N2

B

$$B=\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix} \qquad P=B(B^TB)^{-1}B^T \ (*)$$

$$B^T=\begin{pmatrix}1 & 0 & 1\\ 2 & 1 & 0\end{pmatrix}$$

$$B^TB=\begin{pmatrix}1 & 0 & 1\\ 2 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}2 & 2\\ 2 & 5\end{pmatrix}$$

$$\left(\begin{array}{cc|cc}2 & 2 & 1 & 0\\ 2 & 5 & 0 & 1\end{array}\right)\rightsquigarrow\left(\begin{array}{cc|cc}2 & 2 & 1 & 0\\ 0 & 3 & -1 & 1\end{array}\right)\rightsquigarrow\left(\begin{array}{cc|cc}1 & 0 & 5/6 & -1/3\\ 0 & 1 & -1/3 & 1/3\end{array}\right)$$

$$(B^TB)^{-1}=\begin{pmatrix}5/6 & -1/3\\ -1/3 & 1/3\end{pmatrix}$$

$$(*)\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}5/6 & -1/3\\ -1/3 & 1/3\end{pmatrix}\left(\begin{array}{ccc}1 & 0 & 1\\ 2 & 1 & 0\end{array}\right)=\begin{pmatrix}1/6 & 1/3\\ -1/3 & 1/3\\ 5/6 & -1/3\end{pmatrix}\left(\begin{array}{ccc}1 & 0 & 1\\ 2 & 1 & 0\end{array}\right)=$$

$$=\begin{pmatrix}5/6 & 1/3 & 1/6\\ 1/3 & 1/3 & -1/3\\ 1/6 & -1/3 & 5/6\end{pmatrix}$$

$$P=\begin{pmatrix}5/6 & 1/3 & 1/6\\ 1/3 & 1/3 & -1/3\\ 1/6 & -1/3 & 5/6\end{pmatrix}$$

$$\bar{b}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \bar{b}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{y}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{y}_2 = \bar{b}_2 - \frac{(\bar{b}_2 \bar{y}_1)}{(\bar{y}_1 \bar{y}_1)} \bar{y}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\bar{e}_1 = \begin{pmatrix} \sqrt{2}^{-1} \\ 0 \\ \sqrt{2}^{-1} \end{pmatrix} \qquad \bar{e}_2 = \begin{pmatrix} \sqrt{3}^{-1} \\ \sqrt{3}^{-1} \\ -\sqrt{3}^{-1} \end{pmatrix}$$

$$Q = \begin{pmatrix} \sqrt{2}^{-1} & \sqrt{3}^{-1} \\ 0 & \sqrt{3}^{-1} \\ \sqrt{2}^{-1} & -\sqrt{3}^{-1} \end{pmatrix}$$

$$R = Q^T A = \begin{pmatrix} \sqrt{2}^{-1} & 0 & \sqrt{2}^{-1} \\ \sqrt{3}^{-1} & \sqrt{3}^{-1} & -\sqrt{3}^{-1} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}$$

$$QR = \begin{pmatrix} \sqrt{2}^{-1} & \sqrt{3}^{-1} \\ 0 & \sqrt{3}^{-1} \\ \sqrt{2}^{-1} & -\sqrt{3}^{-1} \end{pmatrix} \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = B$$