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Lent/Easter Terms 2022

Part II Particle and Nuclear Physics
Examples Sheet 1

Matter and Forces

1. Particles, Tripos A-style question.
   Explain the meaning of the terms *quark, lepton, hadron, nucleus* and *boson* as used in the classification of particles.

Relativistic kinematics

2. Natural Units, Tripos A-style question.
   Explain what is meant by *natural* units and the *Heaviside-Lorentz* system.

   (a) The reduced Compton wavelength of a particle can be written in natural units as

   $$\lambda\!\!\!^{-} = \frac{1}{m}$$

   where m is the mass of the particle. Estimate $\lambda\!\!\!^{-}$ for a pion ($m_\pi = 139.6$ MeV/c^2). Quote your answer in natural units and then convert to SI units.

   (b) The total cross-section for e^+e^- annihilation can be written in natural units as

   $$\sigma = \frac{4}{3}\frac{\pi\alpha^2}{s}$$

   where $\alpha = \frac{1}{137}$ is the fine structure constant and $\sqrt{s}$ is the centre-of-mass energy. Estimate σ at a centre-of-mass energy equal to the Z mass ($m_Z = 91.2$ GeV/c^2). Calculate your answer in natural units and then convert to barns.

   (c) Use dimensional analysis to add the appropriate factors of $\hbar$, ϵ_0 and c in the formulae for $\lambda\!\!\!^{-}$ in (a) and for σ in (b), and then do the calculations directly in SI units.

   [Note that $\hbar c = 197$ MeV fm; 1 barn $= 10^{-28}$ m^2]

3. Relativistic Kinematics, Tripos B-style question.
   Consider the decay of a particle X into two particles a and b.

   (a) Show that, in the rest frame of X, the energy of particle a can be written in natural units as

   $$E_a = \frac{m_X^2 + m_a^2 - m_b^2}{2m_X}$$

   where m_i is the mass of particle i. What is the equivalent expression for the energy of particle b ? What is the energy if the final state particles are the same (or antiparticles of each other)?

(b) Show that the magnitude of the momentum of particle a can be written in natural units as

$$p_a = \frac{\sqrt{m_X^4 + m_a^4 + m_b^4 - 2m_X^2 m_a^2 - 2m_X^2 m_b^2 - 2m_a^2 m_b^2}}{2m_X}.$$

What is the equivalent expression for the momentum of particle b ? What is the value of the momentum if the final state particles are the same (or antiparticles of each other) ? Show that if one of the final state particles is massless, e.g. $m_b = 0$, then the expression for the momentum simplifies to

$$p_a = \frac{m_X^2 - m_a^2}{2m_X}$$

[Note: replacing m_X by the centre-of-mass energy, $\sqrt{s}$, in the above gives the equivalent expressions for collision processes.]

(c) The HERA collider at DESY provided head-on collisions between an electron beam of 27.5 GeV and a proton beam of 920 GeV. What energy of electron beam colliding with a fixed target would be required to obtain the same centre-of-mass energy? Would HERA have had sufficient energy to have produced a Higgs boson of mass 125 GeV?

4. Ω Decay, Tripos A-style question.

The figure shows a photograph and line diagram of the event corresponding to the first observation of the Ω^- baryon ($\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$) in a K^-p interaction in a liquid hydrogen bubble chamber (from Barnes *et al.*, Phys. Rev. Lett. **12** (1964) 204):

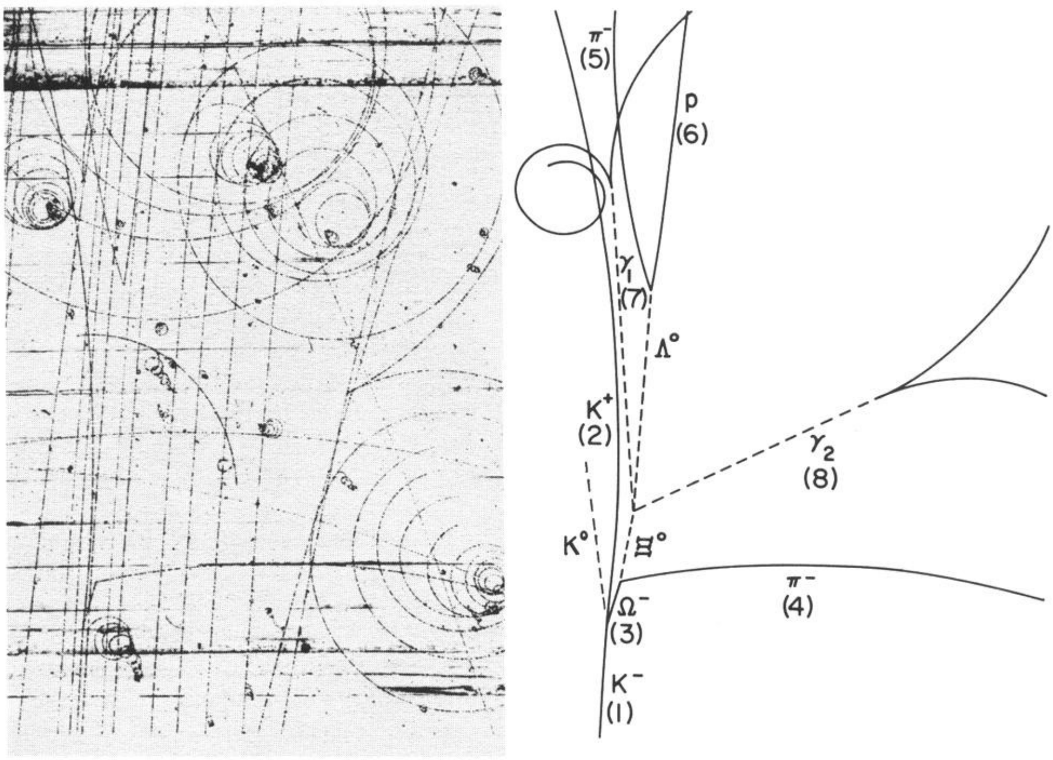

FIG. 2. Photograph and line diagram of event showing decay of Ω^-.

(a) The two photons from the $\pi^0 \to \gamma\gamma$ decay are both seen to convert to e^+e^- pairs. Show that the process $\gamma \to e^+e^-$ is kinematically forbidden *in vacuo*. Explain why the conversion process can take place in the presence of matter, and draw a Feynman diagram representing photon conversion in material, as seen in the figure.

(b) The π^- and Ξ^0 from the Ω^- decay have momenta of 281 MeV/c and 1906 MeV/c respectively. Their spatial opening angle is 71°. Calculate the mass of the Ω^- and compute its momentum.

(c) The length of the Ω^- flight path is 2.5 cm. Calculate the proper lifetime of the Ω^-.

[$m(\pi^-)$=139.6 MeV/c^2, $m(\Xi^0)$=1315 MeV/c^2]

Decays and Reactions

5. Radioactive Decay, Tripos B-style question.
A sample of gold is exposed to a neutron beam of constant intensity such that 10^{10} neutrons per second are absorbed in the reaction

$$\mathrm{n} + {}^{197}\mathrm{Au} \to {}^{198}\mathrm{Au} + \gamma$$

The nuclide ^{198}Au undergoes β decay to ^{198}Hg with a mean lifetime of 4 days.

(a) How many atoms of ^{198}Au will be present after 6 days of irradiation?

(b) How many atoms of ^{198}Hg will be present after 6 days assuming that the neutron beam has no effect on the Hg?

(c) What is the equilibrium number of ^{198}Au nuclei?

6. Caesium Decay, Tripos B-style question.
The decay chain ^{139}Cs→^{139}Ba→^{139}La is observed for an initially pure sample of 1mCi of ^{139}Cs. The half life of ^{139}Cs is 9.5 minutes and that of ^{139}Ba is 82.9 minutes; ^{139}La is stable. Write down the rate equations for this system, and show that the number of Ba atoms present at time t is given by

$$N_{\mathrm{Ba}}(t) = \frac{\lambda_{\mathrm{Cs}} N_{\mathrm{Cs}}(0)}{\lambda_{\mathrm{Ba}} - \lambda_{\mathrm{Cs}}} \left[e^{-\lambda_{\mathrm{Cs}} t} - e^{-\lambda_{\mathrm{Ba}} t}\right]$$

in an obvious notation, where the λ values represent the corresponding decay rates. What is the maximum ^{139}Ba activity (i.e. rate of ^{139}Ba decay), and at what time does it occur?

[1 Ci= 3.7x10^{10} disintegrations per second.]

7. Kaon Decay, Tripos A-style question.

(a) Calculate the branching fraction for the decay $\mathrm{K}^+ \to \pi^+\pi^0$, given that the partial width for this decay is 1.2×10^{-8} eV and the mean lifetime of the K^+ meson is 1.2×10^{-8}s.

(b) A beam of K^+ mesons of momentum 10 GeV is produced. What fraction of them will remain undecayed 100 m downstream?

(c) When the K^+ mesons decay to $\pi^+\pi^0$, what are the minimum and maximum laboratory energies of the produced π^+ mesons?

8. Cross-sections, Tripos B-style question.
Define the terms *total cross-section* and *differential cross-section* for scattering processes.

A beam of neutrons with an intensity 10^5 particles per second traverses a thin foil of ^{235}U with a "thickness" of 10^{-1} kg m^{-2}.[1]

There are three possible outcomes when a neutron interacts with a ^{235}U nucleus:
(i) elastic scattering of the neutron, with a cross-section 10^{-2} b ;
(ii) the capture of the neutron followed by the emission of a γ-ray., with cross-section 70 b;
(iii) the capture of the neutron, followed by the resulting nucleus undergoing fission with cross-section 200 b.
Using this information, determine:

(a) the intensity of the neutron beam transmitted by the foil;

(b) the rate of fission reactions occurring in the foil induced by the incident beam;

(c) the flux of neutrons elastically scattered out of the beam at a point 10 m from the foil, assuming that the neutrons are scattered isotropically.

9. Breit-Wigner Formula, Tripos A-style question.
The Breit-Wigner formula for a reaction cross-section is given by

$$\sigma(E) = \frac{\pi g}{p_i^2} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \Gamma^2/4}.$$

Explain the meaning of the symbols in this equation, and outline its derivation.

The maximum value of the cross-section for radiative capture of neutrons in ^{123}Te (i.e. the process $\mathrm{n} + ^{123}\mathrm{Te} \longrightarrow ^{124}\mathrm{Te} + \gamma$) is 75 kb and is reached at a neutron energy of 2.2 eV, where the elastic width Γ_n is 0.0104 eV and the radiative width Γ_γ is 0.105 eV. The spin of ^{123}Te in its ground state is $J = \frac{1}{2}$. What is the elastic cross-section at resonance and what is the spin of the compound nucleus formed?

[1] To determine the number of particles that interact one must know the density of target particles and the thickness of the target. Instead of giving two numbers which simply have to be multiplied, it is common practice to quote the target thickness multiplied by the target density. This gives a target "thickness" in units of mass/area.

Numerical answers

2. (a) 7.16×10^{-3} MeV^{-1}, 1.41 fm; (b) 2.68×10^{-8} GeV^{-2}, 0.0104 nb
3. (c) $\sqrt{s} = 318$ GeV; 54 TeV.
4. (b) 1689 MeV/c^2, 2015 MeV/c ; (c) 70 ps
5. (a) 2.7x10^{15}; (b) 2.5x10^{15}; (c) 3.46x10^{15}
6. (a) 0.087 mCi; (b) 33.5 minutes
7. (a) 21.8% (b) 0.25, [0.88, 9.18] GeV.
8. (a) 99,311 particles s^{-1}; (b) 510 s^{-1}; (c) 2.03x10^{-5} particles m^{-2} s^{-1}
9. 7.4 kb; $J = 1$

Suggested Tripos Questions

Relativistic Kinematics: 2017 1(a), 2014 3, 2004 (3) C12(b)

Breit-Wigner resonances, production and decay rates: 2018 A1(a), 2015 3, 2005 (3) A3