

Lecture-6

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We continue from Lecture-5, we were considering

the case of semiconductor with uniform concentration of the Dopant & start with a case of an n-type semiconductor with uniform concentration of Dopant.

428 ————— TAVOLE DI MICROELETTRONICA

T. 23.6: Espressioni usate nel calcolo della conc. estrarreca all'equilibrio (tipo n).

phosphorous atoms

Extrinsic Concentration (n-Type) — I

Uniform doping ($N_D = \text{const}$) — Equilibrium case

$$n \doteq N_e/\Omega = (1/\Omega) \int_{E_C}^{E_{CV}} g(E)P(E) dE \neq n_i$$

$$p \doteq N_h/\Omega = (1/\Omega) \int_{E_{VL}}^{E_V} g(E)[1 - P(E)] dE \neq p_i$$

$$n_D \doteq (1/\Omega) \int_{\Delta E_D} g_D(E)P_D(E) dE = N_D - N_D^+$$

$$P(E) = \frac{1}{\exp[(E - E_F)/(k_B T_L)] + 1}, \quad E_F \neq E_{F_i}$$

$$\xi_e = -\frac{\zeta_e}{k_B T_L} = -\frac{E_C - E_F}{k_B T_L}, \quad \xi_h = -\frac{\zeta_h}{k_B T_L} = -\frac{E_F - E_V}{k_B T_L}$$

$$P_D(E) = \frac{1}{(1/d_D) \exp[(E - E_F)/(k_B T_L)] + 1}$$

$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_C m_e^{3/2} \sqrt{E - E_C} \implies n = N_C \Phi_{1/2}(\xi_e)$$

$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_V m_h^{3/2} \sqrt{E_V - E} \implies p = N_V \Phi_{1/2}(\xi_h)$$

$$g_D(E) \simeq \Omega N_D \delta(E - E_D) \implies \begin{cases} n_D = N_D P_D(E_D) \\ N_D^+ = N_D [1 - P_D(E_D)] \end{cases}$$

→ In the Equilibrium Case the method for calculation of ^{Extrinsic} Concentration is identical to the case of Intrinsic Semiconductor

→ we take the Density of states & we multiply by the Fermistatistics and then we integrate over the conduction band in order to find the concentration of electrons in conduction Band of n-type semiconductor

→ we remember, however that Fermilevel is shifted towards the Edge of the conduction Band in n-type semiconductor.

So, obviously the probability of P(E) of occupancy of states will be different, ^{w.r.t. intrinsic case} For this reason the concentration of electrons of the Conduction Band will be different from the intrinsic value (n_i)

\therefore Intrinsic concentration of n-type
Semiconductor is indicated as n

Uniform doping ($N_D = \text{const}$) — Equilibrium case

$$\underline{n} \doteq N_e/\Omega = (1/\Omega) \int_{E_G}^{E_{CV}} g(E) P(E) dE \neq n_i$$

Similarly for p-type semiconductor
The concentration is indicated as 'p'

$$p \doteq N_h/\Omega = (1/\Omega) \int_{E_{VL}}^{E_V} g(E) [1 - \underline{P(E)}] dE \neq p_i$$

↓
probability of
occupancy by a hole.

→ However, in this case we have an
other population i.e. there are also the
states that will be provided by the
Donor atoms as we have seen in earlier
lectures.

The Donor atoms might not have fully
ionized and still might have electrons

in them.

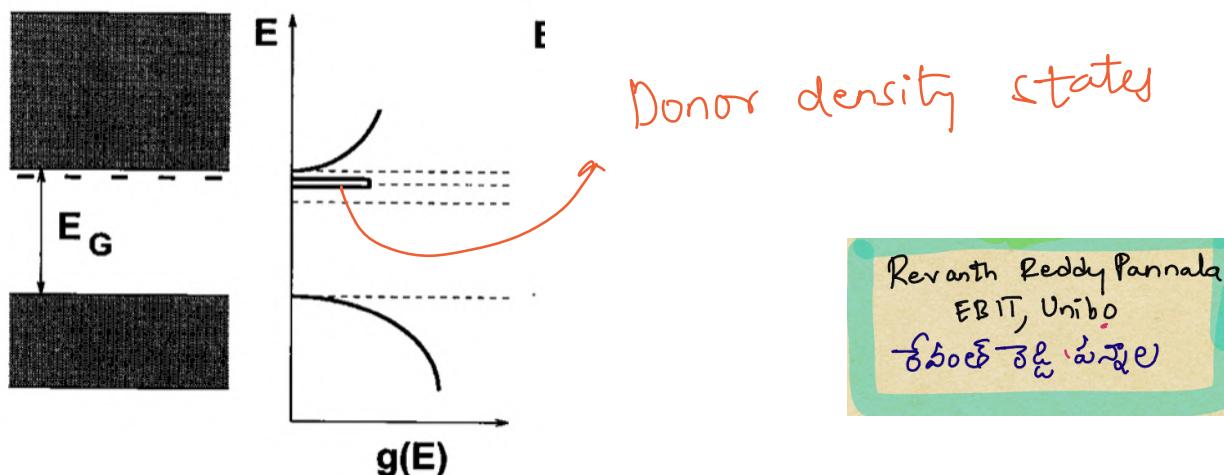
∴ we must also calculate the population of the electrons in Donor state.

The form of calculation is just the same

we take density of states of Donor states.

& multiply it with probability of occupancy
and we integrate over
the small energy difference.

SEMICONDUCTOR
BANDS DENSITY
OF STATES



In conclusion the calculation of Electrons in
Donor state is

$$n_D \doteq (1/\Omega) \int_{\Delta E_D} g_D(E) P_D(E) dE = N_D - N_D^+$$

n_D : Concentration of Donor atoms that are not ionized.

N_D : Total concentration of Donor atoms

N_D^+ : ionized ^{concentration} Donor atoms

\therefore Fermilevel of Intrinsic & Extrinsic Semiconductor are not same

This is because of E_F has shifted towards C.B for n-type semiconductor

$$P(E) = \frac{1}{\exp[(E - E_F)/(k_B T_L)] + 1}, \quad E_F \neq E_{Fi}$$

$$\xi_e = -\frac{\zeta_c}{k_B T_L} = -\frac{E_C - E_F}{k_B T_L}, \quad \xi_h = -\frac{\zeta_h}{k_B T_L} = -\frac{E_F - E_V}{k_B T_L}$$

$$P_D(E) = \frac{1}{(1/d_D) \exp[(E - E_F)/(k_B T_L)] + 1}$$

$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_C m_e^{3/2} \sqrt{E - E_C} \implies n = N_C \Phi_{1/2}(\xi_e)$$

$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_V m_h^{3/2} \sqrt{E_V - E} \implies p = N_V \Phi_{1/2}(\xi_h)$$

$$g_D(E) \simeq \Omega N_D \delta(E - E_D) \implies \begin{cases} n_D = N_D P_D(E_D) \\ N_D^+ = N_D [1 - P_D(E_D)] \end{cases}$$

→ There is a difference though in the expression of fermi statistics for the

Donor states

(Q) What is the reason for this ?

Any The reason is that if we remember when we calculate a statistical distribution it's like having empty Boxes and no of Balls and we have to distribute Balls into the Boxes and we have to calculate all the combinations.

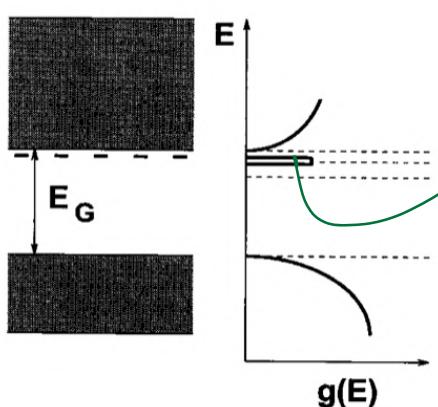
When we do this to the **Donor states** we have Modified Fermi Statistics

* The reason is The Donor States are **localized**

in space . So each Donor State may have either ONE electron or NO

Electron . No other possibility is allowed .

SEMICONDUCTOR
BANDS DENSITY
OF STATES



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→ So the counting of Distribution of electrons in the Donor state is different from counting of the electrons in the Conduction Band.

Because, states in conduction band are distributed in space everywhere in the crystal, They are not localized.

However, the difference is not so big, for the Donor states we have equilibrium probability of occupancy is a Modified fermi statistics in which exponential is multiplied by

$\frac{1}{d_D}$ → Degeneracy coefficient of Donors
Fermi statistics of DONOR STATES

$$P_D(E) = \frac{1}{(1/d_D) \exp[(E - E_F)/(k_B T_L)] + 1}$$

Probability Distribution of Conduction Band

$$P(E) = \frac{1}{\exp[(E - E_F)/(k_B T_L)] + 1}, \quad E_F \neq E_{F_i}$$

d_D

for phosphorous and silicon is
'2'

Given that the calculations
 \rightarrow for concentration of electrons in CB
and holes in the VB is same as
per the Intrinsic case.

Concentration
So, eventually we obtain at the
equilibrium of electrons

$$n = N_c \phi_{1/2}(E_e)$$

effective density of
states of Conduction Band

This is same as for the Intrinsic case.

Q What is the difference b/w n & n_i ?

A The difference is in the numerical value of the parameter (ξ_e)

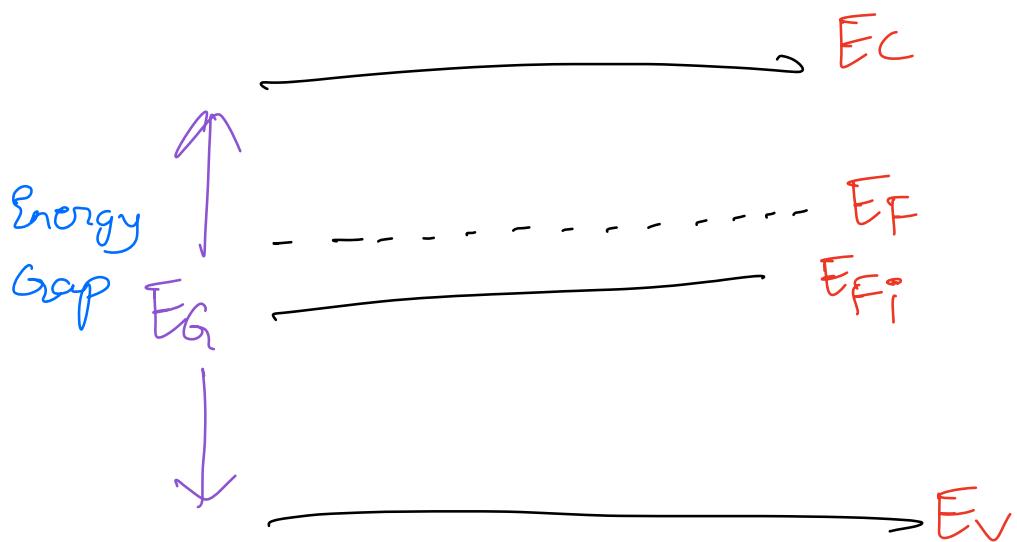
$$\xi_e = \frac{-E_c - E_F}{k_B T_L}$$

For the extrinsic semiconductor
 $E_F \neq E_{F_i}$

\therefore The Numerical value of the parameter is different from Intrinsic case

As usual, we don't know now where the Fermi level is, we just have an idea that it has moved up towards the Conduction Band for n-type.

→ At some point, we shall remember that we are using **Fermilevel** as a symbol and we must be able to calculate its position in the Energy Gap



Similarly the concentration of holes is also same.

$$P = N_v \phi_2(E_h)$$

Here also just E_h varies

→ And Finally, we must be able to calculate the population of electrons in the **Donor states**. We assume that the Donor states are distributed in a tiny space and that the Area is given. and we can replace the Area of the **Donor states** with a **Dirac Delta Function**.

$$\rightarrow \therefore \gamma_D(E)$$

$$g_D(E) = \gamma_D(E)$$

This is same representation

$$g_D(E) = 2 N_D \delta(E - E_D)$$

This Dirac Delta is centered at E_D

↓

This coefficient indicates, if we assume that the probability of occupancy in the Integral is equal to 1

$$n_D = \frac{1}{2} \int_{\Delta E_D} g_D(E) \underline{P_D(E)} dE = N_D - N_D^+$$

→ If $P_D(E) = 1$ we count the states, we impose that every state is filled, so when we count the electrons we count the states too.

so

$$n_D = \int_{\Delta E_D} g_D(E) dE = \frac{\text{no-of Donor states}}{\text{no of electron in Donor atoms}}$$

For $P_D(E) = 1$

∴ in this case we can say that the

Total no. of Donor states

$$= \text{Total no. of Donor atoms}$$

and this Total number of Donor

$$\text{atoms} = \frac{N_D}{V} \Omega \rightarrow \begin{array}{l} \text{volume of} \\ \text{Concentration of} \\ \text{Donor atoms} \end{array} \quad \begin{array}{l} \text{The} \\ \text{crystal} \end{array}$$

$$g_D(E) \approx \Omega N_D \delta(E - E_D)$$

when this is introduced in the integral, the property of the Delta is

$$n_D = \frac{1}{\Omega} \int_{E_D} g_D(E) P_D(E) dE$$

that it will force the other function to be calculated at the place in which δ is centered. So, it is like calculating

$P_D(E)$ at $E = E_D$

\therefore we will get

$$n_D = N_D P_D(E_D)$$

$$N_D^+ = N_D [1 - P_D(E_D)]$$

Note:- Again we have got 3 expression

n , P , N_D^+ in which we have unknown parameters i.e E_{F_e}

$$\therefore E_{F_e} = \frac{E_C - E_F}{k_B T_L}$$

$$E_{F_h} = \frac{E_F - E_V}{k_B T_L}$$

+ $P(E)$

+ $P(E_D)$

all have



Here, we do not know the exact position of the Fermi level & this is rather easy to calculate.

→ For n-type semiconductor.

① At 0°K The conduction Band is Empty

The Donor states & Valence Band will be completely filled.

② If we are providing energy to the n-type crystal to increase the Temperature. This will make some Donor atoms to release some electrons into the Conduction Band. This will create free Donor atoms.

ఎ స్క్రిబ్ ప్రాస్టికో రాపోల్ దుష్టాని
కదా !

ఫ్లైమ్ డానాటమ్ ఇస్ నీ రెలోచె
వ్యాస్పాన్ హెత్ లోట్ కదా !

→ In parallel there will generation of
Electron-Hole pairs from the Valence Bond
The other electrons will also be present
in Conduction Band because they are
coming from Valence Bond.

In Conclusion, the electrons in CB
are from both V.B & Donor atom.

$$\text{Total no. of electrons in } \text{CB} = \text{No. of ionized Donor atoms} + \text{No. of Holes in VB}$$

→ On the other hand we have a uniform semiconductor, so whole properties that relates to Numbers can be Translated into properties related to concentration by simply dividing by

Volume of crystals.

$$\therefore n = p + N_D^+$$

Extrinsic Concentration (n -Type) — II

Uniform doping ($N_D = \text{const}$) — Equilibrium case

$$n = p + N_D^+$$

$$N_C \Phi_{1/2}(\xi_c) = N_V \Phi_{1/2}(\xi_h) + N_D [1 - P_D(E_D)]$$

$$1 - P_D(E_D) = \frac{1}{d_D \exp[(E_F - E_D)/(k_B T_L)] + 1}$$

$$E_F - E_D = \underbrace{E_F - E_C}_{-\zeta_e} + \underbrace{E_C - E_D}_{\zeta_D > 0} \doteq k_B T_L (\xi_e - \xi_D)$$

Non-degenerate case: $\xi < -1$, $\Phi_{1/2}(\xi) \simeq \exp(\xi)$

$$\Rightarrow \begin{cases} \zeta_e > k_B T_L, & E_F < E_C - k_B T_L, & \Phi_{1/2}(\xi_e) \simeq \exp(\xi_e) \\ \zeta_h > k_B T_L, & E_F > E_V + k_B T_L, & \Phi_{1/2}(\xi_h) \simeq \exp(\xi_h) \end{cases}$$

$$np \simeq N_C \exp(\xi_e) N_V \exp(\xi_h) = N_C N_V \exp[-E_G/(k_B T_L)] = n_i^2$$

$$N_C \exp(\xi_e) = N_V \exp(\xi_h) + \frac{N_D}{d_D \exp(\xi_e - \xi_D) + 1}$$

$$s = \exp(\xi_e), \quad N_V \exp(\xi_h) = (1/s) N_V \exp(\xi_h + \xi_e) = \frac{n_i^2}{N_C s}$$

$$N_C^2 s^2 = n_i^2 + \frac{N_C N_D s}{d_D \exp(-\xi_D) s + 1} \implies E_F(T_L, N_D)$$

$$n = P + N_D^+$$

$$\Rightarrow N_C \phi_2(E_c) = N_V \phi_2(E_h) + N_D [1 - P_D(E_D)]$$

$$1 - P_D(E_D) = \frac{d_D \exp(E_F - E_D)/k_B T_L + 1}{}$$

Both E_c & $P_D(E_D)$ have E_F in their respective equations.

$$E_F - E_D = \underbrace{E_F - E_C}_{-\tau_e} + \underbrace{E_C - E_D}_{\tau_D > 0}$$

$$= k_B T_L (E_C - E_D)$$

pronounced as $\text{E}_C - \text{E}_D$

In addition to this relation we can also add the other relation that

$$[T_e + T_h = E_g] \rightarrow \text{Energy gap}$$

$$E_g = E_c - E_h$$

$$= \underbrace{E_c - E_F}_{\text{ }} + \underbrace{E_F - E_h}_{\text{ }}$$

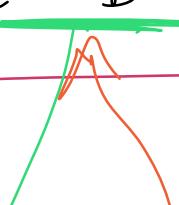
$$\cancel{E_g} \Rightarrow T_e + T_h$$

pronounced at Σ' (as Σ)

And again we have two unknowns

to solve they are E_e & E_h

$$n = p + N_D^+$$

$$N_C \phi_2(E_e) = N_V \phi_2(E_h) + N_D [1 - P_D(E_D)]$$




This has been replaced with

$$d_D \exp\left[\frac{k_B T_L (\epsilon_e - \epsilon_{iD})}{k_B T_L}\right] + 1$$

$$d_D \exp(\epsilon_e - \epsilon_{iD})$$



This difference in extrinsic \rightarrow Intrinsic case is that we cannot make estimate so easily about Fermi level -

We know Qualitatively that E_F

goes towards E_C but don't know how much?

∴ The only solution is The position of Fermi level should be found only Numerically

And we can still do Approximation of Fermi Integrals & it is possible

$$\text{i.e } \phi_{\frac{1}{2}}(\epsilon) \underset{\substack{\longrightarrow \\ \downarrow}}{\approx} \exp(-\epsilon)$$

If this can be done it is known

as Non-degeneracy case

(we can approx. fermi integrals with exponentials)

What is the Condition?

i.e $T_e \Rightarrow$ The distance from Fermi level to E_c must be

large



Non-degenerate case: $\xi < -1$, $\Phi_{1/2}(\xi) \simeq \exp(\xi)$

$$\Rightarrow \begin{cases} \zeta_e > k_B T_L, E_F < E_C - k_B T_L, \Phi_{1/2}(\xi_e) \simeq \exp(\xi_e) \\ \zeta_h > k_B T_L, E_F > E_V + k_B T_L, \Phi_{1/2}(\xi_h) \simeq \exp(\xi_h) \end{cases}$$

$$np \simeq N_C \exp(\xi_e) N_V \exp(\xi_h) = N_C N_V \exp[-E_G/(k_B T_L)] = n_i^2$$

$$N_C \exp(\xi_e) = N_V \exp(\xi_h) + \frac{N_D}{d_D \exp(\xi_e - \xi_D) + 1}$$

$$s = \exp(\xi_e), \quad N_V \exp(\xi_h) = (1/s) N_V \exp(\xi_h + \xi_e) = \frac{n_i^2}{N_C s}$$

$$N_C^2 s^2 = n_i^2 + \frac{N_C N_D s}{d_D \exp(-\xi_D) s + 1} \implies E_F(T_L, N_D)$$



For this condition you can see that exponential Approximation is possible.

for Fermi Integral

$$np = N_C \exp(\xi_e) \cdot N_V \exp(\xi_h)$$

$$np \Rightarrow N_C N_V \exp\left[-\frac{E_G}{k_B T_L}\right] = n_i^2$$

$$\Rightarrow np = n_i^2$$

Conclusion: If the semiconductor is doped and not degenerate then

n is different from n_i^*

$$n \gg n_i^*$$

p is smaller than n_i^*

But, the product
$$np = n_i^{*2}$$

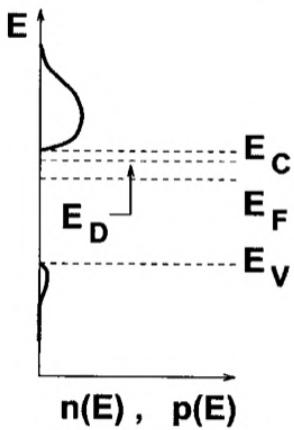
Non degeneracy is far enough from

E_C & far enough from E_V

* Now, let's assume a stricter condition that E_F is also not close to E_D

$$\cancel{f(E)} \times P(E)$$

$$\cancel{f(E)} \times [1 - P(E)]$$



Assume E_F below E_D by atleast one $k_B T_L$

$$1 - P_D(E_D) = \frac{1}{d_D \exp[(E_F - E_D)/(k_B T_L)] + 1}$$

\rightarrow In This equation we have $E_F - E_D$
 if E_F is below E_D by atleast by
 one $k_B T_L$

Then the exponential in the denominator has -1 as its power -

$$1 - P_D(E_D) = \frac{1}{d_D(\exp^{-1}) + 1}$$

~~\downarrow~~
we can neglect exponential

$$\therefore N_D^+ = N_D [1 - P_D(E_D)]$$

as it is 1

$$N_D^+ = N_D$$

i.e all the Donor atoms are Ionized and we will describe this situation as Full ionization.

Therefore, In Nondegenerate & complete Ionization case is $n = p + N_D$

Extrinsic Concentration (n -Type) — III

Uniform doping ($N_D = \text{const}$) — Equilibrium case
Non-degenerate case and complete ionization

$$E_F < E_D - k_B T_L, \quad P_D(E_D) \ll 1 \Rightarrow N_D^+ \simeq N_D, \quad n \simeq p + N_D$$

$$n = N_C s \simeq n_i^2/n + N_D, \quad n^2 - N_D n - n_i^2 = 0$$

$$n = N_D/2 + \sqrt{N_D^2/4 + n_i^2} \simeq N_D, \quad N_D \gg n_i$$

$$p = n_i^2/n \simeq n_i^2/N_D$$

$$N_C \exp(\xi_c) = N_D \Rightarrow E_F = E_C - k_B T_L \log(N_C/N_D) < E_C$$

$$\frac{dE_F}{dN_D} = \frac{k_B T_L}{N_D} > 0, \quad \frac{dE_F}{dT_L} = k_B \log\left(\frac{N_D}{N_C}\right) = -\frac{E_C - E_F}{T_L} < 0$$

$$E_C - E_F = E_C - E_{Fi} + E_{Fi} - E_F = E_C - E_{Fi} + q\varphi_F$$

$$q\varphi_F = -(E_F - E_{Fi})$$

$$n = N_C \exp[-(E_C - E_F)/(k_B T_L)] = n_i \exp[-q\varphi_F/(k_B T_L)]$$

$$n \simeq N_D \Rightarrow E_F - E_{Fi} = -q\varphi_F = k_B T_L \log\left(\frac{N_D}{n_i}\right) > 0$$

$$\varphi_F < 0, \quad E_F > E_{Fi}$$

∴ $n = p + N_D^+$

$$n \Rightarrow p + N_D^+ \quad (\text{Special case})$$

If we bring down the temperature to zero Kelvin. $N_D^+ \cancel{\equiv} 0$

i.e. $n = p$ \rightarrow off course at

$$0^\circ K \quad n = p = 0$$

But we infest the crystal
will become intrinsic at $0^\circ K$

→ Now, if we get back to the case
Uniform Doping,
Equilibrium, Non-degenerate & Complete
Ionization case

- (i) Uniform Doping
- (ii) Equilibrium
- (iii) Non-degenerate
- (iv) Complete Ionization.

In this case it is extremely easy
to calculate the population of the
Electrons and Holes, we can do it
by Algebraic calculations -

$$n = p + N_D$$

$$\therefore N_D^+ \approx N_D$$

↓
Balance Equation

Multiply both sides by ' n '

$$\Rightarrow n^2 = n(p + N_D)$$

$$\Rightarrow n^2 - N_D n - np = 0$$

$$\Rightarrow \underline{n^2 - N_D n - n_i^2 = 0}$$

$$\therefore np = n_i^2$$

$\therefore N_D$ (Doping concentration)

n_i (intrinsic concentration
at a particular Temp.)
is known

$\therefore n$; The population of electrons in conduction band can be simply found by solving the quadratic equation $\underline{n^2 - N_D n - n_i^2 = 0}$

$$n = N_C s \simeq n_i^2/n + N_D, \quad n^2 - N_D n - n_i^2 = 0$$

$$n = N_D/2 + \sqrt{N_D^2/4 + n_i^2} \simeq N_D,$$

$$p = n_i^2/n \simeq n_i^2/N_D$$

$$\overline{N_D \gg n_i}$$

Because in general case

$$N_D \gg n_i$$

$$N_D = 10^{15}/\text{cm}^3$$

$$n_i^2 = 10^{20}$$

$$\Rightarrow N_D^2 = 10^{20} ??$$

∴ If all the 4 cases hold

$$+ N_D \gg n_i$$

Then

$$n \simeq N_D$$

No. of Electrons in Conduction Band is equal
to the Donor ^{atom} concentration.

$$\therefore n_p = n_i^2 \quad \& \quad n \approx N_D$$

$$\therefore P = \frac{n_i^2}{n} = \frac{n_i^2}{N_D}$$

$$n \approx N_D = 10^{15} \quad P = \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{15}} = 10^5$$

$$n = 10^{15}, P = 10^5$$

↳ for N-type
semiconductor

$$n \gg P$$

population of e^- 's in conduction Band

population of holes in the valence Band.

* still considering this situation we may

now try and understand

- Q How the Fermi level behaves
when we change concentration of the
Dopants (Q) When we change
Temperature ?

$$N_C \exp(\xi_c) = N_D \Rightarrow E_F = E_C - k_B T_L \log(N_C/N_D) < E_C$$

$$\frac{dE_F}{dN_D} = \frac{k_B T_L}{N_D} > 0, \quad \frac{dE_F}{dT_L} = k_B \log\left(\frac{N_D}{N_C}\right) = -\frac{E_C - E_F}{T_L} < 0$$

$$E_C - E_F = E_C - E_{Fi} + E_{Fi} - E_F = E_C - E_{Fi} + q\varphi_F$$

$$q\varphi_F = -(E_F - E_{Fi})$$

$$n = N_C \exp[-(E_C - E_F)/(k_B T_L)] = n_i \exp[-q\varphi_F/(k_B T_L)]$$

$$n \simeq N_D \Rightarrow E_F - E_{Fi} = -q\varphi_F = k_B T_L \log\left(\frac{N_D}{n_i}\right) > 0$$

$$\varphi_F < 0, \quad E_F > E_{Fi}$$

$$n = N_C \exp(E_e)$$

but $n = N_D$

concentration of
electrons

$$\Rightarrow N_C \exp(E_e) = N_D = n$$

Density of states

concentration of Donor atoms

(1)

$$E_{F_e} = -\frac{(E_C - E_F)}{k_B T_L}$$

②

From ① & ②

$$N_C \exp(E_{F_e}) = N_D$$

$$\exp(E_{F_e}) = \frac{N_D}{N_C}$$

$$E_{F_e} = \log\left(\frac{N_D}{N_C}\right)$$

$$\frac{-E_C + E_F}{k_B T_L} = \log\left(\frac{N_D}{N_C}\right)$$

$$\Rightarrow \frac{E_C - E_F}{k_B T_L} = -\log\left(\frac{N_D}{N_C}\right)$$

$$\Rightarrow \frac{E_C - E_F}{k_B T_L} = \log\left(\frac{N_D}{N_C}\right)^{-1} = \log\left(\frac{N_C}{N_D}\right)$$

$$\Rightarrow E_F = E_C - k_B T_L \log\left(\frac{N_C}{N_D}\right)$$

$$< \bar{E}_C$$

In this equation we can see that the position of Fermi Level (E_F) is dependent on k_B, T_L & $N_C \& N_D$

To, check the variation w.r.t the variables we need to take the derivatives :

$$\frac{dE_F}{dN_D} = \frac{k_B T_L}{N_D} > 0, \quad \frac{dE_F}{dT_L} = k_B \log\left(\frac{N_D}{N_C}\right) = -\frac{E_C - E_F}{T_L} < 0$$

$$\frac{dE_F}{dN_D} = 0 - k_B T_L \frac{1}{N_C/N_D} \frac{d}{dN_D} \left(\frac{N_C}{N_D} \right)$$

$$\Rightarrow -k_B T_L \frac{N_D \cdot N_C - 1}{N_C^2}$$

$$\frac{dE_F}{dN_D} \Rightarrow \frac{k_B T_L}{N_D} > 0$$

i.e If we increase the Dopeant the Fermi level will go up and comes closer and closer to the edge of the conduction -

Of course, this calculation holds when we have non-degeneracy case but if increase N_D more there won't be a Non-degeneracy case -

$$N_D \uparrow \quad E_F \uparrow$$

∴ For extremely large concentrations of N_D the Fermi level may touch E_C or even cross E_C into the conduction band.

$$\rightarrow \frac{dE_F}{dT_L} = k_B \log\left(\frac{N_D}{n_C}\right)$$

$$\Rightarrow -\frac{(E_C - E_F)}{T_L} < 0$$

∴ If the temperature increase the Fermi level will go down.

$$T \uparrow \quad E_F \downarrow$$

→ In future we will discuss these dependencies with a Figure.

* We will now explore the form of equations that describe transport of charge in semiconductor. We shall see that it is preferable to consider electric potential instead of energies.

We must somehow transform energies in the equations into electric potentials.

$$E_C - E_F = E_C - E_{Fi} + \underline{E_{Fi} - E_F} = E_C - E_{Fi} + q\varphi_F$$
$$q\varphi_F = -(E_F - E_{Fi})$$

$$n = N_C \exp [-(E_C - E_F)/(k_B T_L)] = n_i \exp [-q\varphi_F/(k_B T_L)]$$

$$n \simeq N_D \Rightarrow E_F - E_{Fi} = -q\varphi_F = k_B T_L \log \left(\frac{N_D}{n_i} \right) > 0$$

$$\varphi_F < 0, \quad E_F > E_{Fi}$$

$$q\psi_F = E_{Fi} - \bar{E}_F = -(E_F - E_{Fi})$$

distance between extrinsic

Intrinsic Fermi level with
a minus

Therefore, we will have

$$\bar{E}_C - \bar{E}_F = E_C - E_{Fi} + q\phi_F$$

↑ Fermi potential

Fermi potential multiplied by q

gives distance b/w \bar{E}_F & E_{Fi}
with a minus.

→ If we replace the current expression
into the exponential

$$n = N_C \exp(E_{Fe})$$

when we do so

$$n = N_C \exp \left[\frac{-(E_C - E_F)}{k_B T_L} \right]$$

$\therefore \bar{q}_c = \frac{-(E_C - E_F)}{k_B T_L}$

$$n = N_C \exp \left[\frac{-[(E_C - E_{F_i}) + q\phi_F]}{k_B T_L} \right]$$

$$\Rightarrow N_C \exp \left[\frac{-(E_C - E_{F_i})}{k_B T_L} \right] \cdot \exp \left[\frac{-q\phi_F}{k_B T_L} \right]$$

Intrinsic concentration
of electron in conduction
Band n_i

$$n = n_i \cdot \exp \left[\frac{-q\phi_F}{k_B T_L} \right]$$

In conclusion we have an expression of the concentration of uniform semiconductor

$$n = n_i \exp \left[\frac{-q\phi_F}{k_B T_L} \right]$$

So instead of energies we have potential at the end.

$$q\phi_F = - (E_F - E_{F_i})$$

For a Donor impurity $E_F > E_{F_i}$

$q\phi_F$ is ve in this case

But $n = n_i \exp \left[\frac{-q\phi_F}{k_B T_L} \right]$ This term turns into positive

This concludes the case of Semiconductor

Doped with n-type.

lets goto p-type semiconductor.

Extrinsic Concentration (p-Type) — I

Acceptor Concentration

Uniform doping ($N_A = \text{const}$) — Equilibrium case

$$p \doteq N_h/\Omega = (1/\Omega) \int_{E_{VL}}^{E_V} g(E)[1 - P(E)] dE \neq p_i$$

$$n \doteq N_e/\Omega = (1/\Omega) \int_{E_C}^{E_{CV}} g(E)P(E) dE \neq n_i$$

$$n_A \doteq (1/\Omega) \int_{\Delta E_A} g_A(E)P_A(E) dE = N_A^- \quad ?$$

$$P(E) = \frac{1}{\exp[(E - E_F)/(k_B T_L)] + 1}, \quad E_F \neq E_{Fi}$$

$$\xi_h = -\frac{\zeta_h}{k_B T_L} = -\frac{E_F - E_V}{k_B T_L}, \quad \xi_e = -\frac{\zeta_e}{k_B T_L} = -\frac{E_C - E_F}{k_B T_L}$$

$$P_A(E) = \frac{1}{(1/d_A) \exp[(E - E_F)/(k_B T_L)] + 1}$$

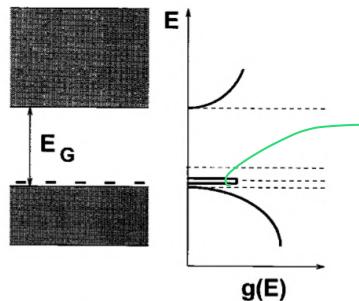
$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_V m_h^{3/2} \sqrt{E_V - E} \implies p = N_V \Phi_{1/2}(\xi_h)$$

$$g(E) \simeq \Omega \frac{\sqrt{2}}{\pi^2 \hbar^3} M_C m_e^{3/2} \sqrt{E - E_C} \implies n = N_C \Phi_{1/2}(\xi_e)$$

$$g_A(E) \simeq \Omega N_A \delta(E - E_A) \implies N_A^- = N_A P_A(E_A)$$

The whole analysis of p-type is dual to that of n-type semiconductor.

SEMICONDUCTOR
BANDS DENSITY
OF STATES



Revanth Reddy Pannala
EBIT, Unibio
శాస్త్రాల ప్రముఖ

Extrinsic Concentration (*p*-Type) — II

Uniform doping ($N_A = \text{const}$) — Equilibrium case

$$p = n + N_A$$

Total no. of holes in VB
no. of e⁻s in CB
No. of Ionized Donor Atoms.

$$N_V \Phi_{1/2}(\xi_h) = N_C \Phi_{1/2}(\xi_e) + N_A P_A(E_A)$$

$$P_A(E_A) = \frac{1}{(1/d_A) \exp[(E_A - E_F)/(k_B T_L)] + 1}$$

$$E_A - E_F = \underbrace{E_A - E_V}_{\zeta_A > 0} + \underbrace{E_V - E_F}_{-\zeta_h} \doteq k_B T_L (\xi_h - \xi_A)$$

Non-degenerate case: $\xi < -1$, $\Phi_{1/2}(\xi) \simeq \exp(\xi)$

$$\Rightarrow \begin{cases} \zeta_h > k_B T_L, & E_F > E_V + k_B T_L, & \Phi_{1/2}(\xi_h) \simeq \exp(\xi_h) \\ \zeta_e > k_B T_L, & E_F < E_C - k_B T_L, & \Phi_{1/2}(\xi_e) \simeq \exp(\xi_e) \end{cases}$$

$$np \simeq N_C \exp(\xi_e) N_V \exp(\xi_h) = N_C N_V \exp[-E_G/(k_B T_L)] = n_i^2$$

$$N_V \exp(\xi_h) = N_C \exp(\xi_e) + \frac{N_A}{(1/d_A) \exp(\xi_h - \xi_A) + 1}$$

$$s = \exp(\xi_h), \quad N_C \exp(\xi_e) = (1/s) N_C \exp(\xi_h + \xi_e) = \frac{n_i^2}{N_V s}$$

$$N_V^2 s^2 = n_i^2 + \frac{N_V N_A s}{(1/d_A) \exp(-\xi_A) s + 1} \implies E_F(T_L, N_A)$$

Extrinsic Concentration (*p*-Type) — III

Uniform doping ($N_A = \text{const}$) — Equilibrium case

Non-degenerate case and complete ionization

$$E_F > E_A + k_B T_L, P_A(E_A) \approx 1 \Rightarrow N_A^- \simeq N_A, p \simeq n + N_A$$

$$p = N_V s \simeq n_i^2/p + N_A, \quad p^2 - N_A p - n_i^2 = 0$$

Holes are
Majority carriers.

$$p = N_A/2 + \sqrt{N_A^2/4 + n_i^2} \simeq N_A, \quad N_A \gg n_i$$

$$n = n_i^2/p \simeq n_i^2/N_A$$

$$N_V \exp(\xi_h) = N_A \Rightarrow E_F = E_V + k_B T_L \log(N_V/N_A) > E_V$$

$$\frac{dE_F}{dN_A} = -\frac{k_B T_L}{N_A} < 0, \quad \frac{dE_F}{dT_L} = k_B \log\left(\frac{N_V}{N_A}\right) = \frac{E_F - E_V}{T_L} > 0$$

$$E_F - E_V = E_F - E_{Fi} + E_{Fi} - E_V = -q\varphi_F + E_{Fi} - E_V$$

$$q\varphi_F = -(E_F - E_{Fi})$$

$$p = N_V \exp[-(E_F - E_V)/(k_B T_L)] = n_i \exp[q\varphi_F/(k_B T_L)]$$

$$p \simeq N_A \Rightarrow E_{Fi} - E_F = q\varphi_F = k_B T_L \log\left(\frac{N_A}{n_i}\right) > 0$$

$$\varphi_F > 0, \quad E_F < E_{Fi}$$

Dependence of Fermilevel on the Dopant (N_A) \hookrightarrow
the Temperature (T)

$$\frac{dE_F}{dN_A} = -\frac{k_B T_L}{N_A} < 0$$

$N_A \uparrow$

$E_F \downarrow$
(goes closer to VB)

$$\frac{dE_F}{dT_L} = k_B \log \left(\frac{N_V}{N_A} \right) = \frac{E_F - E_V}{T_L} > 0$$

i.e $T \uparrow$ $E_F \uparrow$

Fermi level goes nearer to Intrinsic Fermilevel

Fermi Potential

$$E_F - E_V = E_F - E_{Fi} + E_{Fi} - E_V = -q\varphi_F + E_{Fi} - E_V$$

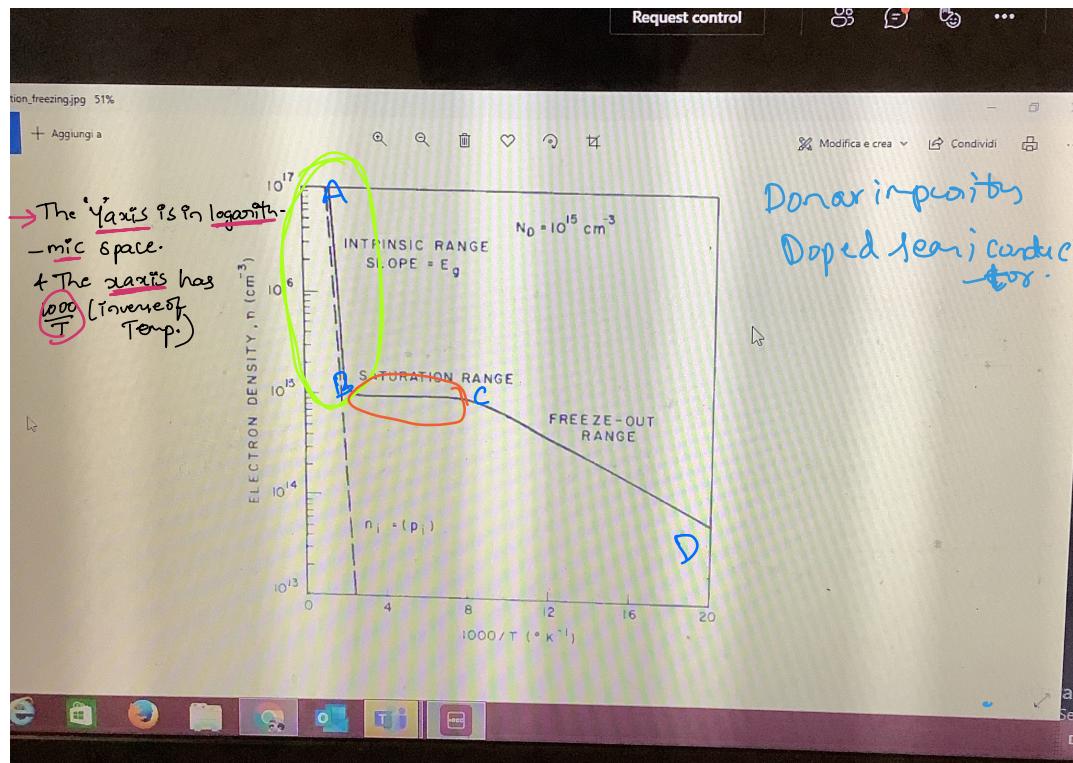
$$q\varphi_F = -(E_F - E_{Fi})$$

$$p = N_V \exp [-(E_F - E_V)/(k_B T_L)] = n_i \exp [q\varphi_F/(k_B T_L)]$$

$$p \simeq N_A \Rightarrow E_{Fi} - E_F = q\varphi_F = k_B T_L \log \left(\frac{N_A}{n_i} \right) > 0$$

$$\varphi_F > 0, \quad E_F < E_{Fi}$$

To understand the equations Graphically
we will see some figures now.



This picture shows the density of electrons in Intrinsic semiconductor and the semiconductor doped of n-type on yaxis & $\frac{1000}{T}$ on xaxis.

- The dotted line represents the dependence of Intrinsic concentration on Temperature.
 - The continuous curve represents the extrinsic case is close to the origin
- At Higher temperatures, both Intrinsic & extrinsic cases have similar behaviour (practically parallel)

Q What is the Reasoning, for similar Behavior of extrinsic & Intrinsic Semiconductors for High Temperatures ?

Ans Consider a Semiconductor at Room Temp. such that all the atoms of **Donor type** are ionized. That means we have $10^{15}/\text{cm}^3$

In parallel there is also contribution from **Silicon atoms**, but at temp. we know that it is small.

→ Let's now consider now an increase in ~~temp.~~

Now the Donor atoms can't provide any extra electrons because they are completely ionized. 2nd Ionization is impossible because Too much Energy would be needed for that.

∴ since we are increasing the Temperature the generation of Electro-Hole pairs by the silicon lattice becomes Richer and Richer.

Note— If you remember at 100°K the Intrinsic concentration of Silicon is $10^{18}/\text{cm}^3$

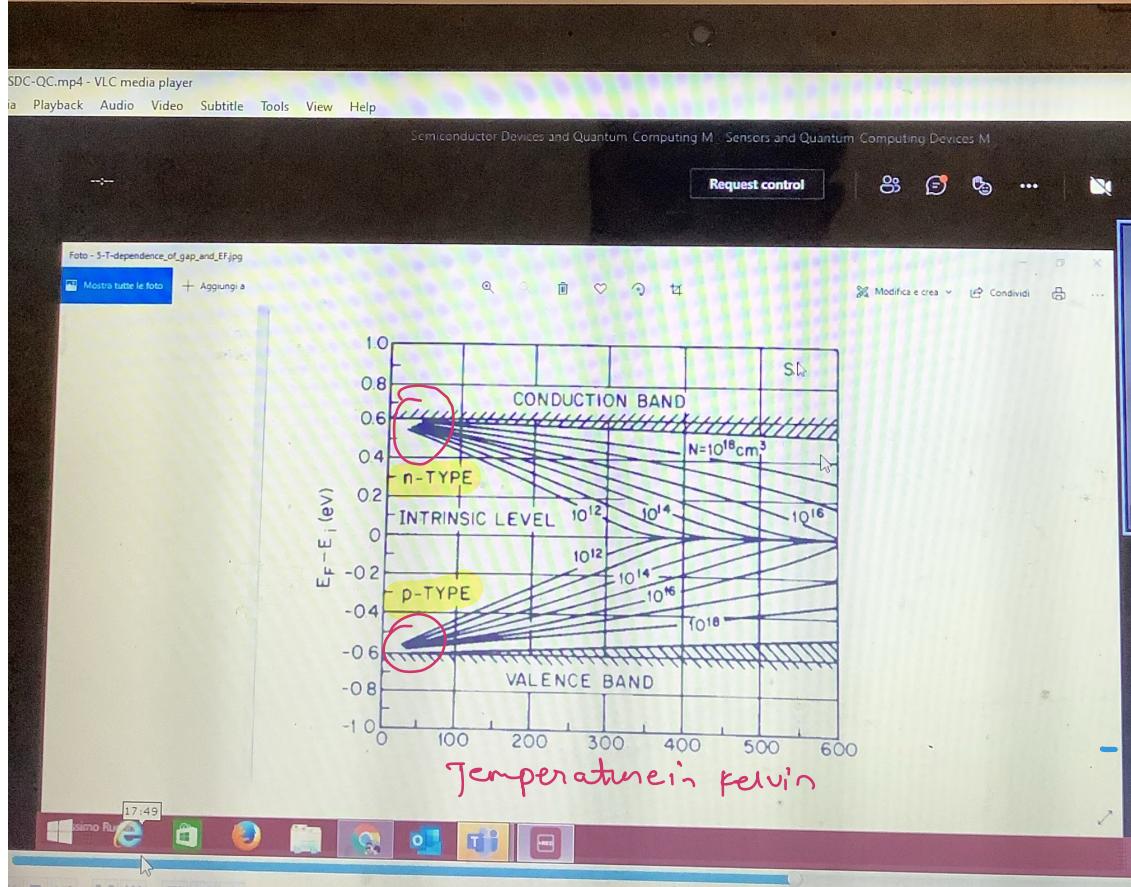
→ So as we increase the temperature the contribution from the lattice will dominate than the contribution from the Donor atoms.

So, this is why slope of Continuous curve is substantially similar to that of Intrinsic [Dotted curve].

→ At Room Temp. The electrons from Dopant Dominate i.e there is a Range of Temperature for which the concentration of the ^{Doped} material will remain constant & it is practically equal to the concentration of Donors.

→ From B to C there is a Saturation Range this is where the device is mostly useful. we could maintain a constant set of ^{Free} electrons in the crystal.

→ From C to D is Freeze-out Region where the Dopants don't have enough energy to ionize.



x-axis: Temperature in Kelvin

y-axis: difference in Fermi level

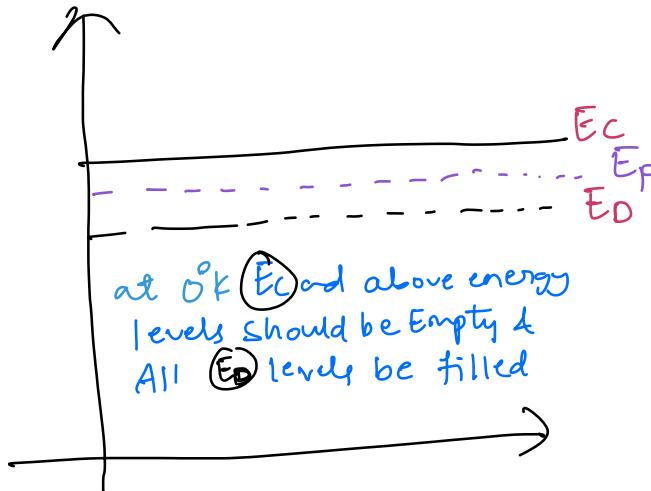
The origin of vertical axis is at Mid Gap

For silicon $E_g = 1.2 \text{ eV}$

→ We have seen for n-type doping that as Temp \uparrow the fermi level will far from E_c down. It also depends on Donor Concentration. for Higher concentration Fermi level will be (E_F)

close to (E_C)

→ But, As the Temperature decreases the Fermi level goes close to the E_C and at absolute 0°K the E_F goes b/w E_C & E_D



Because we remember
that the Fermi
statistics at 0°K
becomes

That's why there is concentration of lines
at the absolute zero in the graph for different
concentrations

The final part of analysis is a case
where we have both Donor & Acceptor
Impurities in the crystal.

In which both Dopants have uniform concentration.

nothing changes we simply have to repeat previous calculations.

Extrinsic Concentration (*n*- and *p*-Type) — I

Uniform doping ($N_D, N_A = \text{const}$) — Equilibrium case

$$n = N_C \Phi_{1/2}(\xi_e), \quad p = N_V \Phi_{1/2}(\xi_h)$$

$$N_D^+ = N_D [1 - P_D(E_D)], \quad N_A^- = N_A P_A(E_A)$$

Balance Equation

$$n + N_A^- = p + N_D^+ \quad \text{"The concentration of electrical charges is same i.e. -ve & +ve"}$$

$$N_C \Phi_{1/2}(\xi_e) + N_A P_A(E_A) = N_V \Phi_{1/2}(\xi_h) + N_D [1 - P_D(E_D)]$$

→ The material is electrically neutral Globally.

Compensation effect *if*

This is similar to Balance condition in *n*-type.

$$n = p + N$$

This is similar to *p*-type semiconductor

$$n - N = p \quad N \doteq N_D^+ - N_A^-$$

$$\text{Non-degenerate case: } \begin{cases} \zeta_e > k_B T_L, E_F < E_C - k_B T_L \\ \zeta_h > k_B T_L, E_F > E_V + k_B T_L \end{cases}$$

$$np \simeq N_C \exp(\xi_e) N_V \exp(\xi_h) = N_C N_V \exp[-E_G/(k_B T_L)] = n_i^2$$

Non-degenerate case and complete ionization

$$\begin{aligned} E_F < E_D - k_B T_L, \quad N_D^+ \simeq N_D \\ E_F > E_A + k_B T_L, \quad N_A^- \simeq N_A \end{aligned} \Rightarrow N \simeq N_D - N_A$$

→ When Non-degenerate case happens the concentration of ionized Donor becomes equal to the concentration of Donors.

$$\left. \begin{aligned} \text{i.e. } N_D^+ &\simeq N_D \\ N_A^- &\simeq N_A \end{aligned} \right\} \Rightarrow N = N_D - N_A$$

$$\begin{aligned} N_D - N_A &\text{ for } n\text{-type} \\ N_A - N_D &\text{ for } p\text{-type} \end{aligned}$$

This occurs, If there is any
difference in N_A & N_D

Extrinsic Concentration (n - and p -Type) — II

Uniform doping ($N_D, N_A = \text{const}$) — Equilibrium case
Non-degenerate case and complete ionization

If $N_D > N_A$

$$n = N_C \exp(\xi_e) \simeq n_i^2/n + N, \quad n^2 - Nn - n_i^2 = 0$$

$$n = N/2 + \sqrt{N^2/4 + n_i^2}$$

If $N_A > N_D$

$$p = N_V \exp(\xi_h) \simeq n_i^2/p - N, \quad p^2 + Np - n_i^2 = 0$$

$$p = -N/2 + \sqrt{N^2/4 + n_i^2}$$

$$N = N_D^+ - N_A^-$$

$$N \gg n_i \implies \begin{cases} n \simeq N \\ p \simeq n_i^2/N \\ -q\varphi_F = k_B T_L \log(N/n_i) > 0 \\ E_F > E_{Fi} \end{cases}$$

$$-N \gg n_i \implies \begin{cases} p \simeq -N = |N| \\ n \simeq n_i^2/|N| \\ q\varphi_F = k_B T_L \log(|N|/n_i) > 0 \\ E_F < E_{Fi} \end{cases}$$

Note: This completes the analysis of Dopant concentration done uniform in space. But, this is something seldom occurs.

In reality it is not the case.



Extrinsic Concentration (*n*- and *p*-Type) — III

Non-uniform doping — Equilibrium case

$$N_D = N_D(\mathbf{r}), \quad N_A = N_A(\mathbf{r})$$

$$P(E, \mathbf{r}) = \frac{1}{\exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$

$$n(\mathbf{r}) = N_C \Phi_{1/2}(\xi_e(\mathbf{r})), \quad p(\mathbf{r}) = N_V \Phi_{1/2}(\xi_h(\mathbf{r}))$$

$$P_D(E, \mathbf{r}) = \frac{1}{(1/d_D) \exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$

$$N_D^+(\mathbf{r}) = N_D(\mathbf{r}) [1 - P_D(E_D, \mathbf{r})]$$

$$P_A(E, \mathbf{r}) = \frac{1}{(1/d_A) \exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$

$$N_A^-(\mathbf{r}) = N_A(\mathbf{r}) P_A(E_A, \mathbf{r})$$

$$n + N_A^- \neq p + N_D^+, \quad \varrho = q (p - n + N_D^+ - N_A^-) \neq 0$$

$\bar{E}_F(\mathbf{r}) \iff$ differential equation.

→ Here, in non-uniform Doping too Fermi statistics apply but they depend on position too, not on energy levels.

$$P(E, \mathbf{r}) = \frac{1}{\exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$

↓

Here the Fermi level is a function
of space because of Non-uniform
Doping.

Note:- We just make small change to the probability of occupancy, and the rest of calculation is almost similar to the uniform distribution case. because we will integrating over Energy.

→ So, when we simply want to calculate the concentration of electrons we simply take $P(E_F(r))$ & multiply by Density of states in every position not influenced by Dopants. Then we integrate of Energy & the function $\bar{E}_F(r)$ is just a parameter. It is not involved in the Integration.

So, The outcome is again a Fermi Integral. The only difference being the Fermi Integral will contain $\bar{E}_F(r)$ so it will also depend on position -

Formally, The concentration of electrons is

Holes in Equilibrium condition with Non-Uniform Doping will be given again by Two Fermi Integral with $\bar{E}_F(r)$

→ ① How $\bar{E}_F(r)$ is dependent on the position?

It is not described now, but will discuss it in the future.

* All the calculations are similar to what we have done in uniform distribution case because we have introduced just $\bar{E}_F(r)$ and $P(E/r)$ is still the probability of the occupancy of the states.

Only here the $P(E/r)$ will depend on the position.

** So, when we want to calculate the concentration of electrons, we simply take $P(E/r)$ and multiply by the density of states, which is same at every position it is not influenced by the presence of dopants.

And then we integrate over the Energy -

→ However, we notice that when we integrate over energy, this function $\bar{E}_F(\mathbf{r})$ is just a parameter.

It is not involved in the Integration.

So the Outcome is again a Fermi Integral.

The rest is all similar to previous case.

$$P_D(E, \mathbf{r}) = \frac{1}{(1/d_D) \exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$
$$N_D^+(\mathbf{r}) = N_D(\mathbf{r}) [1 - P_D(E_D, \mathbf{r})]$$

$$P_A(E, \mathbf{r}) = \frac{1}{(1/d_A) \exp[(E - \bar{E}_F(\mathbf{r})) / (k_B T_L)] + 1}$$
$$N_A^-(\mathbf{r}) = N_A(\mathbf{r}) P_A(E_A, \mathbf{r})$$

$P_D(E, \mathbf{r}) :$ } Modified statistics for Donor
& Acceptor states
 $P_A(E, \mathbf{r}) :$ }

$$N_D^+(r), N_A^-(r)$$

↓
distribution of ionized donors and ionized acceptors.

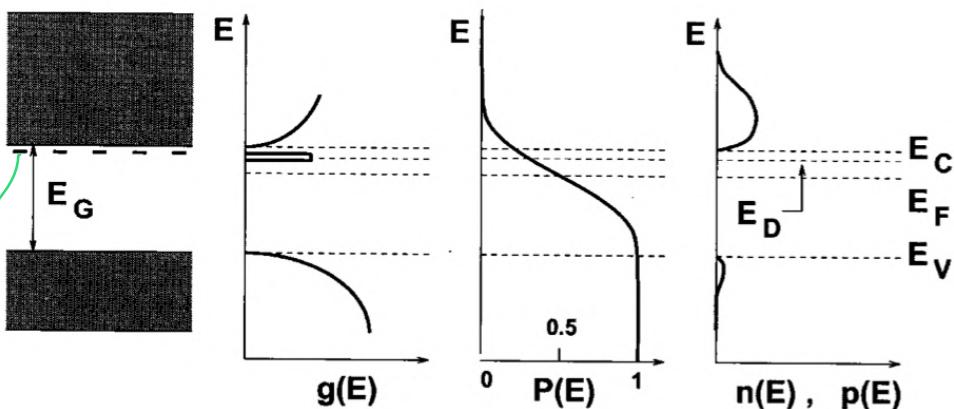
→ At this point we are in usual situation of Uniform Distribution of ionized dopants fulfill a Balance equation.

Then we go from numbers to the concentration by simply dividing by the volume. This is not which can not be done any more if the situation is Non-Uniform.

*

DONOR-TYPE DOPING (n-type semiconductor)

SEMICONDUCTOR BANDS	DENSITY OF STATES	OCCUPATION PROBABILITY	$g(E) \times P(E)$	$g(E) \times [1 - P(E)]$
---------------------	-------------------	------------------------	--------------------	--------------------------



*

These **Horizontal lines** describe the positions of states with respect to the Donor dopants.

Assume, that the distribution is non-uniform in space, there are many of them on the left and not so many on the Right

Ex:

— - - - -

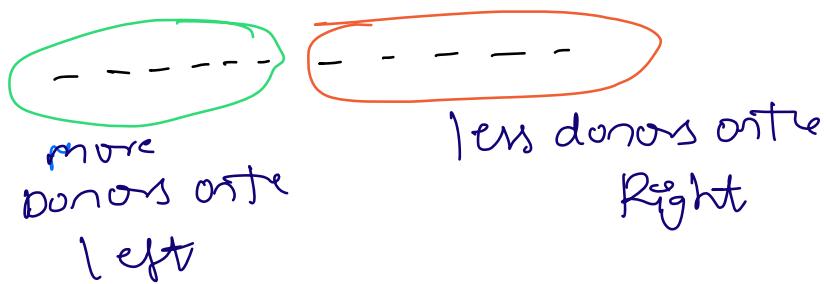
and we start from 0°K and we increase the Temperature, so all the Donor states will start to Ionize and will donate

the electrons to the conduction band.

→ As soon as they ionize there will be e^- s in the conduction band.

However, we remember that the dopant distribution is not uniform.

We have many donors on the left and not so many on the right.



So, there will many electrons donated onto left and not so many onto right.

* ప్రకృతిలో నీడి ఒకే విధంగా ఓండు, వాటి వాటి ప్రకృతుల భాబెల్లు చేసే విధంగా నీడి ప్రతిష్టాపన.

A) మనం ఉన్న దెరికండ్టర్ ను ఐదే
కాల్ కాస్టా కావు నీటి ను నొప్పించాలని
ఏ విశ్లేషణ ను మనం ఉన్న దెరికండ్టర్
ను చేసి అంగుధి ఫాక్టా రెస్టా
కో.

→ The distribution of electrons is
also not uniform from Donor
atoms.

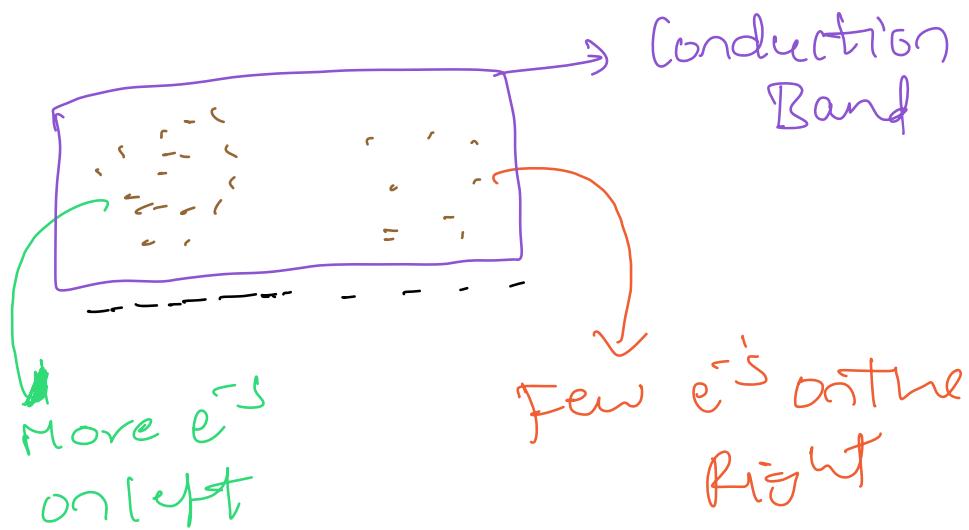
Anyway The electrons of The Conduction
Band
are free to move. So there is a
transport mechanism called Diffusion
(ఎగ్జప్ళిషన్)

And The Diffusion is a mechanism such
that particles that diffuse move
from region their Concentration is
larger to the region where the

Concentration is lower.

కొన్ట్రేషన్, సెంసర్ కోసం నుండి వీళ్లు!

So we have many electrons on the left and fewer on the right

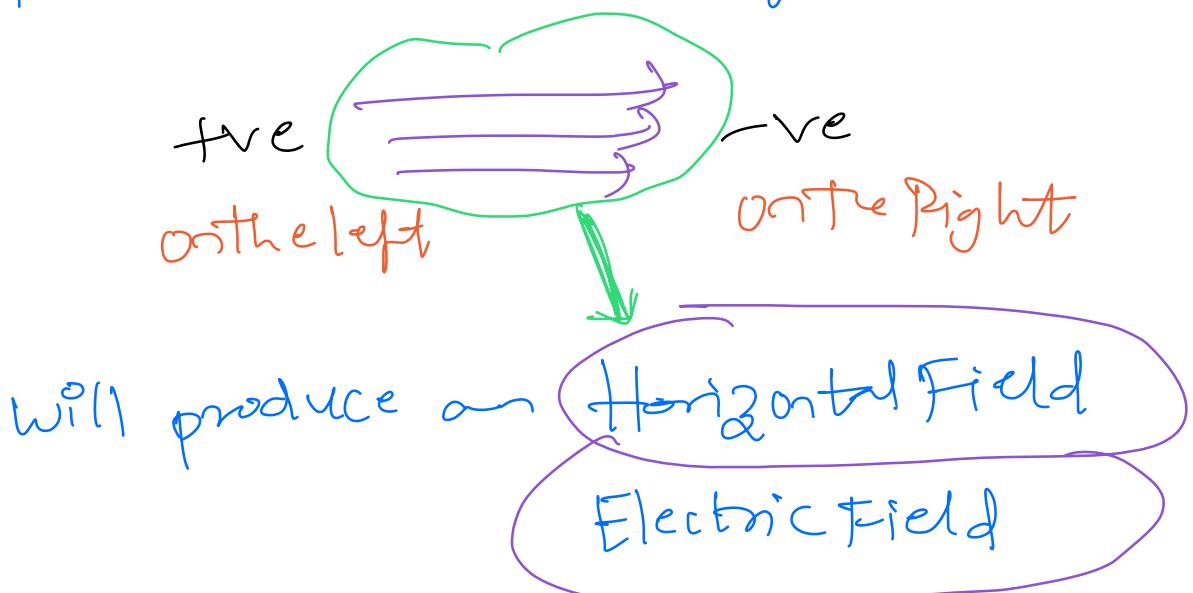


→ So Diffusion Happens from Left to Right.

ఇది నుండి నుండి గుర్తా!

* However, the Ionized Donors that are positive they can not move. So, When the e^- s in CB move Right There will be excess positive charge on the left & excess negative charge on the right of CB.

These Two Excess charges



→ This field will eventually stop the motion of electrons \Rightarrow Equilibrium Condition will be restored.

విద్యుత్, ఇంజనీరింగ్, ఆఫీస్
రమాల్యోడ్ !

→ In Conclusion, we can not anymore establish Balance Equation because the meaning of Balance Equation is at each position the positive charge exactly compensates the Negative charge.

సి. రమా రమా గాన్ నిమిషమ్
Doping S. This is not true anymore in Non-uniform case.



$$n + N_A^- \neq p + N_D^+,$$

$$\varrho = q (p - n + N_D^+ - N_A^-) \neq 0$$

$\bar{E}_F(r) \iff$ differential equation.

Globally, the charge density at each position in the semiconductor is expected to be different from zero.

∴ Balance Equation can not be Applied.

* Therefore, in Non-uniform Doping case given that charge density is Non zero. so that it gives raise to distribution of electric field in the material.

Wenzel solved a differential equation

whose outcome is the Electric Field.

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ఒకొక ప్రశ్నక నుండి వైపులైన్నాడు
సుమారును!

Extrinsic Concentration (*n*- and *p*-Type) — IV

Non-uniform doping — Equilibrium case

$$\zeta_e(\mathbf{r}) = E_C(\mathbf{r}) - \bar{E}_F(\mathbf{r}), \quad \zeta_h(\mathbf{r}) = \bar{E}_F(\mathbf{r}) - E_V(\mathbf{r})$$

$$\zeta_c(\mathbf{r}) + \zeta_h(\mathbf{r}) \doteq -k_B T_L [\xi_e(\mathbf{r}) + \xi_h(\mathbf{r})] = E_G(\mathbf{r})$$

$$E_C(\mathbf{r}) \doteq E_{Ci} - \Delta E_C(\mathbf{r}), \quad E_V(\mathbf{r}) \doteq E_{Vi} + \Delta E_V(\mathbf{r})$$

$\Delta E_C(\mathbf{r})$, $\Delta E_V(\mathbf{r}) > 0$: Band-gap narrowing

$$E_G(\mathbf{r}) = \underbrace{E_{Ci} - E_{Vi}}_{E_{Gi}} - \underbrace{[\Delta E_C(\mathbf{r}) + \Delta E_V(\mathbf{r})]}_{\Delta E_C > 0}$$

$$\bar{E}_F(\mathbf{r}) = -U(\mathbf{r}) + \bar{E}_F^{(0)}, \quad \begin{cases} E_F \doteq \bar{E}_F^{(0)} \\ U(\mathbf{r}) = -q\varphi(\mathbf{r}) \end{cases}$$

$$E_C(\mathbf{r}) - \bar{E}_F(\mathbf{r}) = E_{Ci} - E_F + \frac{U(\mathbf{r}) - \Delta E_C(\mathbf{r})}{U_c(\mathbf{r})}$$

$$\bar{E}_F(\mathbf{r}) - E_V(\mathbf{r}) = E_F - E_{Vi} - \underbrace{[U(\mathbf{r}) + \Delta E_V(\mathbf{r})]}_{U_h(\mathbf{r})}$$

$$\bar{E}_F(\mathbf{r}) = -U(\mathbf{r}) + \bar{E}_F^{(0)}, \quad \begin{cases} E_F \doteq \bar{E}_F^{(0)} \\ U(\mathbf{r}) = -q\varphi(\mathbf{r}) \end{cases}$$

This can be indicated as Electrostatic potential ^{Energy} present in a crystal.

$$V(r) = -q \underbrace{\phi(r)}_{\text{electrostatic potential}}$$

**

$$E_C(\mathbf{r}) - \bar{E}_F(\mathbf{r}) = E_{Ci} - E_F + \underbrace{U(\mathbf{r}) - \Delta E_C(\mathbf{r})}_{U_c(\mathbf{r})}$$

$$\bar{E}_F(\mathbf{r}) - E_V(\mathbf{r}) = E_F - E_{Vi} - \underbrace{[U(\mathbf{r}) + \Delta E_V(\mathbf{r})]}_{U_h(\mathbf{r})}$$

And we get down to a case we have familiar terms

These can also be written as

$$E_C - \bar{E}_F(r) = E_C - E_F + V(r)$$

$$= E_C - E_F - q\phi(r)$$

$$\bar{E}_F(r) - E_V = E_F - E_V - U(r)$$

$$= E_F - E_V + q\phi(r)$$

Extrinsic Concentration (*n*- and *p*-Type) — V

Non-uniform doping — Equilibrium case

Non-degenerate case:

$$\begin{cases} \zeta_e > k_B T_L, \quad \bar{E}_F < E_C - k_B T_L \\ \zeta_h > k_B T_L, \quad \bar{E}_F > E_V + k_B T_L \end{cases}$$

$$np \simeq N_C N_V \exp[-E_G(r)/(k_B T_L)] \doteq n_e^2(r)$$

$$n_e^2(r) = n_i^2 \exp[\Delta E_G(r)/(k_B T_L)] > n_i^2$$

Negligible band-gap narrowing

$$E_C(r) - \bar{E}_F(r) \simeq E_{Ci} - E_F + U(r)$$

$$\bar{E}_F(r) - E_V(r) \simeq E_F - E_{Vi} - U(r)$$

Non-uniform
doping
concentration

$$n = N_C \exp[-(E_{Ci} - E_F + U)/(k_B T_L)]$$

$$p = N_V \exp[-(E_F - E_{Vi} - U)/(k_B T_L)]$$

$$n^{(0)} \doteq N_C \exp[(E_F - E_{Ci})/(k_B T_L)] = n_i \exp[-q\varphi_F/(k_B T_L)]$$

$$p^{(0)} \doteq N_V \exp[(E_{Vi} - E_F)/(k_B T_L)] = n_i \exp[q\varphi_F/(k_B T_L)]$$

Case in which there is atleast a point where $U=0$ which is equal to Intrinsic case

$$n = n^{(0)} \exp[q\varphi/(k_B T_L)] = n_i \exp[q(\varphi - \varphi_F)/(k_B T_L)]$$

$$p = p^{(0)} \exp[-q\varphi/(k_B T_L)] = n_i \exp[q(\varphi_F - \varphi)/(k_B T_L)]$$

Expressing both together to describe Non-uniform
Doping concentration of Electrons & Holes.





If we multiply n & p at Equilibrium
we will still get n_i^2 if the Semiconductor
is Non-degenerate.

$$\Rightarrow n_p = n_i^2 \exp\left(\frac{q(\phi - \phi_F)}{k_B T_L}\right) \times \exp\left(\frac{q(\phi_F - \phi)}{k_B T_L}\right)$$
$$\Rightarrow n_i^2 \exp\left(\frac{q\phi - q\phi_F + q\phi_F - q\phi}{k_B T_L}\right)$$

$$n_p \Rightarrow n_i^2 \cdot 1 *$$

Only in Non-Degenerate Case

* Now, we must figure out **How to**
Calculate Electric potential in a
Semiconductor !

Maxwell's Equations in Semiconductors — I

$$\text{Charge Density} \quad Q = \rho = q(p - n + N), \quad \text{Net concentration of Ionized Dopeants} \quad N = N_D^+ - N_A^-, \quad \text{Displacement vector} \quad \mathbf{D} = \epsilon_{sc} \mathbf{E} = \epsilon_{sc} \mathbf{E}$$

Complete ionization: $N = N(\mathbf{r}) = N_D - N_A$

$$\mathbf{J} = \frac{\rho_p \mathbf{v}_p}{J_p} + \frac{\rho_n \mathbf{v}_n}{J_n} = qp \mathbf{v}_p - qn \mathbf{v}_n$$

Non-degenerate semiconductor — Equilibrium condition

$$\frac{\partial}{\partial t} = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0, \quad \mathbf{E} = -\operatorname{grad} \varphi$$

$$\left. \begin{array}{l} n = n^{(0)} \exp[-q\varphi/(k_B T_L)] \\ p = p^{(0)} \exp[-q\varphi/(k_B T_L)] \end{array} \right\} \Rightarrow \rho = \rho(\varphi, \mathbf{r})$$

Poisson's equation

$$\operatorname{div} \mathbf{D} = \rho \Rightarrow \operatorname{div} (\epsilon_{sc} \mathbf{E}) = -\epsilon_{sc} \nabla^2 \varphi = \rho(\varphi, \mathbf{r})$$

$$-\epsilon_{sc} \nabla^2 \varphi = q \left[p^{(0)} \exp\left(\frac{-q\varphi}{k_B T_L}\right) - n^{(0)} \exp\left(\frac{q\varphi}{k_B T_L}\right) + N(\mathbf{r}) \right]$$

(supplemented with boundary conditions)

$$\mathbf{D} = \frac{\epsilon_{sc}}{J} \mathbf{E} \Leftrightarrow \epsilon_{sc} \mathbf{E}$$

permittivity
of semiconductor

Electric Field
Intensity

We assume that we are in Non-degenerate and complete Ionization Condition. So that the net Dopeant concentration is

$$N = N_D - N_A$$

Current Density $J = J_p + J_n$

$$= \rho_p v_p + \rho_n v_n$$

$$J = neV$$

↓

ρ Charge Density

Drift velocity

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$$J = \rho_p v_p + \rho_n v_n$$

$$J = q_p v_p - q_n v_n$$

" We don't have N_A, N_D because they don't move & can't contribute to the current density "

→ An easy condition to analyze in semiconductor Devices is Equilibrium condition.

In Equilibrium Condition obviously nothing depends on Time i.e

$$\frac{\partial}{\partial t} = 0$$

Also in Equilibrium Condition Current
Densities are zero.

$$\bar{J}_n = \bar{J}_p = 0$$

&

$$\text{Avg. velocity} = 0$$

$$E = E = -\text{grad } \phi$$

ϕ = Electric potential
which is unknown so far.

Non-degenerate semiconductor — Equilibrium condition

$$\frac{\partial}{\partial t} = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0, \quad \mathcal{E} = -\text{grad } \varphi$$

$$\left. \begin{array}{l} n = n^{(0)} \exp[-q\varphi/(k_B T_L)] \\ p = p^{(0)} \exp[-q\varphi/(k_B T_L)] \end{array} \right\} \Rightarrow \rho = \rho(\varphi, \mathbf{r})$$

Poisson's equation

$$\text{div } \mathbf{D} = \rho \quad \Rightarrow \quad \text{div}(\varepsilon_{sc} \mathcal{E}) = -\varepsilon_{sc} \nabla^2 \varphi = \rho(\varphi, \mathbf{r})$$

$$-\varepsilon_{sc} \nabla^2 \varphi = q \left[p^{(0)} \exp\left(-\frac{q\varphi}{k_B T_L}\right) - n^{(0)} \exp\left(\frac{q\varphi}{k_B T_L}\right) + N(\mathbf{r}) \right]$$

(supplemented with boundary conditions)

$$\text{div } \mathbf{D} = \rho$$

$$\begin{aligned} &\Rightarrow \text{div}(\varepsilon_{sc} E) \\ &\Rightarrow \text{div}[\varepsilon_{sc}(-\text{grad } \phi)] \\ &\Rightarrow -\varepsilon_{sc} \nabla^2 \phi \end{aligned}$$

$\therefore E = -\text{grad } \phi$

divergence of a Gradient
is
Laplacian.

$$\therefore \text{div}D = -\epsilon_s \nabla^2 \phi = Q$$

Q : charge density

ϕ : Electric Potential

This is the Poisson's Equation

$$-\epsilon_{sc} \nabla^2 \varphi = q \left[p^{(0)} \exp\left(\frac{-q\varphi}{k_B T_L}\right) - n^{(0)} \exp\left(\frac{q\varphi}{k_B T_L}\right) + N(r) \right]$$

(supplemented with boundary conditions)

This is the differential equation of second order with only one unknown

ϕ .

Int caplination of Maxwell's Equations

Maxwell's Equations — I

First group

$$\text{div } \mathbf{D} = \rho, \quad \text{rot } \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\rho = \rho_1 + \rho_2 + \dots, \quad \mathbf{J} = \rho_1 \mathbf{v}_1 + \rho_2 \mathbf{v}_2 + \dots$$

Second group

$$\text{div } \mathbf{B} = 0, \quad \text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Scalar and vector potential

$$\text{div } \mathbf{B} = 0 \implies \mathbf{B} = \text{rot } \mathbf{A}$$

$$\text{rot } \mathbf{E} + \frac{\partial \text{rot } \mathbf{A}}{\partial t} = \text{rot} \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \implies$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = \text{grad}(-\varphi), \quad \mathbf{E} = -\text{grad} \varphi - \frac{\partial \mathbf{A}}{\partial t}$$

Continuity equation

$$\text{div rot } \mathbf{H} = \text{div} \frac{\partial \mathbf{D}}{\partial t} + \text{div } \mathbf{J} = \frac{\partial \text{div } \mathbf{D}}{\partial t} + \text{div } \mathbf{J}$$

$$\frac{\partial \rho}{\partial t} + \text{div } \mathbf{J} = 0$$

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