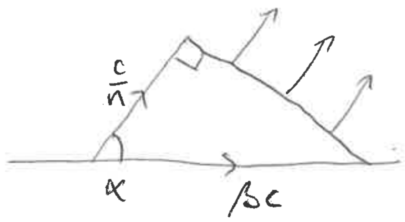
PNP 2014

1) a) Cerenkov photons emitted at angle α to direction of motion of charged particle - relate Cerenkov angle α to β and n



$$\cos\alpha = \frac{c/n}{\beta c} = \frac{1}{n\beta}$$

b) ω^0 meson and π^0 meson have non zero overlap with $u\bar{u}$ wavefunction

ω^0 - $J^P = 1^-$, π^0 - $J^P = 0^-$

can ω^0 decay to $\pi^0\pi^0$ and nothing else?

	Initial	Final	
J	1	$0 + L$	$\Rightarrow$ need $L = 1$
P	-1	$(-1)(-1)(-1)^L$	- $L = 1$ conserves parity

identical bosons in final state - need symmetric wavefunction

$L=1$ state is antisymmetric - not allowed

c) estimate ratio $R = \sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-)$ for $\sqrt{s}$ halfway between threshold for $c\bar{c}$ and $b\bar{b}$ production

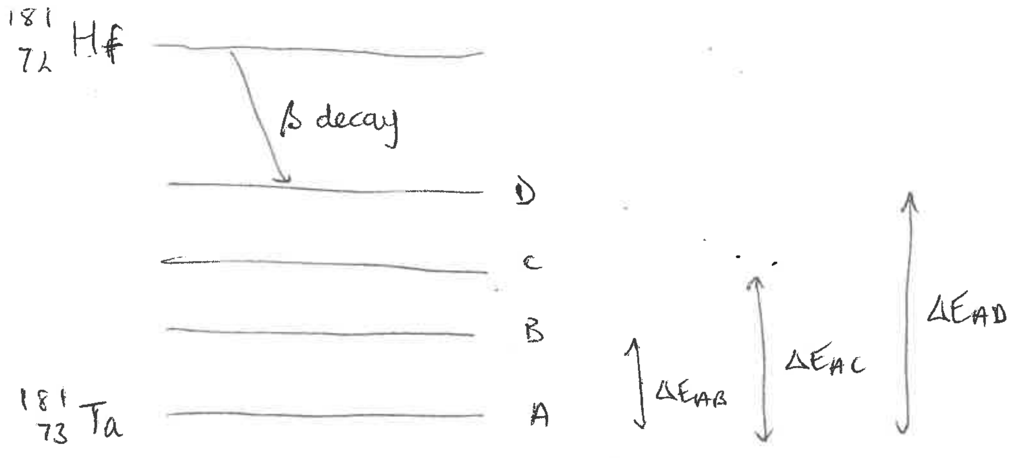
hadron decays - matrix element $M \propto \frac{2}{3}e^2$ for up type quarks

$M \propto -\frac{1}{3}e^2$ for down type quarks

2 possible decays to up type quarks, 2 decays to down type quarks

$\sigma \propto |M|^2 \Rightarrow R = 3\times\left(\frac{2}{3}\right)^2 \times 2 + 3\times\left(\frac{1}{3}\right)^2 \times 2 = \frac{10}{3}$

4) Gamma decays



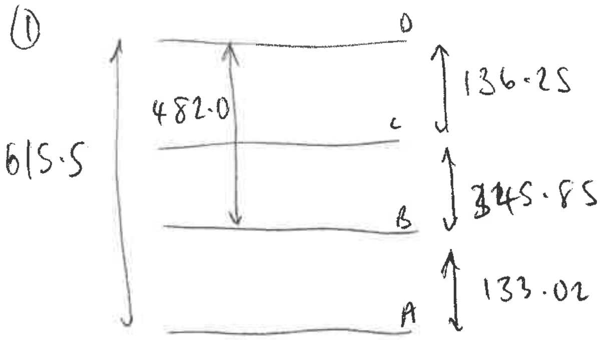
Spectral lines	γ energy/keV	intensity	multipolarity
1	133.02	16.5	E2
2	136.25	2.4	E2 + M1
3	345.85	2.4	E2
4	482.0	14.1	M1 + E2
5	615.5	0.04	M3

a) determine the possible values of $\Delta E_{AB, AC, AD}$

$\Delta E_{AD} = 615.5$ keV

$482 = 136.25 + 345.85 \Rightarrow$ need transitions with $\Delta E = 136.25$, 345.85 keV next to each other

(1)

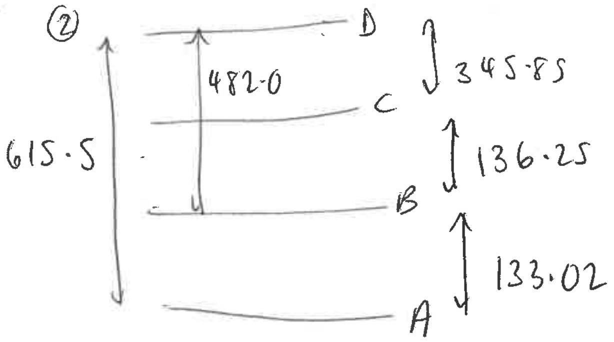


$\Delta E_{AB} = 133.02$

$\Delta E_{AC} = 478.87$

$\Delta E_{AD} = 615.5$ keV

②

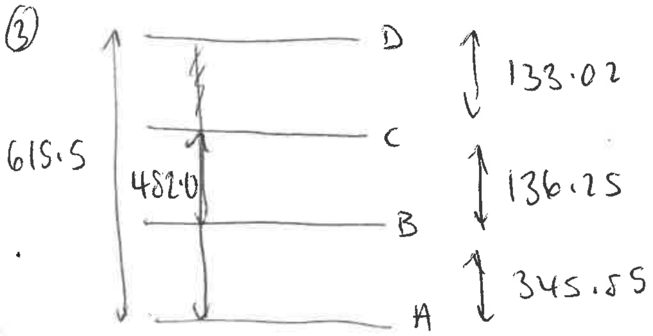


$\Delta E_{AB} = 133.02$

$\Delta E_{AC} = 269.27$

$\Delta E_{AD} = 615.5$ keV

③

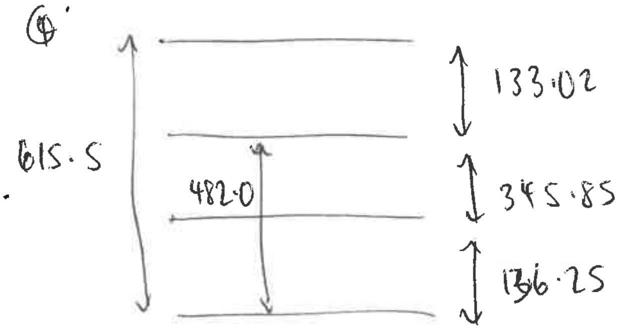


$\Delta E_{AB} = 345.85$

$\Delta E_{AC} = 482.0$

$\Delta E_{AD} = 615.5$ keV

④



$\Delta E_{AB} = 136.25$

$\Delta E_{AC} = 482.0$

$\Delta E_{AD} = 615.5$ keV

b) hierarchy E1 E2 E3
M1 M2 M3

$\longrightarrow$ decreasing rate

E transitions are electric dipole transitions

M transitions are magnetic dipole transitions

Parity change of nucleus in EL transitions with L odd and ML transitions with L even

J^P for gamma radiation :

E1	E2	E3	M1	M2	M3
1^-	2^+	3^-	1^+	2^-	3^+

c) If a decay between 2 states occurs via a process with $l = 3$ for example, with no lower l decays seen, the ~~pm~~ difference in J must be equal to the maximum possible difference (3 in this case) - if there was a smaller difference in J, the decay could occur via a faster process, with $l = 1, 2$

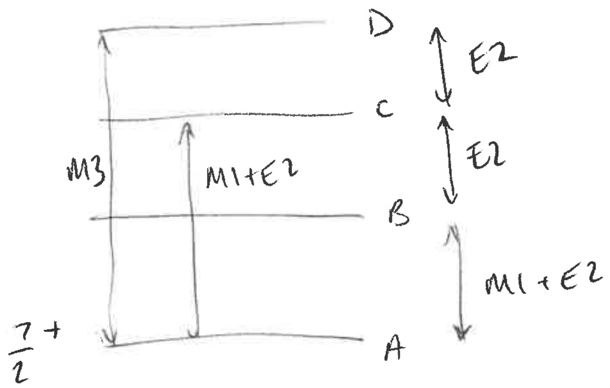
deduce parities - if parity of ~~one~~ initial or final state is known, use the observed transition types to deduce parity of the other state - eg if M1 and E2 transitions are seen, the two states have the same parity, but if E1 is seen, states have opposite parity

In general, if magnetic dipole transitions are seen with l odd or electric dipole transitions are seen with l even, there is no parity change, otherwise parity of initial and final states is opposite

d) ground state $J^P = \frac{7}{2}^+$ ↗ diagram ④

lines 2 and 4 are both decays to ground state

deduce allowed J^P for levels B, C, D



D → A transition

M3, no other transitions seen so need $\Delta J = 3$

$J_D = \frac{1}{2}, \frac{13}{2}$

M3 - no parity change

$J^P(D) = \frac{1}{2}^+, \frac{13}{2}^+$

(If $\Delta J = 0, 1, 2$ could have M1, E2 transitions - not seen)

D → C transition

E2 – $\Delta J = 0, 1, 2$, no parity change

but if $\Delta J = 0, 1$, could have M1 transition – need $\Delta J = 0$

$J_c = \underbrace{\frac{17}{2}, \frac{9}{2}}_{J_D = \frac{13}{2}}$ or $\underbrace{\frac{5}{2}}_{J_D = \frac{1}{2}}$

C → A transition

M1 + E2 – $\Delta J = 0, 1, 2$, no parity change

$J_A = \frac{7}{2} \Rightarrow J_c = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

from D → C and C → A transitions, $J_c = \frac{9}{2}$ $(J_D = \frac{13}{2})$

or $\frac{5}{2}$ $(J_D = \frac{1}{2})$

C → B transition

E2 – $\Delta J = 0, 1, 2$, no parity change

for $J_c = \frac{5}{2}$: $J_B = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$

for $J_c = \frac{9}{2}$: $J_B = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$

but no M1 transition seen ⇒ need $\Delta J = 2$

$J_B = \frac{1}{2}, \frac{9}{2}$ or $\frac{5}{2}, \frac{13}{2}$

B → A transition

M1 + E2 – $\Delta J = 0, 1, 2$, no parity change

$J_A = \frac{7}{2} \Rightarrow J_B = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

from C → B and B → A transitions

$J_B = \frac{5}{2}$ or $\frac{9}{2}$

no D → B transitions ⇒ If $J_D = \frac{13}{2}$, need $J_B = \frac{5}{2}$

If $J_D = \frac{1}{2}$, need $J_B = \frac{9}{2}$

(need large enough ΔJ that transitions not seen) no parity change for M1 / E2 / M3 - all levels have $P = +1$

D	$\frac{1}{2}^+$		$\frac{13}{2}^+$
C	$\frac{5}{2}^+$	or	$\frac{9}{2}^+$
B	$\frac{9}{2}^+$		$\frac{5}{2}^+$
A	$\frac{7}{2}^+$		$\frac{7}{2}^+$

3) $C \to t + B$ $\qquad B \to s + A$

s, t massless
decays isotropic in C, B rest frame

a) i) 3-momentum of C in B rest frame

in C rest frame : $M_C = E_B + E_t$

$$E_t^2 = (M_C - E_B)^2 = p_t^2 = p_B^2$$

as t is massless and $|p_t| = |p_B|$ in C rest frame

$$M_C^2 + E_B^2 - 2M_C E_B = p_B^2$$

$$M_C^2 + M_B^2 - 2M_C E_B = 0$$

$$E_B = \frac{M_C^2 + M_B^2}{2M_C} = \sqrt{p_B^2 + M_B^2}$$

$$p_B^2 = \frac{M_C^4 + M_B^4 + 2M_C^2 M_B^2}{4M_C^2} - M_B^2$$

$$= \frac{M_C^4 + M_B^4 - 2M_C^2 M_B^2}{4M_C^2}$$

- this is equal to the 3-momentum of C in rest frame of B

$$p_C = \left(\frac{M_C^4 + M_B^4 - 2M_C^2 M_B^2}{4M_C^2}\right)^{1/2}$$

ii) 3-momentum of A in rest frame of B

from i) (relabelling) $E_A = \dfrac{M_B^2 + M_A^2}{2M_B}$ in B rest frame

$$p_A = \left(\frac{M_B^4 + M_A^4 - 2M_A^2 M_B^2}{4M_B^2}\right)^{1/2}$$

b) i) $M_{st}^2 = (E_s + E_t)^2 - (p_s + p_t)^2$

$= \cancel{M_s^2} + \cancel{M_t^2} + 2E_sE_t - 2p_sp_t\cos\theta$

$= 2E_sE_t(1-\cos\theta)$

max value when $\cos\theta = -1$

$M_{st}^2 = 4E_sE_t$

min value when $\cos\theta = 1$, $M_{st}^2 = 0$

ii) $M_{At}^2 = (E_A + E_t)^2 - (p_A + p_t)^2$

$= M_A^2 + \cancel{M_t^2}^{\to 0} + 2E_AE_t - \cancel{2E_A}\; 2p_Ap_t\cos\theta$

$= M_A^2 + 2p_t(E_A - p_A\cos\theta)$

If $p_A = 0 \Rightarrow E_A = M_A$

$M_{At}^2 = M_A^2 + 2p_t M_A$

If $p_A \gg M_A \Rightarrow E_A = p_A$

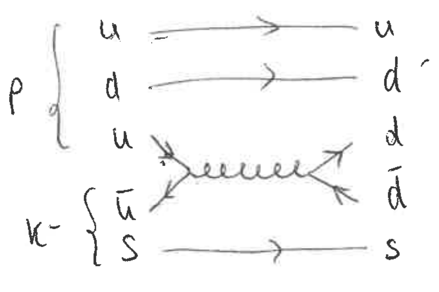
$M_{At}^2 = M_A^2 + 2p_tE_A(1-\cos\theta)$

which has max/min values

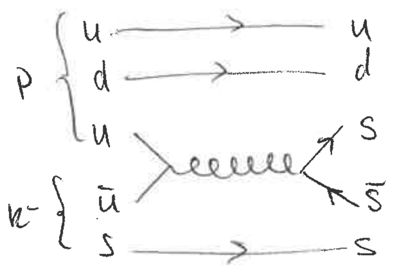
$M_A^2 + 4p_tE_A \sim 4p_tE_A$ and M_A^2

c)

$K^- + p \to \pi^0 + X^0$, $K^- + p \to K^+ + Y^-$ - strong



$d\bar{d} = \pi^0 \Rightarrow X^0 = uds$



$K^+ = u\bar{s} \Rightarrow Y^- = ssd$

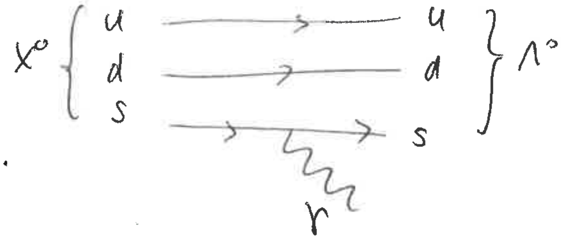
d) $X^0 \to \Lambda + \gamma$, $Y^- \to \Lambda + \pi^-$

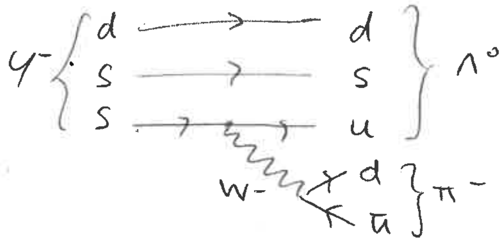
What can be deduced about lifetimes / spins / parities of X^0 and Y^- ?

lifetimes : X^0 decays via EM interaction - fast decay

Y^- must change flavour of ~~s~~ s quarks $\Rightarrow$ weak decay

↳ Y^- has longer lifetime





$X_0 \to \Lambda + \gamma$

J ? $\to \frac{1}{2}^{+}, 1$

P ? $\to (+1)(-1)(-1)^L$

Parity conserved in EM interactions - if L is even, X^0 has $P = -1$, if L is odd, X^0 has $P = +1$

for $L = 0$, $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$ $L = 0$ more likely

$Y^- \to \Lambda + \pi^-$

J ? $\to \frac{1}{2}, 0$

P ? $\to (+1)(-1)(-1)^L$

can't deduce parity - not always conserved in weak interactions

$J = \frac{1}{2} \oplus L = \frac{1}{2}$ for $L = 0$