Complex Variables 2016-03-31

nth term test

If $\sum z_n$ converges then $z_n \to 0$.

absolute converges.

If $\sum |z_n|$ converges then we say that $\sum z$ converges absolutely.

ratio test

$$\lim_{n\to\infty} \left| \frac{z_{n+1}}{z_n} \right| < 1 \Rightarrow \text{converges absolutely}$$

$$\text{" } \quad \text{" } > 1 \to \text{diverges}$$

$$\text{" } \quad \text{" } = 1 \to \text{who knows?}$$

Does $\sum 1/n^{2+i}$ converge?

for $n^{2+i} = \exp((2+i)\operatorname{Log} n)$

$$= \exp(2\ln n + i\ln n)$$

$$= n^2 \cdot (\cos(\ln n) + i\sin(\ln n))$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2(\cos\ln n + i\sin\ln n)}$$

nth term test says nothing.

absolute convergence:

$$\sum_{n=1}^{\infty} \left|\frac{1}{n^{2+i}}\right| = \sum_{n=1}^{\infty} \left|\frac{1}{n^2 \exp(i\ln n)}\right| = \sum \frac{1}{n^2},$$

which converges. In general,

$$\sum_{n=1}^{\infty} \frac{1}{n^{x+yi}}$$ converges absolutely if $x > 1$.

For the ratio! and the root test, if the limits don't exist then you can use the lim sup.

$$L = \limsup_{n\to\infty} \left|\frac{z_{n+1}}{z_n}\right| \quad \text{or} \quad L = \limsup_{n\to\infty} \sqrt[n]{|z_n|}$$

Recall: the geometric series $\sum_{n=0}^{\infty} ar^n$ converges if $|r|<1$ to $a/(1-r)$.

In particular $\sum_{n=0}^{\infty} x^n \to \frac{1}{1-x}$. Radius of convergence $(-1,1)$.

$\sum_{n=0}^{\infty} (-x)^n \to \frac{1}{1+x}$ $\quad |x|<1$ $\quad \Big| \quad$ $\sum_{n=1}^{\infty} x^{2n} \to \frac{1}{1-x^2}$ $\quad |x|<1$

$\sum_{n=0}^{\infty} (-x^2)^n \to \frac{1}{1+x^2}$ $\quad |x|<1$

$$u(x,y) = \frac{1}{1-x-yi} = \frac{1}{1-x-yi} \cdot \frac{1-x+yi}{1-x+yi} = \frac{1-x+yi}{(1-x)^2+y^2}$$

$$u_x = \frac{d}{dx}(1-x+yi)\left((1-x)^2+y^2\right)^{-1}$$

$$= -\frac{1}{(1-x)^2+y^2} - (1-x+yi)(-1)\left((1-x)^2+y^2\right)^{-2}(2-2x)(-1)$$

$$= -\frac{1}{(1-x)^2+y^2} - \frac{(1-x+yi)(2-2x)}{\left((1-x)^2+y^2\right)^2}$$

$$= -\frac{(1-x)^2+y^2+(1-x+yi)(2-2x)}{((1-x)^2+y^2)^2}$$

$$= -\frac{(1-x)^2+y^2+2-2x-2x+2x^2+2yi-2xyi}{((1-x)^2+y^2)^2}$$

$$= -\frac{(1-x)^2+y^2+2(1-x)^2+2yi(1-x)}{((1-x)^2+y^2)^2}$$

$$= -\frac{3(1-x)^2+y^2+2yi(1-x)}{((1-x)^2+y^2)^2}$$

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§7 Taylor Series

Thm Suppose f is analytic in the disk $|z-z_0|<R$. Then f has power series rep. $\sum_{n=0}^{\infty} a_n(z-z_0)^n \quad (|z-z_0|<R)$ where $a_n = \frac{f^{(n)}(z_0)}{n!}$.

HW: p188 #3

Also: prove if $|z_n| \to 0$, then $z_n \to 0$.

Converge or diverge: $\sum_{k=1}^{\infty} \left(\frac{i}{2}\right)^k \qquad \sum_{k=1}^{\infty} (1-i)^k$