UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 29 May 2015 1.30 pm to 3.30 pm

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

QUANTUM CONDENSED MATTER PHYSICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUANTUM CONDENSED MATTER PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) The top of the valence band in GaAs at the centre of the Brillouin zone has $|\mathrm{d}^2E(k)/\mathrm{d}k^2| = 2.44 \times 10^{-38}\mathrm{m}^4\,\mathrm{kg\,s}^{-2}$. Find the corresponding cyclotron frequency for a hole at the top of the valence band in a magnetic field $B = 1$ T. [4]

(b) For a one-dimensional chain of atoms lying along the x-direction, sketch the tight-binding band structure for a band based on p_z orbitals, and also for a band based on p_x orbitals. [4]

(c) The semiconductor germanium has a relative permittivity $\varepsilon_\mathrm{r} = 16$ and an electron effective mass $0.2\,m_\mathrm{e}$. Estimate the n-doping concentration level above which this material has non-zero conductivity even at zero temperature. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) heavy-fermion systems;

(b) the Stoner-Hubbard model for magnetism in metals;

(c) Bloch oscillations.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain why a material with an even number of electrons per unit cell can be a band insulator. [3]

For a free two-dimensional (2D) electron gas, show that the Fermi wavevector is

$$k_{\mathrm{F}} = \sqrt{2\pi n}\,,$$

where n is the electron density. [2]

A 2D electron gas is subject to a periodic potential

$$V(x, y) = -V_0\,[\cos(Qx) + \cos(Qy)]\,,$$

where $Q = \alpha\sqrt{2\pi n}$, with $\sqrt{2} < \alpha < 2$, and $V_0 \geq 0$. Sketch the shape of the 1$^{\mathrm{st}}$ Brillouin zone, clearly labelling its dimensions. [2]

Sketch the Fermi surface for the cases $V_0 = 0$ and $V_0 > 0$, labelling the electron and hole pockets for the case $V_0 > 0$. [4]

For $\alpha = \sqrt{3}$ find the electron and hole carrier densities in the limit when V_0 is very small. Also find the value of α for which the electron and hole carrier densities are equal. [8]

With the help of a sketch of the dispersion relations along different directions, explain the conditions under which the material becomes an insulator. Include in your discussion a rough estimate of the minimum value of V_0 for which this can occur. [6]

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4 *Attempt* **either** *this question* **or** *question 3.*

Show that in a semiconductor in which the charge carriers are non-degenerate the electron and hole concentrations, n and p, satisfy the law of mass action

$$np = A\,(k_\mathrm{B}T)^3 \exp\left(-\frac{E_\mathrm{g}}{k_\mathrm{B}T}\right),$$

where E_g is the band gap energy and A is a constant that you should explicitly calculate. [9]
[*You may find one of the following results useful:* $\int_0^\infty \sqrt{x}\,\mathrm{e}^{-x}\,\mathrm{d}x = \sqrt{\pi}/2$, $\int_0^\infty x^2\,\mathrm{e}^{-x^2}\,\mathrm{d}x = \sqrt{\pi}/4$.]

In a particular material, the chemical potential μ is fixed at the bottom of a conduction band. Other states, not further considered here, ensure that $\mu = 0$ independent of T. The bottom of the conduction band $E_\mathrm{c}(k) = \hbar^2k^2/(2m)$ is separated by E_g from the top of a valence band, $E_\mathrm{v}(k) = -E_\mathrm{g} - \hbar^2k^2/(2m)$.

Explain why in this case the law of mass action is not valid. [2]

Find and sketch the T dependence of both n and p for $\mu = 0$. Without detailed calculations, comment on how the product np differs from the prediction of the law of mass action. [7]

Now suppose that μ is instead fixed at a slightly positive value, $0 < \mu \ll E_\mathrm{g}$. Find n at $T = 0$ and sketch the T dependence of n. [7]

END OF PAPER