NATURAL SCIENCES TRIPOS Part II

Wednesday 27 May 2009 13.30 to 16.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

Candidates offering the whole *of this paper should attempt a total of* six *questions, three from Section A* and *three from Section B. The questions to be attempted are* A1, A2 *and* one *other question from Section A and* B1, B2 *and* one *other question from Section B.*

Candidates offering half *of this paper should attempt a total of* three *questions,* either *three from Section A* or *three from Section B. The questions to be attempted are* A1, A2 *and* one *other question from Section A or* B1, B2 *and* one *other question from Section B. These candidates will leave after* 90 minutes.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains 6 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.

Answers to each *section should be tied up separately, with the numbers of the questions attempted written clearly on the cover sheet.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough workpad
Blue coversheets (2)
Treasury tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

SOFT CONDENSED MATTER AND BIOPHYSICS

A1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) A globular protein of radius 3 nm diffuses in water at room temperature. On average, how far will it move in one hour? [The viscosity of water at room temperature is 8.9×10^{-4} Pa s.] [4]

(b) The hydrocarbon volume of an amphiphile is $0.152\,\mathrm{nm}^3$ and its maximum chain length is 1.2 nm. What is the maximum head group area if the amphiphile is to form a spherical micelle? [4]

(c) The potential at the surface of a colloidal particle is 35 meV, and at a distance of 5 nm from the surface it is 6.6 meV in 0.1 M NaCl aqueous solution. What will the corresponding potential be at this distance from the surface in 0.01 M NaCl? [4]

A2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the Flory χ parameter;

(b) lipid membranes;

(c) viscoelasticity of polymer melts.

A3 *Attempt* **either** *this question* **or** *question A4.*

Describe the packing in a *nematic liquid crystal* and the factors relevant to the formation of a nematic phase. [3]

In a certain model for the nematic phase, the order parameter at the nematic-isotropic transition is 0.44. Explain what this means, and evaluate the mean orientation angle corresponding to this value of the order parameter. [5]

The critical electric field at which the molecules in a twisted nematic cell align is given by

$$E_{\text{crit}} = \frac{\pi}{d}\sqrt{\frac{K_1 + \frac{1}{4}(K_3 - K_2)}{\Delta\epsilon\,\epsilon_0}}.$$

Explain the terms in this equation. [4]

Describe its relevance to the operation of a twisted nematic cell, including an annotated sketch to explain the configuration of the cell. [10]

In a particular realisation of a twisted nematic cell, the nematic has the following properties: $K_1 = 1.1 \times 10^{-11}$ N, $K_2 = 6 \times 10^{-12}$ N, $K_3 = 1.5 \times 10^{-11}$ N, and $\Delta\epsilon = 0.5$. Calculate the threshold voltage required for alignment of the molecules. [3]

A4 *Attempt* **either** *this question* **or** *question A3.*

Show that the entropy S of a random coil is given approximately by

$$S(R, N) = -\frac{3k_B R^2}{2Nl^2} + \text{constant},$$

where R is the end-to-end distance of the chain and N is the number of chain segments each of length l. [7]

Hence show that, for small extensions, a rubber network behaves like a simple spring, and obtain an expression for the corresponding Young's modulus. [7]

How, chemically, can the modulus of a rubber be controlled? [3]

A rubber band with a shear modulus of 20 MPa at 300 K has a natural length of 1 m, width 1 cm and thickness 1 mm. A mass of 1 kg is suspended from one end, the other end being fixed vertically above it. Explain as quantitatively as possible what happens to the system as the temperature is raised from 300 K to 330 K, noting any assumptions made. (The rubber can be taken to be incompressible.) [8]

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SECTION B

QUANTUM CONDENSED MATTER PHYSICS

B1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) The interaction potential between two inert gas atoms is often modelled by a Lennard-Jones 6-12 potential of the form

$$U(r) = -\frac{A}{r^6} + \frac{B}{r^{12}},$$

where r is the distance between the two atoms. Explain the origin of the two terms in the potential, and outline the justification for the r^{-6} dependence of the first term. [4]

(b) Explain the concept of effective mass. Calculate the effective mass for electrons at the bottom of a one-dimensional band which has an energy dispersion relation $E(k) = A(1 - \cos(ka))$, where A is positive. [4]

(c) The Bragg condition for diffraction is $2d \sin\theta = n\lambda$, where θ is the angle between the incident beam and the set of equivalent crystal planes responsible for the diffraction, and d is the perpendicular distance between the planes. Show that the Bragg condition is equivalent to the requirement that the difference between the outgoing and incoming wavevectors of the scattered wave should be a reciprocal lattice vector. [4]

B2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) charge density waves;

(b) the origin of magnetism in metals;

(c) the Hall effect, explaining why the Hall coefficient can have both positive and negative values. The quantum Hall effect should not be discussed.

B3 *Attempt* **either** *this question* **or** *question B4.*
Describe the de Haas-van Alphen effect, outlining the reasoning which leads to the *Onsager relation* for the extremal cross-sectional area A_k of the Fermi surface,

$$A_k = \frac{2\pi e}{\hbar} \frac{1}{\Delta(1/B)},$$

where e is the magnitude of the electronic charge, B is the applied magnetic field, and $\Delta(1/B)$ is the period of the oscillations of the magnetic susceptibility when plotted against $1/B$. [11]

A square lattice of ruthenium (Ru) atoms lying in the xy plane forms the key structural element of the layered compound Sr_2RuO_4. Three of the five Ru d-orbitals are degenerate and contribute to the band structure close to the Fermi energy: $|d_{xy}\rangle$, $|d_{xz}\rangle$ and $|d_{yz}\rangle$. The d_{xy} orbitals hybridise with those of the nearest neighbours in the x- and y- directions, the d_{xz} orbitals only hybridise with those of the nearest neighbours along the x-direction, and the d_{yz} orbitals only hybridise with those of the nearest neighbours along the y-direction. Hybridisation along the z-direction is negligible. Four electrons per Ru atom are distributed equally among the three bands formed by these d-orbitals.

The band formed by the $|d_{xy}\rangle$ orbitals gives rise to the γ Fermi surface sheet, which can be approximated by a cylinder pointing along the z-direction with a circular cross-section in the xy plane.

The de Haas-van Alphen signal of the magnetic susceptibility measured as a function of $1/B$ with B parallel to the z axis has a strong component with a frequency of 1.8×10^4 T, corresponding to the γ Fermi surface sheet. Determine the Fermi wavevector k_F characterising this cylinder and estimate the lattice constant a of the square Ru lattice. [6]

Explain, without detailed derivation, why the energy of states of wavevector $\boldsymbol{k}$ within the band formed by the $|d_{xz}\rangle$ orbitals takes the form

$$E(\boldsymbol{k}) = E_0 + t\cos(k_x a).$$

[2]

Hybridisation between the nearest-neighbour $|d_{xz}\rangle$ orbitals gives rise to a Fermi surface sheet A and hybridisation between the nearest-neighbour $|d_{yz}\rangle$ orbitals leads to a Fermi surface sheet B. Sketch the three Fermi surface sheets A, B and γ, and state their characteristic dimensions. [6]

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B4 *Attempt* **either** *this question* **or** *question B3.*

What is meant by 'screening' of electrostatic fields inside a conductor? Explain how screening arises and why it is not possible for the conductor to perfectly screen an electrostatic field at short ranges. [4]

Outline the approximations made in the Thomas-Fermi theory of screening and explain why the chemical potential μ is expressed in this theory as

$$E_{\mathrm{F}}(\boldsymbol{r}) = \mu + eV(\boldsymbol{r}),$$

where $E_{\mathrm{F}}(\boldsymbol{r})$ is the Fermi energy calculated within the free electron model for the local electron number density $n(\boldsymbol{r})$ at position $\boldsymbol{r}$, e is the magnitude of the electronic charge and $V(\boldsymbol{r})$ is the electrostatic potential at $\boldsymbol{r}$. [5]

Show that within the Thomas-Fermi approximation and for small changes in $n(\boldsymbol{r})$ from its value n_0 found when V is zero, the excess local charge density induced by the electrostatic potential V is given by

$$\rho_{\mathrm{ind}}(\boldsymbol{r}) = -e\left[n(\boldsymbol{r}) - n_0\right] \simeq -\frac{3e^2 n_0}{2E_{\mathrm{F0}}} V(\boldsymbol{r}),$$

where $E_{\mathrm{F0}} = \hbar^2 (3\pi^2 n_0)^{2/3}/(2m)$ is the Fermi energy for electrons of mass m and number density n_0. [6]

Poisson's equation for the electrostatic potential associated with the free charge distribution $\rho_{\mathrm{free}}(\boldsymbol{r})$ in the presence of induced charges $\rho_{\mathrm{ind}}(\boldsymbol{r})$ is given by

$$\nabla^2 V(\boldsymbol{r}) = -\frac{1}{\epsilon_0}\left[\rho_{\mathrm{ind}}(\boldsymbol{r}) + \rho_{\mathrm{free}}(\boldsymbol{r})\right].$$

Show that, within the Thomas-Fermi approximation, a screened potential around a point charge of the form $V(r) \propto 1/r \, \exp(-r/\xi)$ satisfies Poisson's equation and show that the Thomas-Fermi screening length ξ is given by

$$\xi = \sqrt{\frac{2\epsilon_0 E_{\mathrm{F0}}}{3e^2 n_0}}.$$

$$\left[\textit{The radial part of } \nabla^2 \textit{ in spherical polar coordinates is } \quad \nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right).\right]$$

[5]

Evaluate ξ for sodium which has a density of $968\,\mathrm{kg\,m^{-3}}$ and a relative atomic mass of 23. To what extent are the approximations that underlie the Thomas-Fermi theory valid for sodium? [3]

Measurements of phonon frequencies in thin sodium films indicate that the sodium-sodium interlayer force constant for a film just two atomic layers thick is very nearly the same as the average interlayer force constant in a layer 20 atoms thick. Comment on this observation. [2]

END OF PAPER