UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 29 May 2014 9.00 am to 11.00 am

PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

OPTICS AND ELECTRODYNAMICS

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

OPTICS AND ELECTRODYNAMICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) Without being informed which is which, you are presented with two optical elements – a linear polarizer and a quarter-wave plate – and two light sources – one unpolarized and one circularly polarized. Explain how you would distinguish between the two optical elements, and between the two light sources. [4]

(b) A Hertzian dipole lies at the origin O with its axis along the z-axis, Oz, and radiates power P at wavelength λ. A second such dipole lies in the far-field region at $(d, 0, d)$ with its axis parallel to Oz, and is connected to a matched load. Calculate the power delivered to the load. [4]

(c) Explain why a charged particle moving at constant speed in vacuum does not radiate energy, but, for sufficiently high speeds, can radiate when moving through a medium of refractive index n. For $n = 1.5$, estimate the kinetic energy threshold for radiation from a proton. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the Aharonov–Bohm effect;

(b) photonic structures;

(c) synchrotron radiation.

3 *Attempt* **either** *this question* **or** *question 4.*

Explain what is meant by a 4-vector, and a Lorentz invariant. Show that the inner product of two 4-vectors is a Lorentz invariant. [4]

Show that phase is a Lorentz invariant and therefore that, for a wave of angular frequency ω, wavevector $\boldsymbol{k}$ and speed c, $\boldsymbol{K} = (\omega/c, \boldsymbol{k})$ is a true 4-vector. [4]

Hence derive the transformation rules for frequency and wavevector between inertial frames and show that for a photon of angular frequency ω travelling at an angle θ to the x-axis in the laboratory frame S, the angular frequency observed in an inertial frame S′ moving at speed v with respect to S along the x-axis is

$$\omega' = \gamma\omega\left(1 - \frac{v\cos\theta}{c}\right).$$ [4]

Two photons of angular frequency ω_1 and ω_2 collide at an angle α in the laboratory frame. If they are sufficiently energetic and at a suitable angle they can annihilate and produce an electron–positron pair. Explain why the threshold conditions for this process require that, in the inertial frame in which the total 3-momentum is zero, the combined energy of the photons exceeds $2m_e c^2$, where m_e is the rest mass of an electron. [5]

By considering the invariance of the inner product of the photons' 4-wavevectors, show that, for a given ω_1, the threshold frequency for pair production in the laboratory frame is given by

$$\omega_2 = \frac{2m_e^2c^4}{\hbar^2\omega_1(1-\cos\alpha)}\,.$$ [6]

If ω_1 corresponds to the peak of the microwave background radiation with $T \approx 3$ K, estimate the minimum energy (in eV) of photons in the laboratory frame which can produce electron-positron pairs by this mechanism. [2]

4 *Attempt* **either** *this question* **or** *question 3.*

Explain briefly and without mathematical detail the difference between temporal and spatial coherence, and why the degree of lateral coherence depends on the angular profile of the source while the degree of temporal coherence depends on its spectral output. [8]

A standard Young's slit experiment uses an extended source of wavelength λ and angular intensity profile $I(\theta)$ as viewed in the plane of the slits. If $I(\theta)$ is symmetric about the optic axis of the system ($\theta = 0$), show that for slit separation d the fringe visibility in the observation plane is:

$$V(d) \propto \int I(\theta)\, \mathrm{e}^{-\mathrm{i}\theta k d}\, \mathrm{d}\theta$$ [10]

where $k = 2\pi/\lambda$.

Explain why a Michelson stellar interferometer incorporates two well-separated light-collecting mirrors in addition to the standard Young's slit arrangement. [2]

Such an interferometer is used to observe light of wavelength λ from two stars of identical brightness and angular width α, and with angular separation β. Explain, with the aid of a detailed and carefully labelled sketch of the expected fringe visibility as a function of the light-collecting mirror separation Y, how the observations can be used to obtain both α and β. [5]

END OF PAPER