

Lecture-8

"లోగిం స్కూల్ కెపిస్ట్రేజు నుండి"

continuing from lecture 7 we discuss the carrier transport in a Semiconductor

we discussed Drift & Diffusion Equations which we have deduced Heuristically.

i.e. Equation that provides for the current Density of Electrons & holes in the Semiconductor.

→ In doing so we defined **some parameters**

Diffusion coefficients, Mobilities & conductivities

→ It is convenient now to go through these parameters to have an Idea about the **order of Magnitude** of each parameter.

→ First of all the mobilities & diffusion coefficients in the Drift-Diffusion equations are not independent of each other

It can be demonstrated that when we consider the electrons

$$D_n = \frac{k_B T L}{q} M_n$$

Semiconductor Equations — V

Einstein's relationships

Room Temperature

$$D_n = \frac{k_B T_L}{q} \mu_n,$$

$$D_p = \frac{k_B T_L}{q} \mu_p$$

At $T_L = 300$ K one finds for the thermal voltage

$$\frac{k_B T_L}{q} = \frac{1.38 \times 10^{-23} \text{ J/K}}{1.60 \times 10^{-19} \text{ C}} 300 \text{ K} \approx 26 \text{ mV},$$

$$\mu = \frac{q\langle\tau\rangle}{m^*} \approx \frac{1.60 \times 10^{-19} \text{ C}}{0.4 \times 10^{-30} \text{ kg}} \frac{0.25 \times 10^{-12} \text{ s}}{= 10^3 \frac{\text{cm}^2}{\text{Vs}}}$$

we shall see that it appears in the exponent of expression that determines the current of the PN Junction of a Diode

Estimation of Diffusion Constant

$$D = \frac{k_B T_L}{q} \mu \approx 25 \frac{\text{cm}^2}{\text{s}}$$

at $T_L = 300$ K

Conductivity

$$\sigma_n = q\mu_n n \approx 10^{-16} \frac{\text{cm}^2}{\Omega}$$

$$n = \begin{cases} 10^{-12} (\Omega \text{ cm})^{-1}, \\ 10^{-6} (\Omega \text{ cm})^{-1}, \\ 10 (\Omega \text{ cm})^{-1}, \\ 10^6 (\Omega \text{ cm})^{-1}, \end{cases}$$

Concentration of Electrons based on the conductivity of a Material

with $[n] = \text{cm}^{-3}$. It is useful to introduce the net thermal recombination rate $U_{n(p)}$ and the net non-thermal generation rate $G_{n(p)}$, for the conduction (valence) band, such that

$$W_n \doteq G_n - U_n, \quad W_p \doteq G_p - U_p.$$

$$\mathcal{M} = \frac{q \langle \tau \rangle}{m^*}$$

q : Electron charge

$\langle \tau \rangle$: statistical average of the momentum Relaxation Time

m^* : Effective Mass

$\langle \tau \rangle$: Realistic value for this is around pico seconds (10^{-12} s)

m^* : Effective Mass of an Electron is less than the free Electron Mass

So, we take $0.4 \times 10^{-30} \text{ kg}$

Real mass of Electron = $0.9 \times 10^{-30} \text{ kg}$

→ So Globally we can assume that

$$\text{mobility} = 1000 \frac{\text{cm}^2}{\text{Vs}}$$

→ To continue to discuss this model that describes the Transport in a semiconductor.

It is convenient to distinguish some symbols

like we did in the splitting of Global continuity Equation into two individual continuity equations for Holes & Electrons

$$\frac{\partial n}{\partial t} + \operatorname{div}(n v_n) = w_n \quad \text{Electrons}$$

$$\frac{\partial p}{\partial t} + \operatorname{div}(p v_p) = w_p \quad \text{Holes}$$

w_p, w_n net Generation rates of Holes & Electrons

→ These two equations EMBED all the types of Transitions that change the population either of conduction Band or Valence Band in a given VOLUME.

→ In many cases, it's convenient to distinguish what are the reasons for these transitions ?

So, In all cases if we have a Transition of Electron \leftrightarrow Hole from CB to VB
(or) Vice Versa.

or even from a Trap belonging to the Gap into a Band \leftrightarrow Vice Versa

All these Transitions may how imply the exchange of Energy with something else.

→ For several reasons, it is convenient to distinguish based on what entity is exchanging the ENERGY with electrons
(or) Holes

→ Therefore we clarify the transitions depending on other entity which exchanges energy with Electron.

ex:- Exchange of Energy by e^- or hole with nucleus of the atom when there is a collision.

→ This is such an important interaction that it is given a special symbol we split $K_{hv} \downarrow w_p$ into two parts and one part describes the transitions that involves collisions with Nuclei of the crystal.

→ Since the vibrational Energy of Nuclei depends on lattice Temperature, this type

of Transitions are called **THERMAL**
TRANSITIONS.

→ There are also other Transition

Electron - Electron Transition

Electron - Impurity Transition

Electron - photon Transition

All these Transitions are called

Non-THERMAL interactions.

with $[n] = \text{cm}^{-3}$. It is useful to introduce the net thermal recombination rate $U_{n(p)}$ and the net non-thermal generation rate $G_{n(p)}$, for the conduction (valence) band, such that

$$W_n \doteq G_n - U_n, \quad W_p \doteq G_p - U_p.$$

$$w = G_n - U_n$$

→
 Non Thermal Generation Rate Thermal recombination Rate

G_n \Rightarrow Total no. of electrons that enter the Conduction Band per unit volume in Time +

Total no. of no. of electrons that exit the CB per unit volume in Time

(for Reasons that are not collisions with Lattice
i.e Non thermal Collisions)

Up : It's customary to describe as Recombination \therefore we put negative sign in the equations

Total no. of electrons that exit CB due to THERMAL collisions +

Total no. of e⁻s that enter CB due to THERMAL collision

**

Similarly, we define G_p + V_p



By Assuming Quasi-Static Approximation.

Semiconductor Equations — VI

Within the semiconductor region

Poisson's equation split into Two parts

$$\operatorname{div} \mathbf{D} = q(p - n + N), \quad \mathbf{D} = -\varepsilon_{sc} \operatorname{grad} \varphi$$

$$\frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div} \mathbf{J}_n = G_n, \quad \mathbf{J}_n = q\mu_n n \boldsymbol{\mathcal{E}} + qD_n \operatorname{grad} n$$

$$\frac{\partial p}{\partial t} + U_p + \frac{1}{q} \operatorname{div} \mathbf{J}_p = G_p, \quad \mathbf{J}_p = q\mu_p p \boldsymbol{\mathcal{E}} - qD_p \operatorname{grad} p$$

Within the insulator region

$$\operatorname{div} \mathbf{D} = qN_{ox}, \quad \mathbf{D} = -\varepsilon_{ox} \operatorname{grad} \varphi$$

$$n = p = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0$$

They constitute a system of non-linear, partial differential equations of the first order, in the unknowns φ , n , p , and \mathbf{D} , \mathbf{J}_n , \mathbf{J}_p . The coefficients are μ_n , μ_p , D_n , D_p . The data are $N(\mathbf{r})$, $N_{ox}(\mathbf{r})$. In turn, U_n , U_p , G_n , G_p can be expressed as functions of the unknowns. By introducing the expressions of \mathbf{D} , \mathbf{J}_n , \mathbf{J}_p into the divergence operator, one obtains a system of PDEs of the second order.

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We remember we have Poisson's equation that is necessary to calculate the electric potential and the Poisson's equation is what's left of Maxwell's equations. Because we have accepted the Quasi-Static Approximation.



$$\frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div} \mathbf{J}_n = G_n,$$

$$\frac{\partial p}{\partial t} + U_p + \frac{1}{q} \operatorname{div} \mathbf{J}_p = G_p,$$

$$\mathbf{J}_n = q\mu_n n \mathbf{E} + qD_n \operatorname{grad} n$$

$$\mathbf{J}_p = q\mu_p p \mathbf{E} - qD_p \operatorname{grad} p$$

Here, We can see the continuity equation

with \mathbf{J}_n represented as $G_n - U_n$

$$G_n - U_n$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n \mathbf{v}_n) = W_n$$

since

$$\mathbf{J}_n = q n \mathbf{v}_n$$

$$n \mathbf{v}_n = \frac{\mathbf{J}_n}{q}$$

q is ~~five~~ for electron

$$\frac{\partial n}{\partial t} - \frac{1}{q} \operatorname{div}(\mathbf{J}_n) = G_n - U_n$$

$$\Rightarrow \frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div}(\mathbf{J}_n) = G_n$$

As shown in the Book



$$\mathbf{J}_n = \underbrace{q\mu_n n \mathbf{E}}_{\text{Drift}} + \underbrace{qD_n \operatorname{grad} n}_{\text{Diffusion}}$$

$$\mathbf{J}_p = q\mu_p p \mathbf{E} - qD_p \operatorname{grad} p$$

→ we may have some fixed charge in the Insulator

Within the insulator region

$$\operatorname{div} \mathbf{D} = q N_{\text{ox}}, \quad \mathbf{D} = -\varepsilon_{\text{ox}} \operatorname{grad} \varphi$$

$$n = p = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0$$

They constitute a system of non-linear, partial differential equations of the first order, in the unknowns φ , n , p , and \mathbf{D} , \mathbf{J}_n , \mathbf{J}_p . The coefficients are μ_n , μ_p , D_n , D_p . The data are $N(\mathbf{r})$, $N_{\text{ox}}(\mathbf{r})$. In turn, U_n , U_p , G_n , G_p can be expressed as functions of the unknowns. By introducing the expressions of \mathbf{D} , \mathbf{J}_n , \mathbf{J}_p into the divergence operator, one obtains a system of PDEs of the second order.

→ For semiconductors we have ^{system} 6 PDE's

because we have partial derivatives
w.r.t. to space because of Divergence
operators and due to the Gradient operators

We also have PD w.r.t. Time

* The unknowns of the equation are

Electric potential



Scalar
Unknowns

Electron Concentration



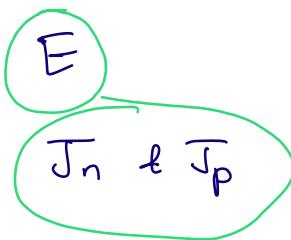
Hole Concentration



& we also have

Electric Field

Current density



→ Then we have coefficients like

μ_n, μ_p, D_n, D_p

→ Data is Dopant profile

$N(r), N_{ox}(r)$

→ U_n, U_p, G_n, G_p which can be expressed in terms of Unknowns



$$\operatorname{div} \mathbf{D} = q(p - n + N), \quad \textcircled{1} \quad \mathbf{D} = -\epsilon_{se} \operatorname{grad} \varphi \quad \textcircled{2}$$

if we substitute $\textcircled{2}$ in $\textcircled{1}$ we get a laplacian which is again scalar and we can eliminate the vectors.

* we can do the same thing with

$$\frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div} \mathbf{J}_n = G_n, \quad \mathbf{J}_n = q\mu_n n \mathbf{E} + qD_n \operatorname{grad} n$$

$$\frac{\partial p}{\partial t} + U_p + \frac{1}{q} \operatorname{div} \mathbf{J}_p = G_p, \quad \mathbf{J}_p = q\mu_p p \mathbf{E} - qD_p \operatorname{grad} p$$

Substitute J_n, J_p in the continuity equations

Note ~~We can reduce the system with~~
6 - 1st order equations to a system

with 3 - 2nd order equations

in which we only have to deal with

ϕ, n, p This is what we follow

to find Numerical Solution of

Equations.

From now we will use this Mathematical Formulations of Semiconductor Devices.

From Now on, we will use these mathematical models to describe the functioning of Semiconductor devices.

→ If we look at the Transport equations which provide current densities.

$$J_n = q \mu_n n E + q D_n \text{grad} n$$

$$J_p = q \mu_p p E - q D_n \text{grad} p$$

They are of the Binomial form

→ It is possible to introduce a manipulation that essentially amounts to changing the unknown functions. To transform Transport Equations into a form called MONOMIAL

* We can do this by introducing Auxillary unknown functions that are called Quasi-Fermi Potentials

These two unknown functions are indicated by

$$\phi_n(r, t), \phi_p(r, t)$$

dependent on space & time

→ We impose that the electron concentration can be written as

Semiconductor Equations — VII

Quasi-Fermi potentials

Define $\varphi_n(r, t), \varphi_p(r, t)$, such that:

$$n = n_i \exp \left[\frac{q(\varphi - \varphi_n)}{k_B T_L} \right],$$

$$p = n_i \exp \left[\frac{q(\varphi_p - \varphi)}{k_B T_L} \right]$$

In equilibrium, $\varphi_n = \varphi_p = \varphi_F$. It is

$$\frac{k_B T_L}{q} \operatorname{grad} n = n \operatorname{grad}(\varphi - \varphi_n), \quad \frac{k_B T_L}{q} \operatorname{grad} p = p \operatorname{grad}(\varphi_p - \varphi)$$

$$\mathbf{J}_n = -q\mu_n n \operatorname{grad} \varphi + q \frac{k_B T_L}{q} \mu_n \operatorname{grad} n = -q\mu_n n \operatorname{grad} \varphi_n$$

$$\mathbf{J}_p = -q\mu_p p \operatorname{grad} \varphi - q \frac{k_B T_L}{q} \mu_p \operatorname{grad} p = -q\mu_p p \operatorname{grad} \varphi_p$$

Q Why do we adopt these new definitions

$$\phi_n(r,t), \phi_p(r,t) ?$$

A If we are in the equilibrium condition we remember ' n ' can be written as

$$n = n_i \exp\left[\frac{q(\phi - \phi_F)}{k_B T_L}\right]$$

Fermi potential

\therefore The two auxiliary functions

$\phi_n(r,t), \phi_p(r,t)$ are new auxiliary unknowns that apply in the general case of the model. They have a property that at

Equilibrium

$$\phi_n = \phi_p = \phi_F$$

Q How are these two additional unknowns useful?

well, by taking expression of ' n ' and take a gradient of it.

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$$n = n_i \exp\left[\frac{q(\phi - \phi_n)}{k_B T_L}\right]$$

$\boxed{\text{grad}(n) = \nabla(n)}$ Similar representation

$$\Rightarrow \text{grad}(n) = n_i \text{ grad} \left[\exp \left[\frac{q(\phi - \phi_n)}{k_B T_L} \right] \right]$$

$$\Rightarrow n_i \exp \frac{q(\phi - \phi_n)}{k_B T_L} \cdot \frac{q}{k_B T_L} \text{ grad}(\phi - \phi_n)$$

$$\Rightarrow \text{grad}(n) = n \cdot \frac{q}{k_B T_L} \text{ grad}(\phi - \phi_n)$$

$$\Rightarrow \boxed{\frac{k_B T_L}{q} \cdot \text{grad}(n) = n \cdot \text{grad}(\phi - \phi_n)}$$

Similarly

$$\boxed{\frac{k_B T_L}{q} \cdot \text{grad}(p) = p \text{ grad}(\phi_p - \phi)}$$

\Rightarrow Now, we simply replace the expression
of $\text{grad}(n)$ into the Drift-Diffusion
equation.

we have from above

$$J_n = q \mu_n n \varepsilon + q D_n \text{grad} n$$

&

$$D_n = \mu_n \frac{k_B T_L}{q}$$

$$\varepsilon = -\text{grad} \phi$$

put $\text{grad}(n)$ & D_n into the current density equation.

$$J_n = -q \mu_n n (\text{grad} \phi) + q \underbrace{\frac{k_B T_L}{q} \mu_n}_{I} \text{grad}(n)$$

$$\Rightarrow J_n = -q \mu_n n (\text{grad} \phi) + q \mu_n \underbrace{\frac{k_B T_L}{q} \text{grad}(n)}_{n \text{ grad}(\phi - \phi_n)}$$

$$J_n \Rightarrow -q \mu_n n \text{grad}(\phi) + q \mu_n n \text{grad}(\underbrace{\phi - \phi_n}_{\downarrow})$$

$$\Rightarrow -q \mu_n n \text{grad}(\phi) - q \mu_n n \text{grad}(\phi_n - \phi)$$

$$\Rightarrow -q \mu_n n \text{grad}(\phi + \phi_n - \phi)$$

$$\mathbf{J}_n \Rightarrow -q\mu_n n \operatorname{grad} \phi_n$$

ప్రాంతము కిణి విభజన అన్న సూక్షు
గమనించలేదు. ఈ SDC subject లోనే
ఉంటే.

SDC ఈ subject ను లభించుటకు
ఏస్ ప్రాంతము.

$$\begin{matrix} \mathbb{B} & \text{Rudan Sir} & \mathbb{B} \mathbb{B} & \text{Rudan} \\ e & & e e & \text{Sir} \end{matrix}$$

→ This is the expression of current density
in a Monomial form as if $-\operatorname{grad}(\phi_n)$
is a pseudo Field: embedding both
drift and diffusion components of
the Current Density.

→ Similarly we can do it Holes

Q

When are there expressions Metal?

They are useful in devices in which one type of current is Dominant which typically happens in MOS Devices.

* For Unipolar Devices : It is preferable to calculate the current using the Quasi Fermi potential
(Dominant carriers)

Bipolar Devices : It is preferable to use customary definition of Drift Diffusion equation.
(Both carriers contribute)

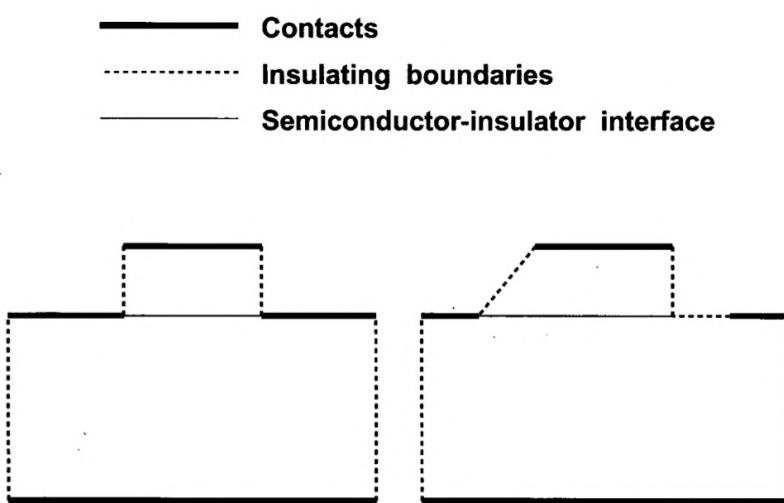
→ Other issue related to Mathematical model of Semiconductor Devices is offcourse the issue of Boundary Condition Because we have Equations that contain the unknown functions and derivatives wrt space.

They also contain derivatives w.r.t.
Time.

* For derivatives w.r.t TIME we must provide Initial conditions every time

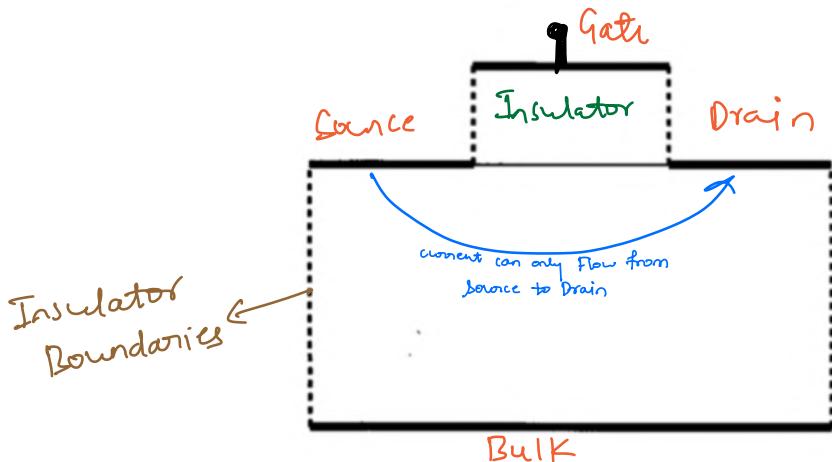
In Space, we have to prescribe the Boundary conditions.

→ To discuss this issue we make use of Typical example of an **MOS Transistor**.

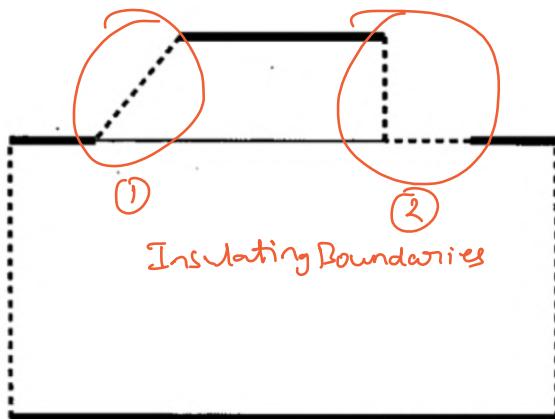




This is very schematic



Realistic Representation



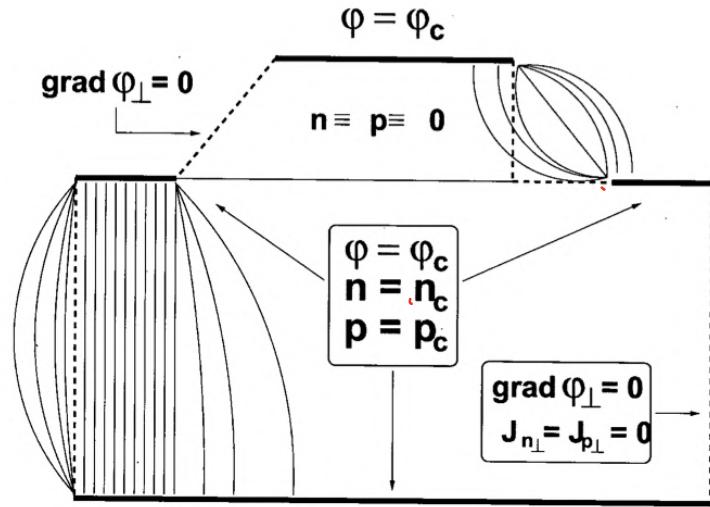
with short Source & Drain contacts

Then insulating boundaries ^{for oxide} has two types

But, we will see that Slope ① is the

correct one to describe Insulator Boundary Condition.

**



CAP.

25: MODELLO MATEMATICO DEI DISPOSITIVI

487

T. 25.25: Esempio di condizioni al contorno per il modello matematico.

We remember when we consider a semiconductor device model.

* we can group the equations in order to have a three 2nd order equations w.r.t space

(i) First Equation

The 1st equation has unknown the electric potential ϕ . This is the poisson's equation.

(ii) Second Equation

It has unknown that's the

electron concentration and

(iii) Third Equation

has unknown the hole
concentration.

→ let's start with Poisson's Equation
which is the 2nd order PDE with
(ϕ) Electric potential as Unknown.

→ we remember from Calculus that when
we have a 2nd order equation over
domain - The mathematical problem is
well posed if the Boundary conditions
are such that the solution exists and
is unique.

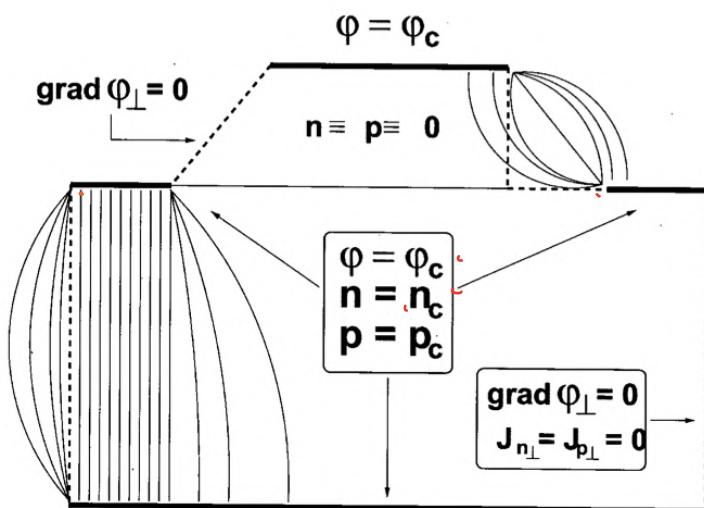
→ A type of Boundary Conditions that
are suitable for Poisson's equation is
like this.

(i) we must prescribe the

value of the unknown along the portion
of a boundary and the normal derivative
of the unknown along the remaining
portion.



Can we do this without
example
of MOS device?



CAP. 25: MODELLO MATEMATICO DEI DISPOSITIVI

T. 25.25: Esempio di condizioni al contorno per il modello matematico.

487

— Contacts
- - - Insulating Boundaries

→ At the contacts we have electric

potentials because we have voltage generators typically applied to the contacts.

The value of Electric potential at the contact is prescribed.

- This is a Good Boundary Condition
Because, The Electric potential is the unknown of the equation and it's prescribed along the contact.
- The contacts are not made of Semiconductors. They are typically made of METALS. There are interfaces b/w Metal & Semiconductor & Metal and Insulator if we consider Gate Contact.
- When we have interface b/w two materials we have a change in Electric potential. → This change is sometimes referred to as the difference in the work functions

of the two materials.

If we take Drain contact and we apply 5V to the semiconductor underneath. Drain contact has the potential that is not equal to 5V. It is equal to

$$5V - \text{Difference in the work function}$$

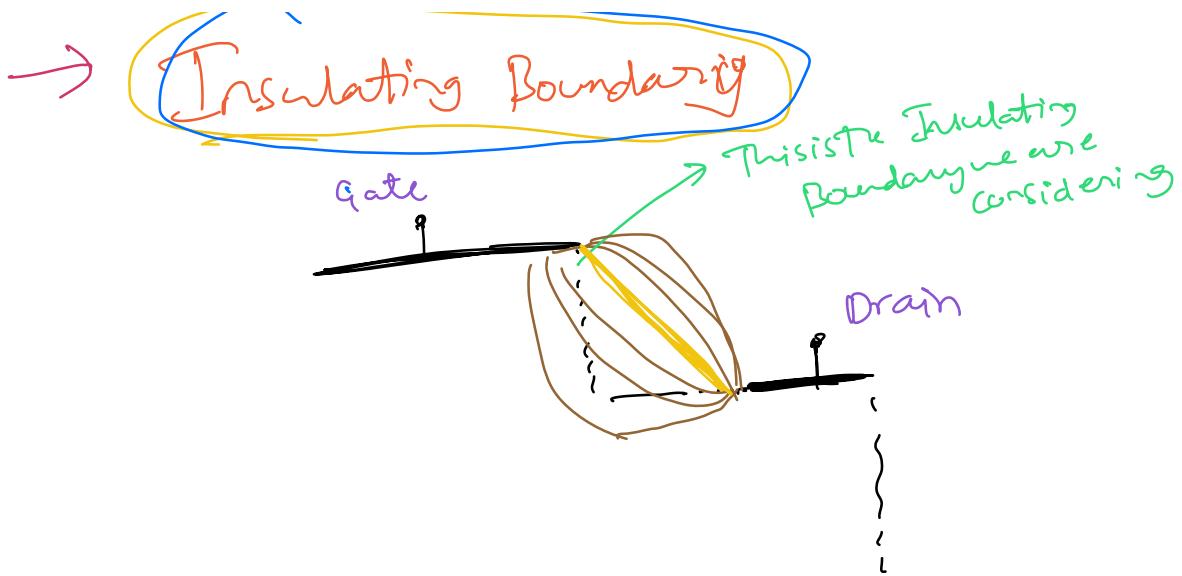
→ But, this difference is given that is known. So it is possible from the contact voltage to also deduce the voltage of the semiconductor that is attached to the contact.

→ So, for contacts we do have a good Boundary Conditions.

and this is indicated as

$$\phi = \phi_c \quad \text{in the figure}$$

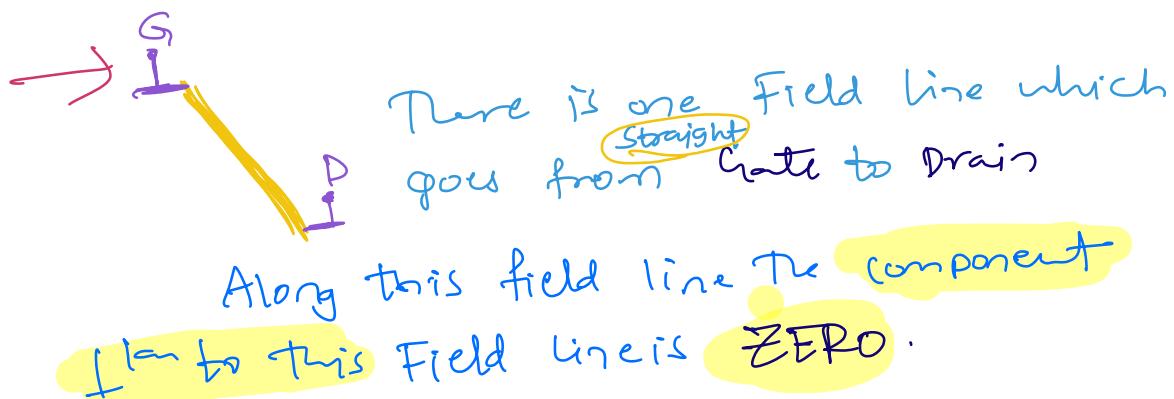
The value of the electric potential of semiconductor underneath the contact is given by the electric potential of the contact.



Assume we apply $2V$ to the Drain

$3V$ to the Gate

The thin lines that we can see are the **Field Lines** in that Region. The start from Gate & close at the **Drain contact**.



i.e. Electric Field is Fully aligned along the straight line.

जो रेखा उसके लिए सीधी है। जो इसके लिए वर्तमान में उपयोग किया जाता है।

स्ट्रेस एड डैग लार्वा लोड एंड कॉम्प्रेस
वर्सिटी एंड बी. # solo

→ It is preferable to select the Boundary along this straight line for the Insulator b/w Gate & drain.

$\text{grad } \phi_{\perp} = 0$

This is also a Good Boundary condition.

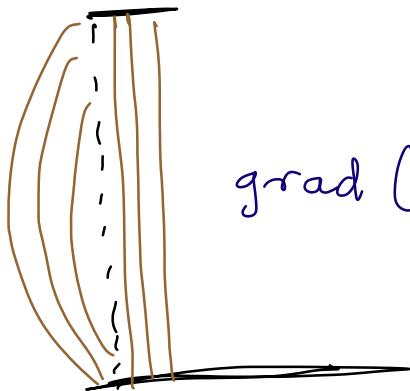
∴ Electric field normal to the straight line is zero. It means that the gradient of Normal component of potential is zero.

→ In mathematical nomenclature prescribing the function at the boundaries is also called as Dirichlet Boundary condition.

Also, prescribing the Normal Derivative at the boundary is also called Neuman Boundary condition.

→ The same analysis is possible along the dashed lines that describe the

Boundaries of the silicon Region



$$\text{grad } (\phi_L) = 0$$

As for the potential we can state that the electric potential is given at the contacts this constitutes a 'Dirichlet's Boundary condition'

$$\text{i.e. } \phi = \phi_C$$

and that we are able to select the insulating boundaries in such a way as to provide homogeneous Neumann condition on those boundaries.

* For the Electric Potential the problem is well posed.

Now we have to discuss the Boundary conditions of the Concentrations - n + p

If, we look at the Oxide insulator

carrier concentrations are identically zero inside the oxide.

For, oxide region there is nothing to solve.



Now, we will discuss what happens to the concentrations of n & p at the contacts Source & Drain and Bulk. Also what happens to the normal derivatives of n & p at the Two insulating Boundaries of the Silicon Region. ?

For contacts: The solution is relatively easy because, the contacts are made of Metals & they are rich of free charge. They have plenty of electrons in them.

If we consider the semiconductor regions that are immediately attached to the contact. If that semiconductor region in some sense tries to depart from the conditions of charge neutrality i.e. charge density

tries to become Non-zero at the contacts

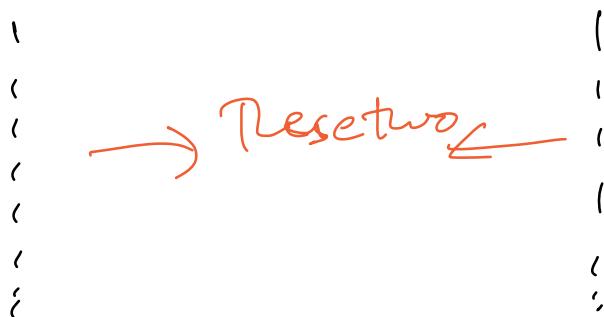
the contact is immediately able to supply charge in order to restore Charge Neutrality.

→ So, we have a condition in which near the Source, Drain & Bulk we may impose a condition of Equilibrium and charge Neutrality to the semiconductor near the contact.

→ If we have these two conditions we can calculate the value of n & p near the Contacts.

For n & p we have Dirichlet's condition at the Contact.

Now, only thing that's left is to discuss the n & p at the Insulating Boundary.



It is obvious that No current crossed the insulating boundary.

So, we may say that Normal component of J_n & J_p equal to zero.

$$J_{n\perp} = J_{p\perp} = 0$$

At the insulating boundaries of semiconductor

T. 25.26: Condizioni al contorno per i contorni isolanti.

$E = E \hat{\nu}$: Electric Field From Source to Bulk
 $\hat{\nu}$: unit vector Normal to the Insulating Boundary

Boundary Conditions for the DD Equations — I

Insulating Boundaries

The geometry of the problem to be solved can always be taken such that at the insulating boundaries $\Gamma_{i1}, \Gamma_{i2}, \dots$ it is

Since E & $\hat{\nu}$ are \perp their Dot product is zero.

$$E \bullet \hat{\nu} = 0 \Rightarrow \frac{\partial \varphi}{\partial \nu} = 0, \quad r \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

At the insulating boundaries it is also $J_n \bullet \hat{\nu} = J_p \bullet \hat{\nu} = 0$ whence, combining with the above,

$$-q\mu_n n \frac{\partial \varphi}{\partial \nu} + qD_n \frac{\partial n}{\partial \nu} = 0 \Rightarrow \frac{\partial n}{\partial \nu} = 0, \quad r \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

and similarly $\partial p / \partial \nu = 0$, $r \in \Gamma_{i1}, \Gamma_{i2}, \dots$. In conclusion, at the insulating boundaries the boundary condition is the same for all the scalar unknowns φ, n, p :

$$\frac{\partial \varphi}{\partial \nu} = 0, \quad \frac{\partial n}{\partial \nu} = 0, \quad \frac{\partial p}{\partial \nu} = 0, \quad r \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

i.e., of homogeneous Neumann type.

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* We have

$$\vec{E} \cdot \vec{J} = 0$$

We know

$$\vec{E} = E = -\nabla \phi$$

It can also be written as

$$\vec{E} = -\nabla \phi = -\nabla \phi$$

$\vec{E} = E =$ electric field from

Source to Bulk

(or)

Drain to Bulk

$\vec{v} =$ unit vector \perp to
to the Insulating
Boundaries

$$\nabla = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right)$$

$$\vec{E} = -\nabla \phi = - \left(\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k} \right)$$

Now

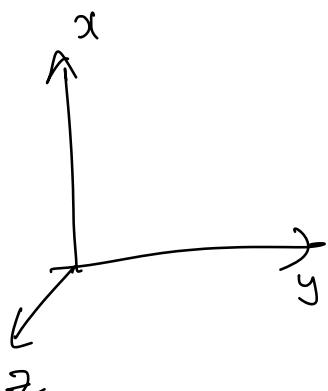
$$\vec{E} \cdot \vec{J} = 0 \text{ implies}$$

$\therefore \vec{v}$ is unit vector \perp to
the Insulating Boundaries

Assume The device in 3D

& \vec{J} should be along y -axis

$\therefore \vec{v}$ is unit vector along y -axis



$$\vec{v} = \hat{j}$$

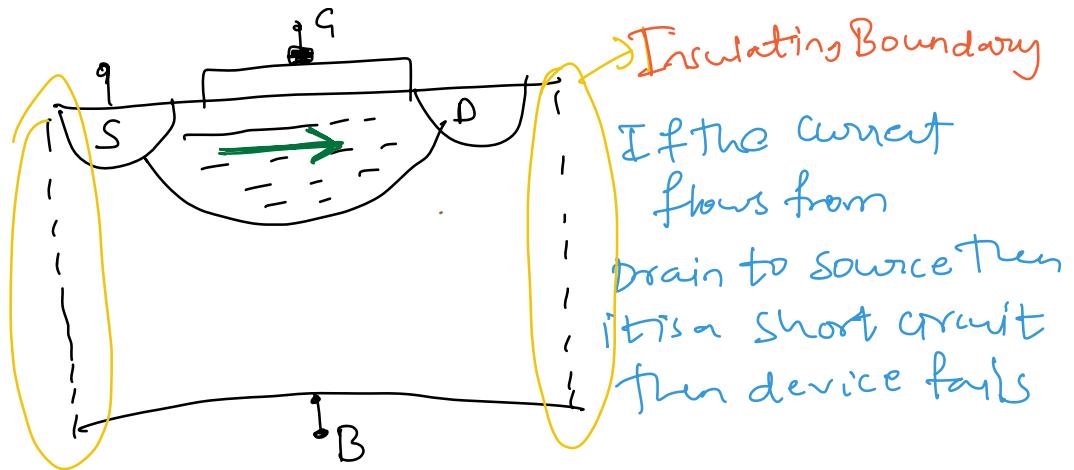
Now

$$\begin{aligned} E \cdot \vec{v} = 0 &\Rightarrow -\left(\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}\right) \cdot \vec{v} \\ &\Rightarrow -\left(\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}\right) \cdot \hat{j} \\ &\Rightarrow -\frac{\partial \phi}{\partial y} (\hat{j} \cdot \hat{j}) \end{aligned}$$

$$\begin{matrix} \hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \\ \hat{j} \cdot \hat{k} = 0 \end{matrix}$$

$$\begin{aligned} &\Rightarrow -\frac{\partial \phi}{\partial y} = 0 \\ \therefore \vec{v} &= \hat{j} \quad \text{dy = dv} \\ &\Rightarrow \frac{\partial \phi}{\partial y} = 0 \\ &\Rightarrow E \cdot \vec{v} = 0 \quad \text{This equation represents variation w.r.t SPACE.} \end{aligned}$$

Note:- In a MOS Device the current should always flow from Source to Drain and only in the channel.



There is small conduction happens in bulk because of minority charge carriers and it quite small

$$-q\mu_n n \frac{\partial \phi}{\partial V} + q D_n \frac{\partial n}{\partial V} = 0 \Rightarrow \frac{\partial n}{\partial V} = 0$$

For an n-channel MOSFET with Bulk p-type

The current from source to Bulk

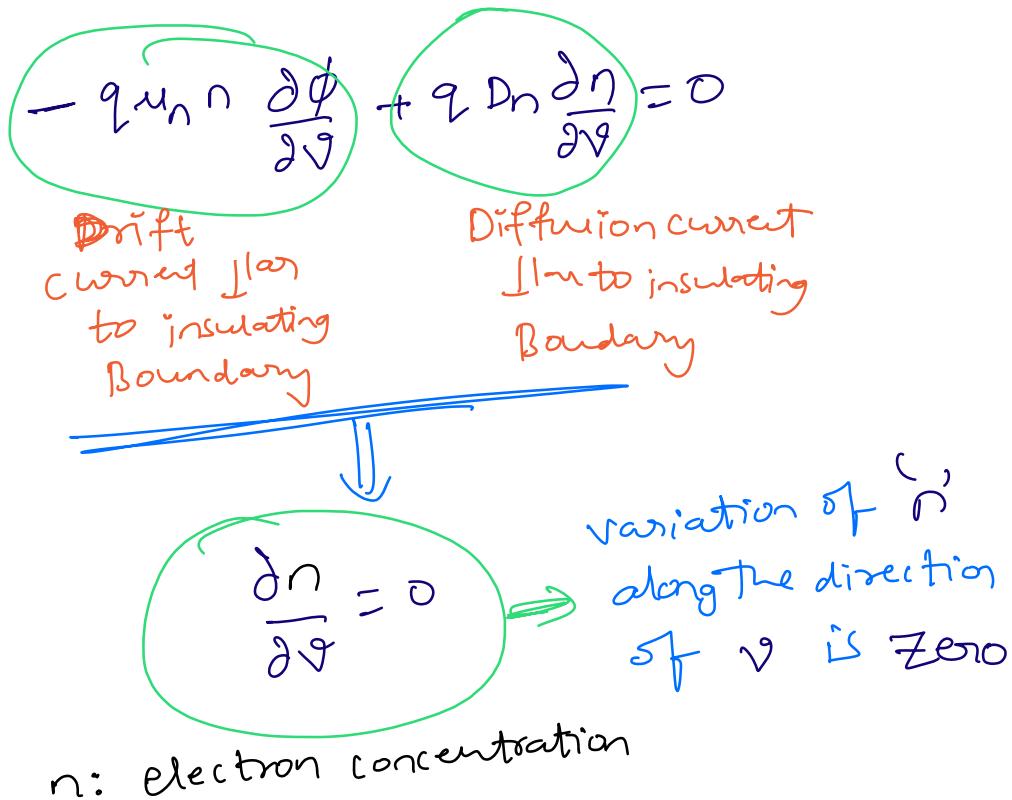
Drain to Bulk

is only from Minority Charge carriers
in this case Electrons of p-type Bulk.

Importantly: Source-Gate } True two Junction
Drain - Gate } are Reverse Biased

That's why only current due to minority carriers.

∴ Drift diffusion current from Drain to Bulk is due to Electrons



Boundary Conditions for the DD Equations — I

Insulating Boundaries

The geometry of the problem to be solved can always be taken such that at the insulating boundaries $\Gamma_{i1}, \Gamma_{i2}, \dots$ it is

$$\mathbf{E} \bullet \mathbf{\nu} = 0 \Rightarrow \frac{\partial \varphi}{\partial \nu} = 0, \quad \mathbf{r} \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

At the insulating boundaries it is also $\mathbf{J}_n \bullet \mathbf{\nu} = \mathbf{J}_p \bullet \mathbf{\nu} = 0$ whence, combining with the above,

$$-q\mu_n n \frac{\partial \varphi}{\partial \nu} + qD_n \frac{\partial n}{\partial \nu} = 0 \Rightarrow \frac{\partial n}{\partial \nu} = 0, \quad \mathbf{r} \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

and similarly $\partial p / \partial \nu = 0, \mathbf{r} \in \Gamma_{i1}, \Gamma_{i2}, \dots$. In conclusion, at the insulating boundaries the boundary condition is the same for all the scalar unknowns φ, n, p :

$$\boxed{\frac{\partial \varphi}{\partial \nu} = 0}, \quad \boxed{\frac{\partial n}{\partial \nu} = 0}, \quad \boxed{\frac{\partial p}{\partial \nu} = 0}, \quad \mathbf{r} \in \Gamma_{i1}, \Gamma_{i2}, \dots$$

i.e., of homogeneous Neumann type.

This whole calculation
is related to
the Insulating
Boundary condition
keep that in Mind -

Conclusion ϕ , n & p have

normal derivative equal to ZERO

at the insulating Boundaries.

i.e Homogenous Neumann
Condition

In other words Normal Derivative along
the unit vector \vec{n} which is
 \perp to the insulating Boundary.

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Another important thing we have to do is describing the coefficients of the mathematical model.

They are

D_n, D_p → Diffusion coefficients

U_n, U_p → Generation & Recombination coefficients

G_n, G_p

→ We have also learned that generation & recombination is of the Thermal & Non-Thermal events separately

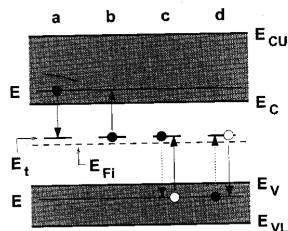
Capitolo 26



Generazione-ricombinazione

Net Thermal Recombination Rate — II

Shockley-Read-Hall theory



$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t,$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t),$$

where N_t is the concentration of traps and P_t is the occupation probability of a trap.

Semiconductor Equations — VI

Within the semiconductor region

Six different 1st order equations that needs to be solved with different unknowns

$$\operatorname{div} \mathbf{D} = q(p - n + N), \quad \mathbf{D} = -\varepsilon_{sc} \operatorname{grad} \varphi \quad (1)$$

$$\frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div} \mathbf{J}_n = G_n, \quad \mathbf{J}_n = q\mu_n n \mathbf{E} + qD_n \operatorname{grad} n \quad (2)$$

$$\frac{\partial p}{\partial t} + U_p + \frac{1}{q} \operatorname{div} \mathbf{J}_p = G_p, \quad \mathbf{J}_p = q\mu_p p \mathbf{E} - qD_p \operatorname{grad} p \quad (3)$$

Within the insulator region

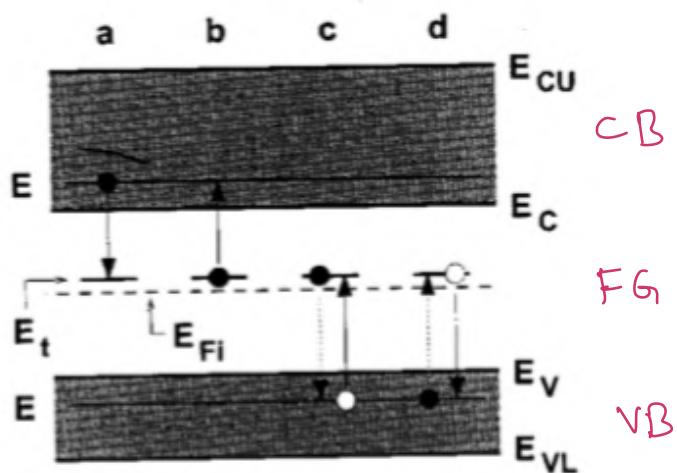
$$\operatorname{div} \mathbf{D} = q N_{ox}, \quad \mathbf{D} = -\varepsilon_{ox} \operatorname{grad} \varphi$$

$$n = p = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0$$

They constitute a system of non-linear, partial differential equations of the first order, in the unknowns φ, n, p , and $\mathbf{D}, \mathbf{J}_n, \mathbf{J}_p$. The coefficients are μ_n, μ_p, D_n, D_p . The data are $N(\mathbf{r}), N_{ox}(\mathbf{r})$. In turn, U_n, U_p, G_n, G_p can be expressed as functions of the unknowns. By introducing the expressions of $\mathbf{D}, \mathbf{J}_n, \mathbf{J}_p$ into the divergence operator, one obtains a system of PDEs of the second order.

* This above figure refers to the Thermal generation & Recombination events. We have already had some short discussion of this issue at much earlier stage when we were discussing the FORM of the Bands.

Now, we go into it in some more detail.



CB : conduction Band

FG : Forbidden Gap

VB : valence Band

→ It may happen that inside the ^(FG) gap of the semiconductor. There are energy

States that are due to existence of some defect of the semiconductor, like Impurities, Defects in the lattice etc.

It is impossible to have semiconductor that is completely free of defects.

→ Defects introduced by Dopants are very special, because states introduced due to Donor dopants are close to CB and the states that are introduced due to Acceptor dopants are close to VB.

Here, we are not considering above states. We are only considering other type of defects such that Energy states introduced by them are near the Middle of FB.

i.e. close to

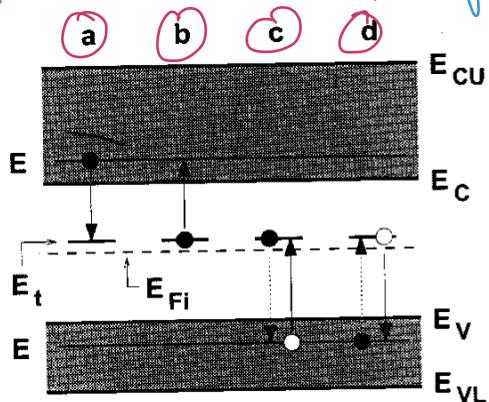
E_F Intrinsic Fermi Level

→ Now, let's consider the transition of one electron from conduction band into the valence band. It is called a Recombination event.

→ We have studied that recombination event may be Direct i.e. e^- jumps from CB to VB, releasing total energy of jumping in one shot. This is called Direct Recombination.

→ The opposite phenomenon is which the e^- in VB absorbs enough Energy to jump into the CB. This is called Direct Generation of Electron-Hole pair

→ There is another phenomenon i.e. also possible. If there is a Trap



case (a) The electron in CB. This electron may release to the lattice enough energy to jump into the Trap with Energy level E_T (Trap energy)

This electron stays for a while in the Trap after if you look at the event called

(c) This e^- makes a 2nd transition

from Trap into the VB

→ So, Globally if event (a) followed by event (c) globally is a Recombination event.

This is called Trap Assisted Recombination.

→ The inverse phenomenon is also possible an e^- initially in VB absorbs energy from the Vibrations of the lattice and goes into the Trap. This event (D)
After this absorbs another block of

energy and goes up into the conduction band
this is called event of Type ①

① followed by ② is called
Generation of an electron
hole pair.

→ All of these events are thermally activated
i.e exchange of energy is with in the
vibrating nuclei of the lattice.

→ ③ which of these events are more probable
Trap Assisted ones vs Direct ones ?

Ans trap assisted ones are MUCH MUCH
likely to occur to the extent that
when we consider thermal generation
and recombination we neglect the
direct events. Because their probability
is negligible.

→ For Description of the Thermally Assisted Generation and Recombination Events it is sufficient to consider only one type of traps. (All of them with same Energy)

This is simplified description but we shall see at the end that it's sufficient

→ But, we remember that when we start discussing about Energy states of the Gap in principle we have two types of them one is Donor Type & The other type is Acceptor Type. like in the Dopants These are not due to Dopants but they behave like Dopants.

They acquire or Release Electrons

→ The Donor Traps have the property that they are Neutral when they are filled with Electron and they are +ve when they are Empty.

→ In contrast the Acceptor traps are neutral when empty and ~~are~~ when filled.

→ So in principle we should consider two more populations those of particles that are trapped in the Acceptor Type and particles that are trapped in Donor type traps

→ Therefore instead of considering only two populations electrons of the CB & holes of the VB, for a moment will consider 4 populations

(i) Electrons of CB

(ii) Holes of VB

(iii) population of Acceptor traps
(considered as electrons)

(iv) population of Donor traps
(considered as populated with holes)

→ In this case we will not write TWO but FOUR continuity equations. and we shall simplify them almost immediately



Net Thermal Recombination Rate — I

Continuity equations for bands and traps

$$\begin{aligned} \frac{\partial n}{\partial t} + U_n - \frac{1}{q} \operatorname{div} \mathbf{J}_n &= G_n, & \frac{\partial p}{\partial t} + U_p + \frac{1}{q} \operatorname{div} \mathbf{J}_p &= G_p \\ \frac{\partial n_a}{\partial t} + U_{na} - \frac{1}{q} \operatorname{div} \mathbf{J}_{na} &= G_{na}, & \frac{\partial p_d}{\partial t} + U_{pd} + \frac{1}{q} \operatorname{div} \mathbf{J}_{pd} &= G_{pd} \end{aligned}$$

Acceptor Traps (Electrons) Donor (Holes)

Let $G_n = G_p = G_{na} = G_{pd} = 0$. Adding over the current densities and observing that $\partial N(\mathbf{r})/\partial t = 0$ yields

$$\begin{aligned} \frac{\partial [q(p + p_d - n - n_a + N)]}{\partial t} + \operatorname{div}(\mathbf{J}_p + \mathbf{J}_{pd} + \mathbf{J}_n + \mathbf{J}_{na}) &= \\ = U_n + U_{na} - (U_p + U_{pd}) &= 0 \quad \Rightarrow \quad U_n + U_{na} = U_p + U_{pd}. \end{aligned}$$

In crystalline silicon it is $\mathbf{J}_{pd} \simeq 0, \mathbf{J}_{na} \simeq 0$:

$$\frac{\partial n_a}{\partial t} = -U_{na}, \quad \frac{\partial p_d}{\partial t} = -U_{pd}.$$

In steady-state conditions this yields $U_{na} = U_{pd} = 0 \Rightarrow$

$$U_n = U_p \doteq U.$$

In equilibrium all continuity equations reduce to the identity $0 = 0$, whence $U_n = U_{na} = U_p = U_{pd} = 0$.

Here, we have four sets of continuity Equations for Bands & Traps.

n_a : Acceptor traps i.e electrons

J_{na} : current Density because of electrons

G_{na} : Generation Term

U_{na} : Recombination Term

→ All these equations are used to determine the expression of Thermal Recombination Term.

so, we simplify the model by considering that there are no Non-Thermal Events

i-e $G_n = G_p = G_{na} = G_{pd} = 0$

→ After this we will add all the 4 equations by taking holes with +ve sign and equations of electrons with -ve sign

$$\frac{\partial [q(p + p_d - n - n_a + N)]}{\partial t} + \text{div}(\mathbf{J}_p + \mathbf{J}_{pd} + \mathbf{J}_n + \mathbf{J}_{na}) = \\ q(U_n + U_{na}) - q(U_p + U_{pd}) = 0 \Rightarrow U_n + U_{na} = U_p + U_{pd}.$$

* Now we have a Global Continuity equation in which we have Time derivative of all contributions to charge density

We have population of holes, electrons of Band & traps.

i.e P, P_d, n, n_a, N
 Concentration of
 Dope

$$\therefore \frac{\partial q}{\partial t} [P + P_d - n - n_a + N]$$

Time derivative of the global charge density with all contributions present.

* $\operatorname{div} (\overline{J_p} + \overline{J_{pd}} + \overline{J_{na}} + \overline{J_h})$

The divergence operator also embeds all possible currents.

So This equation is The continuity equation

Directly from Maxwell's equation ∴

it must be zero.

$$\frac{\partial q}{\partial t} [P + P_d - n - n_a + N] + \operatorname{div} (\overline{J_p} + \overline{J_{pd}} + \overline{J_n} + \overline{J_{n_a}}) = 0$$

$$\Rightarrow q(v_n + v_{n_a}) - q(v_p + v_{p_d}) = 0$$

$$\Rightarrow U_n + U_{na} = U_p + U_{pd}$$

This is a very fundamental result which can be found by reasoning. Instead of doing a calculation.

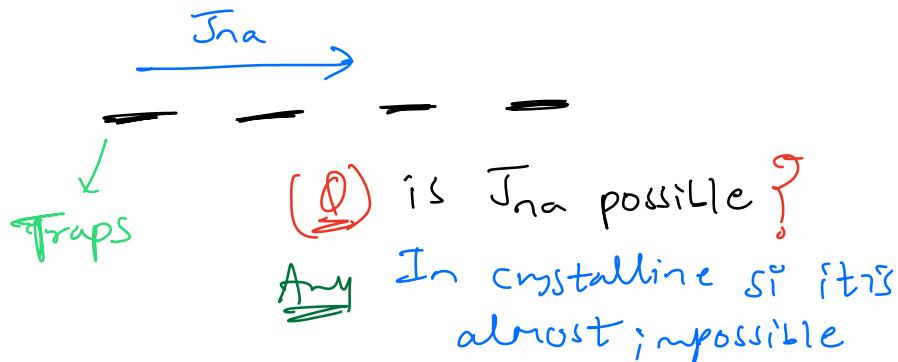
- All these transitions are transition of e^- 's on the left & the transitions of holes on the right.

Transition of electrons must be Transition of holes in opposite way so they must be equal.

- we will introduce a simplification by observing nature of J_{pd} , J_{na}

J_{pd} :

J_{na} : current along the traps



Because the only mechanism by which one electron can move from one Trap to another is the ~~Quadrupole~~ effect called the **Tunnel effect**. The Tunnel effect becomes significant if the traps are closer to each other.

In crystalline Si traps are not so many
 \therefore relative distance b/w traps is large
and this prevents Tunnel effect from occurring

In polycrystalline silicon there are many traps & Tunnel effect is very possible & Non negligible

\therefore we neglect J_{pd} , I_{na} & we have already neglected $q_{na} + q_{pd}$

$$\Rightarrow \frac{dn_a}{dt} + U_{na} = 0$$

$$\Rightarrow \left[\frac{dn_a}{dt} = -U_{na} \right] \text{ and } \left[\frac{dP_d}{dt} = -U_{pd} \right]$$

→ In steady state condition nothing depends on time

$$U_{nr} = U_{pd} = 0$$

because

$$\frac{dn}{dt} = 0$$

$$\frac{dP_d}{dt} = 0$$

Steady state

$$U_n = U_p = U$$

A common symbol
for Thermal Recombination
of electron & hole

* Now, the issue is to calculate the expression for U (Net Thermal Recombination rate)

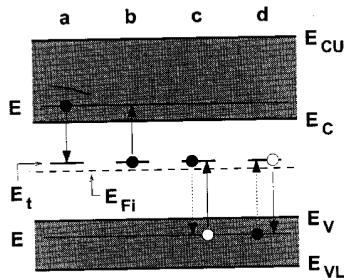
The calculation that provides for the net thermal recombination rate U is due to

SHOCKLEY - READ - HALL THEORY

what we do is treating separately the possible events that concur to determining $U_n + U_p$

In the Figure we assume that there is only one type of traps. The traps have same energy E_t

Shockley-Read-Hall theory



$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t,$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t),$$

where N_t is the concentration of traps and P_t is the occupation probability of a trap.

The Concentration of Traps have a concentration in space i-e N_t

Then we distinguish first event ①

i-e CB to Trap

→ we know electrons don't like to stay together
so for these electrons to make these
Transitions The Trap should be Empty

If the Trap is filled it's impossible for electron
from CB to make this Transition

→ Then we have Transition event ②

→ from Trap to CB, obviously

Total event of Type ① + Total event of

Type b permit volume in time giving you the Net Recombination Rate of the Conduction Band

- These are events of c in which one electron initially belonging to a trap makes a transition into the empty state of the valence band. This is also described as Transition in the opposite direction of a hole (i.e. hole initially in the valence band transitions into a state)
 - Finally, we have an electron in the VB going into an empty trap ^{or} we describe as opposite transition of a hole from the empty trap into the band.
- ∴ events c & d per unit volume in time provide the net recombination rate of the VB

Now, let's give some names to the rates.

$$r_a = \text{No. of events of Type a} / \text{unit volume in time}$$

Similarly r_b, r_c, r_d are described

$$\therefore U_n = r_a - r_b$$

$$U_p = r_c - r_b$$

Remember we are in a Nonequilibrium condition.

In equilibrium $U_n = U_p = 0$ because continuity equations are identically zero.

So the probability of occupation of the electron state cannot be described using Fermi statistics.

The probability of occupation is unknown.

→ Specifically P_t is the probability of electron occupying the traps

P_t is Unknown in Non-equilibrium condition.



$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t,$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t),$$

where N_t is the concentration of traps and P_t is the occupation probability of a trap.

What we will do is find individual expressions for $\tau_a, \tau_b, \tau_c, \tau_d$ and will eventually derive V_n, V_p

→ We find the expressions for $\tau_a, \tau_b, \tau_c, \tau_d$ heuristically.

τ_a : no. of transitions of type a for unit volume in time

No. of electrons that are initially present in the conduction band

→ One may argue that no. of transitions would be proportional to the no. of electrons that are present in the conduction band.

$$\therefore \tau_a \propto n \left(\begin{array}{l} \text{concentration of} \\ \text{electrons in CB} \end{array} \right)$$

→ But also, it is intuitive that no. of transitions are more if the traps are larger.

If we have no traps we have no transitions.

$$\therefore \gamma_a \propto \frac{N_t(1-p_t)}{\text{signifies the traps that are empty}}$$

we have

$$\gamma_a \propto n$$

$$\gamma_a \propto N_t(1-p_t)$$

$$\Rightarrow \boxed{\gamma_a = \alpha_n \cap N_t(1-p_t)}$$

↓
transition coefficient

$\alpha_n \rightarrow$ it describes the interaction
b/w the electron at the trap.

* γ_b : No. of transitions of type b
From the logical reasoning we followed

in the earlier

$$\tau_b \propto N_t P_t$$

No. of filled traps

$\Rightarrow \tau_b \propto$ probability of empty state in CB
which is practically equal to one.

$$\tau_b = e_n N_t P_t$$

Emission coefficient

Similarly for τ_c & τ_d we follow the same logic.

$$\Rightarrow \tau_c = \alpha_p P N_t P_t$$

Hole from VB to Trap

$$\tau_d = e_p N_t (1 - P_t)$$

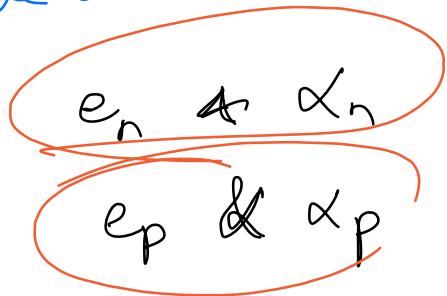
Hole from Trap to VB

$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t,$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t),$$

if $\alpha_n, \alpha_p, e_n, e_p$ are known then
we could easily calculate U_n, U_p

We also have a relationship b/w



→ At equilibrium condition

$$U_n = U_p = 0$$

∴ at equilibrium $r_a = r_b$ &
 $r_c = r_d$

→ On the other hand at equilibrium

n & p are known because

they are equilibrium carrier concentrations of e^- 's & holes

P_t

becomes Fermi Statistics
at Equilibrium

→ This allows us to find relation b/w emission coefficient and Transition coefficient.

$$U_n = \gamma_a - \gamma_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t = 0$$

At equilibrium

$$\Rightarrow \alpha_n n N_t (1 - P_t) = e_n N_t P_t$$

$$\Rightarrow e_n P_t = \alpha_n n^{eq} (1 - P_t)$$

$$e_n = \alpha_n n^{eq} \left(\frac{1}{P_t} - 1 \right)$$

Similarly

$$\alpha_p P^{eq} = e_p \left(\frac{1}{P_t} - 1 \right)$$

Net Thermal Recombination Rate — III

Shockley-Read-Hall theory

In equilibrium it is $U_n = U_p = 0 \Rightarrow$

$$e_n = \alpha_n n^{\text{eq}} \left(\frac{1}{P_t} - 1 \right), \quad \alpha_p p^{\text{eq}} = e_p \left(\frac{1}{P_t} - 1 \right).$$

$$P_t = \frac{1}{(1/d_t) \exp[(E_t - E_F)/(k_B T_L)] + 1} \Rightarrow$$

$$\frac{1}{P_t} - 1 = \frac{1}{d_t} \exp[(E_t - E_F)/(k_B T_L)]$$

$$e_n = \alpha_n \frac{n^{\text{eq}}}{d_t} \exp[(E_t - E_F)/(k_B T_L)] = \alpha_n n_B$$

$$e_p = \alpha_p p^{\text{eq}} d_t \exp[(E_F - E_t)/(k_B T_L)] = \alpha_p p_B$$

α_n & α_p
aren't
independent

In steady-state conditions $U_n = U_p = U \Rightarrow r_a - r_b = r_c - r_d :$

$$\alpha_n n (1 - P_t) - \alpha_n n_B P_t = \alpha_p p P_t - \alpha_p p_B (1 - P_t).$$

One finds

$$P_t = \frac{\alpha_n n + \alpha_p p_B}{D}, \quad 1 - P_t = \frac{\alpha_n n_B + \alpha_p p}{D}$$

$$\text{with } D = \alpha_n (n + n_B) + \alpha_p (p + p_B)$$

E_t : Trap Energy (we assume to know the position of the traps)

d_t : Degeneracy coefficient

n^{eq} : Electron concentration at equilibrium

$$n_B = \frac{n^{\text{eq}}}{d_t} \exp\left[\left(\frac{E_t - E_F}{k_B T_L}\right)\right]$$

Similarly P_B is defined

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→ we know in Non-Equilibrium Condition

$$U_n = U_p$$

There is a provision that the relation between α_n & α_p have been found in Equilibrium condition Q. Are they still valid in Non-Equilibrium Condition?

It can be shown that approx. they are valid.

→ Now in Non-Equilibrium Condition we take

$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t)$$

$$\text{so } U_n = U_p$$

In steady-state conditions $\overbrace{U_n = U_p = U} \Rightarrow r_a - r_b = r_c - r_d$:

$$\alpha_n n (1 - P_t) - \alpha_n n_B P_t = \alpha_p p P_t - \alpha_p p_B (1 - P_t).$$

One finds

$$\boxed{P_t = \frac{\alpha_n n + \alpha_p p_B}{D}} \quad 1 - P_t = \frac{\alpha_n n_B + \alpha_p p}{D}$$

$$\text{with } D = \alpha_n (n + n_B) + \alpha_p (p + p_B)$$

$$P_t = \frac{\alpha_n n + \alpha_p P_B}{D}$$

expression for probability
of occupation of
trap

so we can express P_t without introducing
additional unknowns into the Model.

$$1-P_t = \frac{\alpha_n n_B + \alpha_p P}{D}$$

Then rewrite P_t & $(1-P_t)$ and replace
them back into the below expressions

$$U_n = r_a - r_b = \alpha_n n N_t (1 - P_t) - e_n N_t P_t,$$

$$U_p = r_c - r_d = \alpha_p p N_t P_t - e_p N_t (1 - P_t),$$

where N_t is the concentration of traps and P_t is the occupation probability of a trap.

→ since U_n & U_p are equal eventually
we will obtain a common expression of
for U_n & U_p .

Net Thermal Recombination Rate — IV

Shockley-Read-Hall theory

Replacing P_t and $1 - P_t$ in $U = \alpha_n n N_t (1 - P_t) - \alpha_n n_B N_t P_t$ yields

$$U = N_t \frac{\alpha_n n (\alpha_n n_B + \alpha_p p) - \alpha_n n_B (\alpha_n n + \alpha_p p_B)}{\alpha_n (n + n_B) + \alpha_p (p + p_B)} =$$

$$= \frac{np - n_B p_B}{(n + n_B)/(N_t \alpha_p) + (p + p_B)/(N_t \alpha_n)},$$

with $n_B p_B = n^{\text{eq}} p^{\text{eq}}$. If the semiconductor is non degenerate, it is $n_B p_B = n_i^2$ and

$$n_B = \frac{N_C}{d_t} \exp\left[-\frac{E_C - E_t}{k_B T_L}\right], \quad p_B = N_V d_t \exp\left[-\frac{E_t - E_V}{k_B T_L}\right]$$

In such case the denominator of U has a sharp minimum for

$$E_t = \frac{1}{2}(E_C + E_V) + \frac{1}{2}k_B T_L \log\left(\frac{\alpha_p N_V}{\alpha_n N_C} d_t^2\right) \simeq \frac{1}{2}(E_C + E_V) \simeq E_{Fi},$$

whence $n_B \approx p_B \approx n_i$.

→ In the Non-Degenerate case

$$n^{\text{eq}} = N_C \exp(E_F - E_C)$$

effective density of states of CB

$$e_n = \alpha_n \frac{n^{\text{eq}}}{d_t} \exp[(E_t - E_F)/(k_B T_L)] \doteq \alpha_n n_B$$

$$e_p = \alpha_p p^{\text{eq}} d_t \exp[(E_F - E_t)/(k_B T_L)] \doteq \alpha_p p_B$$

From this we know $n_B \propto p_B$

$$n_B = \frac{n^{eq}}{d_T} \exp \left[\frac{(E_t - E_F)}{k_B T_L} \right]$$

$$\therefore n^{eq} = N_C \exp(E_F - E_C)$$

$$n_B = \frac{N_C \exp(E_F - E_C)}{d_T} \exp \left(\frac{(E_t - E_F)}{k_B T_L} \right)$$

$$\boxed{n_B \Rightarrow N_C \exp \left(\frac{(E_C - E_t)}{k_B T_L} \right)}$$

Similarly

$$P_B = N_v d_T \exp \left[\frac{(E_t - E_V)}{k_B T_L} \right]$$



$$n_B P_B = n_i^2$$

In the non-degenerate case
so, it does not contain the
energy of the Trap anymore

So, Globally The Numerator d_T does not

$$U = N_t \frac{\alpha_n n (\alpha_n n_B + \alpha_p p) - \alpha_n n_B (\alpha_n n + \alpha_p p_B)}{\alpha_n (n + n_B) + \alpha_p (p + p_B)} = \\ = \frac{np - n_B p_B}{(n + n_B)/(N_t \alpha_p) + (p + p_B)/(N_t \alpha_n)},$$

contain Energy of the Trap But the Denominator
does have Trap Energy (E_t)

→ when we started the Analysis we said we always have traps in the Gap. Then Direct transitions would be Negligible because Trap Assisted Transitions are only one that are important.

The point is we made calculations assuming there is only ONE LEVEL OF TRAPS.

మనం ఏది ప్రతిశీలంగా ఒక సెప్చులు లైన్ కు వ్యాపారం చేసి ఉండుకుందా.

ప్రతి జాతుకు మొట్టం లోసు ఇంటి వ్యవసాయాలి.

టర్మినాలు అంచెలంబంగా ప్రత్యేక లైన్లు ఉంచాలి.

ఈ ప్రాణి విధసంస్థ చెలా బాగా ఉన్నాగా ఉంటుంది...!

ఇట్లు మీ

—శ్రీమంతు రాధీ పాండిత

Renuth Reddy Pandit
కృష్ణ నగర పట్టాల

(EBIT)

→ In Reality, we may have many layers of traps with different Trap Energies

Q What should we do Now?

In principle, we should repeat the calculation for each trap Energy that is present in the semiconductor and sum up the result. It is very inconvenient - we are not gonna do it.

→ If we look at the expression

$$U = N_t \frac{\alpha_n n (\alpha_n n_B + \alpha_p p) - \alpha_n n_B (\alpha_n n + \alpha_p p_B)}{\alpha_n (n + n_B) + \alpha_p (p + p_B)} = \\ = \frac{np - n_B p_B}{(n + n_B)/(N_t \alpha_p) + (p + p_B)/(N_t \alpha_n)},$$

we ask ourselves

Q How does U depend on Trap Energy E_t ?

In the expression of n_B & p_B we have

E_t

i.e. n_B & p_B depend on E_t exponentially

More precisely $n_B \propto \exp(-\frac{E_t - E_c}{kT})$

As $E_t - E_c \uparrow n_B \uparrow$

Similarly

$$P_B \propto \exp(-\frac{E_t - E_v}{kT})$$

As $(E_t - E_v)$ becomes more and more -ve
 $P_B \uparrow$

Therefore The Denominator of U in almost all cases depends exponentially on the Traplevd.

→ So, we may assume this analysis where we take value of Trap Energy that provide the minimum of The Denominator for U

so that U will be Maximum.

→ When we go away from this specific E_t for which we have Maximum U , The Denominator will increase exponentially that it will kill ' U ' the Recombination Function.

→ Actually the trap level that is important is the one for which the denominator is minimum

It's easy to calculate the minimum Denominator because they are just exponentials

→ Let take the derivative of the Denominator with respect to E_F and then we put it equal to zero & we get

$$E_F = \frac{1}{2} (E_C + E_V) + \frac{1}{2} k_B T_L \log \left(\frac{\alpha_p N_V}{\alpha_n N_C} \right) d_F^2$$

Small

$$\approx \frac{1}{2} (E_C + E_V) \approx E_{F_i}$$

when $N_B = P_B = n_i$ Intrinsic Fermi Level

So we keep traps at the Mid-Gap as the most efficient ones

→ Given the above simplifications we can still manipulate the expression of ψ a bit.

In the Denominator we have $N_F \alpha_p$ & $N_F \alpha_n$

$$U = N_t \frac{\alpha_n n (\alpha_n n_B + \alpha_p p) - \alpha_n n_B (\alpha_n n + \alpha_p p_B)}{\alpha_n (n + n_B) + \alpha_p (p + p_B)} = \\ = \frac{np - n_B p_B}{(n + n_B)/(N_t \alpha_p) + (p + p_B)/(N_t \alpha_n)},$$

We know N_t : It's a concentration

α_n, α_p are dimensionless volumes over time

So $\left. \begin{array}{c} N_t \alpha_n \\ N_t \alpha_p \end{array} \right\}$ They are inverse of Time

Net Thermal Recombination Rate — V

Shockley-Read-Hall theory

Defining the lifetimes

$$\tau_{p0} = \frac{1}{N_t \alpha_p}, \quad \tau_{n0} = \frac{1}{N_t \alpha_n} \Rightarrow$$

$$U \doteq U_{SRH} = \frac{np - n_i^2}{\tau_{p0}(n + n_B) + \tau_{n0}(p + p_B)}.$$

In equilibrium it is $U = 0$. Recombination (generation) dominates where $U > 0$ ($U < 0$). Because $n_p = n_i^2$

Limiting cases

- Full depletion: $n, p \ll n_i, n_B, p_B \Rightarrow$

$$U \simeq -\frac{n_i^2}{\tau_{p0} n_B + \tau_{n0} p_B} \simeq -\frac{n_i}{\tau_{p0} + \tau_{n0}} \doteq -\frac{n_i}{\tau_g},$$

- Weak injection: $|n - n^{eq}|, |p - p^{eq}| \ll c^{eq}$, where c^{eq} is the equilibrium concentration of the majority carriers.

$U > 0$ Recombination is Dominant

$U < 0$ Generation is Dominant

→ we use the formula for U in future it depends on n & p it does not introduce any additional unknowns
on the other hand U is non-linear w.r.t n & p

→ In analytical calculation it's possible to simplify U in some special cases

Limiting cases

- Full depletion: $n, p \ll n_i, n_B, p_B \Rightarrow$

$$U \simeq -\frac{n_i^2}{\tau_{p0} n_B + \tau_{n0} p_B} \simeq -\frac{n_i}{\tau_{p0} + \tau_{n0}} \doteq -\frac{n_i}{\tau_g},$$

- Weak injection: $|n - n^{eq}|, |p - p^{eq}| \ll c^{eq}$, where c^{eq} is the equilibrium concentration of the majority carriers.

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Net Thermal Recombination Rate — VI

Shockley-Read-Hall theory

- Weak injection, n-type semic.: $|n - n^{eq}|, |p - p^{eq}| \ll n^{eq}$.

$$\begin{aligned} \text{Numerator} &\leftarrow \left\{ np - n_i^2 \simeq n^{eq}(p - p^{eq}) + p^{eq}(n - n^{eq}) \right. \\ \text{Denominator} &\leftarrow \left. \tau_{p0}(n + n_B) + \tau_{n0}(p + p_B) \simeq \tau_{p0}n^{eq} \right\} \Rightarrow \\ U &\simeq \frac{p - p^{eq}}{\tau_{p0}} + \frac{n - n^{eq}}{(n^{eq}/p^{eq})\tau_{p0}}. \end{aligned}$$

$$\tau_n \doteq \frac{n^{eq}}{p^{eq}}\tau_{p0} \gg \tau_p \doteq \tau_{p0} \Rightarrow U \simeq \frac{p - p^{eq}}{\tau_p}$$

- Weak injection, p-type semic.: $|n - n^{eq}|, |p - p^{eq}| \ll p^{eq}$.

$$\begin{aligned} \left\{ np - n_i^2 \simeq n^{eq}(p - p^{eq}) + p^{eq}(n - n^{eq}) \right. \\ \left. \tau_{p0}(n + n_B) + \tau_{n0}(p + p_B) \simeq \tau_{n0}p^{eq} \right\} \Rightarrow \\ U \simeq \frac{p - p^{eq}}{(p^{eq}/n^{eq})\tau_{n0}} + \frac{n - n^{eq}}{\tau_{n0}}. \end{aligned}$$

$$\tau_p \doteq \frac{p^{eq}}{n^{eq}}\tau_{n0} \gg \tau_n \doteq \tau_{n0} \Rightarrow U \simeq \frac{n - n^{eq}}{\tau_n}$$

$$\begin{aligned} &\Rightarrow n_p - n_i^2 \\ &\Rightarrow [(n - n_{eq}) + n_{eq}] [(p - p_{eq}) + p_{eq}] - n_i^2 \\ &\Rightarrow (n - n_{eq})(p - p_{eq}) + p_{eq}(n - n_{eq}) + n_{eq}(p - p_{eq}) \\ &\quad \text{This term is quite small} \\ &\quad + n_{eq}p_{eq} - n_i^2 \quad \text{Both cancel out} \end{aligned}$$

$$\Rightarrow P_{eq}(n - n_{eq}) + n_{eq}(p - p_{eq}) \cong np - n_i^2$$

Numerator.

- Weak injection, n-type semic.: $|n - n^{eq}|, |p - p^{eq}| \ll n^{eq}$.

Denominator

$$\left\{ \begin{array}{l} np - n_i^2 \simeq n^{eq}(p - p^{eq}) + p^{eq}(n - n^{eq}) \\ \tau_{p0}(n + n_B) + \tau_{n0}(p + p_B) \simeq \tau_{p0}n^{eq} \end{array} \right. \Rightarrow$$

we ignore Holes because it is n-type semiconductor

$$U \simeq \frac{p - p^{eq}}{\tau_{p0}} + \frac{n - n^{eq}}{(n^{eq}/p^{eq})\tau_{p0}}$$

$$\tau_n \doteq \frac{n^{eq}}{p^{eq}} \tau_{p0} \gg \tau_p \doteq \tau_{p0} \Rightarrow U \simeq \frac{p - p^{eq}}{\tau_p}$$

Because $n - n^{eq} \geq 0$

(in case of weak Injection)

Conclusion: If we are in weak injection condition and if the material is n-type then the Recombination Rate (U) is entirely dependent on the minority charge carriers i.e. holes

$$U \simeq \frac{p - p^{eq}}{\tau_p}$$

Similarly for P-type Semiconductor

- Weak injection, p-type semic.: $|n - n^{eq}|, |p - p^{eq}| \ll p^{eq}$.

$$\begin{cases} np - n_i^2 \simeq n^{eq}(p - p^{eq}) + p^{eq}(n - n^{eq}) \\ \tau_{p0}(n + n_B) + \tau_{n0}(p + p_B) \simeq \tau_{n0}p^{eq} \end{cases} \Rightarrow$$

$$U \simeq \frac{p - p^{eq}}{(p^{eq}/n^{eq})\tau_{n0}} + \frac{n - n^{eq}}{\tau_{n0}}.$$

$$\tau_p \doteq \frac{p^{eq}}{n^{eq}}\tau_{n0} \gg \tau_n \doteq \tau_{n0} \Rightarrow U \simeq \frac{n - n^{eq}}{\tau_n}$$

Usefulness

→ So, it is linearization of ' U ' and ' U' depends only one on type of carriers because of this it becomes possible we shall see many examples.

Each time we can apply weak injection condition we can decouple the equation of the model from one another because we have ' U' dependent on one type of carriers.

* This concludes the analysis of Thermal Recombination that are the most important ones.

