№2.

A

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P = A(A^TA)^{-1}A^T \quad ②$$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^TA = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \Rightarrow (A^TA)^{-1} = \begin{pmatrix} 0,5 & 0 \\ 0 & 1 \end{pmatrix}$$

$$① \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0,5 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0,5 & 0 \\ 0 & 1 \\ 0,5 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0,5 & 0 & 0,5 \\ 0 & 1 & 0 \\ 0,5 & 0 & 0,5 \end{pmatrix}$$

$$P = \begin{pmatrix} 0,5 & 0 & 0,5 \\ 0 & 1 & 0 \\ 0,5 & 0 & 0,5 \end{pmatrix}$$

$$\bar{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \bar{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{b}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{b}_2 = a_2 - \frac{(\bar{a}_2 \bar{b}_1)}{(\bar{b}_1 \bar{b}_1)} \bar{b}_1 \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{e}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad R = Q^T A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$QR = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = A$$