T-TEST

THE T-TEST

Compares two group means to see if there is a difference

- Compares one group mean to the population mean
- Compares one group mean to that same group's mean at a later time
- Compares one group mean to another group's mean

THE T-TEST

The one-sample t-test

- Compares the mean of the sample to the population mean

The paired-samples t-test (repeated samples)

- Compares mean of one group and time 1 to mean of that group at time 2

The independent samples t-test (two samples)

- Compares mean of one group to mean of another

EDIT: LOGIC OF T-TEST

Total variation across all groups is divisible into two components

- Variation between groups
  - Deviation of group means from one another

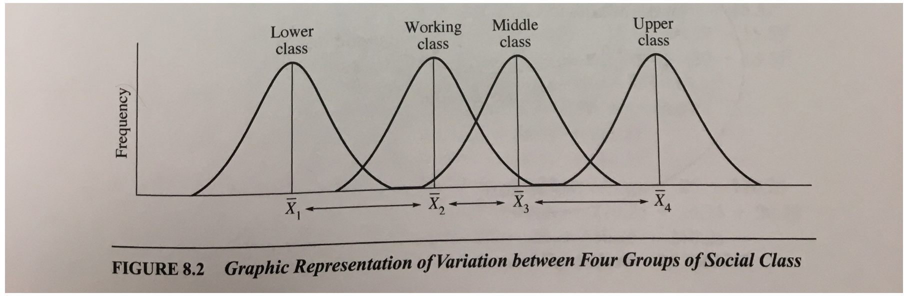


FIGURE 8.2 ***Graphic Representation of Variation between Four Groups of Social Class***

- Variation within groups
  - Deviation of raw scores from their group mean

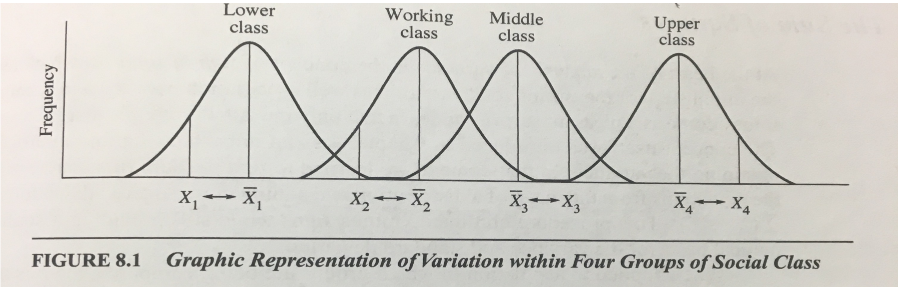


FIGURE 8.1 ***Graphic Representation of Variation within Four Groups of Social Class***

THE T-TEST

But when we substitute Standard Error of the Difference, the distribution becomes a t-distribution, making this a *t*-test:

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{SD_{\bar{X}_1 - \bar{X}_2}} = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\left(\frac{SD_1^2}{N_1}\right) + \left(\frac{SD_2^2}{N_2}\right)}}$$

THE T-TEST

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\left(\frac{SD_1^2}{N_1}\right) + \left(\frac{SD_2^2}{N_2}\right)}}$$

Numerator: difference between means of group 1 and group 2

Denominator: standard error of the difference between means, standardizing the mean difference to see if it is above and beyond chance

LOGIC OF T-TEST

Just like t-test…

- Variation between groups
  - Deviation of group means from one another
- Variation within groups
  - Deviation of raw scores from their group mean

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{SD_{\bar{X}_1 - \bar{X}_2}}$$

- *t* indicates the size of the difference <u>*between*</u> groups relative to the size of the variation <u>*within*</u> each group
- Larger *t* means there is greater variation between groups, and increase the likelihood of rejecting the null hypothesis
- Larger *t* means extreme group differences, beyond what expected by the null hypothesis

INDEPENDENT SAMPLES *T*-TEST (RESEARCH QUESTION)

Is there a mean difference in Y by categories of X?

- Does the mean (Y) vary by group (X)?

INDEPENDENT SAMPLES *T*-TEST (VARIABLE TYPES)

IV: nominal, ordinal (e.g. categorical/discrete)

- Grouping variable
  - Only two groups/samples compared

DV: interval-ratio (e.g. continuous)

INDEPENDENT SAMPLES *T*-TEST (HYPOTHESES)

H_0: No mean difference between two groups / mean of the DV does NOT vary by group

- H_0: $\mu_1 = \mu_2$

H_1: Mean difference between two groups / mean of the DV DOES vary by group

- H_1: $\mu_1 \neq \mu_2$

H_0: NULL HYPOTHESIS

Any observed difference between samples is small and therefore due to chance/sampling error and <u>doesn't represent a true difference between populations</u>

H_1: ALTERNATIVE HYPOTHESIS

Any observed difference between samples is not due to chance/sampling error and <u>does represent a true difference between populations</u>

- <u>Reject the null hypothesis</u> (aka accept the research hypothesis)

Says the difference between samples is TOO BIG/TOO EXTREME to be the result of sampling error

T-TEST ASSUMPTIONS (CANNOT BE VIOLATED)

1. Independence of Observations
   - Groups are not related or dependent upon each other. Case can't be in more than one group. No ties between observations
2. Equal Sample (Group) Sizes
   - The number of cases in each group should be relatively similar.
     - If violated, use "pooled variance" t-test formula
3. Homogeneity of Variance
   - Both groups have approximately equal variances (SD^2). The distributions (or spread) for the groups are approximately equal. Keppel & Zedeck (1989) suggest that variance comparison should not exceed 10:1 ratio.
     - Examine variances/SD in summary table of output
4. Normality of Distribution
   - Distribution must be relatively normal
     - Visual inspection using
       - Histogram
       - Normality (Q-Q) plots
       - Box-and-Whiskers plots
     - If violated, use "unequal variances assumed" formula, otherwise, use "equal variances assumed"

T-TEST AND THE *T*-DISTRIBUTION

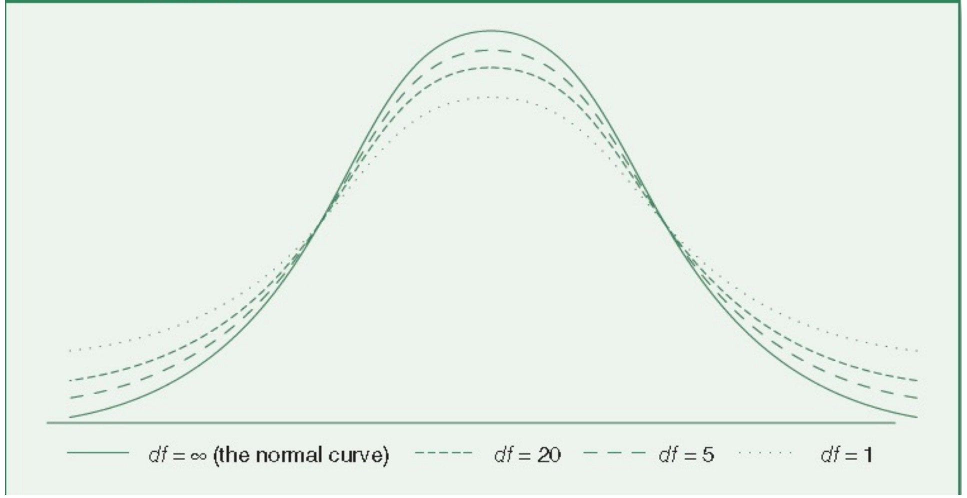
The t-distribution (sort of like normal distribution) has multiple curves

- Each curve based on sample size or degrees of freedom
  - T and Z distributions are equal if sample size is large enough (N ≥ 30 or ≥ 15 per group)

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\left(\frac{SD_1^2}{N_1}\right) + \left(\frac{SD_2^2}{N_2}\right)}}$$

$$df = N_1 + N_2 - 2$$

Figure 9.3 The Normal Distribution and *t* Distributions for 1, 5, 20, and ∞ Degrees of Freedom



IS THE *T*-TEST EXTREME?

Recall:

- If $|Z_{calculated}| \geq |Z_{critical}|$, then…
  - Difference between mean of group 1 and mean of group 2 is so extreme that we can't blame it on sampling error, therefore… H_0 is probably not true, so…

  - $Z_{calculated}$ is in rejection region
  - Reject H_0
  - $p \leq \alpha$
  - Statistically significant difference

IS THE *T*-TEST EXTREME?

So, applied to *t*-Test:

- If $|t_{calculated}| \geq |t_{critical}|$, then…
  - Difference between mean of group 1 and mean of group 2 is so extreme that we can't blame it on sampling error, therefore… H_0 is probably not true, so…
  - $t_{calculated}$ is in rejection region
  - Reject H_0
  - $p \leq \alpha$
  - Statistically significant difference

IS THE *T*-TEST EXTREME?

So, applied to *t*-Test:

- If $|t_{obtained}| \geq |t_{critical}|$, then…
  - Difference between mean of group 1 and mean of group 2 is so extreme that we can't blame it on sampling error, therefore… H_0 is probably not true, so…
  - $t_{obtained}$ is in rejection region
  - Reject H_0
  - $p \leq \alpha$
  - Statistically significant difference

IS THE *T*-TEST EXTREME?

Also recall for Z, if we select $\alpha = .05$:

- If $|Z_{calculated}| \geq |\pm 1.96|$, then…
  - Difference between mean of group 1 and mean of group 2 is so extreme that we can't blame it on sampling error, therefore… H_0 is probably not true, so…
  - $Z_{calculated}$ is in rejection region
  - Reject H_0
  - $p \leq .05$
  - Statistically significant difference

IS THE *T*-TEST EXTREME?

However because t has multiple distributions, based on *df*, if we select $\alpha = .05$:

- We must refer to a table (Appendix T) to figure out what $t_{critical}$ is.
  - Use your chosen $\underline{H_1}$ (directional – one-tailed; non-directional – two-tailed)
  - Your $\underline{\alpha}$
  - Your *df*

- Then, evaluate to see if $| t_{obtained} | \geq | t_{critical} |$

REPORTING T

Report

- The test used
- If you reject or fail to reject the null hypothesis
- The variables used in the analysis
- The degrees of freedom, calculated value of the test, and p-value
  - t(df) = $t_{obtained}$, p-value

- "Using an independent samples t-test, I reject/fail to reject the null hypothesis that there is no difference between group 1's mean and group 2's mean, in the population, t(?) = ?, *p* ? .05"

EXAMPLE: BEERS IN THE HOME FRIDGE FOR UNDERGRADS VS. GRADS

Undergrads: (list of 5 observations below)

- 3, 0, 2, 1, 5

Grads: (list of 5 observations below)

- 1, 6, 17, 9, 2

T-TEST AND CONFIDENCE INTERVALS

We can modify the CI formula to create a confidence interval around the mean difference

- CI = $(\bar{X}_1 - \bar{X}_2) \pm t_{\text{critical}}(SD_{\bar{X}_1 - \bar{X}_2})$