UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 30 May 2012 13.30 to 15.30

EXPERIMENTAL AND THEORETICAL PHYSICS (5)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (5)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

ASTROPHYSICAL FLUID DYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) In a magnetised plasma, the following dispersion relation describes a subset of possible wave modes:

$$(k^2 v_{\rm A}^2 - \omega^2)\boldsymbol{u} + \left(\frac{c_{\rm s}^2}{v_{\rm A}^2} - 1\right) k^2 (\boldsymbol{v}_{\rm A} \cdot \boldsymbol{u})\boldsymbol{v}_{\rm A} = 0,$$

where $\boldsymbol{v}_{\rm A}$ is the Alfvén velocity, $\boldsymbol{u}$ the fluid velocity, k the wavenumber and ω the angular frequency. When does this dispersion relation apply? Discuss the possible wave modes this dispersion relation describes. [4]

(b) The net cooling rate per unit mass in a gas heated by cosmic rays can be written as

$$\dot{Q} = A\rho T^{\alpha} - H,$$

where A and H are constants, ρ the density and T the temperature of the gas. Assuming the gas is at constant pressure, is it thermally stable when cooling is due to optically thin Bremsstrahlung which has $\alpha = 0.5$? [4]

(c) By considering a uniform region of interstellar medium of radius R in which the sound speed is $c_{\rm s}$, estimate the value of R for which the sound-crossing time is equal to the free-fall collapse time. What is the significance of this size? [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) stellar winds;

(b) the Rayleigh–Taylor instability;

(c) Bernoulli's equation.

3 *Attempt* **either** *this question* **or** *question 4.*

Briefly describe how shock waves arise in interstellar space. [3]

Derive the Rankine–Hugoniot conditions at a normal adiabatic shock front

$$\begin{aligned} \rho_2 u_2 &= \rho_1 u_1, \\ p_2 + \rho_2 u_2^2 &= p_1 + \rho_1 u_1^2, \\ \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}u_2^2 &= \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}u_1^2, \end{aligned}$$

where ρ is the density, p the pressure, γ the adiabatic index and u the normal velocity. The subscripts 1 and 2 refer to the upstream and downstream conditions, respectively. [6]

For a strong shock the upstream pressure can be neglected. By eliminating p_2, or otherwise, show that in the limit of a strong shock

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1}.$$

[6]

Two identical gas clouds collide with a relative velocity of $2V$, where V is very much larger than the sound speed in the clouds. A strong, planar, adiabatic shock wave propagates into the clouds away from the surface of contact. Explain why, in the zero-momentum frame, the shocked gas must be stationary, while the unshocked gas continues to travel at its initial speed V. [3]

How fast does the shock travel in the zero-momentum frame? [4]

Without detailed calculation discuss what happens when the shock reaches the side of the cloud distant from the surface of contact. [3]

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4 *Attempt* **either** *this question* **or** *question 3.*

Show that the Lagrangian derivative with respect to time of some quantity Q can be written

$$\frac{\mathrm{D}Q}{\mathrm{D}t} = \frac{\partial Q}{\partial t} + \boldsymbol{u} \cdot \nabla Q,$$

where $\boldsymbol{u}$ is the fluid velocity. [3]

Hence derive Euler's equation for fluid flow

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p - \nabla \phi,$$

where ρ is the density, p the pressure and ϕ the gravitational potential. [4]

Show that for hydrostatic equilibrium of an isothermal, spherical, self-gravitating gas cloud, one possible form for the density profile is $\rho = A/r^2$ where r is the radial coordinate and A is a constant, and show that A is related to the isothermal sound speed c_s in the cloud by

$$A = \frac{c_\mathrm{s}^2}{2\pi G},$$

where G is the gravitational constant. [8]

A cloud of mass M and with this density profile is embedded within a uniform external medium of pressure p_0. Show that the radius of the cloud is $r_0 = GM/2c_\mathrm{s}^2$ and that the external pressure is $p_0 = c_\mathrm{s}^4/2\pi G r_0^2$. [2]

Assuming the adiabatic index of the gas is $5/3$, calculate the internal energy U and gravitational potential energy Ω of the cloud and show that they obey a modified form of the virial equation

$$2U + \Omega = 4\pi r_0^3 p_0.$$

[6]

Comment on the origin of the extra term in the virial equation. [2]

END OF PAPER