Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Lecture - 11

13/10/20

→ Continuing from last class of the calculation of I-V characteristic of a PN Junction

- When we calculated the I-V characteristic of PN Junction we left behind two issues.
  (i) Analysis of the Boundary conditions for Minority carrier concentration.
  (ii) Reasoning for Minority carriers flow in the Quasi Neutral Zone is Diffusion Dominated

(i) Boundary conditions:

We remember that what we did was solving equation for Minority carriers in the Quasi Neutral Zone.

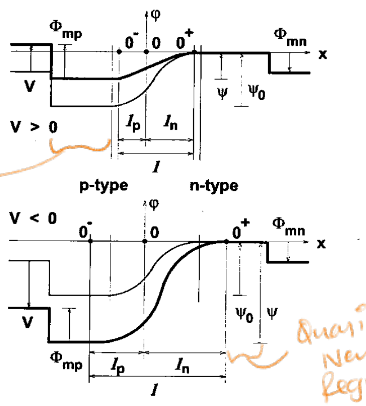
In the P-region we solved the equations b/w the contact & the position 0^-

& similarly for n-type region b/w 0^+ & right contact

& offcourse the sol^n applies both in the Direct bias & the Reverse bias

T. 28.6: Andamento del potenziale nella giunzione *p-n* fuori equilibrio.

p-n Junction in Nonequilibrium — I



Quasi Neutral Region

Quasi Neutral Region

$$\psi + \Phi_{mn} + V - \Phi_{mp} = 0\,, \qquad \psi + V - \underbrace{(\Phi_{mp} - \Phi_{mn})}_{\psi_0} = 0$$

$$\psi = \psi_0 - V\,, \qquad \varphi(\mathrm{O}^+) - \varphi(\mathrm{O}^-) = \psi_0 - V$$

$$l_p = l_p(V)\,, \qquad l_n = l_n(V)$$

→ The eqⁿs that we solved were the combination of continuity eqⁿ for e⁻s and of the transport eqⁿ

i.e.
$$\boxed{\frac{d^2}{dx^2}(n - n_{p_0}) = \frac{n - n_{p_0}}{L_n^2} \qquad L_n = \sqrt{\tau_n D_n}}$$

∵ it is a 2nd order ODE we need two BC one at 0 & another at ∞

* Now, we will solve for the Boundary conditions

$$\boxed{\begin{aligned} n(0) &= n_{p_0} \exp[qV/k_B T_L] \\ n(-\infty) &= n_{p_0} \end{aligned}}$$

Shockley Boundary condition

p-n Junction in Nonequilibrium — V

Steady state, one-dimensional case (Shockley theory)

$$\left\{\begin{array}{l} x < x(\mathrm{O}^-) \\ \text{(weak inj.)} \end{array}\right. \left\{\begin{array}{l} n - n_{p0} > 0 \;\Rightarrow\; \left\{\begin{array}{l} n - n_{p0} \ll p_{p0} \\ n \ll p_{p0} + n_{p0} \simeq p_{p0} \\ n_{p0} < n \ll p_{p0} \end{array}\right. \\ \\ n - n_{p0} < 0 \;\Rightarrow\; \left\{\begin{array}{l} n < n_{p0} \\ n_{p0} \ll p_{p0} \\ n < n_{p0} \ll p_{p0} \end{array}\right. \end{array}\right.$$

$$J_n = -q\mu_n n \frac{d\varphi}{dx} + qD_n \frac{dn}{dx} \simeq qD_n \frac{dn}{dx}$$

$$q\,\frac{n - n_{p0}}{\tau_n} = \frac{dJ_n}{dx} \simeq q\,\frac{d}{dx} D_n \frac{dn}{dx} = qD_n \frac{d^2 n}{dx^2}$$

(Poisson's equation ruled out)

$$\frac{d^2}{dx^2}(n - n_{p0}) = \frac{n - n_{p0}}{L_n^2}\,, \qquad L_n \doteq \sqrt{\tau_n D_n}$$

$$x(\mathrm{O}^-) \doteq 0 \quad\Longrightarrow\quad \left\{\begin{array}{l} n(0) = n_{p0} \exp\left[qV/(k_B T_L)\right] \\ n(-\infty) = n_{p0} \end{array}\right.$$

L_n : minority-carrier diffusion length

→ In order to derive the Boundary we consider the Space charge Region (SCR) in b/w 0^+ & 0^-

- In the SCR we have very large Electric field and very large concentration gradients of both e^{-s} & e^{+s}

So we may think that in the Drift Diffusion eq^n both the Drift & Diffusion terms are very large.

- One the other hand we are considering applied biases (V) which are not so large and thereby Current Density is not so large

→ We also remember that at Equilibrium Drift & Diffusion terms are balanced ∴ We have a zero current

→ When we are in Non-Equilibrium condition The Diffusion & Drift terms unbalance each other but not so much. ∴ The difference is a Non zero Current Density but this Current Density is small.

So the only purpose of determining a BC^s, we make an approximation & we make an observation

p-n Junction in Nonequilibrium — IV

Steady state, one-dimensional case (Shockley theory)

Drift Diffusion Current density

$$\text{S.c.r.:} \begin{cases} -q\mu_p p\mathcal{E} \simeq -qD_p \, dp/dx \gg |J_p| \\ -q\mu_n n\mathcal{E} \simeq +qD_n \, dn/dx \gg |J_n| \end{cases} \implies$$

$$\frac{p(O^+)}{p(O^-)} \simeq \exp\left[\frac{q(V-\psi_0)}{k_B T_L}\right] = \frac{n_i^2}{N_A N_D} \exp\left(\frac{qV}{k_B T_L}\right)$$

$$\frac{n(O^-)}{n(O^+)} \simeq \exp\left[\frac{q(V-\psi_0)}{k_B T_L}\right] = \frac{n_i^2}{N_A N_D} \exp\left(\frac{qV}{k_B T_L}\right)$$

We get this by substituting the expression for Builtin potential ψ_0

$$\psi_0 = \frac{k_B T_L}{q} \log\left(\frac{N_A N_D}{n_i^2}\right)$$

weak injection condition

$$\begin{cases} p\text{-type region:} \quad |p - p_{p0}| \ll p_{p0}\,, \quad p \simeq p_{p0}\,, \quad x < x(O^-) \\ n\text{-type region:} \quad |n - n_{n0}| \ll n_{n0}\,, \quad n \simeq n_{n0}\,, \quad x > x(O^+) \end{cases}$$

$$p(O^-) \simeq p_{p0} \simeq N_A \implies p(O^+) \simeq p_{n0} \exp\left(\frac{qV}{k_B T_L}\right)$$

$$n(O^+) \simeq n_{n0} \simeq N_D \implies n(O^-) \simeq n_{p0} \exp\left(\frac{qV}{k_B T_L}\right)$$

(Shockley's boundary conditions)

If the Drift & Diffusion terms are almost equal this means that they are like in Equillibrium and we know that at Equillibrium case the concentration of e^{-}s & e^{+}s are expressed using exponentials.

$$\psi + \Phi_{mn} + V - \Phi_{mp} = 0\,, \qquad \psi + V - \underbrace{(\Phi_{mp} - \Phi_{mn})}_{\psi_0} = 0$$

$$\psi = \psi_0 - V\,, \qquad \varphi(O^+) - \varphi(O^-) = \psi_0 - V$$

$$l_p = l_p(V)\,, \qquad l_n = l_n(V)$$

** The possibility of neglecting Drift term w.r.t to
(ii) Diffusion term in Neutral Region.

p-n Junction in Nonequilibrium — IX

To better discuss the consequences of the weak-injection condition in a quasi-neutral region, the case of a p-doped region is considered, so that $c^{\mathrm{eq}} = p_{p0}$ and

$$|p - p_{p0}| \ll p_{p0}\,, \qquad |n - n_{p0}| \ll p_{p0}\,.$$

The above may be recast as

$$|p - p_{p0}| \le \alpha\, p_{p0}\,, \qquad |n - n_{p0}| \le \alpha\, p_{p0}\,,$$

with $\alpha \ll 1$. Indicating with p_m, p_M the minimum and maximum values of p imposed by the above, one finds $p_M - p_{p0} = \alpha\, p_{p0}$, $p_{p0} - p_m = \alpha\, p_{p0}$, whence

$$p_M = (1 + \alpha)\; p_{p0}\,, \qquad p_m = (1 - \alpha)\; p_{p0}\,.$$

Similarly, for the minority-carrier concentration one finds $n_M - n_{p0} = \alpha\, p_{p0}$, $n_{p0} - n_m = \alpha\, p_{p0}$, whence

$$n_M = n_{p0} + \alpha\, p_{p0}\,, \qquad n_m = n_{p0} - \alpha\, p_{p0}\,.$$

The maximum absolute variation of p turns out to be:

$$p_M - p_m = 2\alpha\, p_{p0}\,.$$

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

p-n Junction in Nonequilibrium — X

Instead, the maximum variation of n must be treated with some care. In fact, using the non-degenerate case one finds

$$n_M = \frac{n_i^2}{p_{p0}} + \alpha\, p_{p0} = p_{p0} \left(\frac{n_i^2}{p_{p0}^2} + \alpha \right) ,$$

$$n_m = \frac{n_i^2}{p_{p0}} - \alpha\, p_{p0} = p_{p0} \left(\frac{n_i^2}{p_{p0}^2} - \alpha \right) .$$

Even for a relatively low dopant concentration, say, $N_A \simeq p_{p0} = 10^{16}$ cm^{-3}, at room temperature one has $n_i^2/p_{p0}^2 \simeq 10^{-12}$, which is much smaller than the reasonable values of α. It follows $n_M \simeq \alpha\, p_{p0}$, $n_m \simeq 0$, where the limit of n_m must be chosen as such because n is positive definite. In conclusion, the maximum relative variations of p and n with respect to the equilibrium values are given by

$$\frac{p_M - p_m}{p_{p0}} = 2\alpha\, , \qquad \frac{n_M - n_m}{n_{p0}} \simeq \alpha\, \frac{p_{p0}}{n_{p0}} = \alpha\, \frac{p_{p0}^2}{n_i^2} \gg 2\alpha\, .$$

By way of example one may let $\alpha = 10^{-3}$, still with $N_A = 10^{16}$ cm^{-3}. While the maximum relative variation of p is 2×10^{-3}, that of n is 10^9. even if the variation of 'n' fullfills

the constraint of the order 10^9 we are still with in the limits of weak injection condition.

But, the variation of 'n' can be enormous ⟹ The derivative of the 'n' can be enormous. This is why diffusive Term can be large. This justifies the approximation of using only Diffusive Term for the minority carriers in the

Quasi Neutral Region & Now we can complete the Analysis.

p-n Junction in Nonequilibrium — VIII

Total Current density

Steady state, one-dimensional case (Shockley theory)

$$J = J_p(0^+) + J_n(0^-) + J_U = q\underbrace{\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)}_{J_s>0} F + J_U$$

$$J_s = q n_i^2 \left(\frac{\sqrt{D_p/\tau_p}}{N_D} + \frac{\sqrt{D_n/\tau_n}}{N_A}\right), \qquad J_U \doteq \int_{0^-}^{0^+} qU\, \mathrm{d}x$$

Recombination Term

$$J = J_s\left(\exp\left[qV/(k_B T_L)\right] - 1\right) + J_U, \qquad I = AJ$$

$$V = 0 \quad \Rightarrow \quad J = 0$$

$$\frac{qV}{k_B T_L} \gg 1 \quad \Rightarrow \quad J \simeq J_s \exp\left[qV/(k_B T_L)\right]$$

$$\frac{qV}{k_B T_L} \ll -1 \quad \Rightarrow$$

$$J \simeq -J_s + J_U \simeq -J_s + \int_{0^-}^{0^+} q\frac{-n_i}{\tau_g}\,\mathrm{d}x = -J_s - q\frac{n_i}{\tau_g}\, l(V)$$

$$J_U = \int_{0^-}^{0^+} qU\, dx$$

This is the Recombination Term is dependent on the concentration of carriers 'q' in the Space Charge Region. ∴ J_U is not known for the moment.

In any case, we can consider the Total current density eq^n and substitute terms for $J_p(0^+)$, $J_n(0^-)$

$$J = J_p(0^+) + J_n(0^-) + J_v$$

$$= \left(\frac{q D_p p_{n0}}{L_p} + \frac{q D_n n_{p0}}{L_n} \right) F + J_v$$

$$\Rightarrow \underbrace{q \left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n} \right)}_{J_s} F + J_v$$

F: It is a dimensionless quantity $\exp\left(qV/k_B T_L\right) - 1$

$$\boxed{J = J_s\left(\exp\left(\frac{qV}{k_B T}\right) - 1\right) + J_v}$$

& more over another expression for J_s

$$J_s = q n_i^2 \left(\frac{\sqrt{D_p/\tau_p}}{N_D} + \frac{\sqrt{D_n/\tau_n}}{N_A} \right)$$

This expression of J_s is more convenient because N_D, N_A are not dependent on Temperature because we assume these are in complete ionization condition. Diffusion coefficient D_p & D_n are dependent on

Temperature because they too are dependent on Mobility which slightly decrease with ↑ in Temperature

But τ_p & τ_n are also dependent on Temperature.

$$J_s = q n_i^2 \underbrace{\left(\frac{\sqrt{D_p/\tau_p}}{N_D} + \frac{\sqrt{D_n/\tau_n}}{N_A} \right)}_{\text{This term depends very less on Temperature}}$$

But This Term depends exponentially on Temperature

* Essentially J_s depends on Temperature majorly due to the n_i^2 term.

Remember, That n_i^2 is important term in determining the losses of an IC Because in an IC we have always a Junction that is Reverse biased & Reverse bias current is a Loss and we will calculate that the RB current has J_s term

$$\boxed{J = J_s \left(\exp\left(qV/k_BT \right) - 1 \right) + J_U}$$

$I = AJ$

In order to reach to the above expression we made so many approximations & off course in order to check it the first thing to do is to check whether it fullfills the equillibrium condition.

$$\boxed{J = J_s\left(\exp\left[qV/(k_BT_L)\right] - 1\right) + J_U\,,} \qquad I = AJ$$

V=0 (i) put $\Rightarrow V = 0 \quad \Rightarrow \quad J = 0$

V +ve DB (ii) $\dfrac{qV}{k_BT_L} \gg 1 \quad \Rightarrow \quad J \simeq J_s \exp\left[qV/(k_BT_L)\right]$

V -ve RB (iii) $\dfrac{qV}{k_BT_L} \ll -1 \quad \Rightarrow$

$$J \simeq -J_s + J_U \simeq -J_s + \int_{0-}^{0+} q\frac{-n_i}{\tau_g}\,dx = -J_s - q\frac{n_i}{\tau_g}\,l(V)$$

Here SCR will be completely depleted of e^{+s} & e^{-s}

and in full depletion 'U' becomes a constant.

Q) what happens when 'V' is near the origin i.e an intermediate value?

Near the origin there is an approximate method to calculate it but we are not doing it.

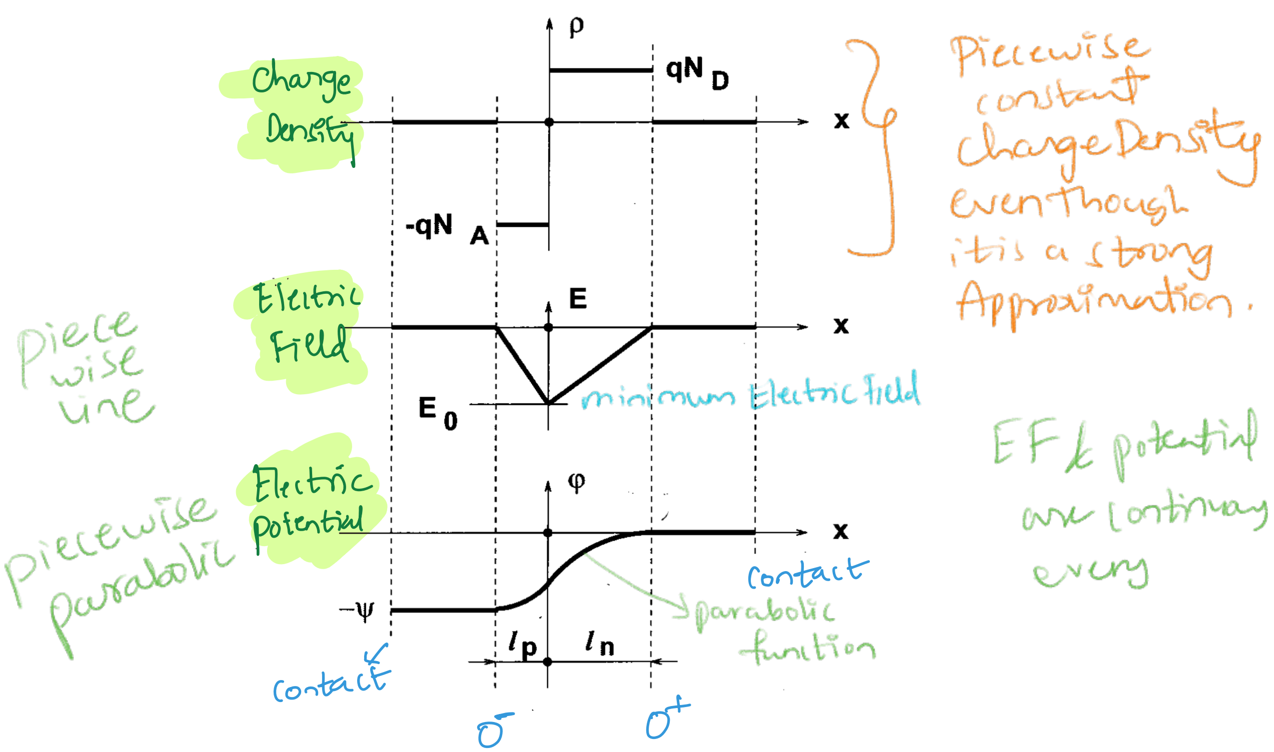
→ If we a region that is fully depleted like Space Charge region, the only contribution to Charge Density is due to the Dopants. But the concentrations of dopants N_A, N_D are given. So we have a 1D problem in which distribution of charges is fully given

So, we can solve analytically the Poisson's Eqn

- So what we do now is considering the RB condition in the approximation of the Depletion of the SCR
→ This approximation can be seen below.

T. 29.1: Approssimazione ASCE nella giunzione p-n.

Abrupt space Charge Edge



→ We will now try and solve the Poisson's Eq^n for the given piecewise constant Charge Density.

• The Relation b/w Electric field & Charge Density according to Maxwell's equation Divergence of Displacement vector is equal to the charge Density.

$$\text{Displacement Vector} = \underbrace{\varepsilon_{sc}}_{\text{permittivity}} \, \mathcal{E}$$

$\mathcal{E}$ → Electric field

$$\boxed{\frac{d(\varepsilon_{sc}\mathcal{E})}{dx} = \rho}$$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx}}$$

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Depletion-Region Width — I

$$\text{Hp:}\begin{cases} V \le 0 \\ \left.\begin{array}{c}\text{Full depletion}\\ \text{ASCE}^\star\text{ approx.}\end{array}\right\} \Rightarrow \varrho = \begin{cases} qN_D & 0 < x < l_n \\ -qN_A & -l_p < x < 0 \end{cases} \\ \text{Global charge neutrality: } \int_{-\infty}^{+\infty} \varrho \, \mathrm{d}x = \int_{-l_p}^{+l_n} \varrho \, \mathrm{d}x = 0 \end{cases}$$

$$\operatorname{div} \boldsymbol{D} = \varrho \quad \Rightarrow \quad \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x} = \frac{\varrho}{\varepsilon_{sc}}$$

electric field

The electric field is continuous because there are no charge layers, hence $\mathcal{E}(l_n^-) = \mathcal{E}(l_n^+)$, $\mathcal{E}(-l_p^+) = \mathcal{E}(-l_p^-)$, $\mathcal{E}_0^+ = \mathcal{E}_0^-$:

$$\frac{\mathcal{E}(l_n) - \mathcal{E}_0}{l_n} = \frac{qN_D}{\varepsilon_{sc}}, \qquad \frac{\mathcal{E}_0 - \mathcal{E}(-l_p)}{l_p} = \frac{-qN_A}{\varepsilon_{sc}}$$

But $\mathcal{E}(l_n) = \mathcal{E}(-l_p)$ due to charge neutrality, and $\mathcal{E}(l_n) = \mathcal{E}(-l_p) = 0$ due to continuity with the neutral regions:

$$\mathcal{E}_0 = -\frac{qN_D}{\varepsilon_{sc}} l_n = -\frac{qN_A}{\varepsilon_{sc}} l_p < 0, \qquad N_D \, l_n = N_A \, l_p .$$

* "Abrupt Space-Charge Edge".

** physical meaning of the equality represents the Charge Neutrality of the Device.

** This relation applies practically to the Distribution of the Dopants that are Piecewise constant.

- But it can also be demonstrated with a bit more complicated expression when the Dopant is Not constant But the idea that Heavily Doped side has less width than the other side

Depletion-Region Width — II

The electric potential is continuous because there are no double layers, and has continuous derivative.

$$\boldsymbol{\mathcal{E}} = -\,\mathrm{grad}\,\varphi \quad \Rightarrow \quad \frac{\mathrm{d}\varphi}{\mathrm{d}x} = -\mathcal{E}$$

Drop in Electric potential in Space Charge Region

$$\psi = \varphi(l_n) - \varphi(-l_p) = \int_{-l_p}^{+l_n} -\mathcal{E}\,\mathrm{d}x = -\frac{1}{2}\,\mathcal{E}_0\,(l_n + l_p)$$

where

$$l_n + l_p = \begin{cases} l_n + (N_D/N_A)\,l_n = N_D l_n\,(1/N_D + 1/N_A) \\ (N_A/N_D)\,l_p + l_p = N_A l_p\,(1/N_D + 1/N_A) \end{cases}$$

In conclusion:

$$\psi = \frac{q}{2\varepsilon_{sc}}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)(N_D l_n)^2 = \frac{q}{2\varepsilon_{sc}}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)(N_A l_p)^2$$

$$l_n = \frac{1}{N_D}\left(\frac{2\varepsilon_{sc}\psi/q}{1/N_D + 1/N_A}\right)^{1/2}, \qquad l_p = \frac{1}{N_A}\left(\frac{2\varepsilon_{sc}\psi/q}{1/N_D + 1/N_A}\right)^{1/2}$$

$$l = l_n + l_p = \left[\frac{2\varepsilon_{sc}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)\psi\right]^{1/2}, \qquad \psi = \psi_0 - V$$

* If $V \ll -\psi_0$ it turns out $l \propto |V|^{1/2}$.

This applies only to Negative 'V'

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నాల

Depletion Capacitance

The charge per unit area in the p and n region is, respectively:

$$Q_p \doteq \varrho_p l_p = -qN_A\, l_p\,, \qquad Q_n \doteq \varrho_n l_n = qN_D\, l_n$$

The capacitance per unit area is defined as

$$\boxed{C \doteq \frac{\mathrm{d}Q_p}{\mathrm{d}V} = \frac{\mathrm{d}Q_n}{-\mathrm{d}V}}$$

$$C = -qN_A\frac{\mathrm{d}l_p}{\mathrm{d}V} = q\,\frac{\mathrm{d}(N_A l_p)}{\mathrm{d}\psi}\,, \qquad C = qN_D\frac{\mathrm{d}l_n}{-\mathrm{d}V} = q\,\frac{\mathrm{d}(N_D l_n)}{\mathrm{d}\psi}$$

whence

$$C = \left[\frac{q\varepsilon_{\mathrm{sc}}/(2\psi)}{1/N_D + 1/N_A}\right]^{1/2} = \frac{[(q\varepsilon_{\mathrm{sc}}/2)/(1/N_D + 1/N_A)]^{1/2}}{[\psi_0\,(1 - V/\psi_0)]^{1/2}}$$

$$C = C_0\left(1 - \frac{V}{\psi_0}\right)^{-1/2}, \qquad C_0 \doteq \left[\frac{q\varepsilon_{\mathrm{sc}}/(2\psi_0)}{1/N_D + 1/N_A}\right]^{1/2}$$

One can observe that

$$\frac{1}{C^2} = \frac{2}{q\varepsilon_{\mathrm{sc}}}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)\psi = \frac{l^2}{\varepsilon_{\mathrm{sc}}^2} \quad \Rightarrow \quad C = \frac{\varepsilon_{\mathrm{sc}}}{l}$$

We have a device i.e. PN Junction in RB condition, The current is very small. In DB The current in exponential w.r.to The applied voltage.

- Assuming we can neglect the current in RB, Now we have two Regions populated by +ve charge on one side and -ve charges on the other side. The charges are equal and opposite to eachother and they depend on the Applied Bias. So essentially we have a Capacitor

- So, it is sensible to calculate the Capacitance of PN Junction in Reverse Bias. This Capacitance is Nonlinear because the dependence of Charge on applied Bias is Non Linear. Any how we can consider the capacitance and from circuit theory we know that when we have a Nonlinear Device that behaves like a Capacitor we must calculate the Differential Capacitance i.e $\frac{dq}{dV}$

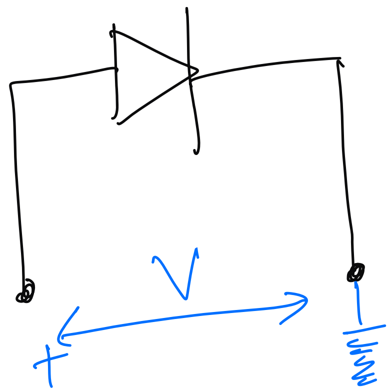
Charge per unit are

$$Q_p = \rho_p l_p = -qN_A l_p \quad , \quad Q_n = \rho_n l_n = qN_D l_n$$

Then Capacitance per unit area: $\boxed{C = \frac{dQ_p}{dV} = \frac{dQ_n}{-dV}}$

Q) what is the reference for the voltage 'V' in the Device ?

Any *



we must consider variation of the charge on the same side of the junction for both regions, if you don't do it, the sign of capacitance would be wrong.

$$C = \frac{dQ_p}{dV} = \frac{dQ_n}{-dV}$$

$$C = -qN_A \frac{dl_p}{dV} = q\,\frac{d(N_A l_p)}{d\psi}$$

$$\psi = \psi_0 - V$$ we know this

Capacitance = $\frac{\text{Permittivity}}{\text{length of SCR}}$

$$C = \frac{\varepsilon_{sc}}{l}$$

$$C = -qN_A\frac{\mathrm{d}l_p}{\mathrm{d}V} = q\,\frac{\mathrm{d}(N_A l_p)}{\mathrm{d}\psi}, \quad C = qN_D\frac{\mathrm{d}l_n}{-\mathrm{d}V} = q\,\frac{\mathrm{d}(N_D l_n)}{\mathrm{d}\psi}$$

whence

$$C = \left[\frac{q\varepsilon_{sc}/(2\psi)}{1/N_D + 1/N_A}\right]^{1/2} = \frac{[(q\varepsilon_{sc}/2)/(1/N_D + 1/N_A)]^{1/2}}{[\psi_0\,(1 - V/\psi_0)]^{1/2}}$$

$$C = C_0\left(1 - \frac{V}{\psi_0}\right)^{-1/2}, \quad C_0 \doteq \left[\frac{q\varepsilon_{sc}/(2\psi_0)}{1/N_D + 1/N_A}\right]^{1/2}$$

One can observe that

$$\frac{1}{C^2} = \frac{2}{q\varepsilon_{sc}}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)\psi = \frac{l^2}{\varepsilon_{sc}^2} \quad\Rightarrow\quad \boxed{C = \frac{\varepsilon_{sc}}{l}}$$

Quantitative Relations in the *p-n* Junction — I

- Taking an abrupt *p-n* junction with $N_A = 10^{16}$ cm^{-3}, $N_D = 10^{15}$ cm^{-3}, and letting $T_L = 300$ K whence $k_B T_L/q \simeq 26$ mV and (for silicon) $n_i \simeq 10^{10}$ cm^{-3}, it follows $n_{p0} = n_i^2/N_A \simeq 10^4$ cm^{-3}, $p_{n0} = n_i^2/N_D \simeq 10^5$ cm^{-3}, and

$$\psi_0 = \frac{k_B T_L}{q} \log\left(\frac{N_A N_D}{n_i^2}\right) \simeq 0.65 \text{ V}.$$

- The experimental minority-carrier mobilities for the doping concentrations and temperature considered here are $\mu_n \simeq 1000$ cm^2/(V s) in the p region and $\mu_p \simeq 500$ cm^2/(V s) in the n region, whence

$$D_n = \frac{k_B T_L}{q}\mu_n \simeq 26\ \frac{\text{cm}^2}{\text{s}}, \qquad D_p = \frac{k_B T_L}{q}\mu_p \simeq 13\ \frac{\text{cm}^2}{\text{s}}.$$

- The experimental minority-carrier lifetimes are $\tau_n = 5 \times 10^{-5}$ s and $\tau_p = 2 \times 10^{-5}$ s, whence the corresponding diffusion lengths

$$L_n = \sqrt{\tau_n D_n} \simeq 360\ \mu\text{m}, \qquad L_p = \sqrt{\tau_p D_p} \simeq 160\ \mu\text{m}.$$

it is useful to remember These when we are Considering Bipolar Transistors.

Quantitative Relations in the *p-n* Junction — II

1. The above data provide for the saturation current

$$J_s = q\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right) \simeq 14\ \frac{\text{pA}}{\text{cm}^2}.$$

2. Using (for silicon) $\varepsilon_{sc} = 11.7 \times \varepsilon_0$, with $\varepsilon_0 \simeq 8.854 \times 10^{-14}$ F/cm, the width of the depletion region at zero bias is found:

$$l_{V=0} = l_n + l_p = \left[\frac{2\varepsilon_{sc}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)\psi_0\right]^{1/2} \simeq 1\ \mu\text{m},$$

with $l_n/l_p = N_A/N_D = 10$. The differential capacitance per unit

3. area at zero bias is

$$\boxed{C_0 = \left[\frac{q\varepsilon_{sc}/(2\psi_0)}{1/N_D + 1/N_A}\right]^{1/2} = \frac{\varepsilon_{sc}}{l_{V=0}} \simeq 11\ \frac{\text{nF}}{\text{cm}^2}.}$$

$\varepsilon_{sc} = \varepsilon_r \varepsilon_0$

Relative permittivity

absolute permittivity

Junction Breakdown due to Impact Ionization — I

(in Reverse Bias)

Hp: $\begin{cases} \text{Steady-state and one-dimensional case} \\ \text{Impact ionization dominates over the other GR processes} \\ \text{Ohmic transport} \end{cases}$

continuity Equation

coefficients of impact ionization

$$-\sigma \operatorname{div} \boldsymbol{J}_n = k_n J_n + k_p J_p , \qquad \sigma \operatorname{div} \boldsymbol{J}_p = k_n J_n + k_p J_p$$

where $\sigma = \mathcal{E}/|\mathcal{E}|$, $\mathcal{E} \neq 0$, and k_n, k_p are the impact-ionization coefficients. In one dimension and steady-state it is $J = J_n(x) + J_p(x) = \text{const}$; in addition, let $\mathcal{E} = \mathcal{E}(x)$ be significant for $a \leq x \leq b$, and let $\mathcal{E}(x) < 0$:

$$\begin{cases} \sigma = -1 \\ J_n = \boldsymbol{J}_n \bullet \boldsymbol{i} = q\mu_n n\mathcal{E} < 0 , \qquad v_n = \boldsymbol{v}_n \bullet \boldsymbol{i} = -\mu_n \mathcal{E} > 0 \\ J_p = \boldsymbol{J}_p \bullet \boldsymbol{i} = q\mu_p p \mathcal{E} < 0 , \qquad v_p = \boldsymbol{v}_p \bullet \boldsymbol{i} = \mu_p \mathcal{E} < 0 \end{cases}$$

$$\frac{\mathrm{d}J_n}{\mathrm{d}x} = k_n J_n + k_p J_p , \qquad \frac{\mathrm{d}J_p}{\mathrm{d}x} = -k_n J_n - k_p J_p ,$$

where k_n, k_p depend on x through the electric field and are determined experimentally. An example is Chynoweth's model:

$$k_n = k_{ns} \exp\left(-|\mathcal{E}_{cn}/\mathcal{E}|^{\beta_n}\right) , \qquad k_p = k_{ps} \exp\left(-|\mathcal{E}_{cp}/\mathcal{E}|^{\beta_p}\right)$$

30/09/24

→ In the RB the EF is so large that Impact Ionization dominates: It is a case where e^- can gain enough energy from the Electric field to able to promote another e^- from VB to CB by giving it the extra energy acquired from the EF and similarly Hole can do the same.

In RB Impact ionization Dominates in this case where two equations:

① continuity Equation in steady state

$$-\sigma \operatorname{div} J_n = k_n J_n + k_p J_p$$

$$\sigma \operatorname{div} J_p = k_n J_n + k_p J_p$$

k_n, k_p → impact ionization coefficients

σ → it is equal to '1' depends on the orientation of unit vectors in the problem.

Now, the purpose is to calculate current in the PN Junction in the above conditions.

In steady state & 1D case, we can say

Total current $J = J_n(x) + J_p(x)$ calculated at the same position and this total current density must be a constant. (The reasoning is same as Shockley Theory)

Here in this case EF has some orientation and current Density is parallel to it because the Drift part of the current Dominates. In the original eqn we replace the $\mathrm{div}\, J_n$ with $\frac{dJ_n}{dx}$ similarly for $\mathrm{div}\, J_p$ with $\frac{dJ_p}{dx}$

$$\therefore \quad \frac{dJ_n}{dx} = K_n J_n + K_p J_p \quad (1)$$

$$\frac{dJ_p}{dx} = -K_n J_n - K_p J_p \quad (2)$$

System of two 1st order Diff. eqns in J_n & J_p

$$\begin{cases} \sigma = -1 \\ J_n = \boldsymbol{J}_n \bullet \boldsymbol{i} = q\mu_n n\mathcal{E} < 0\,, & v_n = \boldsymbol{v}_n \bullet \boldsymbol{i} = -\mu_n \mathcal{E} > 0 \\ J_p = \boldsymbol{J}_p \bullet \boldsymbol{i} = q\mu_p p\mathcal{E} < 0\,, & v_p = \boldsymbol{v}_p \bullet \boldsymbol{i} = \quad \mu_p \mathcal{E} < 0 \end{cases}$$

$$\frac{\mathrm{d}J_n}{\mathrm{d}x} = k_n J_n + k_p J_p\,, \qquad \frac{\mathrm{d}J_p}{\mathrm{d}x} = -k_n J_n - k_p J_p\,,$$

where k_n, k_p depend on x through the electric field and are determined experimentally. An example is Chynoweth's model:

$$k_n = k_{ns} \exp\left(-|\mathcal{E}_{cn}/\mathcal{E}|^{\beta_n}\right)\,, \qquad k_p = k_{ps} \exp\left(-|\mathcal{E}_{cp}/\mathcal{E}|^{\beta_p}\right)$$

k_n, k_p depend on Electric field

& EF depends on 'x'

We know $J = J_n(x) + J_p(x) = \text{constant}$

$$\frac{dJ_n}{dx} = k_n J_n + k_p (J - J_n) = (k_n - k_p) J_n + k_p J$$

This is the standard form of 1st order ODE

$$m = \int_a^x (k_n - k_p)\, dx' \quad , \quad m(a) = 0, \quad \frac{dm}{dx} = k_n - k_p$$

x, a are positions in the region where EF is large

T. 29.10: Integrale di ionizzazione e fattore di moltiplicazione degli elettroni.

calculation of J_n, J_p

Junction Breakdown due to Impact Ionization — II

$$\frac{\mathrm{d}J_n}{\mathrm{d}x} = k_n J_n + k_p (J - J_n) = (k_n - k_p) J_n + k_p J$$

$$m \doteq \int_a^x (k_n - k_p)\,\mathrm{d}x', \qquad m(a) = 0, \qquad \frac{\mathrm{d}m}{\mathrm{d}x} = k_n - k_p$$

$$\frac{\mathrm{d}J_n}{\mathrm{d}x} - \frac{\mathrm{d}m}{\mathrm{d}x} J_n = k_p J = k_n J - \frac{\mathrm{d}m}{\mathrm{d}x} J$$

$$\frac{1}{J}\frac{\mathrm{d}}{\mathrm{d}x}[J_n \exp(-m)] = k_n \exp(-m) - \frac{\mathrm{d}m}{\mathrm{d}x} \exp(-m)$$

where $-(\mathrm{d}m/\mathrm{d}x)\exp(-m) = \mathrm{d}\exp(-m)/\mathrm{d}x$. Integrating: from 'a' to 'b'

$$\frac{J_n(b)}{J} \exp[-m(b)] - \frac{J_n(a)}{J} = Y_n + \exp[-m(b)] - 1,$$

where the electron ionization integral is defined as

$$Y_n \doteq \int_a^b k_n \exp(-m)\,\mathrm{d}x$$

$a = 0^-$
$b = 0^+$

Letting $J = J_n(b) + J_p(b)$ and assuming that $|J_n(b)| \gg |J_p(b)|$ yields

$$1 - \frac{1}{M_n} = Y_n, \qquad M_n \doteq \frac{J_n(b)}{J_n(a)} > 0,$$

where M_n is the electron multiplication factor.

Revanth Reddy Pannala
EBIT, Unibo
రేవంత్ రెడ్డి పన్నల

Junction Breakdown due to Impact Ionization — III

$$\begin{cases} Y_n > 1 & \Rightarrow & M_n < 0 & \text{unphysical} \\ Y_n = 1 & \Rightarrow & M_n = \infty & \text{breakdown (avalanche)} \\ Y_n < 1 & \Rightarrow & M_n < \infty & \text{impact ionization} \end{cases}$$

$$Y_n = \int_a^b k_n \exp(-m)\,\mathrm{d}x = \int_a^b f(\mathcal{E})\,\mathrm{d}x\,, \quad f \doteq k_n \exp(-m)$$

$$\begin{cases} k_n = 0 & \Rightarrow & Y_n = 0 \\ k_n \neq 0\,,\ k_p = 0 & \Rightarrow & Y_n = 1 - \exp[-m(b)] < 1 \\ k_n = k_p \neq 0 & \Rightarrow & m = 0\,, \quad Y_n = \int_a^b k_n\,\mathrm{d}x \end{cases}$$

Example: $\begin{cases} \text{Abrupt } p\text{-}n \text{ junction with a reverse bias } V < 0\,. \\ \text{Strong asymmetry: } N_A \gg N_D \Rightarrow l_p \ll l_n \simeq l\,, \\ \text{whence } a \leftarrow 0\,, b \leftarrow l_n\,. \end{cases}$

$$\mathcal{E} = \mathcal{E}(x, V, N_A, N_D) \simeq \mathcal{E}(x, V, N_D)$$

$$l_n = \frac{1}{N_D}\sqrt{\frac{2\varepsilon_{\mathrm{sc}}\,\psi/q}{1/N_D + 1/N_A}} \simeq \sqrt{\frac{2\varepsilon_{\mathrm{sc}}}{qN_D}\,(\psi_0 - V)} = l_n(N_D, V)$$

The breakdown voltage $V_B > 0$ is the value of $-V$ such that

$$Y_n = \int_0^{l_n(N_D, -V_B)} f(x, N_D, -V_B)\,\mathrm{d}x = 1$$

whence $V_B = V_B(N_D)$ for all $N_D \ll N_A$. In general it is $\mathrm{d}V_B/\mathrm{d}N_D < 0$ (e.g., in the example here $\mathcal{E}_0 \propto \sqrt{N_D}$).

$$\frac{\mathrm{d}J_p}{\mathrm{d}x} = -k_p J_p - k_n\,(J - J_p) = \frac{\mathrm{d}m}{\mathrm{d}x}\,J_p - k_n\,J$$

$$\frac{\mathrm{d}J_p}{\mathrm{d}x} - \frac{\mathrm{d}m}{\mathrm{d}x}\,J_p = -k_n\,J = -k_p\,J - \frac{\mathrm{d}m}{\mathrm{d}x}J$$

$$\frac{1}{J}\,\frac{\mathrm{d}}{\mathrm{d}x}\,[J_p\,\exp(-m)] = -k_p\,\exp(-m) + \frac{\mathrm{d}\exp(-m)}{\mathrm{d}x}$$

$$\frac{J_p(b)}{J}\,\exp[-m(b)] - \frac{J_p(a)}{J} = -\frac{Y_p}{\exp[m(b)]} + \exp[-m(b)] - 1\,,$$

where $Y_p \doteq \int_a^b k_p\,\exp[m(b) - m]\,\mathrm{d}x$ is the hole ionization integral. Letting $J = J_p(a) + J_n(a)\,,\ |J_p(a)| \gg |J_n(a)|$ yields

$$1 - \frac{1}{M_p} = Y_p\,, \qquad M_p \doteq \frac{J_p(a)}{J_p(b)} > 0\,,$$

where M_p is the hole multiplication factor. The breakdown condition $Y_p = 1$ coincides with that of the electrons:

$$Y_n = \int_a^b k_n\,\exp(-m)\,\mathrm{d}x = \int_a^b (k_p + \mathrm{d}m/\mathrm{d}x)\,\exp(-m)\,\mathrm{d}x$$

$$\Longrightarrow \qquad Y_n = \exp[-m(b)]\,Y_p + 1 - \exp[-m(b)]$$

whence $Y_p = 1 \iff Y_n = 1\,.$

T. 29.13: Descrizione schematica del fenomeno della valanga.