21/04/25

Lecture – 22

06/11/20

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→ Stress – strain Tensors :

It follows that the ratio between the components of the elastic tensor Φ^{ij}_{uw} and a length is homogeneous with $\rho\,\ddot{\psi}_u$, whose dimensions are those of a force per unit volume. In conclusion, the dimensions of the elastic tensor are those of a force per unit area, that is, a stress. Defining the *stress tensor* as

$$\sigma_{uw} = \sum_{i,j=1}^{3} \Phi^{ij}_{uw}\,\epsilon_{ij}\,,$$

whose dimensions are the same as those of Φ^{ij}_{uw}, one may recast the long-wavelenght relation for the acoustic modes as

$$\rho\,\ddot{\psi}_u = \sum_{w=1}^{3} \frac{\partial \sigma_{uw}}{\partial x_w}\,,$$

which provides the elasticity law for a continuous medium. The stress tensor is symmetric because Φ^{ij}_{uw} is invariant when u and w are exchanged. The relation between the strain and stress tensors is the generalization of *Hooke's law* to an anisotropic medium.

R. Hooke (1635–1703) first mentioned the law in 1676 as an anagram, *ceiiinosssttuv*, which he explained in 1678 as *ut tensio sic*

Advanced Solid-State Sensors M

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Course Program 2019-2020

1. Introduction – Classification of sensors
2. Main physical effects used in solid-state sensors and simple readout circuits
3. Resolution & Noise
4. Image sensors
5. Mechanical sensors

Mechanical Sensors

Characterization of mechanical sensors

- the measurand and the physical structures

1. Most relevant mechanical signals, their units in the S.I., and ranges for specific applications
2. Definition of the stress and strain tensors
3. Description of the structural elements mainly used in solid-state sensors (beams & plates)

List of mechanical measurands:

subset of six signals covering the most important classes of mechanical microsensors

- Pressure/Stress
- Acceleration/Deceleration
- Displacement
- Flow rate
- Force/Torque
- Position/Angle

Automotive accelerometers

Application	Range
Frontal airbags	+/- 50 g
Lateral airbags	+/- 100 – 250 g
Suspensions	+/- 2 g
ABS (Antilock Braking System)	+/- 1 g

Units are referred to g, with g = 9,8 m/s^2

Pressure units

1 Pa	International System
1 KPa	1e3 Pa
1 kgf/m^2	9,8 Pa (g = 9,8 m/s^2)
1 psi (pound-force per square inch)	6,89e3 Pa (1inch = 25.4mm; 1Kg = 2,2046p)
1 mmHg (= 1 Torr)	1,33e2 Pa (p_{Hg} = 13,59 g/cm^3, liquid)
1 atm	760 Torr = 1,01e5 Pa (equivalent to 10,33 m of water)
1 bar	1e5 Pa

Applications of silicon pressure sensors

Application	Pressure Range
Manifold pressure	15 - 250 kPa
Fuel pressure	15 - 400 kPa
Tyre pressure	1,8 –2,5 bar
Blood pressure (max 120 – 130 mmHg; min 70 – 80 mmHg)	30 - 300 mmHg
Barometric pressure	in hectopascal = mbar o in mmHg

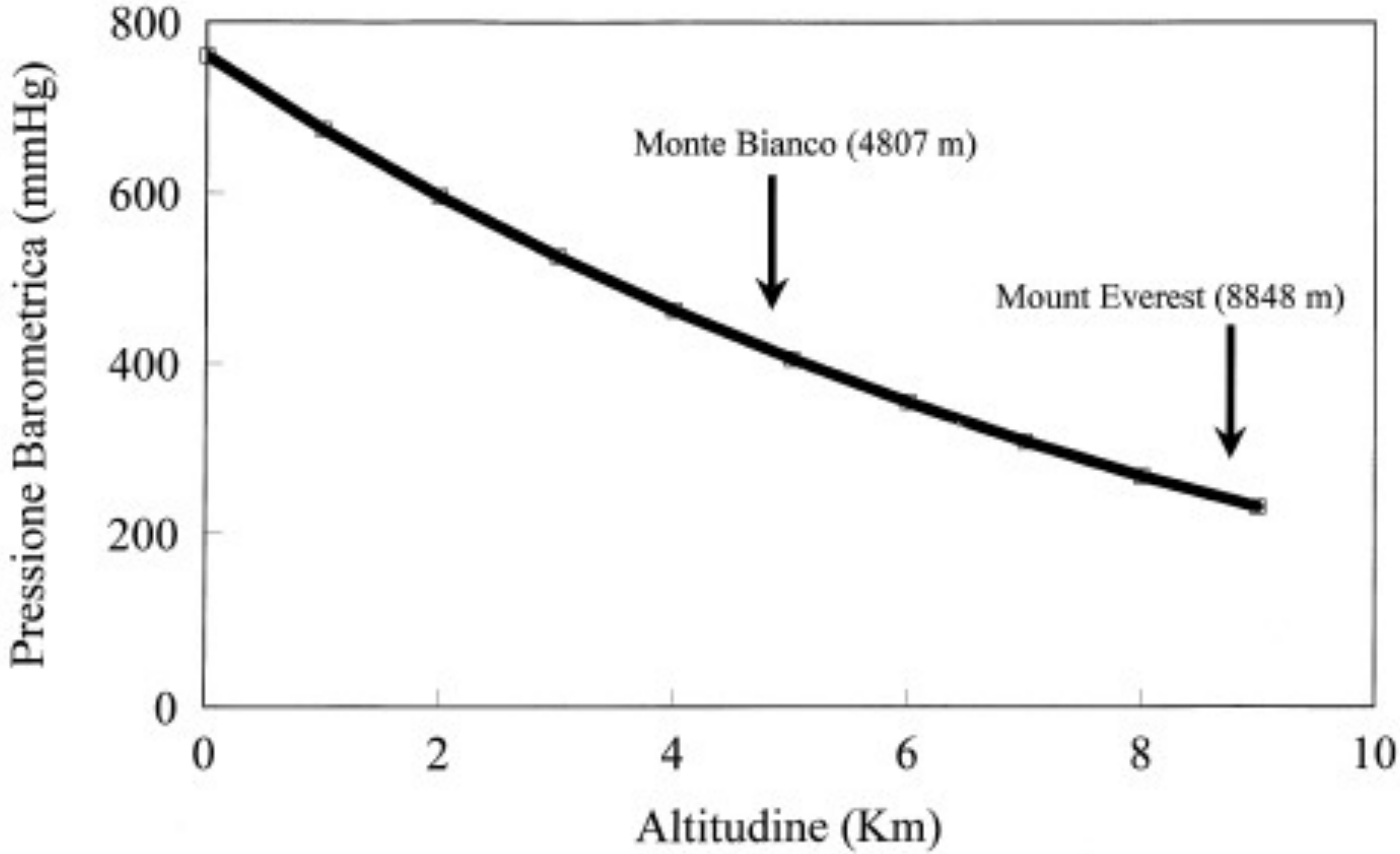

Figura 1. *Modificazione della pressione barometrica in relazione all'altitudine secondo le tabelle dell'Organizzazione dell'Aviazione Civile Internazionale.*

Mechanical Sensors

Characterization of mechanical sensors

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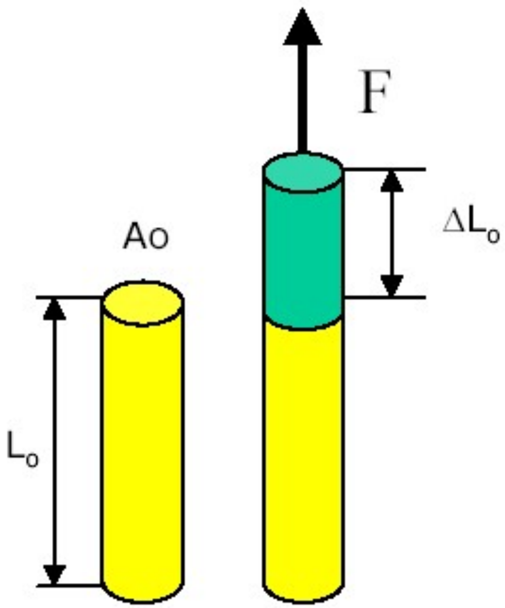
Definition of stress and strain tensors

• one dimensional case of Hook's law



Stress: $\sigma = F/A_0$ [Pa = N/m^2]

Strain: $\varepsilon = \Delta L_0/L_0$

HOOKE's LAW:

$\boxed{E = \sigma / \varepsilon \text{ Young's modulus [Pa]}}$

* we will also see $\boxed{\text{Poisson coefficient}}$ that also describes Thinning of the Bar.

List of Hypothesis we already started examining!

Assumptions

1) • We will consider elastic deformations and linear response → small-deformation hypotesis.

2) • In equilibrium, the resultant force over the body is 0, the resultant moment over the body is 0. (everything is static)

3) • The linear dimensions of the physical system are large enough to consider it as a continuous body (example: lattice distance in silicon a=5,431 Å, MEMS structures with $L_{min} \approx 50$ nm).

4) • The surface elements becomes more and more relevant in small bodies:

l = 1 m → Surface/Volume = 6 m^{-1};

l = 1 cm → Surface/Volume = 6 cm^{-1} = 600 m^{-1}

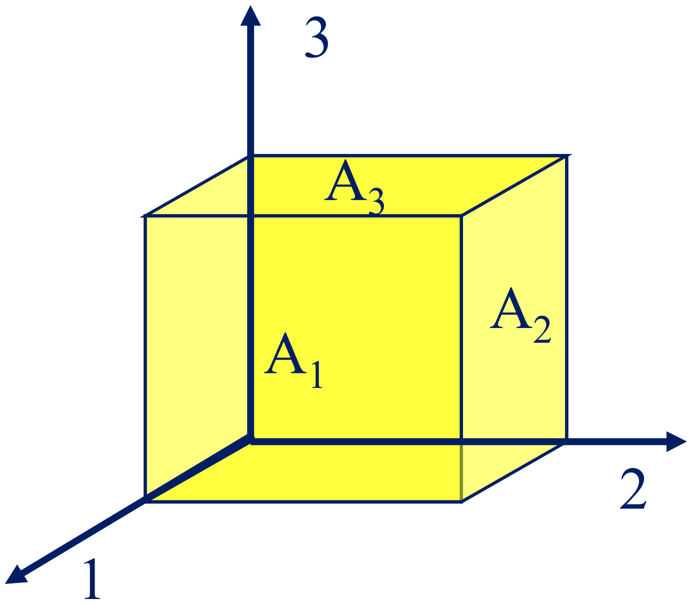
Gravity is a Volume Force

* when the size of the object becomes the Surface Forces dominate the Volume Forces.

→ we consider in our discussions materials of cubic type.

- Forces applied to the body (e.g, gravity)
- Forces applied at the surfaces (e.g., pressure, bending, twisting)

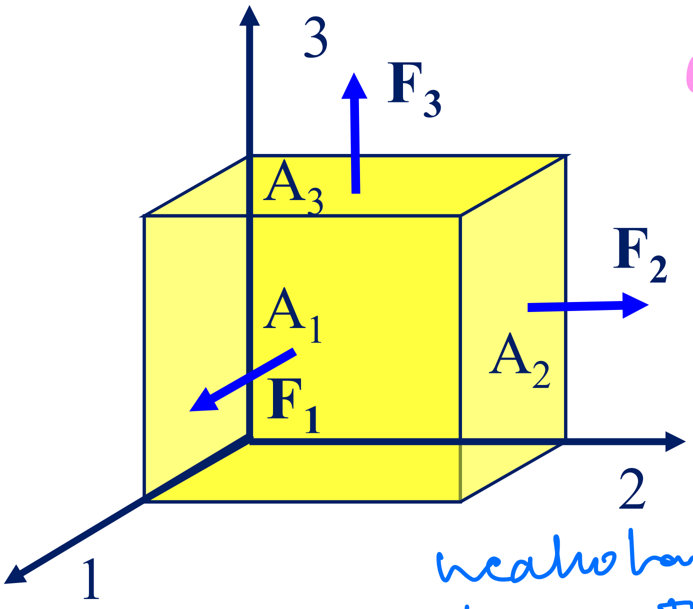
A_1, A_2, A_3 are the Areas of the Faces



- Remember when we apply a Force, there is Force & Torque but the object becomes smaller the Force becomes more Relevant.

- Normal stresses (or *axial*)

F_1, F_2, F_3 are Normal components of the Applied force.



$$\sigma_{11} = F_1/A_1$$

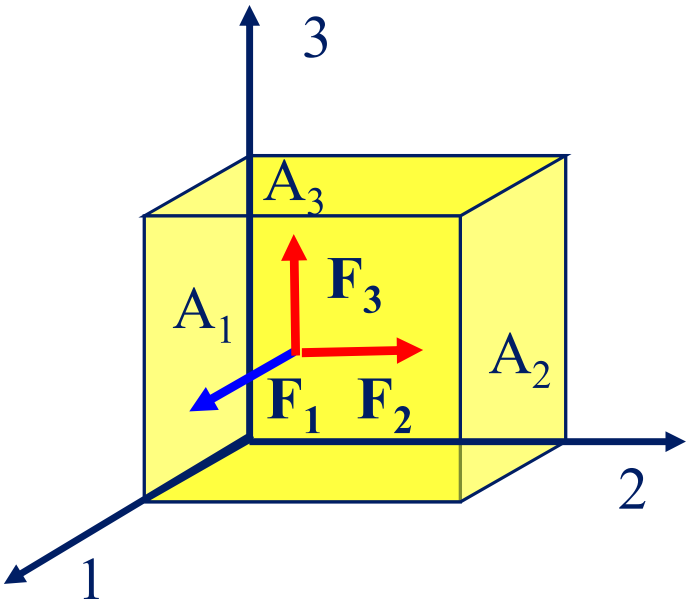
$$\sigma_{22} = F_2/A_2$$

$$\sigma_{33} = F_3/A_3$$

we also have Forces that are acting on the other '3' Faces

In This figure we are considering only Face A_1

- Shear stresses



$\sigma_{31} = \frac{F_3}{A_1}$

$\sigma_{21} = \frac{F_2}{A_1}$

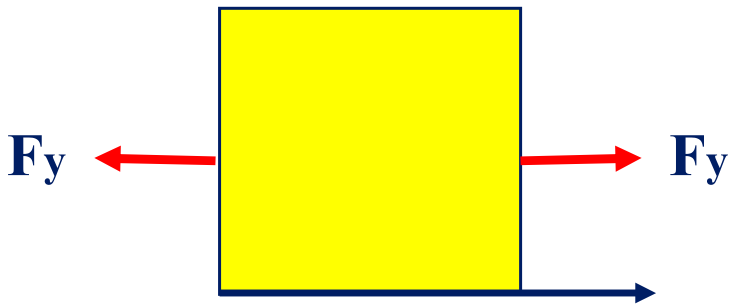
σ_{ij}

This Tensor is Symmetric to Balance Rotational Moments

Equilibrium condition: in order to have no accelerations of the body, all forces and moments must balance:

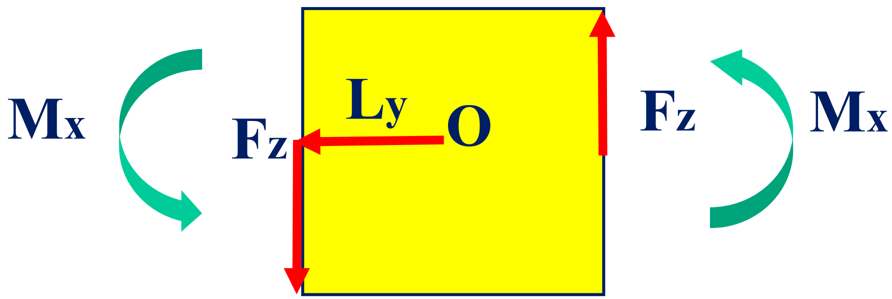
i) The surface axial forces need to be equal on two opposite planes:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$



ii) The shear forces, even if balanced, would induce a rotation moment:

$$\boxed{M_x = 2 * L_y F_z}$$



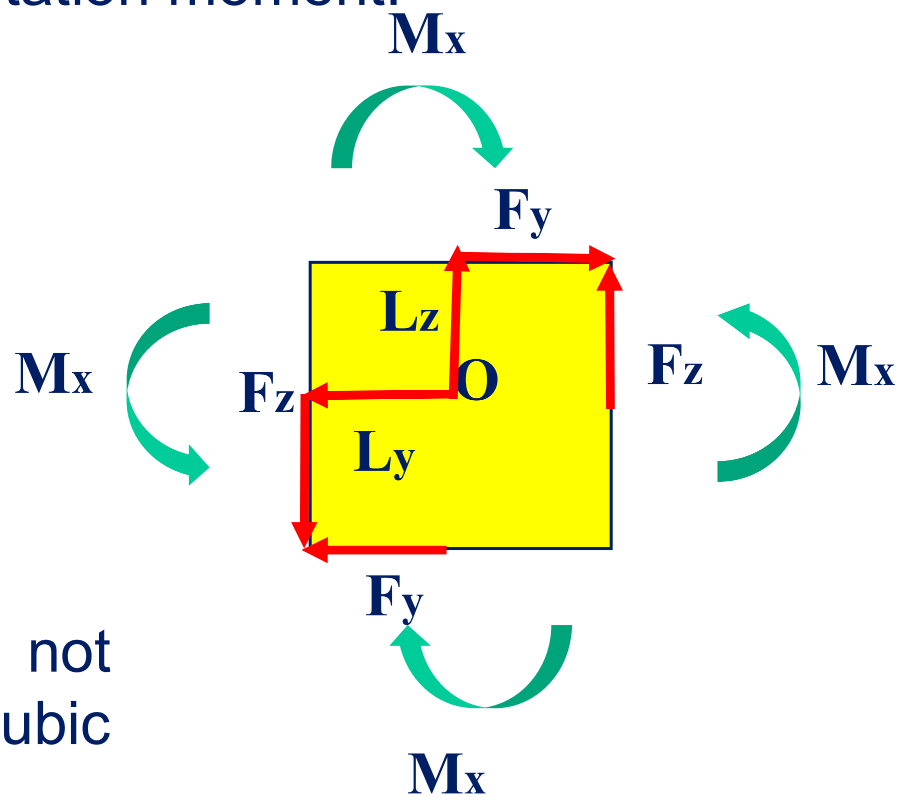
Equilibrium condition:

In order to have no accelerations of the body one must consider the action of a second couple of shear forces on the dual planes, balancing the rotation moment:

$$\boxed{\Sigma M_x = 2 * L_y F_z - 2 * L_z F_y = 0}$$

Balancing Rotational Moments

$$\Rightarrow \boxed{\sigma_{ij} = \sigma_{ji}}$$



(for the above to apply it is not necessary to consider a cubic block)

Note: (1) Does This argument for Equillibrium work only for cubic Block ?

Ans It dosen't depend on the shape, we will see the Answer later.

STRESS TENSOR

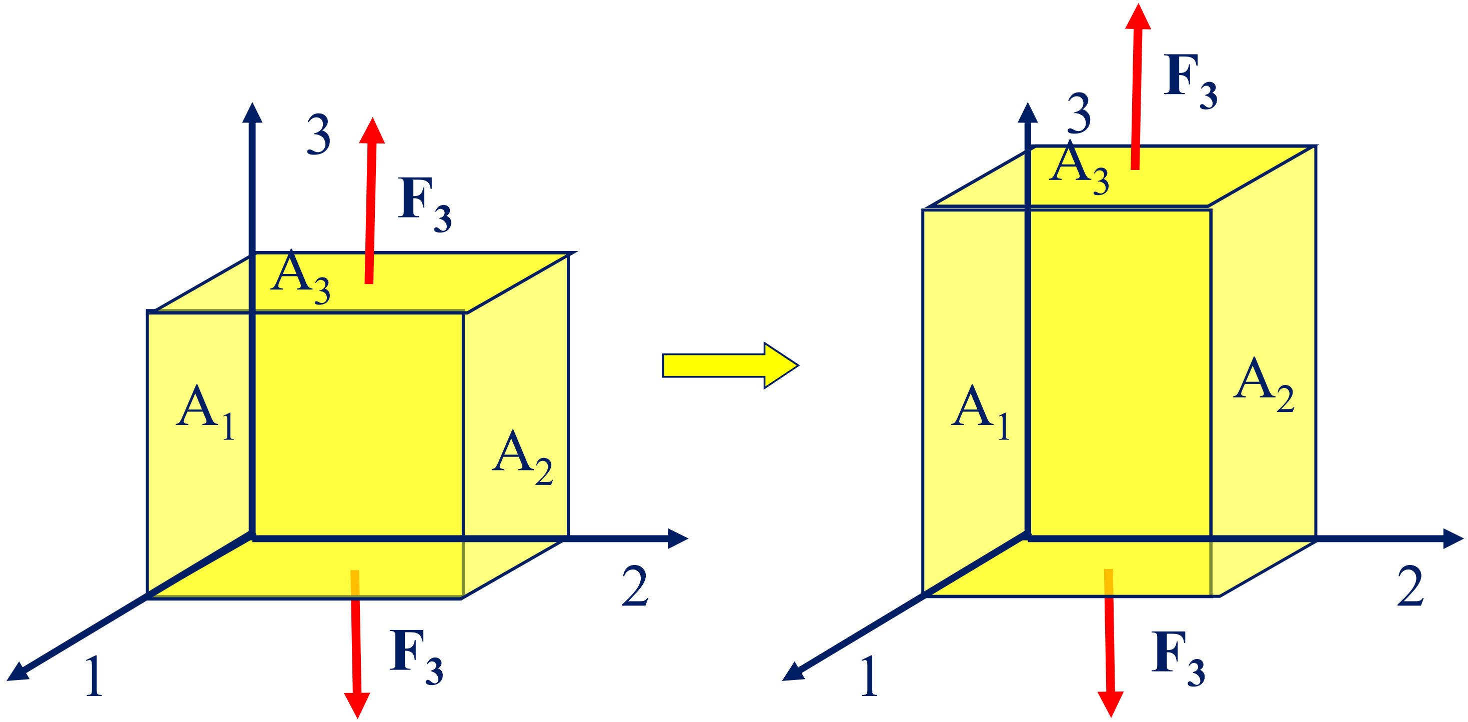
$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

'6' independent components rearranged as vectors

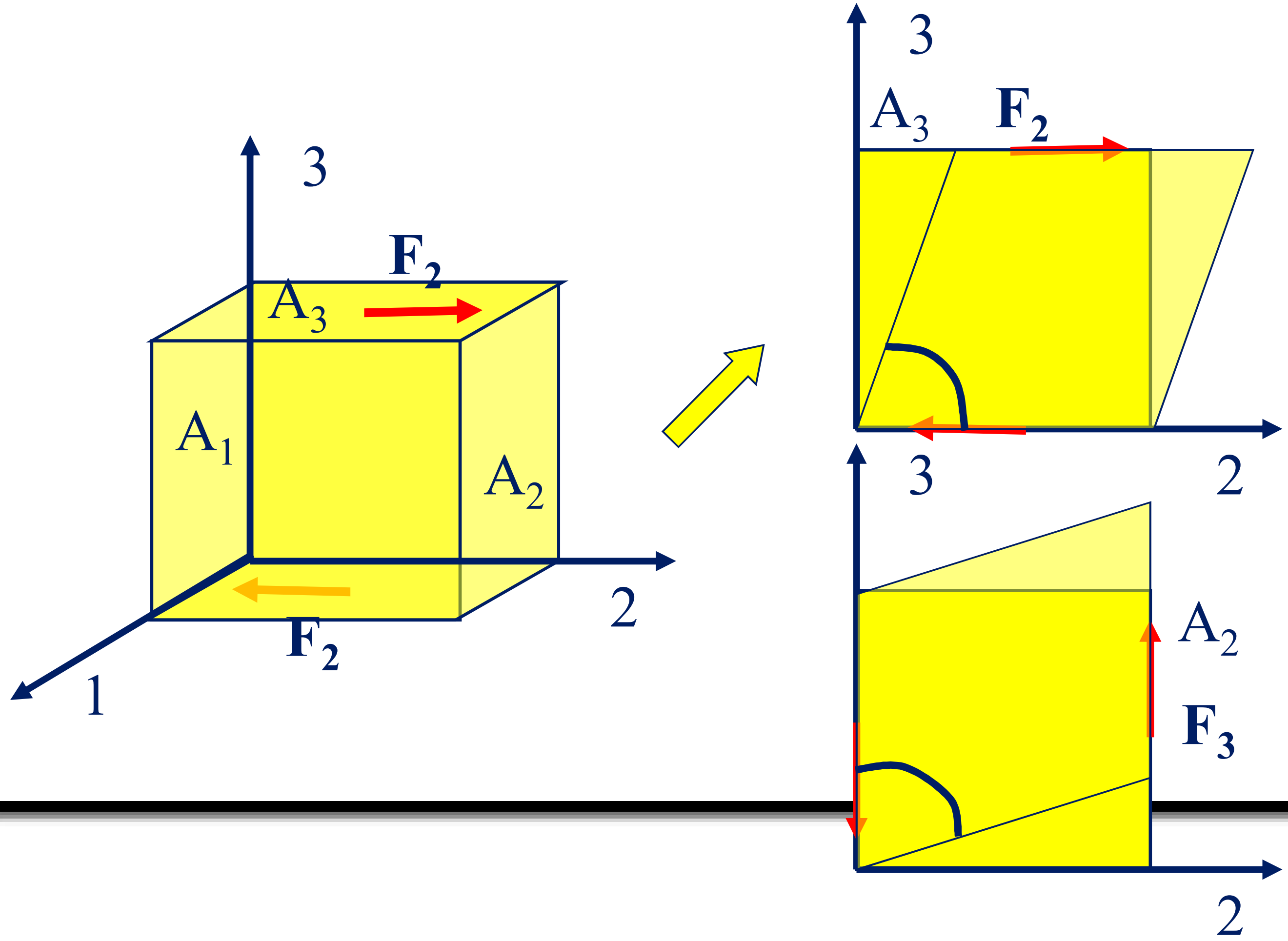
$$\begin{pmatrix} 1 & 6 & 5 \\ & 2 & 4 \\ & & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

Deformations

- Deformations due to axial stresses



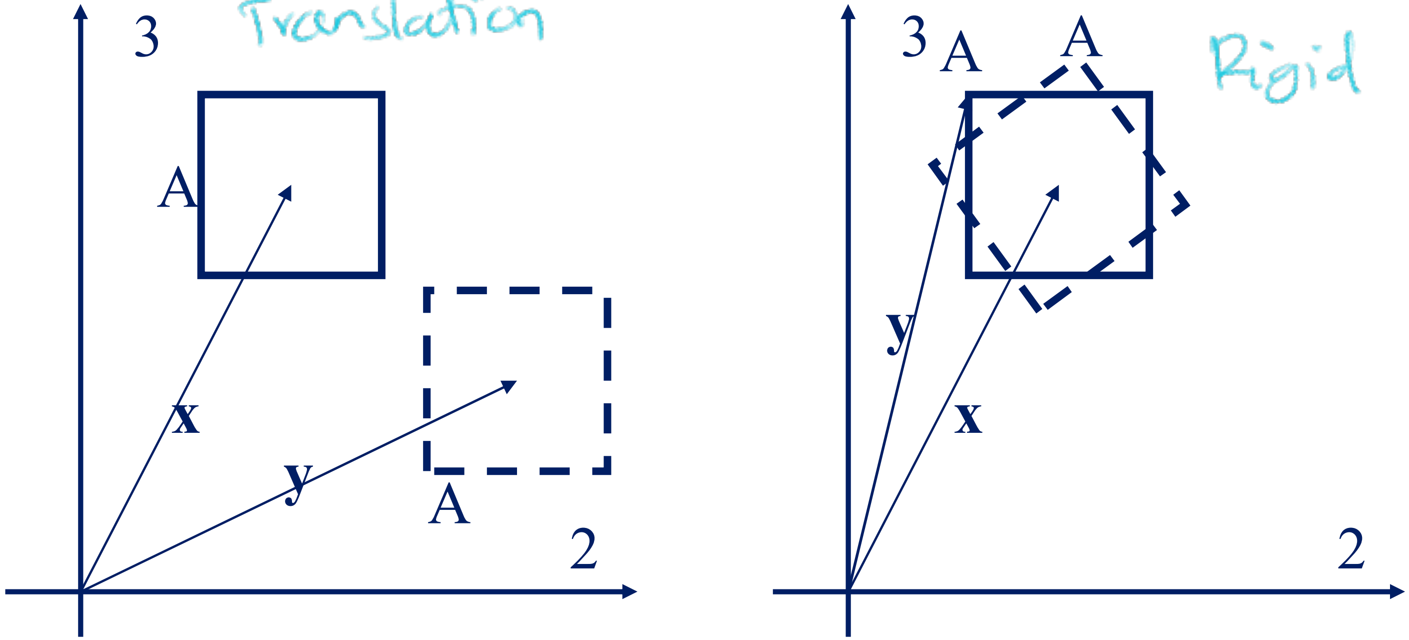
- Deformations due to shear stresses



Strain tensor

Rigid Translation

Rigid Rotation



$$\boxed{t = (y - x)} \qquad r = \omega \wedge (y - x)$$

We ignore this slide because we assumed that the Block of material besides the Deformation the Block of material also undergoes a rigid Translation and or rigid Rotation but we Neglect them.

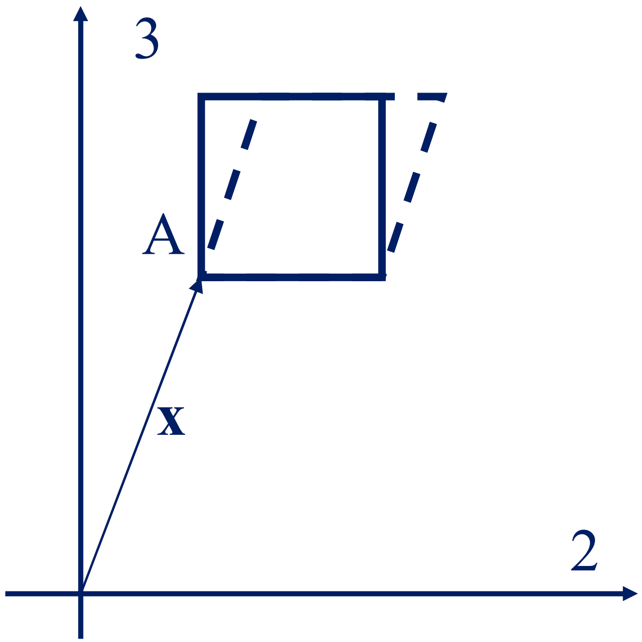
we consider only Local Deformations of the Body.

Traslation and Rotation without deformation

$$\bar{\bar{\omega}} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

$$\boldsymbol{u} = \boldsymbol{dx} + \bar{\bar{\omega}}\boldsymbol{dx}$$

Deformation → Strain tensor



$$\boldsymbol{u} = \boldsymbol{u}_0 + (\nabla_x \boldsymbol{u})d\boldsymbol{x}$$

$$(\nabla_x \boldsymbol{u})_{ij} = \frac{\partial u_i}{\partial x_j} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

Anti Symmetric Tensor that describes Rotation & it is Neglected

$$(\nabla_{\boldsymbol{x}}\boldsymbol{u})_{ij} = \frac{\partial u_i}{\partial x_j} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

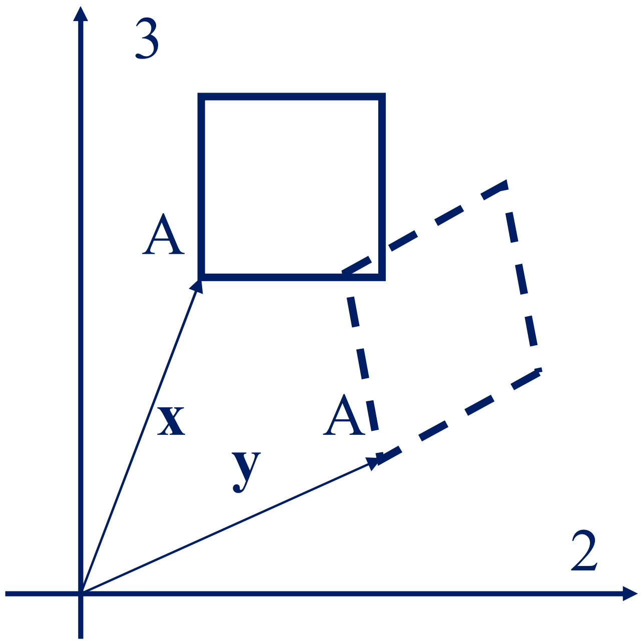
$$(\nabla_{\boldsymbol{x}}\boldsymbol{u})_{ij} = \underline{\varepsilon_{ij}} + \omega_{ij}$$

$$\boxed{\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} \Rightarrow \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{pmatrix}$$

Strain Tensor is also Symmetric

∴ we consider only '6' components

Traslation, Rotation and Deformation



$$u = u_0 + \overline{\overline{\omega}} dx + \overline{\overline{\varepsilon}} dx$$

Note: Remember initially when we introduced Elastic Tensor The total no. of componants were 81 we reduced it to 21 ⇒ 15 ⇒ finally considering the case of Cubic like Crystal ex: Ge down to '3'

* A Vector is equal to a Matrix multiplied by another Vector.

Generalized Hooke's Law

(compliance matrix for isotropic materials)

$$\varepsilon_{hk} = \sum_{ij} b_{hk,ij}\sigma_{ij}$$

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{12} & 0 & 0 & 0 \\ b_{12} & b_{11} & b_{12} & 0 & 0 & 0 \\ b_{12} & b_{12} & b_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{44} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

Here, we have only Three independent componants for Elastic Tensor

Generalized Hooke's Law

- The inverse of the compliance matrix is called *stiffness matrix*.
- Elastic constants in isotropic materials:

$$b_{11} = 1 / E, \qquad b_{12} = -\nu / E, \qquad b_{44} = 1 / G$$

  - E = Young's modulus
  - ν = Poisson's coefficient
  - G = shear modulus

The equations of linear elasticity

$$F_i^b = \int_V f_i^b dV$$

$$F_i^s = \int_S \sigma_{ij} n_j dS = \int_V \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} dV$$

$$\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i^b = \rho \dot{v}_i$$

Mechanical Sensors

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Q) How do we apply the Elastic Tensor concepts in Micro Electronics?

Ans offcourse, we must be able to find Micro Electronic structures in which we can realize

Mechanical things that can realise Mechanical things that can Bend (or) Deform.

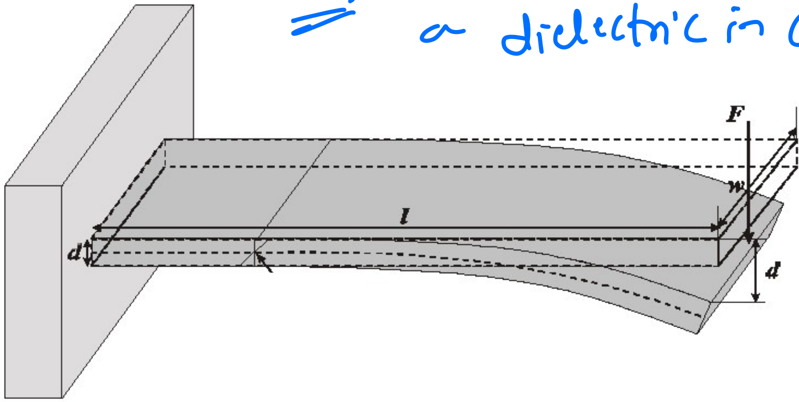
(i) Cantilever Beams

(ii) Membranes / Plates

Mechanical structures used in MEMS sensors

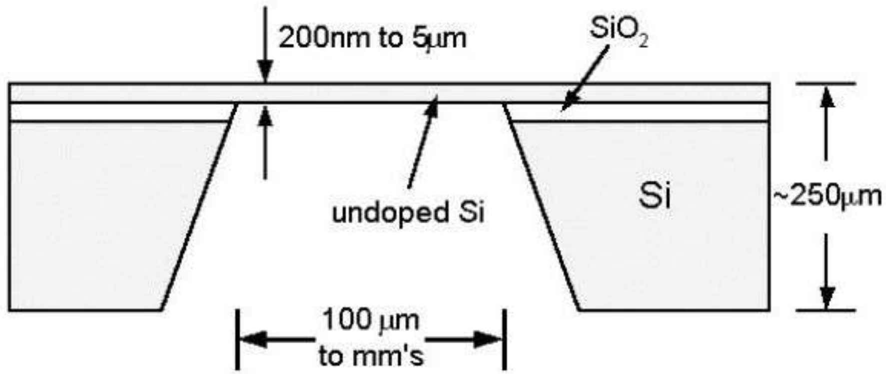
(i) Cantilever beams

eg: cantilever beam used as a dielectric in capacitor, its shape change is reflected in its capacitance



(ii) Membranes/Plates

Both these mechanical structures deform under a force



Cantilever beam (fixed end beam)

Type of loads

a. concentrated load (single force)

b. distributed load (measured by their intensity) :

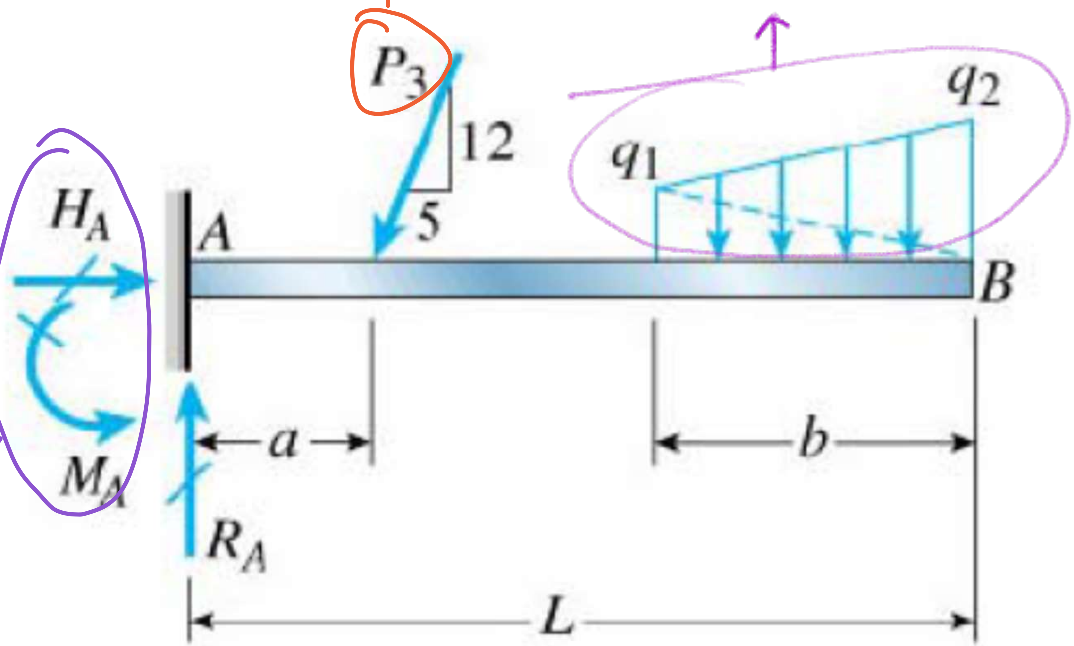
- uniformly distributed load (uniform load)
- linearly varying load

; this is a concentrated load

Distributed Load

Reactions:

The fixed end prevents translations and rotations



No Normal Force and No Rotation ∴ 'A' is Fixed point

Same for R_A.

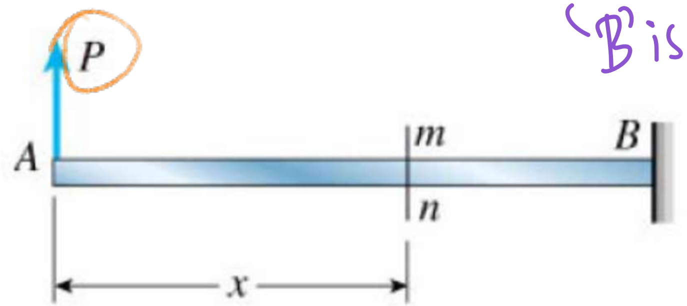
Q) How do we calculate the Shear forces & Moments inside a cantilever?

Ans

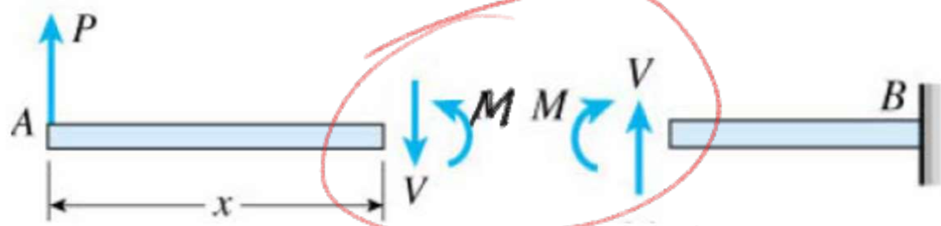
Cantilever beam (fixed end beam)

Shear Forces and Bending Moments

'B' is fixed here



concentrated load P applied at the end



shear force

$$\Sigma F_y = 0 \qquad V = P$$

$$\Sigma M = 0 \qquad M = P\,x$$

To study what happens at 'x' distance from position 'A', we simply cut ideally the cantilever at position 'x' and to each end of seperated we apply Force & Moment s.t Reaction of Force & Moment compensate for the part we have removed.

* Considering finite extension where $V = P$ everywhere

** # Cantilever beam (fixed end beam)

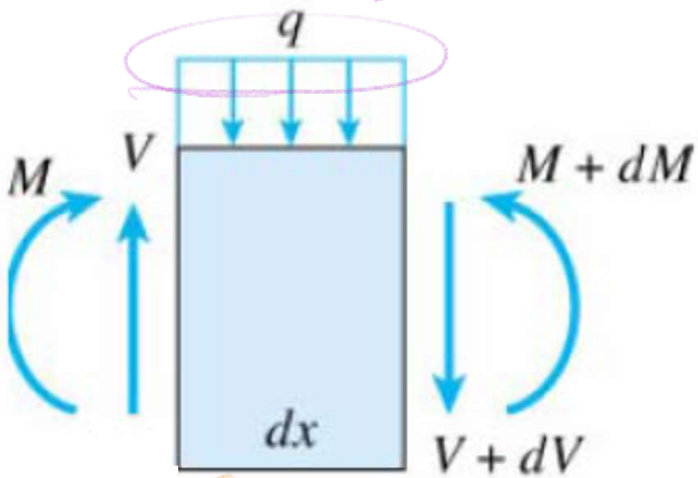
Relationships Between Loads, Shear Forces, and Bending Moments

consider an element of a beam of length dx

subjected to distributed loads q

$\Downarrow$ Force / length

Distributed Load



The extracted Block we consider to be in Equilibrium.

$$\boxed{\begin{array}{c}\Sigma F_y = 0 \\ \text{or}\end{array}}$$

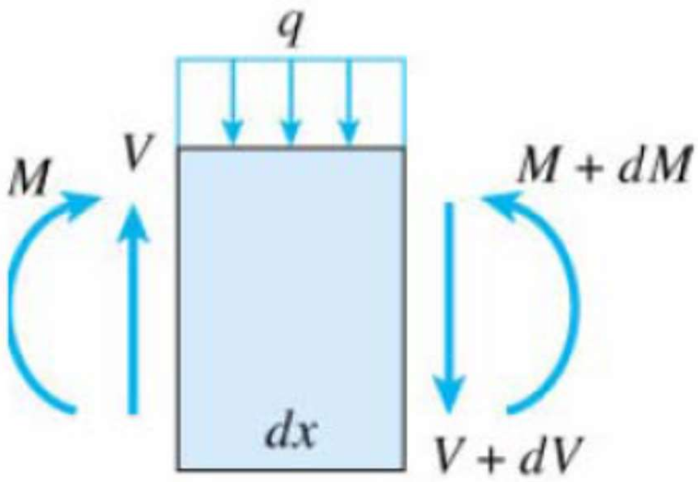
$$\boxed{\begin{array}{c}V - q\,dx - (V + dV) = 0 \\ dV / dx = -q\end{array}}$$

Cantilever beam (fixed end beam)

Relationships Between Loads, Shear Forces, and Bending Moments

consider an element of a beam of length dx

subjected to distributed loads q



we take the origin on the left face ∴

$V(x) = V(0) = 0$

here

$$\Sigma M = 0 \qquad \boxed{-M - q\,dx\,(dx/2) - (V + dV)\,dx + M + dM = 0}$$

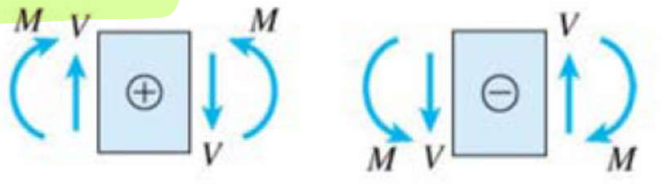
$+V(0)$

or $\quad$ **$dM / dx = V$**

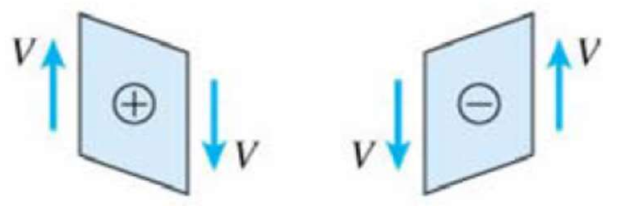
we eliminate the 2nd order terms.

$$\boxed{\frac{d^2M}{dx} = \frac{dV}{dx} = -q}$$

Cantilever beam (fixed end beam) Deformations

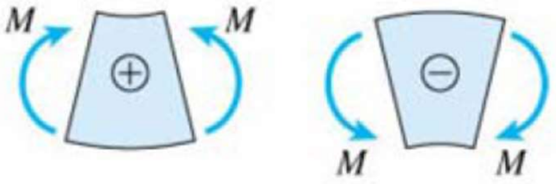


- the shear force tends to rotate the material clockwise is defined as positive



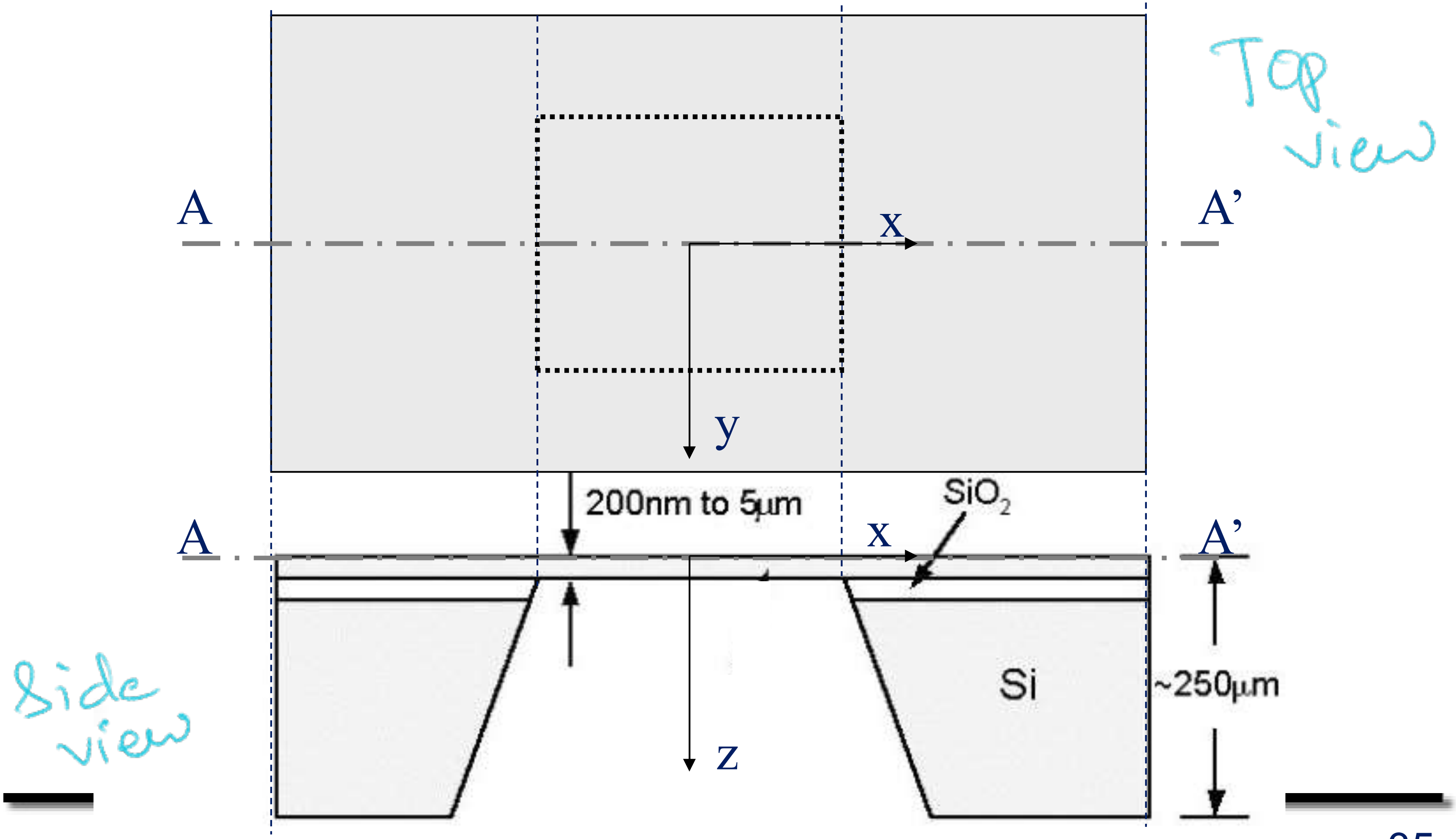
(a)

- the bending moment tends to compress the upper part of the beam and elongate the lower part is defined as positive



(b)

MEMS plates in silicon



Plates (basic concepts of plate bending)

- A plate can be considered the two-dimensional extension of a beam in simple bending.

Both plates and beams support loads transverse or perpendicular to their plane and through bending action.

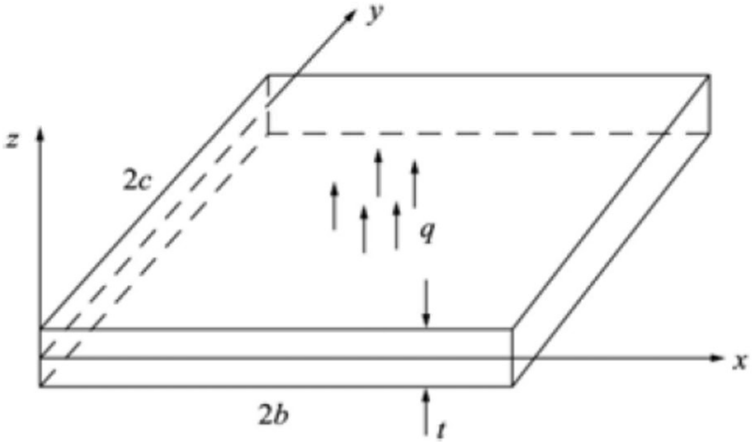
A plate is a flat (if it were curved, it would be a shell).

A beam has a single bending moment resistance, while a plate resists bending about two axes and has a twisting moment.

We will consider the classical thin-plate theory or Kirchhoff plate theory.

Plates (basic concepts of plate bending)

Consider the thin plate in the *x*-*y* plane of thickness *t* measured in the *z* direction shown in the figure below:



The plate surfaces are at $z = \pm t/2$, and its midsurface is at $z = 0$.

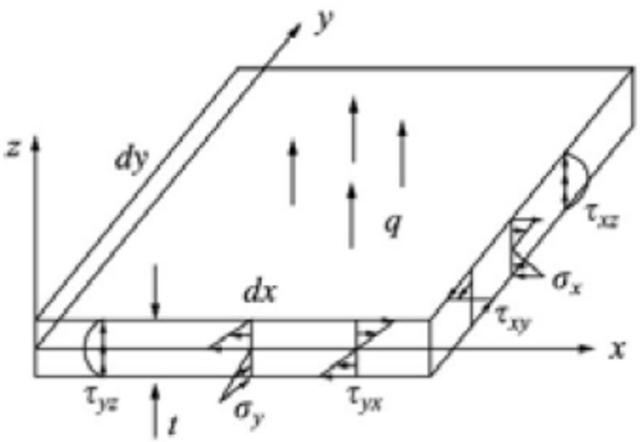
1. The plate thickness is much smaller than its inplane **dimensions** *b* and *c* (that is, $t << b$ or c

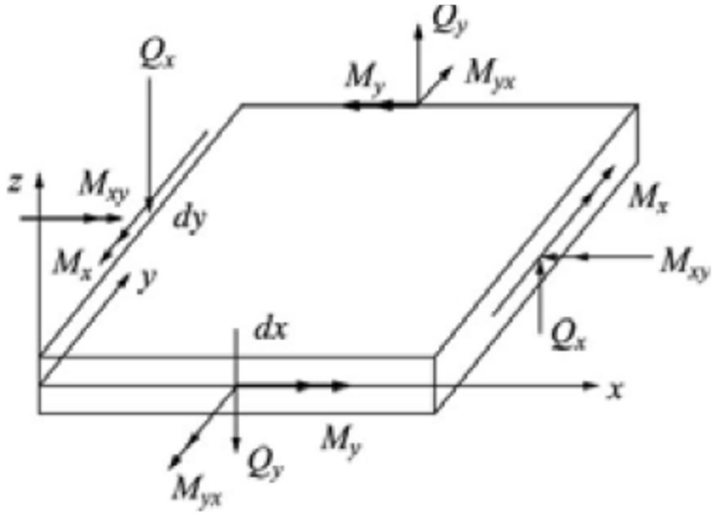
Plates (basic concepts of plate bending)

Kirchhoff Assumptions

1. Normals remain normal. This implies that transverse shears strains $\gamma_{yz} = 0$ and $\gamma_{xz} = 0$. However γ_{xy} does not equal to 0
2. Thickness changes can be neglected and normals undergo no extension. This means that $\varepsilon_z = 0$.
3. Normal stress σ_z has no effect on in-plane strains ε_x and ε_y in the stress-strain equations and is considered negligible.
4. Therefore, the in-plane deflections in the x and y directions at the midsurface, $t = 0$, are assumed to be zero; $u(x, y, 0) = 0$ and $v(x, y, 0) = 0$.

Plates (basic concepts of plate bending)





The governing differential equations are:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$$

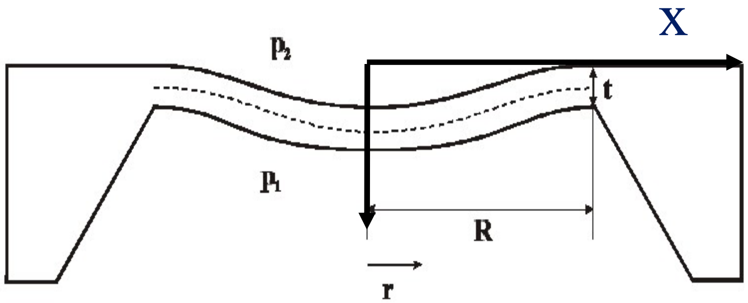
$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$

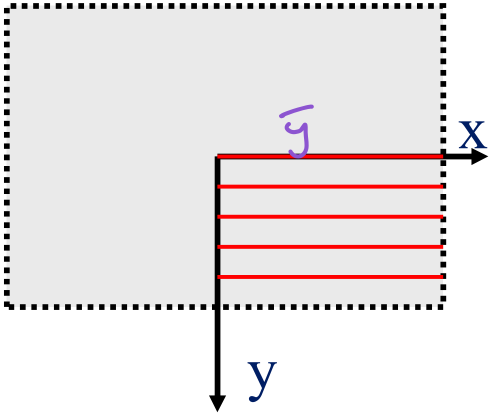
$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0$$

where q is the transverse distributed loading and Q_x and Q_y are the transverse shear line loads.

Solution of the plate bending problem

(S.K.Clark &K.D. Wise, IEEE Tr. ED 26, 1979)





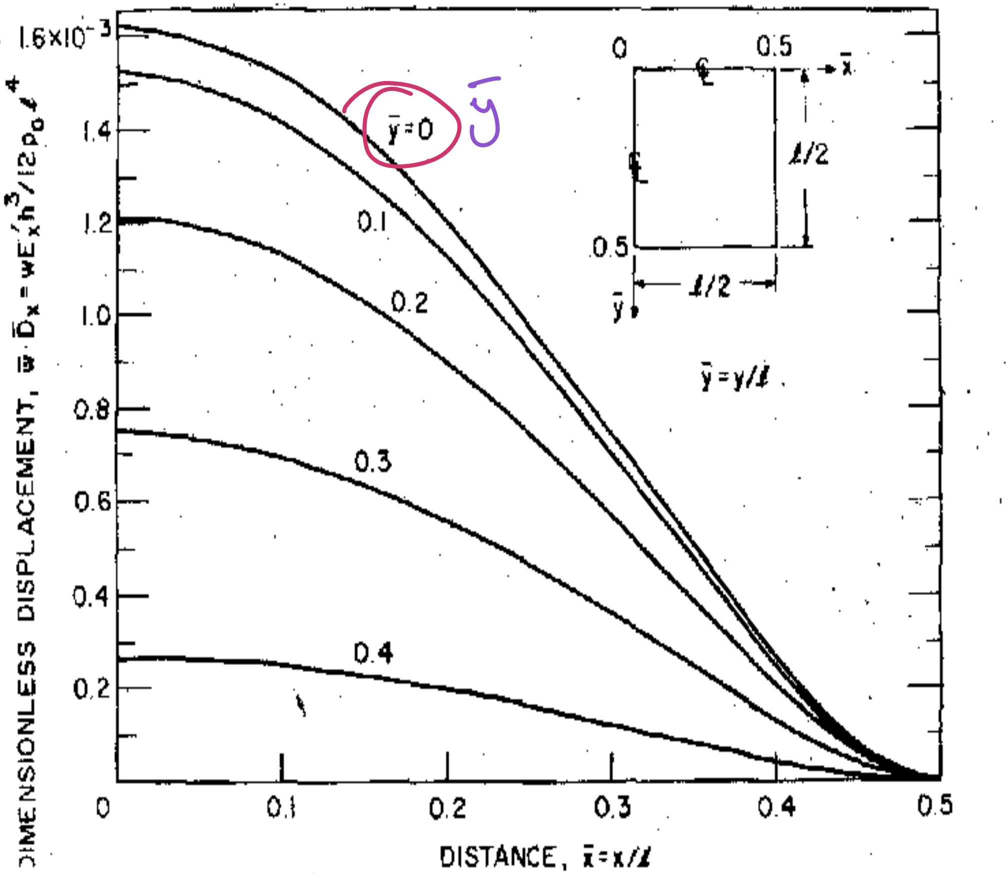
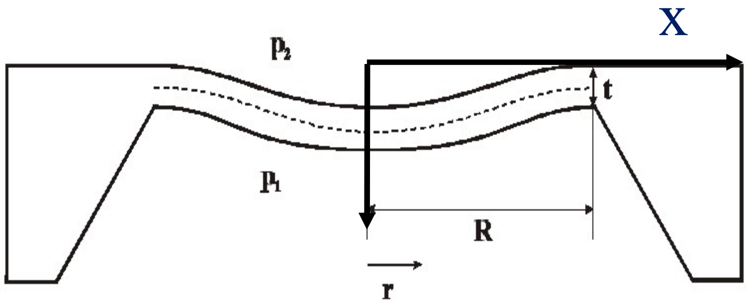
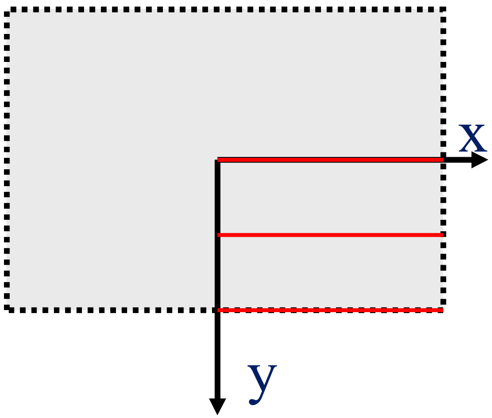


Fig. 4. Dimensionless displacement of a square silicon diaphragm having built-in edges as a function of position on the diaphragm.

Solution of the plate bending problem

(S.K.Clark &K.D. Wise, IEEE Tr. ED 26, 1979)





$$\sigma_{xxmax} = 0.3\ p\ (L/H)^2$$

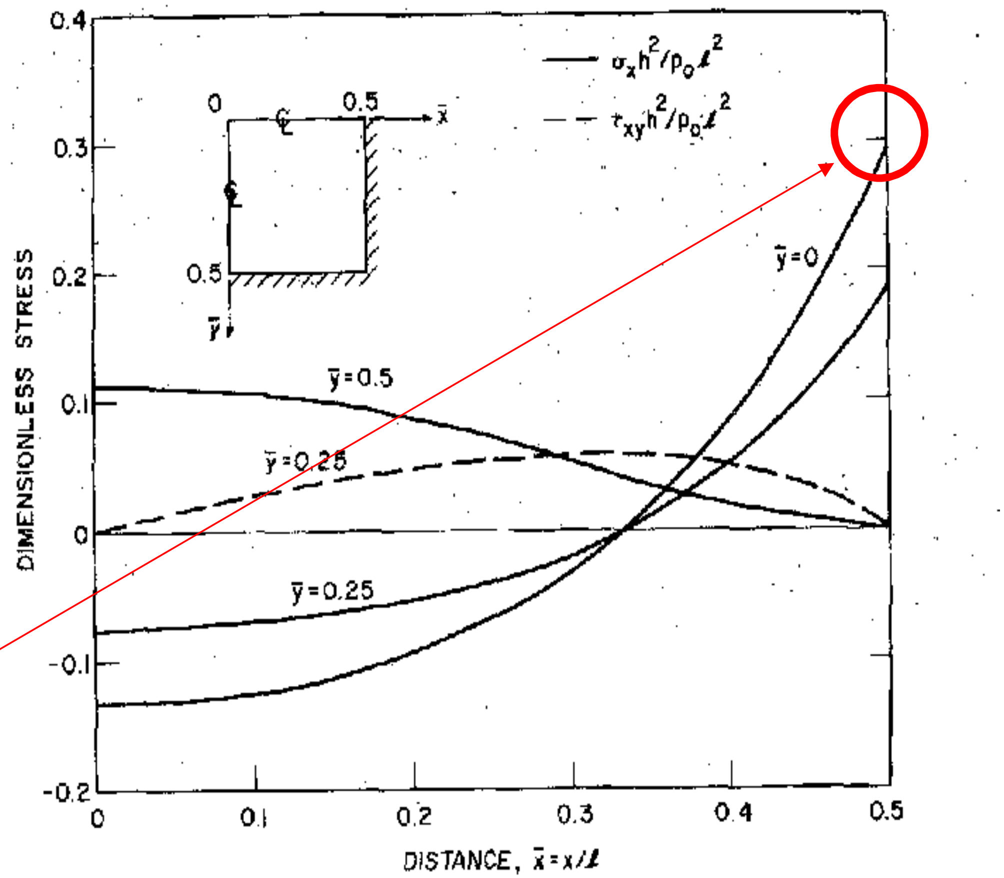
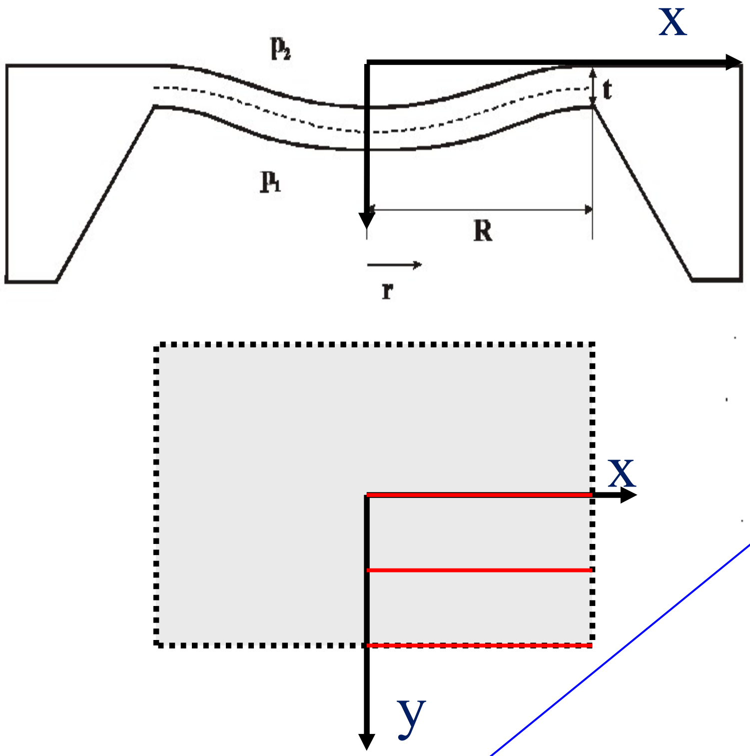
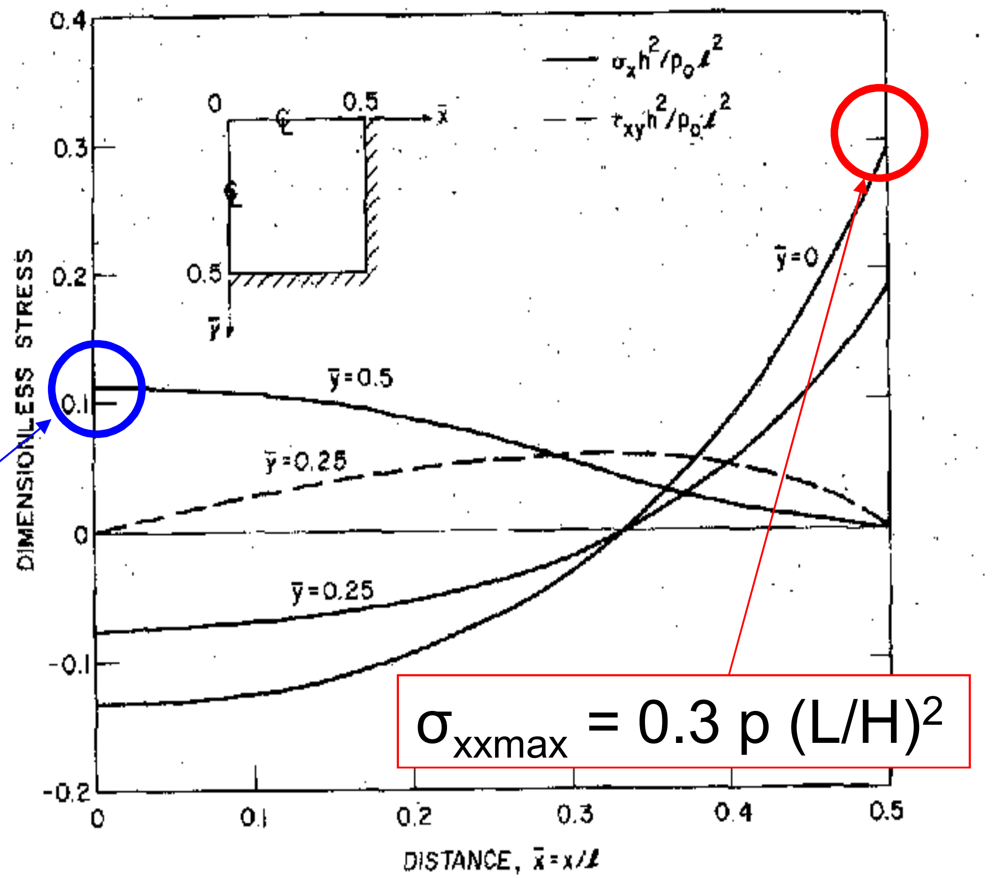


Fig. 5. Dimensionless stress distributions on a silicon diaphragm having built-in edges.

Solution of the plate bending problem

(S.K.Clark &K.D. Wise, IEEE Tr. ED 26, 1979)





σ_{xxmax} = 0.3 p $(L/H)^2$

σ_{yy} = 0.1 p $(L/H)^2$

Fig. 5. Dimensionless stress distributions on a silicon diaphragm having built-in edges.

Advanced Solid-State Sensors M

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Microelectronics Laboratory (main building - third floor)

Course Program 2019-2020

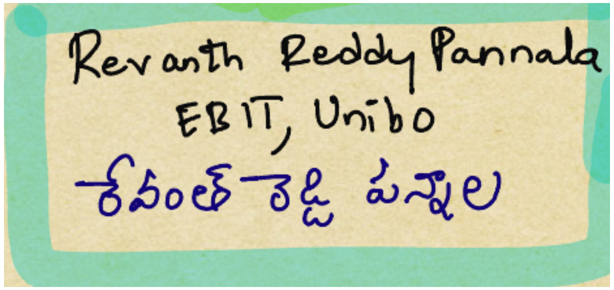
1.Introduction – Classification of sensors

2.Main physical effects used in solid-state sensors and simple readout circuits

3.Resolution & Noise

4.Image sensors

5.Mechanical sensors



Mechanical Sensors

Characterization of mechanical sensors

- MEMS sensors and the different transduction approaches:

    1. Capacitive accelerometers
    2. Piezoelectric sensors
    3. Piezoresistive pressure sensors

22/04/25

Mechanical Sensors

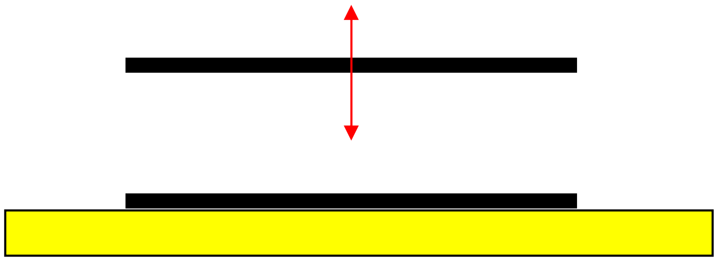
Characterization of mechanical sensors

- MEMS sensors and the different transduction approaches:

    1. Capacitive accelerometers
    2. Piezoelectric sensors
    3. Piezoresistive pressure sensors

Capacitive sensors

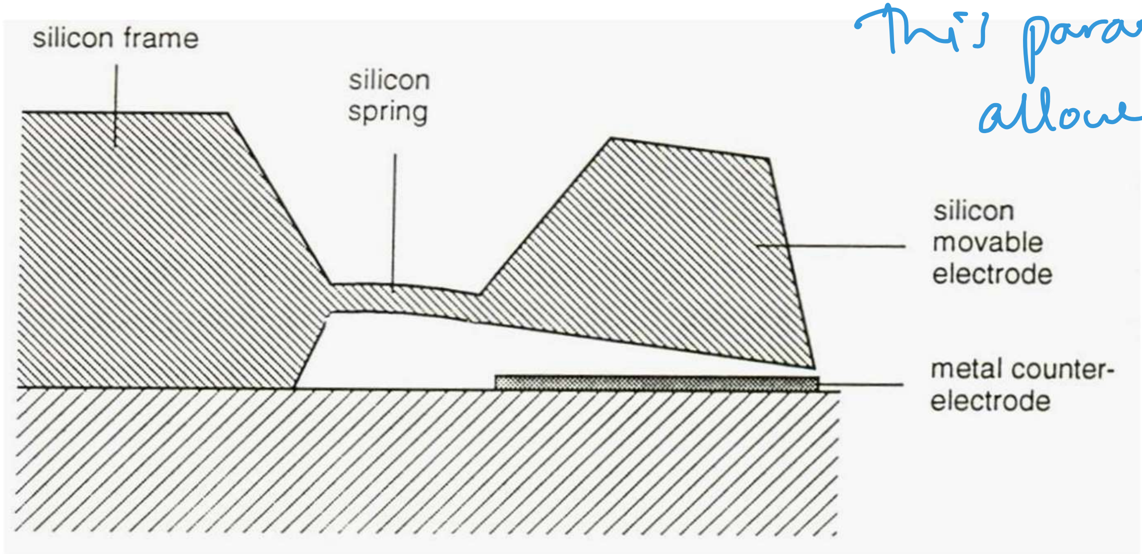
expression of parallel plate capacitor



$$\boxed{C_0 = \epsilon_0 \epsilon \frac{A}{d}} = \epsilon_A \frac{1}{d}$$

This parameter is allowed to change

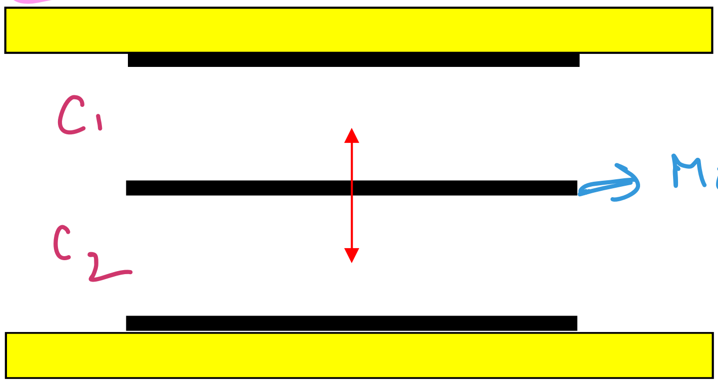
* example of capacitor in which one of the plate moves



Why don't we use THREE PLATES!

Differential capacitance system

upper & lower plates are fixed.



Movable plate

$$C_1 = \epsilon_A \frac{1}{x_1} = \epsilon_A \frac{1}{d+x} = C_0 - \Delta C,$$
$$C_2 = \epsilon_A \frac{1}{x_2} = \epsilon_A \frac{1}{d-x} = C_0 + \Delta C.$$

$$C_2 - C_1 = 2\Delta C = 2\epsilon_A \frac{x}{d^2 - x^2}$$

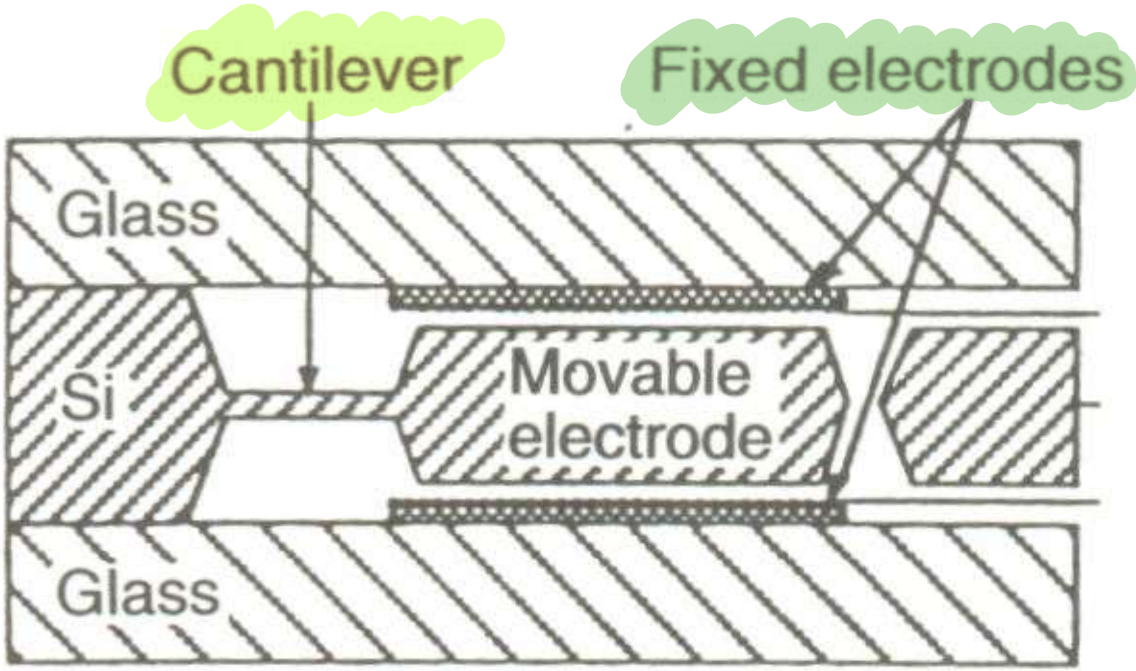
$$\Delta C x^2 + \epsilon_A x - \Delta C d^2 = 0$$

$\Delta C x^2$ is negligible

$$x \approx \frac{d^2}{\epsilon_A}\Delta C = d\frac{\Delta C}{C_0}$$

Conclusion: By measuring ΔC we measure 'x'

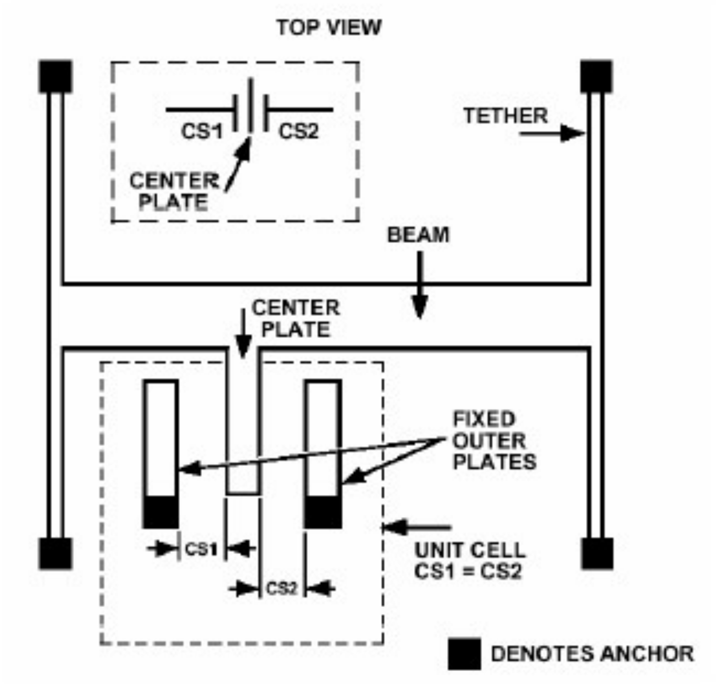
Differential capacitance system: vertical structure :

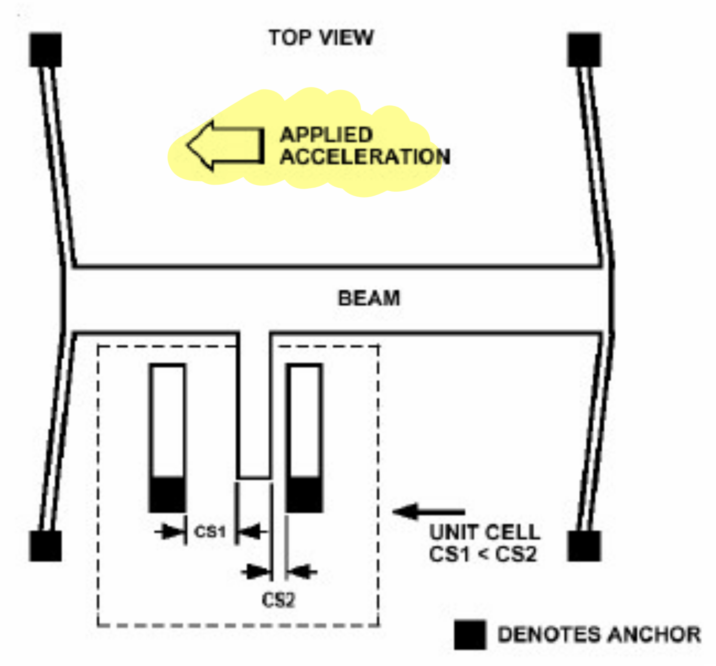


(HITACHI)

Differential capacitance system: lateral structure

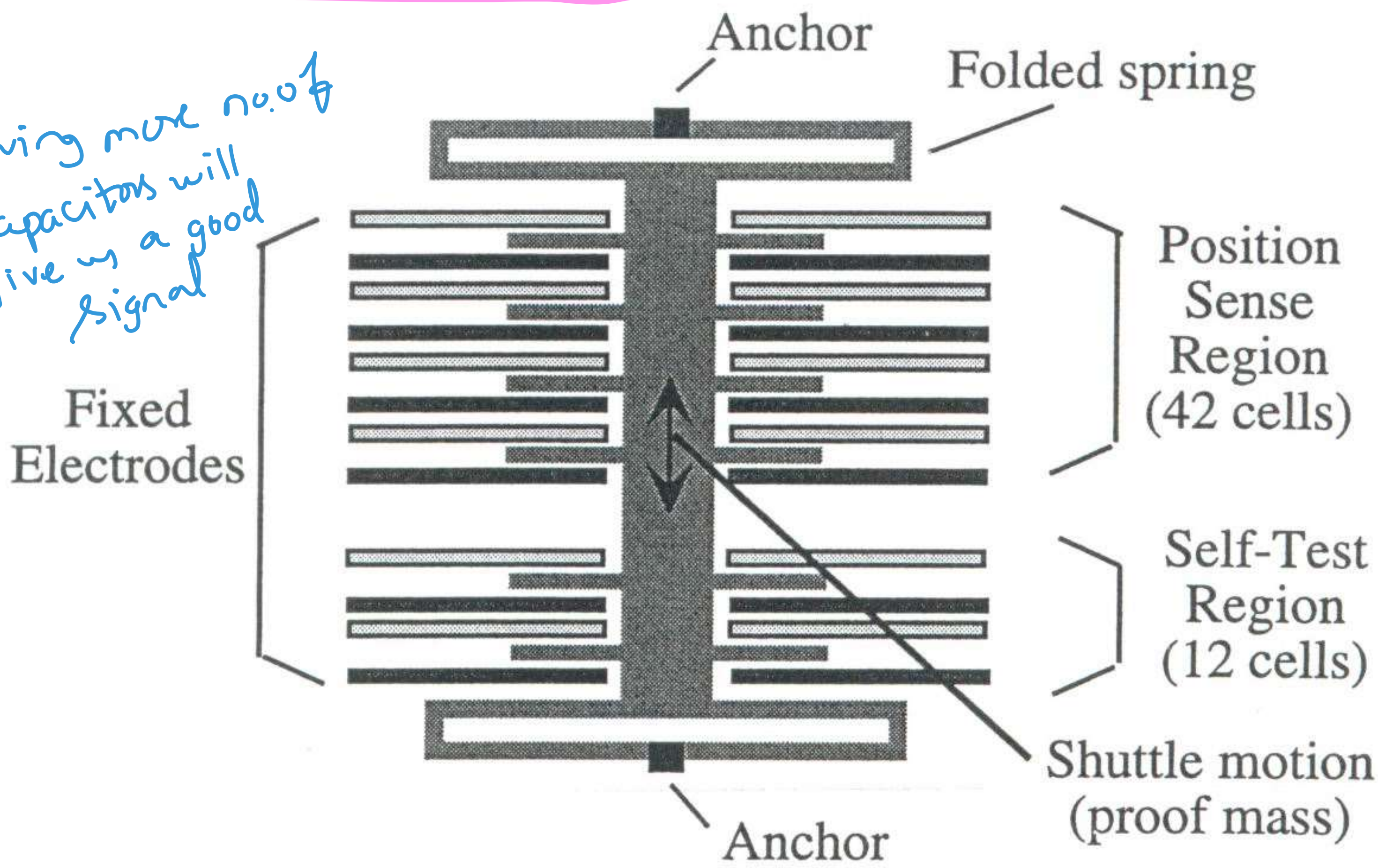
ADXL Differential Capacitive Sensor

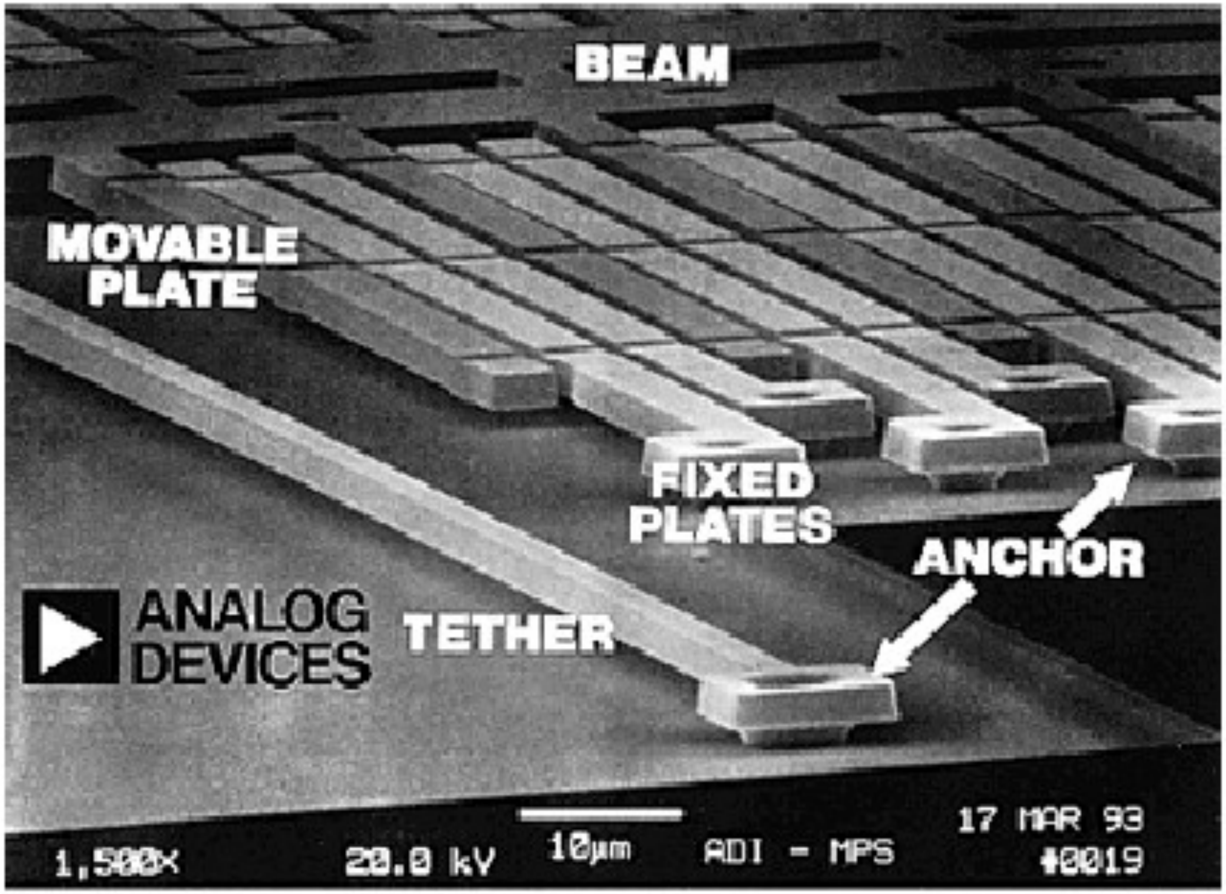




ANALOG DEVICES ADXL

Having more no. of capacitors will give us a good signal



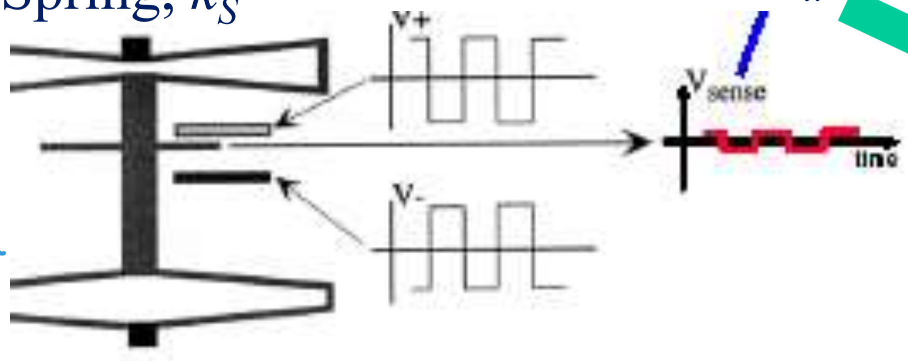

BEAM
MOVABLE PLATE
FIXED PLATES
ANCHOR
TETHER
ANALOG DEVICES
1,500X
20.0 kV
10µm
ADI - MPS
17 MAR 93
#0019

READOUT CIRCUIT (1)

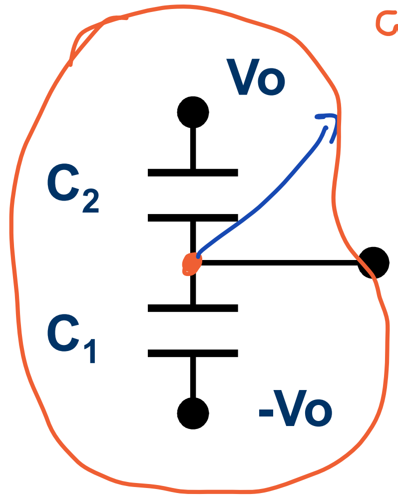
Spring, k_S

compressed

extended



V_x



Total charge 'Q' Globally cannot change because we are measuring V_x with '∞' impedance

Vx voltage of middle plate

k_S is the spring constant → $F_S = k_S x$

$$(V_x + V_0)C_1 + (V_x - V_0)C_2 = 0$$

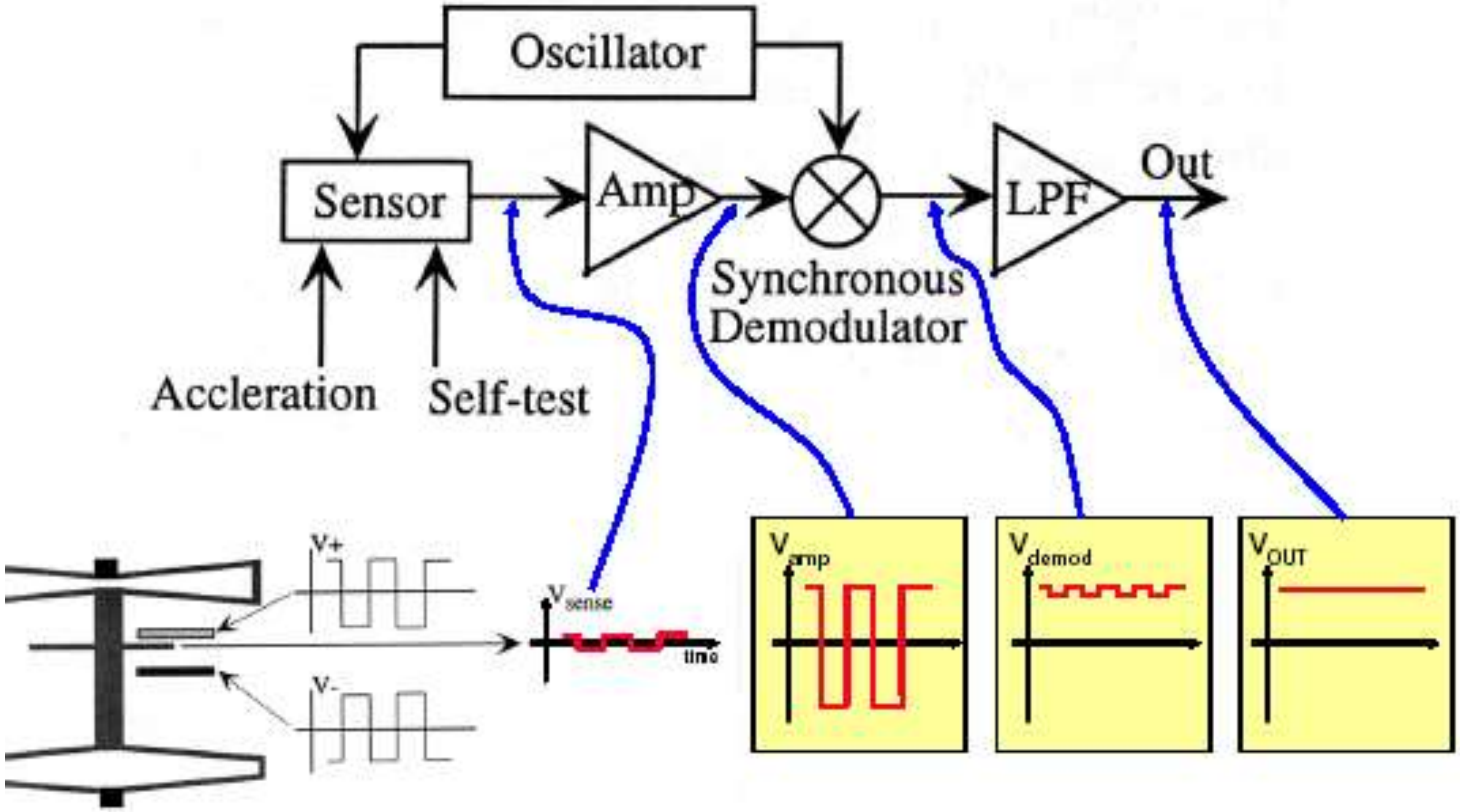
$$V_x = V_0 \frac{C_2 - C_1}{C_2 + C_1} = \frac{x}{d} V_0$$

$$a = \frac{k_S}{m} x$$

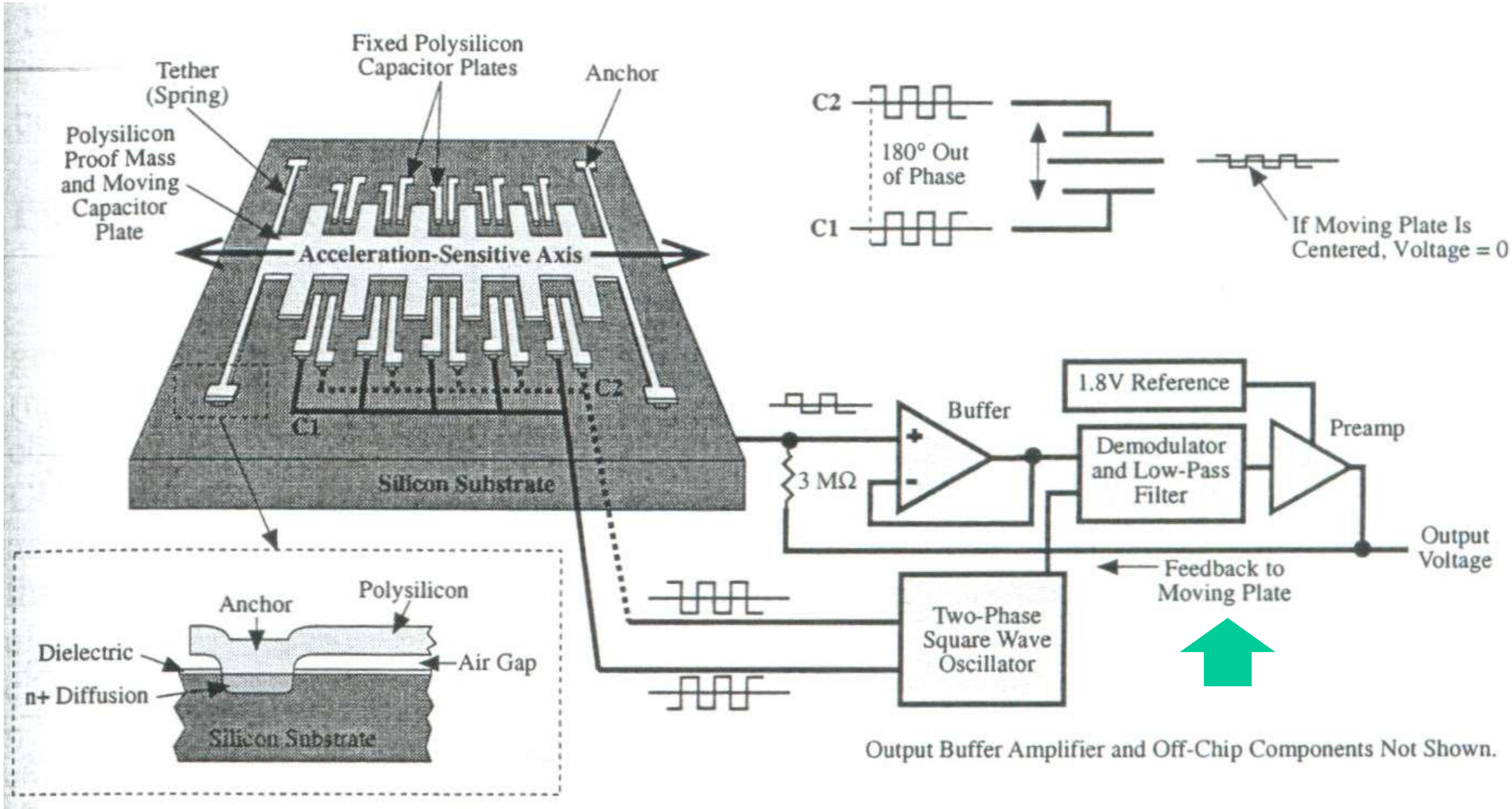
$$a = \frac{k_S d}{m V_0} V_x$$

Readout circuit (1)

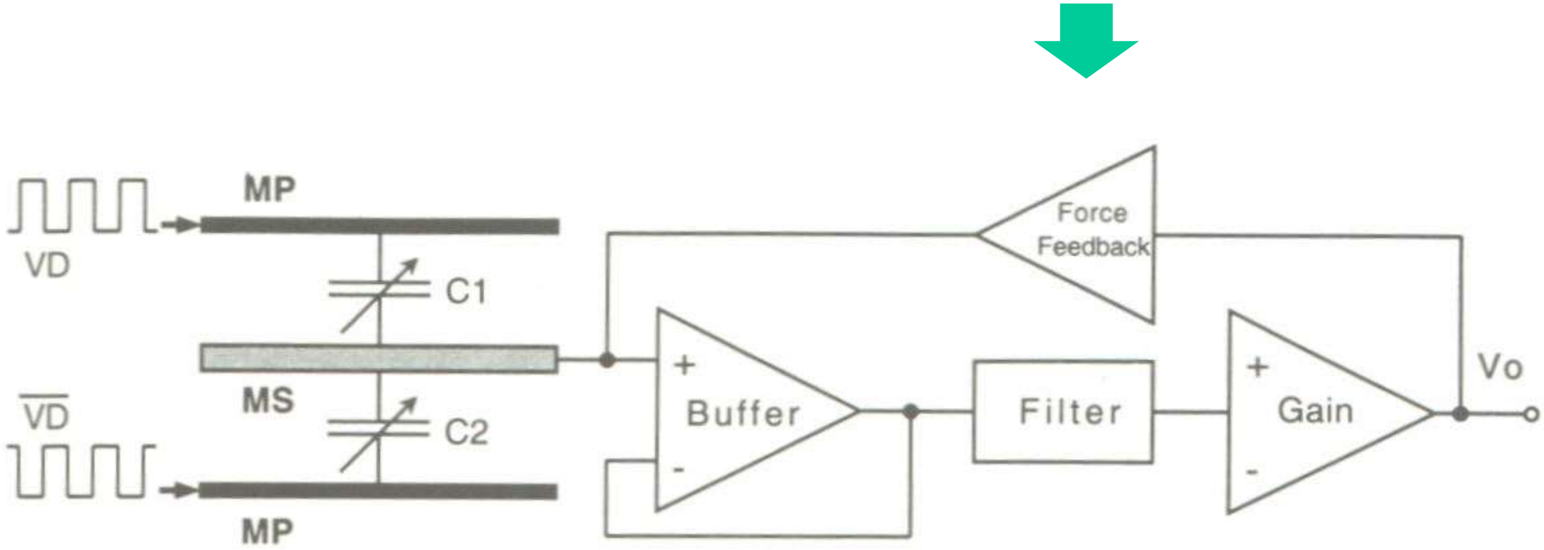
This is how Accelerometers work!



(2) Readout with feedback



(2) Readout with feedback: closed-loop technique with a force-balancing effect



Noise: «Measurements of noise characteristics of MEMS accelerometers» by F. Mohd-Yasin et al., George Washington University, Washington D.C., Solid-State Electronics 2003.

Fig. 2. Noise measurement system for MEMS accelerometers.

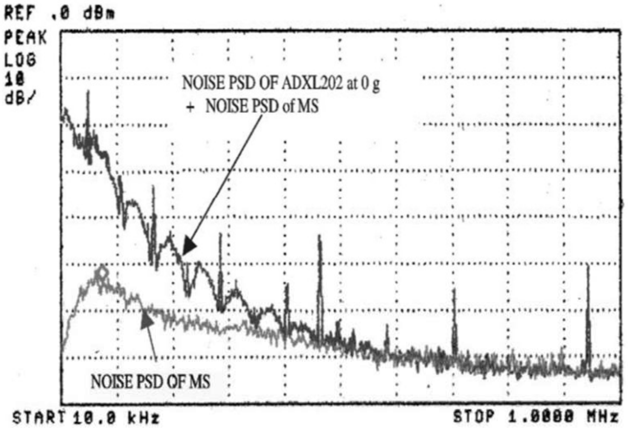


Fig. 3. ADXL202 noise PSD.

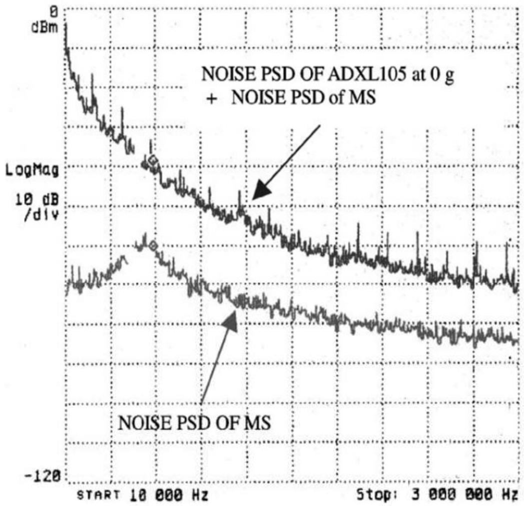


Fig. 4. ADXL105 noise PSD.

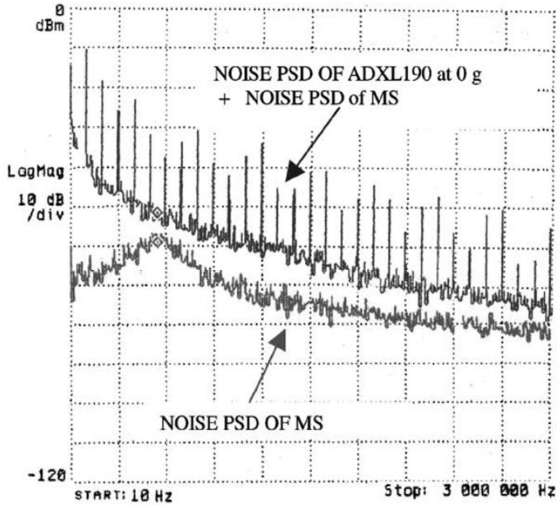


Fig. 5. ADXL190 noise PSD.

Mechanical Sensors

Characterization of mechanical sensors

- MEMS sensors and the different transduction approaches:

  1. Capacitive accelerometers
  2. Piezoelectric sensors
  3. Piezoresistive pressure sensors

Piezoelectric sensors:

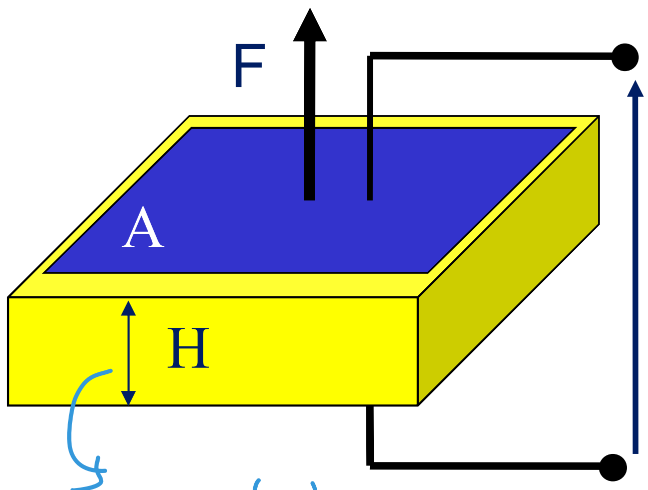
- Generalization of the piezoelectric effect: piezoelectric tensor
- Accelerometer PI-FET proposed by Chen et al. in 1980
- Piezoeletric oxide semiconductor field effect transistor touch sensing devices proposed by R. Dahiya et al. in 2009

Generalization of the piezoelectric effect

From Gauss Theorem

Variation of charge due to application of force 'F'

mechanical stress $= F/A$



Thickness 'H'
Area 'A'

$$\Delta Q = DA = d\sigma A = dF$$

Displacement vector

$$\Delta V = \frac{dF}{\varepsilon A/H} = \frac{dH\sigma}{\varepsilon}$$

Some coefficient

variation of voltage due to application of Force

$$\boxed{D_i = \varepsilon_{ik}E_k + d_{i,kl}\sigma_{kl}}$$

Displacement vector in 'i' direction

Stress Tensor

permittivity in Tensor form

Electric field vector

Generalization of the piezoelectric effect

$$D_i = \varepsilon_{ik} E_k + d_{i,kl} \sigma_{kl}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

	$\varepsilon_{11} = \varepsilon_{22}$	ε_{33}
SiO_2	$4.5\ \varepsilon_0$	$4.6\ \varepsilon_0$
$BaTiO_3$	$1268\ \varepsilon_0$	$1419\ \varepsilon_0$
ZnO	$10.8\text{-}11\ \varepsilon_0$	$10.8\text{-}11\ \varepsilon_0$

Generalization of the piezoelectric effect

$$D_i = \varepsilon_{ik} E_k + d_{i,kl} \sigma_{kl}$$

$\sigma_{kl} \Rightarrow \sigma_m$, $m = 1,\ldots,6$ $\Rightarrow d_{i,kl} \Rightarrow d_{im}$

$$\mathbf{d} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix}$$

'6' independent components

Generalization of the piezoelectric effect

$$\boldsymbol{d} = \begin{bmatrix} d_{11} & d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2\,d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

SiO_2

$d_{11} = -2.31$ pC/N

$d_{14} = -0.727$ pC/N

$BaTiO_3$

$d_{31} = -79$ pC/N

$d_{33} = 191$ pC/N

$d_{15} = 270$ pC/N

ZnO

$d_{31} = -5.1$ pC/N

$d_{33} = 12.3$ pC/N

$d_{15} = -8.3$ pC/N

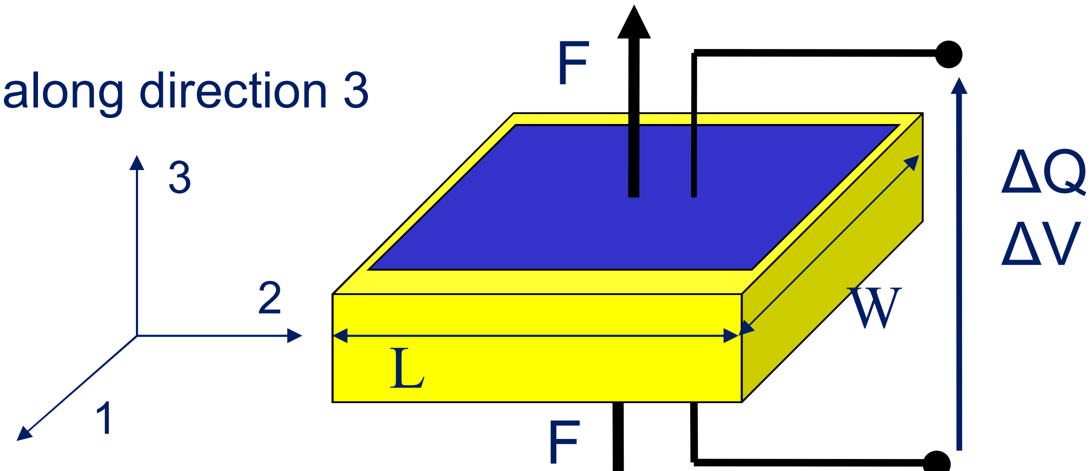
d) How do we measure the coefficients?

Characterization of the piezoelectric coefficients

1) **Longitudinal coefficient**

Example: ZnO measured along direction 3 shown in figure

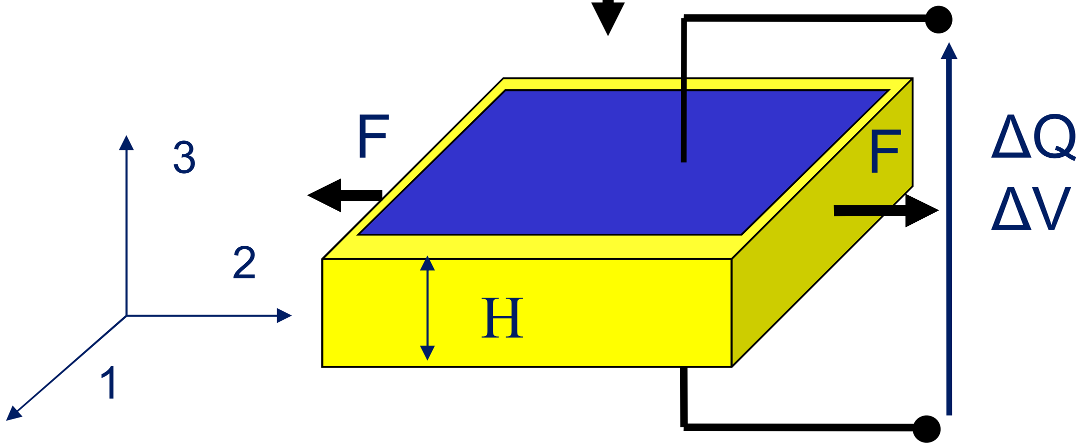
$D_3 = d_{33}$ F/(WL)



2) **Transverse coefficient**

Example: ZnO measured along direction 3 shown in figure

$D_3 = d_{31}$ F/(HW)

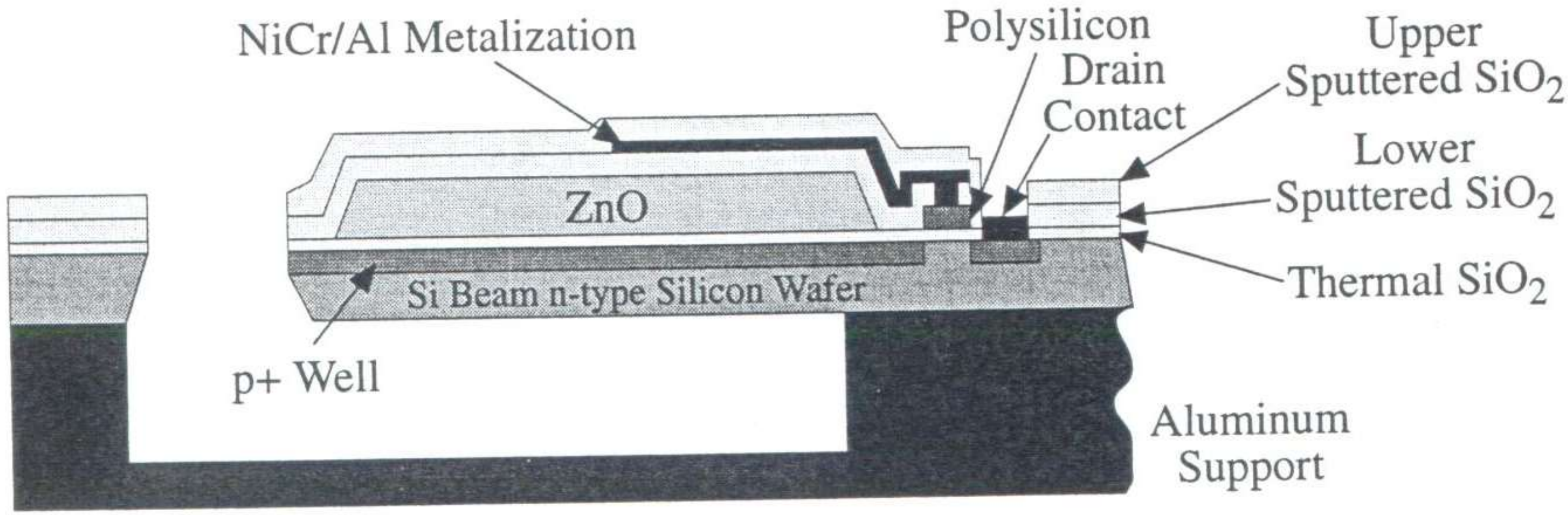


Q) How can we exploit the piezoelectric effect?

Integrated silicon microbeam PI-FET accelerometer

Any

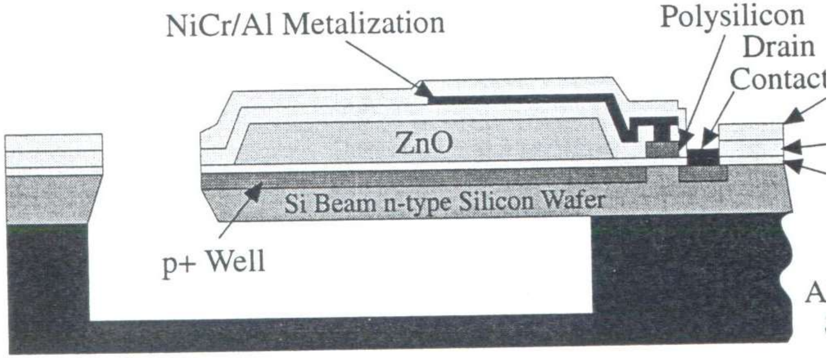
P. Chen et al., IEEE Tr. ED 29, 1982

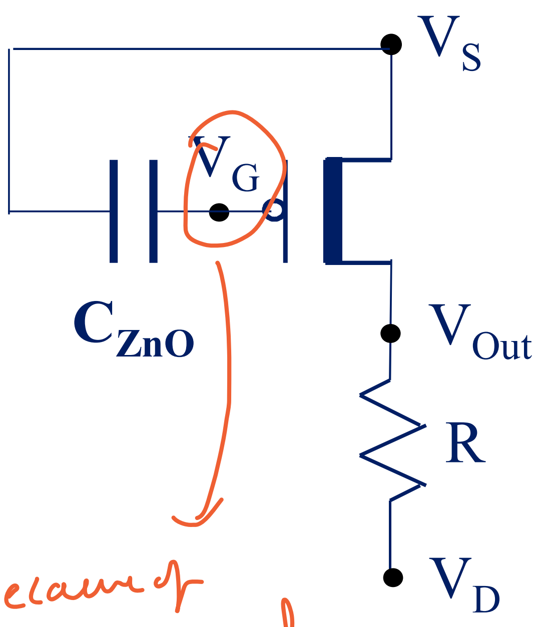


The structure has become a Cantilever

Readout circuit:

1-stage amplifier (depletion p-channel MOSFET)



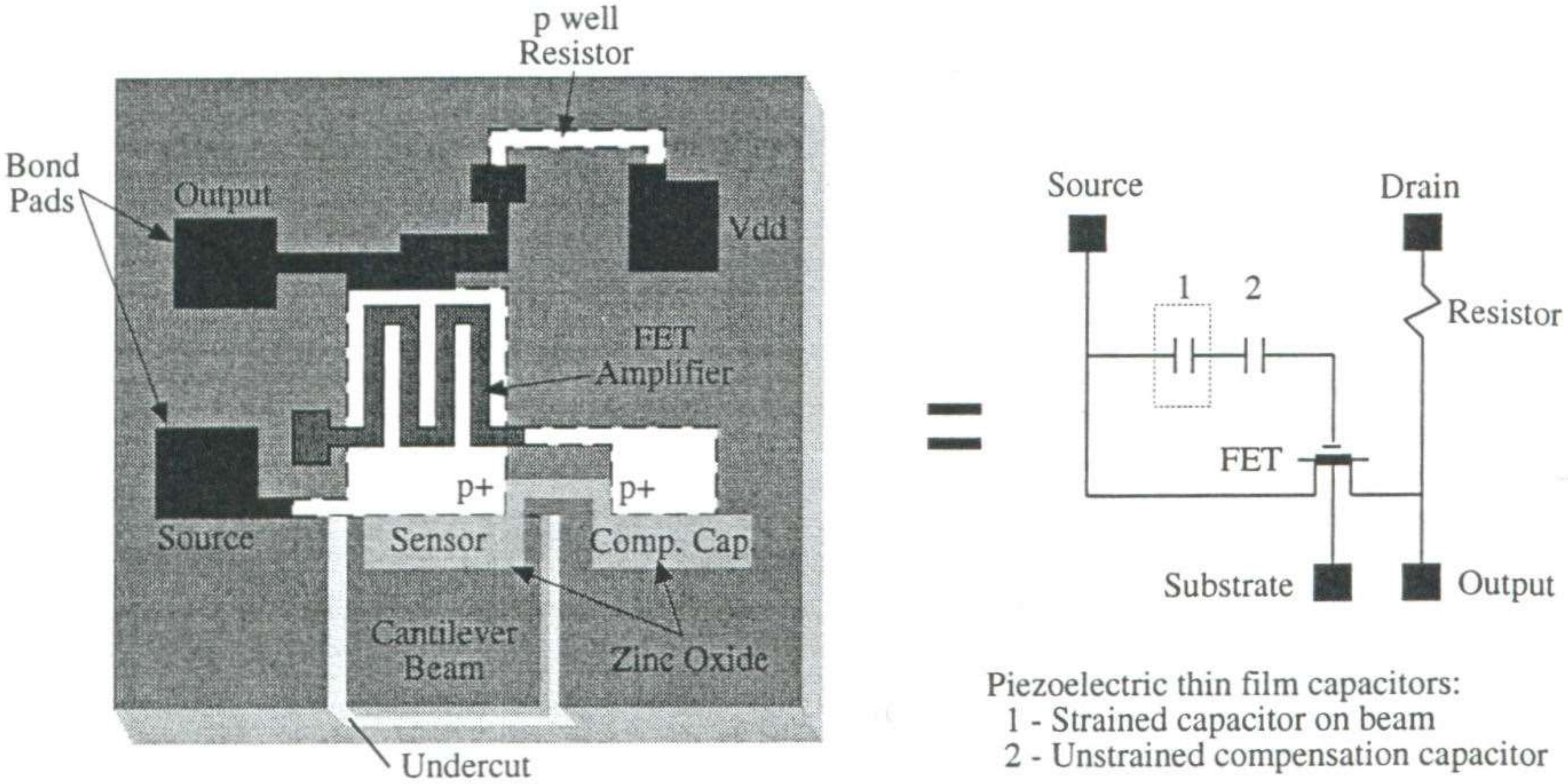


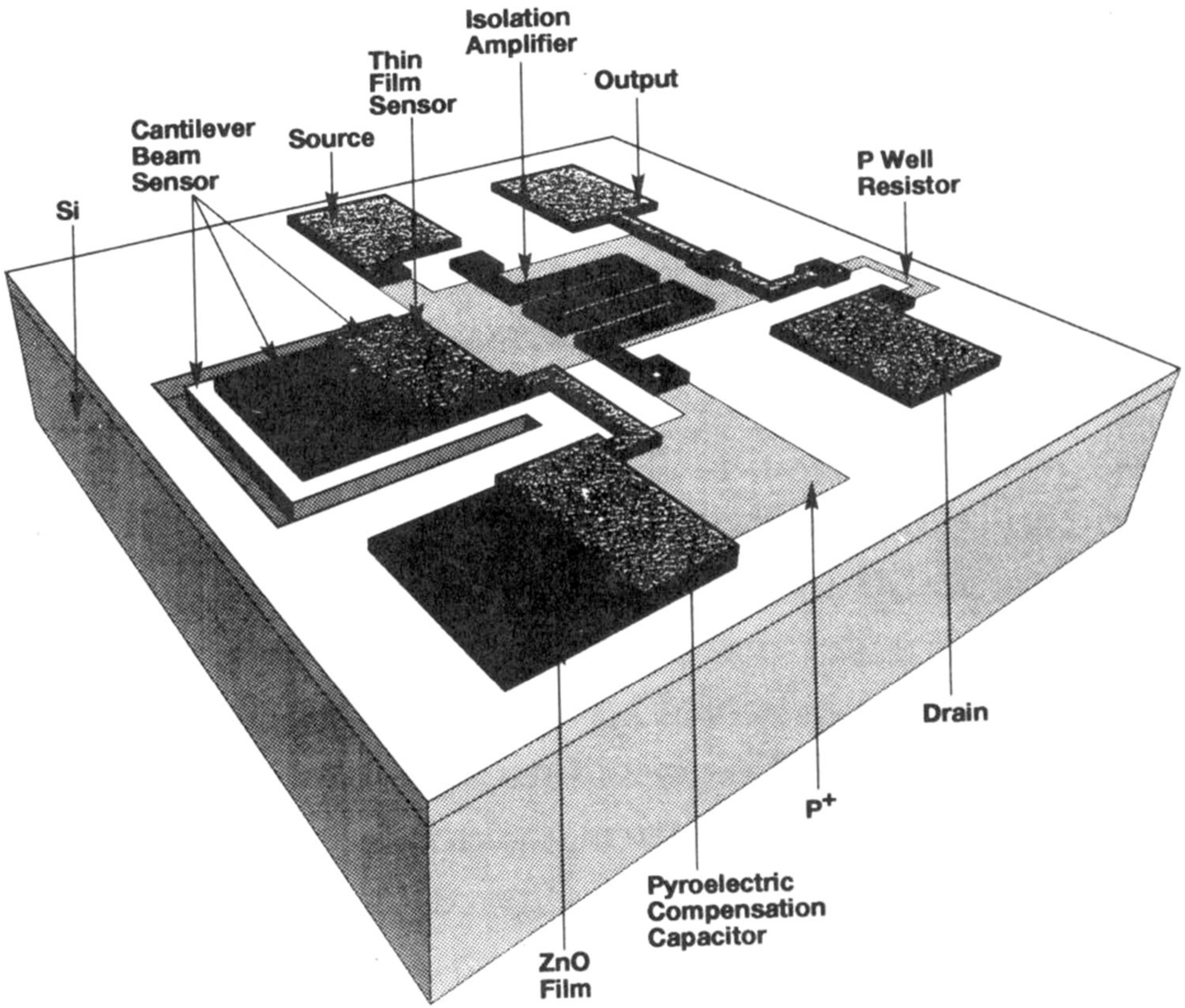
Voltage gain: $A_v = -R\, g_{mp}$

Because of the force applied on the ZnO layer the capacitance varies then by varying V_G

Readout circuit:

2. Pyroelectric compensation capacitor

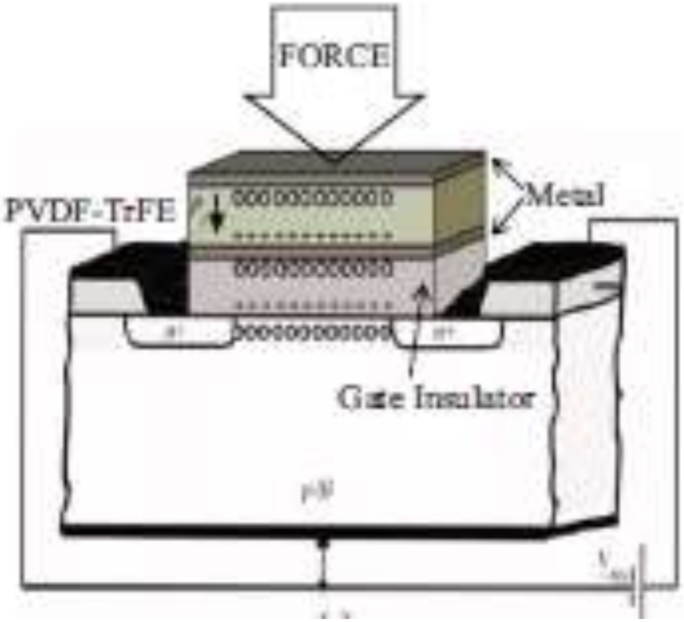


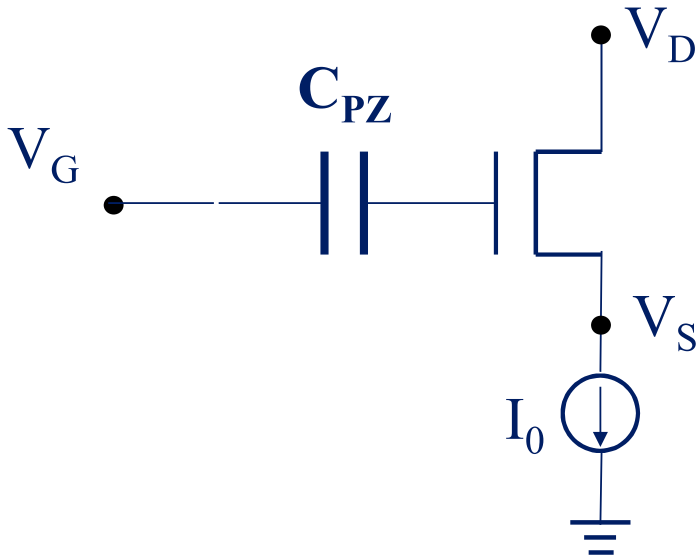

Isolation
Amplifier
Thin
Film
Sensor
Output
Cantilever
Beam
Sensor
Source
P Well
Resistor
Si
Drain
P+
Pyroelectric
Compensation
Capacitor
ZnO
Film

APPLIED PHYSICS LETTERS 95, 034105 (2009)

Piezoelectric oxide semiconductor field effect transistor touch sensing devices

Ravinder S. Dahiya,[1,a] Giorgio Metta,[1,b] Maurizio Valle,[2] Andrea Adami,[3] and Leandro Lorenzelli[3]
[1]RBCS, Italian Institute of Technology, Genoa 16163, Italy
[2]DIBE, University of Genoa, 16145, Italy
[3]Bio-MEMS Group, Fondazione Bruno Kesseler, Povo, Trento 38050, Italy





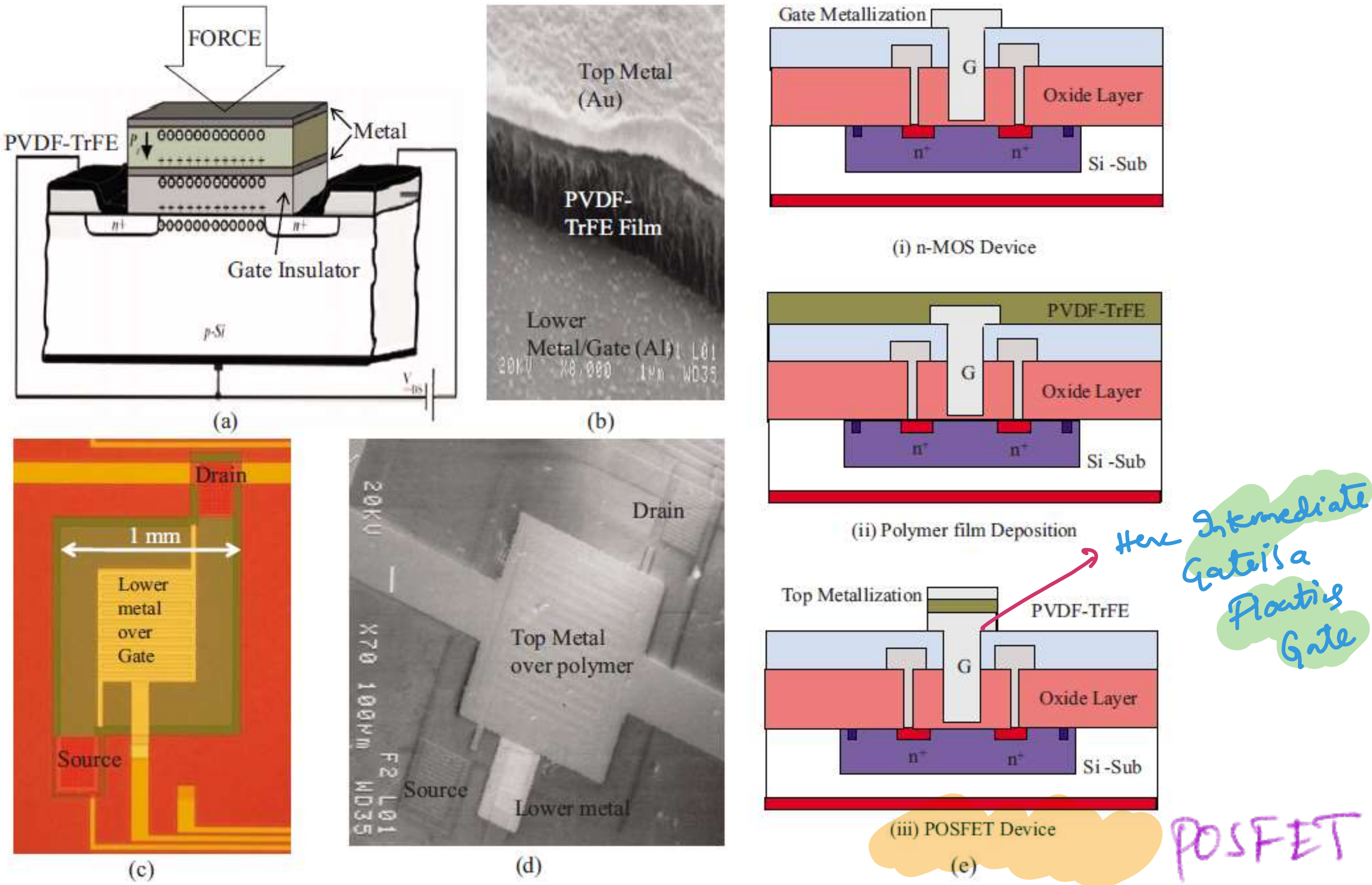
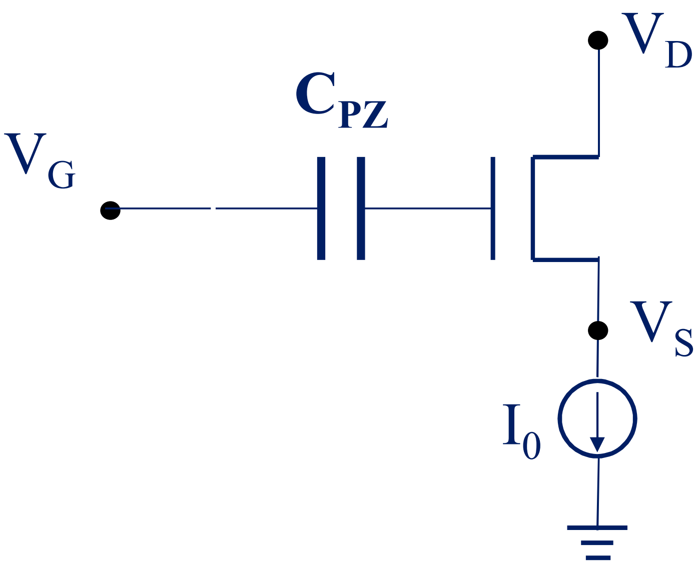
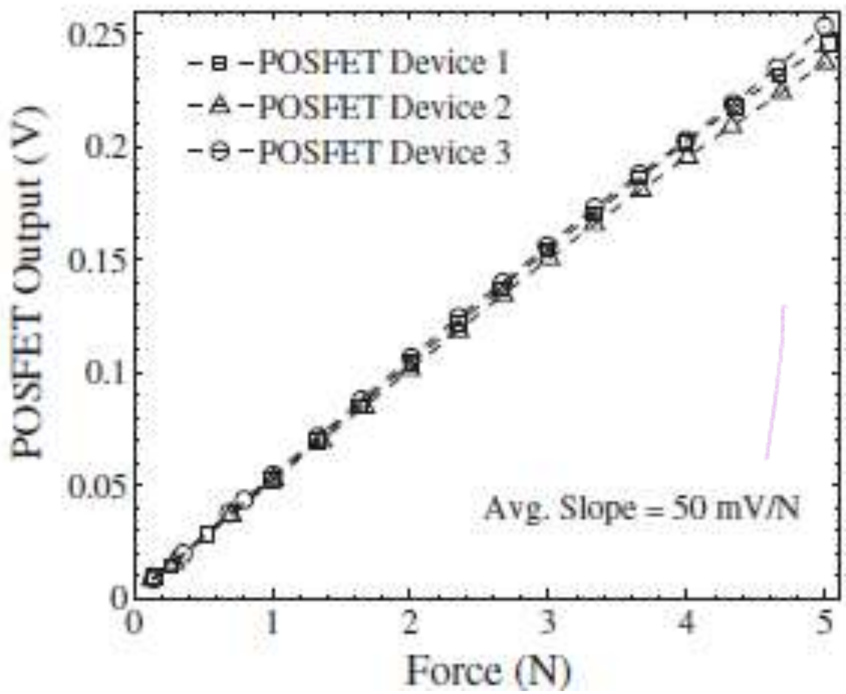


FIG. 1. (Color online) POSFET touch sensing device implementation. (a) Structure and working of POSFET touch sensing device. (b) SEM image of POSFET cross section. (c) POSFET before polymer deposition. (d) SEM image of POSFET after polymer deposition. (e) Fabrication steps of POSFET touch sensing device.



V_D
C_{PZ}
V_G
V_S
I_0



POSFET Device 1
POSFET Device 2
POSFET Device 3
Avg. Slope = 50 mV/N
POSFET Output (V)
Force (N)
0
0.05
0.1
0.15
0.2
0.25
1
2
3
4
5

Mechanical Sensors

Characterization of mechanical sensors

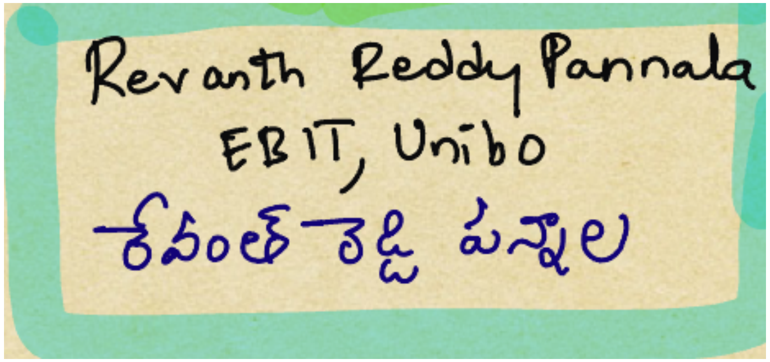
- MEMS sensors and the different transduction approaches:

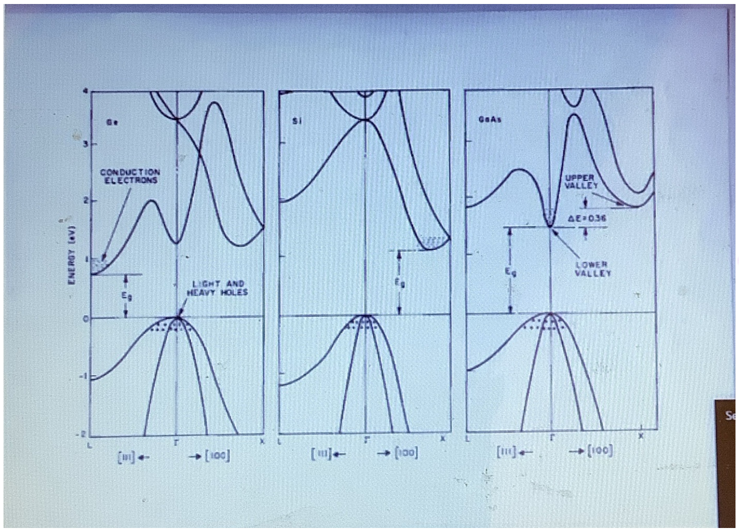
  1. Capacitive accelerometers
  2. Piezoelectric sensors
  3. Piezoresistive pressure sensors

Piezoresistive pressure sensors:

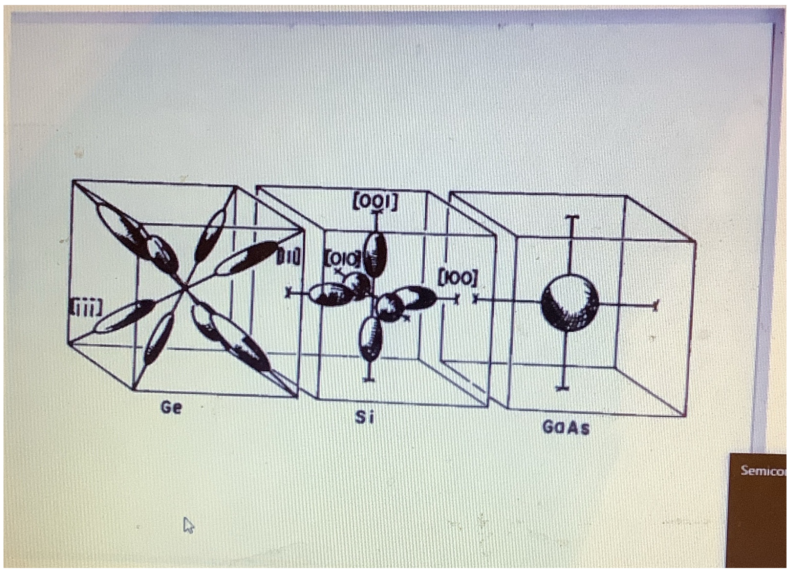
- Generalization of the piezoresistive effect: piezoresistive tensor
- Graphical representation of the piezoresistive coefficients in silicon (Kanda, 1982)

- The MAP sensor by MOTOROLA





- This is a slide showing bands of Si, Ge & GaAs
  for Si has 6 minima in the CB & they are aligned 2x2 in [100] [010] & [001] directions



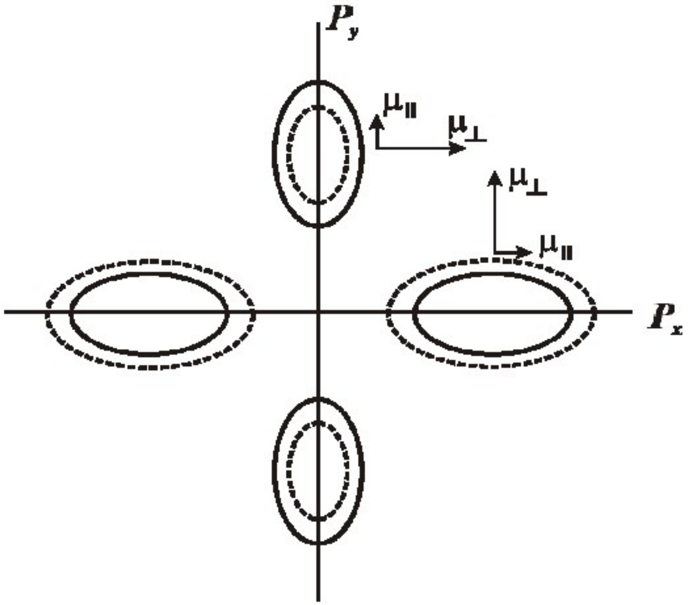
- This figure shows the Surfaces of Equal Energy
- for Si the surface of equal energy are Elipsoids of Rotation
  i.e Two axis are equal and third axis different

- we remember that two axises that are equal are associated to the Transverse Mass of the Mass Tensor
  The other axis of the ellipsoid is Longitudinal Mass.

- In order to describe the Dynamics of electrons of Silicon we describe the relation b/w Force & acceleration using a Scalar Mass that we called Effective Mass This mass is a combination of Transversal & Longitudinal Mass.

Lecture-23 Part-1

Piezoresistive effect



$\Delta\rho/\rho_0 = \pi\ \sigma$

ρ- silicon resistivity

σ- stress

$$\mathbf{E} = \rho_e\ [1 + \mathbf{\Pi}\ \sigma]\ \mathbf{J}$$

Electric field

Resistivity tensor

Stress

Current density

Piezoresistive effect

$$\mathbf{E} = \rho_e \, [1 + \boxed{\mathbf{\Pi}} \, \sigma] \, \mathbf{J}$$

$$\begin{pmatrix} (\Delta\rho/\rho)_{11} \\ (\Delta\rho/\rho)_{22} \\ (\Delta\rho/\rho)_{33} \\ (\Delta\rho/\rho)_{23} \\ (\Delta\rho/\rho)_{13} \\ (\Delta\rho/\rho)_{12} \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

Piezoresistive effect

$$\mathbf{E} = \rho_e \, [1 + \mathbf{\Pi} \, \sigma] \, \mathbf{J}$$

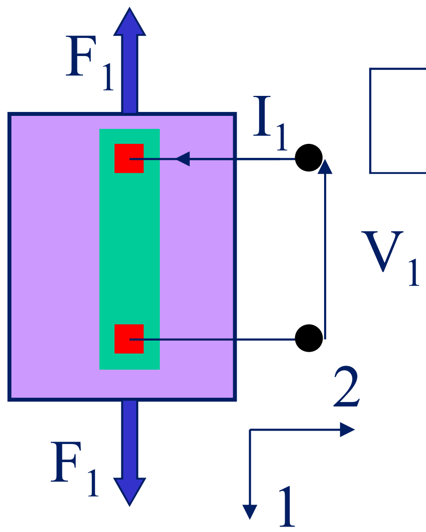
$$\frac{E_1}{\rho_e} = [1+ \pi_{11}\sigma_{11} + \pi_{12}(\sigma_{22} + \sigma_{33})] \, J_1 + \pi_{44}(\sigma_{12}J_2 + \sigma_{13}J_3)$$

$$\frac{E_2}{\rho_e} = [1+ \pi_{11}\sigma_{22} + \pi_{12}(\sigma_{11} + \sigma_{33})] \, J_2 + \pi_{44}(\sigma_{12}J_1 + \sigma_{23}J_3)$$

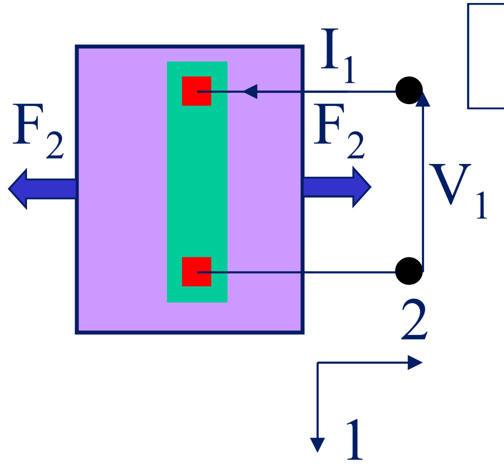
$$\frac{E_3}{\rho_e} = [1+ \pi_{11}\sigma_{33} + \pi_{12}(\sigma_{11} + \sigma_{22})] \, J_3 + \pi_{44}(\sigma_{13}J_1 + \sigma_{23}J_2)$$

$$\frac{E_1}{\rho_e} = [1+ \pi_{11}\sigma_{11} + \pi_{12}(\sigma_{22} + \sigma_{33})]\, J_1 + \pi_{44}(\sigma_{12}J_2 + \sigma_{13}J_3)$$

Type	Resistivity	π_{11}	π_{12}	π_{44}
Units	Ω -cm	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}
n-type	11.7	-102.2	53.4	-13.6
p-type	7.8	6.6	-1.1	138.1



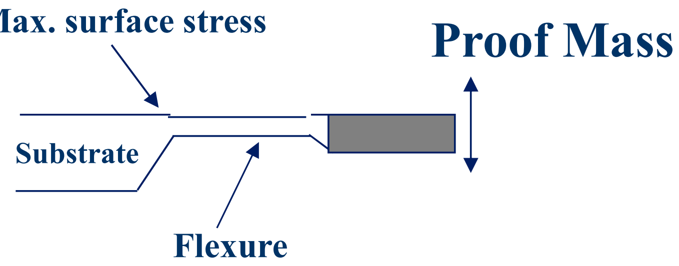
$\pi_l = \pi_{11}$

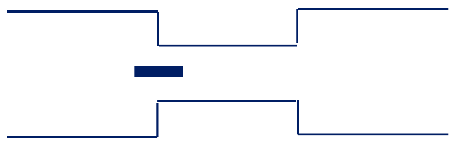


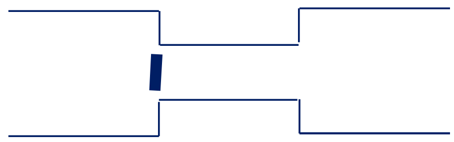
$\pi_t = \pi_{12}$

Concept of a piezoresistive sensing scheme

If piezo-resistor is along [110]:





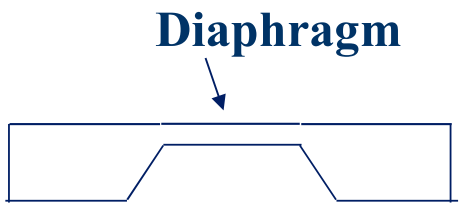


Longitudinal direction	π_l	Transverse direction	π_t
[100]	π_{11}	[010]	π_{12}
[110]	$1/2\,(\pi_{11}+\pi_{12}+\pi_{44})$	[110]	$1/2\,(\pi_{11}+\pi_{12}-\pi_{44})$

Concept of a piezoresistive sensing scheme

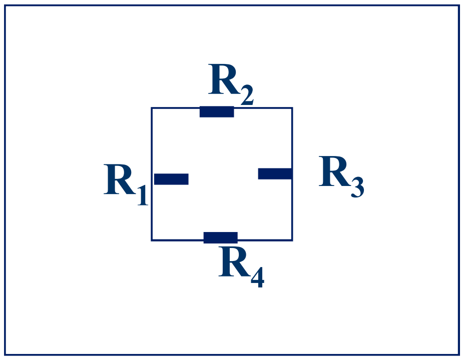
Type	π_l	π_t	orientation
Units	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	
n-type	**-102.2**	**53.4**	[100]
p-type	**6.6**	**-1.1**	[100]
n-type	**-31.2**	**-17.6**	[110]
p-type	**71.8**	**-66.3**	[110]

CROSS-SECTION



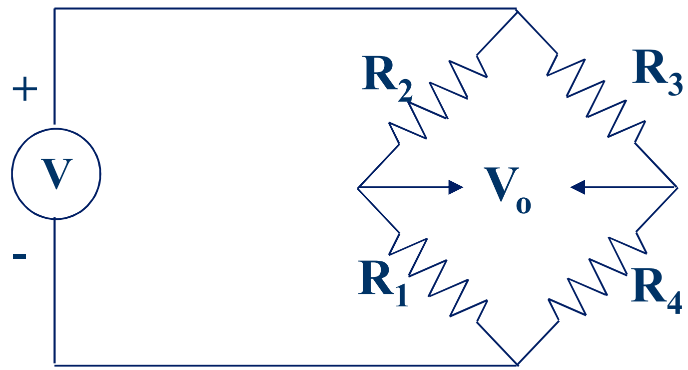
$$\frac{\Delta R_1}{R_1} = \pi_l \sigma_l + \pi_t \sigma_t$$

TOP VIEW



$$\frac{\Delta R_2}{R_2} = \pi_t \sigma_l + \pi_l \sigma_t$$

WHEATSTONE BRIDGE



$$R_1 = R_3 = (1+ \alpha_1)\, R_o$$
$$R_2 = R_4 = (1 - \alpha_2)\, R_o$$

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t \quad \text{l: longitudinal, t: transverse}$$

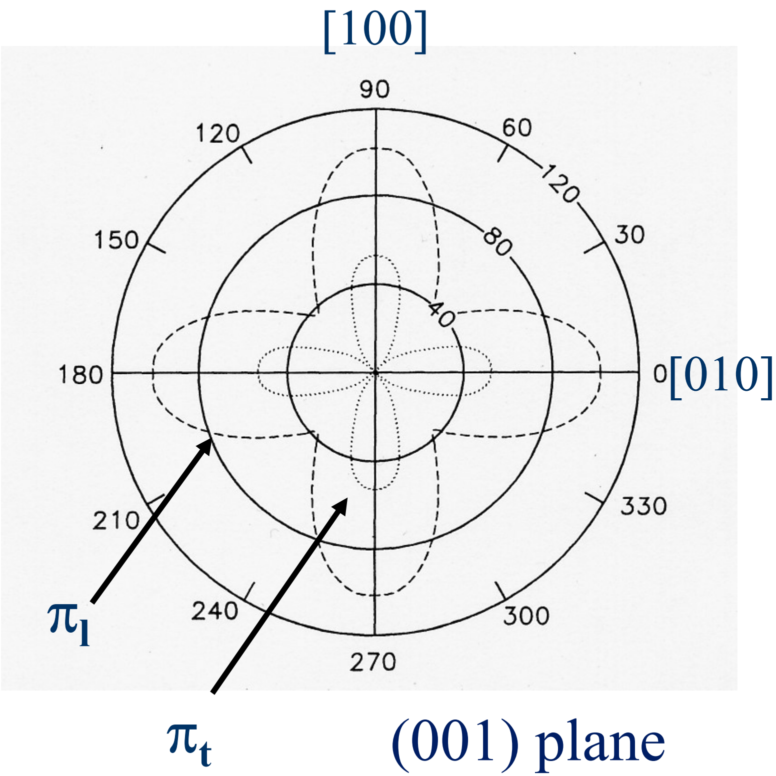
Longitudinal direction	π_l	**Transverse direction**	π_t
[100]	π_{11}	[010]	π_{12}
[110]	**1/2** $(\pi_{11}+\pi_{12}+\pi_{44})$	$[\bar{1}10]$	**1/2** $(\pi_{11}+\pi_{12}-\pi_{44})$

GRAPHICAL REPRESENTATION

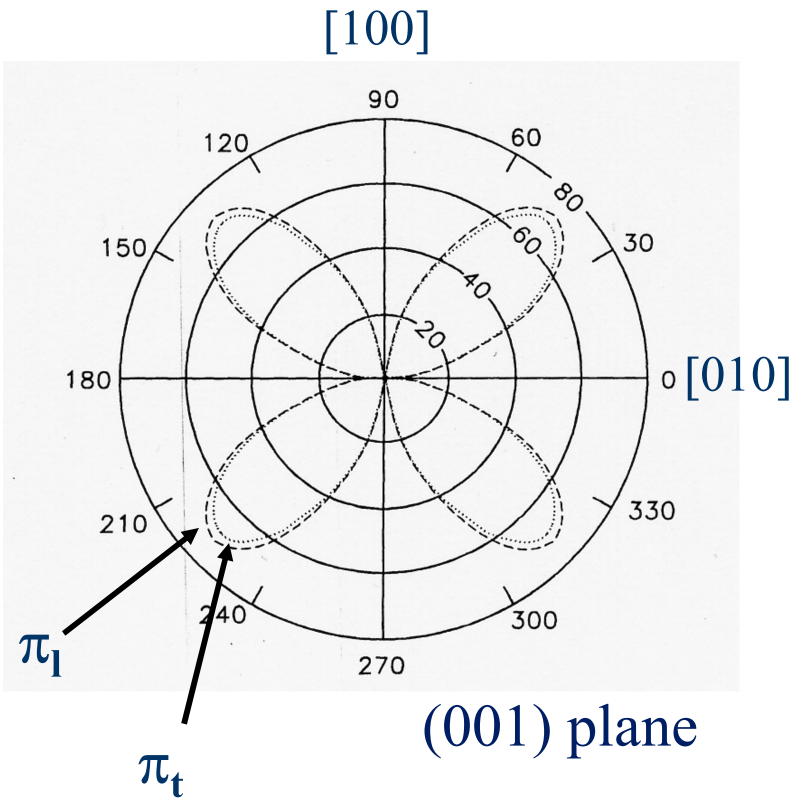
(Y. Kanda, IEEE Tr. ED 29, 1982)

Longitudinal & Transverse piezoresistance coefficients

(Y. Kanda, IEEE Tr. ED 29, 1982)



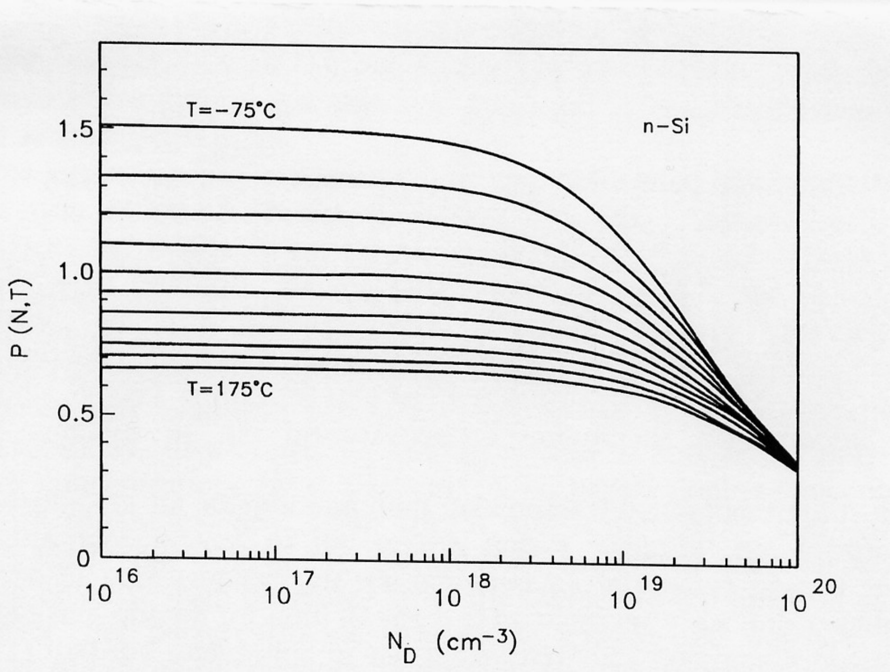
n-type Si

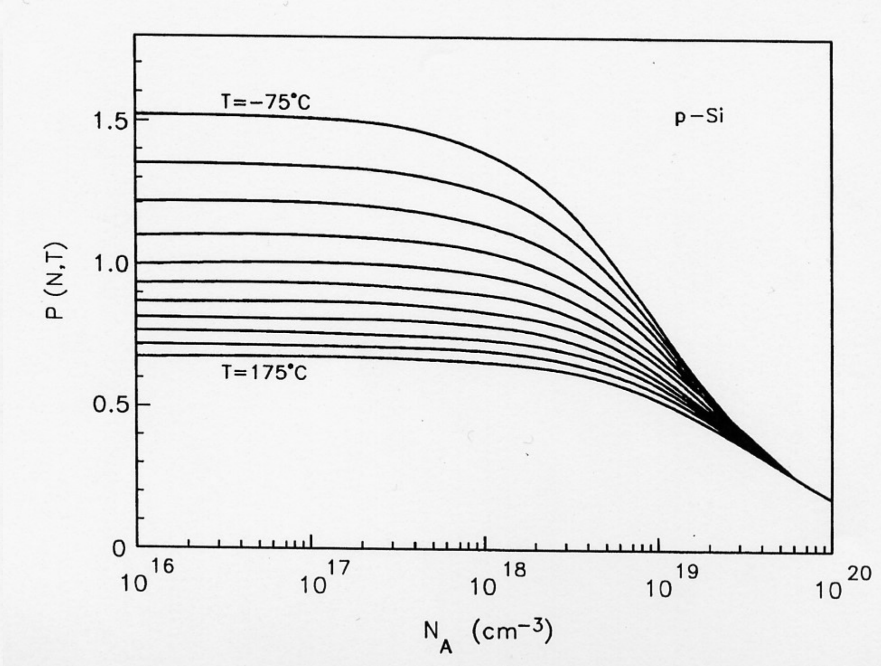


p-type Si

Piezoresistance coefficients as a function of impurity concentration and temperature for n-Si and p-Si

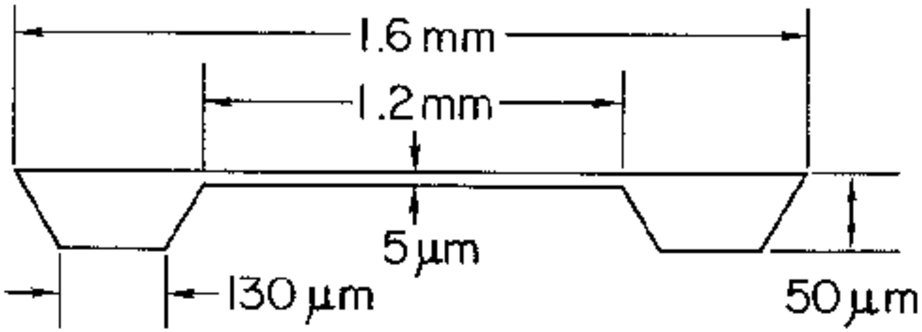
(Y. Kanda, IEEE Tr. ED 29, 1982)



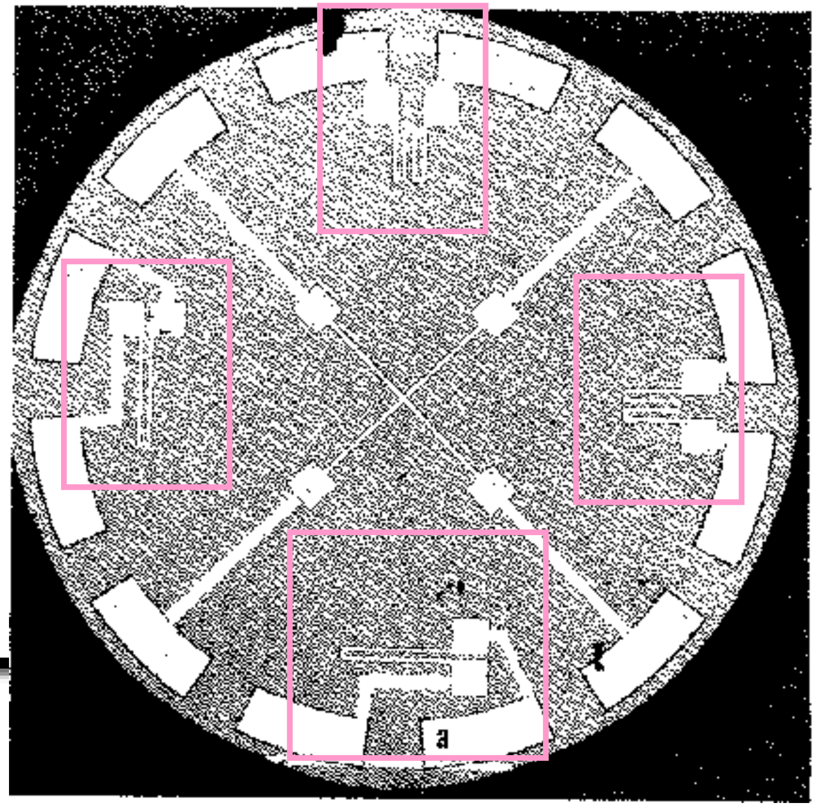


An IC Piezoresistive Pressure Sensor for Biomedical Instrumentation

(S. Samaun et al., ISSCC 1971)

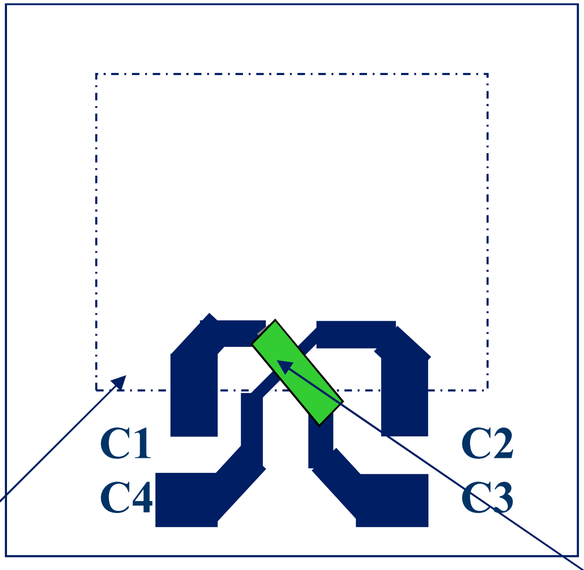


Layout →



The Motorola X-ducer^{TM} piezoresistor

TOP VIEW



DIAPHRAGM

piezoresistor

p+

p