Complex Variables Homework Section 17

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February 1, 2016

Prove $\lim_{z\to\infty} \frac{|z|}{z}$ is not defined.

Theorem 0.1. $\lim_{z\to\infty} \frac{|z|}{z}$ *is not defined.*

Proof. The sequences (n) and (in) both have infinity as their limit, yet

$$\lim_{z\to\infty} \frac{|n|}{n} = \lim_{z\to\infty} \frac{n}{n} = \lim_{z\to\infty} 1$$

$$\lim_{z\to\infty} \frac{|in|}{in} = \lim_{z\to\infty} \frac{n}{in} = \lim_{z\to\infty} \frac{1}{i}\frac{i}{i} = \lim_{z\to\infty} \frac{i}{-1} = -i$$

□

Find $\frac{d}{dz} z^{-1}$.

Theorem 0.2.

$$\frac{d}{dz} z^{-1} = -\frac{1}{z^2}$$

Proof.

$$\begin{aligned}
\frac{d}{dz} z^{-1} &= \lim_{h\to 0} \frac{(z+h)^{-1} - (z)^{-1}}{h} \\
&= \lim_{h\to 0} \frac{\frac{z}{z^2+hz} - \frac{z+h}{z^2+hz}}{h} \\
&= \lim_{h\to 0} \frac{\frac{z-z-h}{z^2+hz}}{h} \\
&= \lim_{h\to 0} \frac{-h}{z^2 h + h^2 z} \\
&= \lim_{h\to 0} \frac{-1}{z^2 + zh} \\
&= -\frac{1}{z^2}
\end{aligned}$$

□