Let the differential equation $(-x^n y')' = \lambda x^m y$ subject to the initial condition $y(0) = 1$ be given. Subtracting the $\lambda x^m y$ term from both sides yields:

$$(-x^n y')' - \lambda x^m y = 0$$

Now, taking the first derivative, we have:

$$-x^n y'' - n x^{n-1} y' - \lambda x^m y = 0$$

Next, let $m = 1$ and $n = 3$. Then, we have:

$$-x^3 y'' - 3x^2 y' - \lambda x y = 0$$

Now, suppose the solutions are of the form $y = x^r$. Then, $y' = r x^{r-1}$ and $y'' = r(r-1)x^{r-2}$. Plugging the derivatives we just took back into the original differential equation, we have:

$$-x^3(r(r-1)x^{r-2}) - 3x^2(r x^{r-1}) - \lambda x(x^r) = 0$$

$$-x^3(-x^2 \cdot x^r \cdot r(r-1)) - 3x^2(x^{-1} \cdot x^r \cdot r) - \lambda x^1 \cdot x^r = 0$$

$$(-x \cdot x^r \cdot r(r-1)) - 3(x \cdot x^r \cdot r) - \lambda x^1 \cdot x^r = 0$$

$$x^r \Big[(-x \cdot r(r-1)) - 3(x \cdot r) - \lambda x \Big] = 0$$

$$r(r-1) + 3r + \lambda = 0, \text{ which is our characteristic equation}$$

Now, in order to find the roots of our characteristic equation, we use the quadratic formula:

$$\frac{-3 \pm \sqrt{3^2 - 4(1)(\lambda)}}{2(1)}$$
$$\implies \frac{-3 \pm \sqrt{9 - 4\lambda}}{2}$$
$$\implies \frac{-3 \pm \sqrt{\frac{9}{4} - \lambda}}{2}$$
$$\implies -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \lambda}$$

Now, if we recall that we supposed that our solutions were of the form $y = x^r$ and denote our two solutions as y_1 and y_2, we have:

$$y_1(x) = x^{r_1} = x^{-\frac{3}{2} + \sqrt{\frac{9}{4} - \lambda}} = x^{-\frac{3}{2}} x^{\sqrt{\frac{9}{4} - \lambda}}$$

$$y_2(x) = x^{r_2} = x^{-\frac{3}{2} - \sqrt{\frac{9}{4} - \lambda}} = x^{-\frac{3}{2}} x^{-\sqrt{\frac{9}{4} - \lambda}}$$

So, in general, the equation for our general solution will take the form:

$$y(x) = c_1 x^{r_1} + c_2 x^{r_2}$$

Substituting back in for x^{r_1} and x^{r_2}, we have:

$$y(x) = c_1 x^{-\frac{3}{2}} x^{\sqrt{\frac{9}{4}-\lambda}} + c_2 x^{-\frac{3}{2}} x^{-\sqrt{\frac{9}{4}-\lambda}}$$

Next, if we use the initial condition $y(0) = 1$, we get:

$$y(0) = c_1 (1)^{-\frac{3}{2}} (1)^{\sqrt{\frac{9}{4}-\lambda}} + c_2 (1)^{-\frac{3}{2}} (1)^{-\sqrt{\frac{9}{4}-\lambda}}$$

$$y(x) = c_1 + c_2$$

Now, consider, specifically, $\sqrt{\frac{9}{4} - \lambda}$:

$$\sqrt{\frac{9}{4} - \lambda} = \sqrt{(-1)\left(\lambda - \frac{9}{4}\right)} = i\sqrt{(-1)\left(\lambda - \frac{9}{4}\right)}$$

In this case, we want to only consider the real roots, so we have to extract them using Euler's formula:

$$\text{let } x^{\alpha i} = (e^{\ln(x)})^{\alpha i} = e^{\alpha \ln(x) i}$$

$$\text{using } e^{\alpha \ln(x) i} \text{ and } \alpha = \sqrt{\lambda - \frac{9}{4}}, \text{ we have:}$$

$$\cos(\alpha \ln(x)) + i \sin(\alpha \ln(x)) = \cos\left(\sqrt{\lambda - \frac{9}{4}} \ln(x)\right) + i \sin\left(\sqrt{\lambda - \frac{9}{4}} \ln(x)\right)$$

Now, earlier, we had shown our two solutions were:

$$y_1(x) = x^{-\frac{3}{2}} x^{\sqrt{\frac{9}{4}-\lambda}}$$

$$y_2(x) = x^{-\frac{3}{2}} x^{-\sqrt{\frac{9}{4}-\lambda}}$$

However, if we now update our solutions using the results from apply Euler's formula, we have:

$$y_1(x) = x^{-\frac{3}{2}} \cos\left(\sqrt{\lambda - \frac{9}{4}} \ln(x)\right)$$

$$y_2(x) = x^{-\frac{3}{2}} sin\left(\sqrt{\lambda - \frac{9}{4}} \ln(x)\right)$$