№1.

$$I_n = \int_0^1 \frac{x^n}{x+\alpha}dx = \int_0^1 x^{n-1} - \alpha x^{n-2} + \dots dx = \int_0^1 x^{n-1}\,dx - \alpha I_{n-1} = \frac{x^n}{n}\Big|_0^1 - \alpha I_{n-1} = \frac{1}{n} - \alpha I_{n-1}$$

$$\begin{array}{r|l} x^n & x+\alpha \\ \hline x^n + \alpha x^{n-1} & x^{n-1} - \alpha x^{n-2} + \dots \\ \hline -\alpha x^{n-1} & \\ -\alpha x^{n-1} - \alpha^2 x^{n-2} & \\ \hline \alpha^2 x^{n-2} & \end{array}$$

$$\alpha I_{n-1} = \int_0^1 \frac{x^{n-1}}{x+\alpha}\,\alpha\,dx$$