

Homework 1 - Math basics & 2D Transformations

CENG315 Computer Graphics – Fall 2017, Abdullah Bulbul

Due date: Tuesday, November 1 - Class hour. You may type or handwrite your work.

Q1) For given vectors $\mathbf{i}=(0.8, 0.6, 0)$, $\mathbf{j}=(-0.6, 0.8, 0)$, and $\mathbf{k}=(1, 0, 0)$ in 3D space; find the results of the following. ($\mathbf{i} \cdot \mathbf{i}$ means the dot product between \mathbf{i} and \mathbf{i} , $\mathbf{i} \times \mathbf{i}$ means the cross product between \mathbf{i} and \mathbf{i})

a) $\mathbf{i} \cdot \mathbf{j} =$

b) $\mathbf{i} \cdot \mathbf{k} =$

c) $\mathbf{i} \times \mathbf{k} =$

d) $\mathbf{i} \times \mathbf{i} =$

e) $||\mathbf{i} \times \mathbf{k}|| =$

f) $2\mathbf{i} \rightarrow \mathbf{k} =$

(projection of $2\mathbf{i}$ onto \mathbf{k})

Q2) Two vectors in the plane, \mathbf{i} and \mathbf{j} , have the following properties: $\mathbf{i} \cdot \mathbf{i} = 1$, $\mathbf{i} \cdot \mathbf{j} = 0$, $\mathbf{j} \cdot \mathbf{j} = 1$.

- a) Is there a vector \mathbf{k} , that is not equal to \mathbf{i} , such that: $\mathbf{k} \cdot \mathbf{k} = 1$, $\mathbf{k} \cdot \mathbf{j} = 0$? What is it? Are there many vectors with these properties?
- b) Is there a vector \mathbf{k} such that: $\mathbf{k} \cdot \mathbf{k} = 1$, $\mathbf{k} \cdot \mathbf{j} = 0$, $\mathbf{k} \cdot \mathbf{i} = 0$? Why not?
- c) If \mathbf{i} and \mathbf{j} were vectors in 3D, how would the answers to the above questions change?

Q3) For three points on the plane (x_1, y_1) , (x_2, y_2) and (x_3, y_3) show that the determinant of

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

is proportional to the area of the triangle whose corners are the three points. If these points lie on a straight line, what is the value of the determinant? Does this give a useful test to tell whether three points lie on a line? Why do you think so?

Q4) The equation of a line in the plane is $ax + by + c = 0$. Given two points on the plane, show how to find the values of a , b , c for the line that passes through those two points. You may find the answer to question 3 useful here.

Q5) Give three different composite 2D transforms so that each one achieves the given transformations below. (Use any number of translations, scales, or rotations for a composite transformation. Use $t(x,y)$ for translation, $s(x,y)$ for scale, and $r(\alpha)$ for rotation. Positive rotation is counter-clockwise. Note that order of transformations is important.)

