UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 2 June 2017 1.30 pm to 3.30 pm

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

QUANTUM CONDENSED MATTER PHYSICS

*Candidates offering this paper should attempt a total of* **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

*The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains* **six** *sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| 2 × 20 Page Answer Book | Mathematical Formulae handbook |
| Rough workpad | Approved calculator allowed |
| Yellow master coversheet | |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUANTUM CONDENSED MATTER PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

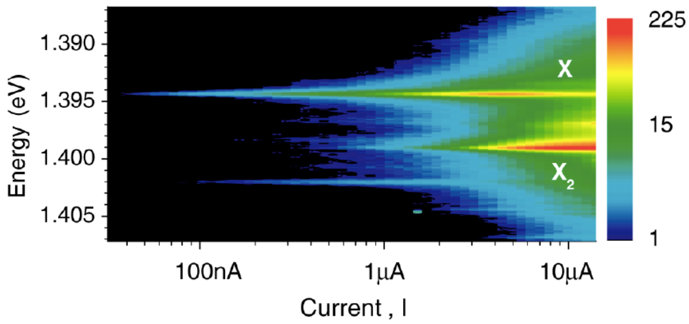
(a) In the Lorentz oscillator model, the electron cloud is described as a damped harmonic oscillator,

$$m\ddot{u} + m\gamma\dot{u} + m\omega_{\mathrm{T}}^2 u = qE\,,$$

where $u$ is the displacement of the cloud, $q$ is the electron charge, $\omega_{\mathrm{T}}$ is the natural frequency and $\gamma$ is the damping rate. Derive the resulting polarisability $\chi_\omega$ and sketch the qualitative behaviour of the real and imaginary parts of the relative permittivity $\epsilon = \chi_\omega + 1$ as a function of $\omega$. [3]

(b) Describe single photon sources based on self-assembled InAs quantum dots, indicating the characteristic dimensions (dot diameter, height, density). Explain briefly the mechanism by which photons are emitted.

The figure below shows emission intensity as a function of current and energy. Explain the origin of the two main features labelled $X$ and $X_2$.




[credit: Z. Yuan et al., Science **295**, 102 (2002)]

Assuming that the emitted photon carries the full energy of an exciton (taken to be in its lowest energy state), that the energy required to create an electron-hole pair in an InAs dot is $E_g = 1.415$ eV and that the dielectric constant is $\varepsilon_r = 15$, compute the reduced effective mass of the exciton. [5]

(c) In the Stoner-Hubbard model, at mean-field level, the energies of the two spin bands are shifted by an applied magnetic field $H$ according to the following equations:

$$\begin{aligned}
\varepsilon_{\boldsymbol{k},\uparrow} &= \varepsilon_{\boldsymbol{k}} + U\,\overline{n}_\downarrow - \mu_0\mu_B H\ , \\
\varepsilon_{\boldsymbol{k},\downarrow} &= \varepsilon_{\boldsymbol{k}} + U\,\overline{n}_\uparrow + \mu_0\mu_B H\,,
\end{aligned}$$

where $U$ is the energy penalty of a doubly occupied site, $\overline{n}_\uparrow$ and $\overline{n}_\downarrow$ are the mean-field densities of carriers in the two spin bands, $\mu_{\mathrm{B}}$ is the Bohr magneton, and all other constants have the usual meaning. Obtain the static magnetic susceptibility as a function of the density of states per atom, which may be assumed constant at its value at the Fermi energy, $g(E_F)$. Describe briefly the Stoner criterion for ferromagnetism. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) screening and Thomas–Fermi theory;

(b) optical phonons in atomic chains;

(c) Fermi liquids.

(TURN OVER

3 *Attempt* **either** *this question* **or** *question 4.*

Explain the difference between direct and indirect band gap semiconductors. Sketch the dispersion near the $\Gamma$ point. What is meant by 'heavy hole band', 'light hole band' and 'split-off hole band'? Why are the split-off hole bands not important in intrinsic semiconductors? [6]

Given the densities of states for the conduction ($e$) and valence ($h$) band

$$g_{\mathrm{e}}(E) = \frac{1}{2\pi^2}\left(\frac{2m_{\mathrm{e}}^*}{\hbar^2}\right)^{3/2}\sqrt{E - E_{\mathrm{c}}}\,, \qquad g_{\mathrm{h}}(E) = \frac{1}{2\pi^2}\left(\frac{2m_{\mathrm{h}}^*}{\hbar^2}\right)^{3/2}\sqrt{E_{\mathrm{v}} - E}\,,$$

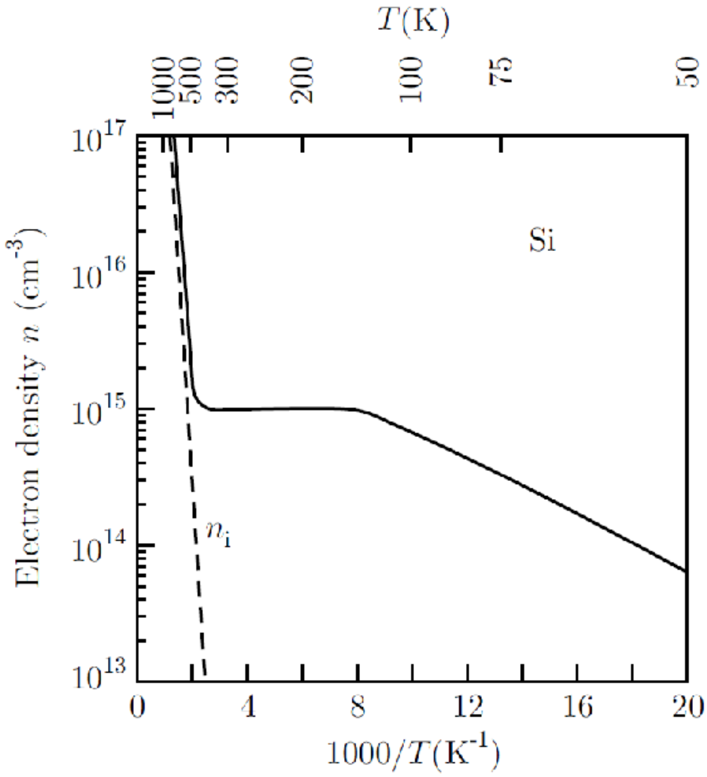
derive the intrinsic carrier concentrations of electrons and holes, and from them illustrate the law of mass action. Here $m_{\mathrm{e}}^*$ and $m_{\mathrm{h}}^*$ are the effective electron and hole masses, and $E_{\mathrm{c}}$ and $E_{\mathrm{v}}$ are the energies of the conduction and valence bands at the $\Gamma$ point. In your calculations, you should consider the case $|E - \mu| \gg k_B T$, where $\mu$ is the chemical potential and $T$ is the temperature of the semiconductor. [8]

[*You may find the following integral helpful in your calculations:*

$$\int_0^\infty \mathrm{d}x\ \sqrt{x}\,\mathrm{e}^{-(x+\alpha)/\beta} = \frac{\sqrt{\pi}}{2}\beta^{3/2}\mathrm{e}^{-\alpha/\beta} \qquad \text{for } \alpha \in \mathbb{R},\ \beta > 0\,.$$
]

Use the result to obtain an expression for the chemical potential of an intrinsic semiconductor as a function of the gap energy $E_{\mathrm{g}} = E_{\mathrm{c}} - E_{\mathrm{v}}$, temperature and the effective electron and hole masses. [3]

When a semiconductor is doped, the temperature dependence of its electron density changes to the typical behaviour illustrated in the figure.




Identify the three different temperature regimes and describe briefly what is happening in each regime. [3]

The figure above refers to Si with a net donor density $N_D - N_A = 10^{15}$ cm$^{-3}$, $m_e^* = 0.98\ m_e$, and $m_h^* = 0.49\ m_e$. Explain how you can determine the gap energy $E_g$ from the figure and estimate its value. Using these parameters or otherwise, estimate the electron and hole densities at room temperature. [5]

4 *Attempt* **either** *this question* **or** *question 3.*

Consider a non-interacting electron with momentum $\boldsymbol{k}$ and energy $\varepsilon_{\boldsymbol{k}}$. By taking the Fourier transform of its time dependent wave function $\psi_{\boldsymbol{k}}(t)$, derive the electron spectral function. [3]

Discuss how the electron spectral function is modified by interactions, commenting in particular on the dispersion relation $\varepsilon_{\boldsymbol{k}}$ and the (quasi)particle lifetime. Derive the corresponding quasiparticle spectral function and show that it can be written as

$$A(\boldsymbol{k},\omega) = -\frac{1}{\pi}\Im\left[\frac{1}{\omega - \frac{\varepsilon_{\boldsymbol{k}}}{\hbar} + \mathrm{i}\Gamma_{\boldsymbol{k}}}\right],$$

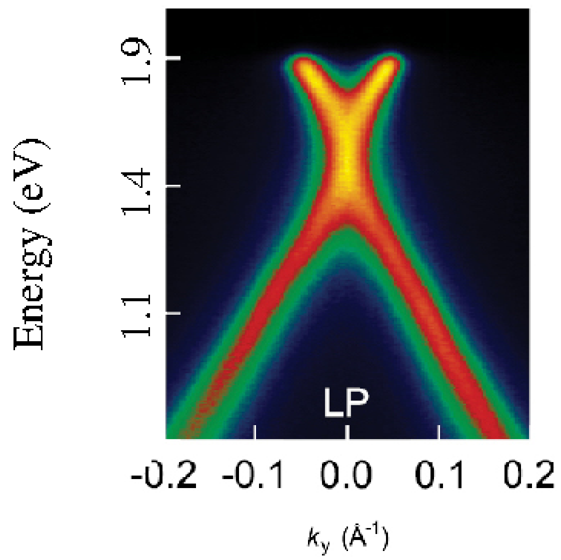
where $\Im$ indicates the imaginary part and $\Gamma_{\boldsymbol{k}}$ is the quasiparticle scattering rate. [5]

Assuming that the renormalised dispersion takes the form $\varepsilon_{\boldsymbol{k}} = \hbar^2 k^2/2m^*$ and that the quasiparticle scattering rate is real, $\Gamma_{\boldsymbol{k}} = \gamma(\boldsymbol{k}) > 0$, show that the spectral function $A(\boldsymbol{k},\omega)$ has a single maximum as a function of $\omega$, and find its position and the Half Width at Half Maximum. [5]

Sketch the behaviour of the electron spectral function for a Fermi gas and for a Fermi liquid. [2]

Discuss how photoemission can be used to measure directly the electron spectral function. You should include equations for the relevant conservation laws and you may use sketches to help illustrate the technique. [6]

Explain how you can use the ARPES measurements illustrated in the figure below to identify the approximate value of the Fermi energy and Fermi velocity in the material, and obtain their values. [4]




[credit: J. Phys.: Condens. Matter 26 (2014) 335501]

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