Homework 2016-04-12

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April 15, 2016

1 Problem 1

Evaluate

$$\int_C \frac{z+1}{z-1}\,dz$$

where C is a positively oriented contour around 1.

2 Problem 2

Find the Laurent series for

$$f(z) = \frac{2}{1-z^2} = \frac{1}{1-z} + \frac{1}{1+z}$$

We utilize example 4 in the text, on page 194.

$$\begin{aligned}
f(z) &= \frac{1}{1-z} + \frac{1}{1+z} \\
&= \sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty} (-z)^n \\
&= \sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty} (-1)^n z^n \\
&= \sum_{n\in 2\mathbb{N}} z^n && \text{(Note the change of index! We assume } 0 \in \mathbb{N}.\text{)} \\
&= \sum_{n=0}^{\infty} z^{2n} \\
&= \sum_{n=0}^{\infty} (z^2)^n \\
&= 2\frac{1}{1-z^2} \\
&= \frac{2}{1-z^2} \\
&= f(z)
\end{aligned}$$