In solving the questions in the assignment, I worked together with my classmate [Shimiao Wang 1004779634]. I confirm that I have written the solutions / code / report in my own words.

Part 1 Q1.1

$$\text{response} = \iint \nabla^2 G(x,y,\delta) \cdot I(x,y)\, dr\, d\theta$$

Let's assume the black part of the filter is 0, white part of the filter is 1.

$$\text{response} = \iint_r^\infty \nabla^2 G(x,y,\delta) \cdot 1 \, dx\, dy + \iint_r^\infty \nabla^2 G(x,y,\delta) \cdot 0 \, dx\, dy$$

$$= \int_0^{2\pi} \int_r^\infty \nabla^2 G(x,y,\delta)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_r^\infty \frac{1}{\pi \sigma^4} \left(\frac{x^2+y^2}{2\sigma^2} - 1 \right) e^{-\frac{x^2+y^2}{2\sigma^2}}\, r\, dr\, d\theta.$$

$$= \int_0^{2\pi} \int_r^\infty \frac{1}{\pi \sigma^4} \left(\frac{r^2}{2\sigma^2} - 1 \right) e^{-\frac{r^2}{2\sigma^2}}\, r\, dr\, d\theta.$$

$$= \frac{1}{\pi \sigma^4} \int_0^{2\pi} \int_r^\infty \left(\frac{r^2}{2\sigma^2} - 1 \right) e^{-\frac{r^2}{2\sigma^2}}\, r\, dr\, d\theta.$$

$$= \frac{1}{\pi \sigma^4} \int_0^{2\pi} \int_r^\infty \frac{r^3}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} - r e^{-\frac{r^2}{2\sigma^2}} \, dr\, d\theta.$$

$$= \frac{1}{\pi \sigma^4} \int_0^{2\pi} \left[\frac{1}{2} r^2 e^{\frac{-r^2}{2\sigma^2}} \right] d\theta$$

$$= \frac{\cancel{2}\cancel{\pi}}{\cancel{\pi}\sigma^4} \frac{1}{\cancel{2}} r^2 e^{\frac{-r^2}{2\sigma^2}} - 0.$$

$$= \frac{r^2}{\sigma^4} e^{\frac{-r^2}{2\sigma^2}}$$

Since $D = 2r$. $\text{response} = \frac{(\frac{1}{2}D)^2}{\sigma^4} e^{-\frac{(\frac{1}{2}D)^2}{2\sigma^2}} = \frac{1}{4\sigma^4} D^2 \cdot e^{-\frac{D^2}{8\sigma^2}}$

D^2

Differentiate $\frac{\partial}{\partial \delta}\left(\frac{D^2}{4\sigma^4} \cdot e^{-\frac{D^2}{8\sigma^2}}\right)$

$$= \frac{D^2}{4\sigma^4} \cdot -\frac{2D^2}{8\sigma^3} e^{-\frac{D^2}{8\sigma^2}} + \frac{4D^2}{4\sigma^5} e^{-\frac{D^2}{8\sigma^2}}$$

$$= \frac{D^2}{\sigma^5} e^{-\frac{D^2}{8\sigma^2}} - \frac{D^4}{16\sigma^7} e^{-\frac{D^2}{8\sigma^2}}$$

$$\Rightarrow \frac{D^2}{\sigma^5} e^{-\frac{D^2}{8\sigma^2}} - \frac{D^4}{16\sigma^7} e^{-\frac{D^2}{8\sigma^2}} = 0$$

$$\Rightarrow \frac{D^2}{\sigma^5} = \frac{D^4}{16\sigma^7}$$

$$\Rightarrow 16\sigma^2 = D^2$$

$$\sigma = \frac{1}{4}D$$

Therefore. $\delta = \frac{1}{4}D$ maximise the magnitude of response

Q1.2. $$\text{response} = \int_0^{2\pi}\int_0^{r} \nabla^2 G(x,y,\delta)\, r\,dr\,d\theta$$

$$= -\frac{r^2}{D^4} e^{-\frac{r^2}{\sigma^2}}$$

$$\Rightarrow \frac{d}{d\sigma}\left(-\frac{r^2}{D^4} e^{-\frac{r^2}{\sigma^2}}\right) = 0$$

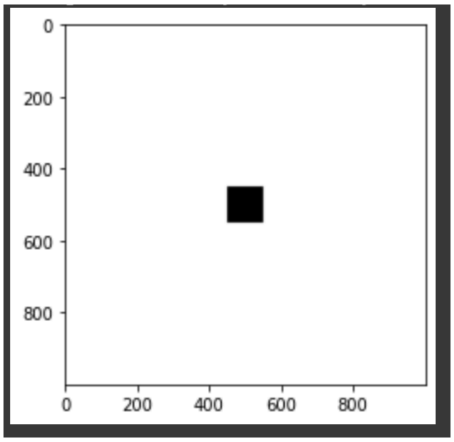
$$\Leftrightarrow -\frac{d}{d\sigma}\left(\frac{r^2}{D^4} e^{\frac{-r^2}{\sigma^2}}\right) = 0$$

$$\Leftrightarrow \frac{d}{d\sigma}\left(\frac{r^2}{D^4} e^{-\frac{r^2}{\sigma}}\right) = 0$$

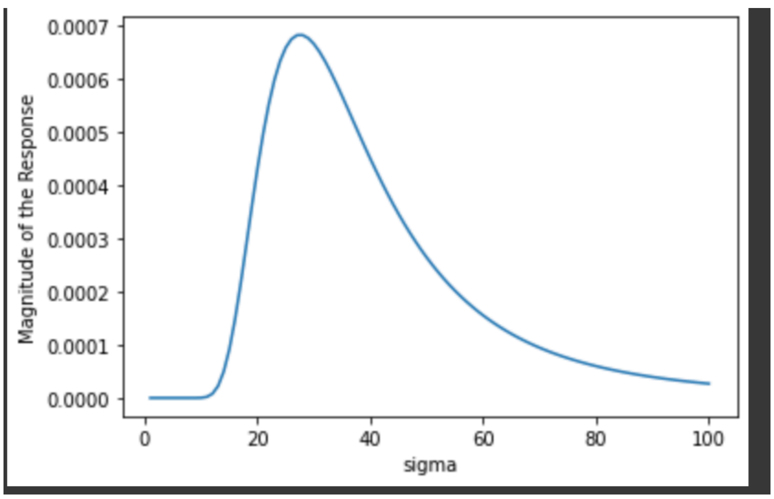
$$\sigma = \frac{1}{4}D$$

Therefore. $\delta = \frac{1}{4}D$ maximise the magnitude of response if we want to detect white circle on a black background.

Q1.3. see CSC420_A3_Q1



The image.



The value of response correspond to sigma value.

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The maximize sigma value is 28
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Q2.1 $\det(N-\lambda I) = \det\begin{pmatrix} I_x^2-\lambda & I_xI_y \\ I_xI_y & I_y^2-\lambda \end{pmatrix}$

$$= (I_x^2-\lambda)(I_y^2-\lambda) - I_xI_yI_xI_y$$

$$= \cancel{I_x^2I_y^2} - I_x^2\lambda - I_y^2\lambda + \lambda^2 - \cancel{I_x^2I_y^2}$$

$$= -I_x^2\lambda - I_y^2\lambda + \lambda^2$$

$\Rightarrow \det(N-\lambda I) = 0$

$$I_x^2\lambda + I_y^2\lambda = \lambda^2.$$

$$\lambda^2 = (I_x^2 + I_y^2)\lambda.$$

$$\lambda_1 = 0, \quad \lambda_2 = I_x^2 + I_y^2$$

Q2.2. if M is positive semi-definite.

Let A be an árbitrary matrix with form $A = (a_1, a_2)$

WTP: $A^T M A \geq 0$.

$$A^T MA = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \sum_x \sum_y w(x,y) N(x,y) (a_1, a_2)$$

$$= \sum_x \sum_y w(x,y) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} N (a_1, a_2)$$

Since $\lambda_1, \lambda_2 \geq 0$. $\quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} N (a_1, a_2) \geq 0$.

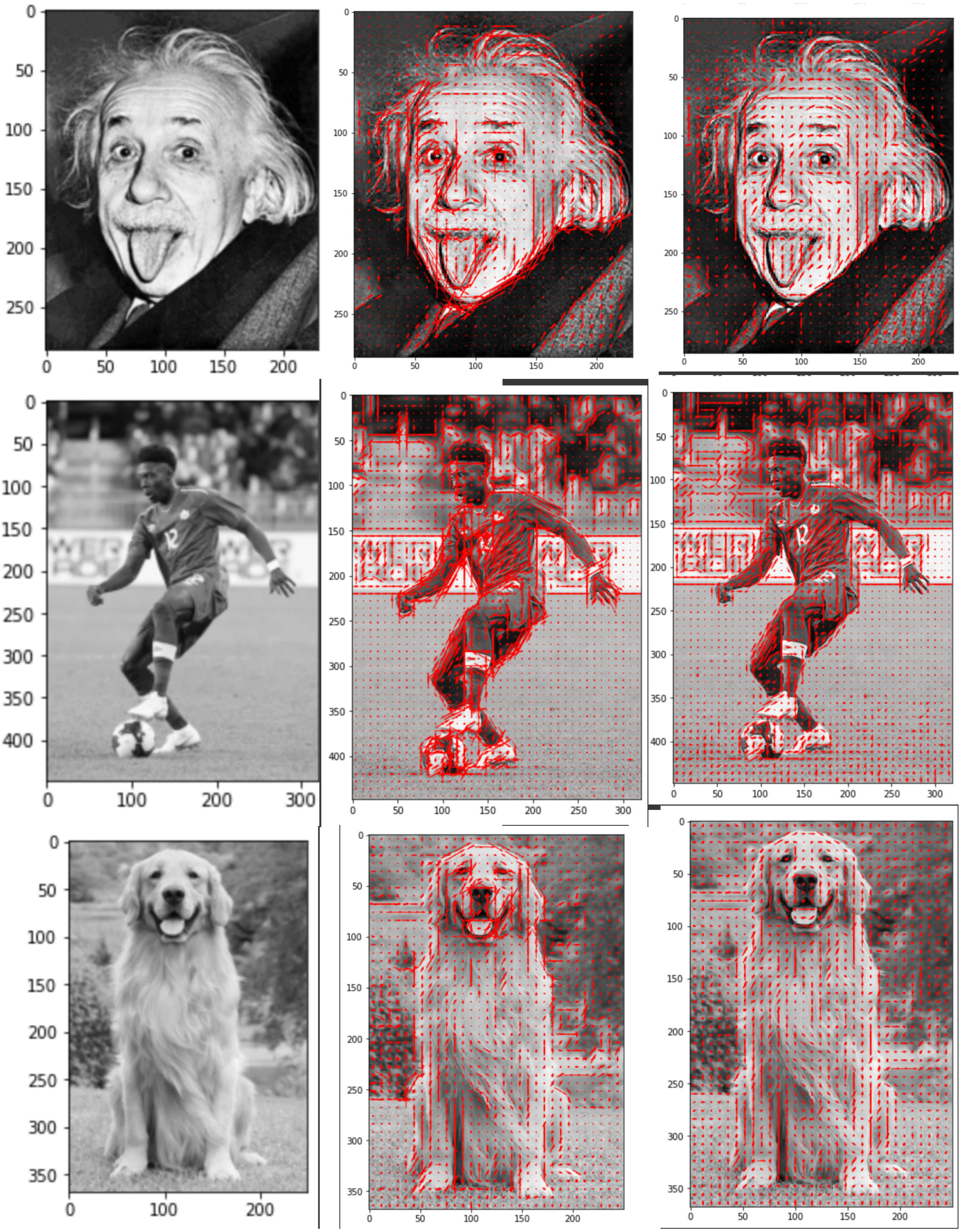
Since $w(x,y) \geq 0$.

$$w(x,y) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} N(a_1, a_2) \geq 0.$$

$$A^T MA \geq 0.$$

Therefore M is positive semi-definite.

Q3.3 original accumulated gradient # of occurance.

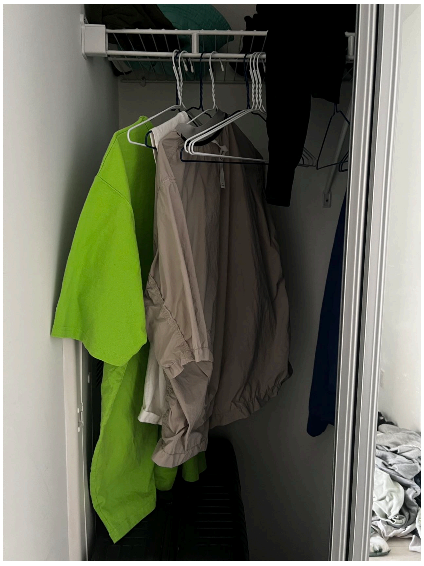


Threshold : 0.03 × Max (gradient magnitude)
Cell size : 8

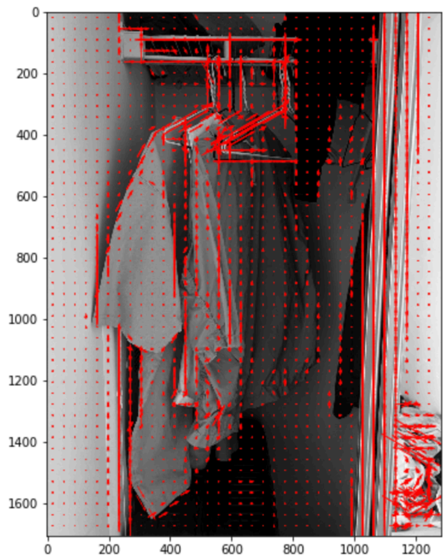
From the above images. we can see that the acamulation method has normally larger vector magnitude compared to the image using occurrance method. Which also interpret that for those more clearly edges (greater gradient difference). The graph will make those edges more noticeble. While for the accumulation method, any edge (no matter how sharp and clear) will possess equal affect on the histogram.

Q3.4.

Image without flashlight



Standard HOG



Nomalized HOG.

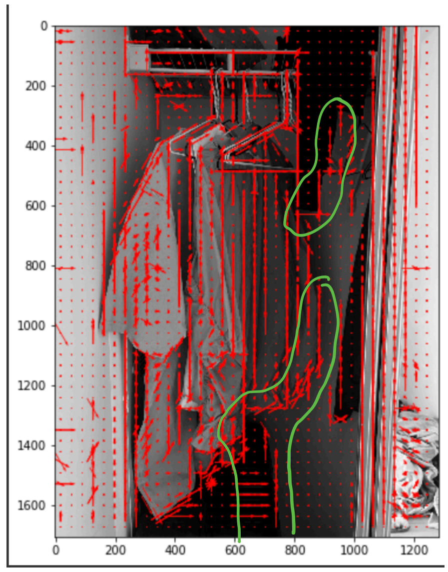
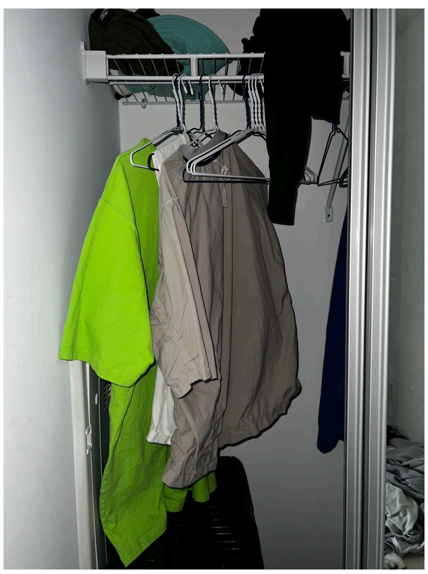
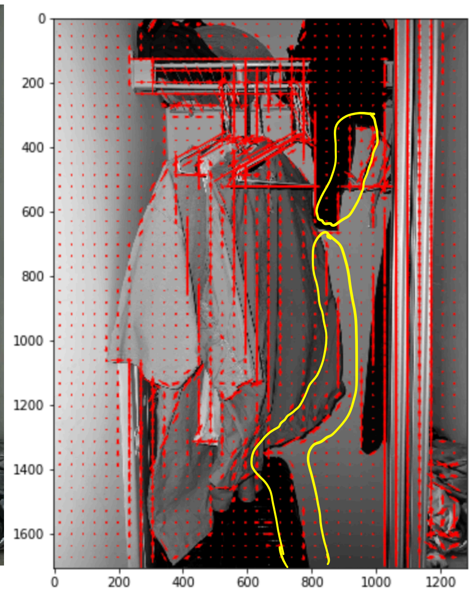


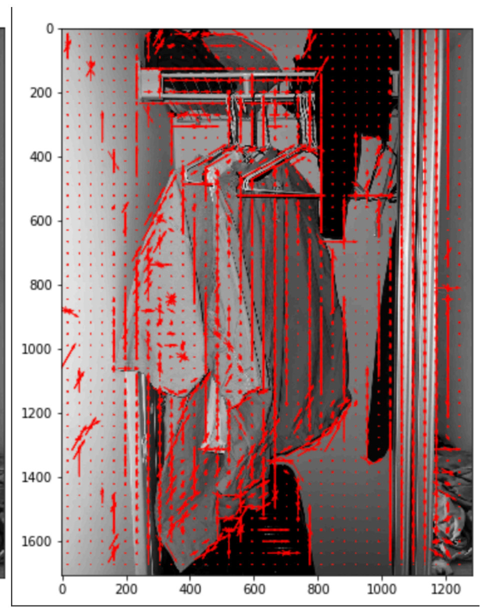
Image with flashlight.



Standard HOG



Nomalized HOG.



For normalized HOG. I used the # of occurrance approach. (since I don't want the extent of difference on edge affect my result).

Threshold : 0.03 × Max (gradient magnitude)

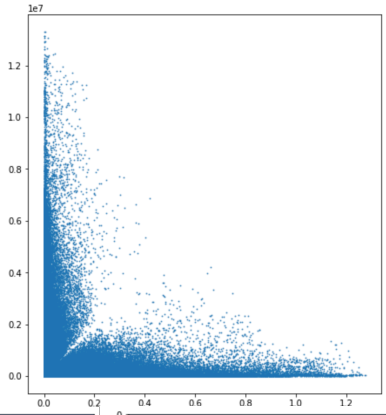
Cell size : 36

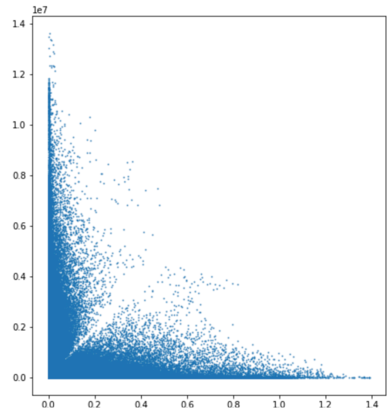
By comparing flash / without flash images. Look at the yellow Circles that I highlighted. Under the flashlight. the edge of Object under a shadow environment will be easily detected due to the light increase the proportion of gradient magnitude on edge.

Now Let's compare the normalized hog of the image without flashlight, By Looking at green area highlighted. The edges we under shadow environment that we cannot detect before are now noticeable. Therefore normalization is a great approach, it can detect more details of edges even under insufficient light environment than the standard HOG.

Q4. 1.2.3.

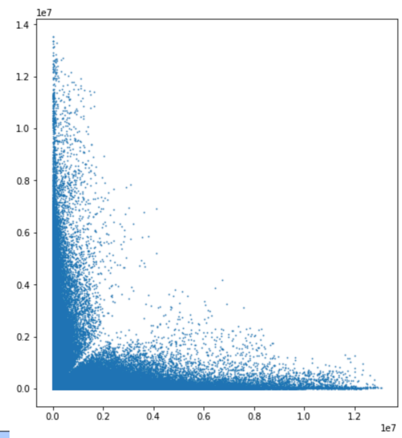
scatter plot $\sigma = 6$

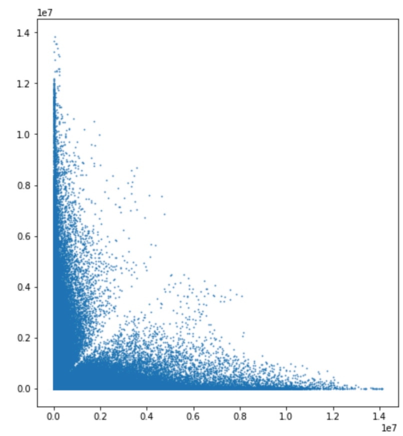


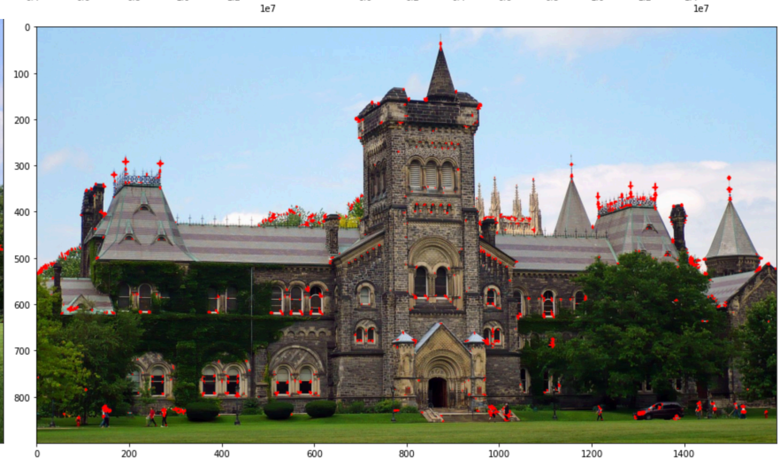


Q4.4.

scatter plot. $\sigma = 60$







The $\sigma=60$ compared with $\sigma=6$ we can see that there are more corner details are detected. Since larger σ means larger size of the blur filter, and make the eigen values scatted more isolated.