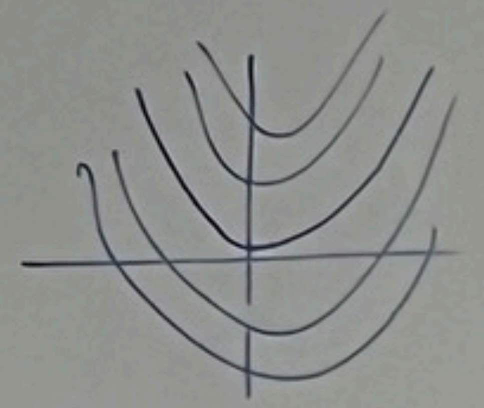
Sayısal integral

anti-türev int.

$$\int f(x)\,dx = F(x) + C$$

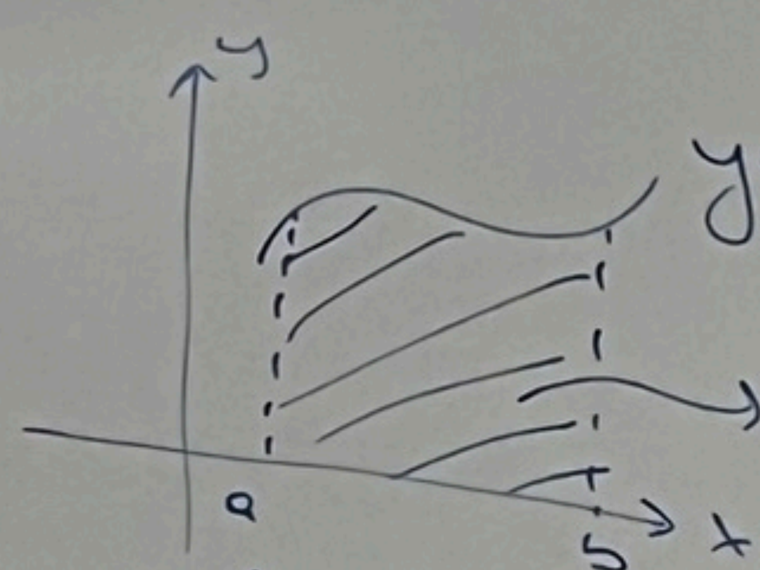
türev

$$\int 2x\,dx = x^2 + C$$

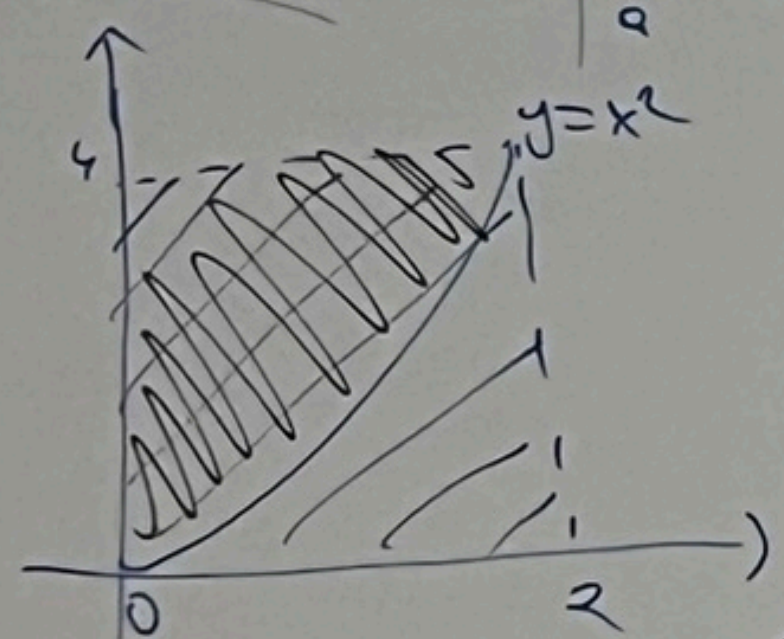


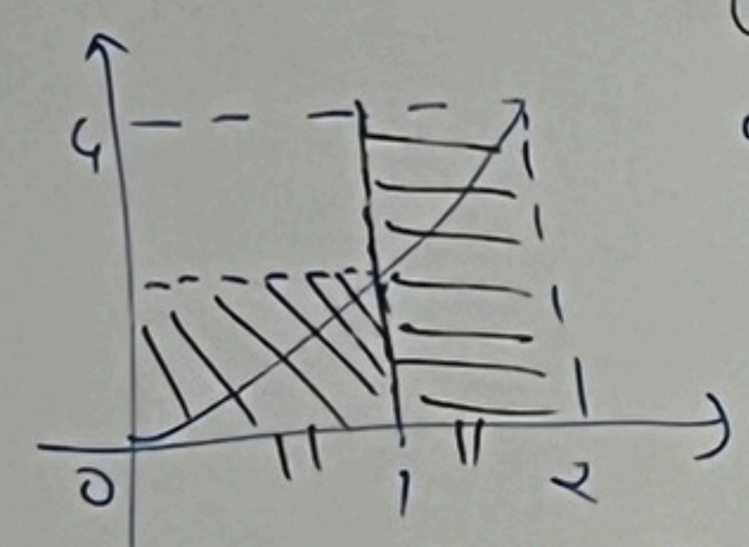
$$\int_a^b f(x)\,dx$$

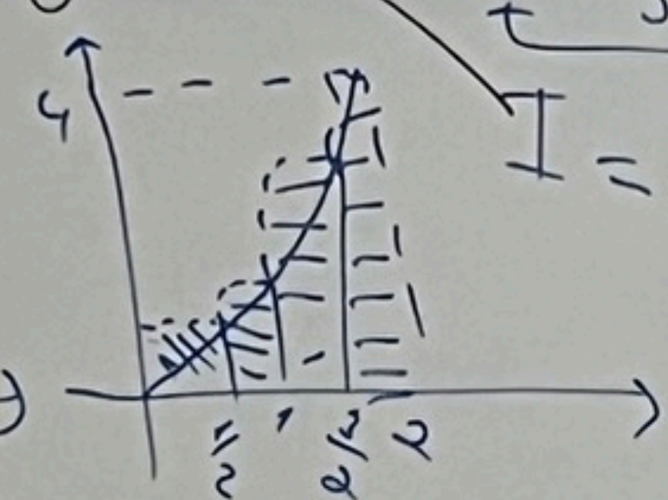
$f(x) > 0$



$\int_a^b f(x)\,dx$





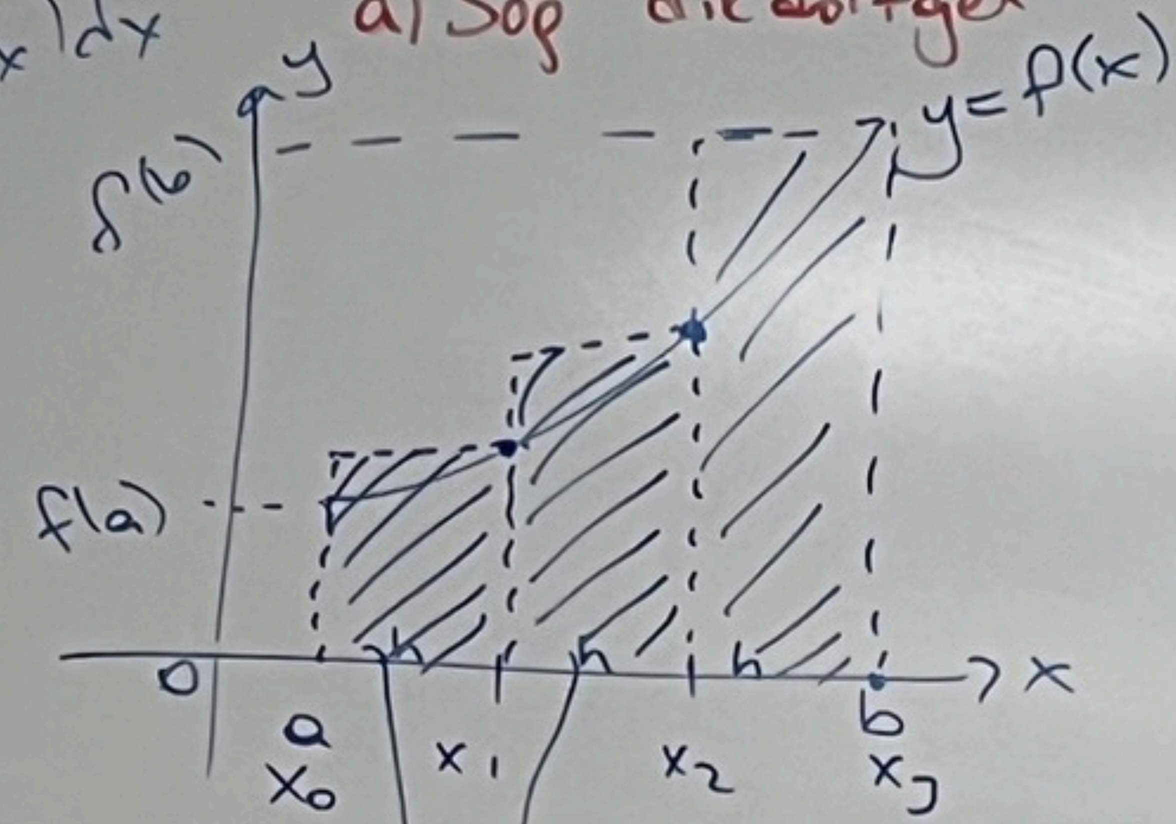


$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$

I. 1 nokta yaklaşımları

$\int_a^b f(x)\,dx$

a) Sağ dikdörtgen



$h = \dfrac{b-a}{n}$

3 aldık

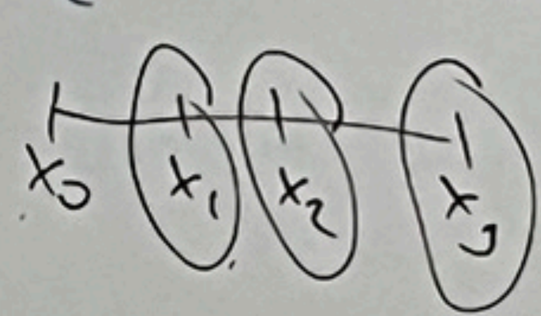
olabilir top. yoktur

$I_1 = h \cdot f(x_1)$

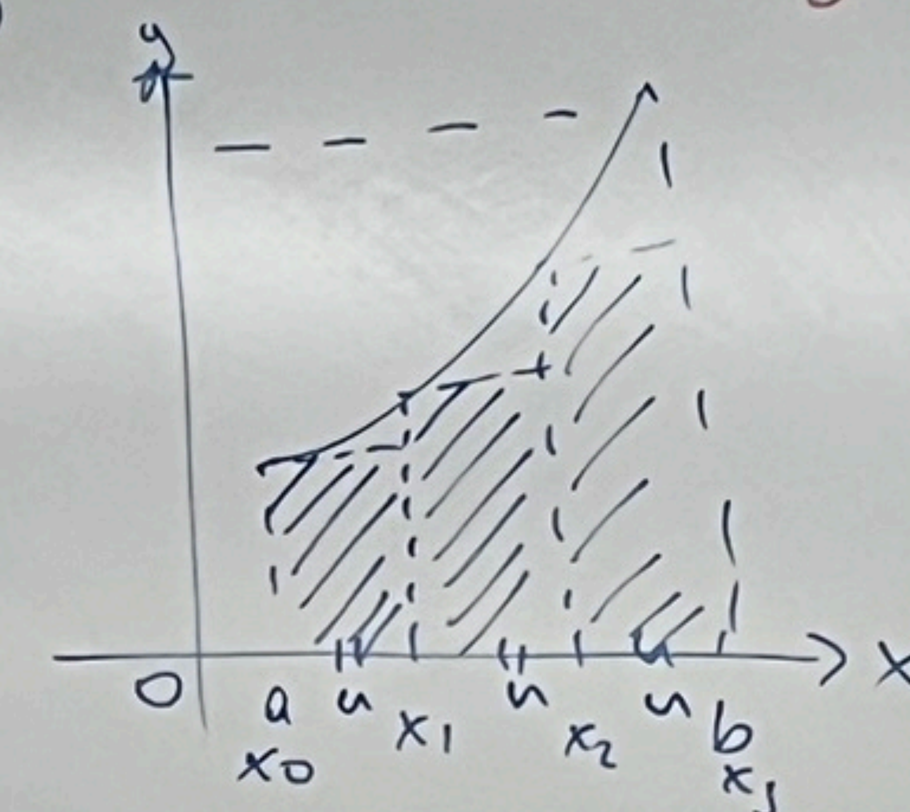
$I_2 = h \cdot f(x_2)$

$I_3 = h \cdot f(x_3)$

$I = h\left[f(x_1) + f(x_2) + f(x_3)\right]$



b) Sol dikdörtgen



$h = \dfrac{b-a}{n}$

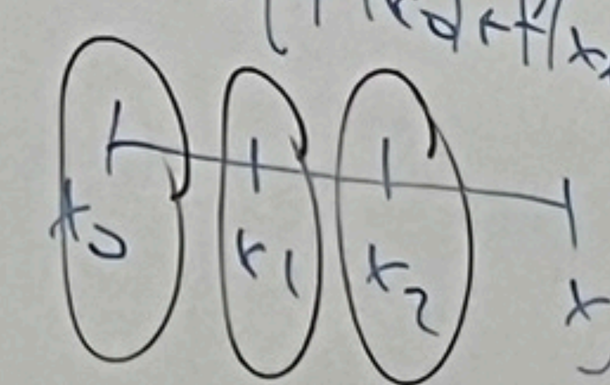
3 aldık

$I_1 = h \cdot f(x_0)$

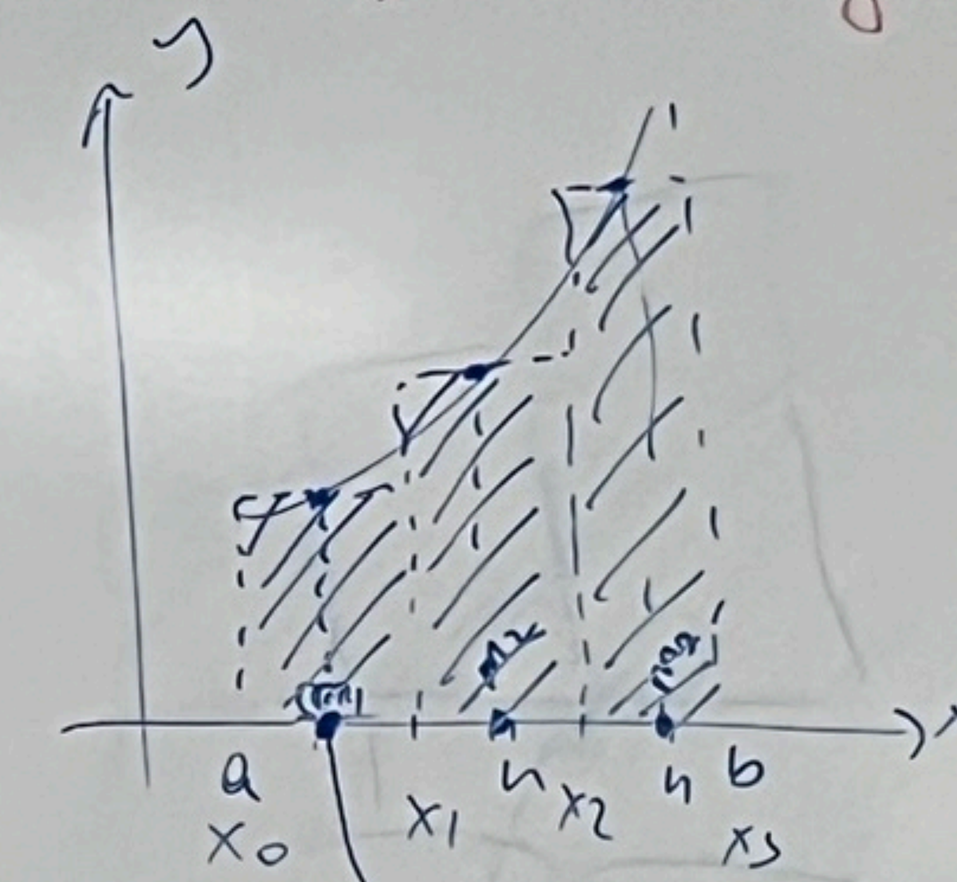
$I_2 = h \cdot f(x_1)$

$I_3 = h \cdot f(x_2)$

$I = h\left[f(x_0) + f(x_1) + f(x_2)\right]$



c) Orta dikdörtgen



$h = \dfrac{b-a}{n}$

3 aldık

$m_1 = \dfrac{x_0 + x_1}{2}$

$m_2 = x_0 + \dfrac{h}{2}$

$m_1 = x_1 - \dfrac{h}{2}$

$I_1 = h f\left(x_0 + \frac{h}{2}\right)$

$I_2 = h f\left(x_1 + \frac{h}{2}\right)$

$I_3 = h f\left(x_2 + \frac{h}{2}\right)$

$I = h\left[f\left(x_0 + \frac{h}{2}\right) \ldots + \right]$

Sayısal integral

ör: $\int_{1}^{8} \frac{x+2}{x^2+2}\,dx$ integrali 7 alt aralık (parçalı)

$f(x)=\frac{x+2}{x^2+2}$ için yaklaşık olarak 1. nokta yaklaşımları ile hesaplayın–

$\frac{8-1}{7} = 1 = h$

↳ noktalar arasındaki eşit uzaklık

1	2	3	4	5	6	7	8
x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7

a) Sağ dikdörtgen

$I_{sağ} = h\left[f(x_1)+f(x_2)+\dots f(x_6)+f(x_7)\right]$

kısa kenarı

uzun kenar.

$=1\left[f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)\right]$

$=1\left[\frac{4}{6}+\frac{5}{11}+\dots+\frac{9}{51}+\frac{10}{66}\right]$

b) Sol dikdörtgen

$I_{sol} = h\left[f(x_0)+f(x_1)+f(x_2)+\dots+f(x_5)+f(x_6)\right]$

$=1\left[f(1)+f(2)+f(3)+\dots f(6)+f(7)\right]$

$=1\left[\frac{3}{3}+\frac{4}{6}+\frac{5}{11}+\dots+\frac{9}{51}\right]$

c) Orta dikdörtgen

$I_{orta} = h\left[f\left(\frac{x_0+x_1}{2}\right)+f\left(\frac{x_1+x_2}{2}\right)+\dots+f\left(\frac{x_6+x_7}{2}\right)\right]$

$=1\left[f(1.5)+f(2.5)+f(3.5)+\dots f(6.5)+f(7.5)\right]$

$=1.\left[\frac{1.5+2}{(1.5)^2+2}+\frac{2.5+2}{(2.5)^2+2}+\dots+\frac{7.5+2}{(7.5)^2+2}\right]$

Sayısal integral

Ör: $\int_{5}^{25} f(x)\,dx$ integrali yaklaşık olarak 5 alt aralık için dikdörtgen yöntemi ile bulun

$$\begin{array}{cccccc} & 7 & 11 & 15 & 19 & 23 \\ 5 & 9 & 13 & 17 & 21 & 25 \\ x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$$\frac{25-5}{5} = 4 = h$$

$* I_{sağ} = h\left[f(x_1)+f(x_2)+\dots+f(x_5)\right]$

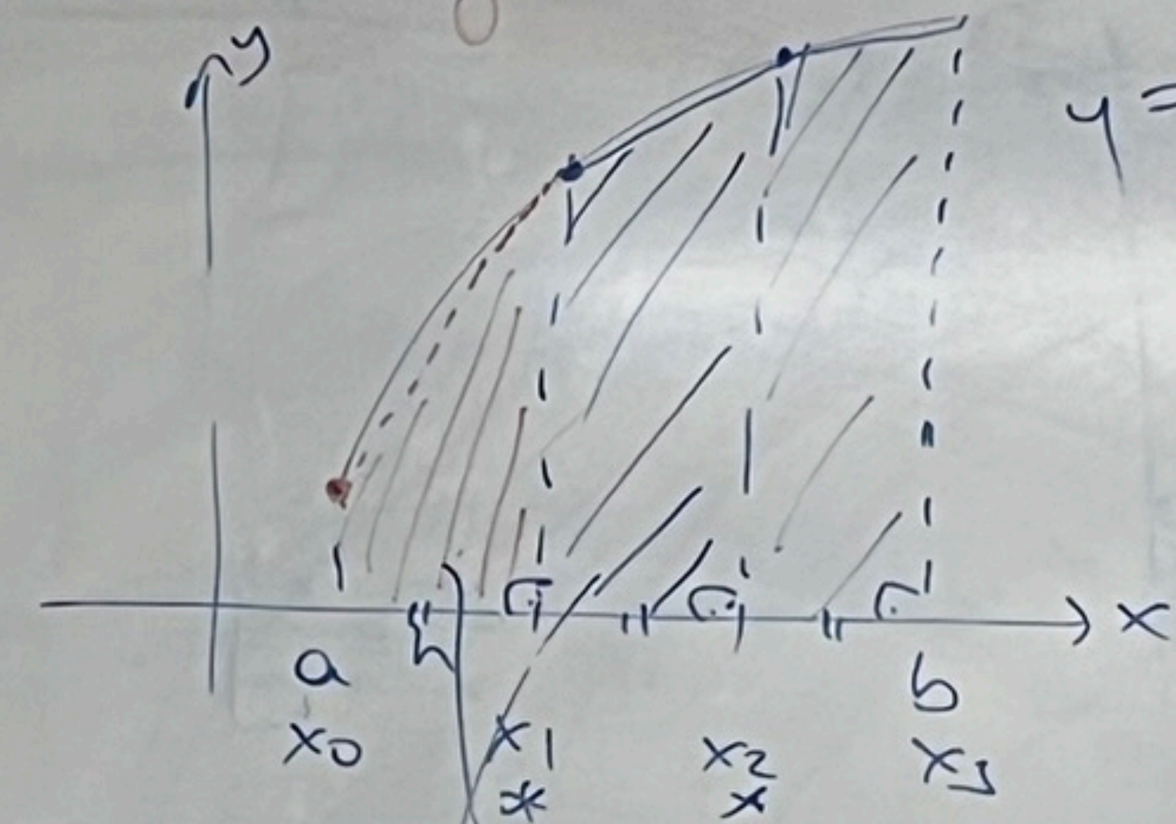
$= 4\left[f(9)+f(13)+f(17)+f(21)+f(25)\right]$

$* I_{sol} = h\left[f(x_0)+f(x_1)+\dots+f(x_4)\right]$

$= 4\left[f(5)+f(9)+f(13)+f(17)+f(21)\right]$

$* I_{orta} = 4\left[f(7)+f(11)+f(15)+f(19)+f(23)\right]$

II 2 nokta yaklaşımı YAMUK/TRAPEZ



$$\frac{b-a}{n} = h$$

$n=3$ ald.

$I_1 = \frac{f(x_0)+f(x_1)}{2}\cdot h$ yamuk

$I_2 = \frac{f(x_1)+f(x_2)}{2}\cdot h$

$I_3 = \frac{f(x_2)+f(x_3)}{2}\cdot h$

$$I_{yamuk} = \frac{h}{2}\left[f(x_0)+2\left(f(x_1)+f(x_2)\right)+f(x_3)\right]$$

n parça

$$x_0 \quad x_1 \quad x_2 \quad \dots \quad x_{n-1} \quad x_n$$

$$I_{yamuk} = \frac{h}{2}\left[f(x_0)+f(x_n)+2\left(f(x_1)+f(x_2)+\dots+f(x_{n-1})\right)\right]$$

Ör: $\int_{5}^{25} f(x)\,dx$ int. yaklaşık olarak 2 nokta yak. ile hesaplayınız (5 parça için)

$$h = \frac{25-5}{5} = 4$$

$$\begin{array}{cccccc} 5 & 9 & 13 & 17 & 21 & 25 \\ x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$$I_{\substack{TRAPEZ \\ Yamuk}} = \frac{h}{2}\left[f(x_0)+f(x_5)+2\left(f(x_1)+f(x_2)+f(x_3)+f(x_4)\right)\right]$$

$$= 2\left[f(5)+f(25)+2\left(f(9)+f(13)+f(17)+f(21)\right)\right]$$

$$I_{TRAPEZ} = \frac{I_{sağ}+I_{sol}}{2}$$

Sayısal integral

Ör: $\int_1^2 \left(x+\frac{1}{x}\right)^2$ integralini $n=1$, $n=2$, $n=4$ için yamuk yön. ile hesaplayınız. $f(x)=\left(x+\frac{1}{x}\right)^2$

$h=\frac{2-1}{1}=1$

1 (x_0) — 2 (x_1)

$n=1$: $I_{trapez}=\frac{1}{2}\left[f(1)+f(2)\right]=5.125$

$n=2$: $x_0=1$, $x_1=1.5$, $x_2=2$

$\frac{2-1}{2}=\frac{1}{2}=h$. $I_{trapez}=\frac{1}{4}\left[f(1)+f(2)+2f(1.5)\right]=4.910$

$n=4$: 1 (x_0), 1.25 (x_1), 1.5 (x_2), 1.75 (x_3), 2 (x_4)

$\frac{2-1}{4}=\frac{1}{4}=0.25$ I_{trapez}

II. 2 nokta yaklaşımı YAMUK/TRAPEZ

$n=4$: $I_{trapez}=\frac{1}{8}\left[f(1)+f(2)+2\left(f(1.25)+f(1.5)+f(1.75)\right)\right]$

⋮

$n=100$ 4.833

$\int_1^2 \left(x+\frac{1}{x}\right)^2 dx = 4.833$