Let the differential equation $(-x^n y')' = \lambda x^m y$ subject to the boundary conditions $y(0) = 0, y(1) = 0$ be given. Subtracting the $\lambda x^m y$ term from both sides yields:

$$(-x^n y')' - \lambda x^m y = 0$$

Now, taking the first derivative, we have:

$$-x^n y'' - nx^{n-1} y' - \lambda x^m y = 0$$

Now, suppose the solutions are of the form $y = x^r$. Then, $y' = rx^{r-1}$ and $y'' = r(r-1)x^{r-2}$. Plugging the derivatives we just took back into the original differential equation, we have:

$$-x^2(r(r-1)x^{r-2}) - nx^1(rx^{r-1}) - \lambda(x^r) = 0$$

$$-x^2(x^{-2} \cdot x^r \cdot r(r-1)) - nx^1(x^{-1} \cdot x^r \cdot r) - \lambda \cdot x^r = 0$$

$$(-x^r \cdot r(r-1)) - n(x^r \cdot r) - \lambda \cdot x^r = 0$$

Factoring out a common factor of x^r and then multiplying through by -1, we have:

$$x^r \Big[(r(r-1) + nr + \lambda \Big] = 0$$

$r(r-1) + nr + \lambda = 0$, which is our characteristic equation

Now, in order to find the roots of our characteristic equation, we use the quadratic formula:

$$\frac{-(n-1) \pm \sqrt{(n-1)^2 - 4(\lambda)}}{2(1)}$$
$$\implies \frac{-n+1 \pm \sqrt{(n-1)^2 - 4\lambda}}{2}$$
$$\implies \frac{1-n \pm \sqrt{(n-1)^2 - 4\lambda}}{2}$$

Now, using $n = 2$, we now have:

$$\implies \frac{1-2 \pm \sqrt{(2-1)^2 - 4\lambda}}{2}$$
$$\implies \frac{-1 \pm \sqrt{1-4\lambda}}{2}$$
$$\implies \frac{-1}{2} \pm \sqrt{\frac{1}{4} - \lambda}$$

Now, if we recall that we supposed that our solutions were of the form $y = x^r$ and denote our two solutions as y_1 and y_2, we have:

$$y_1(x) = x^{r_1} = x^{-\frac{1}{2}+\sqrt{\frac{1}{4}-\lambda}} = x^{-\frac{1}{2}} x^{\sqrt{\frac{1}{4}-\lambda}}$$
$$y_2(x) = x^{r_2} = x^{-\frac{1}{2}-\sqrt{\frac{1}{4}-\lambda}} = x^{-\frac{1}{2}} x^{-\sqrt{\frac{1}{4}-\lambda}}$$

So, in general, the equation for our general solution will take the form:

$$y(x) = c_1 x^{r_1} + c_2 x^{r_2}$$

Substituting back in for x^{r_1} and x^{r_2}, we have:

$$y(x) = c_1 x^{-\frac{1}{2}} x^{\sqrt{\frac{1}{4}-\lambda}} + c_2 x^{-\frac{1}{2}} x^{-\sqrt{\frac{1}{4}-\lambda}}$$

Next, if we use the initial condition $y(0) = 1$, we get:

$$y(0) = c_1 (1)^{-\frac{1}{2}} (1)^{\sqrt{\frac{1}{4}-\lambda}} + c_2 (1)^{-\frac{1}{2}} (1)^{-\sqrt{\frac{1}{4}-\lambda}}$$

$$y(x) = c_1 + c_2$$

Now, consider specifically, $\sqrt{\frac{1}{4} - \lambda}$:

$$\sqrt{\frac{1}{4} - \lambda} = \sqrt{(-1)\left(\lambda - \frac{1}{4}\right)} = i\sqrt{\left(\lambda - \frac{1}{4}\right)}$$

In this case, we want to only consider the real roots, so we have to extract them using Euler's formula:

$$\text{let } x^{\alpha i} = (e^{\ln(x)})^{\alpha i} = e^{\alpha \ln(x) i}$$

$$\text{using } e^{\alpha \ln(x) i} \text{ and } \alpha = \sqrt{\lambda - \frac{1}{4}}, \text{ we have:}$$

$$\cos(\alpha \ln(x)) + i \sin(\alpha \ln(x)) = \cos\left(\sqrt{\lambda - \frac{1}{4}} \ln(x)\right) + i \sin\left(\sqrt{\lambda - \frac{1}{4}} \ln(x)\right)$$

Now, earlier, we had shown our two solutions were:

$$y_1(x) = x^{-\frac{1}{2}} x^{\sqrt{\frac{1}{4}-\lambda}}$$

$$y_2(x) = x^{-\frac{1}{2}} x^{-\sqrt{\frac{1}{4}-\lambda}}$$

However, if we now update our solutions using the results from apply Euler's formula, we have:

$$y_1(x) = x^{-\frac{1}{2}} \cos\left(\sqrt{\lambda - \frac{1}{4}} \ln(x)\right)$$

$$y_2(x) = x^{-\frac{1}{2}} sin\left(\sqrt{\lambda - \frac{1}{4}} \ln(x)\right)$$

Both components of the supposed solution will oscillate and become infinitely large as $x \to 0$