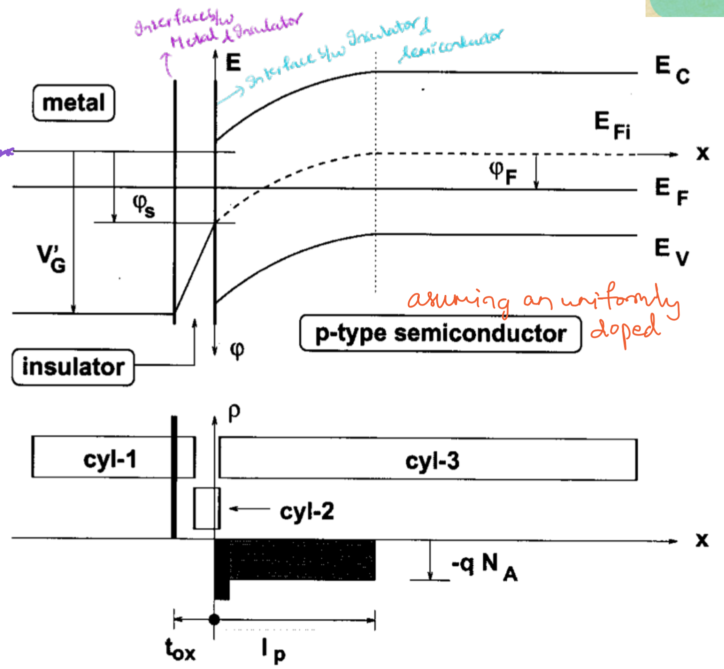
23/03/24

Lecture-13

16/10/20

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Internal zero Reference for Electric potential



→ So, The MOS capacitor is a structure that is made of silicon substrate on which layer of Insulator is deposited or Grown.

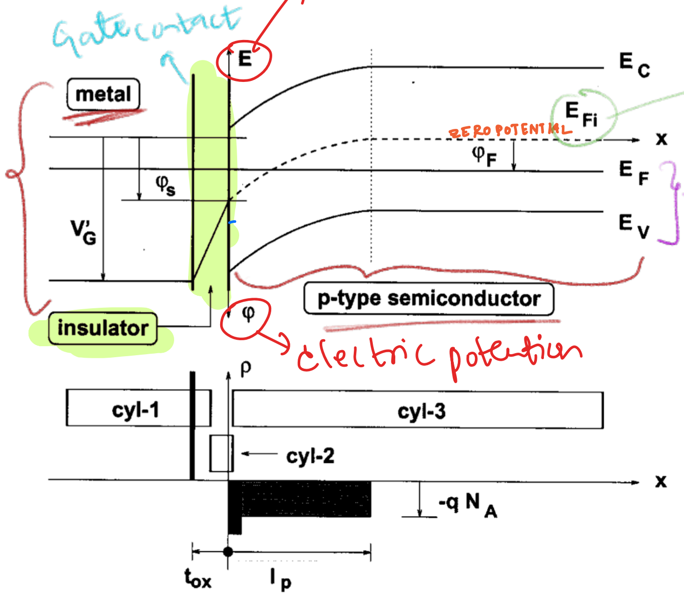
- In the silicon Technology, The insulator is typically made of SiO_2 and This layer is obtained by growing the Oxide on The silicon (by exposing silicon at High temperature to a Gas with some concentration of oxygen)

- If the insulator is made of another material Then it is deposited on top of the silicon.

Then a metal contact is deposited on top of the insulator another contact is deposited on the other side of the semiconductor.

So we have a sandwich. If we look at the figure above we can see it.

energy axis



intrinsic Fermi level

Here E_F is closer to the E_V so it is a P-type semiconductor dopant is of Acceptor type.

$$E \propto -\frac{d\phi}{dv}$$

- In this figure we assume semiconductor is Uniformly with Acceptor impurity, if we look at the position of The Fermi level E_F is below the E_{Fi} (intrinsic Fermi level). because dopant is of Acceptor type

- This diagram mixes Energies & potentials.

The Potential energy of Electron $= -q\phi$

electric potential

i.e The orientation of Energy axis is Opposite to the orientation of Electric potential.

E & ϕ are opposite in The Figure

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

- We assume to apply constant voltage V_G b/w Metal contacts on the left & Right (it is not indicated in the figure)

Left Metal contact $\Rightarrow$ Gate contact

Right Metal contact $\Rightarrow$ Bulk contact

- We assume that this voltage is constant w.r.to time

The point is we have an insulator in b/w, $\therefore$ no current can flow through the device and The device is in equilibrium condition.

- So it is sufficient to use Poisson Eq^n to analyze the Behaviour of device. (MOS)

It is convenient when we study the MOS device by itself, to set the zero of the Electric Potential asymptotically in the semiconductor (i.e inside)

i.e zero of Electric potential coinsides with the horizontal part of dashed line (Mid gap)

- When we apply V_G b/w Gate & Bulk, then we remember that when there is a contact b/w metal & semiconductor, there is a discontinuity in the Electric potential (The difference of work function of Metal & semiconductor)

- But, in the figure we are not considering Bulk contact. We must observe the difference in Electric potential b/w Gate contact & internal zero of the potential is diffent from V_G, because for reference we are using interior of the semiconductor of zero potential.

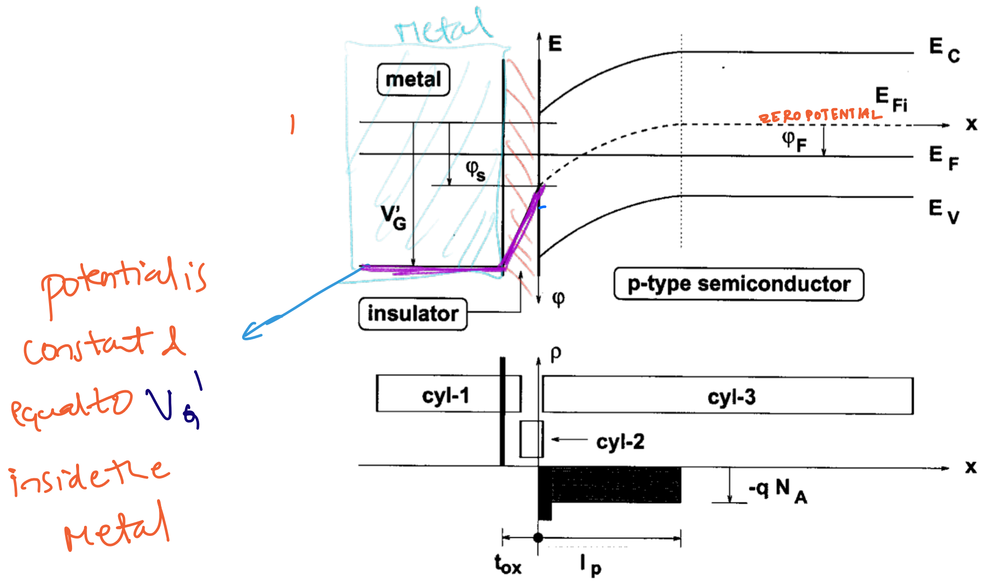
- For This Reason we use another term V_G' b/w

· Gate contact E.P & Internal zero potential as shown in the figure

obviously V_G' differs from actual voltage difference V_G b/w the two metallic plates. by a constant This difference is the difference of work function. we mentioned

→ It is Qualitatively straight forward to study the behaviour of EP in the structure. If

Metal: we start from Metal (Gate) & we apply some V_G' as in the figure. since metals are Equipotential in the interior, because there No difference of charge potential inside the metal.



Insulators The potential inside metal is constant & is equal to V_G' & when we enter the insulator & assume it is a perfect insulator with no charge inside.

When we solve Poisson eqn, the second derivative of electric potential is zero.

$$\nabla^2 \phi = 0$$

$$\Rightarrow \nabla \phi = \text{constant}$$

ϕ = linear   This is indicated in the figure

- We have an angle when we cross the interface b/w metal & insulator, the angle implies, the electric field is discontinuous.

The electric field inside metal = zero, because it has equal potential inside.

Electric field inside the insulator is Non zero, ∵ there is a layer of charge on the interface b/w metal & insulator.

- Going on, when we reach the interface b/w insulator & semiconductor the Electric potential at this point is unknown, it is called the Surface potential (ϕ_s).

→ When we enter the semiconductor, ϕ) what happens in the semiconductor? What's E.P?

It depends on the value of the sign of the Applied voltage.

Ans V_G' is positive

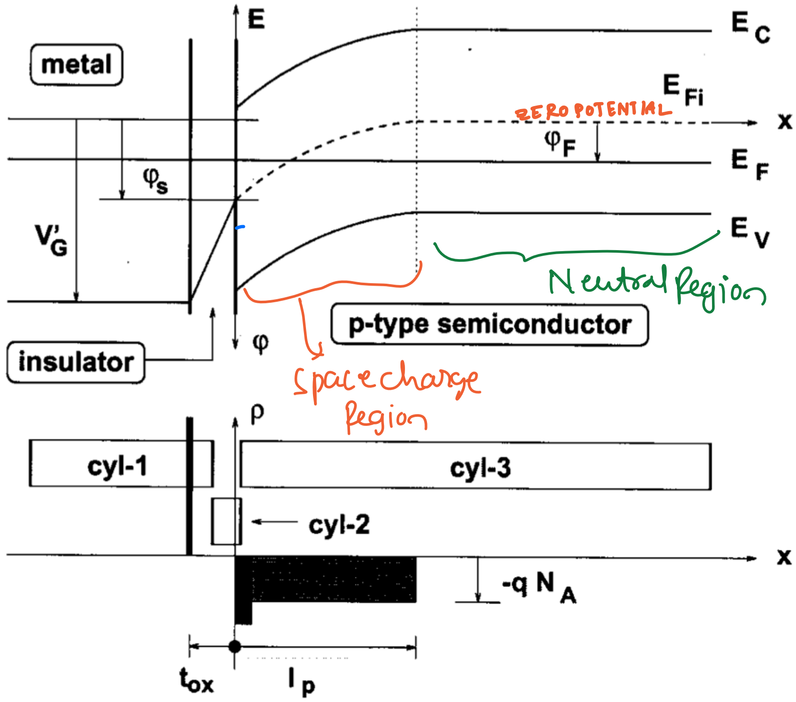
$\boxed{\text{because we assumed the positive direction of electric potential } \phi \text{ is downwards in the figure}}$

The potential inside the p-type semiconductor depends on Voltage Applied ∵ V_G' is +ve So, Qualitatively we expect the application of +ve voltage will push the Holes away from the surface on the interface b/w insulator & semiconductor.

→ The Holes will go to the Right & There is a Region near the interface where there are no Holes, & The charge density here is given by the Acceptor Atoms which are −ve

Charge density = $-qN_A$

In essence we have formed a Space charge Region. near the interface & far from interface the semiconductor retains the Uniformly doped characteristic (essentially it will be Neutral).



→ But, when we go far away from the interface of insulator and semiconductor to the Right. The semiconductor will behave like uniformly doped semiconductor.

- Neutral Region ⇒ Electric potential is constant
- space charge Region → Acceptor impurities $-ve$

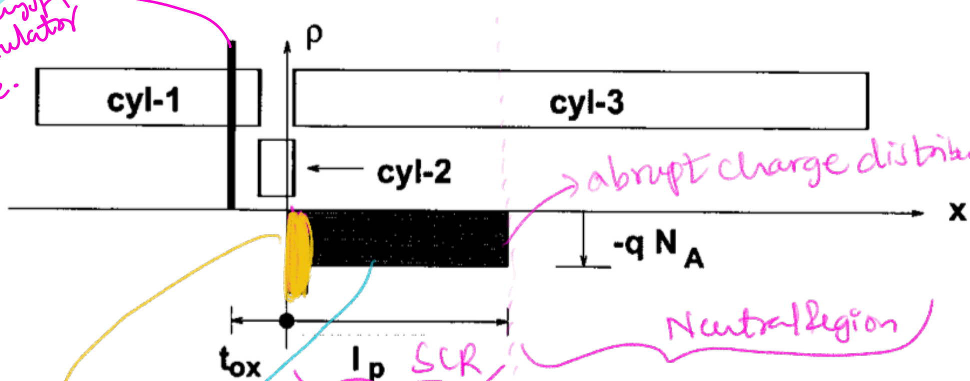
So, This situation is similar to that of the one of the two sides of a PN Junction but there is a difference since we have applied $+ve$ voltage to the Gate, it will not only push holes from the semiconductor Junction but also will attract e^-s. These e^-s can travel till the insulator-semiconductor interface and after that they can't cross because there is an insulator.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

54°

Distribution of charge schematically

$+ve$ charge at The Boundary of metal-Insulator interface.



abrupt charge distribution

Neutral Region

SCR

Wm

504°

layer of e^-s that is much thinner than l_p and the layer of e^- produce another contribution to the $-ve$ charge that is very close to the Interface.

uniform charge Distribution because of Dopants

- Importantly, Globally The MOS device is Neutral so if we have excess -ve charge near the Semiconductor-Insulator interface Then we should have +ve charge on the Metal-Insulator interface.

- If a metal is electrically charged the only possibility is that the charge stays at the Boundary.

t_{ox} = Thickness of The oxide layer.

- Now it is easy to write quantitative relations among these charges using the Gauss Theorem is some what equivalent to the Poisson Eq^n.

** "Gauss Theorem" States that if you take any volume and then integrate the Flux of the Displacement Vector over the Surface of the volume we will obtain Total Charge inside the volume.

- We will take the volumes as cylindrical as shown in the Figure. So the axis of each cylinder is horizontal and ∵ It is assumed as 1D figure the Displacement vector has only Horizontal direction.

→ The Thickness of The charge layer on the Metal is few A°, ex: 5 A°

The Thickness of layer of e⁻s in the Semiconductor if it exists about 10 times Larger Than The Metal side ex: 50 A°

But when considering these for calculations we can use Dirac-Delta function to represent them.

→ Now applying The Gauss theorem for The Cylindrical charges assumed in the figure.

we know Displacement vector ∝ Electric field

$$\boxed{\text{Displacement vector} = \text{Dielectric constant} \times \text{Electric field}}$$

** Displacement vector at different positions horizontally varies with Electric field present at the Different faces of the Cylinder.

→

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

MOS Capacitor at equilibrium — I

density of the charge on the Metal

$$Q_{\rm m} = \int_{-\infty}^{-t_{\rm ox}} \varrho_{\rm m}\, {\rm d}x\,, \quad Q_{\rm ox} = \int_{-t_{\rm ox}}^{0} \varrho_{\rm ox}\, {\rm d}x\,, \quad Q_{\rm sc} = \int_{0}^{\infty} \varrho\, {\rm d}x$$

$$\varrho_{\rm m} = 2\, Q_{\rm m}\, \delta(x + t_{\rm ox})$$

$$\text{Hp: } \varrho_{\rm ox} = 0 \quad \Longrightarrow \quad \mathcal{E}_{\rm ox} = \frac{V_G' - \varphi_s}{t_{\rm ox}}\,, \quad Q_{\rm ox} = 0$$

$$\varrho = q \left[p^{(0)} \exp\left(-q\varphi/(k_B T_L)\right) - n^{(0)} \exp\left(q\varphi/(k_B T_L)\right) + N\right]$$

$$\int_A \boldsymbol{D} \bullet \boldsymbol{\nu}\, {\rm d}A = AQ \quad \Longrightarrow$$

$$A\left(D^{\rm left}\boldsymbol{i} \bullet (-\boldsymbol{i}) + D^{\rm right}\boldsymbol{i} \bullet \boldsymbol{i}\right) = AQ\,, \qquad D^{\rm right} - D^{\rm left} = Q$$

$$D^{\rm left} = \begin{cases} D_{\rm m} = 0 \\ D_{\rm ox} = \varepsilon_{\rm ox}\mathcal{E}_{\rm ox} \\ D_{\rm sc}(0^+) = \varepsilon_{\rm sc}\mathcal{E}_s \end{cases} \qquad D^{\rm right} = \begin{cases} D_{\rm ox} = \varepsilon_{\rm ox}\mathcal{E}_{\rm ox} \\ D_{\rm sc}(0^+) = \varepsilon_{\rm sc}\mathcal{E}_s \\ D_{\rm sc}(\infty) = 0 \end{cases}$$

$$Q = \begin{cases} Q_{\rm m} \\ Q_{\rm ox} = 0 \\ Q_{\rm sc} \end{cases} \quad \Longrightarrow \quad \begin{cases} Q_{\rm m} = \varepsilon_{\rm ox}\mathcal{E}_{\rm ox} \\ \varepsilon_{\rm ox}\mathcal{E}_{\rm ox} = \varepsilon_{\rm sc}\mathcal{E}_s \\ Q_{\rm sc} = -\varepsilon_{\rm sc}\mathcal{E}_s \end{cases}$$

$$\Longrightarrow \qquad Q_{\rm sc} = -Q_{\rm m}\,, \quad Q_{\rm m} + Q_{\rm sc} = 0$$

Q_{ox} = Charge per unit are in the oxide layer

Q_{sc} = charge per unit Area in the semiconductor

$\mathcal{E} = -\frac{dV}{dx}$ electric field

→ The charge in the semicon-ductor, well we are in the

Equilibrium condition as usual we make the assump-tion that the semiconductor is Non-degenerate

- So we can use the exponential expression for the concentrations and in complete Ionization condition.

$$\varrho = q \left[p^{(0)} \exp\left(-q\varphi/(k_B T_L)\right) - n^{(0)} \exp\left(q\varphi/(k_B T_L)\right) + N\right]$$

Net dopant concentration

- $p^{(0)}, n^{(0)}$ are the concentrations at '0' electric potential of the semiconductor.

Since the semiconductor is of p-type

$$p^{(0)} = p_{p_0} \qquad n^{(0)} = n_{p_0}$$

→ Applying Gauss Theorem

$$\boxed{\int_A D \cdot \nu \, dA = AQ}$$

$$\int_A \boldsymbol{D} \bullet \boldsymbol{\nu} \, \mathrm{d}A = AQ \implies$$

$$A\left(D^{\mathrm{left}} \boldsymbol{i} \bullet (-\boldsymbol{i}) + D^{\mathrm{right}} \boldsymbol{i} \bullet \boldsymbol{i}\right) = AQ\,, \qquad D^{\mathrm{right}} - D^{\mathrm{left}} = Q$$

$$D^{\mathrm{left}} = \begin{cases} D_{\mathrm{m}} = 0 \\ D_{\mathrm{ox}} = \varepsilon_{\mathrm{ox}} \mathcal{E}_{\mathrm{ox}} \\ D_{\mathrm{sc}}(0^+) = \varepsilon_{\mathrm{sc}} \mathcal{E}_s \end{cases} \qquad D^{\mathrm{right}} = \begin{cases} D_{\mathrm{ox}} = \varepsilon_{\mathrm{ox}} \mathcal{E}_{\mathrm{ox}} \\ D_{\mathrm{sc}}(0^+) = \varepsilon_{\mathrm{sc}} \mathcal{E}_s \\ D_{\mathrm{sc}}(\infty) = 0 \end{cases}$$

$$Q = \begin{cases} Q_{\mathrm{m}} \\ Q_{\mathrm{ox}} = 0 \\ Q_{\mathrm{sc}} \end{cases} \implies \begin{cases} Q_{\mathrm{m}} = \varepsilon_{\mathrm{ox}} \mathcal{E}_{\mathrm{ox}} \\ \varepsilon_{\mathrm{ox}} \mathcal{E}_{\mathrm{ox}} = \varepsilon_{\mathrm{sc}} \mathcal{E}_s \\ Q_{\mathrm{sc}} = -\varepsilon_{\mathrm{sc}} \mathcal{E}_s \end{cases}$$

Here $E_{ox} \propto \frac{1}{\varepsilon_{ox}}$

electric field is inversely proportional to the dielectric constant

$$\implies \quad Q_{\mathrm{sc}} = -Q_{\mathrm{m}}\,, \quad Q_{\mathrm{m}} + Q_{\mathrm{sc}} = 0$$

The example was taken for the +ve value of the Applied Gate voltage.

16/05/25

MOS Capacitor at equilibrium — II

$$\Rightarrow \quad Q_{sc} = -Q_m = -\varepsilon_{ox}\mathcal{E}_{ox} = -\varepsilon_{ox}\frac{V'_G - \varphi_s}{t_{ox}}$$

$$\Longrightarrow \qquad Q_{sc} = -C_{ox}\,(V'_G - \varphi_s)\ , \qquad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

↳ Capacitance per unit Area

$$\begin{cases} Q_{sc} = -C_{ox}\,(V'_G - \varphi_s) = Q_{sc}(\varphi_s; V'_G) \\ Q_{sc} = -\varepsilon_{sc}\mathcal{E}_s \end{cases}$$

$$\varrho = \varrho(\varphi) \Rightarrow \mathcal{E}_s = \mathcal{E}_s(\varphi_s)$$

$$\left.\begin{array}{l} Q_{sc} = Q_{sc}(\varphi_s; V'_G) \\ Q_{sc} = -\varepsilon_{sc}\mathcal{E}_s(\varphi_s) \end{array}\right\} \quad \Longrightarrow \quad \varphi_s = \varphi_s(V'_G)$$

- $Q_{sc} = -C_{ox}(V'_G - \phi_s)$ This eqn is compact which connects the charge in the semiconductor/unit Area (Q_{sc}) to the surface potential (ϕ_s)

Q) Is the above expression sufficient?

Ans: No, because the Q_{sc} & ϕ_s are unknown

** So we need another relation.

$$Q_{sc} = -\varepsilon_{sc}\mathcal{E}_s$$ Here this is unknown

→ If we solve the poisson eqn in the semi-conductor we can find a relation b/w the surface field & the surface potential.

$$\varrho = \varrho(\varphi) \Rightarrow \mathcal{E}_s = \mathcal{E}_s(\varphi_s)$$

$$\left.\begin{array}{l} Q_{sc} = Q_{sc}(\varphi_s; V_G') \\ Q_{sc} = -\varepsilon_{sc}\mathcal{E}_s(\varphi_s) \end{array}\right\} \quad \Longrightarrow \quad \varphi_s = \varphi_s(V_G')$$

→

MOS Capacitor, p-substrate — I

$$\text{Hp:}\begin{cases} \partial/\partial y = \partial/\partial z = 0 \\ N = -N_A = \text{const} \\ \varphi(\infty) = 0 \\ \varphi'(\infty) = 0 \end{cases} \qquad \text{Def:}\begin{cases} p_{p0} = p(\infty) \\ n_{p0} = n(\infty) \\ u = q\varphi/(k_B T_L) \\ L_A = \sqrt{2\varepsilon_{sc} k_B T_L/(q^2 p_{p0})} \end{cases}$$

$$\varrho = q\,[p_{p0}\exp(-u) - n_{p0}\exp(u) + N]$$

→ expression for charge density in the semiconductor

$$\varrho(\infty) = 0 \Rightarrow N = n_{p0} - p_{p0} \Rightarrow \varrho = -q p_{p0}\, A(u)$$

$$A(u) = (n_{p0}/p_{p0})\,[\exp(u) - 1] + 1 - \exp(-u)$$

$$\begin{cases} n_{p0}/p_{p0} = \exp(-2u_F) \\ u_F > 0 \end{cases} \qquad A(u)\begin{cases} > 0 & u > 0 \\ = 0 & u = 0 \\ < 0 & u < 0 \end{cases}$$

$$-\operatorname{div}(\varepsilon_{sc}\operatorname{grad}\varphi) = \varrho \quad \Longrightarrow \quad u'' = \frac{2}{L_A^2} A(u)$$

$$u'u'' = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}(u')^2 = \frac{2}{L_A^2} A(u)\, u'$$

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

* → Before continuing to solve the Poisson's eq^n we must introduce some useful definitions

(i) We observe that we have a 1D problem
∴ only dependency is on x-coordinate

(ii) Dopant concentration is constant w.r.t to space

i.e $-N_A$

(iii) The Poisson eq^n is a Diff. eq^n of 2nd order so we need two boundary conditions (or) Two initial conditions.

We know them,

- if we go asymptotically to the Right the semiconductor becomes Equipotential

∴ The Electric field at '∞' $= 0$

We also placed potential at $\infty = 0$

$$\Rightarrow \boxed{\underset{\text{Potential}}{\phi(\infty) = 0} \quad \underset{\text{Electric field}}{\phi'(\infty) = 0}}$$

→
$$p_{po} = p(\infty) \simeq N_A$$

$$n_{p_0} = n(\infty) \simeq \frac{n_i^2}{N_A}$$

- Then for convenience we introduce Normalised Electric potential, so instead of using ϕ as the unknown function we use $\boxed{u = \frac{q\phi}{K_B T_L}}$

$$L_A \Rightarrow \sqrt{\frac{2\varepsilon_{sc} K_B T_L}{q^2 p_{p_0}}}$$

→ Charge density ρ

$$\rho = q\left[p_{p_0} \exp(-u) - n_{p_0} \exp(u) + N \right]$$

**

$$\varrho(\infty) = 0 \Rightarrow N = n_{p0} - p_{p0} \Rightarrow \varrho = -q p_{p0} A(u)$$

$$A(u) = (n_{p0}/p_{p0}) \left[\exp(u) - 1\right] + 1 - \exp(-u)$$

- Dimensionless
  - $u=0 \Rightarrow A(u)=0$
  - $u>0,\ A(u)>0$
  - $u<0,\ A(u)<0$

$$\begin{cases} n_{p0}/p_{p0} = \exp(-2u_F) \\ u_F > 0 \end{cases} \qquad A(u) \begin{cases} >0 & u>0 \\ =0 & u=0 \\ <0 & u<0 \end{cases}$$

Normalised Fermi potential

$$-\operatorname{div}(\varepsilon_{sc} \operatorname{grad} \varphi) = \varrho \quad \Longrightarrow \quad \boxed{u'' = \frac{2}{L_A^2} A(u)}$$

Poisson eq^n in 3D & in NonNormalised variables

$$u' u'' = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}x} (u')^2 = \frac{2}{L_A^2} A(u)\, u'$$

Trick to solve the Diff. eq^n

MOS Capacitor, p-substrate — II

$$\int_\infty^0 \frac{\mathrm{d}}{\mathrm{d}x}(u')^2\,\mathrm{d}x = \frac{4}{L_A^2}\int_\infty^0 A(u)\,u'\,\mathrm{d}x$$

$$\int_0^{(u_s')^2} \mathrm{d}(u')^2 = \frac{4}{L_A^2}\int_0^{u_s} A(u)\,\mathrm{d}u$$

because The Integral is Never -ve because if u>0 A(u)>0

$$\left(\frac{q\mathcal{E}_s}{k_B T_L}\right)^2 = \frac{4}{L_A^2}F^2(u_s;u_F)$$

+ve term

$$F^2 = \exp(-2u_F)\,[\exp(u_s) - 1 - u_s] + [\exp(-u_s) - 1 + u_s]$$

$$Q_{sc} = -\varepsilon_{sc}\mathcal{E}_s = \mp\frac{2\varepsilon_{sc}k_B T_L}{qL_A}F(u_s;u_F)$$

$$Q_{sc}\begin{cases} <0 & u_s>0 \\ =0 & u_s=0 \quad \text{(flat band)} \\ >0 & u_s<0 \end{cases}$$

Surface potential

$$Q_m(u_s=0) = -Q_{sc}(u_s=0) = C_{ox}V_G'(u_s=0) = 0$$

$$V_G' = V_G - \text{const} \quad \Longrightarrow \quad V_G(u_s=0) = \text{const} \doteq V_{FB}$$

** This expression gives us the Relation b/w the $\mathcal{E}_s$ (electric field at the surface) as a function of the Surface Potential (u_s)

** when $u_s = 0$ Bands are Flat

Q) what is the Gate voltage for which we have flat Band ?

$$Q_{\mathrm{sc}} = -\varepsilon_{\mathrm{sc}}\mathcal{E}_s = \mp\frac{2\varepsilon_{\mathrm{sc}}k_B T_L}{qL_A}F(u_s;u_F)$$

$$Q_{\mathrm{sc}}\begin{cases} <0 & u_s>0 \\ =0 & u_s=0 \quad \text{(flat band)} \\ >0 & u_s<0 \end{cases}$$

Surface potential

$$Q_m(u_s=0) = -Q_{\mathrm{sc}}(u_s=0) = C_{\mathrm{ox}}V_G'(u_s=0) = 0$$

$$V_G' = V_G - \text{const} \quad \Longrightarrow \quad V_G(u_s=0) = \text{const} \doteq V_{FB}$$

Ans

For flat Band $V_G' = 0$

But $V_G' = V_G - \text{const} = 0$

$\Rightarrow V_G = \text{const} \neq 0 = V_{FB}$ Flat Band voltage

This is the difference in the Work functions of Semiconductor & the Bulk metal.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

MOS Capacitor, p-substrate — III

$$F^2 = \exp(-2u_F)\,[\exp(u_s) - 1 - u_s] + [\exp(-u_s) - 1 + u_s]$$

Approximations of F

$$F \simeq \begin{cases} \exp(-u_s/2) & u_s < 0 \\ \sqrt{u_s} & 0 < u_s < 2u_F \\ \sqrt{n_{p0}/p_{p0}}\,\exp(u_s/2) & 2u_F < u_s \end{cases}$$

Both have exponential Behaviour

$$n_s = n(0) = n_{p0}\exp(u_s) = n_i \exp(u_s - u_F)$$

$$p_s = p(0) = p_{p0}\exp(-u_s) = n_i \exp(u_F - u_s)$$

Surface potential

$$\begin{cases} u_s < 0 & n_s < n_{p0} < n_i < p_{p0} < p_s & \text{accum.} \\ u_s = 0 & n_s = n_{p0} < n_i < p_{p0} = p_s & \text{flat band} \\ 0 < u_s < u_F & n_{p0} < n_s < n_i < p_s < p_{p0} & \text{depletion} \\ u_s = u_F & n_{p0} < n_s = n_i = p_s < p_{p0} & \text{mid gap} \\ u_F < u_s < 2u_F & n_{p0} < p_s < n_i < n_s < p_{p0} & \text{weak inv.} \\ u_s = 2u_F & n_{p0} = p_s < n_i < n_s = p_{p0} & \text{threshold} \\ 2u_F < u_s & p_s < n_{p0} < n_i < p_{p0} < n_s & \text{strong inv.} \end{cases}$$

exponential

$Q \propto \sqrt{u_s}$

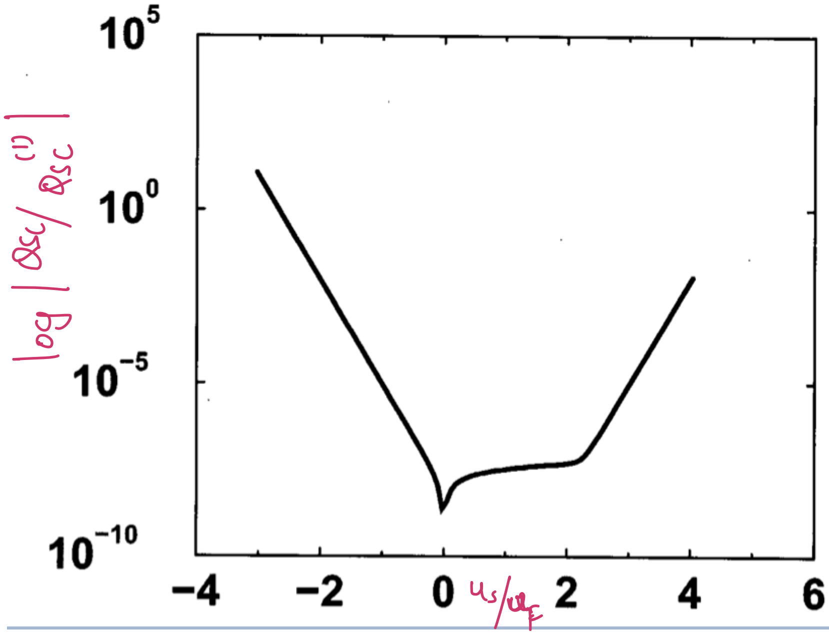
exponential

- $$0 < u_s < 2u_F \implies Q_{\rm sc} \simeq -\frac{2\varepsilon_{\rm sc} k_B T_L}{q L_A}\sqrt{q\varphi_s/(k_B T_L)}$$

$$\Rightarrow \quad Q_{\rm sc} \simeq -\sqrt{2\varepsilon_{\rm sc} q p_{p0} \varphi_s} = -C_{\rm ox}\gamma\sqrt{\varphi_s}\,, \qquad \gamma = \frac{\sqrt{2\varepsilon_{\rm sc} q p_{p0}}}{C_{\rm ox}}$$

- This is the expression for charge in the depletion & weak inversion condition.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

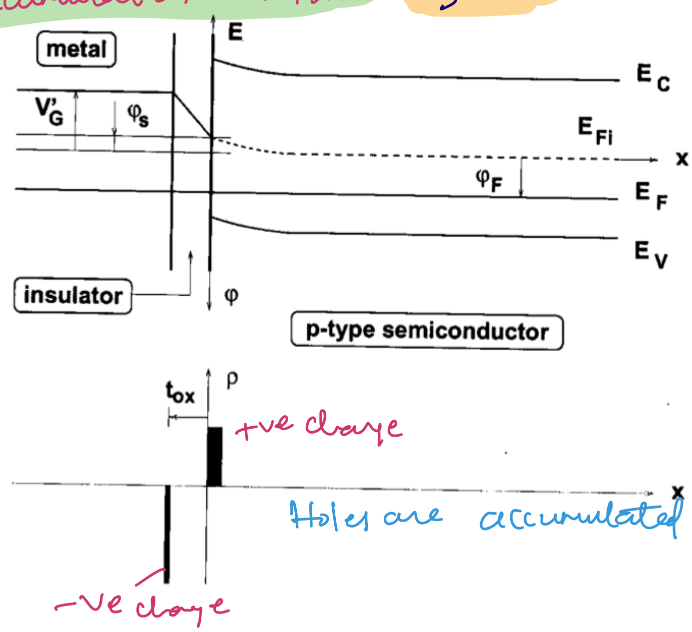


T. 31.7: Condensatore MOS con substrato p — Andamento di $|Q_{sc}|$ in funzione di u_s/u_F.

→ 19|05|25

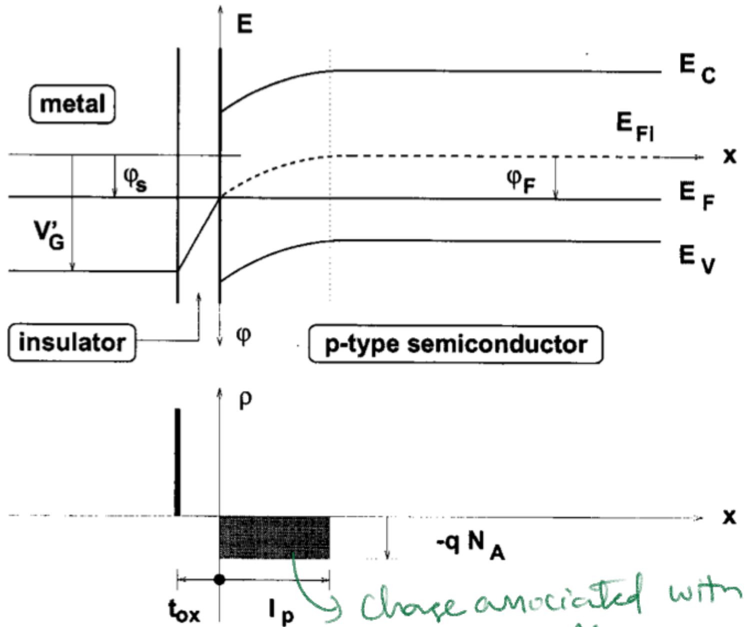
* Accumulation Condition $u_s < 0$

Transistor is OFF



T. 31.8: Andamento delle bande nel condensatore MOS in accumulazione.

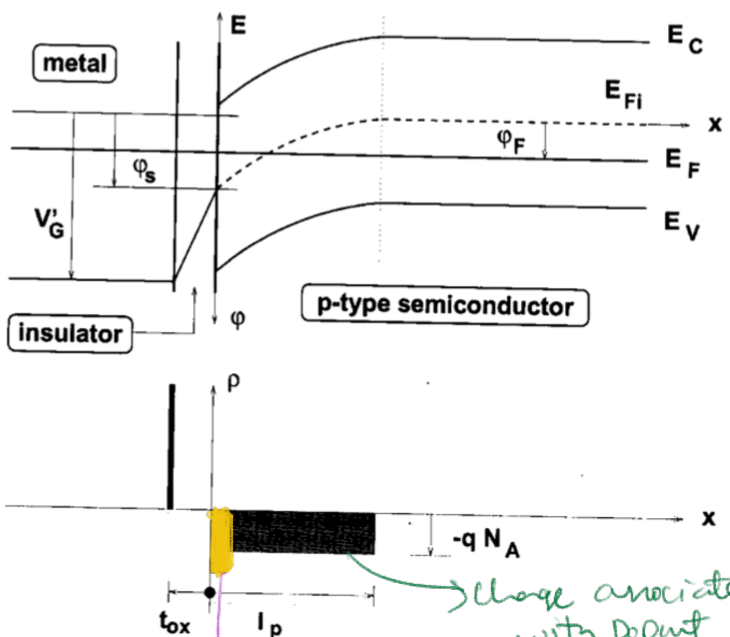
Midgap condition $\phi_s = \phi_F$



Charge associated with Dopant Atoms

T. 31.9: Andamento delle bande nel condensatore MOS nella condizione di *mid-gap.*

At the threshold of Strong Inversion $\phi_s = 2\phi_F$



Charge associated with Dopant Atoms

Charge layer

T. 31.10: Andamento delle bande nel condensatore MOS alla soglia della forte inversione.

MOS Capacitor, p-substrate — IV

excluding Accumulation

$$Q_{sc} = \int_0^{\infty} \varrho \, dx$$

Hp: $\varphi_s > 0$, $\quad \varrho \neq 0$ in $0 < x < l_p = x_d$

$$Q_{sc} \simeq \int_0^{l_p} \varrho \, dx \simeq -q \int_0^{l_p} (n + N_A) \, dx = Q_i + Q_b$$

Inversion charge

Bulk charge

$$Q_i = -q \int_0^{l_p} n \, dx \,, \qquad Q_b = -q \, N_A \, l_p \quad \textbf{(ASCE)}$$

Approximation

$$\boxed{-qn \simeq Q_i \, 2\delta(x)} \quad \Longrightarrow \quad \varphi'' \simeq \frac{qN_A}{\varepsilon_{sc}}, \quad 0 < x < l_p$$

Here we consider Inversion layer as a Dirac Delta condition

The charge over the rest of the Lp because of Dirac Delta approximation of Inversion charge

$$\begin{cases} \varphi(l_p) = 0 \\ \varphi'(l_p) = 0 \end{cases} \quad \Longrightarrow \quad \varphi = \frac{qN_A}{2\varepsilon_{sc}} (x - l_p)^2$$

$$\varphi_s = \frac{qN_A}{2\varepsilon_{sc}} l_p^2 \,, \qquad Q_b = -q \, N_A \, l_p = -\sqrt{2\varepsilon_{sc} q N_A \varphi_s}$$

$$\begin{cases} Q_b = -C_{ox} \gamma \sqrt{\varphi_s} \\ Q_{sc} = Q_i + Q_b \\ Q_{sc} = -Q_m = -C_{ox} \, (V_G' - \varphi_s) \end{cases} \Longrightarrow$$

$$Q_i = Q_{sc} - Q_b = -C_{ox} \left[(V_G' - \varphi_s) - \gamma \sqrt{\varphi_s} \right] < 0$$

$$0 < \varphi_s < 2\varphi_F \quad \Rightarrow \quad Q_{sc} \simeq Q_b \quad \Rightarrow \quad |Q_i| \ll |Q_b|$$

This approximation helps us because solving the Poisson eqn becomes obvious

i.e $\boxed{\phi'' = \frac{qN_A}{\varepsilon_{sc}}}$, $0 < x < L_p$

This is the Poisson's eqn

And the boundary conditions are ; we remember we have charge neutrality after Lp i.e $x > L_p$

i.e $\phi(l_p) = 0$, $\phi'(l_p) = 0$ By continuity

$$\boxed{\phi = \frac{q N_A}{2 \varepsilon_{sc}} (x - l_p)^2}$$

Here using the above sol^n we can immediately calculate the surface potential at $x = 0$

$$\boxed{\phi_s = \frac{q N_A}{2 \varepsilon_{sc}} l_p^2}$$

'l_p' depends on the Applied voltage

✱

$$\begin{cases} Q_b = -C_{\text{ox}} \gamma \sqrt{\varphi_s} \\ Q_{\text{sc}} = Q_i + Q_b \\ Q_{\text{sc}} = -Q_m = -C_{\text{ox}} (V_G' - \varphi_s) \end{cases} \Longrightarrow$$

$$\boxed{Q_i = Q_{\text{sc}} - Q_b = -C_{\text{ox}} [(V_G' - \varphi_s) - \gamma \sqrt{\varphi_s}] < 0}$$

$$0 < \varphi_s < 2\varphi_F \;\Rightarrow\; Q_{\text{sc}} \simeq Q_b \;\Rightarrow\; |Q_i| \ll |Q_b|$$

→ This expression is fundamental, because it's then applied in the calculation of the current of the MOS Transistor.

→ In MOS capacitor there is NO current from Gate into the Bulk because of the Dielectric b/w them but there is equal & opposite charge on both sides which can be treated as capacitance

But it will be a Differential capacitance.

Capacitance of the MOS Structure — I

- The capacitance per unit area of the MOS structure is given by

$$\boxed{C = \frac{dQ_m}{dV_G} = \frac{dQ_m}{dV'_G}},$$

with $V'_G = V_G - V_{FB}$. From $Q_m = C_{ox}(V'_G - \varphi_s)$ and $Q_{sc} = -Q_m$ it follows

$$\boxed{\frac{1}{C} = \frac{dV'_G}{dQ_m} = \frac{d(V'_G - \varphi_s) + d\varphi_s}{dQ_m}} = \frac{1}{C_{ox}} + \boxed{\frac{d\varphi_s}{d(-Q_{sc})}}.$$

For a p-type substrate the semiconductor capacitance per unit area is given by

$$\boxed{C_{sc} \doteq -\frac{dQ_{sc}}{d\varphi_s}} = -\frac{q}{k_B T_L}\frac{dQ_{sc}}{du_s} = \pm\frac{2\varepsilon_{sc}}{L_A}\frac{dF}{du_s} > 0,$$

where the plus (minus) sign holds for $u_s > 0$ ($u_s < 0$). In conclusion, the capacitance is the series of the oxide and semiconductor capacitances:

$$\boxed{\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{sc}}}.$$

C_{sc} will always be +ve

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Capacitance of the MOS Structure — II

Remembering that

$$F^2 = \exp(-2u_F)\,[\exp(u_s) - 1 - u_s] + [\exp(-u_s) - 1 + u_s]\,,$$

the asymptotic behavior is

$$\begin{array}{ll} F \simeq \exp(u_s/2 - u_F) & u_s \gg \;\;1 \\ F \simeq \exp(-u_s/2) & u_s \ll -1 \end{array}$$

For $|u_s| \ll 1$ it is $\exp(\pm u_s) \simeq 1 \pm u_s + u_s^2/2$, whence

$$F^2 \simeq \frac{1}{2}\,[1 + \exp(-2u_F)]\; u_s^2 \simeq \frac{1}{2}\,u_s^2\,, \qquad F \simeq \pm\frac{u_s}{\sqrt{2}}\,.$$

Then,

$$C_{\mathrm{sc}} \simeq \begin{cases} (\varepsilon_{\mathrm{sc}}/L_A)\,\exp(u_s/2 - u_F) & u_s \gg \;\;1 \\ (\varepsilon_{\mathrm{sc}}/L_A)\,\exp(-u_s/2) & u_s \ll -1 \\ (\varepsilon_{\mathrm{sc}}/L_A)\,\sqrt{2} \doteq C_{\mathrm{sc}FB} & |u_s| \ll \;\;1 \end{cases}$$

$$C = \frac{C_{\mathrm{ox}}C_{\mathrm{sc}}}{C_{\mathrm{ox}} + C_{\mathrm{sc}}} \simeq \begin{cases} C_{\mathrm{ox}} & u_s \gg \;\;1 \\ C_{\mathrm{ox}} & u_s \ll -1 \\ C_{FB} < C_{\mathrm{ox}} & |u_s| \ll \;\;1 \end{cases}$$

with $C_{FB} \doteq C_{\mathrm{ox}}C_{\mathrm{sc}FB}/(C_{\mathrm{ox}} + C_{\mathrm{sc}FB})$.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Q) what happens when we apply a large +ve pulse to the Gate of the MOS Transistor (n-type) ?

Ans

you push away the Holes and this action is almost immediate (Time constants of this phenomenon are very short) so we created full Depleted Region.

We donot have immediately an Inversion layer because it takes time to form (remember Generation and Recombination times)

$\therefore$ only at the end of a Transient an Inversion layer forms. $\therefore$ still we can use the eqn

$$Q_i = -C_{ox}\left[V_G' - \phi_s - \gamma\sqrt{\phi_s}\right], \quad \phi_s \geq 0$$

And we can get an expression for ϕ_s

As a consequence, the value of φ_s calculated from

$$Q_i = -C_{\mathrm{ox}}\,[V_G' - \varphi_s - \gamma\sqrt{\varphi_s}]\,, \qquad \varphi_s \geq 0$$

may differ from the equilibrium one. The above still holds in non-equilibrium condition as it is derived from Gauss' theorem and the full-depletion + ASCE approximation. Letting $V_i \doteq V_G' + Q_i/C_{\mathrm{ox}} = V_G' - |Q_i|/C_{\mathrm{ox}}$ the above becomes

$$\varphi_s + \gamma\sqrt{\varphi_s} - V_i = 0 \quad \Longrightarrow \quad \sqrt{\varphi_s} = \sqrt{V_i + \gamma^2/4} - \gamma/2\,.$$

Capacitance of the MOS Structure — III

The calculations leading to the expression of the capacitance per unit area C yields different results when non-static conditions are considered. Taking a p-substrate, let V_G be rapidly brought from V_{FB} to a value above the threshold voltage. The latter is found by setting $u_s = 2u_F$ in

$$Q_{\mathrm{sc}} = -C_{\mathrm{ox}} \left(V_G - V_{FB} - \varphi_s\right) = \mp \frac{2\varepsilon_{\mathrm{sc}}}{L_A} \frac{k_B T_L}{q} F \,,$$

$$F^2 = \exp(-2u_F) \left[\exp(u_s) - 1 - u_s\right] + \left[\exp(-u_s) - 1 + u_s\right] ,$$

namely $-Q_{\mathrm{sc}T} = C_{\mathrm{ox}} \left(u_T - u_{FB} - 2u_F\right) \simeq \left(2\varepsilon_{\mathrm{sc}}/L_A\right) \sqrt{2u_F}$. In transient condition Q_i has not necessarily the equilibrium value. As a consequence, the value of φ_s calculated from

$$Q_i = -C_{\mathrm{ox}} \left[V_G' - \varphi_s - \gamma\sqrt{\varphi_s}\right] , \qquad \varphi_s \geq 0$$

may differ from the equilibrium one. The above still holds in non-equilibrium condition as it is derived from Gauss' theorem and the full-depletion + ASCE approximation. Letting $V_i \doteq V_G' + Q_i/C_{\mathrm{ox}} = V_G' - |Q_i|/C_{\mathrm{ox}}$ the above becomes

$$\varphi_s + \gamma\sqrt{\varphi_s} - V_i = 0 \quad \Longrightarrow \quad \sqrt{\varphi_s} = \sqrt{V_i + \gamma^2/4} - \gamma/2 \,.$$

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Capacitance of the MOS Structure — IV

The surface potential φ_s given by

$$\sqrt{\varphi_s} = \sqrt{V_i + \gamma^2/4} - \gamma/2\,, \qquad V_i = V_G' - |Q_i|/C_{\text{ox}}\,,$$

increases with V_i, hence it increases as $|Q_i|$ decreases. If V_G is rapidly brought from V_{FB} to a value above the threshold voltage, initially $|Q_i|$ is much smaller than in equilibrium. As a consequence,

$$\sqrt{\varphi_s} \simeq \sqrt{V_G' + \gamma^2/4} - \gamma/2 \doteq \sqrt{\varphi_G}$$

is larger than the equilibrium value. In conclusion, a large depleted region is formed in the semiconductor, of width

$$\boxed{x_d = \sqrt{\frac{2\varepsilon_{\text{sc}}}{qN_A}}\sqrt{\varphi_G}}\,,$$

while no inversion layer is present. As the inversion layer is generated, $|Q_i|$ increases and the surface potential decreases towards the equilibrium value. The time necessary to reach the equilibrium condition is determined by the thermal-generation processes.

- In practice, when we apply a Pulse to the Gate of an MOS Transistor we open up a very large Depleted Region & we do not have Initially an Inversion layer.

* This is fundamental for Optical sensors.

because we will ~~try~~ and exploit the very large depleted Region quickly in order to detect some light that comes into the Semiconductor before the Thermal generations fill up the the Inversion layer.

Quantitative relations — I

Taking a p-type substrate with $N_A = 10^{16}\ \mathrm{cm}^{-3}$, and letting $T_L = 300$ K whence $k_B T_L/q \simeq 26$ mV and (for silicon) $n_i \simeq 10^{10}\ \mathrm{cm}^{-3}$, it follows $p_{p0} \simeq N_A$, $n_{p0} = n_i^2/N_A \simeq 10^4\ \mathrm{cm}^{-3}$, $q\,p_{p0} \simeq 1.602 \times 10^{-3}$ C/cm^3, and

$$e^{-2u_F} = \frac{n_{p0}}{p_{p0}} \simeq 10^{-12}, \qquad 2u_F \simeq 27.6\,, \qquad 2\varphi_F \simeq 0.72\ \ \mathrm{V}\,.$$

As $\varepsilon_{\mathrm{sc}} \simeq 11.7 \times 8.854 \times 10^{-14} = 1.036 \times 10^{-12}$ F/cm,

$$L_A = \sqrt{\frac{2\varepsilon_{\mathrm{sc}} k_B T_L}{q^2 p_{p0}}} \simeq 5.8 \times 10^{-2}\ \ \mu\mathrm{m}\,,$$

$$Q_{\mathrm{sc}}^{(1)} \doteq \frac{2\varepsilon_{\mathrm{sc}} k_B T_L}{q L_A} = \sqrt{2\varepsilon_{\mathrm{sc}} k_B T_L\, p_{p0}} \simeq 9.3 \times 10^{-9}\ \ \frac{\mathrm{C}}{\mathrm{cm}^2}\,,$$

where $Q_{\mathrm{sc}}^{(1)}$ is the value of $|Q_{\mathrm{sc}}|$ when u_s is such that $F = 1$. From the derivation of F, the contributions to Q_{sc}^2 of the different types of charges are found.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నల