# Seppi's Topology Notes

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# Contents

# Chapter 1

# Manifolds

## 1.1 Topological Manifolds

## 1.2 Smooth Manifolds

## 1.3 Cobordism

## 1.4 Smooth Maps

If $M$ and $N$ are topological manifolds, the natural notion of a morphism is $f : M \to N$ continuous. Then two topological manifolds $M$, $N$, are equivalent if there exists a homeomorphism $f : M \to N$. Topological manifolds together with continuous maps form a category.

**Definition 1.4.1.** A **category** consists of objects $C$ and arrows $A$. Each arrow goes from some object $C$ (the source) to another object (the target). These arrows are often called **morphisms**. Furthermore, for each object $c \in C$ there exists a morphism $1_c \in A$ such that the source and target of $1_c$ are both $c$. In addition, morphisms can be composed and composition satisfies additivity. For $x, y \in C$ we write $\mathrm{Hom}(x, y) \subset A$ as the set of morphisms from $x$ to $y$. Stated this way, composition becomes

$$\circ : \mathrm{Hom}(x, y) \times \mathrm{Hom}(y, z) \to \mathrm{Hom}(x, z)$$

$$\circ : (f, g) \mapsto g \circ f$$

**Example 1.4.2.** The category of topological manifolds is defined by $C$ being the class of all topological manifolds with arrows $A$ being continuous maps between them.

For smooth manifolds, we need the correct notion of a morphism.

**Definition 1.4.3.** A map $f : M \to N$, $M$, $N$ smooth manifolds, is called **smooth** when for each chart $(U, \varphi)$ for $M$ and each chart $(V, \psi)$ for $N$, the composition

$$\psi \circ f \circ \varphi^{-1} \in C^\infty(\varphi(U), \mathbb{R}^n).$$

The set of smooth maps from $M$ to $N$ is denoted $C^\infty(M, N)$. A smooth map with smooth inverse is called a **diffeomorphism**.

**Lemma 1.4.4.** *If $g : L \to M$ and $f : M \to N$ are smooth maps, then so is $f \circ g : L \to N$.*

# Chapter 2

# Tangent Bundle

# Chapter 3

# Transversality

# Chapter 4

# Vector Fields

## Chapter 5

# Differential Forms