Complex Variables Assignment Section 68

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Show that

$$\int \frac{\sin t}{17 + 8\cos 2t}\, dt = -\frac{1}{12}\tan^{-1}\left(\frac{4}{3}\cos t\right) + C$$

and

$$\int \frac{\cos t}{17 + 8\cos 2t}\, dt = \frac{1}{40}\ln|5 + 4\sin t| - \frac{1}{40}\ln|5 - 4\sin t| + C$$

1 Problem 1

$$\begin{aligned}
\int \frac{\sin t}{17 + 8\cos 2t}\, dt &= \int \frac{\sin t}{17 + 8(2\cos^2 t - 1)}\, dt \\
&= \int \frac{\sin t}{17 - 8 + 16\cos^2 t}\, dt \\
&= \int \frac{\sin t}{9 + 16\cos^2 t}\, dt && \text{(Sub. } u(t) = \cos t,\ u'(t) = -\sin t\text{)} \\
&= -\int \frac{1}{9 + 16u^2} \cdot \left(-\frac{du}{dt}\, dt\right) \\
&= -\int \frac{1}{9 + 16u^2}\, du \\
&= -\frac{1}{16}\int \frac{1}{9/16 + u^2}\, du \\
&= -\frac{1}{16}\left[\frac{1}{3/4}\tan^{-1}\left(\frac{u}{3/4}\right) + C\right] \\
&= -\frac{1}{16}\left[\frac{4}{3}\tan^{-1}\left(\frac{4}{3}u\right) + C\right] \\
&= -\frac{1}{12}\tan^{-1}\left(\frac{4}{3}\cos t\right) + C
\end{aligned}$$

2 Problem 2

$$\begin{aligned}
\int \frac{\cos t}{17 + 8\cos 2t}\, dt &= \int \frac{\cos t}{17 + 8(1 - 2\sin^2 t)}\, dt \\
&= \int \frac{\cos t}{25 - 16\sin^2 t}\, dt \\
&= \int \frac{du/dt}{25 - 16u^2}\, dt && (u = \sin t,\ du/dt = \cos t) \\
&= \frac{1}{16} \int \frac{1}{25/16 - u^2}\, du \\
&= \frac{1}{16} \left[\frac{1}{2(5/4)} \ln \left| \frac{u + 5/4}{u - 5/4} \right| + C \right] \\
&= \frac{1}{40} \ln |u + 5/4| - \frac{1}{40} \ln |u - 5/4| + C \\
&= \frac{1}{40} \ln |\sin t + 5/4| - \frac{1}{40} \ln |\sin t - 5/4| + C
\end{aligned}$$