Homework 1

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1) For class one, there are two distinct functions. For class two, there are two distinct functions. For class three, there are two distinct functions. For class four, there are two distinct functions.

2)
Class 1:
For $S_0(x, y) = 1$, $S_0(-x, -y) = 1$, so the function is even.

For $S_{1,1}(x, y) = \cosh(vx)\cos(vy)$,
$S_{1,1}(-x, -y) = \cosh(-vx)\cos(-vy)$
Since $S_{1,1}(x, y) \neq S_{1,1}(-x, -y)$, the function is odd.

For $S_{1,2}(x, y) = \cos(vx)\cosh(vy)$,
$S_{1,2}(-x, -y) = \cos(-vx)\cosh(-vy)$
Since $S_{1,2}(-x, -y) \neq S_{1,2}(x, y)$, the function is odd.

Class 2:
For $S_3(x, y) = xy$,
$S_3(-x, -y) = -x \cdot -y = xy$
Since $S_3(-x, -y) = S_3(x, y)$, the function is even.

For $S_{2,1}(x, y) = \sinh(vx)\sin(vy)$,
$S_{2,1}(-x, -y) = \sinh(-vx)\sin(-vy)$
Since $S_{2,1}(x, y) \neq S_{2,1}(x, y)$, the function is odd.

For $S_{2,2}(x, y) = \sin(vx)\sinh(vy)$,
$S_{2,2}(-x, -y) = \sin(-vx)\sinh(-vy)$
Since $S_{2,2}(-x, -y) \neq S_{2,2}(x, y)$, the function is odd.

Class 3:
For $S_{3,1}(x, y) = \cosh(vx)\sin(vy)$,
$S_{3,1}(-x, -y) = \cosh(-vx)\sin(-vy)$
Since $S_{3,1}(x, y) \neq S_{3,1}(-x, -y)$, the function is odd.

For $S_{3,2}(x, y) = \cos(vx)\sinh(vy)$,
$S_{3,2}(-x, -y) = \cos(-vx)\sinh(-vy)$
Since $S_{3,2}(x, y) \neq S_{3,2}(-x, -y)$, the function is odd.

Class 4:
For $S_{4,1}(x,y) = \sinh(vx)\cos(vy)$,
$S_{4,1}(-x,-y) = \sinh(-vx)\cos(-vy)$
Since $S_{4,1}(x,y) \neq S_{4,1}(-x,-y)$, the function is odd.

For $S_{4,2}(x,y) = \sin(vx)\cosh(vy)$,
$S_{4,2}(-x,-y) = \sin(-vx)\cosh(-vy)$
Since $S_{4,2}(x,y) \neq S_{4,2}(-x,-y)$, the function is odd.

3)
Class 1:
For $S_0(x,y) = 1$,

$$\frac{\partial S_0}{\partial x} = 0 \quad \frac{\partial S_0}{\partial y} = 0 \quad \frac{\partial S_0}{\partial xx} = 0 \quad \frac{\partial S_0}{\partial yy} = 0 \quad \frac{\partial S_0}{\partial xy} = 0 \quad \frac{\partial S_0}{\partial yx} = 0$$

$$\nabla S_0(x,y) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D^2 S_0(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Delta S_0(x,y) = 0 + 0 = 0$$

For $S_{1,1}(x,y) = \cosh(vx)\cos(vy)$,

$$\frac{\partial S_{1,1}}{\partial x} = 0 = v\sinh(vx)\cos(vy) \quad \frac{\partial S_{1,1}}{\partial y} = -v\cosh(vx)\sin(vy)$$

$$\frac{\partial S_{1,1}}{\partial xy} = -v^2\sinh(vx)\sin(vy) \quad \frac{\partial S_{1,1}}{\partial yx} = -v^2\sinh(vx)\sin(vy)$$

$$\frac{\partial S_{1,1}}{\partial xx} = v^2\cosh(vx)\cos(vy) \quad \frac{\partial S_{1,1}}{\partial yy} = -v^2\cosh(vx)\cos(vy)$$

$$\nabla S_{1,1}(x,y) = \begin{bmatrix} v\sinh(vx)\cos(vy) \\ -v\cosh(vx)\sin(vy) \end{bmatrix}$$

$$D^2 S_{1,1}(x,y) = \begin{bmatrix} v^2\cosh(vx)\cos(vy) & -v^2\sinh(vx)\sin(vy) \\ -v^2\sinh(vx)\sin(vy) & -v^2\cosh(vx)\cos(vy) \end{bmatrix}$$

$$\Delta S_{1,1}(x,y) = v^2\cosh(vx)\cos(vy) - v^2\cosh(vx)\cos(vy) = 0$$

For $S_{1,2}(x,y) = \cos(vx)\cosh(vy)$,

$$\frac{\partial S_{1,2}}{\partial x} = -v\sin(vx)\cosh(vy) \quad \frac{\partial S_{1,2}}{\partial y} = v\cos(vx)\sinh(vy)$$

$$\frac{\partial S_{1,2}}{\partial xy} = -v^2\sin(vx)\sinh(vy) \quad \frac{\partial S_{1,2}}{\partial yx} = -v^2\sin(vx)\sinh(vy)$$

$$\frac{\partial S_{1,2}}{\partial xx} = -v^2\cos(vx)\cosh(vy) \quad \frac{\partial S_{1,2}}{\partial yy} = v^2\cos(vx)\cosh(vy)$$

$$\nabla S_{1,2}(x,y) = \begin{bmatrix} -v\sin(vx)\cosh(vy) \\ v\cos(vx)\sinh(vy) \end{bmatrix}$$

$$D^2 S_{1,2}(x,y) = \begin{bmatrix} v^2\cos(vx)\cosh(vy) & -v^2\sin(vx)\sinh(vy) \\ -v^2\sin(vx)\sinh(vy) & v^2\cos(vx)\cosh(vy) \end{bmatrix}$$

$$\Delta S_{1,2}(x,y) = -v^2\cos(vx)\cosh(vy) + v^2\cos(vx)\cosh(vy) = 0$$

Class 2:
For $S_3(x,y) = xy$,

$$\frac{\partial S_3}{\partial x} = y \quad \frac{\partial S_3}{\partial y} = x \quad \frac{\partial S_3}{\partial xx} = 0 \quad \frac{\partial S_3}{\partial yy} = 0 \quad \frac{\partial S_3}{\partial xy} = 0 \quad \frac{\partial S_3}{\partial yx} = 0$$

$$\nabla S_3(x,y) = \begin{bmatrix} y \\ x \end{bmatrix}$$

$$D^2 S_3(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Delta S_3(x,y) = 0 + 0 = 0$$

For $S_{2,1}(x,y) = \sinh(vx)\sin(vy)$,

$$\frac{\partial S_{2,1}}{\partial x} = v\cosh(vx)\sin(vy) \quad \frac{\partial S_{2,1}}{\partial y} = v\sinh(vx)\cos(vy)$$

$$\frac{\partial S_{2,1}}{\partial xy} = v^2\cosh(vx)\cos(vy) \quad \frac{\partial S_{2,1}}{\partial yx} = v^2\cosh(vx)\cos(vy)$$

$$\frac{\partial S_{2,1}}{\partial xx} = v^2\sinh(vx)\sin(vy) \quad \frac{\partial S_{1,2}}{\partial yy} = -v^2\sinh(vx)\sin(vy)$$

$$\nabla S_{2,1}(x,y) = \begin{bmatrix} v\cosh(vx)\sin(vy) \\ v\sinh(vx)\cos(vy) \end{bmatrix}$$

$$D^2 S_{2,1}(x,y) = \begin{bmatrix} v^2 \sinh(vx)\sin(vy) & v^2\cosh(vx)\cos(vy) \\ v^2\cosh(vx)\cos(vy) & -v^2\sinh(vx)\sin(vy) \end{bmatrix}$$

$$\Delta S_{2,1}(x,y) = v^2\sinh(vx)\sin(vy) - v^2\sinh(vx)\sin(vy) = 0$$

For $S_{2,2}(x,y) = \sin(vx)\sinh(vy)$,

$$\frac{\partial S_{2,2}}{\partial x} = v\cos(vx)\sinh(vy) \quad \frac{\partial S_{2,2}}{\partial y} = v\sin(vx)\cosh(vy)$$

$$\frac{\partial S_{2,2}}{\partial xy} = v^2\cos(vx)\cosh(vy) \quad \frac{\partial S_{2,2}}{\partial yx} = v^2\cos(vx)\cosh(vy)$$

$$\frac{\partial S_{2,2}}{\partial xx} = -v^2\sin(vx)\sinh(vy) \quad \frac{\partial S_{2,2}}{\partial yy} = v^2\sin(vx)\sinh(vy)$$

$$\nabla S_{2,2}(x,y) = \begin{bmatrix} v\cos(vx)\sinh(vy) \\ v\sin(vx)\cosh(vy) \end{bmatrix}$$

$$D^2 S_{2,2}(x,y) = \begin{bmatrix} -v^2\sin(vx)\sinh(vy) & v^2\cos(vx)\cosh(vy) \\ v^2\cos(vx)\cosh(vy) & v^2\sin(vx)\sinh(vy) \end{bmatrix}$$

$$\Delta S_{2,2}(x,y) = -v^2\sin(vx)\sinh(vy) + v^2\sin(vx)\sinh(vy) = 0$$

Class 3:
For $S_{3,1}(x,y) = \cosh(vx)\sin(vy)$,

$$\frac{\partial S_{3,1}}{\partial x} = v\sinh(vx)\sin(vy) \quad \frac{\partial S_{3,1}}{\partial y} = v\cosh(vx)\cos(vy)$$

$$\frac{\partial S_{3,1}}{\partial xy} = v^2\sinh(vx)\cos(vy) \quad \frac{\partial S_{3,1}}{\partial yx} = v^2\sinh(vx)\cos(vy)$$

$$\frac{\partial S_{3,1}}{\partial xx} = v^2\cosh(vx)\sin(vy) \quad \frac{\partial S_{3,1}}{\partial yy} = -v^2\cosh(vx)\sin(vy)$$

$$\nabla S_{3,1}(x,y) = \begin{bmatrix} v\sinh(vx)\sin(vy) \\ v\cosh(vx)\cos(vy) \end{bmatrix}$$

$$D^2 S_{3,1}(x,y) = \begin{bmatrix} v^2\cosh(vx)\sin(vy) & v^2\sinh(vx)\cos(vy) \\ v^2\sin(vx)\cos(vy) & -v^2\cosh(vx)\sin(vy) \end{bmatrix}$$

$$\Delta S_{3,1}(x,y) = v^2\cosh(vx)\sin(vy) - v^2\cosh(vx)\sin(vy) = 0$$

For $S_{3,2}(x,y) = \cos(vx)\sin(vy)$,

$$\frac{\partial S_{3,2}}{\partial x} = -v\sin(vx)\sinh(vy) \quad \frac{\partial S_{3,2}}{\partial y} = v\cos(vx)\cosh(vy)$$

$$\frac{\partial S_{3,2}}{\partial xy} = -v^2\sin(vx)\cosh(vy) \quad \frac{\partial S_{3,3}}{\partial yx} = -v^2\sin(vx)\cosh(vy)$$

$$\frac{\partial S_{3,2}}{\partial xx} = -v^2\cos(vx)\sinh(vy) \quad \frac{\partial S_{3,2}}{\partial yy} = v^2\cos(vx)\sinh(vy)$$

$$\nabla S_{3,2}(x,y) = \begin{bmatrix} -v\sin(vx)\sinh(vy) \\ v\cos(vx)\cosh(vy) \end{bmatrix}$$

$$D^2 S_{3,2}(x,y) = \begin{bmatrix} -v^2\cos(vx)\sinh(vy) & -v^2\sin(vx)\cosh(vy) \\ -v^2\sin(vx)\cosh(vy) & v^2\cos(vx)\sinh(vy) \end{bmatrix}$$

$$\Delta S_{3,2}(x,y) = -v^2\cos(vx)\sinh(vy) + v^2\cos(vx)\sinh(vy) = 0$$

Class 4:
For $S_{4,1}(x,y) = \sinh(vx)\cos(vy)$,

$$\frac{\partial S_{4,1}}{\partial x} = v\cosh(vx)\cos(vy) \quad \frac{\partial S_{4,1}}{\partial y} = v\sinh(vx)\sin(vy)$$

$$\frac{\partial S_{4,1}}{\partial xy} = -v^2\cosh(vx)\sin(vy) \quad \frac{\partial S_{4,1}}{\partial yx} = -v^2\cosh(vx)\sin(vy)$$

$$\frac{\partial S_{4,1}}{\partial xx} = v^2\sin(vx)\cos(vy) \quad \frac{\partial S_{4,1}}{\partial yy} = -v^2\sinh(vx)\cos(vy)$$

$$\nabla S_{4,1}(x,y) = \begin{bmatrix} v\cosh(vx)\cos(vy) \\ -v\sinh(vx)\sin(vy) \end{bmatrix}$$

$$D^2 S_{4,1}(x,y) = \begin{bmatrix} v^2 \sinh(vx)\cos(vy) & -v^2\cosh(vx)\sin(vy) \\ -v^2\sinh(vx)\sin(vy) & -v^2\sinh(vx)\cos(vy) \end{bmatrix}$$

$$\Delta S_{4,1}(x,y) = v^2\sinh(vx)\cos(vy) - v^2\sinh(vx)\cos(vy) = 0$$

For $S_{4,2}(x,y) = \sinh(vx)\cosh(vy)$,

$$\frac{\partial S_{4,2}}{\partial x} = v\cosh(vx)\cosh(vy) \quad \frac{\partial S_{4,2}}{\partial y} = v\sinh(vx)\sinh(vy)$$

$$\frac{\partial S_{4,2}}{\partial xy} = v^2\cosh(vx)\sinh(vy) \quad \frac{\partial S_{4,2}}{\partial yx} = v^2\cosh(vx)\sinh(vy)$$

$$\frac{\partial S_{4,2}}{\partial xx} = v^2\sinh(vx)\cosh(vy) \quad \frac{\partial S_{4,2}}{\partial yy} = v^2\sinh(vx)\cosh(vy)$$

$$\nabla S_{4,2}(x,y) = \begin{bmatrix} v\cosh(vx)\cosh(vy) \\ v\sinh(vx)\sinh(vy) \end{bmatrix}$$

$$D^2 S_{4,2}(x,y) = \begin{bmatrix} v^2\sinh(vx)\cosh(vy) & v^2\cosh(vx)\sinh(vy) \\ v^2\cosh(vx)\sinh(vy) & v^2\sinh(vx)\cosh(vy) \end{bmatrix}$$

$\Delta S_{4,2}(x,y) = v^2\sinh(vx)\cosh(vy) + v^2\sinh(vx)\cosh(vy) =$
$2v^2\sinh(vx)\cosh(vy)$

4)
Class 1:
For $\vec{F_0}(x,y) = \nabla S_0(x,y) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Divergence$=\nabla \cdot \vec{F_0}(x,y)$
Divergence$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot 1 = 0$

Curl$=\nabla \times \vec{F_0}(x,y)$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ 0 - 0 \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\vec{F}_{1,1}(x,y) = \nabla S_{1,1}(x,y) = \begin{bmatrix} v\sinh(vx)\cos(vy) \\ -v\cosh(vx)\sin(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{1,1}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(v\sinh(vx)\cos(vy)) + \dfrac{\partial}{\partial y}(-v\cosh(vx)\sin(vy))$

Divergence=$v^2\cosh(vx)\cos(vy) + (-v^2\cosh(vx)\cos(vy))$

Divergence=$v^2\cosh(vx)\cos(vy) - v^2\cosh(vx)\cos(vy) = 0$

Curl=$\nabla \times \vec{F}_{1,1}(x,y)$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ -v^2\sinh(vx)\sin(vy) - (-v^2\sinh(vx)\sin(vy)) \end{bmatrix}$$

Curl= $\begin{bmatrix} 0 \\ 0 \\ -v^2 \sinh(vx)\sin(vy) + v^2 \sinh(vx)\sin(vy)) \end{bmatrix}$

Curl= $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For $\vec{F}_{1,2}(x,y) = \nabla S_{1,2}(x,y) = \begin{bmatrix} -v\sin(vx)\cosh(vy) \\ v\cos(vx)\sinh(vy) \end{bmatrix}$

Divergence= $\nabla \cdot \vec{F}_{1,2}(x,y)$

Divergence= $\frac{\partial}{\partial x}(-v\sin(vx)\cosh(vy)) + \frac{\partial}{\partial y}(v\cos(vx)\sinh(vy))$

Divergence= $-v^2\cos(vx)\cosh(vy) + v^2\cos(vx)\cosh(vy))$

Divergence=0

Curl= $\nabla \times \vec{F}_{1,1}(x,y)$

Curl= $\begin{bmatrix} 0 \\ 0 \\ \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \end{bmatrix}$

Curl= $\begin{bmatrix} 0 \\ 0 \\ -v^2 \sin(vx)\sinh(vy) - (-v^2 \sin(vx)\sinh(vy)) \end{bmatrix}$

Curl= $\begin{bmatrix} 0 \\ 0 \\ -v^2 \sin(vx)\sinh(vy) + v^2 \sin(vx)\sinh(vy)) \end{bmatrix}$

Curl= $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Class 2:

For $\vec{F_3}(x, y) = \nabla S_3(x, y) = \begin{bmatrix} x \\ y \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F_3}(x, y)$

Divergence=$\dfrac{\partial}{\partial x}(y) + \dfrac{\partial}{\partial y}(x)$

Divergence=$0 + 0$

Divergence=0

Curl=$\nabla \times \vec{F}_{1,1}(x, y)$

Curl=$\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 1 - 1 \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For $\vec{F}_{2,1}(x, y) = \nabla S_{2,1}(x, y) = \begin{bmatrix} v\cosh(vx)\sin(vy) \\ v\sinh(vx)\cos(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{2,1}(x, y)$

Divergence=$\dfrac{\partial}{\partial x}(v\sinh(vx)\cos(vy)) + \dfrac{\partial}{\partial y}(v\cosh(vx)\sin(vy))$

Divergence=$v^2\cosh(vx)\cos(vy) + v^2\cosh(vx)\cos(vy)$

Divergence=$2v^2\cosh(vx)\cos(vy)$

Curl=$\nabla \times \vec{F}_{2,1}(x,y)$

Curl=$\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ v^2 \cosh(vx)\cos(vy) - v^2 \cosh(vx)\cos(vy) \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For $\vec{F}_{2,2}(x,y) = \nabla S_{2,2}(x,y) = \begin{bmatrix} v\cos(vx)\sinh(vy) \\ v\sin(vx)\cosh(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{2,2}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(v\sin(vx)\cosh(vy)) + \dfrac{\partial}{\partial y}(v\cos(vx)\sinh(vy))$

Divergence=$v^2\cos(vx)\cosh(vy) + v^2\cos(vx)\cosh(vy)$

Divergence=$2v^2\cos(vx)\cosh(vy)$

Curl=$\nabla \times \vec{F}_{2,2}(x,y)$

Curl=$\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ v^2 \cos(vx)\cosh(vy) - v^2 \cos(vx)\cosh(vy) \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Class 3:

For $\vec{F}_{3,1}(x,y) = \nabla S_{3,1}(x,y) = \begin{bmatrix} v\sinh(vx)\sin(vy) \\ v\cosh(vx)\cos(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{3,1}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(v\cosh(vx)\cos(vy)) + \dfrac{\partial}{\partial y}(v\sinh(vx)\sin(vy))$

Divergence=$v^2\sinh(vx)\cos(vy) + v^2\sinh(vx)\cos(vy)$

Divergence=$2v^2\sinh(vx)\cos(vy)$

Curl=$\nabla \times \vec{F}_{3,1}(x,y)$

Curl=$\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ v^2\sinh(vx)\cos(vy) - v^2\sinh(vx)\cos(vy) \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For $\vec{F}_{3,2}(x,y) = \nabla S_{3,2}(x,y) = \begin{bmatrix} -v\sin(vx)\sinh(vy) \\ v\cos(vx)\cosh(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{3,2}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(v\cos(vx)\cosh(vy)) + \dfrac{\partial}{\partial y}(-v\sin(vx)\sinh(vy))$

Divergence=$-v^2\sin(vx)\cosh(vy) - v^2\sin(vx)\cosh(vy)$

Divergence=$-2v^2\sin(vx)\cosh(vy)$

Curl=$\nabla \times \vec{F}_{3,2}(x,y)$

Curl=$\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ -v^2\sin(vx)\cosh(vy) - (-v^2\sin(vx)\cosh(vy)) \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ -v^2\sin(vx)\cosh(vy) + v^2\sin(vx)\cosh(vy) \end{bmatrix}$

Curl=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Class 4:
For $\vec{F}_{4,1}(x,y) = \nabla S_{4,1}(x,y) = \begin{bmatrix} v\cosh(vx)\cos(vy) \\ -v\sinh(vx)\sin(vy) \end{bmatrix}$

Divergence=$\nabla \cdot \vec{F}_{4,1}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(-v\sinh(vx)\sin(vy)) + \dfrac{\partial}{\partial y}(v\cosh(vx)\cos(vy))$

Divergence=$-v^2\cosh(vx)\sin(vy) - v^2\cosh(vx)\sin(vy)$

Divergence=$-2v^2\cosh(vx)\sin(vy)$

Curl=$\nabla \times \vec{F}_{4,1}(x,y)$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x}-\dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ -v^2\cosh(vx)\sin(vy)-(-v^2\cosh(vx)\sin(vy)) \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ -v^2\cosh(vx)\sin(vy)+v^2\cosh(vx)\sin(vy) \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\vec{F}_{4,2}(x,y)=\nabla S_{4,2}(x,y)=\begin{bmatrix} v\cosh(vx)\cosh(vy) \\ v\sinh(vx)\sinh(vy) \end{bmatrix}$

Divergence=$\nabla\cdot\vec{F}_{4,2}(x,y)$

Divergence=$\dfrac{\partial}{\partial x}(v\sinh(vx)\sinh(vy))+\dfrac{\partial}{\partial y}(v\cosh(vx)\cosh(vy))$

Divergence=$v^2\cosh(vx)\sinh(vy)+v^2\cosh(vx)\sinh(vy)$

Divergence=$2v^2\cosh(vx)\sinh(vy)$

Curl=$\nabla\times\vec{F}_{4,2}(x,y)$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial g}{\partial x}-\dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ v^2\cosh(vx)\sinh(vy) - v^2\cosh(vx)\sinh(vy) \end{bmatrix}$$

$$\text{Curl}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$