

Lecture-7

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The beginning part of lecture is written in
lecture-6

where we derived the Poisson's equation of
a semiconductor in Equilibrium

Maxwell's Equations in Semiconductors — I

$$\underline{\rho} = q(p - n + N), \quad N = N_D^+ - N_A^-, \quad \mathbf{D} = \epsilon_{sc}\mathbf{E}$$

Complete ionization: $N = N(\mathbf{r}) = N_D - N_A$

Here



Non-degenerate semiconductor — Equilibrium condition

$$\frac{\partial}{\partial t} = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0, \quad \mathbf{E} = -\text{grad } \varphi$$

$$\left. \begin{array}{l} n = n^{(0)} \exp[-q\varphi/(k_B T_L)] \\ p = p^{(0)} \exp[-q\varphi/(k_B T_L)] \end{array} \right\} \implies \rho = \rho(\varphi, \mathbf{r})$$

Poisson's equation

$$\text{div } \mathbf{D} = \rho \implies \text{div}(\epsilon_{sc}\mathbf{E}) = -\epsilon_{sc}\nabla^2\varphi = \rho(\varphi, \mathbf{r})$$

$$-\epsilon_{sc}\nabla^2\varphi = q \left[p^{(0)} \exp\left(\frac{-q\varphi}{k_B T_L}\right) - n^{(0)} \exp\left(\frac{q\varphi}{k_B T_L}\right) + N(\mathbf{r}) \right]$$

(supplemented with boundary conditions)

Anyway, we will again elaborate the above slide

$$Q = P = q(P - n + N)$$

→ we have minus sign because electrons are we adding contribute very to the charge density.

The charge density of the semiconductor

$$Q = q [P - n + N]$$

Holes concentration

electron concentration

No. of ionized
dopant atoms
concentration

we also have from Maxwell's equations

$$D = \epsilon_{sc} E$$

Electric displacement vector

$$E = E$$

Electric field

Electric field

permittivity of the
semiconductor

→ Here the ratio of Displacement Vector and the electric Field we expect to

be Dielectric constant.

However it's the dielectric constant of the vacuum it is for the following reasons.

If, we look at the expression of charge density $\rho = (p - n + N)$

→ we are using the concentrations of holes, electrons & Dopant atoms not the sum of individual charges.

When, we consider concentrations we imply that we have calculated the charges over some volume of the semiconductor i.e. in this case

ex:-

$$n = \frac{\text{no. of electrons}}{\text{cm}^3}$$

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→ Therefore, when we use **current density** we are not using anymore the individual velocity of each particle

we are Using Avg. Velocity of e^- 's at holes.

→ Therefore, it is not correct anymore to use the relation b/w $D \propto E$ the permittivity of vacuum we must use permittivity of semi conductor.

$$D = \epsilon_{sc} E$$

permittivity of semiconductor

→ When, we consider the **complete ionization** condition $N^+ = N_D$, &

$N_A^- = N_A$ so it follows that net Dopant concentration is prescribed. It is the function that we know

Complete ionization: $N = N(r) = N_D - N_A$

Because, we fabricate the profile of the Dopants technologically and we know it at each position in the semiconductor. There is no dependence of Net Dopant Concentration on the Fermi level.

→ When, we consider current density

$$J = \Phi_p v_p + \Phi_n v_n = [q_p v_p - q_n v_n]$$

it is defined like this.

*** Let's Now consider Equilibrium

Condition -

நால்கள் முன் சீர்வு தீவிரமாக

Derivation, equilibrium condition என்று கூறுகின்ற சீர்வு,

என்று, practical Applications என்று

equilibrium condition என்று கூறுகின்ற முன் வரை கணக்கு
ஏன் முன் வரை கணக்கு
சீர்வு.

* 'SDC குழுத் தங்கள் தீவிரமே
நில நடவடிக்கை -'

→ In Equilibrium condition, obviously
there is no dependence on time, So when
time derivatives appear on the equation

they will be equal to zero.

of course, In Equilibrium there are NO currents.

Individually, J_n & \bar{J}_p are equal to zero.

An also Generalization that expresses the electric field i.e

$$E = -\text{grad } \phi$$

$\frac{\partial A}{\partial t}$

Zero
because of Equilibrium

For Equilibrium

→ If in addition we are in Nondegenerate Condition.

which we will always assume from

Now on:

we remember, In Non-Degenerate case of Semiconductor, then the concentration of electrons & the concentration of the holes are defined in terms of Electric Potential through Exponential relations.

i.e

Non-degenerate semiconductor — Equilibrium condition

$$\frac{\partial}{\partial t} = 0, \quad \mathbf{J}_n = \mathbf{J}_p = 0, \quad \mathcal{E} = -\text{grad } \varphi$$

$$\left. \begin{array}{l} n = n^{(0)} \exp[-q\varphi/(k_B T_L)] \\ p = p^{(0)} \exp[-q\varphi/(k_B T_L)] \end{array} \right\} \Rightarrow \rho = \rho(\varphi, \mathbf{r})$$

This only possible in Equilibrium & Non-Degenerate Case

→ Therefore, we we again look at

charge Density

$$Q = P - n + N$$

Here N is a prescribed function of position i.e.

$$\underline{N(r)} = N_D - N_A$$

function of position

P & n are known functions of ϕ

∴ we can say in Equilibrium condition the global charge density of semiconductor is a function that depends on ϕ & r

i.e. $Q = Q(\phi, r)$

$$P = P(\phi, r)$$

$$\textcircled{Q = P}$$

I merely used convenient representation

→ Now, we have time derivatives & current densities zero

$$\frac{\partial}{\partial t} = 0 \quad \& \quad J_n = J_p = 0$$

in Equilibrium case.

We don't need anymore the full set of Maxwell Equations, we only need one i.e

$$\operatorname{div} D = Q$$

Poisson's equation

$$\operatorname{div} D = \rho \implies \operatorname{div} (\varepsilon_{sc} \mathcal{E}) = -\varepsilon_{sc} \nabla^2 \varphi = \rho(\varphi, \mathbf{r})$$
$$-\varepsilon_{sc} \nabla^2 \varphi = q \left[p^{(0)} \exp\left(\frac{-q\varphi}{k_B T_L}\right) - n^{(0)} \exp\left(\frac{q\varphi}{k_B T_L}\right) + N(\mathbf{r}) \right]$$

(supplemented with boundary conditions)

Here concentration of Holes & Electrons has been explicitly expressed as the functions of Electric potential

→ with Boundary conditions we solve the equation for Electric potential ϕ

$-q\phi = \text{Electrostatic Energy of electrons}$

& \bar{E}_F is a constant.

Once, we calculate the electric potential ϕ we can calculate the concentration n & p at any position using.

$$n = n^{(0)} \exp \left[q\phi / k_B T_L \right]$$

$$p = p^{(0)} \exp \left[-q\phi / k_B T_L \right]$$

This is possible because Semiconductor is Non-Degenerate and we don't have to calculate Fermi Integrals,

We get no Exponentials.

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~~*-*~~ If we are in

Non-Equilibrium case we cannot use the relation b/w Electric potential & Concentration of Electrons & Holes.

→ We won't have any Fermi level when we are in Non-Equilibrium case & won't have ^{any more} Fermi Statistics.

We cannot express anymore the concentrations of n & p in terms of Fermi Integrals.

$$n \neq N_c \phi_{\gamma_2}(\epsilon_e), p \neq N_v \phi_{\gamma_2}(\epsilon_h)$$

Semiconductor Equations — I

Non-equilibrium case: $\begin{cases} \partial/\partial t = 0 : \text{ steady-state cond.} \\ \partial/\partial t \neq 0 : \text{ time-dependent cond.} \end{cases}$

$$n = \int_{\text{CB}} Q \Phi d^3k, \quad p = \int_{\text{VB}} Q (1 - \Phi) d^3k,$$
$$n\mathbf{v}_n = \int_{\text{CB}} \mathbf{u} Q \Phi d^3k, \quad p\mathbf{v}_p = \int_{\text{VB}} \mathbf{u} Q (1 - \Phi) d^3k,$$

where Φ differs from the Fermi statistics \Rightarrow

$$n \neq N_C \Phi_{1/2}(\xi_e), \quad p \neq N_V \Phi_{1/2}(\xi_h).$$

The concentrations and fluxes are related by the continuity eqs.

$$\frac{\partial n}{\partial t} + \text{div}(n\mathbf{v}_n) = W_n, \quad \frac{\partial p}{\partial t} + \text{div}(p\mathbf{v}_p) = W_p,$$

where W_n is the difference between the number of electrons entering and leaving the conduction band, per unit volume and time (net generation rate, $[W_n] = \text{cm}^{-3}\text{s}^{-1}$). A similar meaning applies to W_p . In equilibrium, the continuity equations reduce to the identities $0 = 0$.

→ Therefore in Non-Equilibrium case
we must consider, n & p are
unknown themselves.

Also the Avg. Velocities are unknown
themselves.

→ we remember the definition

$$n = \int_{\text{CB}} Q \Phi d^3k, \quad p = \int_{\text{VB}} Q (1 - \Phi) d^3k,$$
$$n v_n = \int_{\text{CB}} u Q \Phi d^3k, \quad p v_p = \int_{\text{VB}} u Q (1 - \Phi) d^3k,$$

$$n = \int_{\text{CB}} Q \Phi d^3k$$
$$p = \int_{\text{VB}} Q (1 - \Phi) d^3k$$

Integration over K-space

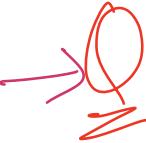
CB: Conduction Band
VB: Valence Band

Note: Here, ϕ = occupation probability

$$n v_n = \int_{\text{CB}} u Q \Phi d^3k$$

Average Velocity of electrons

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 What is the issue with
Non Equilibrium case ?

In this case the 1st issue is try to
separate as much as possible the
expressions related to the CB frontie
expressions related to the VB

For this, we have to start from the
general form of continuity equation

Continuity equation

$$\text{div rot } \mathbf{H} = \text{div} \frac{\partial \mathbf{D}}{\partial t} + \text{div } \mathbf{J} = \frac{\partial \text{div } \mathbf{D}}{\partial t} + \text{div } \mathbf{J}$$
$$\frac{\partial \rho}{\partial t} + \text{div } \mathbf{J} = 0$$

The charge density $\rho = P = q(p_n + N)$
on the other hand if we are in complete

ionization condition.

(N)

The concentration of Dopants is a function of position i.e it is noway a function of Time.

So the Time derivative of concentration of Dopant is Zero.

Essential we will left with time derivatives of q_P & q_N

$$Q = q(P - n + N)$$

$$\frac{dQ}{dt} = \frac{d(qP)}{dt} - \frac{d(qn)}{dt}$$

$$\therefore \frac{dqN}{dt} = 0$$

'N' is only dependent on position.

$$\operatorname{div} \operatorname{rot} H = \operatorname{div} \frac{\partial D}{\partial t} + \operatorname{div} J$$

$\Rightarrow \frac{\partial \operatorname{div} D}{\partial t} + \operatorname{div} J$

$J = J_p + \bar{J}_p$

Correct Density
It's contribution
from J_p & J_n

But, $\operatorname{div} D = P = Q$

$$\Rightarrow \frac{\partial Q}{\partial t} + \operatorname{div} J$$

$$\Rightarrow \frac{\partial}{\partial t} [q_p - q_n] + \operatorname{div} (J_p + \bar{J}_n)$$

$$\Rightarrow \frac{\partial}{\partial t} (q_p) - \frac{\partial}{\partial t} (q_n) + \operatorname{div} [q_p v_p - q_n v_n] = 0$$

$$\Rightarrow \frac{\partial (P)}{\partial t} + \operatorname{div}(Pv_p) = \frac{\partial (\eta)}{\partial t} + \operatorname{div}(nv_n)$$

~~Holes L-H.S~~ ~~electrons R-H.S~~

$$\operatorname{div} J = \operatorname{div}(J_p + J_n) \\ = \operatorname{div} J_p + \operatorname{div} J_n$$

→ Here, we can say we have two partial continuity equations of Holes on LHS & Electrons on RHS and they are equal in each point of space and at each instant of time.

$$-(\frac{\partial \eta}{\partial t} + \operatorname{div} nv_n) + \frac{\partial P}{\partial t} + \operatorname{div}(Pv_p) = 0$$

The concentrations and fluxes are related by the continuity eqs.

$$\frac{\partial n}{\partial t} + \operatorname{div}(nv_n) = W_n, \quad \frac{\partial p}{\partial t} + \operatorname{div}(pv_p) = W_p,$$

~~electrons continuity eq~~

~~Holes continuity Equation~~

The concentrations and fluxes are related by the continuity eqs.

$$\frac{\partial n}{\partial t} + \operatorname{div}(n \mathbf{v}_n) = W_n, \quad \frac{\partial p}{\partial t} + \operatorname{div}(p \mathbf{v}_p) = W_p,$$

where W_n is the difference between the number of electrons entering and leaving the conduction band, per unit volume and time (net generation rate, $[W_n] = \text{cm}^{-3}\text{s}^{-1}$). A similar meaning applies to W_p . In equilibrium, the continuity equations reduce to the identities $0 = 0$.

$$\frac{\partial n}{\partial t} + \operatorname{div}(n \mathbf{v}_n) = W_n$$

$$\frac{\partial p}{\partial t} + \operatorname{div}(p \mathbf{v}_p) = W_p$$

If, we have to integrate over the volume then

* Integration of $\frac{\partial n}{\partial t}$ gives total no. of electrons in the volume.

variation in time gives total no. of

* Integration of $\operatorname{div}(\gamma \mathbf{v}_n)$ over a volume is the total Flux of the particles across the Boundary of the volume.

if $w_n = 0$, which it is not practically, the continuity equation would implies that

$$\text{no. of electrons in volume} = -(\text{Flux})$$

→ The existence of a function that is (w_n) non zero in the equation. This function also must be integrated over the same volume tells us that there is another Reason for the population of Electrons in the Volume.

\therefore in the case of Semiconductors we need to consider an extra property we should consider.

\rightarrow When we consider electrons of CB.
we imply that these electrons have
 \hookrightarrow Energy that belongs to the CB.

\therefore If an Electron without moving in space makes a transition that brings electron into the VB. The electron is a loss for the CB.
So, we should count it as a -ve contribution.

Note:- Here electron is not changing the selected volume in space but changing the Energy to move from CB to VB

& VB to CB

→ similarly. If an electron comes from VB into CB without moving i.e staying within the chosen volume. Then we should include it as the positive contribution to the CB
& as a +ve contribution to function

W_n

Holes.

→ Eventually

$$W_n = \begin{aligned} &\text{no. of electrons} \\ &\text{entering CB} \\ &- \text{no. of electrons} \\ &\text{leaving VB} \end{aligned}$$

w_n = Net Generation Rate of
CB

w_p = Net Generation of VB

no. of holes entering - no. of holes
VB
leaving VB
within the chosen
volume.
within the
chosen volume.

* Relating to Equilibrium case

where $\frac{d}{dt} = 0$ & $J = 0$

$$w_n = w_p = 0$$

only at Equilibrium

Here, we split the Global continuity equation
that comes from Maxwell's Equation into two

for separate populations of particles imply
the need for introducing Net Generation

Rates.

→ So, we must consider at this point
these two continuity equations as two
additional equations ^{that} must be used when
we are in Non-equilibrium Condition.

→  Are W_n & W_p sufficient?

Any No

Because, we have now '4' unknowns
to solve.

n, p & nV_n, pV_p

Concentration
Unknown

Current densities
Unknown

→ So, the equations are welcome but are not sufficient.

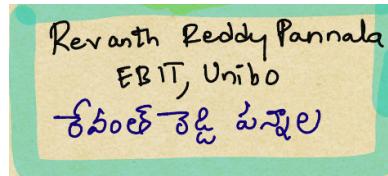
otherwise, we assume the net generation rates w_n & w_p themselves are not unknown.

→ we can split the Non Equilibrium Condition into two SubCases:

Non-Equilibrium case : {

$\frac{\partial}{\partial t} = 0$: steady state
simple case

$\frac{\partial}{\partial t} \neq 0$: time-dependent cond.





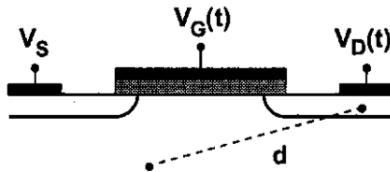
Semiconductor Equations — II

In steady-state conditions it is $\mathbf{E} = -\operatorname{grad} \varphi$, whence Poisson's equation follows (with n and p unknown):

$$\left. \begin{array}{l} \mathbf{E} = -\operatorname{grad} \varphi \\ \operatorname{div} \mathbf{D} = \rho \\ \mathbf{D} = \epsilon \mathbf{E} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} -\epsilon_{sc} \nabla^2 \varphi = q [p - n + N(\mathbf{r})] \\ -\epsilon_{ox} \nabla^2 \varphi = q N_{ox}(\mathbf{r}) \end{array} \right.$$

where the second form holds in the insulator regions.

In time-dependent conditions it is $\mathbf{E} = -\operatorname{grad} \varphi - \partial \mathbf{A} / \partial t$. This introduces a coupling with the rest of Maxwell's equations.



However, the steady-state relation $\mathbf{E} = -\operatorname{grad} \varphi$ still approximately holds if

$$\Delta t = \frac{d}{u_f} \ll T_{\min} = \frac{1}{\nu_{\max}}, \quad \nu_{\max} \ll \frac{u_f}{d} \approx \frac{10^{10} \text{ cm/s}}{10^{-4} \text{ cm}} = 10^{14} \text{ Hz}$$

which is true in the realistic cases.

I care
 In steady state we can still express the electric field $E = -\operatorname{grad} \phi$ because time derivative of magnetic potential is zero. Because we are in

Steady state.

→ If the Electric field (E) depends on electric potential we substantially split the set of Maxwell's equations into two parts and keep using only the Poisson's equation.

In the steady state, Non equilibrium case out of the maxwells equation we merely poissn's equation.

Q What is the difference b/w
Non-Equilibrium Steady state
&
Equilibrium case ?

A In Equilibrium the concentration of holes & electrons are function (P) (N)

of electric potential

* But In the nonequilibrium condition the concentrations are Unknown You have the poisson Equation .

In steady-state conditions it is $\mathbf{E} = -\text{grad } \varphi$, whence Poisson's equation follows (with n and p unknown):

$$\left. \begin{array}{l} \mathbf{E} = -\text{grad } \varphi \\ \text{div } \mathbf{D} = \rho \\ \mathbf{D} = \epsilon \mathbf{E} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} -\epsilon_{sc} \nabla^2 \varphi = q [p - n + N(\mathbf{r})] \\ -\epsilon_{ox} \nabla^2 \varphi = q N_{ox}(\mathbf{r}) \end{array} \right. \quad \begin{array}{l} (1) \\ (2) \end{array}$$

poisson's eqn for Dielectric in MOS

In Semiconductor devices like MOS

devices there is essentially a space in the volume which has Dielectric Insulator. and there are no mobile charges

In principle the Insulator should have No carriers. But it may have some fixed charge for some reason.

So, 2nd equation is for insulating
MOS semiconductor device.

1st equation is for semiconductor
obviously.

$$-\epsilon_{ox} \nabla^2 \phi = q N_{ox}(r)$$

ϵ_{ox} : permittivity of SiO_2

Case - I Dynamic Condition

(as)

Time dependent
Condition

In time-dependent conditions it is $\mathbf{E} = -\text{grad } \phi - \partial \mathbf{A} / \partial t$. This introduces a coupling with the rest of Maxwell's equations.

The Electric field

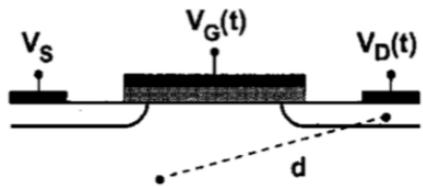
$$\mathbf{E} = -\text{grad } \phi - \frac{\partial \mathbf{A}}{\partial t}$$

This is very inconvenient to analyze from a practical viewpoint because when we have the magnetic potential, then we must couple together all the Maxwell's equations.

→ we cannot avoid using the full set of Maxwell equations. This is quite a Hard Task.

∴ we will have to look for some approximation that avoids using the time derivative of the Magnetic potential.

→ Obviously, we can say if we go to the limit of steady state condition we go to the limit of the steady state condition of course derivative of A w.r.t. time is zero.



However, the steady-state relation $\mathcal{E} = -\text{grad } \varphi$ still approximately holds if

$$\Delta t = \frac{d}{u_f} \ll T_{\min} = \frac{1}{\nu_{\max}}, \quad \nu_{\max} \ll \frac{u_f}{d} \approx \frac{10^{10} \text{ cm/s}}{10^{-4} \text{ cm}} = 10^{14} \text{ Hz}$$

which is true in the realistic cases.

In dynamic state

If the change w.r.t. time is slow, then the time derivative of A w.r.t. 't' will be small which can be neglected.

→ So, we must seek for a condition in Dynamic case where $\frac{\partial A}{\partial t}$ is small and can be neglected.

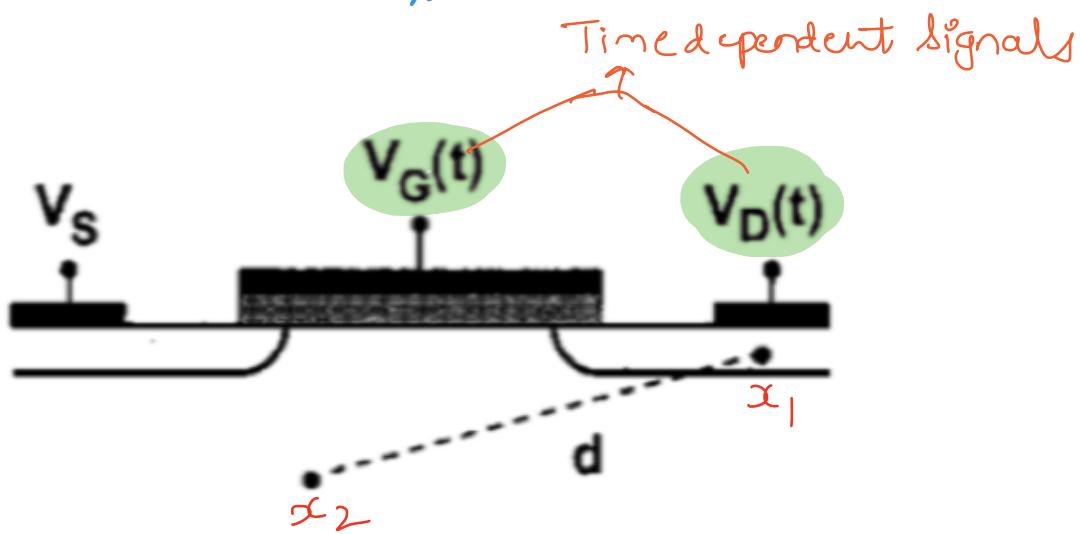


Why is magnetic potential changes with Time?

If some of the boundary conditions of

the problem changes with Time.

So, will analyze an MOS device



→ In Digital circuits assume they are pulses which are dependent on time and vary with time.

So, $V_D(t)$ has a spectrum of signals and now consider the **largest frequency** of the spectrum. This large Frequency is called v_{max} (pronounced as ω) ("knee")

$$\Delta t = \frac{d}{u_f} \ll T_{\min} = \frac{1}{\nu_{\max}}, \quad \nu_{\max} \ll \frac{u_f}{d} \approx \frac{10^{10} \text{ cm/s}}{10^{-4} \text{ cm}} = 10^{14} \text{ Hz}$$

minimum period

Maximum Frequency

→ Now we take two points in the MOS device x_1 & x_2

x_1 : very near to $V_D(t)$

x_2 : far from $V_D(t)$

Now $V_D(t)$ changes V_{x_1} also changes instantly with same rate.

→ But V_{x_2} will not change instantly. The electro magnetic field at x_2 will change after a time. Because Electromagnetic perturbation will take some time to propagate from drain contact to

the interior of the Material.

→ So, we can calculate the time delay that is needed for the propagation to occur.

$$\Delta t = \text{Time delay}$$

$$\Delta t = \frac{d}{v_f}$$

→ distance
→ propagation velocity of
Electro magnetic field at
the frequency (ν_{\max})

$$\Delta t = \frac{d}{v_f} \ll T_{\min} = \frac{l}{\nu_{\max}}$$

If this condition holds then we can infer that the propagation is Instantaneous.

& x_1 & x_2 experience same voltage change as $V_o(t)$

→ This situation we can consider approx. steady state condition.

$$\Delta t = \frac{d}{u_f} \ll T_{\min} = \frac{1}{\nu_{\max}}, \quad \nu_{\max} \ll \frac{u_f}{d} \approx \frac{10^{10} \text{ cm/s}}{10^{-4} \text{ cm}} = 10^{14} \text{ Hz}$$

for 1 micron
which is a old technology

The ν_{\max} is very large when compared to the frequencies which are given to the Mprocessors

→ Modern MP have channel length around 7nm to 20nm which will also increase the limit of T_{\min}

In conclusion; The propagation delay is practically negligible microprocessor with MOS technology.

→ Fortunately, we can safely neglect the time derivative of the magnetic potential - even in dynamic situation.

This is called Quasi-static Approx.

→ Now, we are left with the problem that the two continuity equations we have derived above are not sufficient because they are two equations and

4 unknowns

→ Because Quasi-static approximation does not apply to the two terms $\frac{\partial \mathbf{P}}{\partial t}$ because phenomena that determine the variation in time of \mathbf{n} & \mathbf{P} are much much slower than propagation times.

∴ In dynamic condition $\frac{\partial n}{\partial t}$ & $\frac{\partial p}{\partial t}$ must be kept in the equations.

→ So, we must find extra equations to solve two individual continuity equations.

For instance we must extra equation able to describe fluxes - The Fluxes are

$$J_n \propto n v_n \quad J_p \propto p v_p$$

$$\bar{J}_n = q n v_n \quad \bar{J} = q p v_p$$

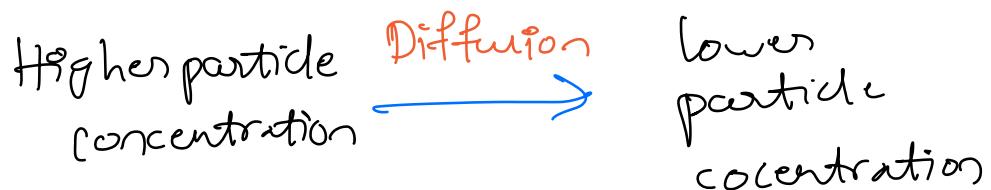
→ providing an expression to current density is essentially describing the transport of carriers into materials.

& There are two fundamental mechanisms by which carriers can move in the semiconductor.

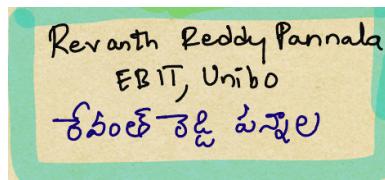
→ (i) One mechanism is obvious : Electrons and Holes are charged particles if there is an Electric field this field will force e's & Holes to move.

This kind of transport is called **Drift**

(ii) The second mechanism is called **Diffusion**
this can happen without Electric field
and also of Non-charged particles.



** As the particles go from higher concentration to lower concentration the concentrations on both sides tend to equalize and eventually diffusion comes to halt.



Semiconductor Equations — III

Expressions for $n\mathbf{v}_n$ and $p\mathbf{v}_p$ in the continuity equations.

Purely diffusive flux

$$(n\mathbf{v}_n)_{\text{diff}} = -D_n \text{grad } n,$$

$$(p\mathbf{v}_p)_{\text{diff}} = -D_p \text{grad } p,$$

where $D_{n(p)}$ is the electron (hole) diffusion coefficient, $[D] = \text{cm}^2 \text{s}^{-1}$.
It follows

$$\begin{cases} (\mathbf{J}_n)_{\text{diff}} = -(qn\mathbf{v}_n)_{\text{diff}} = qD_n \text{grad } n \\ (\mathbf{J}_p)_{\text{diff}} = (qp\mathbf{v}_p)_{\text{diff}} = -qD_p \text{grad } p \end{cases}$$

Purely ohmic flux

$$\begin{cases} -(qn\mathbf{v}_n)_{\text{drift}} = (\mathbf{J}_n)_{\text{drift}} = \sigma_n \mathbf{E} \\ (qp\mathbf{v}_p)_{\text{drift}} = (\mathbf{J}_p)_{\text{drift}} = \sigma_p \mathbf{E} \end{cases}$$

where $\sigma_{n(p)}$ is the electron (hole) conductivity, $[\sigma] = \Omega^{-1} \text{cm}^{-1}$.

If the perturbation from equilibrium is small, superposition yields the *drift-diffusion transport equations* for electrons and holes:

$$\begin{cases} \mathbf{J}_n \doteq (\mathbf{J}_n)_{\text{drift}} + (\mathbf{J}_n)_{\text{diff}} \\ \mathbf{J}_p \doteq (\mathbf{J}_p)_{\text{drift}} + (\mathbf{J}_p)_{\text{diff}} \end{cases}$$

In equilibrium, the drift and diffusion terms balance each other.

→ when we have purely diffusive flux we have the particles move i.e. There is a nonzero flux because concentration is Non-uniform



What is the indicator that tells us that concentration of particle is Non-Uniform?

The indicator is obviously Concentration Gradient

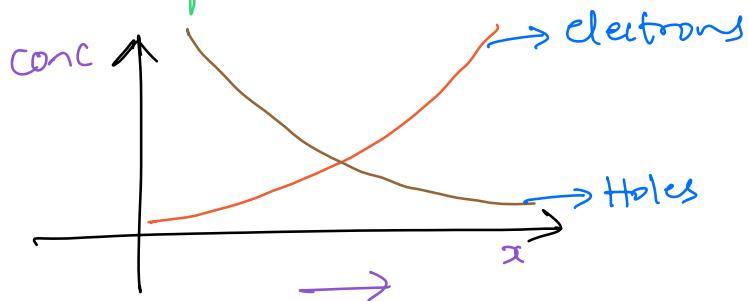
i.e. concentration changes with position.

We assume, a very simple model that the Flux not only depends on the Gradient of the concentration & the dependence is LINEAR.

$$\rightarrow \therefore \text{Flux} = \left(n v_n \right)_{\text{diff}} = -D_n \text{grad}(n)$$

$$= \left(p v_p \right)_{\text{diff}} = -D_p \text{grad}(p)$$

$D_p \& D_n$ → Diffusion Coefficients of Holes & Electrons.
coefficients are positive



The Flux flows in the opposite direction of gradient \therefore we have -ve signs in the expression.

Purely diffusive flux

$$(n\mathbf{v}_n)_{\text{diff}} = -D_n \text{grad } n, \quad (p\mathbf{v}_p)_{\text{diff}} = -D_p \text{grad } p,$$

where $D_{n(p)}$ is the electron (hole) diffusion coefficient, $[D] = \text{cm}^2 \text{s}^{-1}$. It follows

$$\begin{cases} (\mathbf{J}_n)_{\text{diff}} = -(qn\mathbf{v}_n)_{\text{diff}} = qD_n \text{grad } n \\ (\mathbf{J}_p)_{\text{diff}} = (qp\mathbf{v}_p)_{\text{diff}} = -qD_p \text{grad } p \end{cases}$$

Drift & Ohmic Flux

Purely ohmic flux

$$\begin{cases} -(qn\mathbf{v}_n)_{\text{drift}} = (\mathbf{J}_n)_{\text{drift}} = \sigma_n \mathbf{E} \\ (qp\mathbf{v}_p)_{\text{drift}} = (\mathbf{J}_p)_{\text{drift}} = \sigma_p \mathbf{E} \end{cases}$$

Sigma = electron conductivity
Linear Relation b/w \mathbf{J} & \mathbf{E}

where $\sigma_{n(p)}$ is the electron (hole) conductivity, $[\sigma] = \Omega^{-1} \text{cm}^{-1}$.

If the perturbation from equilibrium is small, superposition yields the **drift-diffusion transport equations** for electrons and holes:

$$\begin{cases} \mathbf{J}_n \doteq (\mathbf{J}_n)_{\text{drift}} + (\mathbf{J}_n)_{\text{diff}} \\ \mathbf{J}_p \doteq (\mathbf{J}_p)_{\text{drift}} + (\mathbf{J}_p)_{\text{diff}} \end{cases}$$

In equilibrium, the drift and diffusion terms balance each other.

→ Assume that Non-Equilibrium state that we are considering is not far from the Equilibrium state we are considering. & also assume that

The two phenomena Diffusion & Drift occur together in the material.

- If material is not too far from equilibrium it can be demonstrated that we can simply add two effects together using superposition principle.

<

$$\begin{cases} \mathbf{J}_n = (\mathbf{J}_n)_{\text{drift}} + (\mathbf{J}_n)_{\text{diff}} \\ \mathbf{J}_p = (\mathbf{J}_p)_{\text{drift}} + (\mathbf{J}_p)_{\text{diff}} \end{cases}$$

If semiconductor goes back to equilibrium

$$\bar{J}_n = \bar{J}_p = 0$$

This does not mean $(\bar{J}_n)_{\text{drift}}$ & $(\bar{J}_p)_{\text{diff}}$

$(\bar{J}_p)_{\text{drift}}$ & $(\bar{J}_p)_{\text{diff}}$ are equal to zero.

They can also be

$$(\bar{J}_n)_{\text{drift}} = (\bar{J}_n)_{\text{diff}}$$

This can happen in the case of non-uniform semiconductor

Note:- Non-uniformity will introduce concentration gradient and therefore Diffusion Current.

But, Non-uniformity of charge will also introduce a Net charge density at each position in the semiconductor that gives raise to Drift current.

→ Now, we will try and extract Heuristically from some Microscopic considerations. The expressions of conductivities.

Semiconductor Equations — IV

Microscopic derivation of the drift term at low fields

$$\mathbf{F} = m^* \mathbf{a} \implies \frac{d\mathbf{u}}{dt} = \pm \frac{q}{m^*} \boldsymbol{\mathcal{E}}, \quad \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}(t)$$

$$\mathbf{u}(t) - \mathbf{u}(t - \tau) = \pm \frac{q}{m^*} \int_{t-\tau}^t \boldsymbol{\mathcal{E}}(t') dt' \simeq \pm \frac{q}{m^*} \boldsymbol{\mathcal{E}} \tau,$$

where τ is the time-after-collision of the carrier. Then

$$\langle \mathbf{u}(t) \rangle - \langle \mathbf{u}(t - \tau) \rangle = \pm \frac{q}{m^*} \boldsymbol{\mathcal{E}} \langle \tau \rangle, \quad \text{with}$$

$$\langle \mathbf{u}(t) \rangle = \begin{cases} (\mathbf{v}_n)_{\text{drift}} \\ (\mathbf{v}_p)_{\text{drift}} \end{cases} \quad \langle \tau \rangle = \begin{cases} \tau_{pn} \\ \tau_{pp} \end{cases} \quad m^* = \begin{cases} m_n \\ m_p \end{cases},$$

and $\langle \mathbf{u}(t - \tau) \rangle \simeq 0$ if $\boldsymbol{\mathcal{E}}$ is small. It follows

$$\begin{cases} (\mathbf{v}_n)_{\text{drift}} = -\mu_n \boldsymbol{\mathcal{E}} \\ (\mathbf{v}_p)_{\text{drift}} = \mu_p \boldsymbol{\mathcal{E}} \end{cases} \quad \begin{cases} \mu_n = q\tau_{pn}/m_n \\ \mu_p = q\tau_{pp}/m_p \end{cases}$$

$$\begin{cases} (\mathbf{J}_n)_{\text{drift}} = -(qn\mathbf{v}_n)_{\text{drift}} = q\mu_n n \boldsymbol{\mathcal{E}} \\ (\mathbf{J}_p)_{\text{drift}} = (qp\mathbf{v}_p)_{\text{drift}} = q\mu_p p \boldsymbol{\mathcal{E}} \end{cases} \quad \begin{cases} \sigma_n = q\mu_n n \\ \sigma_p = q\mu_p p \end{cases}$$

In the above, $\mu_{n(p)}$ is the electron (hole) mobility, $[\mu] = \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$.

→ Let us consider for now a single particle it could be one electron or one hole.

We remember, the inertia of \dot{e} or hole in a semiconductor is determined by the effective mass.

Let's use for effective m^*

$$F = m^* a$$

$$\Rightarrow \boxed{\frac{du}{dt} = \pm \frac{q}{m^*} E}$$

+ for Hole
- for Electron

$$F = qE \text{ or } qE$$

Forced due to electric field

$$du = \pm \frac{q}{m^*} E dt$$

Now by integrating w.r.t time

$$F = m^* a \implies \frac{du}{dt} = \pm \frac{q}{m^*} \mathcal{E}, \quad \mathcal{E} = \mathcal{E}(t)$$

$$u(t) - u(t - \tau) = \pm \frac{q}{m^*} \int_{t-\tau}^t \mathcal{E}(t') dt' \simeq \pm \frac{q}{m^*} \mathcal{E}\tau,$$

τ is positive

→ Integration time is from $t - \tau$ to t

Now, let's assume that τ is very short such that Electric field does not change too much along time τ

Q What is this τ ?

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To describe τ we must figure out what is the typical trajectory of one electron (or hole) in a semiconductor.

Consider, one electron that travels in space and it is subjected to Electric Field after a while this electron has a collision with something else (some other electron or Nucleus etc)

So, we can figure out ^{The} Trajectory (or) the dynamic evolution of electron inside a crystal as alternation of what we call Free flights during which the Electron

is subjected to only external fields and collisions.

- So, This is enough to specify the time t & $t-\tau$
- t : Anytime during Freeflight
- τ : Time interval b/w last collision event & 't'
- Typical time interval b/w two successive collisions is smaller than **Pico seconds**

At this point we take expression

$$\{u(t)\} - \{u(t-\tau)\} = \pm \frac{q \cdot E \cdot \{\tau\}}{m^*}$$

for whole set of Electrons in conduction Band.

↓
Statistical Average

Momentum relaxation time of electron momentum τ_{pn} electron

Momentum Relaxation Time of holes τ_{ph} holes

where τ is the time-after-collision of the carrier. Then

$$\langle \mathbf{u}(t) \rangle - \langle \mathbf{u}(t - \tau) \rangle = \pm \frac{q}{m^*} \mathcal{E} \langle \tau \rangle, \quad \text{with}$$

$$\langle \mathbf{u}(t) \rangle = \begin{cases} (\mathbf{v}_n)_{\text{drift}} \\ (\mathbf{v}_p)_{\text{drift}} \end{cases} \quad \langle \tau \rangle = \begin{cases} \tau_{pn} \\ \tau_{pp} \end{cases} \quad m^* = \begin{cases} m_n \\ m_p \end{cases}$$

and $\langle \mathbf{u}(t - \tau) \rangle \approx 0$ if \mathcal{E} is small. It follows

$$\begin{cases} (\mathbf{v}_n)_{\text{drift}} = -\mu_n \mathcal{E} \\ (\mathbf{v}_p)_{\text{drift}} = \mu_p \mathcal{E} \end{cases} \quad \begin{cases} \mu_n = q\tau_{pn}/m_n \\ \mu_p = q\tau_{pp}/m_p \end{cases}$$

$$\begin{cases} (\mathbf{J}_n)_{\text{drift}} = -(qn\mathbf{v}_n)_{\text{drift}} = q\mu_n n \mathcal{E} \\ (\mathbf{J}_p)_{\text{drift}} = (qp\mathbf{v}_p)_{\text{drift}} = q\mu_p p \mathcal{E} \end{cases} \quad \begin{cases} \sigma_n = q\mu_n n \\ \sigma_p = q\mu_p p \end{cases}$$

In the above, $\mu_{n(p)}$ is the electron (hole) mobility, $[\mu] = \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$.

→ we can say that statistical Avg. of
 $u(t - \tau)$ i.e. $\{u(t - \tau)\} \approx 0$ for smaller
 electric field

The collisions are randomizing the motion
 of a particle - when particle completes
 a collision its velocity may be oriented
 anywhere in space.

→ So, when we take statistical average of
 a particle that can be oriented anywhere in
 space.

The statistical Avg. Velocity will be
 practically equal to zero.

Eventually, what we obtain is that

$v_n \propto v_p$ would be proportional to Electric field.

$$v_n \propto -E \quad v_p \propto E$$

$$v_n = \frac{q\tau_{np}}{m_n} \cdot E$$

This constant is called Mobility

Drift Velocity $\left\{ \begin{array}{l} (\mathbf{v}_n)_{\text{drift}} = -\mu_n \mathbf{E} \\ (\mathbf{v}_p)_{\text{drift}} = \mu_p \mathbf{E} \end{array} \right. \quad \left\{ \begin{array}{l} \mu_n = q\tau_{pn}/m_n \\ \mu_p = q\tau_{pp}/m_p \end{array} \right.$

$$\left\{ \begin{array}{l} (\mathbf{J}_n)_{\text{drift}} = -(qn\mathbf{v}_n)_{\text{drift}} = q\mu_n n \mathbf{E} \\ (\mathbf{J}_p)_{\text{drift}} = (qp\mathbf{v}_p)_{\text{drift}} = q\mu_p p \mathbf{E} \end{array} \right. \quad \left\{ \begin{array}{l} \sigma_n = q\mu_n n \\ \sigma_p = q\mu_p p \end{array} \right.$$

In the above, $\mu_{n(p)}$ is the electron (hole) mobility, $[\mu] = \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$.

Expression of Conductivity
of Holes & Electrons.

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