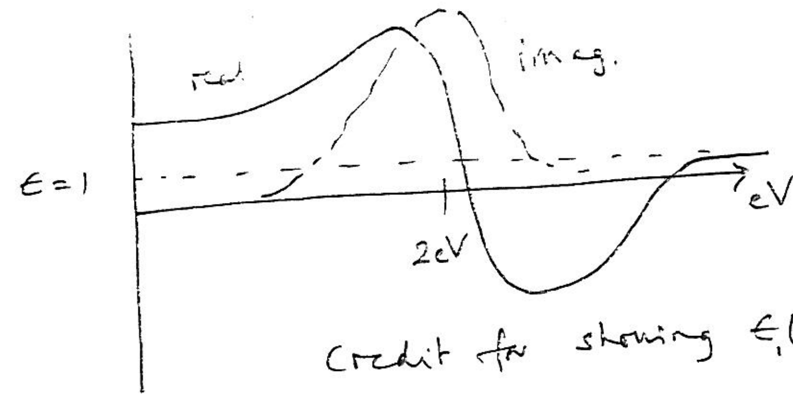
31 10|1|15

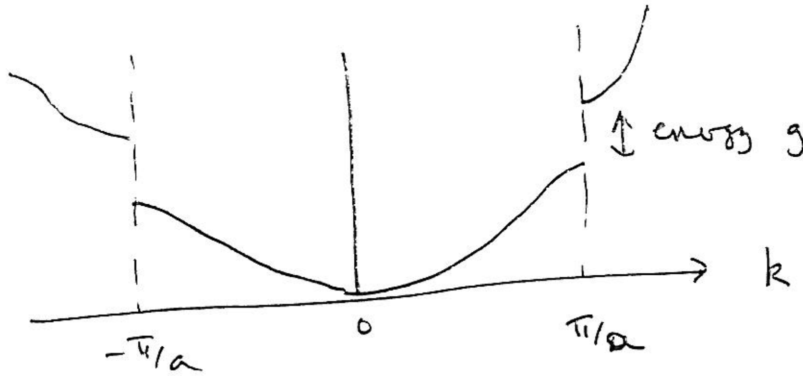
QCM Paper 7 2013

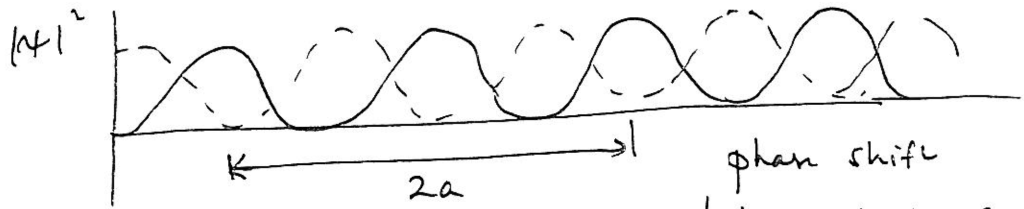
1 (a) Lorentz oscillator at 2eV



Credit for showing $\epsilon_1(\omega = 0) > \epsilon_1(\omega = \infty) = 1$

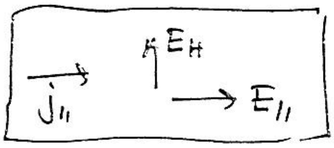
(b)





phase shift between upper and lower band states

(c)



$j_{\parallel} = \sigma E_{\parallel} \qquad \sigma = ne\mu$

$\Rightarrow j_{\parallel} = ne\mu E_{\parallel}$

$E_H = R_H j_{\parallel} B \qquad R_H = \frac{1}{ne}$

$\Rightarrow E_H = \frac{j_{\parallel} B}{ne} = \mu E_{\parallel} B$

$\mu = 0.2 \ m^2/Vs$

$\Rightarrow B = \frac{10^{-2}}{0.2} = 5 \times 10^{-2} \ T$

QCM Paper 7 2013

2 (a) Heat capacity of non-metallic solids and metallic solids

non-metallic – due to lattice vibrations

- equipartition gives 3R (or 3k)
- Debye treats sound waves, gives $C \sim T^3$ at low T, 3R above Θ_{Debye}
- full treatment of phonon modes slight deviation from Debye.

metallic, as above, but linear T term from thermal excitation of electron gas, and $\propto$ density of states at E_F.

[Extra credit if mention magnetic contribution]

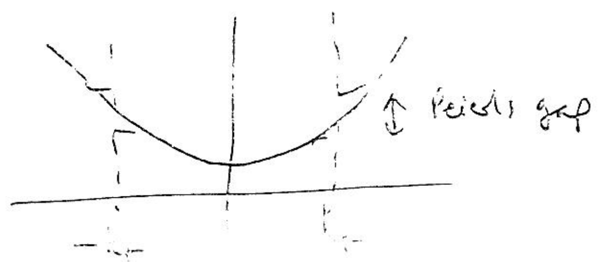
2 (b) Peierls Distortion

1-d metal, a single periodic distortion with wavevector $2k_F$ opens complete gap at E_F. This lowers band energy and causes static distortion with periodicity $2k_F$.

At higher temperatures thermal excitation across gap weakens distortion, to give semiconductor to metal transition

$2k_F$ may be incommensurate with lattice (charge density waves)

credit for a sketch.



QCM Paper 1, 2013

2 (c)

Fermi Liquids

Include interactions between electrons, then Fermi gas of 'non-interacting' particles remarkably survives but with renormalized parameters, including effective mass near E_F
"electrons" replaced by "quasi-particles" which ~~~~ have a long life time near E_F

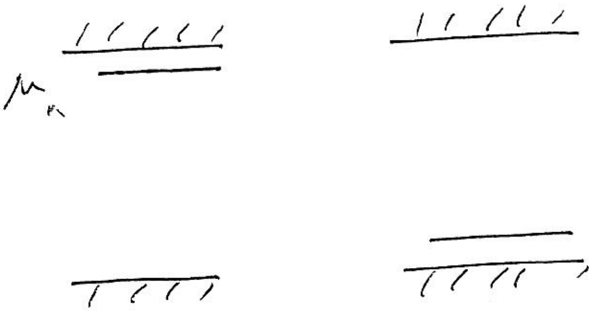
credit for examples:

He^3 - not exactly electrons, but covered in lectures

Heavy fermion metals, for which $m_{\text{effective}} \sim 10 - 100\ m_e$

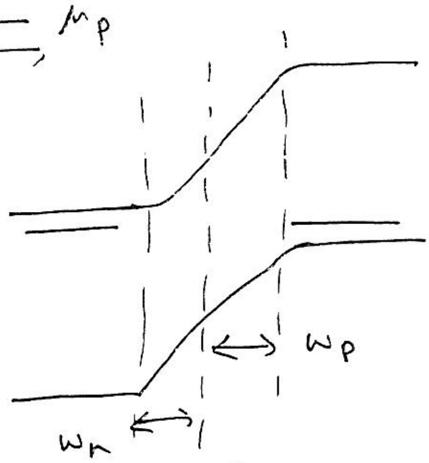
QCM Paper 7 2013

3. (Bookwork) Substitutional doping: replace e.g. Si with P
one extra electron → conduction band
replace Si with Ga
one missing electron; from valence band

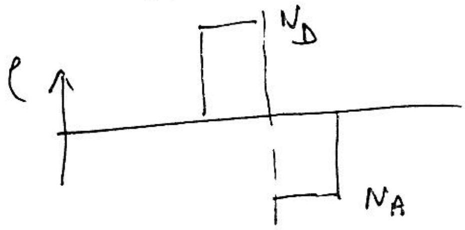


(5)

bring the two into contact, equalise μ ⟹



depletion regions
Space charge from dopants - no free charges.



$$w_n e N_D = w_p e N_A$$

(charge neutrality)

(5)

solve Poisson's equation in space charge region

$$\frac{d^2\phi}{dx^2} = \rho/\epsilon\epsilon_0$$

$$\phi = \text{const} + \rho x^2/2\epsilon\epsilon_0$$

Set $\phi = 0$ at $x = 0$

at $x = +w_p$, $\phi = \frac{eN_A}{2\epsilon\epsilon_0} w_p^2$

$x = -w_n$, $\phi = -\frac{eN_D}{2\epsilon\epsilon_0} w_n^2$

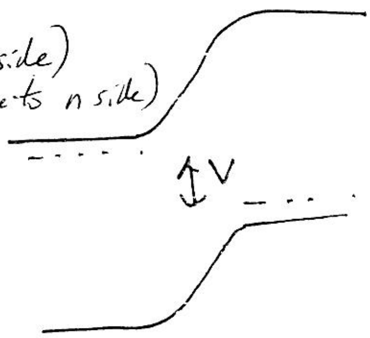
5 (cont.)

$$\Delta\phi_0 = \frac{eN_A}{2\epsilon\epsilon_0} w_p^2 + \frac{eN_D}{2\epsilon\epsilon_0} w_n^2 = \frac{e}{2\epsilon\epsilon_0} w_p^2 \left(N_A + N_D \cdot \frac{N_A^{2}}{N_D}\right)$$

$$= \frac{e w_p^2}{2\epsilon\epsilon_0} \frac{N_A}{N_D} (N_A + N_D) \quad \text{hence } w_p = \left\{\frac{2\epsilon\epsilon_0 N_D \,\Delta\phi_0}{e N_A (N_A + N_D)}\right\}^{1/2}$$

similarly for w_n 4/5

Add extra voltage, V (to p side) (relative to n side)



Add V to $\Delta\phi_0$ for new 'equilibrium' 3 2

$$Q/\text{unit area} = e N_D w_n = e N_A w_p$$

$$\frac{\partial Q}{\partial V} = e N_A \frac{\partial}{\partial V}\left(\left\{\frac{2\epsilon\epsilon_0 N_D}{N_A (N_A + N_D)} (\Delta\phi_0 - V)\right\}^{1/2}\right)$$

$$= \left\{\frac{\cancel{2}\,\epsilon\epsilon_0\, e N_A N_D}{2 (N_A + N_D)(\Delta\phi_0 - V)}\right\}^{1/2}$$ 3

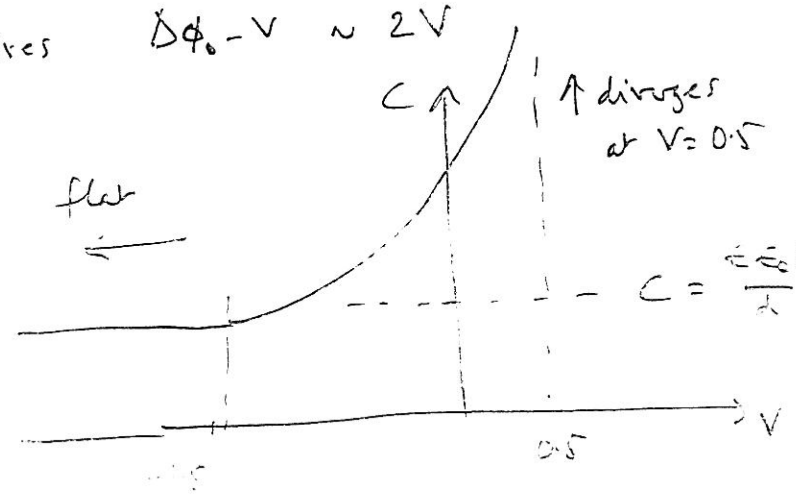
(Unseen) $A = 10^{-7}\,m^2$, n and p thickness 10^{-5} m, $N_A = N_D = 2\times10^{19}\,m^{-3}$

$\Delta\phi_0 = \frac{1}{2}$V, $\epsilon = 12$

Voltage at which full depletion is achieved:

$$w_p^2 = w_n^2 = \frac{\epsilon\epsilon_0 (\Delta\phi_0 - V)}{e N_A}$$

gives $\Delta\phi_0 - V \sim 2V$

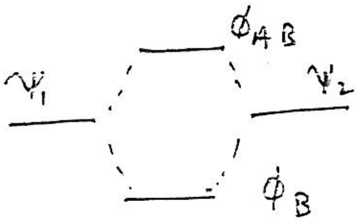


5

QCM Paper 7 2013

4. (Bosworth) Degenerate perturbation theory: use to construct linear combinations of states in presence of perturbation potential.

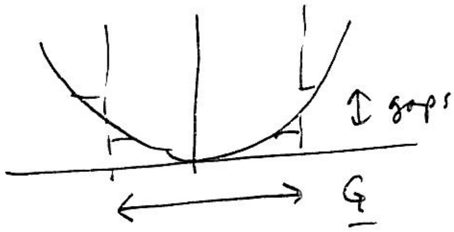
LCAO for H_2: s wavefunctions for each H; ψ_1, ψ_2



$\phi_B = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$

$\phi_{AB} = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$ 4

NFE: presence of Brillouin zone boundary mixes k states modulo k, so take linear combination of ψ_k with $\sum_G \psi_{k+G}$



4

total 8

Fe, Co, Ni two bands are s and d character 2

$|\psi_k\rangle = \alpha|\phi_k\rangle + \beta|\chi_k\rangle$

hybrid s band d band

$H|\psi_k\rangle = E_k|\psi_k\rangle \quad \times \langle\phi_k|, \langle\chi_k|$

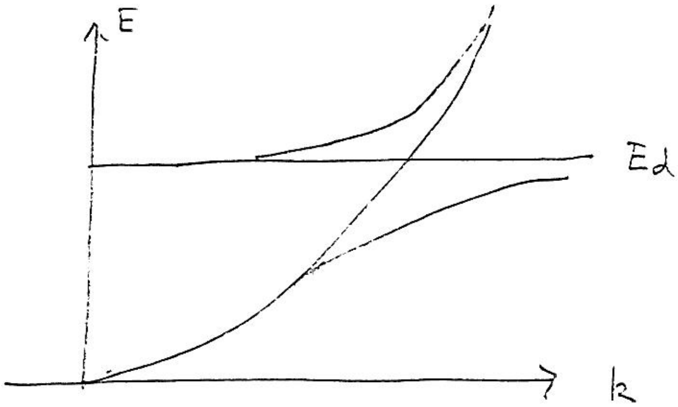
$\alpha E_1(k) + \beta V = \alpha E(k)$

$\alpha V^* + \beta E_2(k) = \beta E(k) \quad ; \quad E_2(k) = E_d$

$(E - E(k))(E_d - E) - |V|^2 = 0$;

hence $E = \frac{E_1(k) + E_d}{2} \pm \left\{ \left(\frac{E(k) - E_d}{2}\right)^2 + |V|^2 \right\}^{1/2}$ 3

4 (cont.)



(4)

(Unseen)

[Magnetic) metal has (single) occupancy of both bands with $|V| = 0$
when $|V|$ greater than zero, just the lower band occupied,
so k_F increases by $2^{1/3}$

$\hbar v_F = \partial E / \partial k$ at E_F

$$\frac{\partial E}{\partial k} = \frac{1}{2}\frac{\partial E_1}{\partial k}\left(1 - \frac{(E_1 - E_d/2)}{\left\{\left(\frac{E_1 - E_d}{2}\right)^2 + V^2\right\}^{1/2}}\right)$$

now $k_F = 2^{1/3} k_F'$, $E_1(k_F) = 2^{2/3} E_d$, $\frac{E_1 - E_d}{2} = \left(\frac{2^{2/3} - 1}{2}\right) E_d$
$= \delta E_d$

$V \ll E_d, \ \delta E_d$

$$\frac{\partial E}{\partial k} = \frac{1}{2}\frac{\partial E_1}{\partial k}\left(1 - \frac{\delta E_d}{\delta E_d \left(1 + V^2/\delta^2 E_d^2\right)^{1/2}}\right) \approx \frac{1}{2}\frac{\partial E_1}{\partial k}\cdot\frac{1}{2}\frac{V^2}{\delta^2 E_d^2}$$

$$\frac{\partial E}{\partial k} = \frac{1}{2}\frac{\hbar^2 k_F}{m}\frac{V^2}{\delta^2 E_d^2} = \frac{1}{2}\cdot 2^{1/3}\,\hbar v_F \frac{V^2}{\delta^2 E_d^2}$$

$$\Rightarrow v_F = v_F^{(1)} \cdot 2^{1/3} / \left(2^{2/3} - 1\right)^2$$

(4)

small v_F implies high density of states
so this is boosted when a d band
crosses the Fermi energy

(2)

$\rightarrow$ + say why magnetic