09/04/25

Lecture-18

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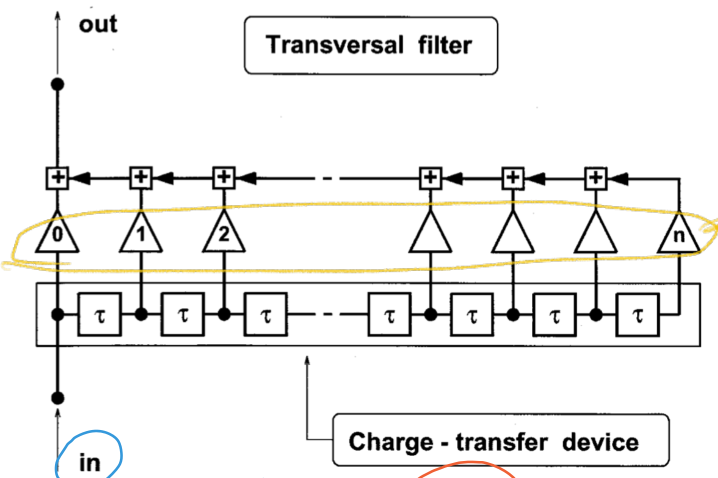
* Application of CCD's apart from being Image Sensors

- We can use CCD as a Delay Line and There by as a component in Filter Design.



Amplifiers assisted by Floating Gates for each CCD Gate

Here input is not light but some Analog Signal which will be Transformed into a charge packet. This Charge packet enters CCD from the left.

τ : Time delay

- Similarly we can inject as many input signals as possible by shifting the already existing charge packets to the right by onestep each time.

** By controlling The Gains of Amplifiers We will be building a filter to our specifications

Use of the CCD as a Filter

- The filtering operation is based on the properties of the Fourier transform of a function $v(t)$,

$$\boxed{V(\omega) = \mathcal{F}v \doteq \int_{-\infty}^{+\infty} v(t)\, \exp(-j\omega t)\, \mathrm{d}t\,,}$$

- in particular on the convolution integral

Transfer function

$$u(t) = h(t) * v(t) \doteq \frac{1}{2\pi}\int_{-\infty}^{+\infty} h(t')\, v(t-t')\, \mathrm{d}t' = \mathcal{F}^{-1}(HV)\,,$$

with $H(\omega) = \mathcal{F}h$. If H is a transfer function, it must be $h(t') = 0$ for $t' < 0$, whence $\int_{-\infty}^{+\infty} \leftarrow \int_{0}^{+\infty}$. The discrete form of the convolution integral is

Filtered signal

$$\boxed{u(t) = \sum_{i=0}^{\infty} \simeq \sum_{i=0}^{n} h(t_i')\, v(t - t_i')\, \Delta t'\,.}$$

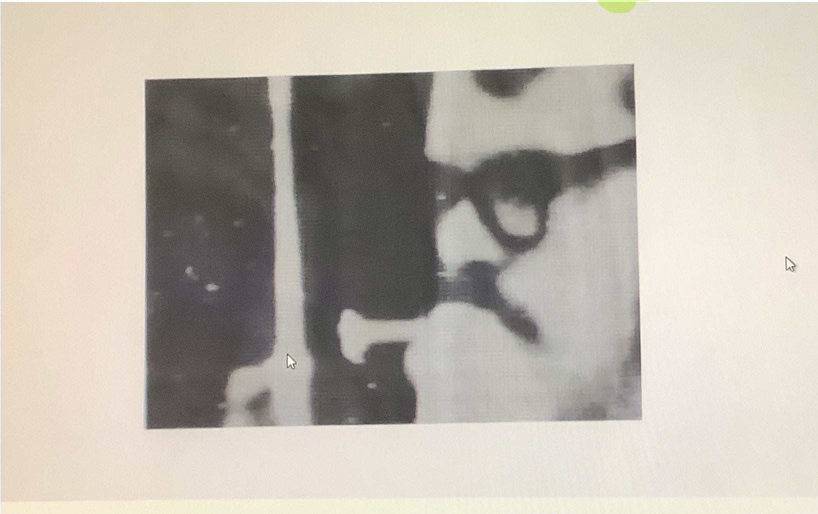
Letting $\tau \doteq \Delta t'$, $\boxed{a_i \doteq h(t_i')\,\tau}$, $t_i' = i\tau$, the above becomes

Finite delay of one block of CCD to the next

$$u(t) = \sum_{i=0}^{n} a_i\, v(t - i\tau) = a_M \sum_{i=0}^{n} \frac{a_i}{a_M}\, v(t - i\tau)\,,$$

with $a_M = \max(a_i)$. With a suitable choice of the attenuations a_i/a_M , this expression can be implemented by a CCD used as a delay line.

→ phenomenon of Blooming



- Here, we can see a person lighting a cigarette.
- But the spot where the flame is so bright that the no. of charges generated by CCD is so large. That they cannot be accomodated at the same location of CCD and they spread. They typically spread along same CCD so they are spread vertically in the figure. This is called Blooming.

- one possible solution to Blooming is to realize extra Gates on the two sides of the CCD.

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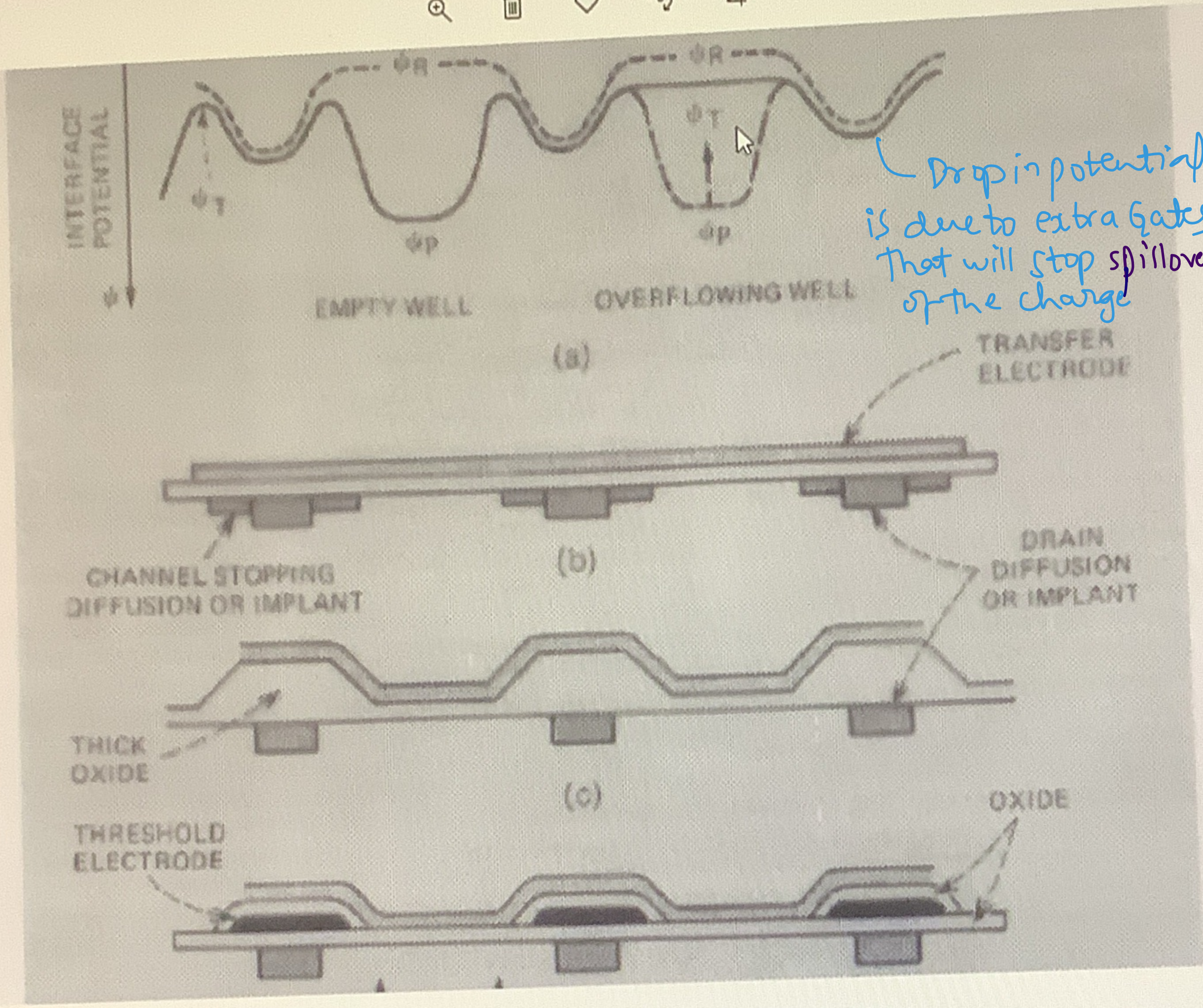


Drop in potential is due to extra Gates That will stop spillover of the charge

The Drift-Diffusion Transport Equation — IV

The contribution of the term containing the magnetic field to the current density of the ath band, when the magnetic field is sufficiently small, is $\hat{\mu}_a (\boldsymbol{B} \wedge \boldsymbol{g}_a)$. The mobility tensor $\hat{\mu}_a = \tau_{pn}/\hat{m}_a$ is diagonal and, in the conduction band of silicon, has one of the following forms:

$$\begin{bmatrix} \mu_l & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_t \end{bmatrix}, \qquad \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_l & 0 \\ 0 & 0 & \mu_t \end{bmatrix}, \qquad \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_l \end{bmatrix},$$

with $\mu_l \doteq \tau_{pn}/m_l$, $\mu_t \doteq \tau_{pn}/m_t$. In turn, $\boldsymbol{g}_a$ has the form $\boldsymbol{g}_a = \hat{\mu}_a/(M_C \mu_n)\, \boldsymbol{g}$. Adding over the valleys with $M_C = 6$

$$\sum_{a=1}^{M_C} \hat{\mu}_a (\boldsymbol{B} \wedge \boldsymbol{g}_a) = \frac{\mu_t\,(\mu_t + 2\mu_l)}{3\mu_n} \boldsymbol{B} \wedge \boldsymbol{g} = a_n\, \mu_n\, \boldsymbol{B} \wedge \boldsymbol{g},$$

with $a_n \doteq \mu_t\,(\mu_t + 2\mu_l)/(3\mu_n^2)$. In conclusion, the current density of the electrons in the conduction band reads

$$\boldsymbol{J}_n = q\mu_n\, n\, \boldsymbol{\mathcal{E}} + qD_n \operatorname{grad} n + q a_n\, \mu_n\, \boldsymbol{B} \wedge (\mu_n n\, \boldsymbol{\mathcal{E}} + D_n \operatorname{grad} n),$$

which is called ***drift-diffusion transport equation.***

* We begin by investigating the behaviour of the semiconductor when we have the presence of a Magnetic Field in the semiconductor.

In general when we consider the motion of charges in a semiconductor in principle we should use All equations belonging to Maxwell equations but at some point we introduced the (QSA) Quasi Static Approximation that is applicable when the typical time in which there is a variation of the Applied voltage to the device is much much longer than the propagation time of the Radiation.

$$J_n = q\mu_n n \mathcal{E} + qD_n \operatorname{grad} n + qa_n \mu_n B \wedge (\mu_n n \mathcal{E} + D_n \operatorname{grad} n) ,$$

which is called *drift-diffusion transport equation.*

QSA

i.e In the expression of the Electric field we only consider the Gradient of the Electric potential and we neglect the Time derivative

- However, when it comes to sensors we may wish to exploit the properties of semiconductor in order to detect the Magnetic field that is in the Environment

- To build a Magnetic sensor we have to consider another contribution from external force acting on the e^-s (or) Holes in the semiconductor and this other contribution is of Magnetic type.

If we limit ourselves to Magnetic effect on e^-s and the Drift Diffusion Model we obtain the

below eq^n
Drift Diffusion external Magnetic field

$$\boldsymbol{J}_n = q\mu_n n \boldsymbol{\mathcal{E}} + qD_n \operatorname{grad} n + qa_n \mu_n \boldsymbol{B} \wedge (\mu_n n \boldsymbol{\mathcal{E}} + D_n \operatorname{grad} n) ,$$

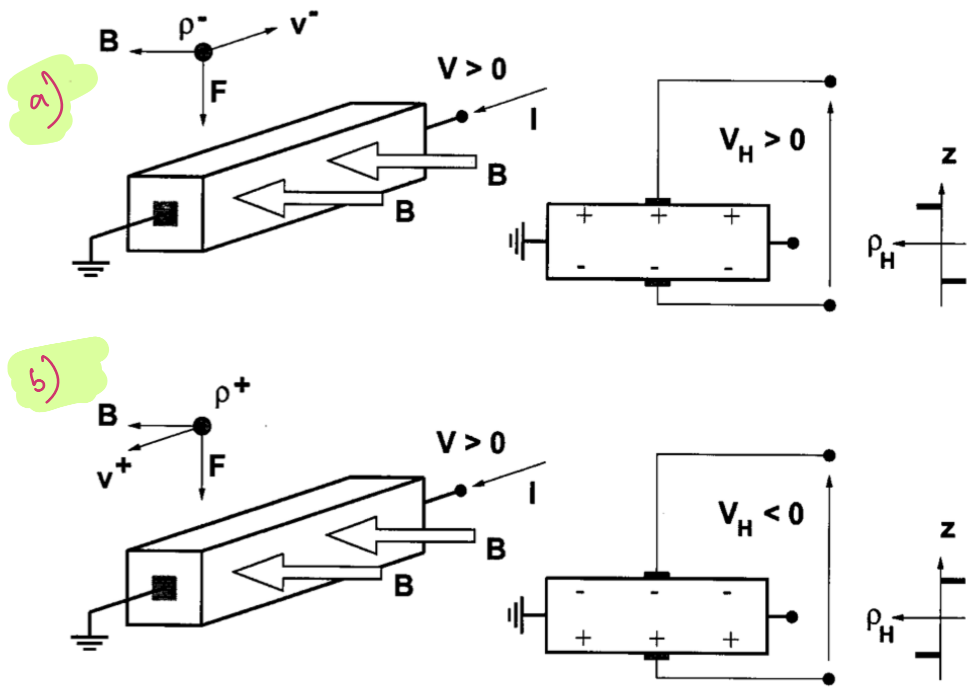
which is called **drift-diffusion** transport equation.

" a_n " coefficient is a Dimensionless quantity and is a complicated combination of the terms of the Mass Tensor

- In practice the sensors that are devoted to the detection of the Magnetic field are Uniform in space. This makes it possible simplify some how the eq^n above, because we don't have the Gradient of the concentration. So we can neglect the Diffusion Term and we consider only Drift term.

→ We shall examine one (or) two important effects that we can obtain in a semiconductor in presence of a Magnetic Field. The first one is a so called Hall Effect

HALL EFFECT



T. 8.25: Misura dell'effetto Hall.

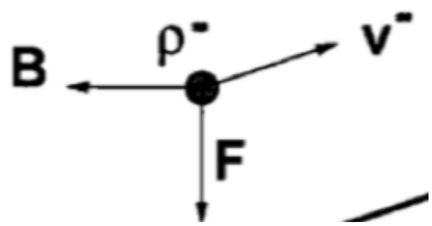
- consider figure-a A uniform Bar of semiconductor the semiconductor is shown in the middle part of the figure from the side
- since the voltage V>0 the current will enter the Bar from the right contact and flows towards Left contact.
- we assume that the device is long & Thin so more con len we can assume that the flow lines of the current density are parallel to the axis of the device and also parallel to each other

- At this point assume that you apply a magnetic field normal to the lateral face of the device 'B' indicated in the figure.

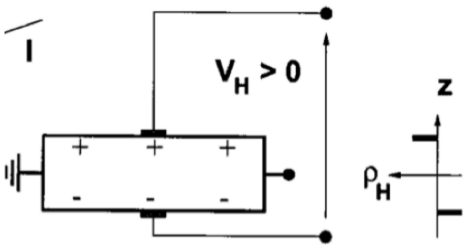
- Considering the case where electric current is carried by the Electrons. So if the current goes from Right to left that means the Average velocity of the electrons is opposite to it. So if you look at the little diagram that is in the upper left corner of the page we see a V^- which indicates orientation of the Average velocity of e^-s, and Charge density is also called ρ^-.

- Now, the magnetic field produces a Lorentz force that acts onto the Electrons in the verticle direction

* Lorentz force is the Vector product of B & Charge (Here is -ve) ∴ Force is downwards



- So when we apply Magnetic field the flow lines of current density are pushed down wards near the lower face of the device.



- It's as if you have accumulation of excess negative charge near the lower face. ∵ Device is globally neutral we have +ve charge on the upper face

* when we have this two seperate layers of charge one +ve & one −ve we have an Electric field that is verticle in the Device it's oriented from +ve charge to −ve charge. we assume the field to be Uniform

we know $E = -\frac{dv}{dr}$

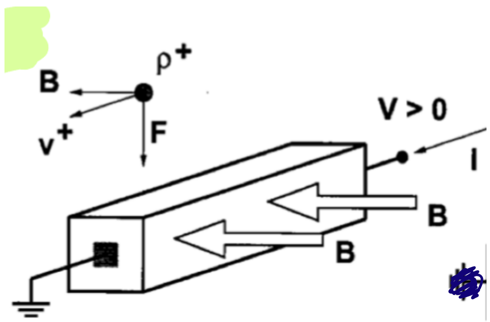
∴ if we multiply 'E' by the Thickness of the Device you have a voltage drop

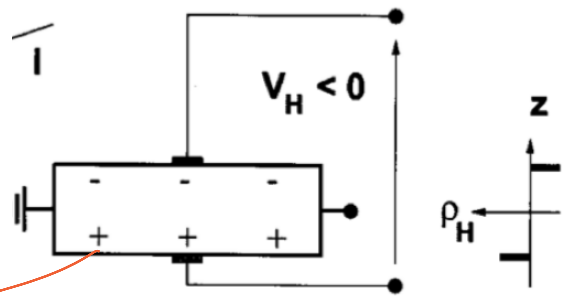
- This voltage is V_H whole voltage.
  we can measure the V_H as shown in the figure from the contacts in the middle with

a measuring equipment of Infinite Impedence.
i.e you measure the voltage without absorbing any current.

- offcourse, we can expect the whole voltage to be dependent on the Intensity of the Applied Magnetic field and some other physical and Geometrical properties of the device.

** → We repeat the experiment on a semiconductor with p-type doping i.e figure-b





Accumulation of +ve charges on the lower edge of the Device

Q) How can we exploit the above properties to measure external Magnetic field ?

Measurements Using the Hall Effect — I

In a uniformly-doped semiconductor sample, slightly perturbed from equilibrium, the contribution of the diffusive effect to the charge transport is negligible, whence

Drift term

$$\boldsymbol{J}_n = q\mu_n n \boldsymbol{\mathcal{E}} - q a_n \mu_n^2 n \boldsymbol{\mathcal{E}} \wedge \boldsymbol{B},$$

$$\boldsymbol{J}_p = q\mu_p p \boldsymbol{\mathcal{E}} + q a_p \mu_p^2 p \boldsymbol{\mathcal{E}} \wedge \boldsymbol{B}.$$

The total current density $\boldsymbol{J} = \boldsymbol{J}_n + \boldsymbol{J}_p$ then reads

$$\boxed{\boldsymbol{J} = \sigma \boldsymbol{\mathcal{E}} + r\,\sigma^2 \boldsymbol{\mathcal{E}} \wedge \boldsymbol{B}},$$

where $\boxed{\sigma = q\mu_p p + q\mu_n n}$ is the electrical conductivity and

$$r \doteq \frac{q}{\sigma^2}\left(a_p \mu_p^2 p - a_n \mu_n^2 n\right) = \frac{a_p \mu_p^2 p - a_n \mu_n^2 n}{q\left(\mu_p p + \mu_n n\right)^2}$$

is the ***Hall coefficient***. The two quantities σ and r can be measured independently. While σ is positive definite, r has a sign. In particular, the following limiting cases hold:

$$\begin{cases} p \gg n & \Rightarrow \quad \sigma \simeq q\mu_p p, \quad r \simeq a_p/(qp) > 0 \\ n \gg p & \Rightarrow \quad \sigma \simeq q\mu_n n, \quad r \simeq -a_n/(qn) < 0 \end{cases}$$

This very easy method to calculate concentration of Maj carriers in uniform materials & also the Mobility

**

Measurements Using the Hall Effect — II

- From the limiting cases, the majority-carrier concentration and mobility can easily be determined as functions of, e.g., lattice temperature and dopant concentrations:

$$\begin{cases} p \gg n & \Rightarrow \quad p = a_p/(qr)\,, \quad \mu_p = \sigma r/a_p \\ n \gg p & \Rightarrow \quad n = -a_n/(qr)\,, \quad \mu_n = -\sigma r/a_n \end{cases}$$

- The measurement of σ and r is easily carried out by applying $\boldsymbol{J} = \sigma\boldsymbol{\mathcal{E}} + r\,\sigma^2\boldsymbol{\mathcal{E}} \wedge \boldsymbol{B}$ to a prismatic sample with $L \gg H, W$, in which the bias is applied between the edges normal to $\boldsymbol{i}_L$, and $\boldsymbol{B} = B\boldsymbol{i}_W$. Observing that $\mathcal{E}_W = 0$, $J_W = 0$, and $\boldsymbol{\mathcal{E}} \wedge \boldsymbol{B} = \mathcal{E}_L B\,\boldsymbol{i}_H - \mathcal{E}_H B\,\boldsymbol{i}_L$, it follows

$$J_L = \sigma\mathcal{E}_L - r\,\sigma^2\mathcal{E}_H B\,, \qquad J_H = \sigma\mathcal{E}_H + r\,\sigma^2\mathcal{E}_L B\,.$$

Assuming we still have two components of the current.
They are longitudinal & verticle components

- As B is small and $J_H = 0$, the above become

no current in verticle direction

$$\boxed{J = J_L \simeq \sigma\mathcal{E}_L}\,, \qquad \boxed{\mathcal{E}_H = -r\,\sigma\mathcal{E}_L B \simeq -rJB}\,.$$

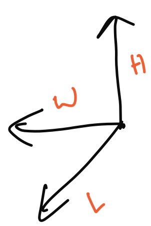
- As $\mathcal{E}_L \simeq V_L/L$, $\mathcal{E}_H \simeq V_H/H$, $J = I/(WH)$, whence $V_H = -rBI/W$, $I/(WH) = \sigma V_L/L$. In conclusion,

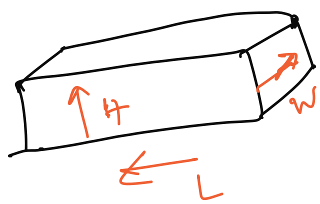
$$\boxed{\sigma = \frac{LI}{WHV_L}}\,, \qquad \boxed{r = -\frac{WV_H}{BI}}\,.$$

from Geometrical properties of the semiconductor

Conductivity

Hall Coefficient





L >> H, W

*

$$\mathcal{E} = \mathcal{E}_L \hat{i}_L + \mathcal{E}_H \hat{i}_H + \mathcal{E}_W(0)$$

$$B = 0\,\hat{i}_L + 0\,\hat{i}_H + B\,i_W$$

$$\mathcal{E} \wedge B = \begin{vmatrix} i_L & i_W & i_H \\ \mathcal{E}_L & 0 & \mathcal{E}_H \\ 0 & B & 0 \end{vmatrix}$$

$$= i_L\,(-B\mathcal{E}_H) + i_H\,\mathcal{E}_L B$$

$$\Rightarrow -\mathcal{E}_H B i_L + \mathcal{E}_L B i_H$$

$$\boxed{\mathcal{E} \wedge B \Rightarrow \mathcal{E}_L B i_H - \mathcal{E}_H B i_L}$$

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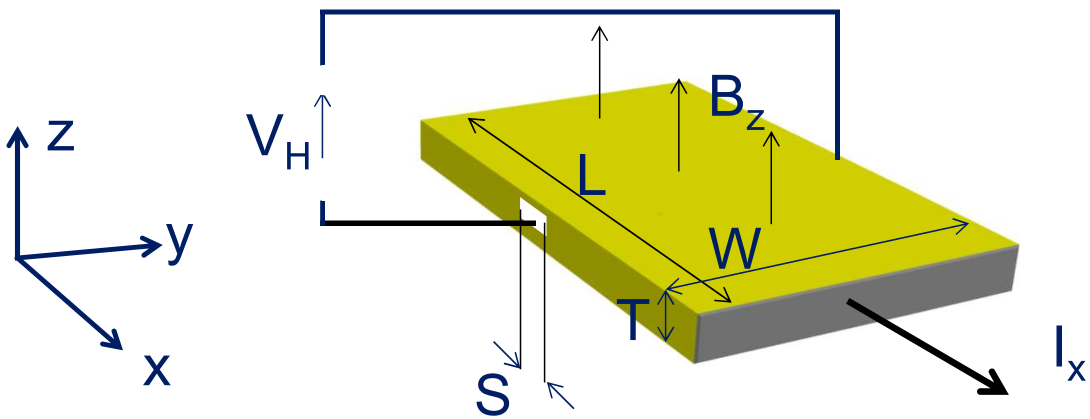
Review of the Hall effect

In a conducting material with a magnetic induction $\boldsymbol{B}_z$ applied in the direction normal to the electric current flux $\boldsymbol{I}_x$, a voltage difference V_y is created across the conductor transverse to $\boldsymbol{I}_x$ and perpendicular to $\boldsymbol{B}_z$.

Ideal plate: $T \ll W \ll L$, Hall probes: $S \ll W$.

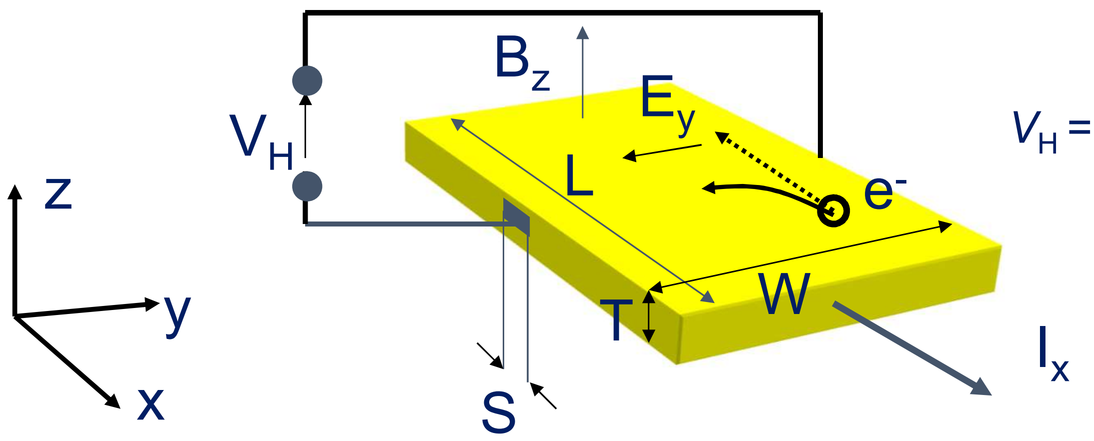
$V_H = R_H \, I_X \, B_Z \, / \, T$ with R_H the Hall coefficient (in cm^3/C).



Explanation of the Hall effect - I

Lorentz's force: $\boldsymbol{F} = q\,\boldsymbol{v} \times \boldsymbol{B} = -q\, v_x B_z\, \boldsymbol{i}_y$

$q\boldsymbol{E} = \boldsymbol{F} \rightarrow E_y = v_x B_z = R_H J_x B_z = R_H I_x B_z/(TW)$, $E_y = V_H/W$



$$\Downarrow$$

$$V_H = R_H I_x B_z/T$$

Explanation of the Hall effect - III

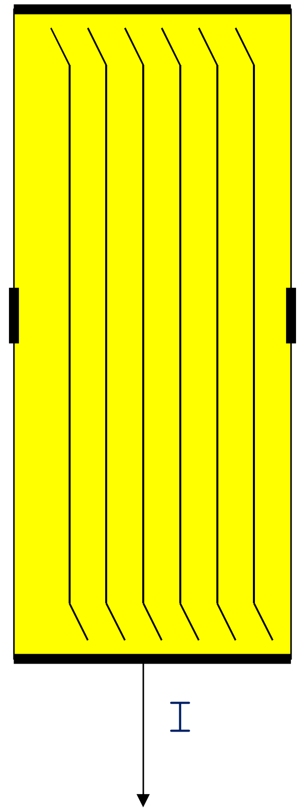
$$\boldsymbol{J}_n = \sigma_{nB}(\boldsymbol{E} + \mu_n^* \boldsymbol{B} \times \boldsymbol{E})$$

$$J_{ny} = 0$$

$$E_y + \mu_n^* B_z E_x = 0$$

$$E_y = -\mu_n^* B_z E_x = -\mu_n^* B_z J_{nx}/\sigma_n$$

$$R_H = -\mu_n^*/\sigma_n = -r_n/qn$$



MAGNETORESISTANCE EFFECT

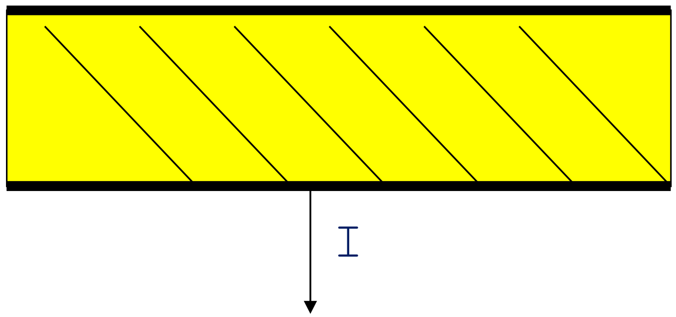
$$\boldsymbol{J}_n = \sigma_{nB}(\boldsymbol{E} + \mu_n^* \boldsymbol{B} \times \boldsymbol{E})$$

$$E_y = 0$$

$$J_{nx} = \sigma_{nB} E_x$$

$$\sigma_{nB} = \frac{\sigma_n}{1 + \mu_n^{*2} B_z^2}$$

$$\frac{\rho_{nB}}{\rho_n} = 1 + \mu_n^{*2} B_z^2$$



Magnetic Field Sensors (MFSs)

1. Measured quantities (H and B)

Measurement Units for the magnetic field (H) and induction (B)

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} + \mathbf{J}_I$$

H – magnetic field: A/m

$$\mathbf{B} = \mu_r \, \mu_0 \, \mathbf{H}$$

B – magnetic induction: Tesla

μ_r is the relative permeability of the specific medium;

$\mu_0 = 4\pi\ 10^{-7}$ H/m is the vacuum permeability

T = 1 Weber/m^2 = 10^4 Gauss

Magnetic Field Sensors (MFSs)

2. Applications:
   - Earth magnetic field measurements
   - Reading of magnetic types and disks
   - Recognition of magnetic ink patterns in banknotes
   - Reading of credit cards

   Indirect applications:
   - Contactless switching
   - Detection of a current through H
   - Linear and angular displacement detection through the mechanical displacement of a magnet

Range of interesting values for the magnetic induction field B

geomagnetic field	30-60 μT
data storage (hard-disk, floppy-disk, videotapes)	$\approx$ 10μT-10mT
reading of magnetic patterns (credit cards)	$\approx$ 10μT-10mT
reading of magnetic ink of banknotes	$\approx$ 10μT-10mT
permanent magnets used in switches and sensors	5 $\div$100 mT
permanent magnets used in measuring instrumentations	4 $\div$ 5 T