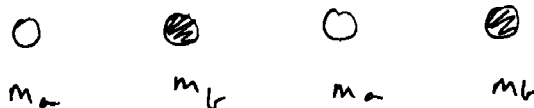


B1

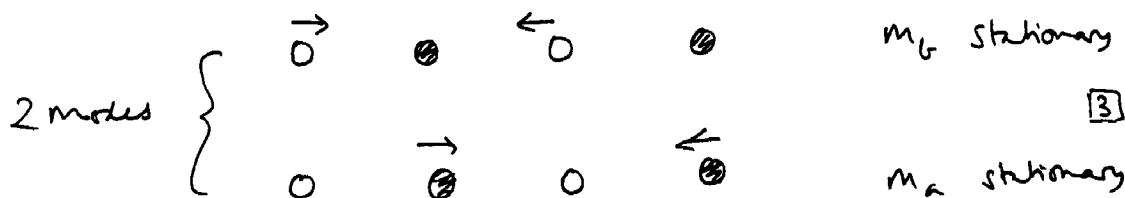
diatomic linear chain

(Quantum condensed matter physics answers

$\leftarrow a \rightarrow$



$$k = \pi/2a \Rightarrow \lambda = 4a$$



$$\text{ratio of frequencies} = \sqrt{\frac{m_a}{m_b}} \quad [1]$$

B2

ITO transparent from red end of visible

$$\text{set } \omega_{\text{plasma}} = 700 \text{ nm}$$

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m}$$

$$\text{set } m = \text{electron mass} \quad [2]$$

$$\lambda = 700 \text{ nm}, \omega = \frac{2\pi c}{\lambda}$$

$$n = \left(\frac{2\pi \times 3 \times 10^8 \times \omega}{7 \times 10^{-7} \times 1.6 \times 10^{-19}} \right)^2 \cdot 8.85 \times 10^{-12} \times 9 \times 10^{-31} \text{ m}^{-3}$$

$$= (1.68 \times 10^{34})^2 \times 8.85 \times 10^{-12} \times 9 \times 10^{-31} = 2.2 \times 10^{27} \text{ m}^{-3} \quad [2]$$

B3

$\tau = 10^{-182}$ secs. Compare with cyclotron frequency

$$\text{for free electron, } \omega_c = \frac{eB}{m} \quad [2]$$

$$\text{set } \omega_c \tau = 1 \Rightarrow B = \frac{m}{e\tau} = \frac{9 \times 10^{-31}}{1.6 \times 10^{-19} \times 10^{-12}}$$

$$B = 5.6 \text{ T} \quad [2]$$

Part II Quantum Condensed Matter (Paper 4B)

Model answers for A2

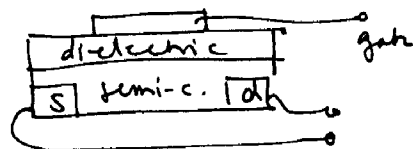
(a) Fermi liquids

- concept of particle-like excitations for a correlated electron system
- renormalised parameters, such as mass
- useful description when 'particle' lifetime is long, so low temperatures and energies close to E_{Fermi}
- measurement of F -surfaces even for strongly-interacting systems

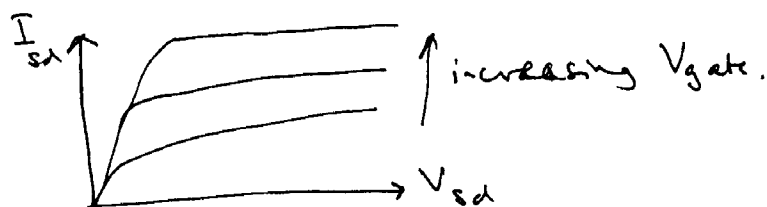
(b) Field Effect Transistor

- field effect = charge induced to either side of dielectric when gate voltage set up
- inversion or accumulation in semiconductor

- credit for diagram of structure



- credit for characteristics e.g. pinch-off



(c) Coordination number

- Covalent - fixed by valency. E.g. carbon in diamond (4) or III-V such as GaAs (4) Generally low values
- Ionic - fixed by Madelung electrostatic energy - long range effects important. Generally, high values
- van der Waals Nearest-neighbour interaction dominates, so close-packed (12)
- metallic Not usually directional, so high values close to close-packed (12) e.g. f.c.c. copper

Bloch's theorem

$$\psi(k, r) = u_k(r) e^{ikr}$$

where $u_k(r)$ is periodic with the lattice,

$$u_k(r) = u_k(r+T), \quad T = \text{lattice vector}$$

1-d chain, repeat distance a

choice of Bloch wavefunction:

$$\psi_k(x) = \sum_n e^{ikna} \phi(x-na)$$

where $\phi(x)$ is p_z wavefunction (4)

p_z orbitals experience potential from adjacent carbon sites,

$$V(r) = V_{\text{atomic}} + \Delta V(r)$$

$$\begin{aligned} \text{so, } H_{\text{el}} \sum_n e^{ikna} \phi(x-na) + \Delta V \sum_n e^{ikna} \phi(x-na) \\ = E(k) \sum_n e^{ikna} \phi(x-na) \end{aligned}$$

Multiply by $\phi^*(x)$ and integrate over all x :

$$\int \phi^*(x) H_{\text{el}} \sum_n e^{ikna} \phi(x-na) dx$$

$$+ \int \phi^*(x) \Delta V \sum_n e^{ikna} \phi(x-na) dx = E(k)$$

$$E_{\text{atomic}} + (e^{ikn} + e^{-ikn}) \int \phi^*(n) \Delta V \phi(n+a) dx$$

$$+ \int \phi^*(n) \Delta V \phi(n) dx = E(k)$$

$$\text{define } t = - \int \phi^*(n) \Delta V \phi(n+a) dx$$

$$B = - \int \phi^*(n) \Delta V \phi(n) dx$$

$$E(k) = E_{\text{atomic}} - 2t \cos(ka) - B$$

(4)

N carbon atoms with cyclic boundary conditions

1st Brill zone runs from $-\pi/a$ to π/a

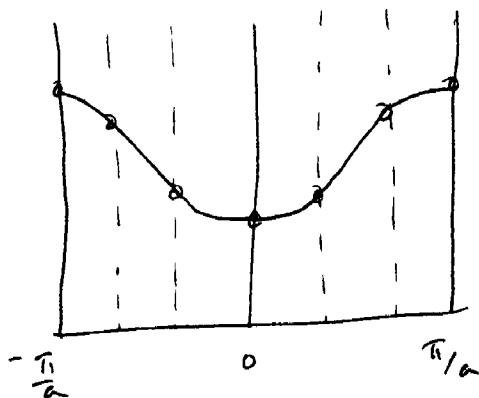
boundary conditions require $p \frac{2\pi}{a} = Na$ p integer

$$\text{so, } k = \frac{2\pi}{Na} p$$

$$\text{total number of states} = \frac{2\pi}{a} / \frac{2\pi}{Na} = N$$

(2)

Benzene allowed values of k are $0, \pm \frac{\pi}{3a}, \pm \frac{2\pi}{3a}, \pm \frac{\pi}{a}$

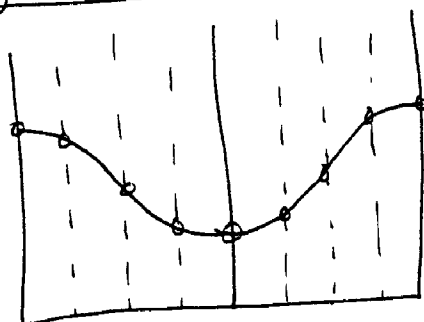


$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \uparrow \text{gap} = 2t \\ \begin{array}{c} 2t \\ t \\ -t \\ -2t \end{array} \end{array}$$

[credit for stating that $+\pi/a$ and $-\pi/a$ describe same state]

(4)

Ring with $N=8$ allowed values of k are $0, \pm \frac{\pi}{4a}, \pm \frac{\pi}{2a}, \pm \frac{3\pi}{4a}, \pm \frac{\pi}{a}$



$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \uparrow \text{gap} = 0 \\ \begin{array}{c} 2t \\ t \\ -t \\ -2t \end{array} \end{array}$$

← 2 electrons
4 states available
so gap = 0

(4)

Dimerization of chain. Example of Peierls distortion,

sets up energy gap at Fermi energy, lowering the energy of the occupied (lower) band. [2] ~~Benzene~~

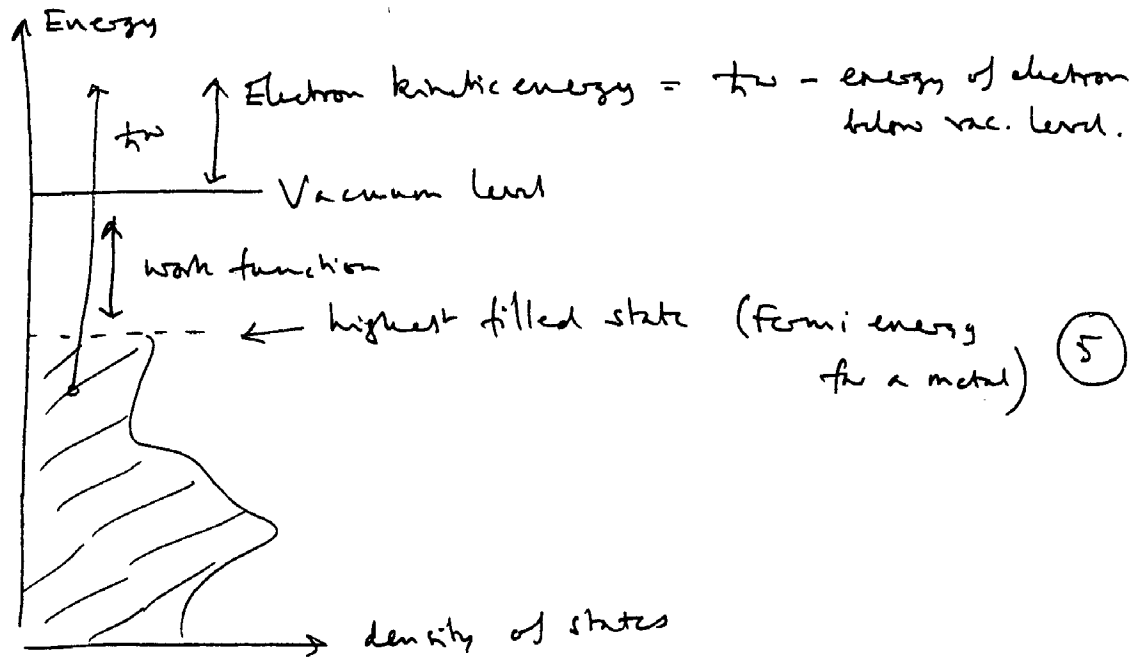
Benzene has large gap between filled and empty states anyway so little driving force. [1] $N=8$ has degeneracy at 'Fermi energy', so dimerisation lowers energy of filled state [1]

(4)

Carotene $N=22$ so ~~states~~ $k = p \frac{\pi}{11a}$. Highest filled at $k = \frac{5\pi}{11a}$
lowest empty at $k = \frac{6\pi}{11a}$. Energy difference = $3 \left(\cos\left(\frac{5\pi}{11}\right) - \cos\left(\frac{6\pi}{11}\right) \right) = 0.84 \text{ eV}$
gap is larger (2.3 eV) because (i) boundary conditions at chain ends and (ii) dimerisation.

(3)

photoelectric effect: $h\nu$ photon promotes electron from valence band state to state above the vacuum level, so electron escapes from surface of metal. ~~The~~ Threshold for escape when photon energy is equal to work function of the material.



density of valence band states probed, as evident in diagram.

Two other experimental measurements: list includes:

- optical absorption - 'vertical' transition between k states, direct gap versus indirect gap semiconductors etc.
- magnetic quantisation, de Haas van Alphen etc reveals Fermi surface (6)
- effective mass of carriers (including negative mass for 'holes') in semiconductors, Hall effect
- tunnelling current-voltage characteristic.

Optical transition is vertical ~~for~~ so $k_{\parallel, \text{final}} = k_{\parallel, \text{initial}}$
 Here, the final state is a free electron outside
 the surface, so $k_{\parallel, \text{final}}$ gives the real momentum
 in the plane of the surface.

Measure experimentally E_{kin} and θ

$$E_{\text{kin}} = \frac{\hbar^2 k^2}{2m} \quad \hbar k = \sqrt{2mE}$$

$$\hbar k_{\parallel} = \sqrt{2mE} \sin \theta \quad (4)$$

for ΓK spectra, zone boundary reached near E_F at 55°

$$E (\text{photoemitted electron}) \sim 21.2 - 4.7 = 17 \text{ eV}$$

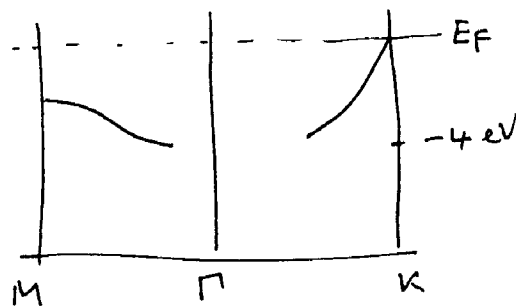
(workfunction)

$$\sqrt{2mE} = (2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19})^{1/2} = 2.2 \times 10^{-24} \text{ m}^{-1}$$

$$\hbar k_{\parallel} \text{ at 2B max} = \frac{10^{-34} \cdot 4.8}{3 \times 2.46 \times 10^{-10}} = 1.7 \times 10^{-24} \text{ m}^{-1}$$

$$\Rightarrow \sin \theta = 1.7 / 2.2, \quad \theta = 51^\circ \quad (4)$$

Energy band plot:



Note that valence band only reaches the Fermi energy near the K point (well below at M)
 \Rightarrow consistent with semimetal.

(3)