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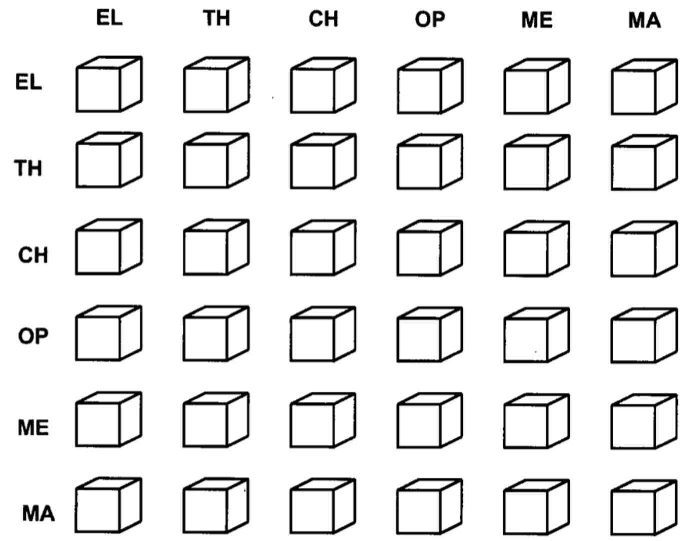
Lecture-15 (MOS is written in previous Lecture)

Sensors Introduction

Sensor classification implemented using Solid State devices

Electrical
Thermal
Chemical
optical
Mechanical
Magnetic



T. 34.1: Schematizzazione di possibili trasduttori ("cubo di Middelhoek").

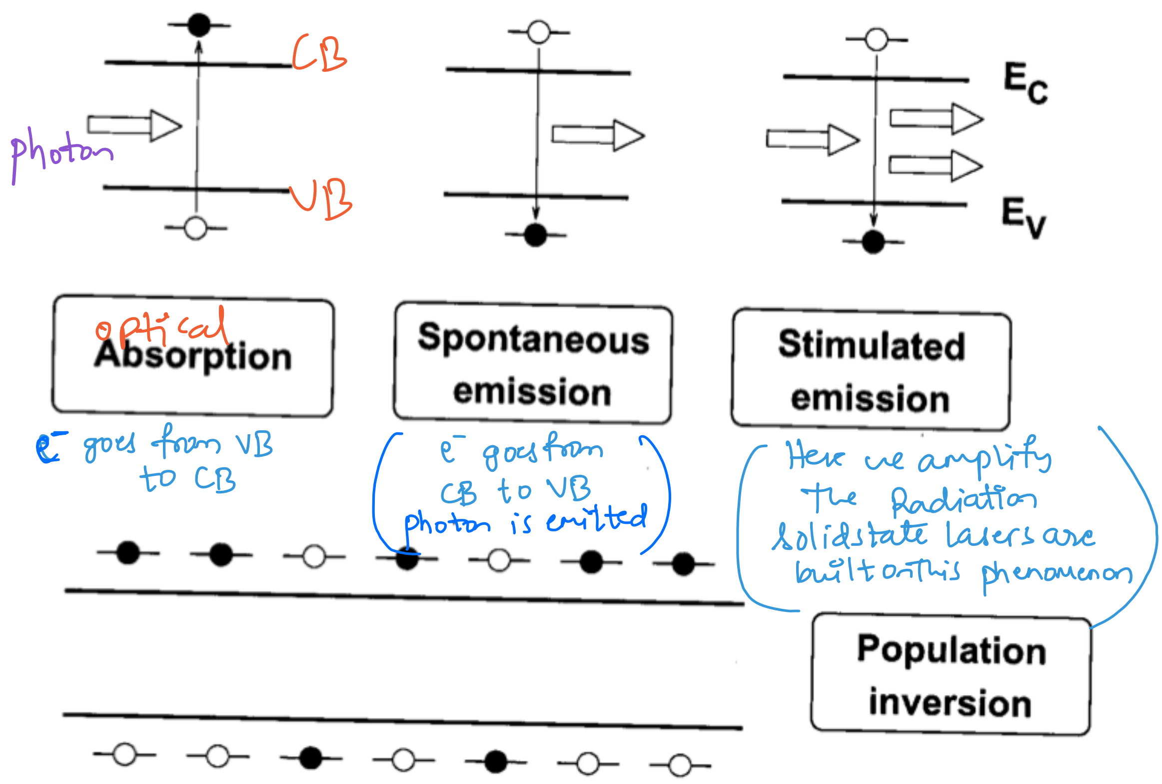
Rows → Signal to be detected

columns indicate type of signal that is provided to the User.

→ Firstly, we will study optical sensors, in which we exploit the ability of the Solid state matter to absorb visible radiation and provide some kind of signal.

- In a semiconductor when we consider valance & conduction Band, we already analyzed the Optical absorption

SENSOR: Sensor is a device that acquires an information from the environment about some physical property and provides a signal to the user to tell the user that certain physical property is detected.



T. 34.2: Rappresentazione delle transizioni elettroniche assistite da fotoni.

→ Here we are going to investigate only sensors not LASER

- In case of Optical Absorption, we see e^- go from VB to CB in this case e^- is free to conduction, so current is the way to deduce whether the material is subjected to radiation.

- When an uniform material is illuminated then the no. of e^-s in CB & no. of holes in VB increases which will result in the increase of the current. This is the case of Photo Resistor. It is just a

Resistor made of Semiconductor & when we apply a voltage we see a current and when the Photo Resistor is illuminated the Current Increases.
This not an efficient Solution because there is current is present even when there is no illumination i.e we are simply consuming power for nothing

- when light impinges on photo Resistor the current has a variation so we must detect the variation from the non illuminated current, this variation might not be large so it can be subjected to Noise. We can improve the situation using a Photo Diode

Photo Diode

A PhotoDiode is a PN Junction

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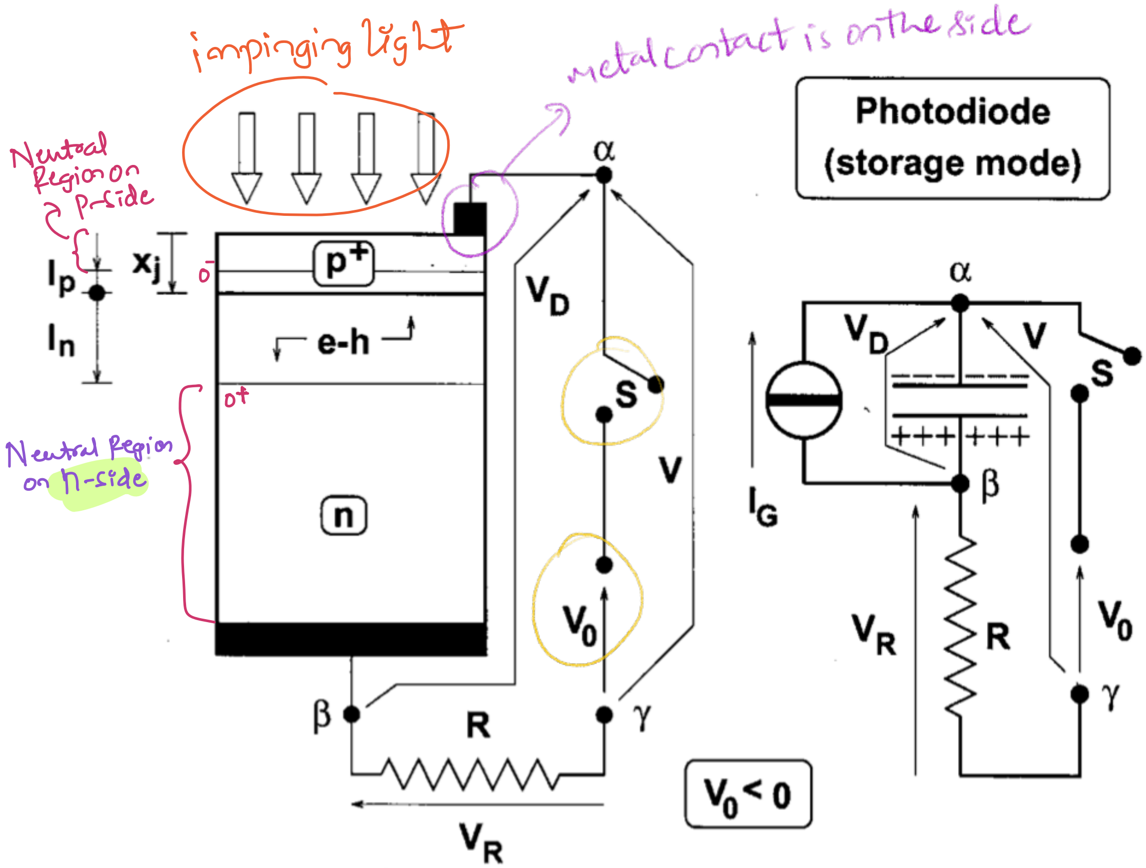

T. 34.3: Funzionamento in storage mode del fotodiodo e circuito equivalente.

→ Assume the switch 'S' is closed and the voltage V_0 is applied and V_0 is -ve ∴ The PN Junction is in Reverse Bias ⇒ There is small current when there is No light impinging on the photo Diode.

- The Metal contact is on the side because we don't want the light to be Reflected.

- The Neutral Region on the P-side should be maintained of small width because the incoming photons will create e-H pairs and they decrease exponentially in the Material because in Neutral Region EF is negligible and the e-H pairs move by diffusion and while they move they Recombine Because when There are excess of e^-'s & H's they move parallel to each other & recombine. (The optically generated e-H pairs are lost)

- Photons that survive the absorption in p-type enter the Space charge Region and in this region they continue to generate e-H pairs. But in Space charge Region There is a Strong EF and since the applied Bias is Reverse i.e $V_D < 0$
electric field

e-H pairs are seperated Immediately ∴ They do not Recombine but They contribute to current.

The photons that enter the Neutral Region of n-type are also lost e-H pairs are also lost. So in order to avoid this the Space charge Region has to be Big i.e The Boundary l_n must be low. This can be done by introducing Assymmetric Doping. i.e p^+, n

p-region Heavily doped

The current produced by optical generation would contribute to The already existing Thermal current i.e Reverse Saturation current

This make photodiode a Good Sensor because current because of optical generation is significantly more than The Thermal current.
∴ We have much better SNR ratio w.r.t to photo Resistor.

The Resistance at the bottom is an equipment that detects The current.

This mode of operation is called Continuous Mode. it is an improvement wrto photo Resistor but we can improve more.

Thermal Recombination

Photodiode in Continuous Mode

optical Generation

The hole-continuity equation reads

derivative of Hole concentration

$$\boxed{\frac{\partial p}{\partial t} + U + \frac{1}{q}\,\mathrm{div}\,\boldsymbol{J}_p = G}$$

which, assuming a one-dim. and steady-state condition, yields

$$\boxed{J_p(\mathrm{O}^+) - J_p(\mathrm{O}^-) \simeq -J_p(\mathrm{O}^-) = \int_{\mathrm{O}^-}^{\mathrm{O}^+} q\,(G-U)\,\mathrm{d}x\,.}$$

- For a monochromatic radiation it is $G = G_0 \exp[-k(s+x)] \doteq G_s \exp(-kx)$, with $s = x_j - l_p$. As $U \simeq -n_i/\tau_g$, $J = \mathrm{const} = J_p(\mathrm{O}^-) + J_n(\mathrm{O}^-) \simeq J_p(\mathrm{O}^-)$, it follows

$$-J = q\frac{G_s}{k}\left[1 - \exp(-kl)\right] + q\frac{n_i}{\tau_g}l \simeq q\frac{G_s}{k} + q\frac{n_i}{\tau_g}l > 0\,.$$

- For a wavelength $\lambda = 0.5\ \mu\mathrm{m}$ it is $1/k \sim 1\ \mu\mathrm{m}$, whence $kl > 1$. Defining the ratio $r \doteq \tau_g G_s/(kl n_i)$ between the optical and thermal current-densities, one finds $G_s = rkl\,n_i/\tau_g$. As $\eta \sim 1$ and $s \simeq x_j$, the incident power per unit area reads

$$h\nu\,\Phi_0 = \frac{h\nu}{\eta k}G_0 = \frac{h\nu}{k}G_s \exp(ks) \simeq r\,h\nu \exp(kx_j)\frac{n_i}{\tau_g}l\,.$$

Assuming One dimentional in the figure the x-axis is vertical and we also assume steady state condition

It is not really necessary to completely solve the model of semiconductor devices, it is sufficient to calculate the current using the below eqn

$$J_p(0^+) - J_p(0^-) \simeq -J_p(0^-) = \boxed{\int_{0^-}^{0^+} q(G-U)\,dx}$$

We calculate the value of the current by Integrating over Space charge Region.

- We also make another observation that the Holes as soon as they are generate they go upwards towards Edge 0^-, instead the e^-s are oriented by the Field towards the lower edge of the SCR i.e 0^+

∴ When we calculate the current of the Holes the only contribution to the Hole current will be given at 0^- because practically there are no Holes crossing 0^+

So when we integrate the eq^n

$$\underbrace{J_p(0^+)}_{\text{neglected}} - J_p(0^-) \simeq - J_p(0^-)$$

In one dimention divergence is Derivative

$$\therefore -q\left(\frac{1}{q}\,\frac{d}{dx}\,J_p(0^-)\right) = q\,(G-U)$$

I. OBS

$$\boxed{-J_p(0^-) = \int_{0^-}^{0^+} q(G-U)\,dx}$$

→ We remember that Optical Generation decays exponentially when we enter the Material

- So for a Monochromatic Radiation with fixed absorption coefficient 'k'

$$G = G_0 \exp[\underbrace{-k(s+x)}] =$$

Here we have $-k(s+x)$ because we assume in the structure that zero coincides with 0^-

- So the photons have already crossed a piece of semiconductor ~~and~~ we had an absorption in the initial region of the semiconductor. This absorption is lost. Q) So how long is the initial piece of the semiconductor?

Ans It is equal to $x_j - l_p$ (from the figure)

- We can write

$$G = G_0 \exp[-k(s+x)] = G_s \exp(-kx)$$

Here we simply modified the constant to incorporate e^{-ks}

- We also have 'U' it's a net Recombination Term in depleted region of the junction that's RB so we can assume full depletion condition.

$$\therefore \quad U \simeq -n_i / \tau_g$$

generation lifetime

Intrinsic concentration

- Finally, we remember in steady state of one dimention. The total current is equal to the sum of the Hole current & Electron current at the same position. for example at 0^-

At 0^- Hole current dominates

$$\therefore \quad J_p(0^-) \simeq J$$

$$\Rightarrow \quad -J \simeq q \frac{G_s}{K}\left[1 - \exp(-Kl)\right] + q \frac{n_i}{\tau_g} l$$

$$\simeq \quad q \frac{G_s}{K} + q \frac{n_i}{\tau_g} l > 0$$

→ Now, we can make an estimate we can take a monochromatic radiation that is in the centre of visible

Spectrum i.e $\lambda = 0.5\mu m$ (Green)

- For a wavelength $\lambda = 0.5$ μm it is $1/k \sim 1$ μm, whence $kl > 1$. Defining the ratio $r \doteq \tau_g G_s/(kln_i)$ between the optical and thermal current-densities, one finds $G_s = rkl\, n_i/\tau_g$. As $\eta \sim 1$ and $s \simeq x_j$, the incident power per unit area reads

$$h\nu\,\Phi_0 = \frac{h\nu}{\eta k} G_0 = \frac{h\nu}{k} G_s \exp(ks) \simeq r\, h\nu \exp(kx_j)\, \frac{n_i}{\tau_g}\, l\,.$$

for Green Wave

$$\text{penetration length} = \frac{1}{k} \simeq 1\mu m$$

$\therefore$ $kl > 1$ $\therefore$ we can ignore $\exp(-kl)$

$$\therefore \quad -J \simeq q\frac{G_s}{k} + q\frac{n_i}{\tau_g} l$$

This expression is Good

does not depend on the Bias

In practicle situation if semiconductor is very Good not with many defects Then τ_g is Large

$\therefore$ above term is small

In the above conditions we know $G_s \propto G_0$ and G_0 is the flux at the surface of the semiconductor (immediately below the surface)

∴ if there is Large illumination
G_s is Large

$$h\nu\phi_0 = \frac{h\nu\, G_0}{\eta K} = \frac{h\nu}{K} G_s \exp(Ks)$$

flux of the photons

$$\eta \simeq 1$$

∴ We can assume in the current equation Generation term dominates

→ 'J' is practically proportional to the Flux (flux of photons) which is very good it is a simple solution.

Q) Can we improve the photo Diode further?
Can we manipulate the Device to produce Larger current?

Ans YES in STORAGE MODE

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Photodiode in Storage Mode — I

The photodiode response can be improved by adopting the storage mode. At $t = 0$ the switch is opened. The effect of the generation processes for $t > 0$ is to modify the stored charge Q. This in turn changes the voltage drop $V_D = V$ across the photodiode according to the relation

$$\mathrm{d}Q = C\,\mathrm{d}V_D\,, \qquad C \simeq \frac{C_0}{(1 - V_D/\psi_0)^{1/m}}\,, \quad m \geq 2\,.$$

In the above it is $\mathrm{d}Q = -J\,\mathrm{d}t$ with $-J \simeq qG_s/k + qln_i/\tau_g \simeq qG_s/k = \text{const}$. Letting $w \doteq 1 - V_D/\psi_0$ yields

$$\frac{-J}{C_0\psi_0}\,\mathrm{d}t = -w^{-1/m}\,\mathrm{d}w\,.$$

Defining $\tau_0 \doteq -C_0\psi_0/J > 0$, $\alpha \doteq 1 - 1/m \geq 1/2$ and integrating from $t = 0$ yields

$$\alpha\frac{t}{\tau_0} = w_0^\alpha - w^\alpha\,, \qquad w_0 \doteq 1 - V_0/\psi_0\,.$$

As $V_0 \leq V_D \leq 0$ it is $w_0 \geq w > 0$. The switch is closed after the integration time T_i. This suddenly brings back to V_0 the voltage of the upper contact with respect to ground.

→ Storage Mode:

In storage mode we keep the switch 'S' open. So no current can flow through the circuit. But light keeps impinging on the Device ∴ optical generations continue to happen But it's impossible for Generated charges to flow through the external circuit.

Conclusion: The Generated Charges accumulate inside the device. This is called Storage Mode.

→ The interval of time during which switch gets opened in which storage takes place is called the Integration Time. When the Integration time is finished we will close the switch and a much larger current will flow through the external circuit.

Q) How long the switch be closed?

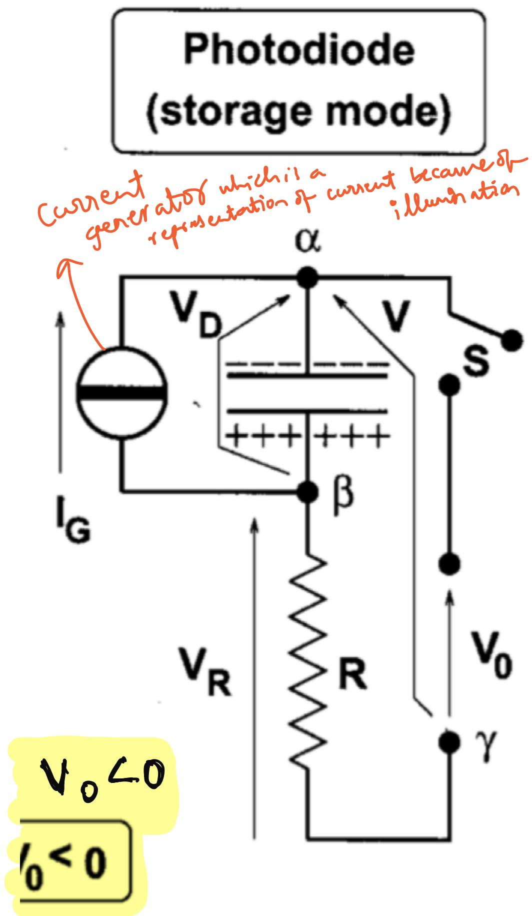
Ans It's the time needed to restore initial conditions. i.e for the total charge to be discharged.

Q) Why is the storage Mode Better? even though we don't have a continuous description of light impinging on the device

Ans We can live with non continuous description of information.

ex: Picture on a TV is not shown continuously but frame by frame with each frame taking some ms of time

Lecture-16 (continuation of photo Diode in Storage Mode)



Current generator which is a representation of current because of illumination

$V_0 < 0$

- When the switch 'S' is switched off. But the current generator keeps functioning because we have illumination.

  ∴ I_G flows through the capacitor (photodiode in RB) not through the External circuit.

- What happens in the capacitor is that the charge initially stored in the capacitor because of Acceptor & Donor atoms in the respective regions. and the charge is equal and opposite.

→ when we have a current I_G which flows through the capacitor this current essentially brings positive charges bottom up which means the Total amount of +ve charge on the lower electrode of the capacitor decreases. and similarly the -ve electric charge on the upper electrode also decreases. This voltage drop is called V_D

- initially before we open the switch the voltage drop V_D is forced by V_0 and

V_0 is $-ve$

But during the Integration time we are bringing $+ve$ charges from the lower electrode to the upper electrode the $|V_D|$ decreases but V_D is $-ve$ & V_D during integration time becomes less Negative

- During the Integration time the External circuit with switch 'S' is Open. So there is no possible current across the Resistance 'R' during the Integration time.

$\therefore$ There is no voltage drop across 'R'

& $V_R = 0$

$\Rightarrow$ potential at 'α' & potential at 'β' are equal to each other.

$\rightarrow$ At the end of the Integration Time the switch 'S' is Closed.

In This case 'α' goes in zero time from The value V_D to The value V_0

$$|V_D| < |V_0|$$

So The voltage at 'α' has a discontinuity in zero time & a Capacitor will not allow for sudden change in the voltage.

- The change in 'α' That has become more negative reflects into an equal change at 'β' so $\beta = 0$ originally because There was no current flowing through the Resistance 'R' so at the time when we close the switch The potential at $\beta = 0$ Then it becomes +ve

- Due to This we will have a Peak current Through the resistance which is Large. The voltage drop V_R across the resistance.

Note: What we gain from The Storage Mode is The current we get through The Resistor when we close The switch is Larger Than The Continuous Mode.

The gain can be of order 20 (or) 30 times.

→ After the peak current the charges at the plates of the capacitor should go back to the steady state value, and this time taken is called Transient time.

Photodiode in Storage Mode — I

- The photodiode response can be improved by adopting the storage mode. At $t = 0$ the switch is opened. The effect of the generation processes for $t > 0$ is to modify the stored charge Q. This in turn changes the voltage drop $V_D = V$ across the photodiode according to the relation

$$\boxed{\mathrm{d}Q = C\,\mathrm{d}V_D}\,, \qquad C \simeq \frac{C_0}{(1 - V_D/\psi_0)^{1/m}}\,, \qquad m \geq 2\,.$$

In the above it is $\boxed{\mathrm{d}Q = -J\,\mathrm{d}t}$ with $\boxed{-J \simeq qG_s/k + qln_i/\tau_g \simeq}$ $qG_s/k = \mathrm{const}$. Letting $w \doteq 1 - V_D/\psi_0$ yields

current only due to Optical Generation

$$\boxed{\frac{-J}{C_0\psi_0}\,\mathrm{d}t = -w^{-1/m}\,\mathrm{d}w}\,.$$

Defining $\tau_0 \doteq -C_0\psi_0/J > 0$, $\alpha \doteq 1 - 1/m \geq 1/2$ and integrating from $t = 0$ yields

$$\alpha\frac{t}{\tau_0} = w_0^\alpha - w^\alpha\,, \qquad w_0 \doteq 1 - V_0/\psi_0\,.$$

As $V_0 \leq V_D \leq 0$ it is $w_0 \geq w > 0$. The switch is closed after the integration time T_i. This suddenly brings back to V_0 the voltage of the upper contact with respect to ground.

We know PN Junction in RB is a Non linear capacitor so we must use the Differential Capacitance

$$Q = CV$$

$$\boxed{dQ = C\, dV}$$

'Q' = charge per unit Area

'C' = capacitance per unit Area

$$\boxed{C \simeq \frac{C_0}{\left(1 - V_D/\psi_0\right)^{1/m}}, \quad m \geq 2}$$

Differential capacitance

C_0 : value of capacitance when applied voltage is zero i.e at equillibrium.

V_D : Here is the applied voltage

ψ_0 : Builtin potential

- We have earlier calculated the capacitance of a PN Junction considering an Abrupt Junction & $m = 2$
- However in real life the Junctions are not abrupt if we calculate the capacitance of a PN Junction in which the substrate has a Uniform Doping and the SCR is obtained by Diffusion. 'm' is larger than '2'

In the above slide we arrive at the eq^n

$$dQ = \boxed{\frac{-J}{c_0 \varphi_0} dt = -w^{-1/m} dw}$$

where $w = 1 - V_D/\varphi_0$

from $dQ = C\, dV_D$

$dQ = -J\, dt$

$\Rightarrow -J\, dt = C\, dV_D$

$$C = \frac{C_0}{(1 - V_D/\varphi_0)^{1/m}}$$

$\Rightarrow -J\, dt = \frac{C_0}{(w)^{1/m}} (-dw)\varphi_0$

let $\left(1 - \frac{V_D}{\varphi_0}\right) = w$

D.O.B.S

$\Rightarrow dw = \frac{-dV_D}{\varphi_0}$

$\Rightarrow dV_D = -dw\,\varphi_0$

$$\Rightarrow \boxed{\frac{-J\, dt}{C_0 \varphi_0} \Rightarrow -\left(w^{-1/m}\right) dw}$$

$\rightarrow$ Defining $T_0 = -C_0 \varphi_0 / J > 0$, $\alpha = 1 - 1/m \geq 1/2$

and integrating from $t = 0$

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Defining $\tau_0 \doteq -C_0\psi_0/J > 0$, $\alpha \doteq 1 - 1/m \geq 1/2$ and integrating from $t = 0$ yields

$$\boxed{\alpha \frac{t}{\tau_0} = w_0^\alpha - w^\alpha}, \qquad w_0 \doteq 1 - V_0/\psi_0 .$$

value of 'w' at t=0

As $V_0 \leq V_D \leq 0$ it is $w_0 \geq w > 0$. The switch is closed after the integration time T_i. This suddenly brings back to V_0 the voltage of the upper contact with respect to ground.

$V_0 \leq V_D \leq 0$

initial the voltage across the Junction is -ve

As time evolves in storage mode the voltage across Junction will be much less Negative.

Photodiode in Storage Mode — II

T_i is in the range of few ms

As typically $T_i \ll \tau_0$, the RHS can be expanded to first order: $w_0^\alpha - w^\alpha \simeq \alpha w_0^{-1/m} (w_0 - w)$, to yield, with $V_i = V_D(T_i)$,

$$\frac{T_i}{\tau_0} \simeq w_0^{-1/m} \frac{V_i - V_0}{\psi_0} = -\frac{C(V_0)}{C_0} \frac{V_0 - V_i}{\psi_0} .$$

- In the load resistor, the bottom contact is grounded, while the upper one is brought at $t = T_i$ from ground to $V_0 - V_i < 0$. This gives rise to a peak current in the load resistor

$$I_R = \frac{V_0 - V_i}{R} = -\frac{T_i}{RC(V_0)} \frac{C_0\psi_0}{\tau_0} = -\frac{T_i}{RC(V_0)} (-J) .$$

The current gain of the storage-mode operation is defined as

$$\boxed{\frac{I_R}{A_e J} \doteq \frac{T_i}{RA_e C(V_0)}} .$$

A_e = Area

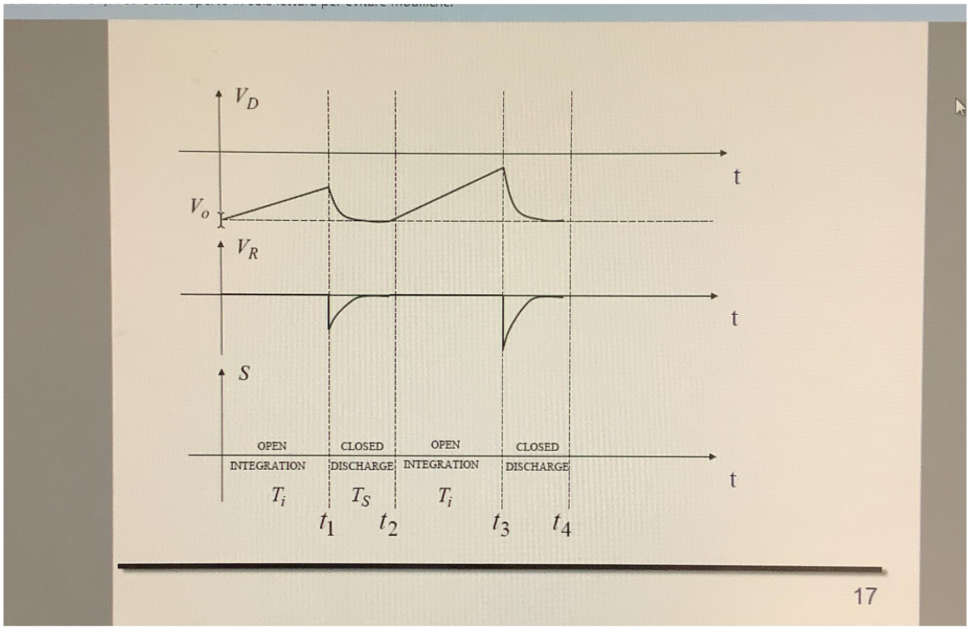
Typical values are $T_i \sim 1$ ms, $R \sim 1\,\mathrm{k}\Omega$, $A_e C(V_0) \sim 1$ pF, whence $I_R/(A_e J) \sim 10^6$. For $t > T_i$ a new transient occurs described by

$$I_R = A_e \frac{\mathrm{d}Q}{\mathrm{d}t} \implies \boxed{\frac{V_0 - V_D}{R} = A_e C(V_D) \frac{\mathrm{d}V_D}{\mathrm{d}t}} .$$

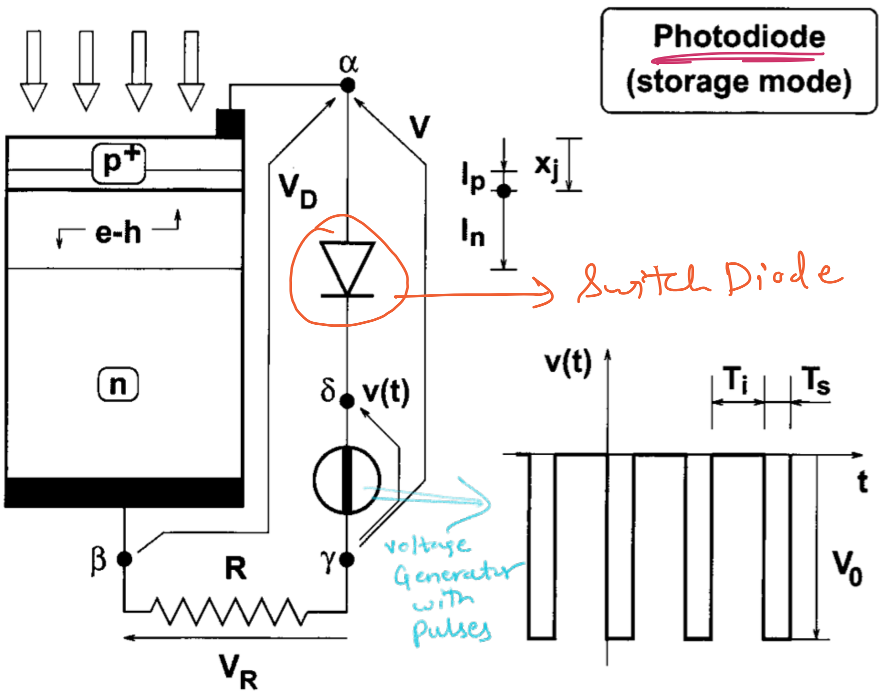
This is a Non Linear Differential eqn because 'C' depends on 'V_D' in a Nonlinear way

Note: Integration Time = when Switch is OPEN

Sampling Time = when Switch is Closed



→



- Switch diode in place of the switch is protected from a Illumination.

Another property of a diode that is used as a switch is that its cross sectional area is less than the Area of the Photo Diode

- Because in the circuit there is always a rearranging of the charge we don't want the charge of switch Diode to collude with the Charge of the Photo Diode.

- Voltage Generator (VG) is in series to the (SD) Switch Diode, (VG) applies pulses.

Assume VG is in series to the switch Diode is suddenly brought to a very −ve value.

('γ' is a reference voltage ∴ 'δ' goes to a very −ve value)

- The n-side of SD goes to −ve and it is in Direct bias (Forward Bias) Then 'α' follows i.e 'α' will become $V_\delta + V_{THSD}$

Threshold of switch diode

∴ 'α' will also become $-ve$ then the Photo Diode is biased in the Reverse condition

In this case the loop is closed and current flows through the circuit.

- Then after some time VG brings $V(t) = 0$ then 'δ' goes to voltage $= 0$ and 'γ' is $-ve$ then SD has no time to rearrange.

So essentially the SD becomes with

n-side voltage $= 0 = V_\delta$

p-side voltage $= V_\alpha = -ve$

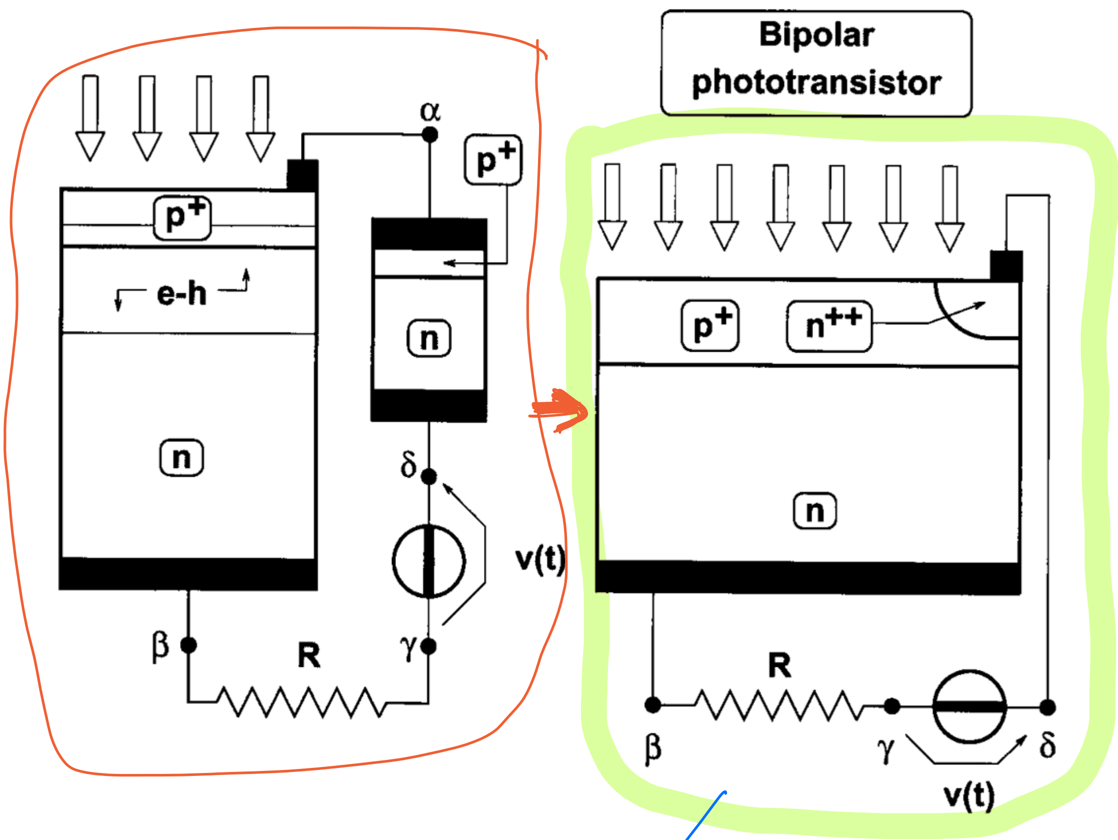
∴ $V_A = 0 \Rightarrow$ Switch OPEN $\Rightarrow$ Integrating Time

$V_A = -ve \Rightarrow$ Switch Closed $\Rightarrow$ Sampling Time

Q) Since 'P' side of SD & 'P' side of Photo Diode are shorted in the figure, why not merge them?

Ans

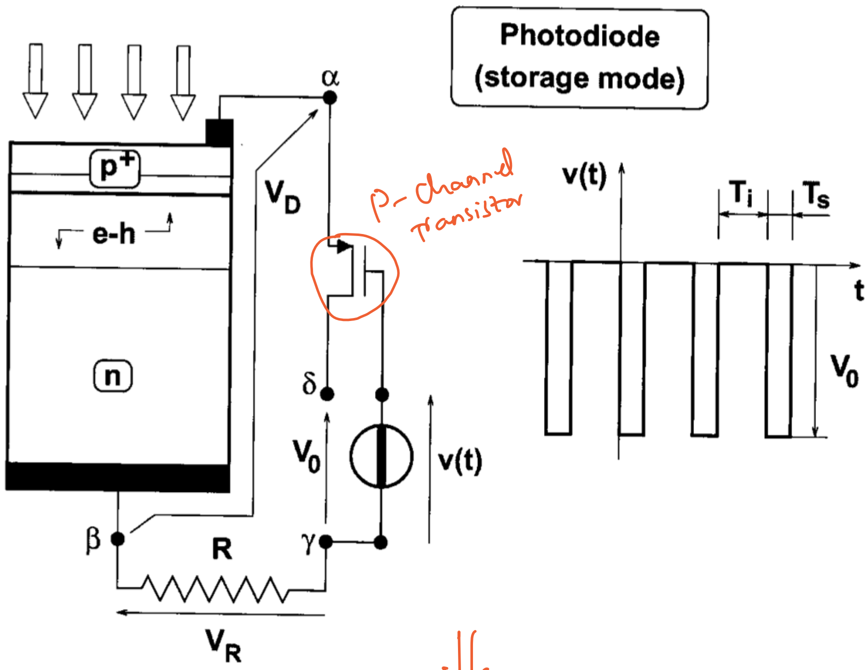
Yes, we will merge them and form what is known as Photo Transistor



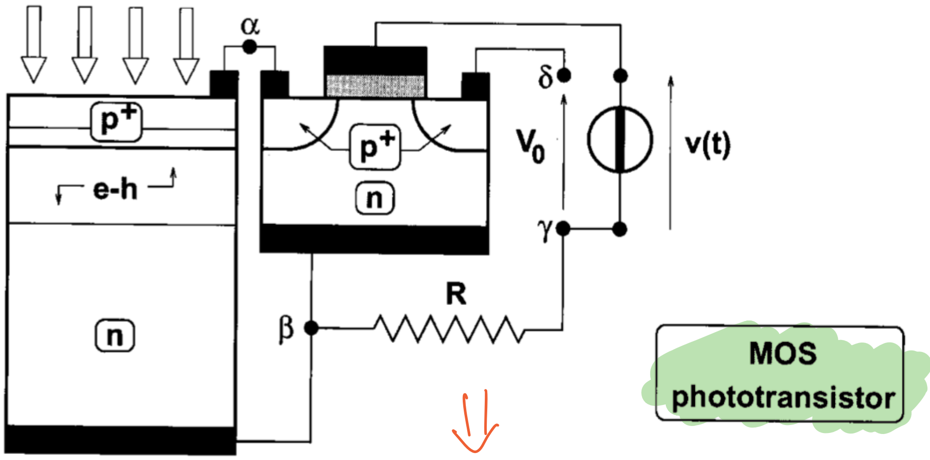
T. 34.9: Fototransistore bipolare.

→ This is an NPN which is a Bipolar Transistor

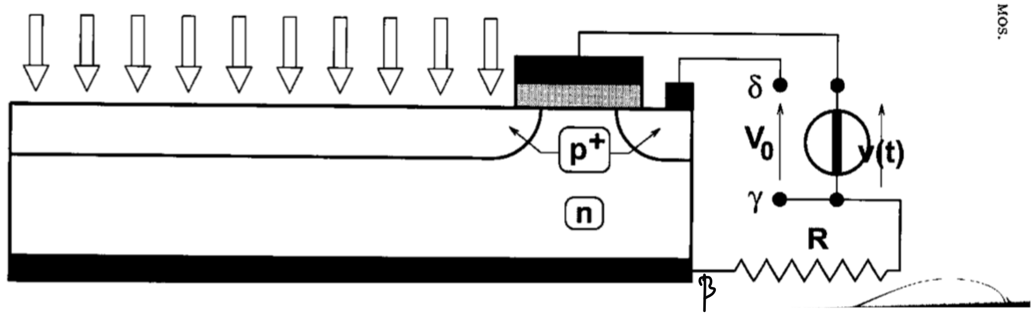
MOS Photo Transistor



T. 34.8: Controllo del fotodiodo per mezzo di un transistore MOS.

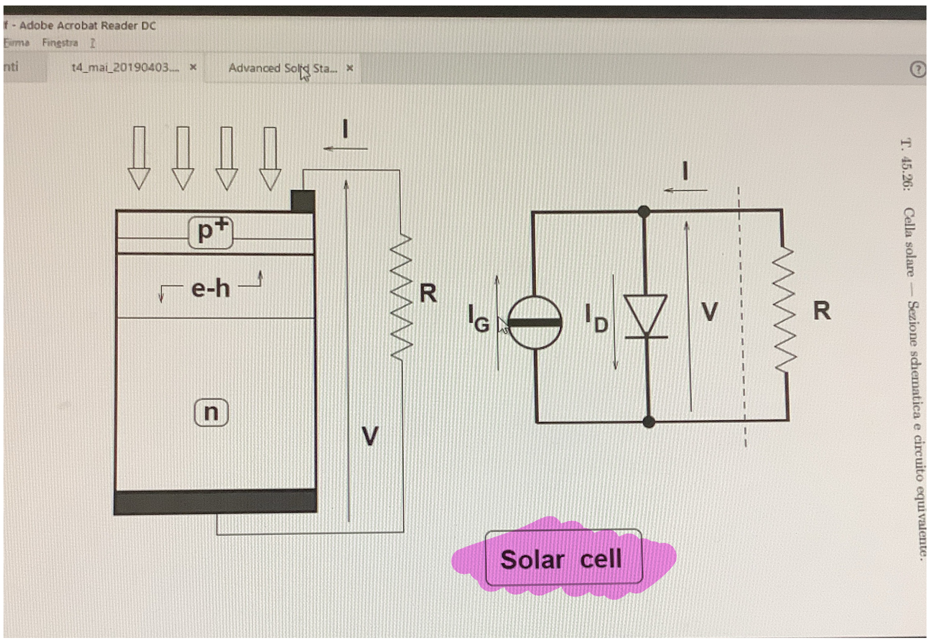


T. 34.10: Fototransistore MOS.



(The reasoning for MOS photo transistor is similar to all the above reasonings we did, so I have not explicitly written the explination)

→ Before continuing with the Optical sensors we will examine a possible application still in the optical field in broder sense of PN Junction so we consider the Solar cell.



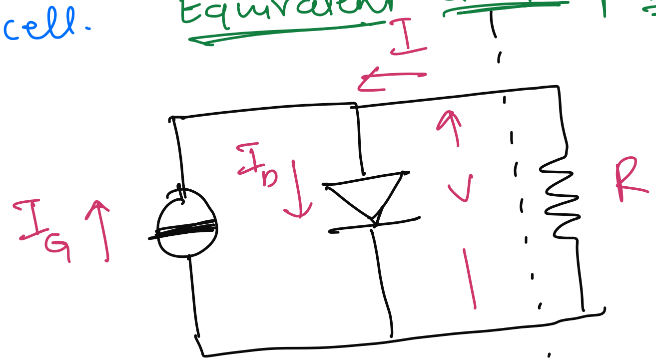
→ However, The application of Solar cell is not detecting of light at some position but the aim is The Absorption of The Energy.

- Solar cell has The same structure of The PhotoDiode it's a PN Junction very shallow, so we have minimization Electron, Hole recombination in The Neutral of the P-type. Generation (or) ?

The only difference is we don't have a bias we simply attach a Load Resistor.

- When light impinges on the solar cell we know it is a photodiode, the action of external light is essentially like placing a current generator in parallel to the cell.

Equivalent Circuit of Solar Cell



Here I, V have orientation we have assumed

Q) what is the difference Here (solar cell) from photodiode?

Ans We don't apply RB in case of solar cell

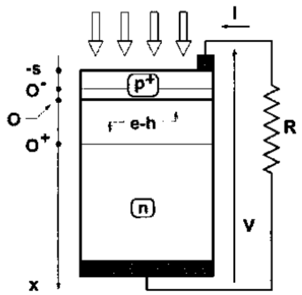
∴ We don't know apriori whether the voltage may appear at the contacts of the solar cell is either +ve (or) -ve.

We will calculate it, but we don't know!

Then we cannot represent PN Just as a capacitor
(PN Junction acts as a capacitor in FB)

→ $\boxed{\text{Power} = VI}$

Solar Cell — I



Neutral Region of p-type

$-s \le x \le x(O^-) \doteq 0$:

$U \simeq (n - n_{p0})/\tau_n$ Recombination Rate

$-s \le x < \infty$:

Monochromatic rad.

$$G = G_0 \exp[-k(x+s)]$$
$$G_s \doteq G_0 \exp(-ks)$$

$G = G_s e^{-kx}$

continuity of Minority carrier in the P-region

$$\frac{dJ_n}{dx} = qU - qG\,, \qquad J_n \simeq qD_n \frac{dn}{dx} = qD_n \frac{d(n-n_{p0})}{dx}$$

$$\frac{d^2(n-n_{p0})}{dx^2} = \frac{n-n_{p0}}{L_n^2} - \frac{G_s}{D_n}\exp(-kx)\,. \qquad \text{Hp: } \frac{1}{k} \neq L_n \,:$$

$$n - n_{p0} = A_n \exp(x/L_n) + B_n \exp(-x/L_n) + C_n \exp(-kx)$$

$$\frac{d^2(n-n_{p0})}{dx^2} - k^2 C_n \exp(-kx) = \frac{n - n_{p0} - C_n \exp(-kx)}{L_n^2}$$

$$-\frac{G_s}{D_n} - k^2 C_n = -\frac{C_n}{L_n^2} \quad \Rightarrow \quad \left(\frac{1}{L_n^2} - k^2\right) C_n = \frac{G_s}{D_n}$$

$$C_n = \tau_n G_s/(1 - k^2 L_n^2)$$

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$$\frac{dJ_n}{dx} = qU - qG$$ (1)

Continuity eq^n for minority carriers in p-region

We assume J_n is mostly Diffusive current.

At this point we have Diff. eq^n

$$J_n \simeq qD_n \frac{dn}{dx} = qD_n \frac{d(n-n_{p0})}{dx}$$ (2)

(2) in (1)

$$\Rightarrow \boxed{\frac{d^2(n-n_{p0})}{dx^2} = \frac{n-n_{p0}}{L_n^2} - \frac{G_s}{D_n}\exp(-kx)}$$

Here we have a Non Homogenous equation.

Hp: $1/k \neq L_n$

Solution of the differential eq^n

$$n - n_{p0} = A_n \exp(x/L_n) + B_n \exp(-x/L_n) + C_n \exp(-kx)$$ (a)

$$\frac{d^2(n-n_{p0})}{dx^2} - k^2 C_n \exp(-kx) = \frac{n - n_{p0} - C_n \exp(-kx)}{L_n^2}$$ (b)

$$-\frac{G_s}{D_n} - k^2 C_n = -\frac{C_n}{L_n^2} \Rightarrow \left(\frac{1}{L_n^2} - k^2\right) C_n = \frac{G_s}{D_n}$$

$$C_n = \tau_n G_s / (1 - k^2 L_n^2)$$

A_n, B_n are found by applying Boundary condition

C_n constant is for the extra term which makes the eq^n non-Homogeneous. Cannot be found from Boundary conditions

Substituting (a) in diff. eq^n we get (b)

$$\frac{d^2(n-n_{p0})}{dx^2} = \frac{d^2}{dx^2}\left\{ A_n \exp(x/L_n) + B_n \exp(-x/L_n) + C_n \exp(-Kx) \right]$$

$$\Rightarrow \frac{A_n \exp(x/L_n)}{L_n^2} + \frac{B_n \exp(-x/L_n)}{L_n^2} + K^2 C_n \exp(-Kx)$$

$$\Rightarrow \frac{d^2(n-n_{p0})}{dx^2} - K^2 C_n \exp(-Kx) = \frac{A_n \exp(x/L_n) + B_n \exp(-x/L_n)}{L_n^2}$$

$$\Rightarrow \frac{(n-n_{p0}) - C_n \exp(-Kx)}{L_n^2}$$

$$\Rightarrow \frac{d^2(n-n_{p_0})}{dx^2} - k^2 C_n(\exp(-kx)) = \frac{(n-n_{p_0}) - C_n \exp(-kx)}{L_n^2}$$

From This eq^n we will find 'C_n' & other Quantities.

$$C_n = \tau_n G_s / (1 - k^2 L_n^2)$$

→ Since we found C_n we will find A_n B_n using Boundary condition

Boundary conditions

$x = x(0^-) = 0$: $n = n_{p_0} \exp[qV/k_B T_L]$

0^- : is The edge of the SCR on the P-side
(Space charge Region)

So the value of the e^- concentration at that point is given by Shockley Boundary conditions at 0^-

i.e $n = n_{p_0} \exp(qV/k_B T_L)$

Solar Cell — II

- $x = x(O^-) = 0 : \qquad n = n_{p0}\,\exp[qV/(k_B T_L)] \quad \Rightarrow$

$$n_{p0}F = A_n + B_n + C_n\,, \qquad \boxed{F = \exp[qV/(k_B T_L)] - 1}$$

$$x = -s : \qquad n = n_{p0} \quad \Rightarrow$$

- $A_n \exp(-s/L_n) + B_n \exp(s/L_n) + C_n \exp(ks) = 0$

$$s \ll L_n\,, \frac{1}{k} : \qquad A_n(1 - s/L_n) + B_n(1 + s/L_n) + C_n(1 + ks) = 0$$

$$\frac{s}{L_n}(A_n - B_n) = A_n + B_n + C_n(1 + ks) = n_{p0}F - C_n + C_n(1 + ks)$$

$$\Rightarrow \qquad \frac{A_n - B_n}{L_n} - kC_n = \frac{n_{p0}F}{s}$$

$$\frac{J_n}{qD_n} = \frac{A_n \exp(x/L_n) - B_n \exp(-x/L_n)}{L_n} - kC_n \exp(-kx)$$

$$\boxed{J_n(O^-) = qD_n\left[\frac{A_n - B_n}{L_n} - kC_n\right] = q\frac{D_n n_{p0}}{s}F}$$

$$\boxed{\frac{\mathrm{d}J_n}{\mathrm{d}x} = qU - qG \quad \Rightarrow \quad J_n(O^+) - J_n(O^-) = J_U - J_G}$$

$$\boxed{J_G \doteq \int_{O^-}^{O^+} qG\,\mathrm{d}x = q\frac{G_s}{k}(1 - \exp[-kx(O^+)]) \simeq q\frac{G_s}{k}}$$

we have at $x = x(0^-) = 0$

$n - n_{p_0} = A_n + B_n + C_n$

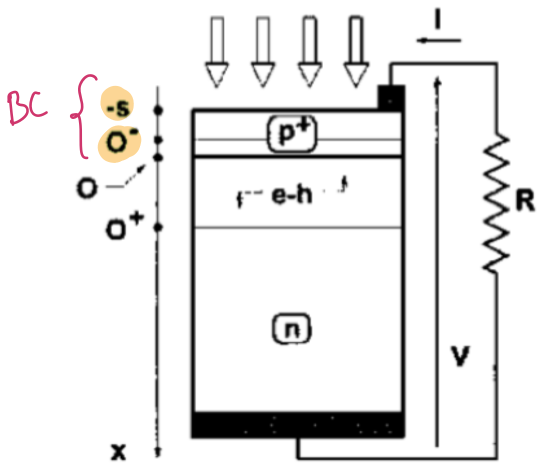
& substitute $n = n_{p_0} \exp[qV/k_BT_L]$

$$\Rightarrow \quad n_{p_0} \exp\left[qV/k_BT_L\right] - n_{p_0} = A_n + B_n + C_n$$

$$\rightarrow \quad n_{p_0} F = A_n + B_n + C_n \quad \text{--- (1)}$$

where $F = \exp\left[qV/k_B T_L \right] - 1$

We have to solve the Minority carrier eq^n in the Neutral region of the p-type from $-S$ to 0 (This is from the figure)



- Since we had imposed BC at $x = x(0^-) = 0$
  Now we must impose BC at $x = -S$
  But here we have a difference w.r.to the standard analysis of a PN Junction

- If you remember in the Analysis of PN Junction apart from '0' the other Boundary is taken at ∞.

- But in this case the Junction is very thin $\therefore$ We cannot take $-S$ as $-\infty$ since $-S$ is very close to the origin.
  $\therefore$ In solar cell we don't have any asymptotic

condition

$\therefore$ @ $\boxed{x = -s \qquad n = n_{p0}}$ — (2)

Here we took the condition of equillibrium but at a finite distance from the origin

→ Actually we can exploit the fact that Junction is very shallow by simplifing the exponentials.

's' is much smaller than the Diffusion length.

and 's' is much smaller than $1/k$

$$\boxed{s << L_n , \frac{1}{k}}$$

$\therefore$'s' is small we can linearize the exponentials

- $A_n \exp(-s/L_n) + B_n \exp(s/L_n) + C_n \exp(ks) = 0$

$s \ll L_n , \frac{1}{k}$: $\quad A_n(1 - s/L_n) + B_n(1 + s/L_n) + C_n(1 + ks) = 0$

$$\frac{s}{L_n}(A_n - B_n) = A_n + B_n + C_n(1 + ks) = n_{p0}F - C_n + C_n(1 + ks)$$

$$\Rightarrow \qquad \frac{A_n - B_n}{L_n} - kC_n = \frac{n_{p0}F}{s}$$

Linearized eq^n

These calculations are elementary to find A_n, B_n

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→ At some point you calculate J_n at 0^- because we need it in the standard theory of the PN Junction.

$$J_n = q D_n \frac{dn}{dx}$$

we know

$$\Rightarrow \frac{J_n}{qD_n} = \frac{dn}{dx} = \frac{A_n \exp(x/L_n) - B_n \exp(-x/L_n)}{L_n} - kC_n \exp(-kx)$$

$$\Rightarrow J_n(0^-) = qD_n\left[\frac{A_n - B_n}{L_n} - kC_n\right] = q\frac{D_n n_{p0}}{s}F$$

remember at 0^-, $x=0$

$$\frac{J_n}{qD_n} = \frac{A_n \exp(x/L_n) - B_n \exp(-x/L_n)}{L_n} - kC_n \exp(-kx)$$

$$J_n(O^-) = qD_n\left[\frac{A_n - B_n}{L_n} - kC_n\right] = q\frac{D_n n_{p0}}{s}F$$

$$J_n(0^-) = q\frac{D_n n_{p0}}{s}F$$

→ Now by taking the continuity eq^n of the Minority carriers in the Neutral Region 'p'

Continuity eq^n

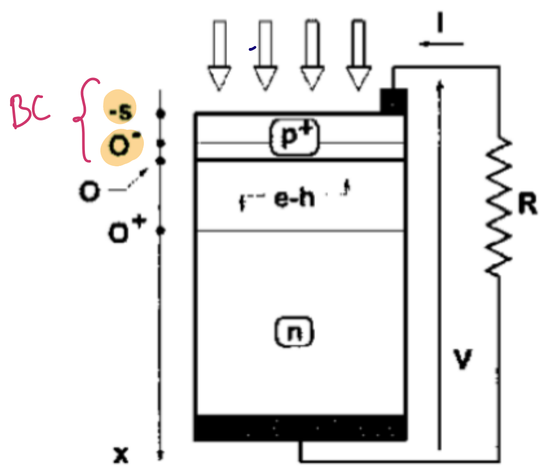
Integrating over SCR results in this eq^n

$$\frac{dJ_n}{dx} = qU - qG \Rightarrow J_n(O^+) - J_n(O^-) = J_U - J_G$$

$$J_G \doteq \int_{O^-}^{O^+} qG\,dx = q\frac{G_s}{k}(1 - \exp[-kx(O^+)]) \simeq q\frac{G_s}{k}$$

$\because$ $x(O^+)$ is large $\exp(-k\,x(O^+))$ can be negligible

→ Now we go to the other side of the Junction i.e n-side $x > x(O^+)$



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Solar Cell — III

$$x > x(O^+): \quad \frac{\mathrm{d}J_p}{\mathrm{d}x} = qG - qU \simeq -qU = -q\frac{p-p_{n0}}{\tau_p}$$

n-type

$$\frac{\mathrm{d}^2(p-p_{n0})}{\mathrm{d}x^2} = \frac{p-p_{n0}}{L_p^2}$$

$$p - p_{n0} = A_p \exp(-x/L_p) + B_p \exp(x/L_p)$$

$$x \to \infty: \qquad p = p_{n0} \quad \Rightarrow \quad B_p = 0$$

$$x = x(O^+) = 0: \qquad p = p_{n0}\,\exp[qV/(k_B T_L)] \quad \Rightarrow$$

$$p_{n0}F = A_p, \qquad \boxed{F = \exp[qV/(k_B T_L)] - 1}$$

$$J_p = -qD_p\frac{\mathrm{d}p}{\mathrm{d}x} = q\frac{D_p p_{n0}}{L_p}F\exp(-x/L_p)$$

$$\boxed{J_p(O^+) = q\frac{D_p p_{n0}}{L_p}F}$$

$$J = J_p(O^+) + J_n(O^+) = J_p(O^+) + J_n(O^-) + J_U - J_G$$

$$J_s' \doteq q\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{s}\right)$$

$$J = (J_s'F + J_U) - J_G = J_D - J_G$$

In n-type region we can actually simplify the analysis because the photon flux actually doesn't penetrate that deep. ($\therefore$ optical generation term is ignored)

$$\therefore \frac{dJ_p}{dx} = qG - qU \simeq -qU = -q\left(\frac{P - P_{no}}{\tau_p}\right)$$

in doing so we are left with standard eqn that is exactly the standard eqn of the n-region in a PN Junction

$$\frac{d^2(p - p_{n0})}{dx^2} = \frac{p - p_{n0}}{L_p^2}$$

$$p - p_{n0} = A_p \exp(-x/L_p) + B_p \exp(x/L_p)$$

Asymptotic condition at $x \to \infty$: $p = p_{n0} \Rightarrow B_p = 0$

- Assumption
$x = x(0^+) = 0$

$$x = x(O^+) = 0 : \qquad p = p_{n0} \exp[qV/(k_B T_L)] \quad \Rightarrow$$

$$p_{n0} F = A_p , \qquad \boxed{F = \exp[qV/(k_B T_L)] - 1}$$

$$J_p = -q D_p \frac{\mathrm{d}p}{\mathrm{d}x} = q \frac{D_p p_{n0}}{L_p} F \exp(-x/L_p)$$

$$\boxed{J_p(O^+) = q \frac{D_p p_{n0}}{L_p} F}$$

- Total Current Density

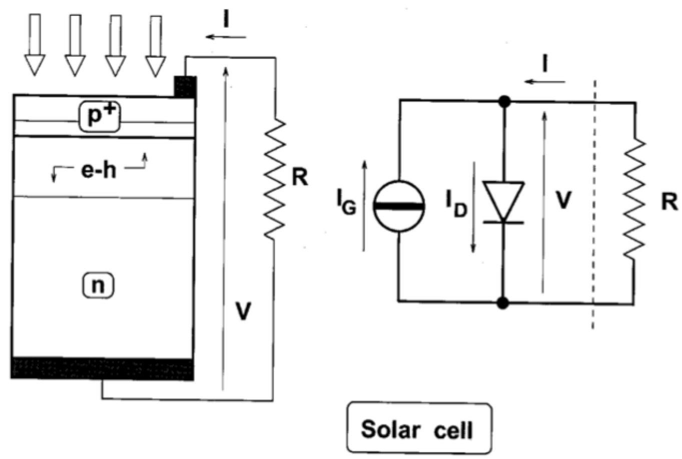
$$J = J_p(O^+) + J_n(O^+) = J_p(O^+) + J_n(O^-) + J_U - J_G$$

$$J_s' \doteq q \left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{s} \right)$$

$$\boxed{J = (J_s' F + J_U) - J_G = J_D - J_G}$$

Global current density eqn

This analysis we can say that it is correct to describe the SOLAR CELL as a parallel of a Diode & a Current Generator.



But we still don't know the polarity of the Voltage 'V'

→ Q) What if the impinging Radiation is Not monochromatic ?

Ans Since Generation & Absorption coefficients depend on the Frequency.

$$G_0 = G(\nu) \qquad K = K(\nu)$$

Solar Cell — IV

- If the radiation has a spectrum, it is $\boxed{G_0 = G_0(\nu)}$, $\boxed{k = k(\nu)}$

$$G_s \exp(-kx) \Leftarrow \int_0^\infty g_s(\nu) \exp[-k(\nu)x]\, d\nu$$

$$C_n \exp(-kx) \Leftarrow \int_0^\infty c_n(\nu) \exp[-k(\nu)x]\, d\nu$$

$$C_n = \tau_n G_s/(1-k^2L_n^2) \Leftarrow c_n = \tau_n g_s/(1-k^2L_n^2)$$

$$\frac{A_n - B_n}{L_n} - kC_n \Leftarrow \frac{A_n - B_n}{L_n} - \int_0^\infty kc_n\, d\nu$$

- We take $G_s \exp(-kx)$ & $C_n \exp(-kx)$ as the Integration of elementary generation $g_s(\nu)$. s.t $g_s(\nu)\, d\nu$

The no. of generations per unit Volume & Time at the frequency ν

- Current density of Generation $J_G = q \int_0^\infty (g_s/k)\, d\nu$

Total current Density $J = (J_s' F + J_U) - J_G = J_D - J_G$

- $I = A_e J = I_D - I_G$, $\boxed{I_D = I_s' (\exp[qV/(k_BT_L)] - 1) + I_U}$

Cross sectional Area of Device

$$I_{sc} \doteq I(V=0) = I_U - I_G \simeq -I_G < 0$$

$$V_{oc} \doteq V \ni I = 0 \Rightarrow I_D(V_{oc}) = I_G$$

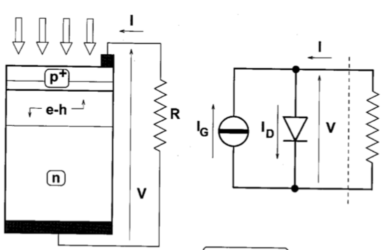
$$V_{oc} \simeq \frac{k_BT_L}{q} \log\left(\frac{I_G}{I_s'} + 1\right) > 0$$

$$0 \le V \le V_{oc} \iff -I_G \le I \le 0$$

$$\forall R: \quad V = -RI \Rightarrow V > 0,\ I < 0,\ P = VI < 0$$

- Here in the eq^n at the moment we still yet don't know the sign & the value of the voltage 'V'

- But we can define some interesting characteristic points, we can see that since current due to the Generator is a constant. The I-V curve of the SOLAR CELL is simply equal to the standard characteristic of a Diode shifted by constant value (I_G)



Solar cell

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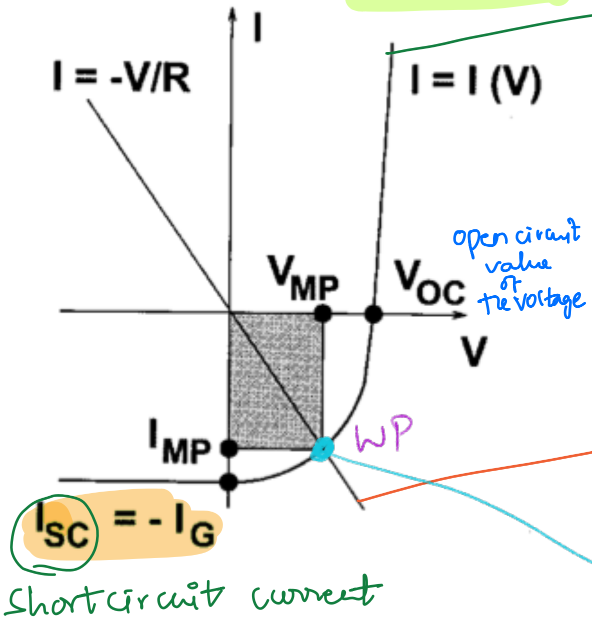
I-V Characteristic of a Diode



Here in the figure the I-V curve of solar cell is shifted below by the constant value I_G

$$I = \frac{-V}{R}$$

Load characteristic of Resistance

IV curve of Resistance

Interception of IV curve of Resistance & IV curve of Solar Cell is the Working Point

- And we can see immediately that the WP corresponds to a +ve value of the voltage and to a -ve value of the current.
- Since 'V' is +ve the Solar Cell works in FB condition. So it self polarizes due to the current of Generator I_G

  ∴ 'I' is -ve, so the actual current goes from solar cell to resistance

$\therefore$ Power = VI goes from SC to R

$$I = A_e J = I_D - I_G\,, \qquad \boxed{I_D = I_s'\left(\exp[qV/(k_BT_L)] - 1\right) + I_U}$$

cross sectional Area of Device

$$I_{sc} \doteq I(V=0) = I_U - I_G \simeq -I_G < 0$$

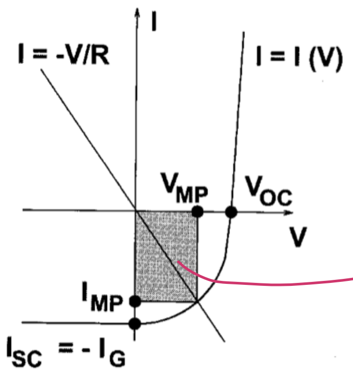
$$V_{oc} \doteq V \ni I = 0 \quad \Rightarrow \quad I_D(V_{oc}) = I_G$$

$$V_{oc} \simeq \frac{k_BT_L}{q}\log\left(\frac{I_G}{I_s'} + 1\right) > 0$$

$$0 \le V \le V_{oc} \quad \Longleftrightarrow \quad -I_G \le I \le 0$$

$$\forall R: \quad V = -RI \quad \Rightarrow \quad V > 0\,,\ I < 0\,,\ P = VI < 0$$

→



- This Area is the Power that Solar Cell gives to the Load.
- By changing 'R' we can optimize the power received by 'R' $\therefore$ we seek the position of Intercept that Maximizes the Area of the Rectangle.

Solar Cell — V

eq^n for the optimization of Power transfer to the load

$$\boxed{\frac{\mathrm{d}P}{\mathrm{d}V} = I + V\frac{\mathrm{d}I}{\mathrm{d}V}\,, \qquad I = I(V)}$$

- $P(V=0) = P(V=V_{\mathrm{oc}}) = 0$: $\quad \frac{\mathrm{d}P}{\mathrm{d}V} = 0 \quad \Rightarrow$

$$V = V_{\mathrm{mp}}\,, \quad 0 < V_{\mathrm{mp}} < V_{\mathrm{oc}}\,, \qquad V_{\mathrm{mp}} \approx 0.5\,\mathrm{V}$$

$$I_{\mathrm{mp}} = I(V_{\mathrm{mp}}) < 0\,, \quad -V_{\mathrm{mp}}I_{\mathrm{mp}} = \max|P|$$

$$R_{\mathrm{mp}} \doteq -\frac{V_{\mathrm{mp}}}{I_{\mathrm{mp}}}$$

Fill Factor $\quad r \doteq \frac{V_{\mathrm{mp}}I_{\mathrm{mp}}}{\int_0^{V_{\mathrm{oc}}} I\,\mathrm{d}V}\,, \qquad \eta = \frac{|V_{\mathrm{mp}}I_{\mathrm{mp}}|/A_c}{\mathrm{d}P_{\mathrm{in}}/\mathrm{d}S} \doteq \eta'\eta''$

efficiency $\Rightarrow$ $\dfrac{\text{Power per unit Area produced by the cell}}{\text{Power per unit Area entering the cell}}$

- Where $\eta'' \approx 0.5$ is due to reflexion, recombination, lateral resistance in the semiconductor, and series resistance of the contacts. The latter cover about 5% of A_e.

	η'		η	
T_L	20 °C	100 °C	20 °C	100 °C
Si	0.22	0.11	0.10	0.05
GaAs	0.28	0.14	0.15	0.08

→ $\eta' \Rightarrow$ It is due to the Interaction of Radiation Through the semiconductor

- for instance not all photons will be producing e^--hole pairs. for instance Photons with smaller frequency have energy less than the Band Gap can't produce $(e^- - o)$ pairs

- Some photons may interact with e^-s already in the CB so they increase the energy but don't contribute to their number.

- Some photons may also interact with nuclei.

- Also Increase in Temperature the efficiency decreases because it Redistributes the carriers.

	η'		η	
T_L	20 °C	100 °C	20 °C	100 °C
Si	0.22	0.11	0.10	0.05
GaAs	0.28	0.14	0.15	0.08

→

$$\boxed{\eta = \frac{|V_{mp} I_{mp}| / Ae}{dP_{in}/dS} = \eta' \eta''}$$

power that is coming from the Sun

→

Solar Cell — VI

$$\frac{dP^\star_{in}}{dS} = \frac{dE_{in}}{dt\,dS} = \left(\frac{r_S}{r_0}\right)^2 \frac{dE_{out}}{dt\,dS} = \left(\frac{r_S}{r_0}\right)^2 4\sigma T_S^4$$

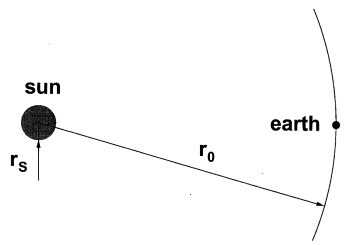
where $\sigma = 5.67\ 10^{-12}\ \mathrm{W\,cm^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant and $T_S = 5800$ K is the temperature at the surface of the sun. From $r_0/r_S \simeq 435$ it follows

$$\boxed{\frac{dP^\star_{in}}{dS} \simeq 135\ \mathrm{mW\,cm^{-2}}}$$

for the normally-impinging power per unit area outside the atmosphere. It is

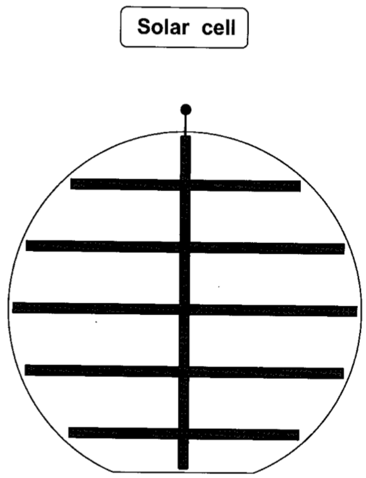
$$\boxed{\frac{dP_{in}}{dS} < \frac{dP^\star_{in}}{dS}}$$

due to the angle of incidence and to the absorption by the atmosphere.



- Here Sun is considered as a Black Body as in Thermodynamics

→ During disussion of n'' we had impact of Lateral Resistance of a Semiconductor



- Top view of Solar cell

The Black lines are upper contacts and it is of the shape show because Semiconductor has a series Resistance that opposes the motion of the carrier in reaching the contact

- The contacts cannot cover whole SC because then most of the Radiation is Reflected

4 The contacts can't be too thin because the Resistance increases. and they should branch as much as possible inorder to increase the chance of carrier reaching it.

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