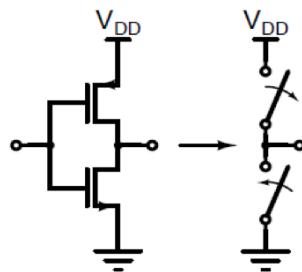


Lecture-3

- In the last lecture we studied the possible amplification effect of a MOS Transistor.
- It is different what MOS operation we see in Digital Circuits. In Digital circuits the Transistor is either **ON or OFF**

MOSFET in Digital Circuits

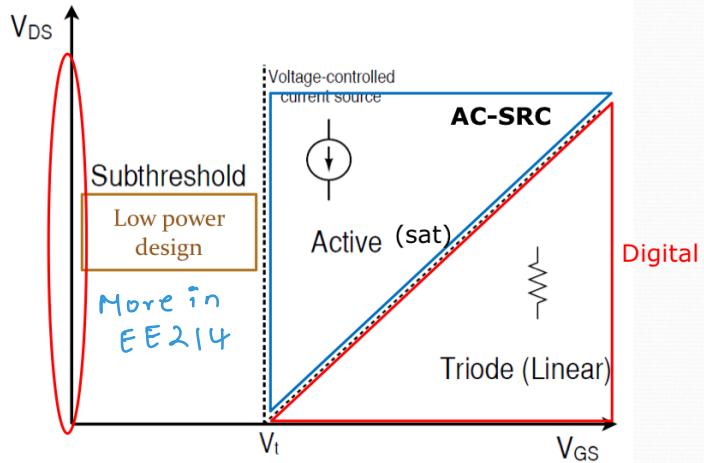
- MOSFET is treated as a switch in digital design.
- Below threshold there is no current. Above threshold a sufficiently large V_{GS} can push the device into triode, where it is a resistor.
- This resembles a switch which has infinite resistance when off and finite resistance when on.
- Unlike analog where we use the device mostly in forward active region, in digital circuits the device toggles between off and linear regions.



- In Analog circuits we mostly operate in the saturation region.

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Pannala
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MOS regions usage



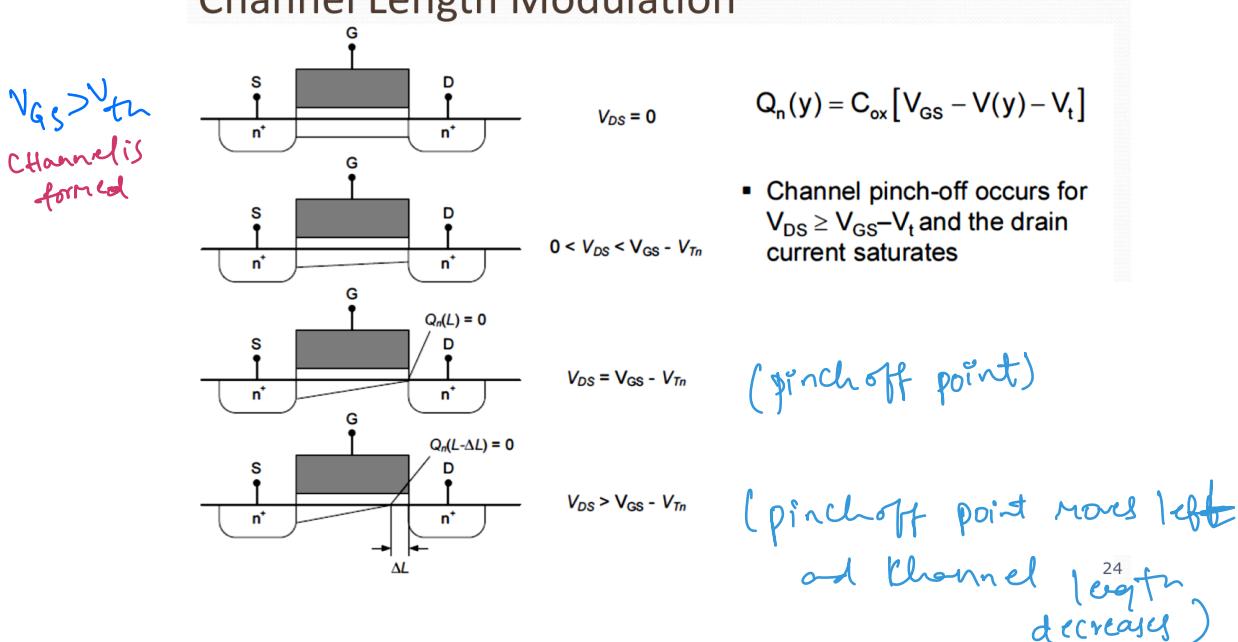
M. Hekmat EE114S

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CHANNEL LENGTH MODULATION & Drain Current Saturation

Drain current Saturation and Channel Length Modulation

2



Q How to Model the Impact of Channel length Modulation in the Saturation Region ?

For this we make use of concept

Taylor's Expansion

we have saturation current (I_D) given by

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DS} - V_t)^2$$

when we are in saturation
as V_{BL} increases "L" decreases.

→ and the ^{new} "length of channel" "L" is a function of V_{DS}

$$\Rightarrow L' = f(V_{DS})$$

∴ we can say that I_D will also be a function of V_{DS} in Saturation Region.

Taylor Series

If we a function $y = f(x)$

Then we can define the function as

$$y = f(x) + (x - x_0) \frac{df}{dx} + \dots$$

Here x_0 : operating point

→ In this case we can express I_D in terms of Taylor series expansion

$$\Rightarrow I_D = I_{D_0} + \frac{\partial I_D}{\partial V_{DS}} (V_{DS} - V_{DS_0})$$

Operating point

let us choose operating

V_{DS_0} = limit of the saturation region

$$= V_{GS} - V_{th}$$

overdrive voltage.

→ ∴ we can write

$$I_{D_0} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

const

$$\propto \frac{W}{L} (V_{GS} - V_t)^2$$

→ we have I_{D_0} & V_{DS_0} what we need

to find is the partial derivative

$$\frac{\partial I_D}{\partial V_{DS}}$$

$$\therefore \frac{\partial I_D}{\partial V_{DS}} = \frac{B^1}{2} w (V_{GS} - V_T)^2 \frac{\partial (\frac{1}{L'})}{\partial V_{DS}}$$

$$\therefore \frac{\partial (\frac{1}{L'})}{\partial V_{DS}} = \frac{\partial L'}{\partial V_{DS}} \cdot \frac{\partial (\frac{1}{L'})}{\partial L'}$$

$$\frac{\partial (\frac{1}{L'})}{\partial V_{DS}} = \frac{\partial L'}{\partial V_{DS}} \cdot -\frac{1}{(L')^2}$$

$$\Rightarrow \therefore \frac{\partial I_D}{\partial V_{DS}} = -\frac{1}{(L')^2} \cdot \frac{B^1}{2} \cdot w \cdot (V_{GS} - V_T)^2 \cdot \frac{\partial L'}{\partial V_{DS}}$$

L ①

we have earlier that

$$I_D = \frac{B^1}{2} \frac{w}{L'} (V_{GS} - V_T)^2$$

$$\Rightarrow I_D L' = \frac{B^1}{2} w (V_{GS} - V_T)^2$$

②

put ② in ①



$$\frac{\partial I_D}{\partial V_{DS}} = -\frac{1}{(L')^2} I_0 L' \cdot \frac{\partial L'}{\partial V_{DS}}$$



$$\frac{\partial I_D}{\partial V_{DS}} = -\frac{I_0}{L'} \cdot \frac{\partial L'}{\partial V_{DS}}$$

When $V_{DS} \uparrow$ $L' \downarrow$

∴ (as L' decreases
as V_{DS} increases)

we can say that the
The quantity is $-ve$

as

$$\frac{\partial L'}{\partial V_{DS}} < 0$$



$$\frac{\partial I_D}{\partial V_{DS}} > 0$$

which states that the
Saturation current Increases



Now lets call

$$-\frac{1}{L'} \frac{\partial L'}{\partial V_{DS}} = \lambda$$

Channel length modulation coefficient
it is a parameter that depends mostly on technology and on Biasing conditions



we would have

$$\therefore I_D = \frac{P}{2} \cdot \frac{W}{L} (V_{GS} - V_t)^2 [1 + \lambda (V_{DS} - V_{DS0})]$$

expression of ' I_D ' including CLM.

usually this quantity is neglected because it will be of range $0.1 - 0.2$ V



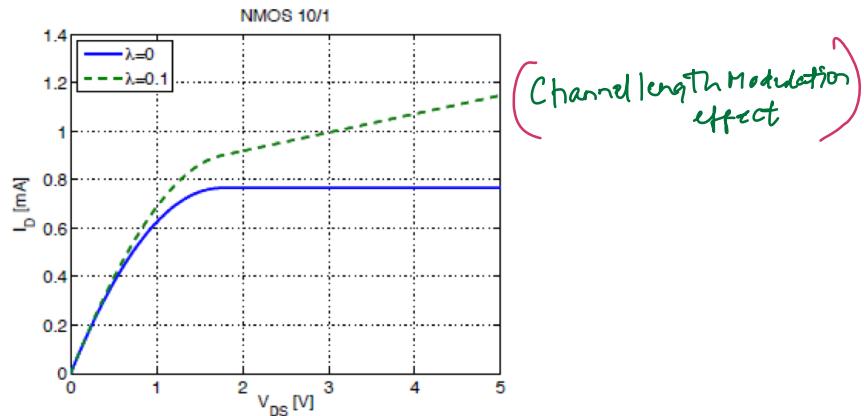
$$\Rightarrow I_D = \frac{P}{2} \cdot \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$V_{DS0} = V_{GS} - V_t \quad (\text{usually the } V_{GS} \text{ is kept little higher than } V_t)$$

$\therefore V_{DS0}$ value is very small

Drain current Saturation and Channel Length Modulation

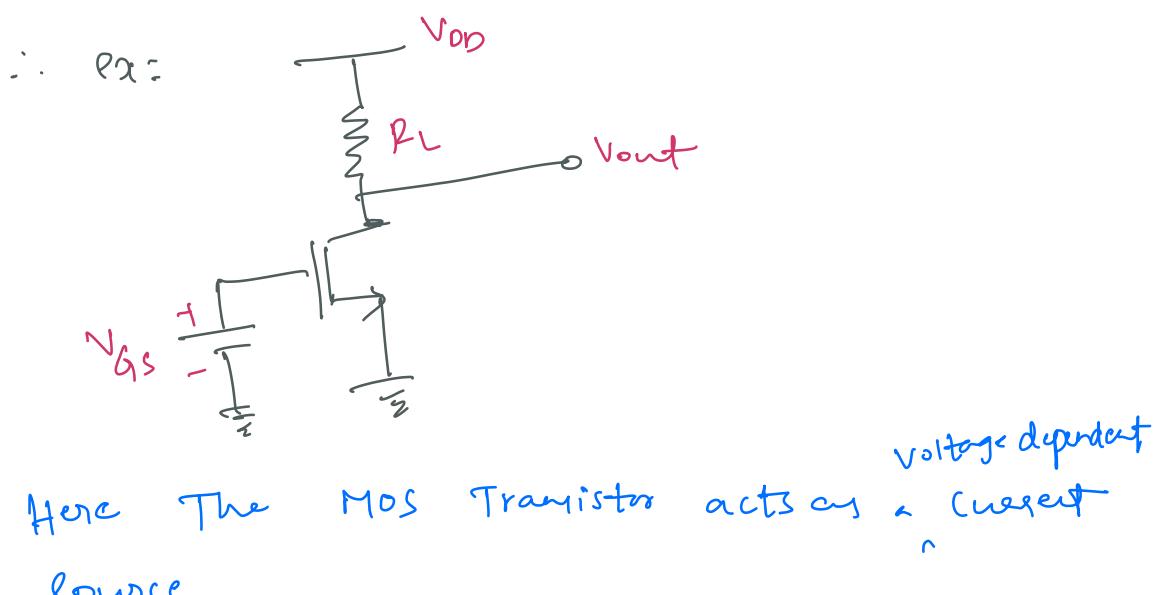
$$\Rightarrow I_D \simeq \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 [1 + \lambda V_{DS}]$$



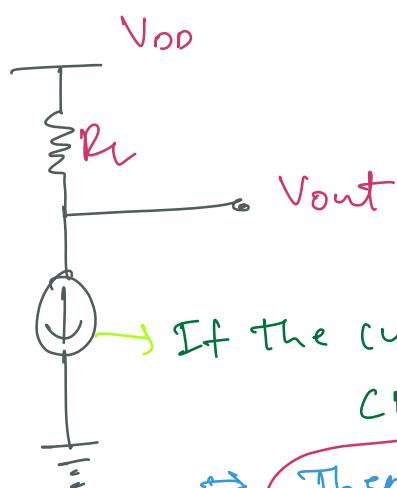
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∴ The effect of CLM is that the current increases even in the saturation region where it is ideally assumed to be constant.



→ Ideally we want the current source to remain constant.



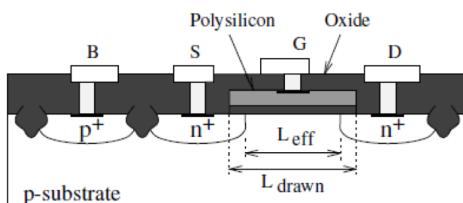
If the current increases because of CLM.

⇒ Then the current through R_L increases ↑

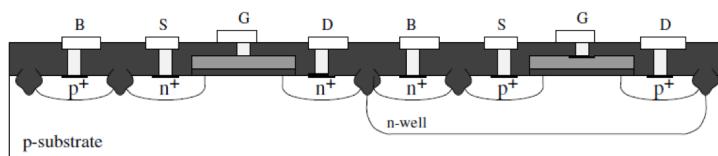
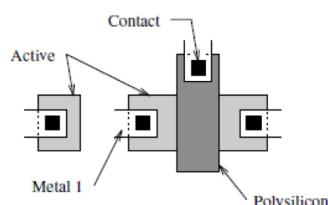
$$\text{as } V_{out} = V_{DD} - I_D R_L$$

∴ Value of V_{out} ↓

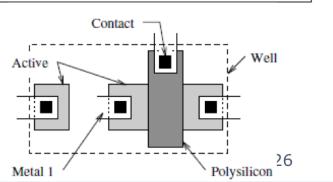
MOSFET technology



REAL device

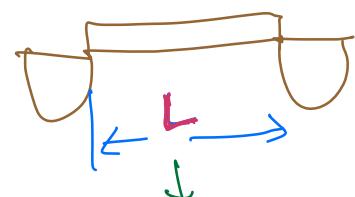


Transistor structure in a CMOS process with an n -well



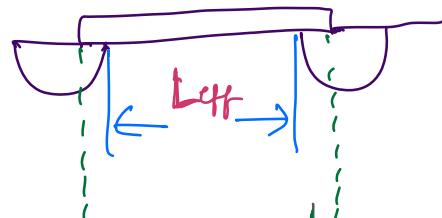
→ The models that we have considered so far even though enriched with channel length Modulation are very rough approximations of what a **REAL device** is.

→ An Ideal device we see a perfectly divided channel regions from source diffusion & Drain Diffusions may be without any overlap b/w Oxide Gate and Metalization of the Gate w.r.t the Source and Drain diffusion



L : Ideal length

L_{eff} : Effective length because of Non Idealties



overlap of
drain diffusion
with channel.

→ There are several Non Idealties which are intrinsic to the Manufacturing process of the Device.

→ On Top of that we have multiple Geometry connected effects which we cannot Neglect.

→ we should consider all these Non-Idealities before Analyzing the performance of the device in the Field.

→ Note:- Another important contact which is to be taken into effect is the Body Contact

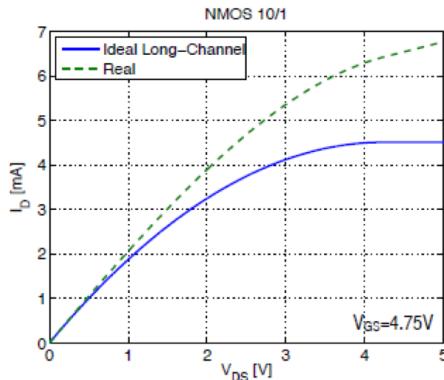
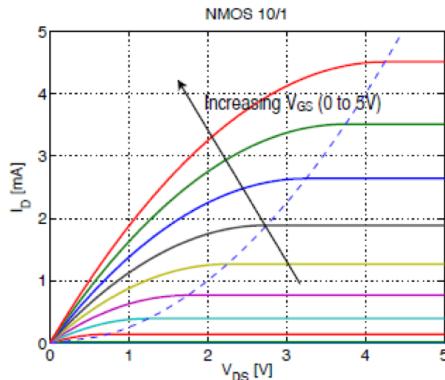
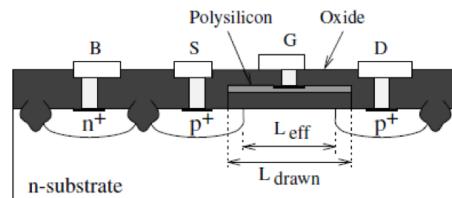
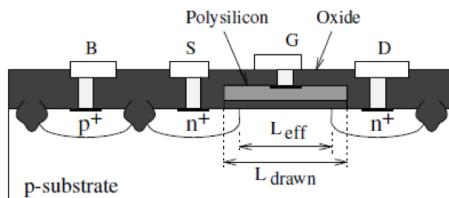
An important we should take into account is whether we have or not the possibility to keep the bulk to source voltage zero.

It is ^{not} always possible.

Note:- In our course we would consider the Bulk contact of all NMOS devices on a chip to be Low.

& Bulk contact of all PMOS devices on a chip to be High.

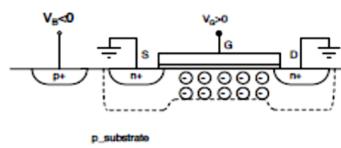
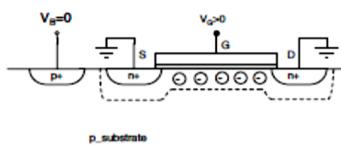
MOSFET: I_d - V_{ds} Curves



* Dashed blue line denotes the border between active and linear regions.

Substrate Voltage & MOS Device

Substrate Voltage and MOS Device



- Applying a voltage to bulk can modulate the depletion width.
- Positive V_{SB} draws more holes to the substrate, widening the depletion region.
- The increased amount of fixed negative ions in the depletion region, makes it harder for electrons to form the channel.
 - Larger V_{GS} is required to reach inversion.
 - V_t increases.

- As introduced earlier the Substrate voltage (V_{SB}) is the Bulk voltage (V_B). The Body voltage plays a crucial role in the operation of the MOSFET.
- Because Essentially MOS is a 4 Terminal Device.
- Bulk voltage plays an important role because
 (V_{SB})
 Source to bulk voltage

Backgate Effect

- The backgate effect is the change in threshold voltage of the device due to change in the bulk potential.
- One can think of the substrate terminal as an additional gate for the transistor as it can control the channel in somewhat the same way.
- Change in V_t means change in I_D .
- Define small-signal backgate transconductance.
 - Remember that g_m embodies changes of I_D due to V_{GS} variations.

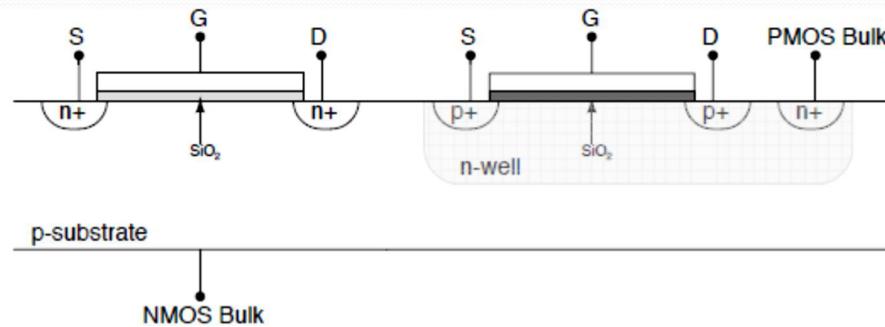
$$\begin{aligned} g_{mb} &= \frac{\partial I_D}{\partial V_{BS}} = -\frac{\partial I_D}{\partial V_{SB}} \\ &= \mu C_{ox} \frac{W}{L} (V_{GS} - V_t) \times \left(-\frac{\partial V_t}{\partial V_{BS}}\right) \end{aligned}$$

$$\frac{\partial V_t}{\partial V_{BS}} = -\frac{\gamma}{2\sqrt{(2\phi_f + V_{SB})}} \Rightarrow g_{mb} = g_m \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

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Bulk connections



The MOSFET is a 4-terminal device.

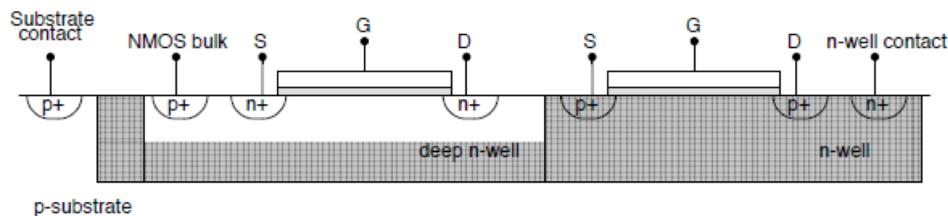
~~So far we have assumed that the p-substrate is grounded, and all PMOS bulks are connected to V_{DD} . But these assumptions are not binding.~~

The connection of the bulk can affect the behavior of the device.

In our technology, PMOS devices have isolated bulk connections but all NMOS bulk nodes are connected through the substrate.

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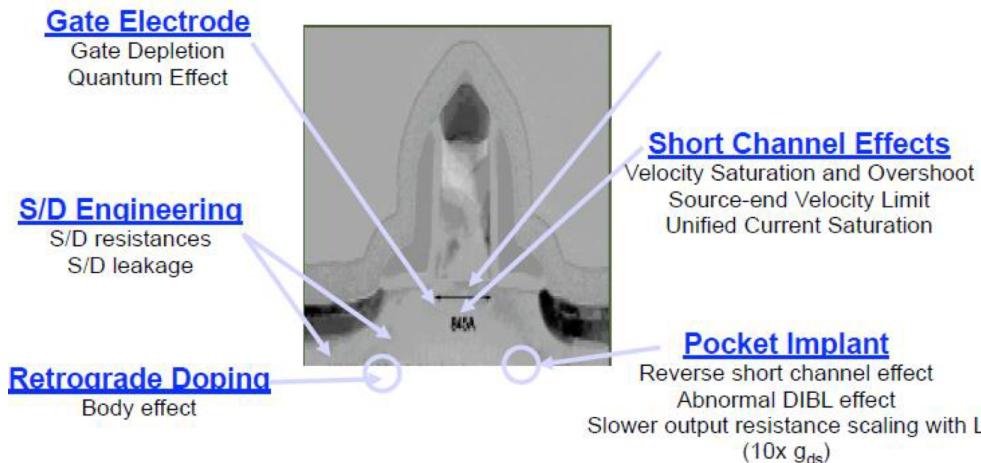
Triple-Well CMOS Technology



- More advanced technologies (e.g., $0.13\mu\text{m}$ and beyond) typically offer separate bulk connection for all devices.
- The separate connection of NMOS and p-substrate helps reducing noise coupling between different parts (specially between analog and digital sections).

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A Real Transistor



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Some additional complications...

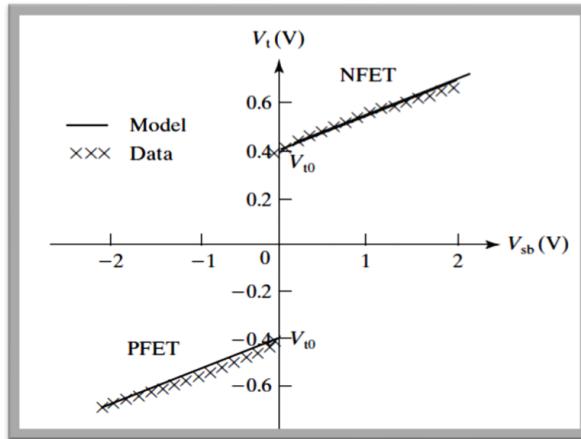
Effective mobility

$$\mu_{ns} = \frac{540 \text{ cm}^2/\text{Vs}}{1 + \left(\frac{V_{gs} + V_t + 0.2 \text{ V}}{5.4 T_{oxe}} \right)^{1.85}}$$

Retrograde Doping

$$V_t(V_{sb}) = V_{t0} + \frac{C_{dep}}{C_{oxe}} V_{sb} = V_{t0} + \alpha V_{sb}$$

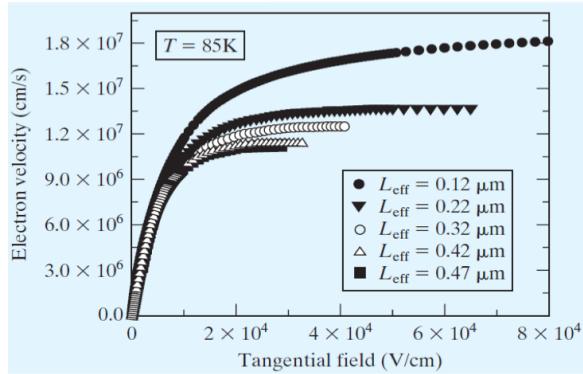
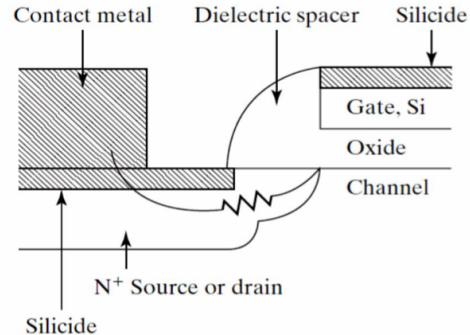
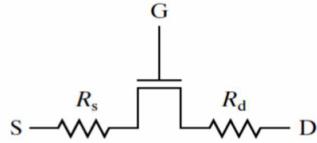
$$\alpha = C_{dep}/C_{oxe} = 3T_{oxe}/W_{dmax}$$



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Some additional complications...

$$I_{\text{dsat}} = \frac{I_{\text{dsat}0}}{1 + R_s I_{\text{dsat}0} / (V_{\text{gs}} - V_t)}$$



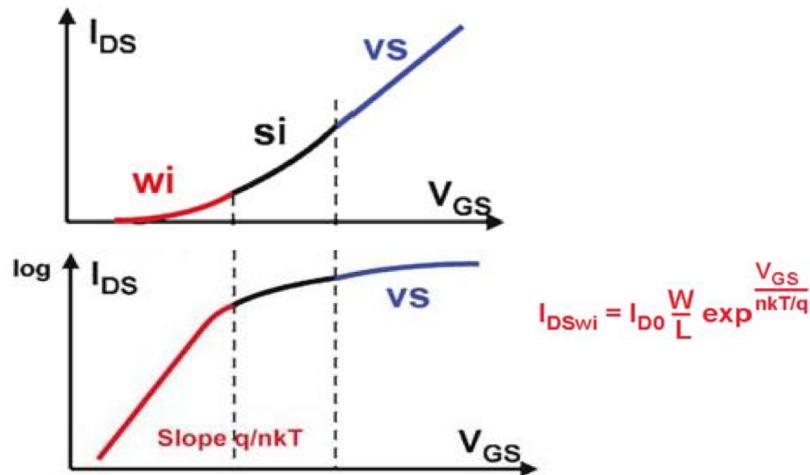
Velocity saturation

$$v = \frac{\mu_n s \mathcal{E}}{1 + \mathcal{E}/\mathcal{E}_{\text{sat}}}$$

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Sub Threshold Conduction

Some additional complications...



Current in Weak inversion

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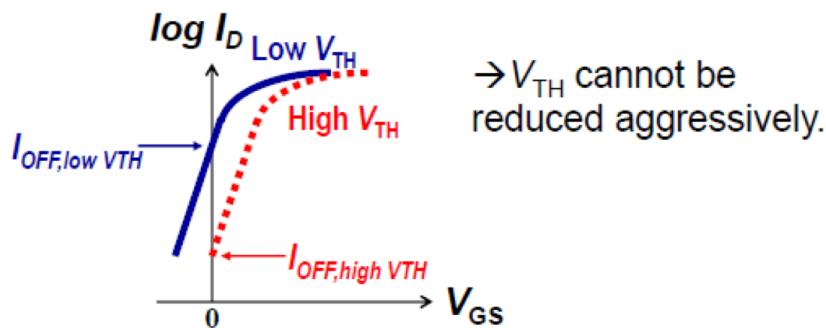
V_{TH} Design Trade-Off

- Low V_{TH} is desirable for high ON-state current:

$$I_{D,sat} \propto (V_{DD} - V_{TH})^\eta \quad 1 < \eta < 2$$

Velocity saturation

- But high V_{TH} is needed for low OFF-state current:



Prof. Wu, UC Berkeley

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What are μC_{ox} ("KP") and λ ("LAMBDA") for our Technology?

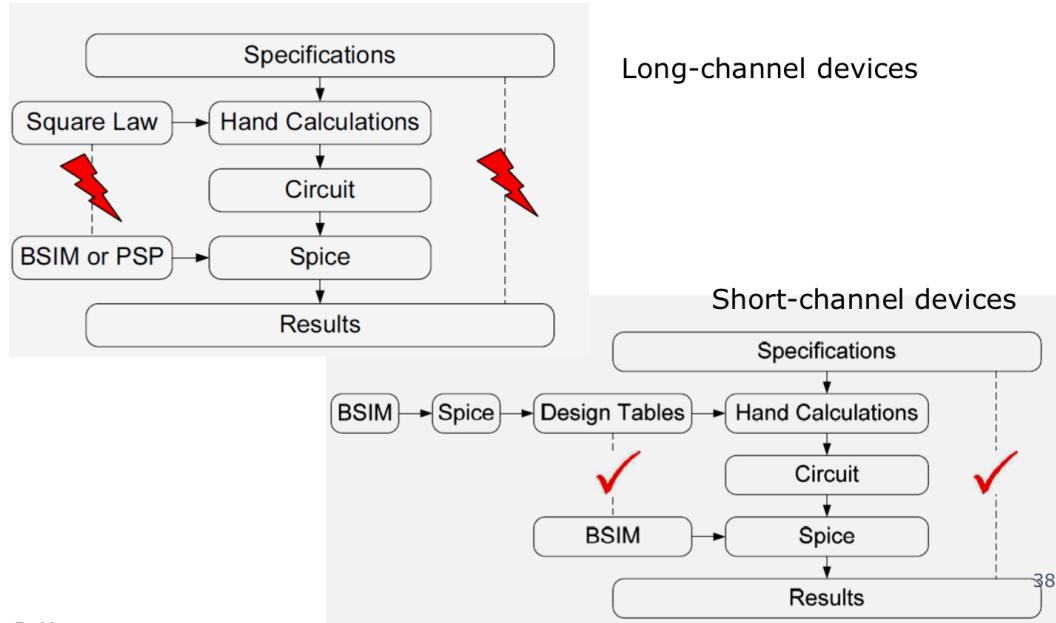
```
.MODEL nmos214 nmos
+NM = 3 RDIF = 0.32E-6 LLEVEL = 49
+VDSIION = 3.1 THOM = 27 TOT = 4.15E-9
+XZ = 1E-7 NCH = 2.3548C17 VDD0 = 0.3031397
+XL = 0.5916055 K2 = 3.2251598E-3 K3 = 1E-3
+XC = 2.3938862 W0 = 1E-7 NUE = 1.7762688E-7
+DPDNW = 0 DVT1M = 0 DPT1W = 0
+DPDW = 1.31273488 DVT1L = 0.3876801 DPT1L = 0.02387708
+DD = 256.74593 DA = -1.585658E-9 US = 7.592753E-18
+DC = 5.182125E-11 VSAT = 1.00326885 AD = 1.9815392
+AWE = 0.4347252 DO = 4.999268E-7 UI = 5E-6
+XTRA = -0.088408E-3 AI = 6.164523E-4 A2 = 0.3088917
+DSD = 128.705483 PRM = 0.5 PWS = -0.2
+WR = 1 WINT = 0 LDRT = 1.6173116E-8
+XL = 0 XM = -1E-8 DM = -5.383412E-9
+XC = 9.111767E-9 YOFF = -0.0858324 NDUCTOR = 2.12425572
+CT = 0 CGDC = 2.4E-4 CMCD = 0
+CGDCB = 0 ETAD = 2.991159E-3 ETBD = 0.239344E-6
+DEBS = 0.0159732 PCLM = 0.7245544 PDEBLCI = 0.1568183
+VFBLC2 = 2.543351E-3 VFBLCB = -0.1 DRDUT = 0.4445011
+VFB31 = 8E10 VFB32 = 1.976443E-9 PFG = 1.230284E-5
+SHDFA = 0.01 RSS = 6.6 NMOD = 1
+VRF = 0 UTR = -1.5 RTR = -0.11
+XTL = 0 RT2 = 0.022 UAL = 4.13E-9
+XRI = -7.61E-18 UC1 = -0.68E-11 AT = 3.35E
+XL = 0 WIN = 1 WW = 0
+WW = 1 WNL = 0 LL = 0
+LW = 1 LW = 0 LM = 1
+LM = 0 CARMOD = 2 XNART = 1
+COBO = 4.8E-10 OSGO = 4.9E-10 COBO = 1E-12
+CJ = 0.632022E-4 FD = 0.8 MJ = 0.3234899
+CMW = 2.326445E-10 PRSW = 0.8 MJSW = 0.1253135
+CMWS = 3.3E-10 PRSMW = 0.8 MJSMW = 0.1253135
+CF = 0 PVTHD = -1.714081E-4 PRGDN = -2.5877257
+FD = 0.61599E-4 WEXTA = -1.060423E-4 LDGTA = -5.373522E-3
+FUD = 4.576089E-14 PDR = 1.4693038E-14 PIR = 1.733193E-23
+FVDET = 1.1977483 PFTAO = 0.968409E-5 PFTA = -2.51194E-3
+XLEV = 3 xf = 0.5e-25
```

- The HSpice model for an NMOS device in our technology is shown to the left
- This is a 110-parameter BSIM3v3 model
 - More recent models may require even more parameters (e.g. PSP, BSIM6)
 - KP and LAMBDA are nowhere to be found
- It turns out that the I-V characteristics of a modern MOSFET cannot be accurately described by the square law

B. Murmann

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Design strategies

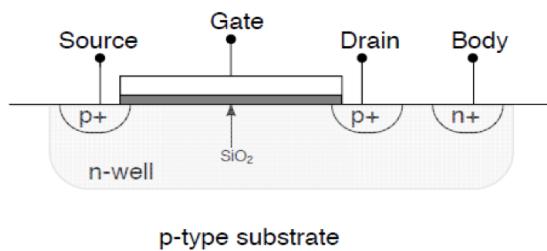


B. Murmann

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PMOS vs NMOS

If we reverse all voltages and switch all dopings their opposite type, we will have a PMOS.



Body should be connected to the highest potential in the circuit.

For PMOS, V_t is negative.

Typically, provides lower current for the same voltage due to holes having a smaller μ than electrons.

Everything we have derived so far is equally applicable to PMOS. You only need to put absolute values around every term in all our equations.

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PMOS I-V Equations

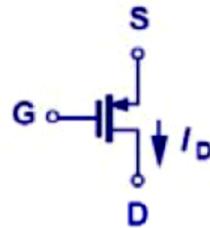
$$I_{D,ni} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} [2(V_{SG} - V_{TH})V_{DS} - V_{DS}^2] \quad DS \leftrightarrow SD \quad GS \leftrightarrow SG$$

$$= \frac{1}{2} \mu_p C_{ox} \frac{W}{L} [2(|V_{GS}| - |V_{TH}|)|V_{DS}| - V_{DS}^2]$$

Long Channel:

$$I_{D,sat} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{SG} - V_{TH})^2 [1 + \lambda (V_{SD} - V_{SD,sat})]$$

$$= \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (|V_{GS}| - |V_{TH}|)^2 [1 + \lambda (|V_{DS}| - |V_{D,sat}|)]$$



Short Channel:

$$I_{D,sat} = v_{sat} W C_{ox} (V_{SG} - V_{TH}) [1 + \lambda (V_{SD} - V_{SD,sat})]$$

$$= v_{sat} W C_{ox} (|V_{SG}| - |V_{TH}|) [1 + \lambda (|V_{DS}| - |V_{D,sat}|)]$$

Note: $V_{GS} < 0, V_{DS} < 0, V_{D,sat} < 0, V_{TH} < 0$ in PMOS

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Taylor series

- Single variable:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots$$

$f(x) \approx f(a) + f'(a)(x-a)$ linear approximation

- Multiple variables (linear approximation):

$$f(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b)$$

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→ Let's try to apply this concept to the saturation current that we have in our MOSFET device.

* $y = f(x)$ → If we have a function like this

$$y_0 + \Delta y = f(x_0) + f'(x_0) \Delta x$$

for small variations

where

$$y_0 = f(x_0)$$

$$\Delta y = f'(x_0) \Delta x$$

what if

$$y = f(x, z)$$

then

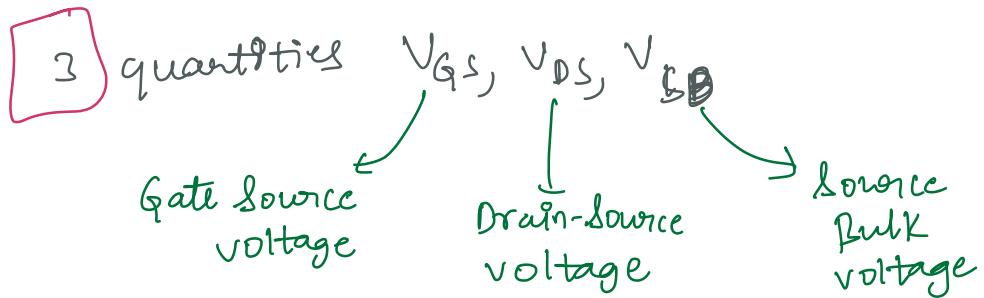
$$y_0 + \Delta y = f(x_0, z_0) + \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial z} \Delta z$$

→ Applying this concept of Multivariable calculus to the saturation current equation

$$I_D = \frac{B}{2} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$V_t = V_{T0} + \gamma \left(\sqrt{2\phi_F + V_B} - \sqrt{2\phi_F} \right)$$

→ ∵ The saturation current is dependent on



$$\therefore I_D = I_{D_0}(V_{GS}, V_{DS}, V_{SB})$$

According to Taylor's series Expansion

$$I_D = I_{D_0} + \text{Variations w.r.t. 3 quantities}$$

\downarrow
Nominal Value

w.r.t. V_{GS}, V_{DS}
 V_{SB}

$$\therefore I_D = I_{D_0} + \underbrace{\delta I_D}_{\text{Variation of current around Nominal value}}$$

Similarly

$$V_{GS} = V_{GS_0} + \underbrace{\delta V_{GS}}_{\text{variation of Gate-source voltage}}$$

These are called
Small signals/ variations

$$V_{DS} = V_{DS_0} + \delta V_{DS}$$

$$V_{BS} = V_{BS_0} + \delta V_{BS}$$

→ If we combine all this using Taylor series Expansion -

$$I_{D_0} = I_D(V_{GS0}, V_{DS0}, V_{BS0})$$

$$\dot{i}_d = \left(\frac{\partial I_D}{\partial V_{GS}} \right) \cdot \dot{V}_{GS} + \frac{\partial I_D}{\partial V_{DS}} \cdot \dot{V}_{DS} + \frac{\partial I_D}{\partial V_{BS}} \cdot \dot{V}_{BS}$$

This formula is the most important in our course

Because the small signal Model is derived from the taylor Expansion.

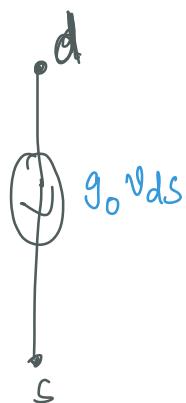
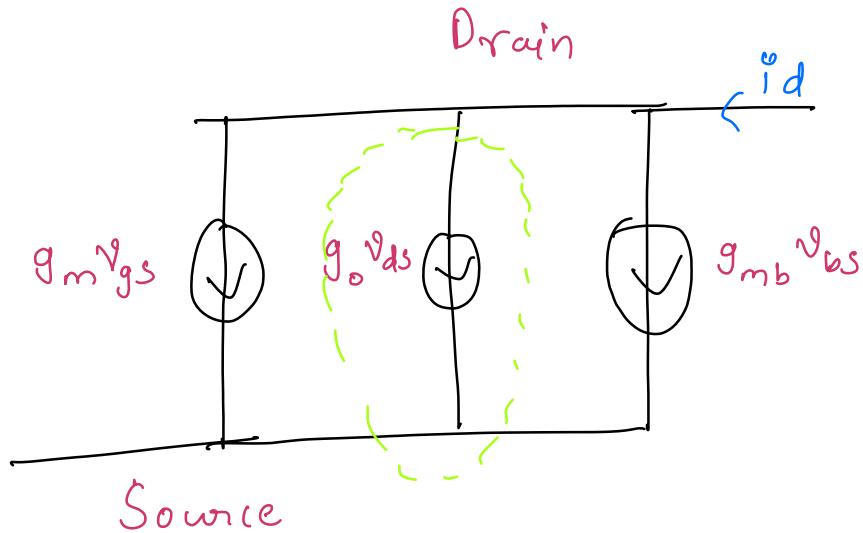
$$\dot{i}_d = g_m \cdot \dot{V}_{GS} + g_o \cdot \dot{V}_{DS} + g_{mb} \cdot \dot{V}_{BS}$$

(Transconductance)
(Ω^{-1})

Bulk Transconduc-tance

→ we can turn this Algebraic expression into a Circuit Equivalent. Small signal Model

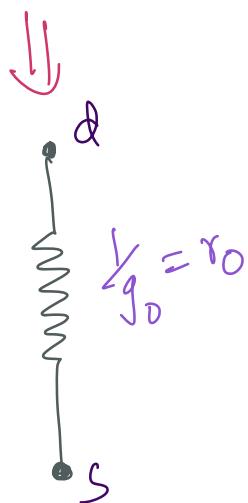
because it maps small changes in the drain current with respect to variations in V_{GS} , V_{DS} , V_{BS}



\therefore the current through this C-S is controlled by voltage across V_{DS} .

\therefore It can be Replace by a Resistor with Resistance

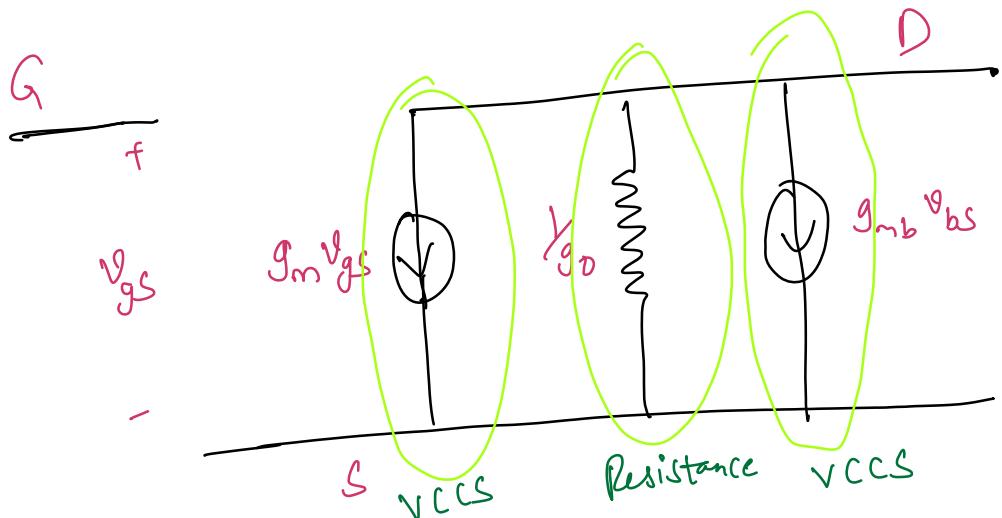
$$r_0 = \frac{1}{g_o}$$



"But this is not the case for $g_m V_{GS}$, $g_{mb} V_{BS}$

Because they appear L/w DS and are dependent on V_{GS} and V_{BS} not V_{DS}

∴ The small signal Model would be



∴ If we are in the neighbourhood of our operating point (at operating point, that variations are small). Then we can transform our Non linear circuit into combination of linear elements.

→ The above simple sub-circuit describes the behaviour of Transistor. we have drawn a linearised circuit based on the partial differential equation

$$\hat{i}_d = \frac{\partial I_d}{\partial v_{gs}} i_{gs} + \frac{\partial I_d}{\partial v_{bs}} v_{ds} + \frac{\partial I_d}{\partial v_{rb}} v_{rb}$$

g_m g_o g_{mb}

→ we know that g_m , g_o , g_{mb} play a critical role in the function of the MOSFET.

Q How can we play with g_m , g_o , g_{mb} possibly with the tools we have or redesign?

→ First of all lets compute this quantity so that we can infer how can these quantities define the operation of MOSFET.

$$g_m = \frac{dI_D}{dV_{GS}}$$

This derivative must be calculated at a specific operating point in Saturation Region.

∴ we have $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \alpha V_{DS})$

$$I_D = \frac{\beta}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \alpha V_{DS})$$

For the sake of calculation of g_m we neglect this term.

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

usually $0.1 \leq \alpha \leq 0.2$

if we compute this derivative we get

$$g_m = 2(V_{GS} - V_T) \frac{\beta}{2} \frac{w}{L}$$

$$\therefore g_m = \frac{\beta}{2} \frac{w}{L} (V_{GS} - V_T)$$

$$g_m = \beta (V_{GS} - V_T)$$

g_m can also be written as using I_D expression

$$\Rightarrow g_m = \beta (V_{GS} - V_T)$$

from $I_D = \frac{1}{2} \beta (V_{GS} - V_T)^2$

$$\Rightarrow \sqrt{\frac{2I_D}{\beta}} = V_{GS} - V_T$$

$$\Rightarrow g_m = \beta \sqrt{\frac{2I_D}{\beta}}$$

$$g_m = \sqrt{2I_D \beta}$$

we can also write

$$\beta = \frac{2I_D}{(V_{GS} - V_T)^2}$$

If we substitute β in $g_m = \sqrt{2I_D\beta}$

then

$$g_m = \sqrt{2I_D\beta}$$

$$= \sqrt{2I_D \left(\frac{2I_D}{(V_{GS} - V_T)^2} \right)}$$

$$g_m \propto \frac{2I_D}{V_{GS} - V_T}$$

$$\therefore g_m = (V_{GS} - V_T)\beta = \sqrt{2I_D\beta} = \frac{2I_D}{V_{GS} - V_T}$$

Here we have

$$\begin{aligned} g_m &\propto \sqrt{I_D} \\ k & \\ g_m &\propto I_D \end{aligned}$$

which one is correct?

Ans:- Both are correct and it depends on the

other variables which ones are constant
which ones are NOT.

→ To calculate

$$g_o = \frac{\partial I_D}{\partial V_{DS}}$$

$$\Rightarrow I_D = I_{D_0} (1 + \alpha V_{DS})$$

$$\Rightarrow \frac{\partial I_D}{\partial V_{DS}} = I_{D_0} \cdot \alpha$$

The units of

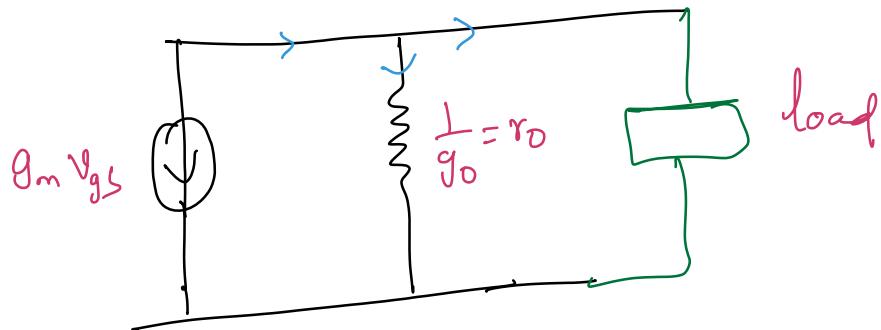
$$\alpha \text{ is } \frac{1}{\text{volt}}$$

→ The calculation of $g_{mb} = \frac{\partial I_D}{\partial V_{CB}}$ is in the slide
above Back gate Effect

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\phi_F + V_{CB}}}$$

g_{mb} is order of the magnitude $\left(\frac{1}{10}\right)$ of
 g_m ∴ g_{mb} is mostly Ignored.

→ ∴ The new small signal can be drawn as

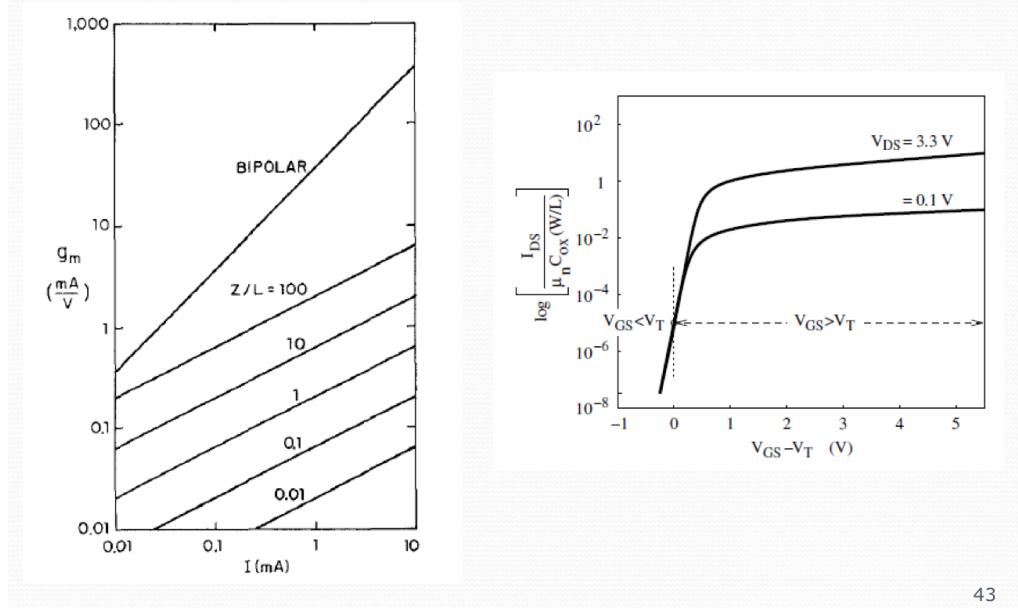


→ Ideally we would like all the current generated by the VDCS to the load and for this to happen g_o should be low so that the r_o will be high (CLM parameter). And all the current flows to the load.

∴ g_m should be Larger

g_o should be as low as possible

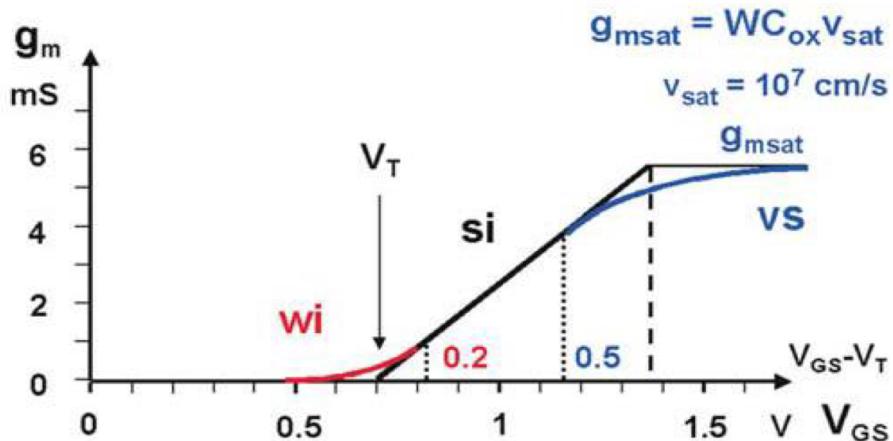
MOS vs BJT transconductance



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→ In case of BJT we have large Transconductance when compared to MOSFET

Transconductance and MOS non-idealities



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→ g_m saturates and can't increase further because of velocity saturation.

$$g_m \propto (V_{GS} - V_T)$$

But as $V_{GS} \uparrow$ the g_m increase but after certain point it saturates.

By

Revanth Reddy
Pannala

EBIT, UnibO