PNP 2011

1) a) . Λ^0, Σ^0 baryons both uds

ud part can be flavour symmetric or antisymmetric

$\psi_{ud} = \frac{1}{\sqrt{2}}(ud \pm du)$

for antisymmetric ψ_{baryon}, $\psi_{spin}\psi_{flavour}$ must be symmetric

- Λ^0 has symmetric $\psi_{flavour}$ (ud part) $\Rightarrow$ symmetric ψ_{spin}

Σ^0 has antisymmetric ψ_{spin}

$\therefore$ Σ^0 has greater mass - baryon mass formula depends on spin interactions

Σ^0 can decay to Λ^0 via EM interaction - fast decay, short lifetime

Λ^0 is the lightest uds state - can only decay via much slower weak interaction - longer lifetime.

b) $^{15}_{7}N$ - unpaired proton in $1p_{1/2}$

$j = \frac{1}{2}$, $l = 1$ $\Rightarrow$ $P = (-1)^1 = -1$

$J^P = \frac{1}{2}^-$

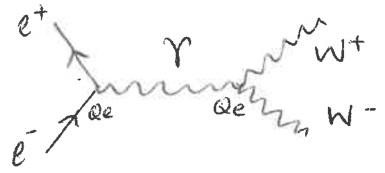
$^{6}_{2}He$ even-even nucleus - $J^P = 0^+$

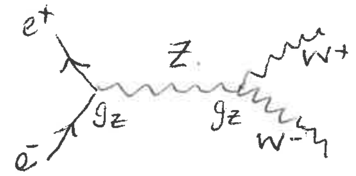
$^{19}_{10}Ne$ - unpaired neutron in $1d_{5/2}$

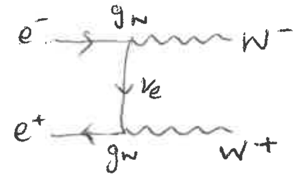
$j = \frac{5}{2}$, $l = 2$ $\Rightarrow$ $P = (-1)^2 = +1$

$J^P = \frac{5}{2}^+$

3) $e^+ e^- \to W^+ W^-$







positron beam incident on fixed target containing electrons

$m_W = 80.4$ GeV

min $\sqrt{s} = 2m_W$

$s = (E_+ + E_-)^2 - (p_+ + p_-)^2$

$p_- = 0 \quad - \quad E_- = m_e$

$E_+ \gg m_e \quad - \quad p_+ = E_+$

$s = (E_+ + m_e)^2 - E_+^2 \quad = \quad m_e^2 + 2m_eE_+$

$m_e^2 + 2m_eE_+ = 4m_W^2$

$$E_+ = \frac{4m_W^2 - m_e^2}{2m_e} \sim \frac{2m_W^2}{m_e} = 2.5 \times 10^7 \text{ GeV}$$

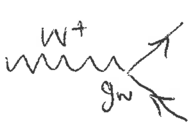
100 GeV e^+/e^- beams colliding head on

$e^\pm$ beams : $\sqrt{s} = 2E$, $E = 100$ GeV

$2E = 2E_W \quad - \quad E_W = E = 100$ GeV

$P_W = \sqrt{E_W^2 - m_W^2} = 59.5$ GeV

Branching ratio for $W^+ \to e^+ \nu_e$



u	c	e^+	μ^+	τ^+
$\bar{d}'$	$\bar{d}'$	ν_e	ν_μ	ν_τ
$\times 3$	$\times 3$			

for colour

each decay involves same coupling constant g_W
9 possible decay products (including factor of 3 for colour for hadronic decays)

$B_{e\nu} = \frac{1}{9}$

min and max e^+ energies

W bosons produced with energy 100 GeV, momentum 59.5 GeV

$\gamma = \frac{E}{m} = \frac{100}{80.4} = 1.244$

$\beta = \sqrt{1 - 1/\gamma^2} = 0.595$

in lab frame max/min energies are $E = \gamma(E_{cm} - \beta p_{cm})$

in cm frame

$E_W = E_e + E_\nu$

$E_W = m_W, \quad E_\nu = p_\nu = p_e$

$E_\nu^2 = (m_W - E_e)^2 = p_e^2$

$m_W^2 + E_e^2 - 2m_W E_e = E_e^2 - m_e^2$

$E_e = \frac{m_W^2 + m_e^2}{2m_W} \sim \frac{1}{2} m_W$

$p_e \sim E_e$

$E_{max} = 1.244 \cdot \frac{1}{2} m_W (1 + 0.595) = 79.8\,\text{GeV}$

$E_{min} = 1.244 \cdot \frac{1}{2} m_W (1 - 0.595) = 20.3\,\text{GeV}$

total decay width $= 2.1\,\text{GeV}$

Mean lifetime $\tau = \frac{1}{\Gamma} = 0.476 (\text{GeV})^{-1} = 3.12 \times 10^{-25}\,\text{s}$

in lab frame lifetime $= \gamma\tau$

speed $= \beta c$

average distance travelled in lab frame $= \gamma\tau\beta c$

$= 6.9 \times 10^{-17}\,\text{m}$

typical separation of W bosons when they decay

$= 2 \times 6.9 \times 10^{-17}\,\text{m} = 1.4 \times 10^{-16}\,\text{m}$

range of strong force $\sim 10^{-15}\,\text{m} > $ W separation

quarks produced in W decays can interact via strong force – hadronisation.

4) fission of nucleus $[A, Z]$ into fragments $[\alpha A, \alpha Z]$ and $[(1-\alpha)A, (1-\alpha)Z]$

energy released $E_0 = m(A, Z) - m(\alpha A, \alpha Z) - m((1-\alpha)A, (1-\alpha)Z)$

$$E_0 = m_p[Z - \alpha Z - (1-\alpha)Z] + m_n(A-Z)[1-\alpha-(1-\alpha)] - a_V A[1-\alpha-(1-\alpha)]$$
$$+ a_S A^{2/3}[1 - \alpha^{2/3} - (1-\alpha)^{2/3}] + \frac{a_C Z^2}{A^{1/3}}[1 - \alpha^{5/3} - (1-\alpha)^{5/3}]$$
$$+ a_A \frac{(A-2Z)^2}{A}[1-\alpha-(1-\alpha)]$$

$$E_0 = a_S A^{2/3}(1 - \alpha^{2/3} - (1-\alpha)^{2/3}) + \frac{a_C Z^2}{A^{1/3}}(1 - \alpha^{5/3} - (1-\alpha)^{5/3})$$

max energy release - $\frac{\partial E_0}{\partial \alpha} = 0$

$$\frac{\partial E_0}{\partial \alpha} = a_S A^{2/3}\left[-\frac{2}{3}\alpha^{-1/3} + \frac{2}{3}(1-\alpha)^{-1/3}\right] + \frac{a_C Z^2}{A^{1/3}}\left[-\frac{5}{3}\alpha^{2/3} + \frac{5}{3}(1-\alpha)^{2/3}\right] = 0$$

$$\frac{2}{3}a_S A^{2/3}[\alpha^{-1/3} - (1-\alpha)^{-1/3}] = \frac{5 a_C Z^2}{3A^{1/3}}[(1-\alpha)^{2/3} - \alpha^{2/3}]$$

for $\alpha = 1-\alpha \Rightarrow \alpha = \frac{1}{2}$, LHS and RHS $= 0$

- max energy released for $\alpha = \frac{1}{2}$

$$E_0^{max} = a_S A^{2/3}\left[1 - \left(\frac{1}{2}\right)^{2/3} - \left(\frac{1}{2}\right)^{2/3}\right] + \frac{a_C Z^2}{A^{1/3}}\left[1 - 2\left(\frac{1}{2}\right)^{5/3}\right]$$
$$= -0.260 a_S A^{2/3} + 0.370 a_C \frac{Z^2}{A^{1/3}}$$

min value of $\frac{Z^2}{A}$ for which fission should be energetically possible - need +ve energy released, $E_0 > 0$

$$-0.260 a_S A^{2/3} + 0.370 a_C \frac{Z^2}{A^{1/3}} = 0$$

$$\frac{Z^2}{A} = \frac{0.260 a_S}{0.370 a_C} = 17.6$$

In practise, only have spontaneous fission for much heavier nuclei - increase in surface energy when deforming nucleus before Coulomb energy is reduced by splitting into smaller nuclei - tunnelling energy difference between nucleus [A, Z] and 2 nuclei [1/2 A, 1/2 Z] when daughter nuclei just touch at their surfaces

mass difference $= -0.260 A^{2/3} + 0.370 a_c \frac{Z^2}{A^{1/3}}$

2 nuclei just touching - extra energy difference due to Coulomb repulsion

potential $V = \frac{q}{4\pi\varepsilon_0 r}$ with $q = \frac{1}{2} Ze$, $r = 2R_0\left(\frac{A}{2}\right)^{1/3}$

potential energy $= \frac{Z^2 e^2/4}{4\pi\varepsilon_0\left(2R_0\left(\frac{A}{2}\right)^{1/3}\right)} = \frac{Z^2\alpha}{8R_0(A/2)^{1/3}}$

total energy difference $\Delta E = -0.260 a_s A^{2/3} + 0.370 a_c \frac{Z^2}{A^{1/3}} - \frac{Z^2\alpha}{8R_0(A/2)^{1/3}}$

threshold for spontaneous fission - $\Delta E = 0$

$$0.260 a_s = 0.370 a_c \frac{Z^2}{A} - \frac{\alpha}{R_0} 2^{-8/3} \frac{Z^2}{A}$$

$$\frac{Z^2}{A} = \frac{0.260 a_s}{0.370 a_c - 2^{-8/3}\alpha/R_0} = 60$$