UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 2 June 2017 9.00 am to 11.00 am

PHYSICS (5)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (5)

ASTROPHYSICAL FLUID DYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

ASTROPHYSICAL FLUID DYNAMICS

1 *Answer* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) In a particular model of the Earth's atmosphere, the temperature within the troposphere can be approximated by the relation $T(z) = T_0 - \alpha z$, where T_0 is the temperature at sea level, z is the altitude and $\alpha = 6.5\ \mathrm{K\,km^{-1}}$. Find an expression for the air density as a function of altitude and calculate the ratio of the density at the top of the troposphere ($z = 11$ km) to that at sea level. Assume $T_0 = 300$ K and a mean molecular weight of the atmosphere $\mu = 28$. [4]

(b) Show that an ideal gas in a constant gravitational field is convectively stable if

$$\left|\frac{\mathrm{d}T}{\mathrm{d}z}\right| < \left(1 - \frac{1}{\gamma}\right) \frac{T}{p} \left|\frac{\mathrm{d}p}{\mathrm{d}z}\right| ,$$

where T is the gas temperature, p is the gas pressure, γ is the adiabatic index and z is the spatial coordinate in the opposite direction to the gravitational acceleration. [4]

(c) Two media, A and B, are non-dispersive and behave as perfect gases. They are in pressure equilibrium and have the same mean molecular weight. A sound wave propagates in the direction perpendicular to the boundary between the two media. Derive the amplitude of the transmitted wave relative to the incident wave as a function of the density contrast between the two media $\delta = \rho_{\mathrm{B}}/\rho_{\mathrm{A}}$ (where ρ_{A} and ρ_{B} are the densities of the two media, respectively). [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) plasmas and their behaviour in the presence of magnetic fields;

(b) accretion discs: their properties, evolution and emitted radiation;

(c) description of stars as self-gravitating polytropes.

3 *Attempt* **either** *this question* **or** *question 4.*
Answer all parts of this question.

(a) Derive the three Rankine–Hugoniot relations for the physical properties of a medium across a shock discontinuity. [9]

(b) A jet with cross-sectional area S, velocity v and density ρ hits and shocks a slab of gas. Assume an ideal-gas equation of state and that upstream and downstream media have the same mean molecular weight μ. For an adiabatic shock in the strong shock limit, determine the heating rate of the gas in the slab as a function of v, ρ, μ and γ, where the latter is the adiabatic coefficient. [*You may assume that the ratio between the pressure in the downstream and upstream media is* $M^2 2\gamma/(\gamma + 1)$, *where M is the Mach number in the upstream medium.*] [8]

(c) A galaxy can be approximated as a spherically symmetric system with a gravitational potential given by $\psi = -Ar^{-1/2}$ (A being a constant). The gas in the galaxy has a mean molecular weight μ, is isothermal with temperature T and is in pressure equilibrium at a radius r_0 with an external medium, whose pressure is p_0. Find an expression for the gas density inside the galaxy as a function of p_0, T and r. [5]

(d) A much smaller galaxy collides radially with the larger galaxy of the previous problem. The mass and size of the small galaxy can be neglected relative to those of the larger galaxy. At any time the distance between the small galaxy and the centre of the large galaxy is $d(t)$. Assuming the motion of the small galaxy is governed solely by the gravitational interaction with the larger galaxy (and neglecting shocks), determine an expression for the ram pressure exerted by the gas in the large galaxy on the smaller galaxy, as a function of d. The radial velocity of the small galaxy at the distance $d = r_0$ can be assumed to be zero.

[3]

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4 *Attempt* **either** *this question* **or** *question 3.*
Answer all parts of this question.

(a) Derive Bernoulli's equation for a gas with a barotropic equation of state. [6]

(b) Show that the flux of the vorticity $\boldsymbol{w}$ over a surface S within a barotropic fluid is conserved, i.e.

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_S \boldsymbol{w}\cdot d\boldsymbol{S} = 0\ .$$

[*Use Helmholtz's equation:* $\frac{\partial \boldsymbol{w}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{w})$.] [7]

(c) A star of mass M produces an isothermal wind with temperature T_0 and mass outflow rate $\dot{M}$. The wind reaches a maximum velocity at a distance r_M from the star. Estimate the gas density at r_M as a function of T_0, $\dot{M}$ and M. Assume that the effect of radiation pressure is negligible, that the wind is always subsonic, and that the mass of the star always dominates the gravitational field. [7]

(d) In the previous part now include the effect of radiation pressure. The stellar luminosity is L_*, and the radiation absorption cross section of the particles in the wind is σ. The source of radiation can be approximated as a point source. The wind now reaches a maximum velocity at a distance r'_M. Determine r'_M as a function of T_0, M, L_* and σ. [*The force exerted by the radiation pressure on each particle is $\sigma S/c$, where S is the radiation flux per unit area and c is the speed of light.*] [3]

(e) In the previous part assume that the wind behaves as a perfect gas, that it is composed of neutral atomic hydrogen and that at a distance r_m from the star the hydrogen atoms combine to form molecular hydrogen. The energy resulting from the formation of molecules is radiated away. Describe the change of the properties of the gas (density, velocity, temperature and pressure) as it passes through this transition radius.

[2]

END OF PAPER