UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Wednesday 29 May 2019 1.30 pm to 3.30 pm

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of ***five*** *questions:* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains ***five*** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof.*

A1 A plane monochromatic electromagnetic wave propagating in free space is incident normally on the plane of a mirror. Relative to a stationary observer, the electric field of the incident wave is given by the real part of $E_x^0 e^{i(kz-\omega t)}$, where z is the coordinate along the surface normal. Obtain the frequency of the reflected wave, with respect to the observer, if the mirror is moving with velocity u along the positive z direction. [4]

A2 A quarter-wave plate is used to convert light of a particular wavelength between linear and circularly polarised light. What optical properties does it utilise to achieve phase retardation? [2]

Polarised light passing through a quarter-wave plate is reflected off a metallic mirror and then passes back through the wave plate. Determine the polarisation state of the light upon the second pass of the wave plate. [2]

A3 The Sun is approximately 1.5×10^8 km from the Earth. It has a diameter of approximately 1.4×10^6 km. If you assume it emits light at 500 nm wavelength (at the peak of its spectrum), determine the transverse coherence length of sunlight on earth. [2]

Assuming the Sun's surface temperature is 5778 K, and its spectrum spans between 275 nm and 875 nm (without atmospheric absorption), find the longitudinal coherence length of sunlight on earth. [2]

SECTION B

Attempt **two** *questions from this Section*

B4 A particle of charge q is moving non-relativistically with velocity $\boldsymbol{u} = \boldsymbol{\beta}c$. The retarded Liénard-Wiechert potentials for this particle could be written as

$$\varphi(\boldsymbol{r}, t) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})R}\right]_{\text{ret}}, \quad \boldsymbol{A}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi r}\left[\frac{q\,\boldsymbol{u}}{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})R}\right]_{\text{ret}},$$

with the unit vector $\boldsymbol{n}$ from the charge towards the observer.

Outline how the expressions for the electric and magnetic field, $\boldsymbol{E}$ and $\boldsymbol{B}$, of this charge take the far-field forms

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 c}\left[\frac{\boldsymbol{n}\times(\boldsymbol{n}\times\dot{\boldsymbol{\beta}})}{R}\right] \text{ and } \boldsymbol{B} = \frac{\mu_0 q}{4\pi}\left[\frac{\boldsymbol{n}\times\dot{\boldsymbol{\beta}}}{R}\right].$$ [6]

Considering the Poynting vector, determine the time-averaged power radiated per unit solid angle, $\mathrm{d}P/\mathrm{d}\Omega$. [3]

Find the expression for $\mathrm{d}P/\mathrm{d}\Omega$, if the particle moves as $z(t) = a\cos(\omega t)$, with $a << c/\omega$, and sketch the angular distribution of radiation in this case. [4]

Find $\mathrm{d}P/\mathrm{d}\Omega$ for circular motion of radius R in the xy plane with constant angular frequency ω, and sketch the angular distribution of radiation in this case. [6]

B5 Briefly explain what is Rayleigh scattering, and show why the scattering cross-section $\sigma = \langle P \rangle / \langle S \rangle$, which is the ratio of time-averaged total radiated power and the time-averaged Poynting vector of the incident wave, is inversely proportional to the fourth power of wavelength. [3]

Explain what is Thomson scattering, and why is the scattering cross-section in this case is not a function of wavelength. [3]

Consider a general case of classical electrodynamic response of a medium to an incident plane-polarised light wave, when the electrons in the medium respond according to the equation $m\,\ddot{\boldsymbol{x}} + k\,\boldsymbol{x} = q\,\boldsymbol{E}\,e^{-i\omega t}$, where k is the spring constant of the binding potential. Derive the expression for the resulting scattering cross-section, and (ignoring the resonant scattering) demonstrate the Rayleigh and the Thomson limits of this expression. [8]

Compton scattering occurs in the quantum regime when $\hbar\omega \geq mc^2$. What should be the strength of electron bond (expressed by the spring constant k) for the Rayleigh scattering to directly transform into Compton scattering, bypassing the classical Thomson regime? [5]

[*Take the mass of electron to be* $m = 10^{-30}$ kg.]

B6 Show that $\boldsymbol{E}^2 - c^2\boldsymbol{B}^2$ and $\boldsymbol{E} \cdot \boldsymbol{B}$ are invariant under the Lorentz transformation. [10]

If $\boldsymbol{k}$ is the wave vector in the direction of propagation of a plane electromagnetic wave, then in classical electrodynamics: $\boldsymbol{k} \cdot \boldsymbol{E} = \boldsymbol{k} \cdot \boldsymbol{B} = 0$. Show that this statement is invariant under the Lorentz transformation by demonstrating its equivalence to the manifestly Lorentz-invariant relation $n^\mu F_{\mu\nu} = 0$, where n^μ is a four-vector in the direction of propagation of the wave, and $F_{\mu\nu}$ is the field strength tensor [9]

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix}$$

[*The Lorentz transformations for the* $\mathbf{E}$ *and* $\mathbf{B}$ *fields between frames* S *and* S' *may be written*

$$\boldsymbol{E}'_{\parallel} = \boldsymbol{E}_{\parallel}, \;\; \boldsymbol{B}'_{\parallel} = \boldsymbol{B}_{\parallel}, \;\; \boldsymbol{E}'_{\perp} = \gamma\left(\boldsymbol{E}_{\perp} + \boldsymbol{u} \times \boldsymbol{B}_{\perp}\right), \;\; \boldsymbol{B}'_{\perp} = \gamma\left(\boldsymbol{B}_{\perp} - \frac{1}{c^2}\boldsymbol{u} \times \boldsymbol{E}_{\perp}\right),$$

where the fields are decomposed into components parallel to and perpendicular to the relative motion of the two frames.]

END OF PAPER