Complex Variables Homework Section 53

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March 23, 2016

1 Problem 1

Evaluate

$$\int_C \frac{e^{2z}}{z^{k+1}}\, dz$$

for $k = -1, 0, 1, 2, 3$ and C the unit circle.

For $k = -1$, then we have

$$\int_C \frac{e^{2z}}{z^0}\, dz = \int_C e^{2z}\, dz = 0$$

Since in this case e^{2z} is entire, the integral is 0.

For $k = 0$,

$$\int_C \frac{e^{2z}}{(z-0)^1}\, dz = \frac{2\pi i}{0!} f^{(0)}(z_0) = 2\pi i e^{2(0)} = 2\pi i$$

For $k = 1$,

$$\int_C \frac{e^{2z}}{(z-0)^2}\, dz = \frac{2\pi i}{1!} f'(0) = 2\pi i \cdot 2e^{2(0)} = 4\pi i$$

For $k = 2$,

$$\int_C \frac{e^{2z}}{(z-0)^3}\, dz = \frac{2\pi i}{2!} f''(0) = \frac{2\pi i}{2} \cdot 4e^{2(0)} = 4\pi i$$

For $k = 3$,

$$\int_C \frac{e^{2z}}{(z-0)^4}\, dz = \frac{2\pi i}{3!} f^{(3)}(0) = \frac{2\pi i}{6} \cdot 8e^{2(0)} = \frac{8\pi i}{3}$$

2 Problem 2

Evaluate

$$\int_C \frac{z^2 + \sin z}{(z-1)^3}\, dz$$

where C is any contour around 1.

$$\int_C \frac{z^2 + \sin z}{(z-1)^3}\, dz = \frac{2\pi i}{2!} f''(1) = \pi i\, [2 - \sin 1] = 2\pi i - \pi i \sin(1)$$