NATURAL SCIENCES TRIPOS Part II

Friday 28 May 2010 9.00 am to 12.00 noon

EXPERIMENTAL AND THEORETICAL PHYSICS (3)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (3)

Candidates offering the **whole** *of this paper should attempt a total of* **six** *questions, three from Section A* **and** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A and* **B1**, **B2** *and* **one** *other question from Section B.*

Candidates offering **half** *of this paper should attempt a total of* **three** *questions,* **either** *three from Section A* **or** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A or* **B1**, **B2** *and* **one** *other question from Section B. These candidates will leave after* **90 minutes**.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **6** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

A separate Answer Book should be used for each section.

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

PARTICLE AND NUCLEAR PHYSICS

A1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) With reference to the Semi-Empirical Mass Formula, explain why it is possible for the nuclide ${}^{40}_{19}\mathrm{K}$ to undergo both β^+ and β^- decay. [4]

(b) Using the uncertainty principle, or otherwise, estimate the minimum velocity of an α-particle confined in a uranium nucleus and the time it takes to traverse the nucleus. [4]

(c) Draw leading-order Feynman diagrams for the following decays, and use them to rank the decays in order of decay rate: [4]

$$\mathrm{B}^- \to \mathrm{K}^-\mathrm{D}^0; \quad \mathrm{B}^- \to \mathrm{K}^-\overline{\mathrm{D}}^0; \quad \mathrm{B}_s^0 \to \mu^+\mu^-.$$

[*The meson quark compositions are:* $\mathrm{B}^-(\mathrm{b\bar{u}})$*;* $\mathrm{B}_s^0(\mathrm{s\bar{b}})$*;* $\mathrm{D}^0(\mathrm{c\bar{u}})$*;* $\mathrm{K}^-(\mathrm{s\bar{u}})$.]

A2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) Quantum Electrodynamics and its experimental verification;

(b) measurement of nuclear sizes;

(c) nuclear fission and its applications.

A3 *Attempt* **either** *this question* **or** *question A4.*

What is meant by *magic numbers* in the context of nuclear structure? Give an account of the shell model of nuclei, and explain how it may be used to predict the spins and parities of nuclear ground states. [9]

According to a particular shell-model calculation, the nucleon energy levels are, in order of increasing energy:

$$1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2}, \cdots$$

Show that the shell model predicts the ground state of $^{13}_{6}C$ to have spin and parity $\frac{1}{2}^-$. What does it predict for the ground-state spins and parities of the following nuclides

$$^{14}_{7}N, \quad ^{30}_{14}Si, \quad ^{39}_{16}S \quad ?$$ [6]

A study is made of the γ-radiation emitted in transitions involving the ground state and first three excited states of $^{13}_{6}C$. Only the following γ emissions are observed:

(a) E1 radiation at 0.17, 0.59 and 3.09 MeV;

(b) a mixture of M1 and E2 radiation at 3.68 MeV;

(c) E2 radiation at 0.76 MeV;

(d) faint radiation of unidentified type at 3.85 MeV.

Establish a level scheme for this nuclide consistent with these data, explaining your reasoning carefully. What is the likely nature of the 3.85 MeV radiation? [10]

A4 *Attempt* **either** *this question* **or** *question A3.*

Outline the evidence for the existence of quarks as constituents of hadrons. [4]

Explain how the quark model, when applied to particles involving the three lightest flavours of quarks, predicts the existence of nonets of mesons, and octets and decuplets of baryons. [6]

Show how the quark model may be used to estimate the masses of the mesons forming the two lightest nonets. Show that the masses are predicted to satisfy the relationship:

$$\frac{\rho - \pi}{K^* - K} = 2\left(\frac{3K^* + K}{3\rho + \pi}\right) - 1,$$

where the symbols represent the masses of the corresponding vector mesons, $\rho^0(776)$ and $K^{*0}(896)$, and pseudoscalar mesons, $\pi^0(135)$ and $K^0(498)$. [9]

What does each side of the equation represent? Test this mass relation for these mesons. Replace the kaons by the corresponding charmed mesons $D^0(1865)$ and $D^{*0}(2007)$ and comment on your results. [6]

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SECTION B

ASTROPHYSICAL FLUID DYNAMICS

B1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) A supernova injects 10^{44} J into an interstellar medium of density 10^6 hydrogen atoms per cubic metre. Assuming that the initial mass of the explosion can be ignored, estimate the radius of the blast wave one thousand years later. [4]

(b) By considering a uniform region of interstellar medium of radius R, estimate the value of R for which the sound-crossing time is equal to the collapse time. What is the physical significance of this distance? [4]

(c) The dispersion relation for a magnetised ionised medium is

$$-\omega^2 \boldsymbol{u}_1 + (c_s^2 + v_A^2)(\boldsymbol{k} \cdot \boldsymbol{u}_1)\boldsymbol{k} + (\boldsymbol{v}_A \cdot \boldsymbol{k})\left[(\boldsymbol{v}_A \cdot \boldsymbol{k})\boldsymbol{u}_1 - (\boldsymbol{v}_A \cdot \boldsymbol{u}_1)\boldsymbol{k} - (\boldsymbol{k} \cdot \boldsymbol{u}_1)\boldsymbol{v}_A\right] = 0,$$

where $\boldsymbol{v}_A \equiv \boldsymbol{B}_0/\sqrt{\mu_0 \rho_0}$, $\boldsymbol{u}_1$ is the velocity perturbation from the equilibrium conditions, and the other terms have their usual meanings. Interpret the propagating modes described by this equation. [4]

B2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) thermal instabilities;

(b) shocks with strong cooling;

(c) the evolution of viscous accretion disks.

B3 *Attempt* **either** *this question* **or** *question B4.*

For a star in hydrostatic equilibrium, show that the density $\rho(r)$ satisfies $\mathrm{d}P = -\rho\,\mathrm{d}\Phi$, where P is the pressure and Φ is the gravitational potential. [2]

For a spherically-symmetric polytropic star with $P = K\rho^{1+1/n}$, indicate how this leads to the Lane–Emden equation

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right) + \theta^n = 0,$$

where $\theta^n = \rho/\rho_\mathrm{c}$, ρ_c is the central density, $\xi = \alpha r$ and α is a constant. Show that

$$\alpha^2 = \frac{4\pi G\rho_\mathrm{c}^2}{(n+1)P_\mathrm{c}},$$

where P_c is the central pressure. [11]

For $n = 5$, show that

$$\frac{\rho}{\rho_c} = \left(1 + \frac{\xi^2}{3}\right)^{-5/2}$$

is a solution of the Lane–Emden equation. [3]

Hence show that the total mass of the star is given by

$$\mathcal{M} = \frac{18}{\sqrt{2\pi}}\left(\frac{P_\mathrm{c}}{G}\right)^{3/2}\rho_\mathrm{c}^{-2}.$$

[4]

Let R be the radius at which $\rho/\rho_\mathrm{c} = 10^{-3}$. Assuming the stellar material satisfies the perfect-gas law, show that the central temperature T_c is proportional to $G\mathcal{M}/R$. [5]

[$\int_0^\infty x^2(1+x^2)^{-5/2}\,\mathrm{d}x = 1/3$.]

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B4 *Attempt* **either** *this question* **or** *question B3.*

A star produces a steady spherically symmetric wind. The velocity of the wind at radius r is $v(r)$, the density is $\rho(r)$ and the pressure is $P(r)$. The ratio of specific heats, $C_p/C_V = \gamma$. Write down the continuity equation and Bernoulli's equation for this case, and give the physical interpretation of each. [7]

The wind satisfies $P = K\rho^{3/2}$, where K is a constant. Show that the Mach number M and the adiabatic sound speed c at distance r from the centre of the star are related by

$$c^5 M\xi^2 = c\rho K^2\gamma^2 = A$$
$$\frac{1}{2}c^2M^2 + 2c^2 - c_0^2\xi^{-1} = E,$$

where $\xi = r/R$, $c_0^2 = G\mathcal{M}/R$, $\mathcal{M}$ is the mass of the star, and A, E and R are constants. [6]

By eliminating c from these equations, or otherwise, show that

$$f(M^2) = Bf(\xi/\xi_0),$$

where $f(x) \equiv (x + 4)x^{-1/5}$, $B^5 = Ec_0^8/8A^2$, and ξ_0 is a constant which you should determine in terms of the other constants of the problem. [5]

By considering the shape of the curve $f(\xi)$, or otherwise, show that the wind can only make a transition from subsonic to supersonic flow if $B = 1$ and $M = 1$ at $\xi = \xi_0$. Show that, in this case, $M \sim (\xi/\xi_0)^{1/2}$ as $\xi/\xi_0 \to 0$. [7]

[*You may assume that the mass of the wind is negligible compared with the mass of the star.*]

END OF PAPER