NATURAL SCIENCES TRIPOS Part II

Tuesday 27th May 2008 9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (1)

Attempt the **whole** *of Section A, the* **whole** *of Section B,* **one** *question from Section C and* **one** *question from Section D.*
Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to **each** *question from Sections B, C and D should be tied up separately, with the number of the question written clearly on the cover sheet.*
Sections A and B each carry approximately a quarter of the total marks. The **approximate** *number of marks allocated to each part of questions in Sections C and D is indicated in the right margin. This paper contains 6 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work paper
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.

A1 Use the Clausius-Clapeyron equation to show that the pressure p of an ideal vapour in equilibrium with its liquid phase is given by

$$p \propto \exp[-L/k_{\rm B}T],$$

where the latent heat of vaporisation, L, is independent of temperature, and the liquid is much denser than the vapour.

A2 The Gibbs free energy of a system is given by

$$G(T,p,N) = Nk_{\rm B}T\ln(ApT^{-\alpha}),$$

where A and α are constants. Calculate the heat capacity at constant pressure.

A3 What factors determine the universality class of a continuous phase transition?

A4 For a system of particles write down a Lorentz invariant from the 4-momentum and use it to derive an expression relating total energy and total momentum.

A5 By considering the Poynting flux explain why the effective area of a loop antenna can be larger than its geometrical area.

A6 A beam of charged particles fired through a glass block of refractive index n=1.5 generates Cerenkov radiation only when the beam energy exceeds 700 MeV. What is the rest mass of the particles?

SECTION B

Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.

B7 **Either** (a) Write an essay on ideal (perfect) and imperfect classical gases. Your answer should include a discussion of the nature of the interaction potential in real gases, contributions to the heat capacity, equations of state, and the virial expansion.

Or (b) Write an essay on ensembles in statistical mechanics. Your answer should include descriptions of the microcanonical, canonical and grand canonical ensembles and their probability distributions.

B8 Write brief notes on **two** of the following:

(a) synchrotron radiation;

(b) the use of birefringence in the construction of optical waveplates;

(c) Rayleigh scattering from the sky;

(d) the use of Jones matrices.

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SECTION C

C9 Write down the expression for the classical partition function Z_{cl} of a particle moving in one dimension with Hamiltonian $H(p, x)$. [2]

Evaluate the classical partition function of a one-dimensional harmonic oscillator whose Hamiltonian is $H = \frac{p^2}{2m} + cx^2$. [4]

The Hamiltonian of a one-dimensional classical anharmonic oscillator is

$$H = \frac{p^2}{2m} + cx^2 + gx^3.$$

The effect of the cubic term can be taken to be small.

(a) Show that Z_{cl} for this system, retaining the term in g to lowest order, is given by

$$Z_{\text{cl}} = \frac{\pi}{h\beta}\sqrt{\frac{2m}{c}}\left(1 + \frac{15g^2}{16\beta c^3}\right).$$

[6]

(b) Evaluate the internal energy and heat capacity of the system, retaining the term in g to lowest order. [6]

(c) Evaluate the mean position of the particle, again retaining the term in g to lowest order. [4]

[*You may use the results that* $\int_{-\infty}^{\infty} x^n \exp\left(-ax^2\right) dx$ *is equal to* $\sqrt{\frac{\pi}{a}}$ *for* $n = 0$, $\frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$ *for* $n = 4$, *and* $\frac{15}{8a^3}\sqrt{\frac{\pi}{a}}$ *for* $n = 6$.]

C10 The density of states of a system is

$$\mathcal{D}(\epsilon) = \begin{cases} A & 0 < \epsilon < \Delta \\ 0 & \text{otherwise.} \end{cases}$$

The system contains N particles and is in thermal equilibrium.

(a) Write down equations relating N, $\mathcal{D}(\epsilon)$, and the chemical potential μ, valid when the particles are (*i*) fermions and (*ii*) bosons. [2]

(b) Find expressions for the chemical potential of the system at zero temperature when the particles are (*i*) fermions and (*ii*) bosons. [4]

(c) Consider such a system with $N \ll A\Delta$ at high temperatures where the distinction between fermionic and bosonic statistics can be neglected. Show that the chemical potential is given by

$$\mu = \frac{1}{\beta}\ln\left[\frac{\beta N}{A\left(1 - \exp[-\beta\Delta]\right)}\right].$$

[5]

(d) Obtain expressions for the internal energy and heat capacity of the system in the high-temperature regime. [5]

(e) Sketch the high-temperature forms of (*i*) the internal energy and (*ii*) the heat capacity as a function of temperature for finite and infinite Δ, and comment on the results. [6]

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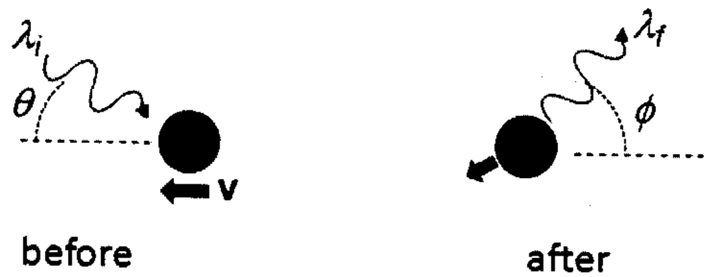
SECTION D

D11 Explain both Compton scattering and inverse Compton scattering processes, highlighting the differences between them. In what context might one be likely to encounter these processes? [6]



A photon of wavelength λ_i collides with a massive particle travelling at $v = \beta c$, as in the sketch above. Write down the 4-momentum of both particles before and after the collision. Hence by eliminating the momentum of the massive particle after the collision, show that the wavelength of the scattered photon is given by

$$\lambda_f = \frac{\lambda_i(1+\beta\cos\phi) + \lambda_p\left[1-\cos(\phi+\theta)\right]}{1+\beta\cos\theta},$$

and define λ_p. [8]

For what collision geometry is the shortest wavelength photon produced? For this case, how does the emitted wavelength depend on the incident wavelength in the limits of large β and small β? [4]

By focussing an intense laser beam of wavelength λ=1048nm onto an electron beam, the photon wavelength can be shifted into the X-ray region. Calculate the shortest X-ray wavelength produced for electrons travelling at $0.98c$. [2]

Explain how you might use this process to measure the speed of electrons in a linear accelerator, and comment on the range of velocities you could measure. [2]

D12 Explain how the vector and scalar potentials are modified when transforming to a frame travelling at velocity $\boldsymbol{v}$.

Describe what happens to the field lines around a charged particle moving at an appreciable fraction of the speed of light in free space (no proof is required). [5]

Two insulating uniformly-charged plates are separated by d and charged to a potential V. The plates are at rest in S with their faces perpendicular to the y axis. Transform the 4-potential to a frame S' moving at u along the x'-axis. Hence calculate the electric and magnetic fields in this moving frame. [6]

A relativistic charged particle is midway between the two plates at time $t = t' = 0$ travelling parallel to their surfaces. Using the above transformation into the rest frame S' of the particle, discuss the particle trajectory and calculate the times in both S and S' when the particle hits one of the plates. [5]

Explain how the difference in these times in S and S' can be understood. [2]

By transforming the potential of this moving charge q into the rest frame of the plates, calculate the change in the potential at a point on the plates directly above the particle at time $t = 0$. Explain your result. [4]

END OF PAPER