when we take the Relation b/w velocity & $\sqrt{\frac{2}{M}}\sqrt{E-V(x)}$

i.e $\dot{x} = \sqrt{\frac{2}{M}}\sqrt{E-V(x)}$

$\dot{x} = (\pm)\sqrt{\frac{2}{M}}\sqrt{E-V(x)}$

double

So in principle for the same Total Energy 'E' we have two possibilities of velocity one +ve & another -ve

& two average moment (P)

i.e Two opposite value of Average momentum correspond to same energy and that means that Energy is even w.r.to momentum.

→ So, substantially we found an important result that if we ~~had~~ consider the Average Momentum we find constant of motion like in the case of Free particle.

'Average Momentum is Related to Energy only'.

'Energy is Even w.r.to not only momentum (p) but also Average Momentum (P)

The only thing that changes is that the relation b/w "Avg. Momentum & Energy" is not Quadratic any more.

Q. What we analized till now, is it sufficient?

When we consider a semiconductor crystal, which is obviously periodic, so the reasoning of "Periodic potential energy" applies to that. But the device has contacts and we also apply voltages to the contacts. So the issue is that Potential energy that acts on the electrons in the crystal is not periodic any more.

The new Potential Energy

It is the superposition of the P.E imposed by the crystal and the external non periodic energy imposed by external voltage applied to the semiconductor crystal.

रेवन्त रेड्डी [illegible], Rexarith, EBIT

However, we can still consider the Average Momentum because the total P.E that exists in a crystal when we apply external voltages to the contacts is made of two contributions. One is periodic energy of atoms and that is very strong and rapidly varying function (i.e it is the function which is as same order as lattice constant)

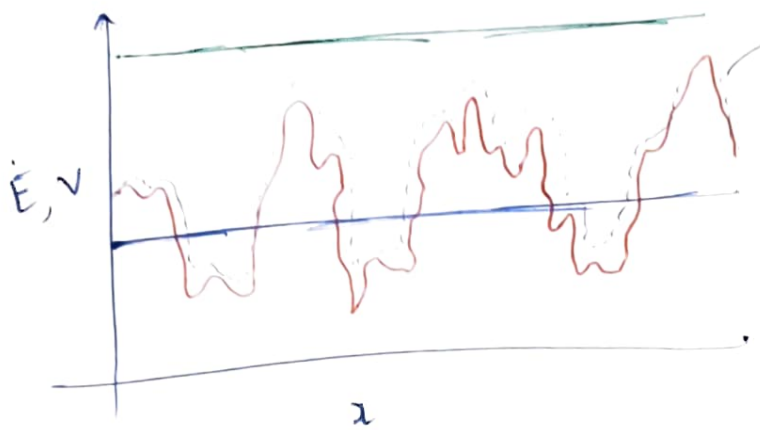
That means forces due to that are enormous.

Forces due to external voltages are small and they are non-periodic.

So we can try to continue on the same line as before that the external forces can be considered as small perturbations.

to the existing Periodic Potential Energy due to atoms.

And we can see it in the figure



→ we assume that we superimpose to the periodic P.E $V(x)$ a small non periodic potential cx because of external force

→ And external force produced by cx is a derivative with negative sign, it is like we are adding a constant force due to a linear potential energy. So we consider this as small perturbation.

Then we repeat the calculation by calculating again the average momentum incorporating into the calculation the small perturbation.

Small perturbation is called cx. 4 and for the equation it is written as

$$T(s) = \sqrt{\frac{M}{2}} \int_s^{s+a} \frac{dx}{\sqrt{E - V(x) - \delta V(x)}}$$

$$\bar{p}(s) = \sqrt{\frac{2M}{a}} \int_s^{s+a} \frac{\cancel{dx}}{\sqrt{E - V(x) - \delta V(x)}}\, dx$$

→ δV is small w.r.to V (i.e δV is almost constant over a period)

and δV has small dependence on 'x' and $V(x)$ has strong dependence on 'x'.

∴ we can consider

$$\sqrt{E-V-\delta V} \simeq \sqrt{E-V}\left(1-\frac{1}{2}\frac{\delta V}{E-V}\right)$$

Taylor expansion ~~[illegible]~~

$$\hat{\tau}(s) = \sqrt{\frac{m}{2}}\int_{s}^{s+a}\frac{dx}{\sqrt{E-V(x)-\delta V(x)}}$$

It is dependent on 's' now because $V(x)$ is not periodic any more

If δV is almost constant over a period

Then we can replace ~~[illegible]~~ $\delta V(x)$ with $\delta V(s)$.

Then what we do is calculate the Integral.

Then we find

$$\boxed{\hat{p}(s) = p - \frac{\tau}{a}\,\delta V(s)}$$

$\hat{p}(s)$: perturbed average momentum

p: Average Momentum of

$$\boxed{\frac{\hat{p}(s+a)-\hat{p}(s)}{\tau} = -\frac{\delta V(s+a)-\delta V(s)}{a}}$$

The variation of Average Momentum in Time necessary for particle to cross one period

Variation of the external potential energy devided by the Period a.

[illegible] Revanth, EBIT

→ The relation tells us that in cases where internal force is much weaker than the periodic force and also the external force changes little over a period

→ we can find that changes in Average momentum is only due to external perturbation ($\delta V(x)$)

∴ we can say that the average force acting on a particle is external force. This is a very fundamental result because it gives us the possibility to describe the dynamics of a particle inside a crystal by considering only the External forces.

The conclusions we draw are:

Average momentum in place of real Momentum

External Forces instead of Total Forces

provided External forces are constant and not too strong which is the case for the typical operating conditions of semiconductor devices.

Essentially when we use Avg. Momentum in a Crystal (or Periodic structure)

⟹

It's like we are considering a particle in Free space with an External Force.

It's like crystal has disappeared.

→ we also deducted that the Relation b/w energy and Average momentum is not Quadratic any more this is the price we have to pay to eliminate the action of the lattice that acts on the particle. ~~The periodic po.~~

$$P_1 = \frac{1}{a}\int_{s}^{s+a} p\,dx = \frac{\sqrt{2m}}{a}\int_{s}^{s+a}\sqrt{E - V(x)}\,dx$$

→ all the above analysis is done using ~~Quantum~~ Classical Mechanics.

→ But to obtain appropriate results we should use Quantum Mechanics.

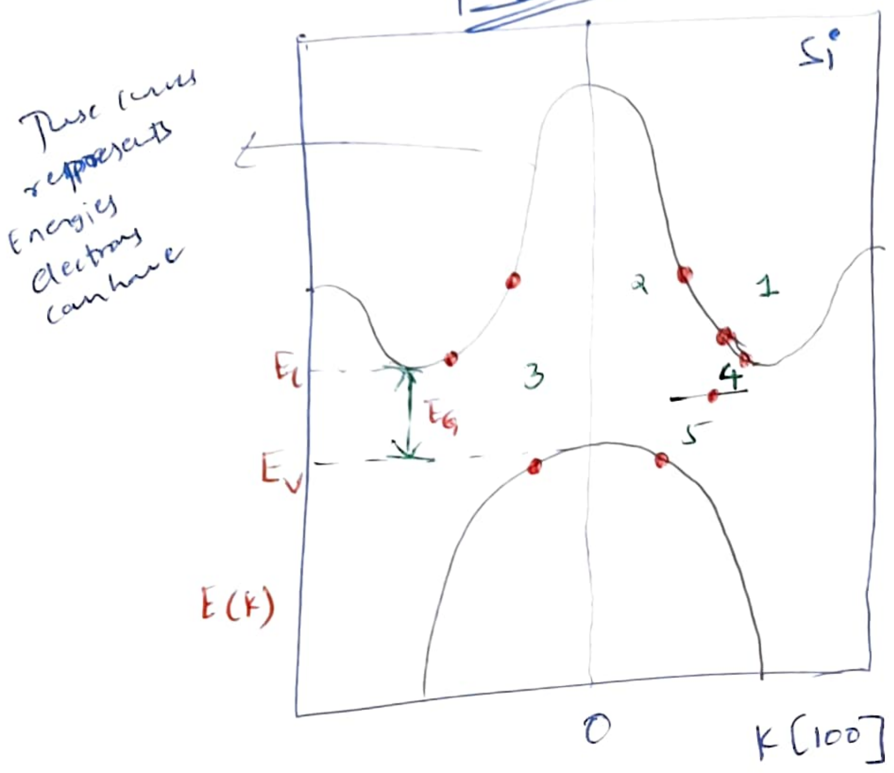
** Now we jump back to "Tavole di Elettronica Book"

The difference b/w using QM & CM and the interpretation of the results.

→ we shall still use in QM case Average Momentum this Average should be considered in time it is a Statistical Average.

→ The Average Momentum of a particle in a Crystal according to QM is proportional to the vector belonging to the 'scaled reciprocal lattice' that we call 'k'

Fig. 3



Transition

1: Intra-valley
2: Inter-valley
3: Inter-band
1,2: Intra-band
4+5: Trap-assisted

There're several differences in the Results we obtain using QM. on a crystal w.r.t to the case of a treatment based on classical mechanics.

→ The most important results are:

we shall use Average Momentum, this (Avg. P) is not calculated in time, it is a statistical Average

→ The Average momentum of a particle in a crystal is proportional to the vector belonging to the scaled Reciprocal Lattice. That we call 'k'.

The vector 'k' we studied earlier had only ~~Geogr~~ Geometrical meaning of a Scaled Reciprocal Lattice.

$k \propto$ Avg. Momentum of Quantum mechanical treatment of the particle.

→ Because of this then we can expect, Energy when we take averages, Energy depends on Avg. Momentum

that means E is expected to depend on K

i.e $\boxed{E = P(x)}$ [Energy as a function of 'P' in classical mechanics]

↓

This is Replaced by

$\boxed{E = K(x)}$ {energy as a function of 'K' in QM treatment]

↓

(This relation is not Quadratic).

when we learned SRL (Scaled Reciprocal lattice) we also observed periodicity in that case.

So, one of the outcomes of the QM treatment that eventually gives the form of the Relation b/w Energy and Momentum i.e $E = K(x)$ shows us that This relation is periodic.

** So "Energy is a periodic function of Momentum" in QM very different from "Quadratic term we observed in Classical Mechanics".

→ When we have a periodic function it is sufficient to consider one period. and one period is as we remember Brillouin Zone

"E & Momentum" from now will be considered in one period [Brillouin Zone]

Fig. 3 looks strange but it's one of the Results using QM for a crystal lattice

Relation b/w E & k in QM interpretation is a many valued function.

To each 'k' there correspond many possible energies.

So, we find that is made of several or infinite branches. "Thats QM for You?" Hahahaha.

If we consider one branch at a time it gonna take forever.

∴ The standard method is to select one specific element in space and remember that in cubic lattice

'k_1' component is alligned with (100) direction

k_2 — (010)

k_3 — (001)

→ We simply consider the (100) direction i.e K_1 and show the total energy in verticle direction w.r.to $'K_1'$ as shown in Fig.3 (Energy as function of 'K'

& 'K' is a function of average momentum)