UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Thursday 31 May 2012 09.00 to 11.00

EXPERIMENTAL AND THEORETICAL PHYSICS (6)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (6)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20 Page Answer Book	Mathematical Formulae handbook
Metric graph paper	Approved calculator allowed
Rough workpad	
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

PARTICLE AND NUCLEAR PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) Estimate the range (in fm) of the interaction between hadrons via pion exchange. [$m_\pi \approx 140$ MeV.] [4]

(b) A high energy electron collides with an atomic electron. Calculate the total energy of the incident electron (in MeV) at the threshold for production of an e^+e^- pair. [$m_e = 0.511$ MeV.] [4]

(c) Use Bose-Einstein statistics to show that three $J^P = 2^+$ quanta of nuclear vibration (phonons) can couple to give total $J^P = 0^+, 2^+, 3^+, 4^+$ or 6^+. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the role of the Higgs boson in the Standard Model;

(b) β-decay in nuclear and particle physics;

(c) the size and shape of nuclei.

3 *Attempt* **either** *this question* **or** *question 4.*

Outline how Feynman diagrams are used to calculate particle scattering and decay processes. [4]

Describe the various vertices which arise in Feynman diagrams involving the electromagnetic, weak and strong interactions. [6]

For each of the following processes, either draw the leading-order Feynman diagram(s), or explain why the process is forbidden. Within each group of reactions, rank the reactions in order of decreasing rate, giving your reasons.

(a)

$$D^0 \to \pi^+\pi^- \; ; \; D^0 \to K^+K^- \; ; \; D^0 \to K^+\pi^- \; ; \; D^0 \to \pi^+K^- \; ;$$

[4]

(b)

$$\Xi^0 \to \Lambda^0\pi^0 \; ; \; \Xi^0 \to \Lambda^0\overline{K}^0 \; ; \; \Xi^0 \to \Lambda^0\gamma \; ;$$

[3]

(c)

$$\eta'^0 \to \rho^0\gamma \; ; \; \eta'^0 \to \pi^0\gamma \; ; \; \eta'^0 \to \pi^0\rho^0 \; ;$$

[3]

(d)

$$e^+e^- \to \mu^+\mu^- \; ; \; e^+e^- \to W^+W^- \; ; \; e^+e^- \to \nu_e\overline{\nu}_e \; .$$

In this last group, briefly state how your answer depends on the collision energy. [5]

[*You may find some of the following particle properties useful:*

Particle	composition	J^P	mass /MeV/c^2
e^-		$\frac{1}{2}^+$	0.511
μ^-		$\frac{1}{2}^+$	105.7
π^+	$u\overline{d}$	0^-	139.6
π^0	$(u\overline{u} - d\overline{d})/\sqrt{2}$	0^-	135.0
η^0	$(u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6}$	0^-	547.5
η'^0	$(u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3}$	0^-	957.8
ρ^0	$(u\overline{u} - d\overline{d})/\sqrt{2}$	1^-	775.5
K^+	$u\overline{s}$	0^-	493.7
K^0	$d\overline{s}$	0^-	497.6
D^0	$c\overline{u}$	0^-	1864.5
Λ^0	uds	$\frac{1}{2}^+$	1115.7
Ξ^0	uss	$\frac{1}{2}^+$	1314.8
W^+		1^-	80403

]

4 *Attempt* **either** *this question* **or** *question 3.*

The Breit-Wigner formula gives the cross-section for the resonant formation of a compound nucleus in the reaction $x + X \to Z^* \to y + Y$ as:

$$\sigma = \frac{\pi g}{p_*^2} \frac{\Gamma_x \Gamma_y}{(E - E_0)^2 + \frac{1}{4}\Gamma^2} \quad \text{with} \quad g = \frac{2J_{Z^*} + 1}{(2J_x + 1)(2J_X + 1)} \; ,$$

where p_* is the centre-of-mass momentum of the reactants.

(a) Explain the meanings of Γ_x, Γ_y and Γ. [3]

(b) Derive the form of the denominator $(E - E_0)^2 + \frac{1}{4}\Gamma^2$. [4]

(c) Justify the expression given for g. [3]

Assume that all particles are non-relativistic, that the target X is initially at rest, and that x has kinetic energy T_x in the laboratory frame. Show that the total kinetic energy of the reactants in the centre-of-mass frame is given by $T_* = \mu T_x / m_x$, where m_x is the mass of particle x and μ the reduced mass of x and X. Also obtain an expression for p_* in terms of μ and T_*. [6]

The reaction

$$\alpha + {}^{12}\mathrm{C} \to {}^{16}\mathrm{O} + \gamma$$

exhibits a resonance when the α-particle has laboratory kinetic energy of 10.10 MeV. The only other decay mode of the resonance is to p + ^{15}N where the ^{15}N nucleus is in its $J^P = \frac{1}{2}^+$ ground state. The resonance has a total width of 0.20 MeV and partial widths to $\alpha + {}^{12}\mathrm{C}$ and ${}^{16}\mathrm{O} + \gamma$ final states of 0.15 MeV and 0.10 eV respectively.

(i) Determine the centre-of-mass momentum (in MeV/c) for the $\alpha + {}^{12}\mathrm{C}$ system at resonance. [2]

(ii) Determine the laboratory kinetic energy at which the resonance is excited in p + ^{15}N collisions. [3]

(iii) Evaluate the ratio of resonant cross-sections

$$\frac{\sigma(\alpha + {}^{12}\mathrm{C} \to {}^{16}\mathrm{O} + \gamma)}{\sigma(\mathrm{p} + {}^{15}\mathrm{N} \to {}^{16}\mathrm{O} + \gamma)}$$

[4]

The masses of the nuclides (in MeV/c^2) are:

p	938.26
α	3728.34
^{12}C	11177.76
^{15}N	13973.30

END OF PAPER