Complex Variables Homework Section 29

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p. 92 #6. Find $\log(-1/\sqrt{2} + i/\sqrt{2})$.

1 Problem 6

Theorem 1.1.

$$\left|\exp(z^2)\right| \leq \exp(|z|^2)$$

Proof. Consider an arbitrary $z = x + yi$.

$$\begin{aligned}
\left|\exp(z^2)\right| &= \left|\exp(x^2 - y^2 + xyi)\right| \\
&= \left|\exp(x^2 - y^2)\right| \cdot |\cos(xy) + i\sin(xy)| \\
&= \exp(x^2 - y^2) \cdot \sqrt{\cos^2(xy) + \sin^2(xy)} \\
&= \exp(x^2 - y^2) \cdot 1 \\
&= \exp(x^2 - y^2) \\
&\leq \exp(x^2 + y^2) && \text{(exp is increasing on } \mathbb{R}\text{)} \\
&= \exp(|z|^2)
\end{aligned}$$

□

2 Finding logarithm

$$\begin{aligned}
\log(-1/\sqrt{2} + i/\sqrt{2}) &= \ln\left|\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right| + i\left(\frac{3\pi}{4} + 2\pi k\right) \\
&= \ln\sqrt{\frac{1}{2} + \frac{1}{2}} + i\left(\frac{3\pi}{4} + 2\pi k\right) \\
&= \ln 1 + i\left(\frac{3\pi}{4} + 2\pi k\right) \\
&= \frac{3}{4}\pi i + 2\pi i k
\end{aligned}$$