PNP 2013

i) a) $H \to \gamma\gamma$

γ energies 68 GeV, 73 GeV, separated by angle $\theta = 128°$

energies known to 10% precision, no uncertainty in angle

$$s = m_H^2 = (E_1+E_2)^2 - (p_1+p_2)^2$$
$$= E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2p_1p_2\cos\theta$$
$$= 2E_1E_2(1-\cos\theta)$$
$$= 16040 \text{ GeV}^2$$

$m_H = 126.65$ GeV

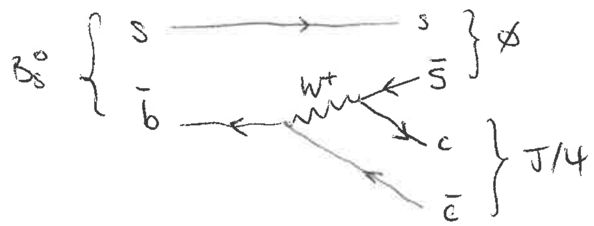
$$\sigma_{m_H}^2 = \left(\frac{m_H}{2E_1}\sigma_{E_1}\right)^2 + \left(\frac{m_H}{2E_2}\sigma_{E_2}\right)^2 = 80.2 \text{ GeV}^2$$

$\sigma_{m_H} = 8.96$ GeV

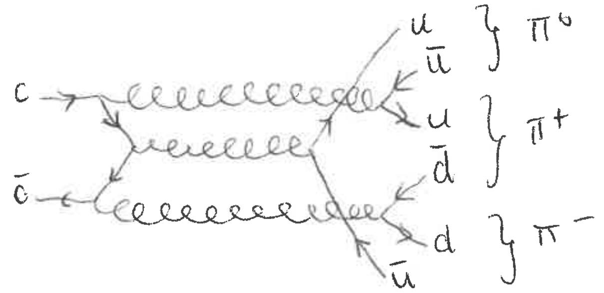
$m_H = 126.7 \pm 9.0$ GeV

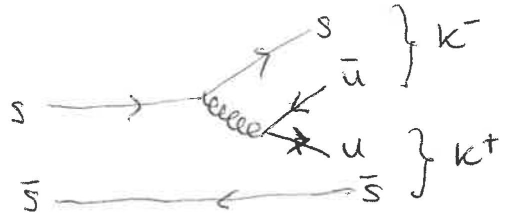
b) $B_s^0 \to J/\psi\,\phi$

$B_s^0 = \bar{b}s$



J/ψ and ϕ decay modes





c) ^{239}Pu - α emitter with $\tau_{1/2} = 24120\,yr$

initial activity of 1kg of ^{239}Pu?

$$A = \left|\frac{dN}{dt}\right| = \lambda N(t) = \lambda N(0) e^{-\lambda t} = \lambda N(0) \text{ at } t=0$$

$$\lambda = \frac{1}{\tau} = \frac{\ln 2}{\tau_{1/2}}, \quad N(0) = \frac{1kg}{239 \times 1.67 \times 10^{-27}} = 2.51 \times 10^{24}$$

$$A = \frac{\ln 2}{\tau_{1/2}} \times 2.51 \times 10^{24} = 2.28 \times 10^{12}\, s^{-1}$$

3) Explain how hadron wavefunction leads to an octet of spin-$\frac{1}{2}$ states and a decuplet of spin $\frac{3}{2}$ states for lowest mass baryons formed from u, d, s quarks

baryons are fermions - need ψ_{baryon} overall antisymmetric under exchange of any 2 quarks

$\psi_{spatial}$ is symm for $L=0$ baryons

ψ_{colour} always antisymm

require $\psi_{spin}\psi_{flavour}$ ~~a~~ symm under quark exchange

ψ_{spin}: $J = 3/2$ - symmetric under exchange of any 2 quarks

$J = 1/2$ states can be symm or antisymm under exchange of quarks 1 and 2

3 like quarks - ~~[illegible]~~ uuu ddd sss

symm under exchange of any 2 quarks - $J=\frac{3}{2}$ states only

2 like quarks uud, uus, ddu, dds, ssu, ssd

ψ flavour symm under exchange of quarks 1 and 2

- need ψ spin symm under same exchange - $J = \frac{3}{2}$ or $\frac{1}{2}$

all different quarks uds

If ud part is flavour symmetric, need ψspin symm under ud exchange - $J=\frac{1}{2}$ or $J = \frac{3}{2}$ (one state each)

If ud part is flavour antisymm, need 4 spin antisymm under ud exchange - 1 $J=\frac{1}{2}$ state

So $J=\frac{1}{2}$ states are uud, uus, ddu, dds, ssu, ssd + 2 x uds - total 8

$J=\frac{3}{2}$ states are uuu, ddd, sss, uud, uus, ddu, dds, ssu, ssd, uds - total 10

mass formula - sum of quark masses + contribution from spin-orbit coupling which shifts the energy by an amount $A\frac{S_1 \cdot S_2}{m_1 m_2}$, sum over spin contributions from all pairs of quarks

spin $\frac{3}{2}$ $\uparrow\uparrow\uparrow$ - total spin of any 2 quarks is 1

$$S_i \cdot S_j = \frac{1}{2}[S^2 - S_i^2 - S_j^2] = \frac{1}{2}[S(S+1) - S_i(S_i+1) - S_j(S_j+1)]$$

with $S_i = S_j = \frac{1}{2}$, $S=1$

$S_i \cdot S_j = \frac{1}{4}$ for any pair of quarks

$$M_{uus} = 2m_u + m_s + A'\left[\frac{S_1 \cdot S_2}{m_u^2} + \frac{S_2 \cdot S_3}{m_u m_s} + \frac{S_3 \cdot S_1}{m_u m_s}\right]$$

$$= 2m_u + m_s + A'\left[\frac{1/4}{m_u^2} + \frac{2 \cdot 1/4}{m_u m_s}\right]$$

$$= 2m_u + m_s + A'\left[\frac{1}{4m_u^2} + \frac{1}{2m_u m_s}\right]$$

spin $\frac{1}{2}$ - total spin of uu pair is 1 $\Rightarrow S_{u_1} \cdot S_{u_2} = \frac{1}{4}$

total $S = \frac{1}{2}$

$$S^2 = S_{u_1}^2 + S_{u_2}^2 + S_s^2 + 2\left[\underbrace{S_{u_1} \cdot S_{u_2}}_{1/4} + S_{u_1} \cdot S_s + S_{u_2} \cdot S_s\right)$$

$$\frac{3}{4} = 3 \cdot \frac{1}{2} \cdot \frac{3}{2} + \frac{1}{2} + 2(S_{u_1} \cdot S_s + S_{u_2} \cdot S_s)$$

$$S_{u_1} \cdot S_s + S_{u_2} \cdot S_s = -1$$

for spin $\frac{1}{2}$, $m_{uus} = 2m_u + m_s + A'\left[\frac{S_{u_1}\cdot S_{u_2}}{m_u^2} + \frac{S_{u_1}\cdot S_s + S_{u_2}\cdot S_s}{m_u m_s}\right]$

$$= 2m_u + m_s + A'\left[\frac{1/4}{m_u^2} - \frac{1}{m_u m_s}\right]$$

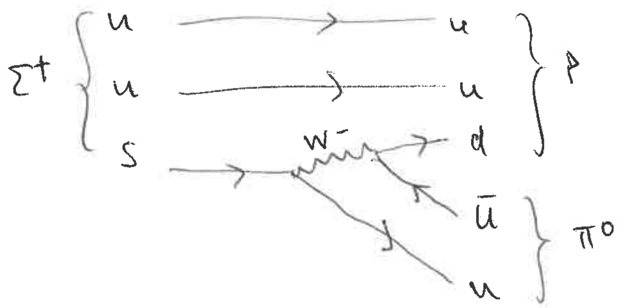
Σ^{*} $(J^P = \frac{1}{2}^+)$, Σ^{*+} $(J^P = \frac{3}{2}^+)$, both uus

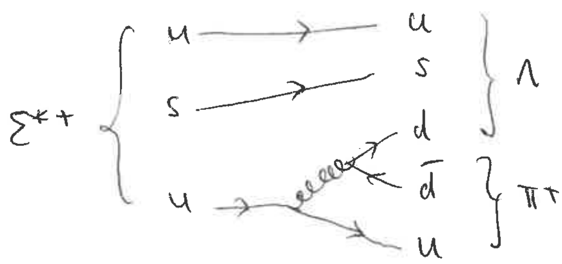
predicted masses

$1.177\ \mathrm{GeV}/c^2$, $1.377\ \mathrm{GeV}/c^2$

$\Sigma^+ \to p\,(uud) + \pi^0\,(u\bar{u}/d\bar{d})$.

$\Sigma^{*+} \to \Lambda\,(uds) + \pi^+\,(u\bar{d})$





$\Gamma \propto |M|^2 \propto |V_{us} V_{ud}\, g_W^2|^2$

$\Gamma \propto |M|^2 \propto \alpha_s$

Σ^{*+} decay can occur via strong interaction with much larger coupling constant than for weak interaction, $\Gamma \propto (\text{coupling const.})^2 \Rightarrow \Gamma$ much larger for Σ^{*+} decay

Σ^+ decay also Cabibbo suppressed - weakens coupling

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$\boxed{3.}$

(a)

For 3 spin $\frac{1}{2}$ particles the possible eigen-functions are: $\{\}$ denotes all permutations

m_s^{tot}			
$\frac{3}{2}$	$\uparrow\uparrow\uparrow$		
$\frac{1}{2}$	$\uparrow\uparrow\downarrow$	$\uparrow\downarrow\uparrow$	$\downarrow\uparrow\uparrow$
$-\frac{1}{2}$	$\uparrow\downarrow\downarrow$	$\downarrow\uparrow\downarrow$	$\downarrow\downarrow\uparrow$
$-\frac{3}{2}$	$\downarrow\downarrow\downarrow$		

So the linear combinations of definite symmetry are (unnormalised):

$$\uparrow\uparrow\uparrow \; ; \; (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) ; (\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow) ; \downarrow\downarrow\downarrow$$

$\Rightarrow$ symmetric; forms $J = \frac{3}{2}$ subgroup

~~The antisymmetric combinations are under exchange of particle 2&3 are:~~

$\cancel{\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow \; ; \; \uparrow\downarrow\downarrow}$

$\Rightarrow$ Since overall wavefunction must be anti-symmetric need $\psi_{space}\,\psi_{flavour}\,\psi_{colour}$ to be anti-symmetric

for $J = \frac{3}{2}$.

- Hadrons are colour singlets $\Rightarrow$ anti-symmetric for baryons $(rgb + gbr + brg - grb - \dots)$

- Consider ground state $l = 0 \Rightarrow$ ~~parity~~ $\Rightarrow \psi_{spatial}$ = symmetric

$\Rightarrow$ need flavour to be symmetric under any two particle exchanges

$\Rightarrow$ possibilities: uuu, ddd, sss;

$\boxed{3}$

(b)

Spin-Spin coupling

(c)

$$(\underline{S}_1+\underline{S}_2+\underline{S}_3)^2 = \underline{S}_1^2+\underline{S}_2^2+\underline{S}_3^2+2\underline{S}_1\cdot\underline{S}_2+2\underline{S}_2\cdot\underline{S}_3+2\underline{S}_1\cdot\underline{S}_3 \quad (\mathrm{I})$$

$\cancel{\underline{S}_1\underline{S}_4}$ $(\underline{S}_1+\underline{S}_2)^2 = \underline{S}_1^2+\underline{S}_2^2+2\underline{S}_1\cdot\underline{S}_2$

$$\Rightarrow \underline{S}_1\cdot\underline{S}_2 = \frac{1}{2}\left(\underline{S}_{12}^2-\underline{S}_1^2-\underline{S}_2^2\right) =$$

$$= \frac{1}{2}\left(s_{12}(s_{12}+1) - 2s(s+1)\right) \overset{s=\frac{1}{2}}{=}$$

$$= \frac{1}{2}\left(s_{12}(s_{12}+1) - \frac{3}{2}\right)$$

Also from (I):

$$\sum_{i<j}\underline{S}_i\cdot\underline{S}_j = \underline{S}_1\cdot\underline{S}_2+\underline{S}_2\cdot\underline{S}_3+\underline{S}_1\cdot\underline{S}_3 = \frac{1}{2}\left(S^{tot}(S^{tot}+1) - 3s(s+1)\right) =$$

$$= \frac{1}{2}\left(S^{tot}(S^{tot}+1) - \frac{9}{4}\right)$$

$$= \begin{cases} \frac{3}{4} & \text{for } J^P = \frac{3}{2}^+ \\ -\frac{3}{4} & \text{for } J^P = \frac{1}{2}^+ \end{cases} \quad (*)$$

Now since $1=u;\ 2=u;\ 3=\cancel{d}$:

$$\underline{S}_1\cdot\underline{S}_3 = \underline{S}_2\cdot\underline{S}_3 = \underline{S}_u\cdot\underline{S}_{\cancel{d}}$$

not symm.

$$\underline{S}_u\cdot\underline{S}_u = \frac{1}{2}\left(s_{uu}(s_{uu}+1) - \frac{3}{2}\right) = \begin{cases} -\frac{3}{4} & S_{uu}=0 \\ \frac{1}{4} & S_{uu}=1 \end{cases}$$

$$\underline{S}_u\cdot\underline{S}_d = \frac{1}{2}\left(s_{ud}(s_{ud}+1) - \frac{3}{2}\right) = \begin{cases} -\frac{3}{4} \\ \frac{1}{4} \end{cases} \quad S_{ud} =$$

(c) cont.

$$\underline{S}_u \cdot \underline{S}_u = \frac{1}{2}\left(s_{uu}(s_{uu}+1) - \frac{3}{2}\right) = \begin{cases} -3/4 & s_{uu}=0 \\ 1/4 & s_{uu}=1 \end{cases}$$

$$\underline{S}_u \cdot \underline{S}_d = \frac{1}{2}\left(s_{ud}(s_{ud}+1) - \frac{3}{2}\right) = \begin{cases} -3/4 & s_{ud}=0 \\ 1/4 & s_{ud}=1 \end{cases}$$

For $\cancel{\boxed{J^P = \frac{3}{2}}}$ ⇒ symmetric under any particle exchange: $s_{uu} = s_{ud} = 1$

⇒

$$M_{uus} = 2m_u + m_s + A'\left[\frac{1}{4m_u^2} + \frac{1}{4m_u m_d} + \frac{1}{4m_d m_u}\right] = \ldots$$

$\boxed{J^P = \frac{1}{2}}$

⇒ symmetric under exchange of like particle $u \leftrightarrow u$

⇒ $s_{uu} = 1$ ⇒ $\underline{S}_u \cdot \underline{S}_u = \frac{1}{4}$

⇒ from $2S_{ud} = -\frac{3}{4} - \frac{1}{4} = -1$

$$\Rightarrow M_{uus} = 2m_u + m_s + A'\left(\frac{1}{4m_u^2} - \frac{1}{2m_u m_d} - \frac{1}{2m_d m_u}\right)$$