Lecture-10

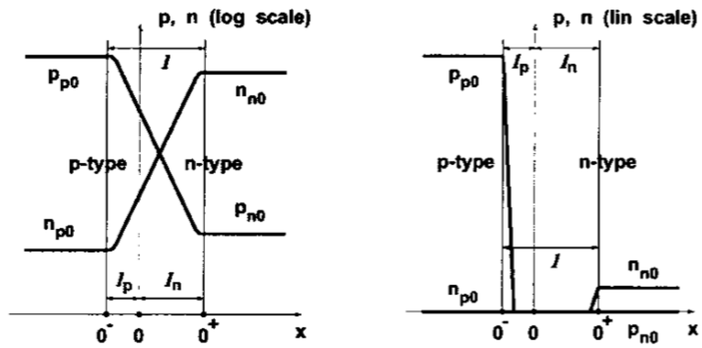
* "ఓం శ్రీం గ్లౌం గణాధ్యక్షాయ నమః" 08/10/20

Poisson's equation

$$\frac{d^2\varphi}{dx^2} = \frac{q}{\varepsilon_{sc}}\left[n^{(0)}\exp\left(\frac{q\varphi}{k_B T_L}\right) - p^{(0)}\exp\left(\frac{-q\varphi}{k_B T_L}\right) - N(x)\right]$$

$$N(x) = \begin{cases} -N_A\,, & x < 0 \\ N_D\,, & x > 0 \end{cases} \qquad \text{B.c.s :} \quad \begin{cases} \varphi(-\infty) = -\psi_0 \\ \varphi(+\infty) = 0 \end{cases}$$

p-n Junction in Equilibrium — IV



Example:

$$\left.\begin{array}{l} N_A = 10^{16}\ \text{cm}^{-3} \\ N_D = 10^{15}\ \text{cm}^{-3} \\ T_L = 300\ \text{K} \\ n_i \simeq 10^{10}\ \text{cm}^{-3} \end{array}\right\} \quad \Longrightarrow \quad \left\{\begin{array}{l} p_{p0} \simeq N_A = 10^{16}\ \text{cm}^{-3} \\ n_{p0} \simeq n_i^2/N_A = 10^4\ \text{cm}^{-3} \\ n_{n0} \simeq N_D = 10^{15}\ \text{cm}^{-3} \\ p_{n0} \simeq n_i^2/N_D = 10^5\ \text{cm}^{-3} \end{array}\right.$$

$$-l_p < x < l_n \ : \quad \varrho \simeq qN = \begin{cases} -qN_A\,, & x < 0 \\ qN_D\,, & x > 0 \end{cases}$$

$$\begin{cases} -q\mu_p p\mathcal{E} = -qD_p\,dp/dx > 0\,, & J_p = 0 \\ -q\mu_n n\mathcal{E} = +qD_n\,dn/dx > 0\,, & J_n = 0 \end{cases}$$

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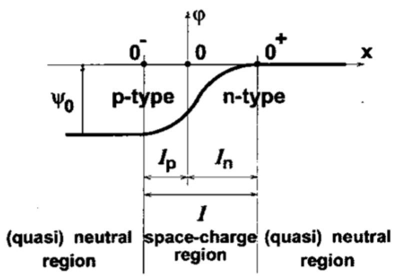
→ In the solution (poisson's Equation)

i.e values of electric potential at each point in the semiconductor and then use the Equilibrium expressions of the concentrations of electrons and Holes in exponential form

Because, we are assuming Non-Degeneracy condition. Then the concentration of electrons is exponential of ϕ

→ If we draw a graph instead of concentration of 'n' we use logarithm of 'n' we expect to obtain a curve which is similar to figure below.



and if we make another graph describing Hole concentration again in logarithmic scale then we find the behaviour with electric potential as −ve

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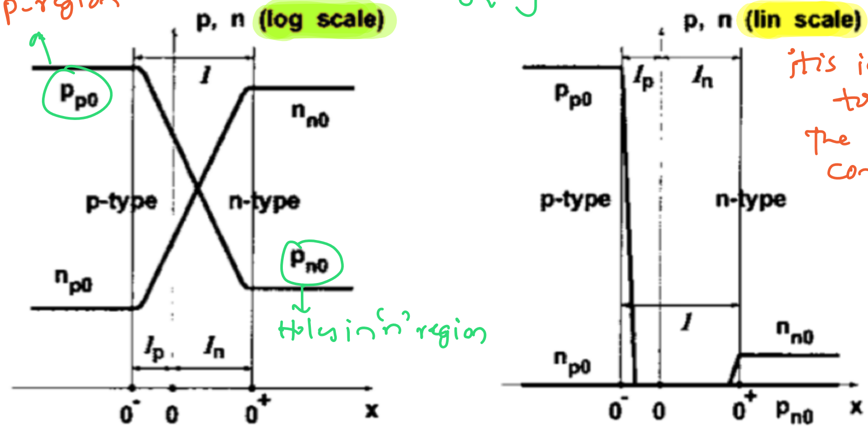
This is what we are analyzing

Holes in p-region

linear scale



it is impractical to analyze the carrier concentrations.

Example:

The conentration of Holes in p-region is practically equal to N_A Conc. Acceptor impurities.

$$\left.\begin{aligned} N_A &= 10^{16}\ \text{cm}^{-3} \\ N_D &= 10^{15}\ \text{cm}^{-3} \\ T_L &= 300\ \text{K} \\ n_i &\simeq 10^{10}\ \text{cm}^{-3} \end{aligned}\right\} \implies \left\{\begin{aligned} p_{p0} &\simeq N_A = 10^{16}\ \text{cm}^{-3} \\ n_{p0} &\simeq n_i^2/N_A = 10^{4}\ \text{cm}^{-3} \\ n_{n0} &\simeq N_D = 10^{15}\ \text{cm}^{-3} \\ p_{n0} &\simeq n_i^2/N_D = 10^{5}\ \text{cm}^{-3} \end{aligned}\right.$$

Conc. of e^- in n-region is practically equal to N_D Donor impurities

$$-l_p < x < l_n \ : \ \varrho \simeq qN = \begin{cases} -qN_A\,, & x < 0 \\ qN_D\,, & x > 0 \end{cases}$$

$$\begin{cases} -q\mu_p p\mathcal{E} = -qD_p\,\mathrm{d}p/\mathrm{d}x > 0\,, & J_p = 0 \\ -q\mu_n n\mathcal{E} = +qD_n\,\mathrm{d}n/\mathrm{d}x > 0\,, & J_n = 0 \end{cases}$$

→ In the linear scale we can see that

p_{p_0} immediatly goes to zero after the begining of space charge region

& Similarly n_{n0} goes to zero.

& n_{p0} & p_{n0} are practically equal to zero in the linear scale.

** From this linear scale we can say that the

Space Charge Region has mobile charges
which are substantially negligible.

Q Who is contributing to Charge density in

Space Charge Region?

Ans The only possible contribution comes from charges that cannot move. and those are charges associated to Donor & Acceptor atoms.

→ In the Abrupt Junction the Donor Atoms are on the Right of origin

i.e in b/w 0 & 0^+

charge density due to Donor Atoms

$$-l_p < x < l_n : \varrho \simeq qN = \begin{cases} -qN_A\,, & x<0 \\ qN_D\,, & x>0 \end{cases}$$

Charge density in space charge region

→ If we look at the profiles of Holes & Electrons in the logarithmic scale. There is a Gradient that represents the change in concentrations of Holes & electrons in the 3 regions i.e P, space charge

& n.

→ We also remember that there is a transport phenomenon called diffusion such that particles that are able to move go from the regions where concentration is higher to the regions where concentration is smaller.

∴ There is possible contribution to the Diffusion term. This Diffusion flux of Holes & electrons is not there after certain limit because it is counter Balanced by Electric field and we have no current.

$$\begin{cases} -q\mu_p p\mathcal{E} = -qD_p\,\mathrm{d}p/\mathrm{d}x > 0\,, & J_p = 0 \\ -q\mu_n n\mathcal{E} = +qD_n\,\mathrm{d}n/\mathrm{d}x > 0\,, & J_n = 0 \end{cases}$$

Drift Diffusion

Both Balance each other. at Equillibrium.

→ So far we have neglected the fact that at the end of n-type & p-type regions we have contacts.

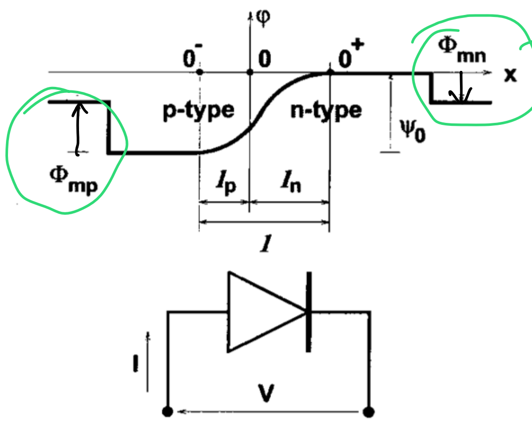
These contacts are made up of metals and This goes against our assumption of an Infinite semiconductor on Right and left.

→ There contacts introduce a discontinuity in Electric potential this is because of the Function Difference b/w two materials.

In principle it's similar to what happens in the P-N Junction.

T. 28.5: Andamento del potenziale ai contatti nella giunzione *p-n*.

p-n Junction in Equilibrium — V



→ small change in Electric potential

This can be treated as discontinuity

Balance equation.

$$\psi_0 + \Phi_{mn} - \Phi_{mp} = 0$$

(contacts made of the same material)

$$\Phi_{mp} = \Phi_{mp}(N_A)\,, \qquad \Phi_{mn} = \Phi_{mn}(N_D)$$

Depends on Dopant Concentration.

Hp: $\begin{cases} \Phi_{mp}\,, \Phi_{mn} \text{ independent of } I \text{ (ideal ohmic contacts)} \\ \text{Neutral and space-charge regions hold also for } V \neq 0 \end{cases}$

→ In P-N Junction eventhough the material was similar but the dopings were different for two sides.

Similarly for semiconductor - Metal contacts there is a change in Electric potential. But there is one difference though i.e the change occurs in a very very small region.

→ In many situations we can replace the change in Electric potential with Discontinuity & This is done in the figure shown above.

$\phi_{mn} \rightarrow$ metal - ntype "Here it goes from 0 to ϕ_{mn}"

$\phi_{mp} \rightarrow$ metal - ptype "Here it goes from $-\phi$ to ϕ_{mp}"

work function

∵ we are in equillibrium there is voltage applied to p-N Junction.

we can also see that ϕ_{mn} & ϕ_{mp} are aligned Horizontally.

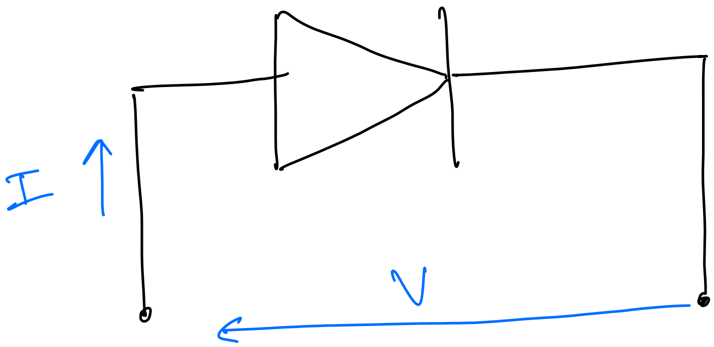
→ Balance Equation

$$\psi_0 + \Phi_{mn} - \Phi_{mp} = 0$$

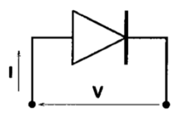
$$\Rightarrow \psi_0 = \Phi_{mp} - \Phi_{mn}$$

& $\Phi_{mp} = \Phi_{mp}(\underline{N_A})$ $\quad \Phi_{mn} = \Phi_{mn}(\underline{N_D})$

depends on Dopant concentration



Symbol of PN Junction Diode & Indicating voltage & currents in standard form.



Balance equation.

$$\psi_0 + \Phi_{mn} - \Phi_{mp} = 0$$

(contacts made of the same material)

$$\Phi_{mp} = \Phi_{mp}(N_A)\,, \qquad \Phi_{mn} = \Phi_{mn}(N_D)$$

Depends on Dopant concentration.

Hypothesis → Hp: $\begin{cases} \Phi_{mp}, \Phi_{mn} \text{ independent of } I \text{ (ideal ohmic contacts)} \\ \text{Neutral and space-charge regions hold also for } V \neq 0 \end{cases}$

→ There are two conditions which we assume the PN Junction follows even in Non Equilibrium condition.

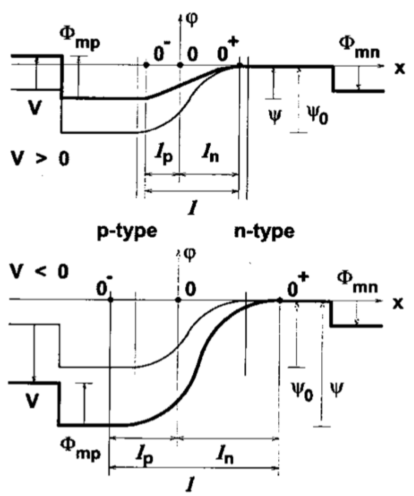
(i) i.e the Φ_{mp} & Φ_{mn} are Independent of the current. (contacts are Ideal)

(ii) Even in the Nonequilibrium condition the partition of PN Junction into 3 regions is still valid.

→

T. 28.6: Andamento del potenziale nella giunzione p-n fuori equilibrio.

p-n Junction in Nonequilibrium — I



$$\psi + \Phi_{mn} + V - \Phi_{mp} = 0\,, \qquad \psi + V - \underbrace{(\Phi_{mp} - \Phi_{mn})}_{\psi_0} = 0$$

$$\psi = \psi_0 - V\,, \qquad \varphi(\mathrm{O}^+) - \varphi(\mathrm{O}^-) = \psi_0 - V$$

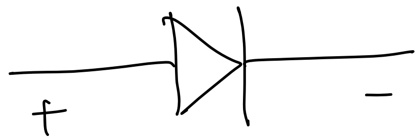
$$l_p = l_p(V)\,, \qquad l_n = l_n(V)$$

The above figure depicts the How the shape of the electric potential changes? in the Direct Bias & Reverse Bias.

Qualitatively, when we are in Non-Equillibrium condition with a Direct Bias.

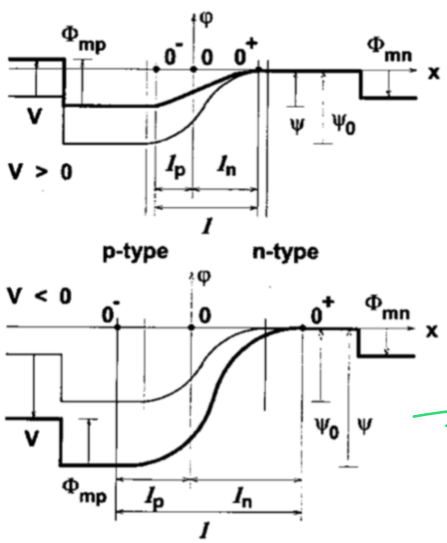
$V = 0$ at equillibrium

$V = +ve$ Non-Equillibrium for Direct Bias



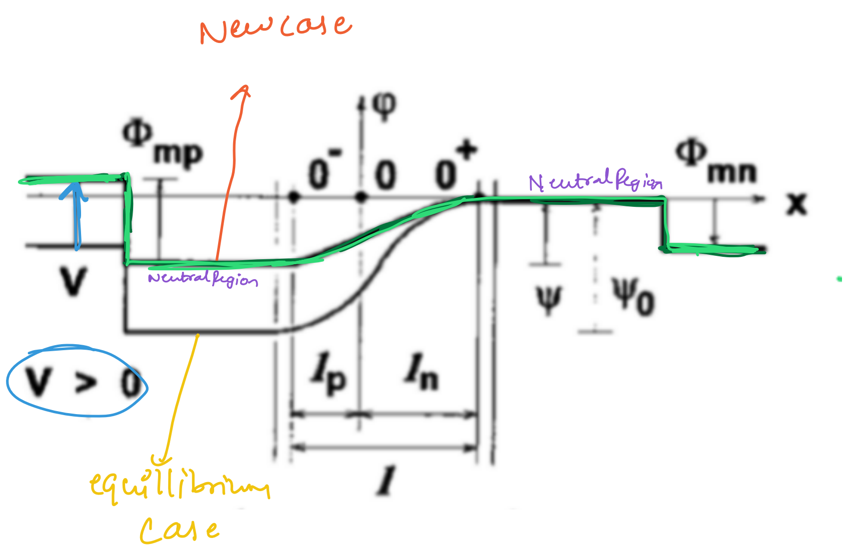
Direct Bias

p-n Junction in Nonequilibrium — I



Forward (or) Direct Bias

Reverse Bias



→ we have applied +ve voltage V to the pside
& as assumed Φ_{mp} will stay constant

The new case is Highlighted in Green colour.

Q Does the extent of space charge Region changes because of the voltage V applied to PN Junction ?

Ans It changes, in particular when PN Junction is in Direct Bias then the space charge region shrinks a little.

→ In the n-type neutral Region the shape of the graph stays the same

∴ Globally, on the Right Hand side the profile of the potential does not change because of Equillibrium. But on the left Hand side the graph shifts upwards.

∴ The Drop in potential from 0^- to 0^+ is ψ $\boxed{\psi < \psi_0}$

∴ The potential drop across the spacecharge region due to the application of a Direct Bias is such that across the space charge Region the potential difference is small.

→ On the other Hand the length of spacecharge region is almost similar to that of Equillibrium

We can say that we have a substantial change in potential drop over more or less the same length.

Conclusion being The Electric field will decrease.

→ If the Electric field becomes smaller w.r.to equillibrium the drift term will become smaller & weaker.

Q What happens to the Diffusion term?

Any This Term does not change because substantially the concentrations of Holes and e^- on two sides do not change.

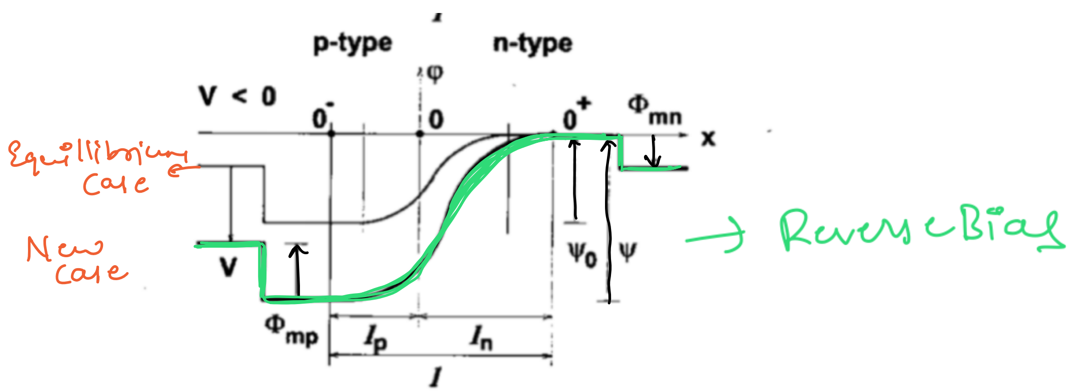
They are dictated essentially by dopants.

$\therefore$ Diffusive terms of the transport Equation are dominant when compared to Drift.

→ It follows we shall have an injection of Holes from left to Right and flux of e^- from Right to left.

→ We expect a significant current and The current will flow from Left to Right because Holes are positive

→ Reverse Bias



→ In Reverse bias the extent of space charge region increases. This is significant change.

→ In RB the total drop of potential (ψ) is larger than (ψ_0)

In this electric field obviously increases because we have larger potential drop with increase in space charge region.

→ But, we will see that EF (electric field) will increase in space charge region.

This strong EF will act on the insignificant free charges in the space charge region.

→ If we go around the loop in Nonequilibrium condition we will have a Balance Equation.

$$\psi + \phi_{mn} + V - \phi_{mp} = 0$$

$$\Rightarrow \quad \psi + V - \underbrace{(\phi_{mp} - \phi_{mn})}_{\psi_0} = 0$$

$$\psi + \Phi_{mn} + V - \Phi_{mp} = 0\,, \qquad \psi + V - \underbrace{(\Phi_{mp} - \Phi_{mn})}_{\psi_0} = 0$$

$$\boxed{\psi = \psi_0 - V\,, \qquad \varphi(0^+) - \varphi(0^-) = \psi_0 - V}$$

$$\underline{l_p} = l_p(V)\,, \qquad \underline{l_n} = l_n(V)$$

lengths depends on applied voltage.

→ Q can we apply voltage (V) larger than Builtin potential of the Junction?

If it is too large the current (I) depends exponentially on the voltage and This current may damage the device.

→ So, Direct Bias cannot be Too Large

on the other Hand Reverse Bias can be Large. Which also has a restriction i.e till avalanche Breakdown which will destroy the device.

→ We now proceed to the analysis of PN Junction to calculate the current - voltage Relation.

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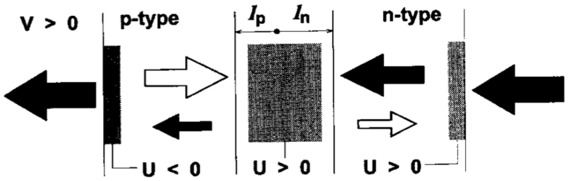
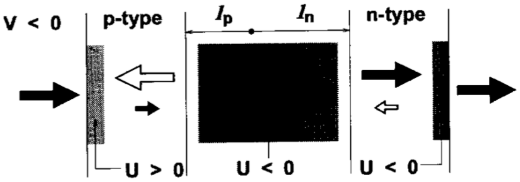


It is an illustration of fluxes in the device.

$$\text{S.c.r.:}\ \begin{cases} -qD_p \mathrm{d}p/\mathrm{d}x > -q\mu_p p\mathcal{E} > 0\,, & \boldsymbol{J}_p \bullet \boldsymbol{i} = \ \ qpv_p > 0 \\ +qD_n \mathrm{d}n/\mathrm{d}x > -q\mu_n n\mathcal{E} > 0\,, & \boldsymbol{J}_n \bullet \boldsymbol{i} = -qnv_n > 0 \end{cases}$$



$$\text{S.c.r.:}\ \begin{cases} -q\mu_p p\mathcal{E} > -qD_p \mathrm{d}p/\mathrm{d}x > 0\,, & \boldsymbol{J}_p \bullet \boldsymbol{i} = \ \ qpv_p < 0 \\ -q\mu_n n\mathcal{E} > +qD_n \mathrm{d}n/\mathrm{d}x > 0\,, & \boldsymbol{J}_n \bullet \boldsymbol{i} = -qnv_n < 0 \end{cases}$$

** Now we perform some calculations assuming some simplified Hypothesis. i.e Steadystate, onedimentional case.

p-n Junction in Nonequilibrium — III

Steady state i.e No dependence on time

Steady state, one-dimensional case (Shockley theory)

General form of continuity equation

$$\boxed{\frac{\partial \varrho}{\partial t} + \mathrm{div}\,\boldsymbol{J} = 0} \Rightarrow \frac{\mathrm{d}J}{\mathrm{d}x} = 0\,, \quad J = J_p(x) + J_n(x) = \text{const}\ \forall x$$

$$G = 0: \begin{cases} \mathrm{d}J_p/\mathrm{d}x = -qU\,, & J_p(a) - J_p(b) = \int_a^b qU\,\mathrm{d}x \\ \mathrm{d}J_n/\mathrm{d}x = +qU\,, & J_n(b) - J_n(a) = \int_a^b qU\,\mathrm{d}x \end{cases}$$

$\forall a, b$ ($U(x)$ not known)

$$J_U \doteq \int_{0^-}^{0^+} qU\,\mathrm{d}x = \begin{cases} J_p(0^-) - J_p(0^+) \\ J_n(0^+) - J_n(0^-) \end{cases} \Longrightarrow$$

$$\begin{cases} J = J_p(0^-) + J_n(0^-) = J_p(0^-) + J_n(0^+) - J_U & \text{(maj.)} \\ J = J_p(0^+) + J_n(0^+) = J_p(0^+) + J_n(0^-) + J_U & \text{(min.)} \end{cases}$$

$$\text{Hp: weak injection} \Rightarrow \begin{cases} U \simeq (n - n_{p0})/\tau_n\,, & x < x(0^-) \\ U \simeq (p - p_{n0})/\tau_p\,, & x > x(0^+) \end{cases}$$

$$\begin{cases} \mathrm{d}J_n/\mathrm{d}x = +q\,(n - n_{p0})/\tau_n\,, & x < x(0^-) \\ \mathrm{d}J_p/\mathrm{d}x = -q\,(p - p_{n0})/\tau_p\,, & x > x(0^+) \end{cases}$$

$\Longrightarrow$ { The minority-carrier continuity equations are decoupled in the neutral regions

→ $$\boxed{\frac{\partial \rho}{\partial t} + \operatorname{div} J = 0}$$ This is the General form of the Continuity Equation.

In equillibrium case

$$\frac{\partial \rho}{\partial t} = 0$$ i.e Rate of Change of Charge Density for steady state

For one dimentional case

$$\operatorname{div} J = 0 \Rightarrow \frac{dJ}{dx} = 0$$

$$\boxed{J = J_p(x) + J_n(x) = \text{constant} \quad \forall x}$$

Total current density; Current density becoz of Holes; current density Because of e^-'s

This constant is not known but is constant at each position.

i.e if we succeed in calculating Total current density (J) in one position then it will be the same in all other positions.

→ Now, we will assume continuity equations for electrons & Holes seperately and we assume that there is only one type of Generation & recombination phenomenon - i.e Thermal (U) and keeping all the

Generation + recombination terms to zero.
i.e Auger, optical etc.

→ $J = \underbrace{J_p(x) + J_n(x)}$ = constant $\forall x$

Individually $J_p(x)$, $J_n(x)$ depend on the position but the sum is independent.

→ We consider only one kind of Generation and Recombination phenomenon i.e the Thermal one. U

$$G = 0 : \begin{cases} \mathrm{d}J_p/\mathrm{d}x = -qU \, , & J_p(a) - J_p(b) = \int_a^b qU \, \mathrm{d}x \\ \mathrm{d}J_n/\mathrm{d}x = +qU \, , & J_n(b) - J_n(a) = \int_a^b qU \, \mathrm{d}x \end{cases}$$

$\forall a, b$ ($U(x)$ not known)

So, two continuity equations in steady state J_p, J_n become

$$G=0 : \begin{cases} dJ_p/dx = -qU \\ dJ_n/dx = +qU \end{cases}$$

→ We can formally integrate these two equations over any domain.

If we take one value of $x = a$
& another value of $x = b$

Then we can integrate the two equations $J_n(x)$, $J_p(x)$ from a to b

$$\boxed{\begin{aligned} J_p(a) - J_p(b) &= \int_a^b qU\,dx \\ J_n(b) - J_n(a) &= \int_a^b qU\,dx \end{aligned}}$$

$\forall$ a, b U(x) not know

This expression is useful although we are not able to calculate the integral.

Because U(x) depends on the concentrations and the concentrations so far are not known the dependence on 'x' is not found yet. So we cannot do the integral.

Q Now, How do we select the point a & b ?

Ans We select them at the edges of the space charge region.

Because the assumption that the PN Junction is partitioned into 2 regions is still valid.

we let $b = 0^+$ & $a = 0^-$

So we obtain

This integral is a Current Density by construction

$$J_U \doteq \int_{0^-}^{0^+} qU\,\mathrm{d}x = \begin{cases} J_p(0^-) - J_p(0^+) \\ J_n(0^+) - J_n(0^-) \end{cases} \implies$$

$$\begin{cases} J = J_p(0^-) + J_n(0^-) = J_p(0^-) + J_n(0^+) - J_U & \text{(maj.)} \\ J = J_p(0^+) + J_n(0^+) = J_p(0^+) + J_n(0^-) + J_U & \text{(min.)} \end{cases}$$

Recombination current density for which we yet don't know the value.

→ we know that total current J is equal to $J_p(x) + J_n(x)$.

∴ For x let's take 0^- for 'x'

Then $J = J_p(x) + J_n(x)$

$$J = J_p(0^-) + J_n(0^-)$$

But $J_U = J_n(0^+) - J_n(0^-)$

$$\Rightarrow J_n(0^-) = J_n(0^+) - J_U$$

$$\Rightarrow \boxed{J = J_p(0^-) + J_n(0^+) - J_U}$$

This manipulation indicates Majority carriers.

$$\boxed{J = J_p(0^+) + J_n(0^-) + J_U}$$

minority carriers

Here 'J' is expressed in terms of minority carriers calculated at specific positions.

→ Now we introduce the Hypothesis of Weak injection.

we remember, that in weak injection the actual concentration of electrons and Holes depart from the Equillibrium values little w.r.to the Majority carrier Concentration.

Q What is the importance of weak injection condition?

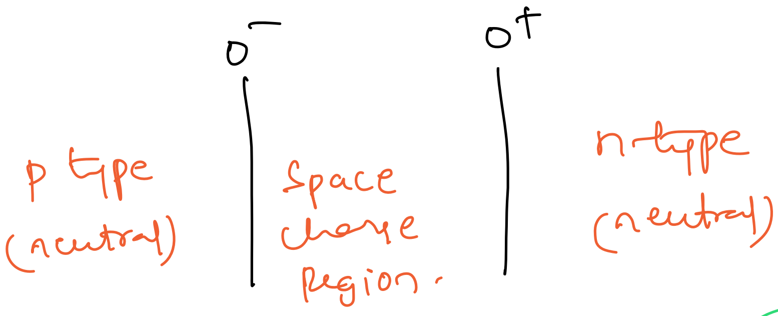
Ans If we are in WI condition then we can simply very much the form of net Recombination term because it depends only on the minority carrier concentration.

$$\text{Hp: weak injection} \Rightarrow \begin{cases} U \simeq (n - n_{p0})/\tau_n\,, & x < x(0^-) \\ U \simeq (p - p_{n0})/\tau_p\,, & x > x(0^+) \end{cases}$$

$$\begin{cases} \mathrm{d}J_n/\mathrm{d}x = +q\,(n - n_{p0})/\tau_n\,, & x < x(0^-) \\ \mathrm{d}J_p/\mathrm{d}x = -q\,(p - p_{n0})/\tau_p\,, & x > x(0^+) \end{cases}$$

$$\Longrightarrow \begin{cases} \text{The minority-carrier continuity equations} \\ \text{are decoupled in the neutral regions} \end{cases}$$

→ So, in WI is taken we can say that in the neutral region of p-type i.e left of 0^-



$$\text{Hp: weak injection} \Rightarrow \begin{cases} U \simeq (n - n_{p0})/\tau_n\,, & x < x(0^-) \text{ p-type} \\ U \simeq (p - p_{n0})/\tau_p\,, & x > x(0^+) \text{ n-type} \end{cases}$$

τ_n lifetime of electrons

τ_p lifetime of Holes

Q Why are the above equations so important ?

Ans Because it decouples the continuity equation after introducing these simplification i.e extremely important. The continuity eqn for electrons of minority carriers on LHS of 0^- i.e ptype

$$dJ_n/dx = +q(n - n_{p0})/\tau_n$$

⇓

It depends only on minority carriers

Similarly in the Neutral region on the Right of 0^+ i.e n type

$$\boxed{dJ_p/dx = -q(p-p_{no})/\tau_p}$$

This is the fundamental advantage of using the minority carriers in the analysis of PN Junction

→ we may argue that everytime we may not be in Weak Injection condition. But it shall be be possible to extract from this condition a large range of operating Conditions of the device.

→ We shall now proceed by solving the continuity equation for minority carriers in the Neutral regions.

Offcourse we have a price to pay. That we shall be able to find e^-'s only in the Neutral region on the left ptype and then we should be able to find the behaviour of the Holes on the Neutral region on the Right ntype and then some we shall be forced to put these Regions together. But this can be done. we will see it down the line

Art of convenience

మన సౌఖ్యం, సౌలభ్యం, స్వార్థం చూసుకోవాలి SDC Analysis లో

→ For the moment we have to solve the continuity equations and offcourse we have to couple them with Transport Equations of the electrons and Holes

T. 28.9: Condizioni al contorno di Shockley.

→ మైనారిటీ eq^n కొరకు డిఫ్యూజన్ eq^n రెండూ కలిపి డిఫరెన్షియల్ eq^n సృష్టిస్తాం.

p-n Junction in Nonequilibrium — IV

Steady state, one-dimensional case (Shockley theory)

$$\text{S.c.r.:} \begin{cases} -q\mu_p p\mathcal{E} \simeq -qD_p\, \mathrm{d}p/\mathrm{d}x \gg |J_p| \\ -q\mu_n n\mathcal{E} \simeq +qD_n\, \mathrm{d}n/\mathrm{d}x \gg |J_n| \end{cases} \implies$$

$$\frac{p(\mathrm{O}^+)}{p(\mathrm{O}^-)} \simeq \exp\left[\frac{q(V-\psi_0)}{k_B T_L}\right] = \frac{n_i^2}{N_A N_D} \exp\left(\frac{qV}{k_B T_L}\right)$$

$$\frac{n(\mathrm{O}^-)}{n(\mathrm{O}^+)} \simeq \exp\left[\frac{q(V-\psi_0)}{k_B T_L}\right] = \frac{n_i^2}{N_A N_D} \exp\left(\frac{qV}{k_B T_L}\right)$$

$$\begin{cases} p\text{-type region: } |p - p_{p0}| \ll p_{p0}\,, \quad p \simeq p_{p0}\,, \quad x < x(\mathrm{O}^-) \\ n\text{-type region: } |n - n_{n0}| \ll n_{n0}\,, \quad n \simeq n_{n0}\,, \quad x > x(\mathrm{O}^+) \end{cases}$$

$$p(\mathrm{O}^-) \simeq p_{p0} \simeq N_A \implies p(\mathrm{O}^+) \simeq p_{n0} \exp\left(\frac{qV}{k_B T_L}\right)$$

$$n(\mathrm{O}^+) \simeq n_{n0} \simeq N_D \implies n(\mathrm{O}^-) \simeq n_{p0} \exp\left(\frac{qV}{k_B T_L}\right)$$

(Shockley's boundary conditions)

Now, we have to solve eqn for e^{-}'s

$$\boxed{dJ_n/dx = +q(n-n_{p0})/\tau_n}$$

$x < x(0^-)$
p-type

This equation must be solved b/w left contact and the abssisa of the 0^-

The above eqn by itself is insufficient because we have 2 unknowns i.e J_n, n

∴ We must couple this equation with transport eqn for electrons and it is of drift diffusion type in one dimension.

p-n Junction in Nonequilibrium — V

Steady state, one-dimensional case (Shockley theory)

$$\begin{cases} x < x(\mathrm{O}^-) \\ \text{(weak inj.)} \end{cases} \begin{cases} n - n_{p0} > 0 \Rightarrow \begin{cases} n - n_{p0} \ll p_{p0} \\ n \ll p_{p0} + n_{p0} \simeq p_{p0} \\ n_{p0} < n \ll p_{p0} \end{cases} \\ n - n_{p0} < 0 \Rightarrow \begin{cases} n < n_{p0} \\ n_{p0} \ll p_{p0} \\ n < n_{p0} \ll p_{p0} \end{cases} \end{cases}$$

$$J_n = -q\mu_n n \frac{\mathrm{d}\varphi}{\mathrm{d}x} + qD_n \frac{\mathrm{d}n}{\mathrm{d}x} \simeq qD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

→ This eqn has electric potential, so it couples with corresponding continuity equation with a poisson's equation.

$$q\frac{n - n_{p0}}{\tau_n} = \frac{\mathrm{d}J_n}{\mathrm{d}x} \simeq q\frac{\mathrm{d}}{\mathrm{d}x}D_n\frac{\mathrm{d}n}{\mathrm{d}x} = qD_n\frac{\mathrm{d}^2 n}{\mathrm{d}x^2}$$

(Poisson's equation ruled out)

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(n - n_{p0}) = \frac{n - n_{p0}}{L_n^2}, \qquad L_n \doteq \sqrt{\tau_n D_n}$$

$$x(\mathrm{O}^-) \doteq 0 \quad \Longrightarrow \quad \begin{cases} n(0) = n_{p0}\exp\left[qV/(k_B T_L)\right] \\ n(-\infty) = n_{p0} \end{cases}$$

L_n : minority-carrier diffusion length

However, in This case of PN Junction we can introduce another simplification i.e when we consider minority carriers in neutral region, Now we are Neutral region of p-type

$\therefore$ They are e^-'s

So, The transport eq^n for minority carriers in neutral region is such that the diffusive term is dominating. i.e it's much larger Than Drift-term.

$$\therefore \quad J_n = -q\mu_n n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

$$\Downarrow$$

$$J_n \simeq qD_n \frac{dn}{dx}$$

Note: only for minority carriers in the Quasi Neutral Region.

Since, electric potential (ϕ) doesn't appear anymore it is not necessary to solve poisson eq^n.

→ Now we have $J_n \simeq qD_n \dfrac{dn}{dx}$ — ①

$$\frac{dJ_n}{dx} = +q\,(n-n_{po})/\tau_n \quad -②$$

we put these two eqn's together

So that we have

$$J_n = -q\mu_n n \frac{d\varphi}{dx} + qD_n \frac{dn}{dx} \simeq qD_n \frac{dn}{dx}$$

$$q\frac{n-n_{p0}}{\tau_n} = \frac{dJ_n}{dx} \simeq q\frac{d}{dx}D_n\frac{dn}{dx} = qD_n\frac{d^2n}{dx^2}$$

(Poisson's equation ruled out)

combining ① & ②

$$\boxed{\frac{d^2}{dx^2}(n-n_{p0}) = \frac{n-n_{p0}}{L_n^2}, \qquad L_n \doteq \sqrt{\tau_n D_n}}$$

2nd order Diffn eqn

$$x(0^-) \doteq 0 \implies \begin{cases} n(0) = n_{p0}\exp[qV/(k_BT_L)] \\ n(-\infty) = n_{p0} \end{cases}$$

L_n : minority-carrier diffusion length

Q

$$q\,\frac{n-n_{po}}{\tau_n} = \frac{dJ_n}{dx} \simeq q\,\frac{d}{dx}D_n\frac{dn}{dx}$$

Q is diffusion coefficient (D_n) independent of position?

No, we can take it out of derivative

$D_n \propto \mu$ (mobility)

Mobility depends on T (temperature)

in this it is constant so D_n is constant.

$\therefore \Rightarrow \boxed{q\left(\frac{n-n_{po}}{\tau_n}\right) = \frac{dJ_n}{dx} = q D_n \frac{d^2 n}{dx}}$

Mobility is also depends on Interfaces but in PN Junction device The Transport occurs far from the Boundary.

$$\Rightarrow \frac{d^2}{dx^2}(n - n_{p_0}) = \frac{n - n_{po}}{L_n^2}$$

Here we took instead of n we took $n - n_{p_0}$

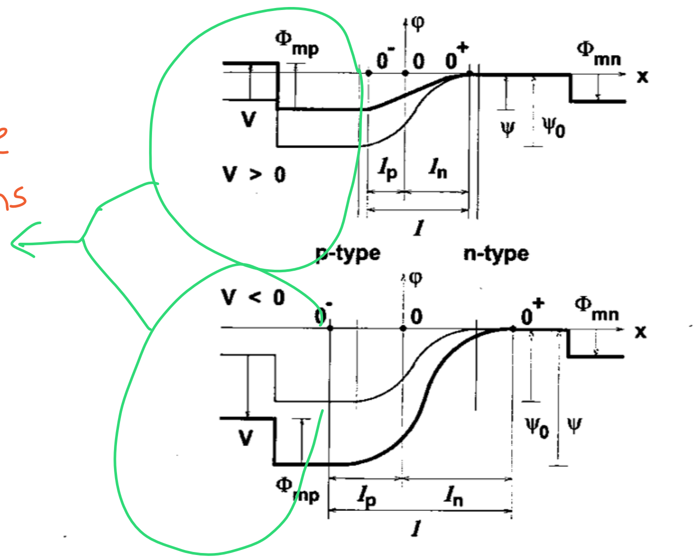
Because n_{p_0} is a constant and it's derivative is zero.

In conclusion when we consider Minority carriers in the Neutral region in this case p-type we are able to reduce the problem to a 2nd order Differential Eq^n with constant coefficients.

→ Since we have a 2nd order diff eq^n we need two boundary conditions and we find them in the Neutral region of p-type.

p-n Junction in Nonequilibrium — I

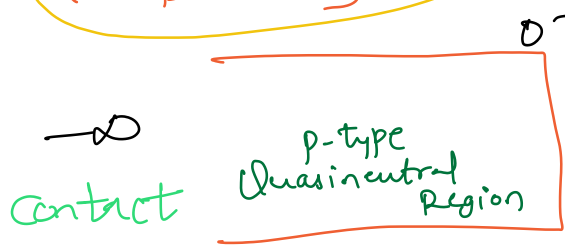
p-type regions



So, the Boundary conditions are boundaries of p-type Regions.

One Boundary is Infinity $(-\infty)$ because the contact is very far. i.e we go away from the Junction the semiconductor concentrations will behave like in Equilibrium.

→ 2nd Boundary is 0^-



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→ Now we can observe in all these calculations it was never necessary to prescribe the origin of the reference. we assume origin is in the Junction.

→ But it is easy to understand if we repeat all the reasoning we have done so far after changing the position of the origin. The current would not change.

The current would not depend on the position of the origin !

So, it is legitimate to position Origin at a convenient position i.e at O^-

Boundary conditions

Asymptotic concentration of minority carriers

$$x(O^-) \doteq 0 \implies \begin{cases} n(0) = n_{p0} \exp[qV/(k_B T_L)] \\ n(-\infty) = n_{p0} \end{cases}$$

L_n : minority-carrier diffusion length

uniform region because we are far from the Junction.

Shockley Boundary condition

∴ We can now solve the diff. Eqⁿ.

Obviously, The solution is linear combination of exponentials

T. 28.12: Densità di corrente dei minoritari ai bordi della regione di carica spaziale.

Unknown in diff eqⁿ is $n - n_{p_0}$

This unknown is a Linear combination of exponentials with Undetermined coefficients.

p-n Junction in Nonequilibrium — VII

Steady state, one-dimensional case (Shockley theory)

p-type neutral region: $x < x(\mathrm{O}^-)$, $x(\mathrm{O}^-) \doteq 0$

$$n = n_{p0} + A_n \exp(x/L_n) + B_n \exp(-x/L_n)$$

$$n(-\infty) = n_{p0} \quad \Rightarrow \quad B_n = 0$$

$$n(0) = n_{p0} \exp[qV/(k_B T_L)] \Rightarrow \begin{cases} A_n = n_{p0} F \\ F = \exp[qV/(k_B T_L)] - 1 \end{cases}$$

$$J_n = qD_n \frac{\mathrm{d}n}{\mathrm{d}x} = q\frac{D_n n_{p0}}{L_n} F \exp(x/L_n) = J_n(\mathrm{O}^-) \exp(x/L_n)$$

n-type neutral region: $x > x(\mathrm{O}^+)$, $x(\mathrm{O}^+) \doteq 0$

$$p = p_{n0} + A_p \exp(-x/L_p) + B_p \exp(x/L_p)$$

$$p(+\infty) = p_{n0} \quad \Rightarrow \quad B_p = 0$$

$$p(0) = p_{n0} \exp[qV/(k_B T_L)] \Rightarrow \begin{cases} A_p = p_{n0} F \\ F = \exp[qV/(k_B T_L)] - 1 \end{cases}$$

$$J_p = -qD_p \frac{\mathrm{d}p}{\mathrm{d}x} = q\frac{D_p p_{n0}}{L_p} F \exp(-x/L_p) = J_p(\mathrm{O}^+) \exp(-x/L_p)$$

p-type neutral region: $x < x(\mathrm{O}^-)$, $x(\mathrm{O}^-) \doteq 0$

$$n = n_{p0} + A_n \exp(x/L_n) + B_n \exp(-x/L_n)$$

$$n(-\infty) = n_{p0} \quad \Rightarrow \quad B_n = 0$$

$$n = n_{p_0} + A_n \exp(x/L_n) + B_n \exp(-x/L_n)$$

A_n, B_n — coefficients

Q How do we find coefficients using Boundary conditions?

Ans put $x = -\infty$ Then

$$n = n_{p_0} + \underbrace{A_n \exp(-\infty/L_n)}_{\text{This term goes to zero}} + \underbrace{B_n \exp(\infty/L_n)}_{\text{This term diverges}}$$

If we let x go to $-\infty$

This is not acceptable we can not accept divergent solution. ∴ The B_n term should be equated to zero.

$$\therefore \boxed{n(-\infty) = n_{p_0}} \Rightarrow B_n = 0$$

Because we can not accept divergent solutions

So now we have fulfilled Boundary condition on the left.

→ Now for Boundary condition on the Right

$$n(0) = n_{p_0} \exp\left(qV/k_BT_L\right) \Rightarrow \begin{cases} A_n = n_{p_0} F \\ F = \exp\left[qV/k_BT_L\right] - 1 \end{cases}$$

$$J_n = qD_n \frac{dn}{dx} = \frac{q D_n n_{p_0}}{L_n} F \exp(x/L_n)$$

$$= J_n(0^-) \exp(x/L_n)$$

Note

similarly for minority carriers in n-type neutral region

n-type neutral region: $x > x(0^+)$, $x(0^+) \doteq 0$

$$\boxed{p = p_{n0} + A_p \exp(-x/L_p) + B_p \exp(x/L_p)}$$

$$p(+\infty) = p_{n0} \quad \Rightarrow \quad B_p = 0$$

$$p(0) = p_{n0} \exp\left[qV/(k_BT_L)\right] \Rightarrow \begin{cases} A_p = p_{n0}\, F \\ F = \exp\left[qV/(k_BT_L)\right] - 1 \end{cases}$$

$$J_p = -qD_p \frac{\mathrm{d}p}{\mathrm{d}x} = q\frac{D_p p_{n0}}{L_p} F \exp(-x/L_p) = J_p(0^+) \exp(-x/L_p)$$

Boundary 1 : 0^+

Boundary 2 : $+\infty$

p-n Junction in Nonequilibrium — VI

Steady state, one-dimensional case (Shockley theory)

$$\begin{cases} x > x(0^+) \\ \text{(weak inj.)} \end{cases} \begin{cases} p - p_{n0} > 0 \;\Rightarrow \begin{cases} p - p_{n0} \ll n_{n0} \\ p \ll n_{n0} + p_{n0} \simeq n_{n0} \\ p_{n0} < p \ll n_{n0} \end{cases} \\ \\ p - p_{n0} < 0 \;\Rightarrow \begin{cases} p < p_{n0} \\ p_{n0} \ll n_{n0} \\ p < p_{n0} \ll n_{n0} \end{cases} \end{cases}$$

diffusion dominates

$$\boxed{J_p = -q\mu_p p \frac{\mathrm{d}\varphi}{\mathrm{d}x} - qD_p \frac{\mathrm{d}p}{\mathrm{d}x} \simeq -qD_p \frac{\mathrm{d}p}{\mathrm{d}x}}$$

$$-q\,\frac{p - p_{n0}}{\tau_p} = \frac{\mathrm{d}J_p}{\mathrm{d}x} \simeq -q\,\frac{\mathrm{d}}{\mathrm{d}x} D_p \frac{\mathrm{d}p}{\mathrm{d}x} = -qD_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2}$$

(Poisson's equation ruled out)

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(p - p_{n0}) = \frac{p - p_{n0}}{L_p^2}, \qquad L_p \doteq \sqrt{\tau_p D_p}$$

$$x(0^+) \doteq 0 \quad \Longrightarrow \quad \begin{cases} p(0) = p_{n0} \exp\left[qV/(k_B T_L)\right] \\ p(+\infty) = p_{n0} \end{cases}$$

L_p : minority-carrier diffusion length

The calculation is similar to the Minority carrier calculation in Quasineutral Region of p-type.

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