

Complex Variables Section 28 Homework

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Suppose $f(z)$ is analytic on D . Prove that $F(z) = \overline{f(\bar{z})}$ is also analytic on D .

Proof. Suppose $f(z)$ is analytic on some domain D . Then letting $F(z) = Q(x, y) + iW(x, y)$ and $f(z) = u(x, y) + iv(x, y)$,

$$F(z) = \overline{f(\bar{z})}$$

$$Q(x, y) + iW(x, y) = \overline{f(x - iy)}$$

$$Q(x, y) + iW(x, y) = \overline{u(x, -y) + iv(x, -y)}$$

$$Q(x, y) + iW(x, y) = u(x, -y) - iv(x, -y)$$

Hence,

$$Q(x, y) = u(x, -y) \quad W(x, y) = -v(x, -y)$$

So

$$Q_x = u_x$$

$$Q_y = -u_y$$

$$W_x = -v_x$$

$$W_y = v_y$$

Thus, since $u_x = v_y$ by the fact that f is analytic on D ,

$$Q_x = u_x = v_y = W_y$$

Similarly, since $u_y = -v_x$,

$$Q_y = -u_y = -(-v_x) = -W_x$$

And thus $F(z)$ also satisfies the Cauchy-Riemann equations on D , and so it is analytic on D also. \square