong

iv) $\tau^- \to \nu_\tau \pi^-$



weak

b) derive momentum spectrum in nuclear beta decay and show that total decay rate $\propto$ fifth power of energy released

$E_0 = E_\nu + E_e + T_{nucleus}$, neglect $T_{nucleus}$

$dE_0 = dE_\nu = dp_\nu$ for fixed E_e

$$\frac{dN}{dp_\nu} = \frac{dN}{dE_0} = \frac{p_\nu^2}{(2\pi^3)} d\Omega_\nu \frac{p_e^2}{(2\pi)^3} d\Omega_e \, dp_e$$

assuming isotropic decay, $d\Omega \to 4\pi$

$E_e \sim p_e$, $p_\nu = E_0 - p_e$

$$\frac{dN}{dE_0} = \frac{(4\pi)^2}{(2\pi)^6} (E_0 - p_e)^2 p_e^2 \, dp_e$$

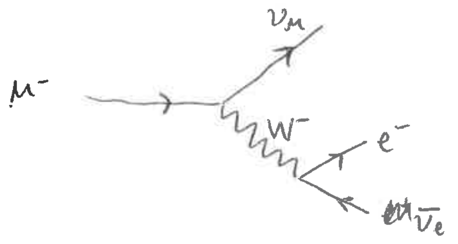
total decay rate $\propto 2\pi |M|^2 \rho(E_f) \propto \int_0^{E_0} (E_0 - p_e)^2 p_e^2 \, dp_e$

$$\Gamma \propto \int_0^{E_0} E_0^2 p_e^2 - 2E_0 p_e^3 + p_e^4 \, dp_e$$

$$\propto \left[\frac{1}{3} E_0^2 p_e^3 - \frac{1}{2} E_0 p_e^4 + \frac{1}{5} p_e^5 \right]_0^{E_0} = \frac{1}{30} E_0^5$$

$\Gamma \propto E_0^5$

c) Feynman diagram for dominant muon decay



$$\Gamma_\mu = \frac{G_F^2}{192\pi^3} M_\mu^5$$

- Fermi theory - matrix element $M \propto G_F$

Standard model $M \propto \frac{g_w^2}{q^2 - M_W^2}$, but ~~when~~ $M_\mu << M_W$

so Fermi theory can be applied to muon decay

muon lifetime

$$\Gamma_\mu = \frac{(1.166\times10^{-5})^2}{192\pi^3}(0.106)^5 = 3.056\times10^{-19} \text{ GeV}$$

$$\tau_\mu = \frac{1}{\Gamma_\mu} = 3.27\times10^{18} \text{ GeV}^{-1}$$

$$\times \hbar = 2.15\times10^{-6} \text{ s}$$

d) lifetimes of τ lepton, c and b quarks

$$\Gamma_\tau = \frac{\Gamma_{\tau\to e\nu}}{B_{e\nu}}, \quad \tau_\tau = \frac{B_{e\nu}}{\Gamma_{\tau\to e\nu}} = B_{e\nu}\left(\frac{M_\mu}{M_\tau}\right)^5 \tau_\mu$$

possible τ decays

$\to e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$

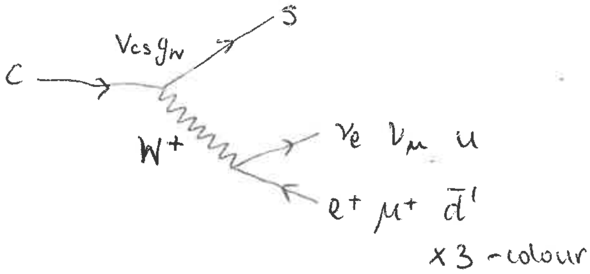
$d\bar{u}$, $s\bar{u}$

$$\Gamma_{e\nu}(2 + \underset{\pi}{3|V_{ud}|^2} + \underset{K}{3|V_{us}|^2}) = \Gamma$$

$$B_{e\nu} = \frac{\Gamma_{e\nu}}{\Gamma} = 20.13\%$$

$$\tau_\tau = \tau_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 \cdot 0.2013 = 3.27\times10^{-13}\,s$$

c quark:



$M \propto V_{cs} g_W^2$, $\Gamma \propto |V_{cs}|^2 g_W^4$

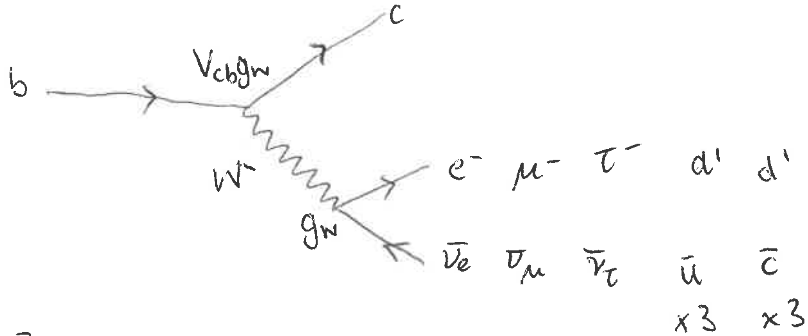
$\therefore$ for each possible decay

$B_{e\nu} = \frac{1}{5}$

$$\Gamma_c = 5\Gamma_{e\nu} = 5\left(\frac{G_F^2}{192\pi^3}\right) m_c^5 |V_{cs}|^2 .$$

$$\tau_c = \frac{1}{5}\tau_\mu \left(\frac{m_\mu}{m_c}\right)^5 \frac{1}{|V_{cs}|^2} = 8.05\times10^{-13}\,s$$

b quark:



$B_{e\nu} = \frac{1}{9}$

$\Gamma_{e\nu} \propto |V_{cb}|^2 g_W^4$

$$\tau_b = \frac{1}{9}\tau_\mu \left(\frac{m_\mu}{m_b}\right)^5 \frac{1}{|V_{cb}|^2} = 6.39\times10^{-13}\,s$$

4) a) deuteron has $J^P = 1^+$

assuming $L=0$ state dominates, and noting that no excited states exist, what can be concluded about the nature of the p-n force and the existence of pp and nn bound states

If $J^P = 1^+$, n-p force must be strongest when n and p spins aligned parallel - not enough binding energy to form $S=0$ bound states with spins antiparallel

nn/pp bound states - to bind together need spins parallel, $S=1$ is $n\downarrow n\downarrow$ or $n\uparrow n\uparrow$ - but this violates Pauli's exclusion principle - 2 neutrons or 2 protons in same quantum state

-so nn/pp bound states don't exist.

b) binding energy

$M_d = M_p + M_n - B \Rightarrow B = M_p + M_n - M_d = 2.224\ \text{MeV}$

c) deuteron magnetic moment $\mu = 0.857\ \mu_N$

$\mu_n = -1.913\ \mu_N$, $\mu_p = 2.793\ \mu_N \Rightarrow$ ~~μ_d~~ $\mu_n + \mu_p = 0.88\ \mu_N$

$\mu_d < \mu_n + \mu_p$

expect μ_d = sum of intrinsic proton, neutron dipole moments for $l=0$ - must have small amount of $l=2$ in wavefunction - nucleus not spherically symmetric, this also accounts for non-zero electric quadrupole moment.

d) spherically symmetric square well with depth V_0 , radius b

radial wave equation

$$-\frac{\hbar^2}{2\mu}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r)$$

$$\frac{d^2u}{dr^2} = -\frac{2\mu}{\hbar^2}(E+V_0)u(r) \quad 0<r<b$$

$$u(r) = A\sin k_1 r + B\cos k_1 r \quad , \quad k_1^2 = \frac{2\mu}{\hbar^2}(E+V_0)$$

need $R(r)$ finite at $r=0 \Rightarrow u(0)=0 \quad \Rightarrow B=0$

$$R(r) = \frac{A}{r}\sin k_1 r \quad 0<r\leq b$$

$$\frac{d^2u}{dr^2} = -\frac{\hbar^2}{2\mu}Eu(r) \quad b<r$$

$$u(r) = Ce^{k_2 r} + De^{-k_2 r} \quad , \quad k_2 = \sqrt{\frac{-2\mu E}{\hbar^2}}$$

$$u(\infty) = 0 \Rightarrow C=0$$

$$R(r) = \frac{D}{r}e^{-k_2 r} \quad r>b$$

$u(r)$ continuous at $r=b$ (and $\frac{du}{dr}$)

$$A\sin k_1 b = De^{-k_2 b}$$

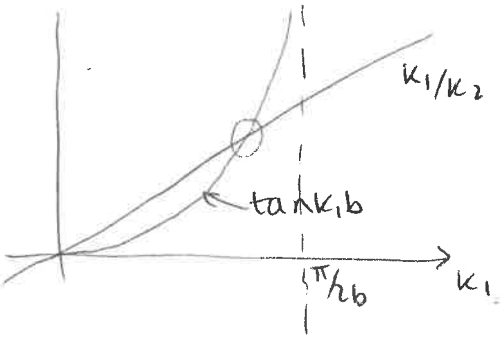
$$k_1 A\cos k_1 b = -k_2 De^{-k_2 b}$$

$$\frac{1}{k_1}\tan k_1 b = -\frac{1}{k_2}$$

$$\sqrt{\frac{-2\mu E}{\hbar^2}}\tan\left[b\sqrt{\frac{2\mu}{\hbar^2}(E+V_0)}\right] = -\sqrt{\frac{2\mu}{\hbar^2}(E+V_0)}$$

$E=$ binding energy B

find V_0



need $k_1 b = \frac{\pi}{2}$

$$\left(\frac{\pi}{2b}\right)^2 = \frac{2\mu}{\hbar^2}(E+V_0) \sim \frac{2\mu V_0}{\hbar^2}$$

if $B << V_0$

$$V_0 = \frac{\hbar^2 \pi^2}{8\mu b^2}$$

$\mu = \frac{m_n m_p}{m_n + m_p}$, $b = 2.1\,fm \Rightarrow V_0 = 23.1\,MeV$

e) reaction $np \to d\gamma$

if neutron and proton assumed to be at rest, photon energy slightly smaller than deuteron binding energy

Explain why and calculate $B - E_\gamma$ - recoil energy of deuteron

$$m_p + m_n = E_d + E_\gamma$$

$$E_d^2 = (m_p + m_n - E_\gamma)^2$$

$$m_d^2 + \cancel{p_d^2} = (m_p + m_n)^2 + \cancel{E_\gamma^2} - 2E_\gamma(m_p + m_n)$$

$$E_\gamma = \frac{(m_p + m_n)^2 - m_d^2}{2(m_p + m_n)} = \frac{(m_p + m_n)^2 - (m_p + m_n - B)^2}{2(m_p + m_n)}$$

$$E_\gamma = \frac{2B(m_p + m_n) - B^2}{2(m_p + m_n)} = B - \frac{B^2}{2(m_p + m_n)}$$

$$B - E_\gamma = \frac{B^2}{2(m_p + m_n)} = \frac{2.224}{2(938.272 + 939.565)} = 0.001317\,MeV$$

$$= 1.317\,keV$$