

22/05/25

Lecture-14

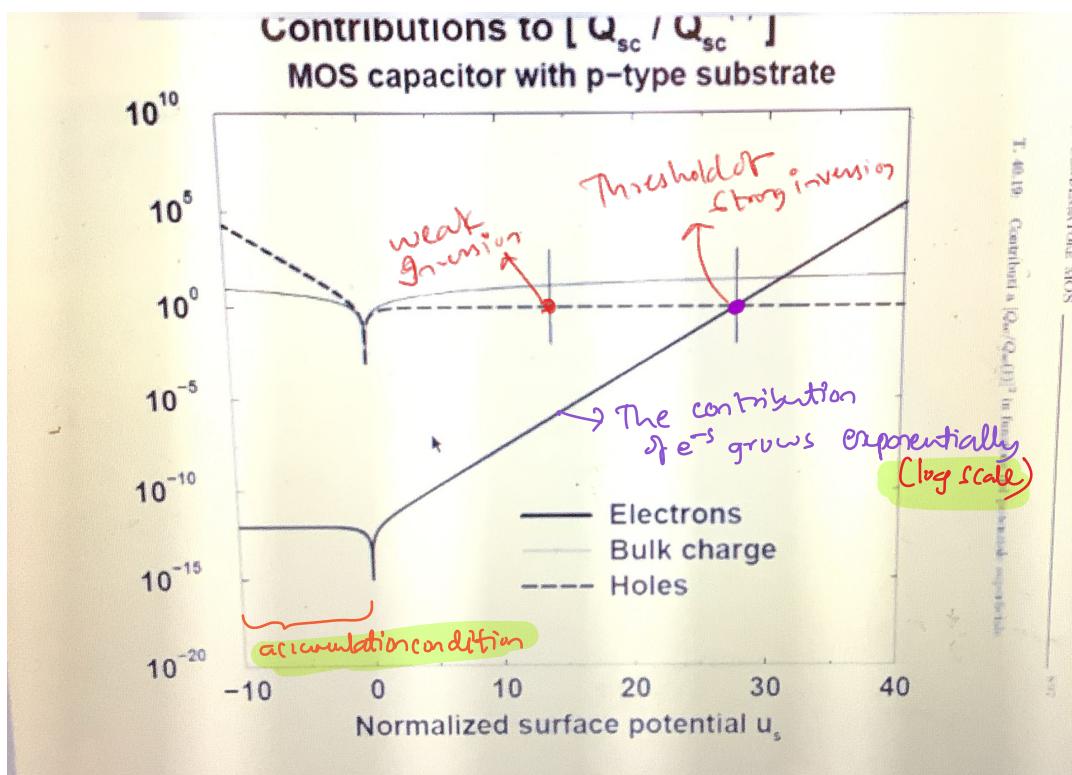
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Contributions to

$$\left[\frac{Q_{sc}}{Q_{sc}^{(1)}} \right]^2$$

MOS capacitor with p-type substrate



This is graph with Normalised surface Potential (u_s) on the x-axis

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Capacitance of the MOS Structure

p-type substrate, quantitative relations

The normalized capacitance reads

$$\boxed{\frac{C}{C_{ox}} = \frac{1}{1 + C_{ox}/C_{sc}}} \quad C_{sc} = \pm \frac{2\epsilon_{sc}}{L_A} \frac{dF}{du_s} > 0,$$

where the plus (minus) sign holds for $u_s > 0$ ($u_s < 0$), and $C_{scFB} = \sqrt{2}\epsilon_{sc}/L_A$. Using

$$\frac{dF^2}{du_s} = 2F \frac{dF}{du_s},$$

$$\frac{dF^2}{du_s} = \exp(-2u_F) [\exp(u_s) - 1] + 1 - \exp(-u_s) = A(u_s; u_F)$$

yields, with $r = \epsilon_{sc}t_{ox}/(\epsilon_{ox}L_A)$,

$$C_{sc} = \pm \frac{\epsilon_{sc}}{L_A} \frac{A}{F} > 0, \quad \frac{C_{ox}}{C_{sc}} = \pm \frac{\epsilon_{ox}L_A}{\epsilon_{sc}t_{ox}} \frac{F}{A} = \pm \frac{F}{rA}.$$

The relation $C(V_G)$ is found by eliminating u_s from

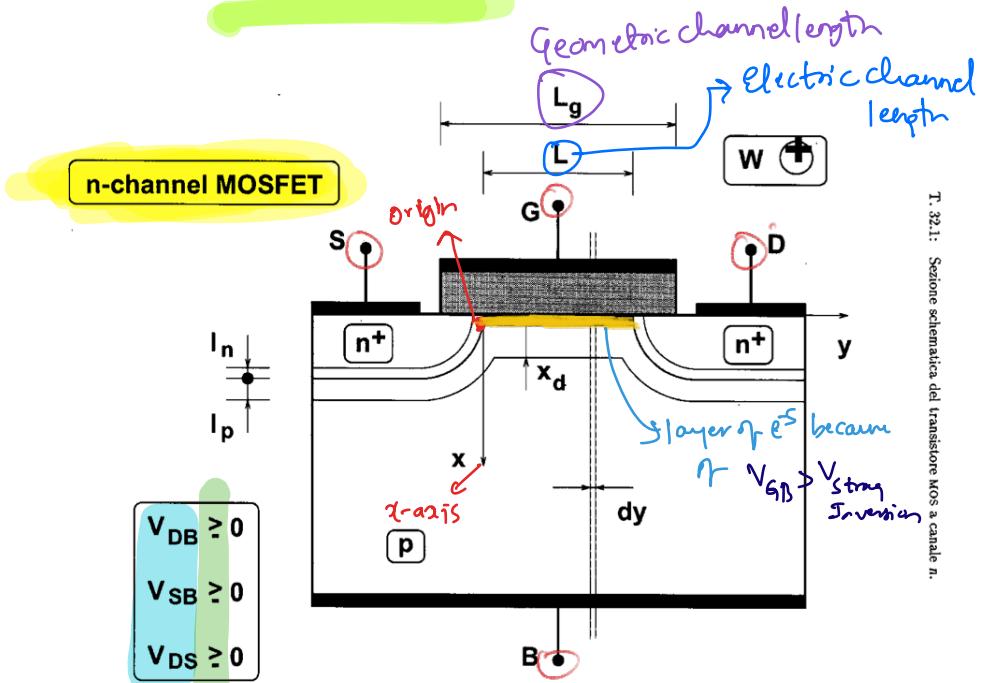
$$u_s' = u_s \pm 2r F = -\frac{C}{C_{ox}} = \frac{1}{1 \pm F/(rA)}.$$

In particular, $C(V_G = 0) = C_{ox}/[1 + 1/\sqrt{2r}]$.





MOSFET



T 32.1: Sezione schematica del transistore MOS a canale n.

↳ Should always
be greater than zero

→ MOS Transistor is the most important device in the IC technology

- The current flowing from 'D to S' depends on the no. of e^- in the Inverted layer.

no. of e^- \propto Gate voltage
(current) (control port)



we want to calculate the current flowing
from D to S

MOSFET, n-channel — I

• electron current density

Hole current density

But $\begin{cases} \mathbf{J}_n = J_{nx} \mathbf{i} + J_{ny} \mathbf{j} \\ \mathbf{J}_p = J_{px} \mathbf{i} + J_{py} \mathbf{j} \\ J_{nx} = J_{px} = 0 \end{cases}$

$|J_{ny}| \gg |J_{py}| \Rightarrow \mathbf{J} = \mathbf{J}_n + \mathbf{J}_p \simeq \mathbf{J}_n = J_{ny}(x, y) \mathbf{j}$

for a well formed channel

Flux of \mathbf{J}_n

drift Fermi potential

Simpler expression of the current

unit vector of y-axis

Hp: $\begin{cases} \partial/\partial z = 0 \\ \partial/\partial t = 0 \\ V_{GS} \geq V_T \end{cases}$

No current in 'z' direction

steady state condition

at each point we have strong Inversion condition i.e. channel is well formed

Only this component of the current survives - (Unipolar Device)

Recombination Rate is zero!

i.e. J_n is solinoidal

The flux J_n does not depend on the position.

$$\begin{aligned} & \text{div } \mathbf{J} = 0 \Rightarrow \text{div } \mathbf{J}_n = 0, U = 0 \\ & \Rightarrow I_A - I_B = \iint_A \mathbf{J}_n \cdot \mathbf{n} dx dz - \iint_B \mathbf{J}_n \cdot \mathbf{n} dx dz = 0 \\ & I = \text{const} = \int_{z=0}^W \int_{x=0}^{\infty} \mathbf{J}_n \cdot \mathbf{j} dx dz = W \int_0^{\infty} J_{ny} dx \end{aligned}$$

$$\begin{cases} \mathbf{J}_n = -q\mu_n n \text{ grad } \varphi_n \\ J_{nx} = -q\mu_n n \partial \varphi_n / \partial x = 0 \\ J_{ny} = -q\mu_n n \partial \varphi_n / \partial y \end{cases} \Rightarrow \begin{cases} \partial \varphi_n / \partial x = 0 \\ \varphi_n = \varphi_n(y) \\ \partial \varphi_n / \partial y = d\varphi_n / dy \end{cases}$$

$$\begin{cases} n = n(x, y) \\ \mu_n = \mu_n(x, y) \\ \infty \leftarrow x_d \end{cases}$$

∴ We can calculate the Current of the channel by simply calculating the flux of the current density at any position 'y' in the channel.

$\not\rightarrow$

$$I = w \frac{d\phi}{dy} \int_0^\infty -q\mu_n n dx$$

we can restrict the upper limit of the Integral
to x_d

MOSFET, n-channel — II

Inversion charge per unit Area

- $Q_i(y) = \int_0^{x_d} -q n dx < 0$
- $\mu_e(y) = \frac{\int_0^{x_d} -q \mu_n n dx}{\int_0^{x_d} -q n dx} > 0$
- $I = W \frac{d\varphi_n}{dy} \mu_e(y) Q_i(y) = \text{const} \Rightarrow \varphi_n(y) \Leftrightarrow y(\varphi_n)$

Average Mobility

$\Rightarrow \int_0^L I dy = LI = W \int_{\varphi_n(0)}^{\varphi_n(L)} \mu_e(\varphi_n) Q_i(\varphi_n) d\varphi_n$

$\frac{d\mu_e}{dy} \simeq 0 \Rightarrow I = \frac{W}{L} \mu_e \int_{\varphi_n(0)}^{\varphi_n(L)} Q_i(\varphi_n) d\varphi_n$

Def: $\varphi_s(y) = \varphi(x=0, y)$, Hp: $\varphi_n(\varphi_s) \Leftrightarrow \varphi_s(\varphi_n)$

Mobility degradation is almost zero!

The surface potential at it's two dimensional

$I = \frac{W}{L} \mu_e \int_{\varphi_s(0)}^{\varphi_s(L)} Q_i(\varphi_s) \frac{d\varphi_n}{d\varphi_s} d\varphi_s$

Φ_n is a Monotonic function of y

To solve this we introduce an Approximation called Gradual Channel Approximation

ASCE \Rightarrow Abrupt Space charge edge

MOSFET, n-channel — III

Gradual-channel approximation (GCA)

Inversion charge

$$\left\{ \begin{array}{l} \text{I. Parallel-plate capacitor } \forall dy \\ \text{II. Full depletion + ASCE} \\ \text{III. Ohmic transport (strong inversion } \forall y) \end{array} \right.$$

$\left\{ \begin{array}{l} \text{I+II. } Q_i = -C_{\text{ox}} [(V'_{GB} - \varphi_s) - \gamma \sqrt{\varphi_s}] \quad \forall y \\ \text{III. } \varphi_s = \varphi_n + \varphi_F \text{ constant} \end{array} \right.$

\rightarrow expression from MOS capacitor

- III.: $\frac{d\varphi_n}{dy} = \frac{d\varphi_s}{dy} \Rightarrow J_{ny} = -q\mu_n n \frac{d\varphi_s}{dy} = q\mu_n n \mathcal{E}_{sy}$
- $\varphi_s = (\varphi_n - \varphi_F) + 2\varphi_F \Rightarrow \left\{ \begin{array}{l} \varphi_s(0) = V_{SB} + 2\varphi_F \\ \varphi_s(L) = V_{DB} + 2\varphi_F \end{array} \right.$
- $I_D = -I = \beta \int_{V_{SB}+2\varphi_F}^{V_{DB}+2\varphi_F} [(V'_{GB} - \varphi_s) - \gamma \sqrt{\varphi_s}] d\varphi_s$

$$\beta = \frac{W}{L} \mu_e C_{\text{ox}}, \quad V_{DS} = V_{DB} - V_{SB} > 0 \Rightarrow I_D > 0$$

In this expression for Drain current, it depends on three independent voltages V_{DB} , V_{SB} , V'_{GB} and would be Nonlinear upto V_{DB}, V_{SB} .

→ Next step is to calculate the Differential conductances for this Device i.e $\frac{\partial I_D}{\partial V_{GB}}$, $\frac{\partial I_D}{\partial V_{DB}}$, $\frac{\partial I_D}{\partial V_{SB}}$



MOSFET, *n*-channel — IV

Gradual-channel approximation (GCA)

Drain conductance

$$g_D = g_o = \left(\frac{\partial I_D}{\partial V_{DB}} \right)_{V_{SB}, V_{GB}} = \left(\frac{\partial I_D}{\partial V_{DS}} \right)_{V_{BS}, V_{GS}} \text{ Constants}$$

Saturation surface potential

$$\begin{aligned} g_D &= \beta \left[(V'_{GB} - V_{DB} - 2\varphi_F) - \gamma\sqrt{V_{DB} + 2\varphi_F} \right] = \\ &= \beta \left[(V'_{GB} - \varphi_s(L)) - \gamma\sqrt{\varphi_s(L)} \right] = \frac{W}{L} \mu_e [-Q_i(L)] \end{aligned}$$

Def: $\varphi_s^{\text{sat}} = \varphi_s(L) \ni Q_i(L) = 0$, $\begin{cases} V_{DB}^{\text{sat}} = \varphi_s^{\text{sat}} - 2\varphi_F \\ V_{DS}^{\text{sat}} = V_{DB}^{\text{sat}} - V_{SB} \end{cases}$

$$\begin{aligned} g_D(V_{DS} = 0) &= \beta \left[(V'_{GB} - V_{SB} - 2\varphi_F) - \gamma\sqrt{V_{SB} + 2\varphi_F} \right] = \\ &= \beta \left[(V_{GS} - V_{FB} - 2\varphi_F) - \gamma\sqrt{2\varphi_F - V_{BS}} \right] = \\ &= \beta (V_{GS} - V_T), \quad V_T = V_{FB} + 2\varphi_F + \gamma\sqrt{2\varphi_F - V_{BS}} \end{aligned}$$

This has effect on V_T

GCA: $\begin{cases} V_{GS} \geq V_T \\ V_{GS} < V_T \end{cases} \quad \begin{cases} 0 \leq V_{DS} \leq V_{DS}^{\text{sat}} & \text{linear region} \\ V_{DS}^{\text{sat}} < V_{DS} & \text{saturation region} \\ & \text{off state} \end{cases}$

Not completely OFF we have Subthreshold current

- V_{BS} can be made constant (on source & bulk shorted).

MOSFET, n -channel — V

Gradual-channel approximation — Linear region

$$I_D = \beta \int_{V_{SB}+2\varphi_F}^{V_{DB}+2\varphi_F} [(V'_{GB} - \varphi_s) - \gamma \sqrt{\varphi_s}] d\varphi_s$$

• $\int_{V_{SB}+2\varphi_F}^{V_{DB}+2\varphi_F} V'_{GB} d\varphi_s = V'_{GB} V_{DS}$

• $\int_{V_{SB}+2\varphi_F}^{V_{DB}+2\varphi_F} \varphi_s d\varphi_s = \frac{1}{2} [(V_{DB} + 2\varphi_F)^2 - (V_{SB} + 2\varphi_F)^2] =$
 $= \frac{1}{2} [(V_{DS} + V_{SB})^2 - V_{SB}^2] + 2\varphi_F V_{DS} =$
 $= \frac{1}{2} V_{DS}^2 + (V_{SB} + 2\varphi_F) V_{DS}$

$$V'_{GB} V_{DS} - \frac{1}{2} V_{DS}^2 - (V_{SB} + 2\varphi_F) V_{DS} =$$

linear &
parabolic dependence
on V_{DS}

$$= (V_{GS} - V_{FB} - 2\varphi_F) V_{DS} - \frac{1}{2} V_{DS}^2$$

• $\int_{V_{SB}+2\varphi_F}^{V_{DB}+2\varphi_F} \sqrt{\varphi_s} d\varphi_s = \frac{2}{3} [(V_{DB} + 2\varphi_F)^{3/2} - (V_{SB} + 2\varphi_F)^{3/2}]$

MOSFET, n -channel — VI

Gradual-channel approximation — Linear region

*elementary
stdnd eqn for I_D*

$$I_D = I'_D - I''_D$$

$$I'_D = \beta \left[(V_{GS} - V_{FB} - 2\varphi_F) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

*linear parabolic
dependence on V_{DS}*

$$I''_D = \beta \frac{2}{3} \gamma \left[(V_{DS} + 2\varphi_F - V_{BS})^{3/2} - (2\varphi_F - V_{BS})^{3/2} \right]$$

$$g_m = \left(\frac{\partial I_D}{\partial V_{GB}} \right)_{V_{SB}, V_{DB}} = \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}} = \beta V_{DS}$$

$$g_B = \left(\frac{\partial I_D}{\partial V_{BS}} \right)_{V_{DS}, V_{GS}}$$

$$I''_D \ll I'_D \Rightarrow \begin{cases} I_D \simeq \beta \left[(V_{GS} - V_{FB} - 2\varphi_F) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \\ g_D \simeq \beta (V_{GS} - V_{FB} - 2\varphi_F - V_{DS}) \\ V_{DS}^{\text{sat}} \simeq V_{GS} - V_{FB} - 2\varphi_F \\ V_T \simeq V_{FB} + 2\varphi_F \\ g_B \simeq 0 \end{cases}$$

*for small I_D
carle ignored*

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MOSFET, n-channel — VII

Gradual-channel approximation — Saturation region

$$V_{GS} > V_T, \quad V_{DS}^{\text{sat}} < V_{DS}$$

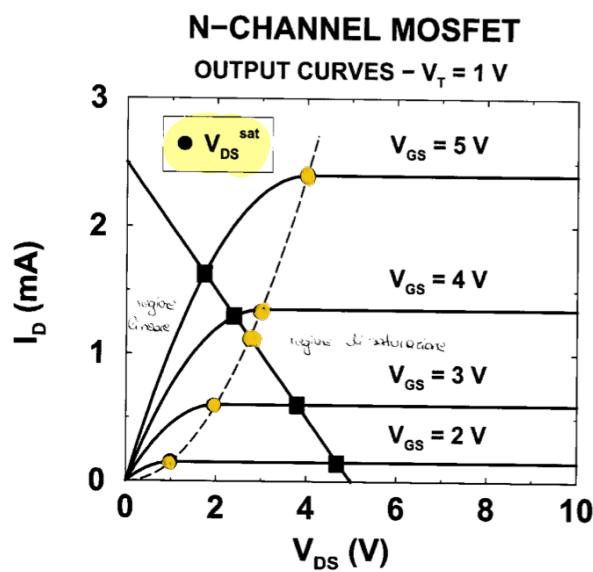
$$I_D \simeq \beta \left[\left(V_{DS}^{\text{sat}} \right)^2 - \frac{1}{2} \left(V_{DS}^{\text{sat}} \right)^2 \right] =$$

$$= \frac{1}{2} \beta (V_{GS} - V_{FB} - 2\varphi_F)^2 \simeq \frac{1}{2} \beta (V_{GS} - V_T)^2$$

$$\begin{cases} g_D \simeq 0 \\ g_m = \beta (V_{GS} - V_{FB} - 2\varphi_F) \simeq \beta (V_{GS} - V_T) \\ g_B \simeq 0 \end{cases}$$

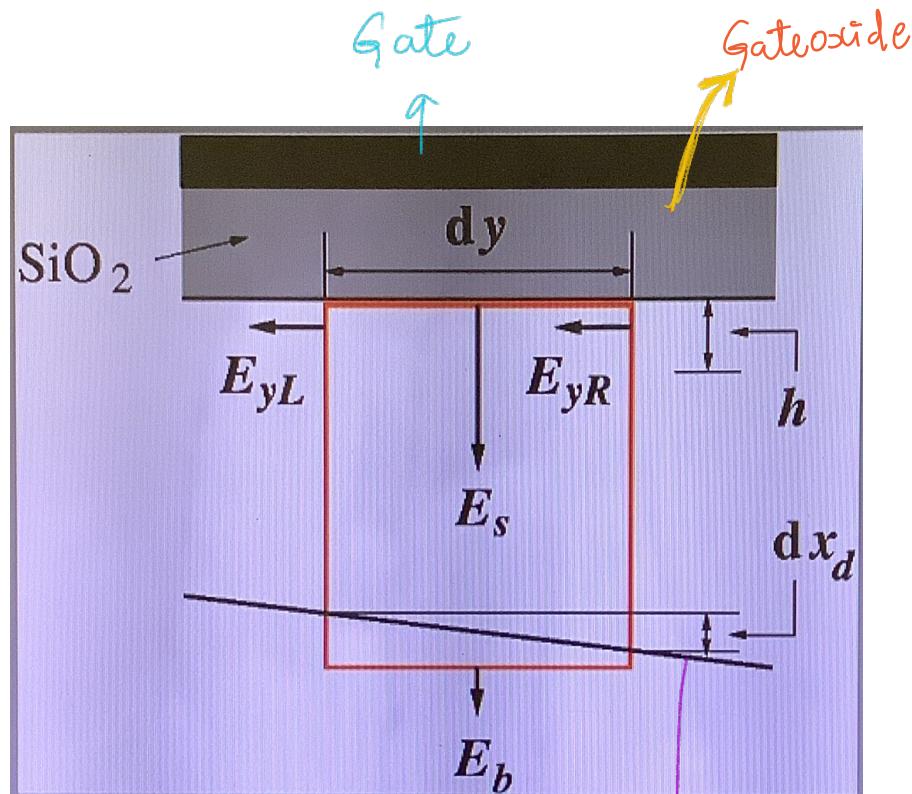
Gradual-channel approximation — Off state

$$V_{GS} < V_T, \quad I_D \simeq 0$$



T. 32.10: Caratteristiche di uscita del transistore MOS a canale n.

$\rightarrow *$



edge of The Depleted Region

- E_s : Field Normal to the Surface (interface between insulator & semiconductor)

$$E_b \approx 0$$

- Interestingly it's Not Horizontal Anymore due to the application of a Drain to source voltage V_{DS} we also have lateral component of the Electric field

→ E_y is the Horizontal field that is oriented from Right (Drain) to Left (source)

E_{yL} , E_{yR} if these two components are equal to each other they would compensate exactly. Because one of them is oriented inside the cylinder

If the other is oriented on the external side of the cylinder.

∴ In this case there is no contribution to the charge from the Electric displacement in the y' direction.

The right boundary is large by $d \approx d$

• For approximation we ignore the $d \approx d$ component.

• $E_{yR} \approx E_{yL}$ (in the linear region of operation)

$$E_s \gg E_{yR} \approx E_{yL}$$

$$V_D < V_{DSAT}$$

After these simplifications we are back to the case of the MOS capacitor the only difference is that surface potential is different at the interface between oxide layer & the semiconductor (which we have already accounted for)

→ The above proves the Parallel Plate capacitor Approximation. & the Full depletion + ASCE follow the parallel plate Approximation.

→ Now proving Ohmic Transport (strong Inversion Hy)

We have to demonstrate

$$\phi_s = \phi_n + \phi_F \quad \forall y$$

This is equivalent to stating the Horizontal variation in the Electric field (E) from left to right over the infinitesimal length dy is negligible.

we know $E = \frac{d\phi}{dy}$

$$\Rightarrow \frac{dE}{dy} = \frac{d^2\phi}{dy^2} = \text{very small}$$

Variation of
Electric field

We shall exploit this to demonstrate Ohmic Transport.

Poisson's Eq. in n-Channel MOSFETs — I

Hypothesis \Rightarrow

Hp: $\begin{cases} \partial/\partial z = 0 \\ N = -N_A = \text{const} \\ \varphi(\infty, y) = 0 \\ \varphi'(\infty, y) = 0 \end{cases}$

Def: $\begin{cases} p_{p0} = p(\infty, y) \\ n_{p0} = n(\infty, y) \\ u = q\varphi/(k_B T_L) \\ u_{n,p} = q\varphi_{n,p}/(k_B T_L) \\ L_A = \sqrt{2\varepsilon_{sc} k_B T_L / (q^2 p_{p0})} \end{cases}$

Majority carrier in the Bulk

Minority carriers in the Bulk

Normalized Electric potential

Since we are not in equilibrium

we are Normalised drain fermi potentials u_n, u_p

The expressions of electron concentration in the Non equilibrium condition at we also assume that we are in Non-degenerate condition. So we are the exponentials in Normalised form

$n = n_i \exp(u - u_n) = n_i \exp(u - u_F + u_F - u_n)$

$p = n_i \exp(u_p - u) = n_i \exp(u_p - u_F + u_F - u)$

whence, letting $\xi_n \doteq \varphi_n - \varphi_F$, $\xi_p \doteq \varphi_p - \varphi_F$ and $\chi_n \doteq q\xi_n/(k_B T_L)$, it is

At Equilibrium both go to zero

Normalized quantities corresponding to ξ_n, ξ_p

$\rho = q [p_{p0} \exp(\chi_p - u) - n_{p0} \exp(u - \chi_n) + N]$.

$\rho(\infty) = 0 \Rightarrow N = n_{p0} - p_{p0} \Rightarrow \rho = -qp_{p0} A(u, \chi_n, \chi_p)$

$A = (n_{p0}/p_{p0}) [\exp(u - \chi_n) - 1] + 1 - \exp(\chi_p - u)$

$n_{p0}/p_{p0} = \exp(-2u_F)$, $u_F > 0$.

$-\operatorname{div}(\varepsilon_{sc} \operatorname{grad} \varphi) = \rho \Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2}{L_A^2} A$.

Charged density $\xrightarrow{\text{Poisson Eq}}$

Mechanics to solve it 2D

- Φ_n & Φ_p i.e. ξ_n & ξ_p do not depend on the 'x' co-ordinate They depend only on 'y' at the interface b/w oxide layer & the Bulk

- Now we introduce the simplifications when we have a well formed channel $E_S \gg E_y$ & $E_{yL} = E_{yR}$ which says $\frac{\partial^2 u}{\partial y^2} = 0$ which we proved above
- So we have only derivative w.r.t. 'y' to solve

for. (only for a fully formed channel)

so we can neglect the hole terms from the charge density eq? because of strong inversion

$$\begin{aligned} \rho &= q [p_{p0} \exp(\chi_p - u) - n_{p0} \exp(u - \chi_n) + N] . \\ \rho(\infty) &= 0 \Rightarrow N = n_{p0} - p_{p0} \Rightarrow \rho = -q p_{p0} A(u, \chi_n, \chi_p) \\ A &= (n_{p0}/p_{p0}) [\exp(u - \chi_n) - 1] + 1 - \exp(\chi_p - u) \\ n_{p0}/p_{p0} &= \exp(-2u_F), \quad u_F > 0 . \\ -\operatorname{div}(\epsilon_{sc} \operatorname{grad} \varphi) &= \rho \quad \Rightarrow \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2}{L_A^2} A . \end{aligned}$$

$$\begin{aligned} \therefore A &= \left(\frac{n_{p0}}{p_{p0}} \right) \left[\exp(u - \chi_n) - 1 \right] + 1 - \exp(\chi_p - u) \\ &= \exp(-2u_F) \quad \text{very large} \quad \text{ignored} \end{aligned}$$

Poisson's Eq. in n-Channel MOSFETs — II

- If the accumulation condition is not considered, the contribution of holes to A can be neglected:

$$A \simeq \exp(u - \chi_n - 2u_F) + 1 > 0 ,$$

- with $u = u(x, y)$, $\chi_n = \chi_n(y)$. If in addition the applied biases are such that $|\partial \mathcal{E}_x / \partial x| \gg |\partial \mathcal{E}_y / \partial y|$, then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \simeq \frac{d^2 u}{dx^2} .$$

- The dependence of A on y can then be treated parametrically in

$$\frac{d^2 u}{dx^2} \simeq \frac{2}{L_A^2} A .$$

In conclusion, remembering that $u_s \geq 0$ because accumulation is excluded, the same procedure as for the MOS capacitor yields

$$\begin{aligned} u' u'' &= \frac{1}{2} \frac{d}{dx} (u')^2 = \frac{2}{L_A^2} A(u) u' \\ \int_0^{(u_s')^2} d(u')^2 &= \frac{4}{L_A^2} \int_0^{u_s} A(u) du > 0 , \quad u_s = u_s(y) . \end{aligned}$$

Poisson's Eq. in n-Channel MOSFETs — III

$$\left(\frac{q\mathcal{E}_s}{k_B T_L}\right)^2 = \frac{4}{L_A^2} F^2(u_s; \chi_n, u_F),$$

$$F^2 = \exp(-\chi_n - 2u_F) [\exp(u_s) - 1] + u_s,$$

$$Q_{sc} = -\varepsilon_{sc} \mathcal{E}_s = -\frac{2\varepsilon_{sc} k_B T_L}{q L_A} F.$$

$$Q_{sc} \begin{cases} < 0 & u_s > 0 \\ = 0 & u_s = 0 \end{cases} \quad (\text{flat band})$$

we have two
expressions for Q_{sc}

Combining with $Q_{sc} = -C_{ox} (V'_{GB} - \varphi_s)$ yields the relation $\chi_n = \chi_n(u_s)$, with $u'_{GB} \doteq qV'_{GB}/(k_B T_L)$:

$$C_{ox} (u'_{GB} - u_s) = \frac{2\varepsilon_{sc}}{L_A} \sqrt{\exp(-\chi_n - 2u_F) [\exp(u_s) - 1] + u_s}$$

Surface potential

The threshold condition is then $u_{sT} = 2u_F + \chi_n$, which generalizes the equilibrium case $u_{sT} = 2u_F$. Due to the exponential dependence one may also assume that in strong inversion $u_s \simeq u_{sT}$. In conclusion, from $\chi_n = u_n - u_F$ it follows

Normalized
Fermi
potential
or \tilde{e}^s

$$u_s \simeq u_F + u_n, \quad \varphi_s \simeq \varphi_n + \varphi_F = \xi_n + 2\varphi_F, \quad 0 \leq y \leq L.$$

→ This proves that after reaching the threshold condition the surface potential varies little if we try to increase further the gate voltage