Complex Variables Section 57 Homework

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1 Problem 1

$$f(z) = \begin{cases} z^{-1}\sin(z) & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

On $\mathbb{C} \setminus \{0\}$,

$$\begin{aligned} f(z) &= z^{-1}\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+1}}{(2n+1)!} \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{z^{2n}}{(2n+1)!} \\ &= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cdot\frac{z^{2n}}{(2n)!} \end{aligned}$$

which can be naturally extended to all of $\mathbb{C}$.

2 Problem 2

In the first derivative, the first term drops out.

$$\begin{aligned} f'(z) &= \sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}\cdot\frac{z^{2n-1}}{(2n-1)!} \\ f^{(2)}(z) &= \sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}\cdot\frac{z^{2n-2}}{(2n-2)!} \end{aligned}$$

In the third derivative, another term is forced to drop out.

$$\begin{aligned} f^{(3)}(z) &= \sum_{n=2}^{\infty}\frac{(-1)^n}{2n+1}\cdot\frac{z^{2n-3}}{(2n-3)!} \\ f^{(4)}(z) &= \sum_{n=2}^{\infty}\frac{(-1)^n}{2n+1}\cdot\frac{z^{2n-4}}{(2n-4)!} \end{aligned}$$

Thus, $f^{(4)}(0) = (-1)^2(2(2)+1)^{-1}(0!)^{-1} = 1/5 = 0.2$.