UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Monday 06 June, 2022 9.00 to 11.00

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

Quantum Condensed Matter Physics

Candidates offering this paper should attempt a total of ***five*** *questions: all* ***three*** *questions from Section A and* ***two*** *questions from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains six *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2 × 20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(1st May 2022)

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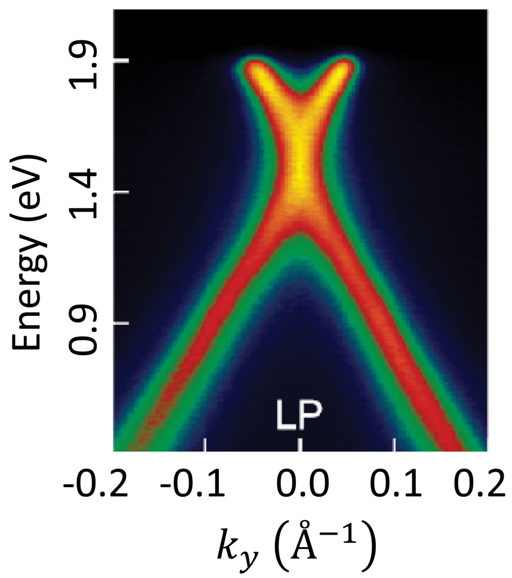
SECTION A
*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 Give a brief explanation of the concepts of drift velocity, electron mobility, and effective mass, as used in solid state physics. [4]

2 Consider the Stoner instability of free conduction electrons of total density $n = n_\uparrow + n_\downarrow$ in a metal with Fermi energy E_F and density of states $g(\epsilon)$. Electrons of opposite spin interact via a short-range repulsive interaction U, giving rise to a total interaction energy of $E_{\mathrm{IA}} = U n_\uparrow n_\downarrow$. Derive an expression for the critical value U_c of the interaction, above which a ferromagnetic state has a lower energy compared to an unpolarised state with $n_\uparrow = n_\downarrow$. [4]

3 The picture below shows an ARPES measurement of a doped graphene sample. Describe how one can extract the Fermi energy and Fermi velocity from this measurement and obtain approximate values. [4]



SECTION B

Attempt two questions from this section

4 This question is about the optical response of insulators and simple metals. We will start with *insulating* materials, where bound electrons respond to an optical light field $E = E_0 e^{-i\omega t}$ according to the Lorentz model.

(a) State the relevant differential equation for the oscillating displacement $u(t) = u_0 e^{-i\omega t}$ considering a single atomic transition of frequency ω_0 and explain the physical origin of all the terms. [3]

(b) Show that this differential equation gives rise to the following frequency-dependent permittivity,

$$\epsilon_\omega = 1 + n \frac{q^2}{m\epsilon_0(\omega_0^2 - \omega^2 - i\omega\gamma)},$$

where m and q denote the mass and charge of the electron, ϵ_0 is the vacuum permittivity, n the density of electrons, and γ is a phenomenological damping rate. [5]

(c) Sketch the real and imaginary parts of the permittivity and discuss their physical significance in the case of a weakly absorbing medium. [3]

(d) Repeat the above sketch for the case of several atomic transitions and discuss the connection to normal and anomalous dispersion in optics. [3]

From here on, consider the situation of a simple metal where the conduction electrons can be seen as free electrons.

(e) Discuss how the model above needs to be modified for simple metals and sketch the resulting permittivity. [3]

(f) Describe the frequency-dependent reflectivity of a good metal. [2]

5 In this question we will adapt the concept of Linear Combination of Atomic Orbitals to analyse the hybridisation between two coupled bands.

(a) Describe the concept of linear combination of atomic orbitals (LCAO) applied to the case of a *diatomic molecule* with one orbital per atom. [4]

Now consider a three dimensional lattice containing one highly dispersive, approximately parabolic band $E_p = m_p \hbar^2 k^2$, where $\boldsymbol{k}$ is the wavevector of the electron and $k = |\boldsymbol{k}|$, and one almost flat band $E_{\mathrm{fl}} = const > 0$. Assume that the bands are coupled with a momentum-independent coupling strength $\alpha \geq 0$.

(b) Give an example of how such a situation can arise in real solids. [2]

(c) Sketch the two bands for $\alpha = 0$. [2]

(d) Sketch the dispersion of the hybridised bands for the cases $0 < \alpha \ll E_{\mathrm{fl}}$ and $\alpha \approx E_{\mathrm{fl}}$. Can such a material be insulating and if yes, for which filling? [3]

(e) Show that the dispersion of the hybridised bands can be written as [5]

$$E_{1,2}(k) = \frac{1}{2}\left(E_p + E_{\mathrm{fl}} \pm \sqrt{(E_p - E_{\mathrm{fl}})^2 + 4\alpha^2}\right).$$

(f) Assume now that the almost flat band is also parabolic but with a much higher effective mass. Assume a metallic state for $\alpha = 0$ with the same number of electrons in both bands. By how much does the Fermi momentum change when a finite coupling ($\alpha > 0$) is introduced? You can ignore possible intersections with the Brillouin zone boundary. [3]

6 Start by considering a homogeneous bulk semiconductor in three dimensions. Assume that the dispersion relations for electrons in the conduction band and holes in the valence band are given by

$$\epsilon_c(k) = \epsilon_c + \frac{\hbar^2 k^2}{2m_e^*} \quad \text{and} \quad \epsilon_v(k) = \epsilon_v - \frac{\hbar^2 k^2}{2m_h^*}.$$

(a) Show that the densities of states for those bands per unit volume are given by [4]

$$g_e(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(\epsilon - \epsilon_c)^{1/2} \quad \text{and} \quad g_h(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}(\epsilon_v - \epsilon)^{1/2}.$$

(b) Use the above results to show the law of mass action, i.e., show that the product of intrinsic carrier concentrations $n_i^2 = n\,p$ is given by

$$n_i^2 = \frac{1}{2}(m_e^* m_h^*)^{3/2}\left(\frac{k_B T}{\pi\hbar^2}\right)^3 \exp\left(-\frac{\epsilon_c - \epsilon_v}{k_B T}\right),$$

where n (p) denotes the density of electrons (holes). You can assume the semiconductor to be non-degenerate, i.e., the Fermi distribution can be approximated to $f(\epsilon) \approx \exp\left(-\frac{\epsilon-\mu}{k_B T}\right)$. [5]

(c) Discuss why semiconductor devices can operate only in a limited temperature regime. [3]

(d) Sketch the current through a *pn* junction as a function of applied voltage and discuss how the temperature dependence of n_i^2 calculated above can be extracted from such measurements. [3]

(e) Briefly discuss the classical Hall effect and explain why the Hall coefficient can change sign as a function of temperature in doped semiconductors. [4]

[*You might find the following integrals useful:* $\int_0^\infty \sqrt{x}e^{-x}dx = \sqrt{\pi}/2$, $\int_0^\infty x^2 e^{-x^2}dx = \sqrt{\pi}/4$.]

END OF PAPER