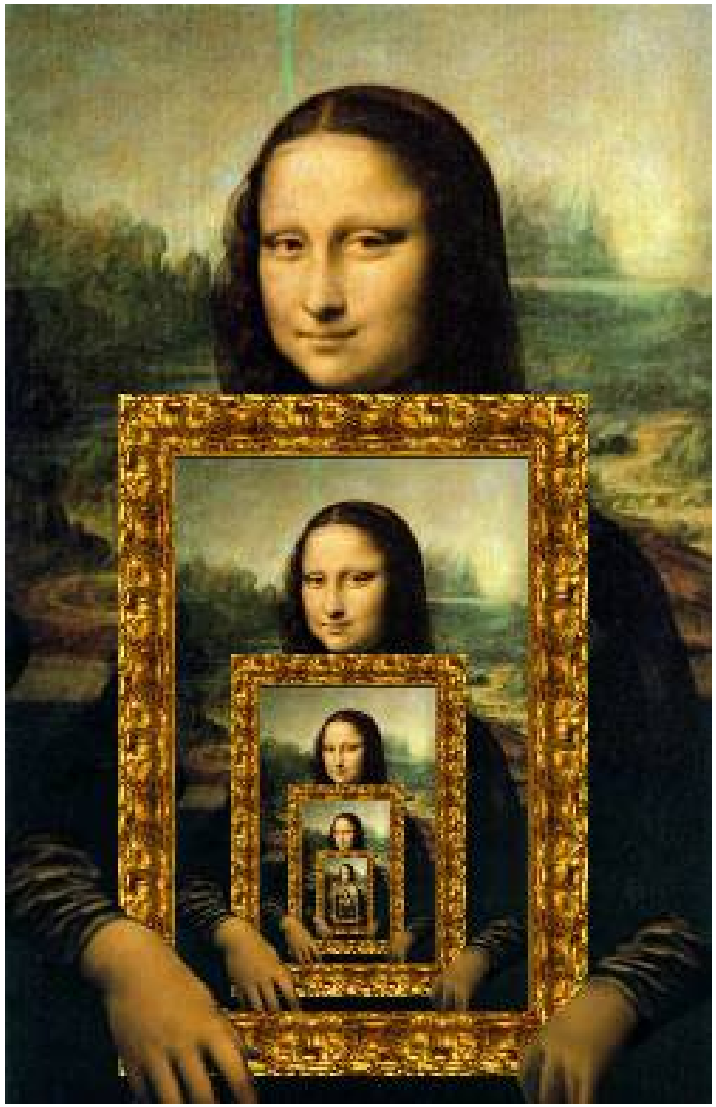
Recursão

Marcelo Hashimoto

Recursão



Um algoritmo é recursivo quando chama a si próprio.

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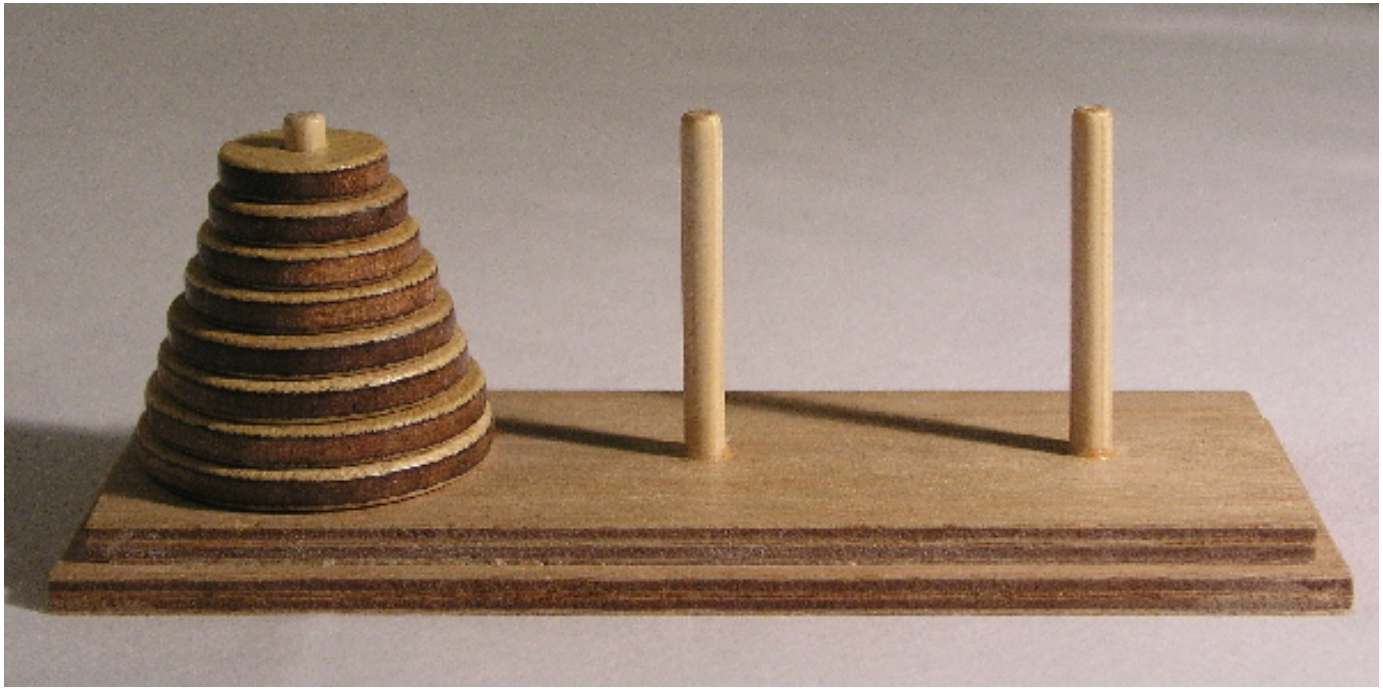
(mais especificamente, versões menores de si próprio)

Torres de Hanói



Torres de Hanói

- 3 torres e 64 discos de tamanhos diferentes.

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- Todos os discos começam na primeira torre. (origem)

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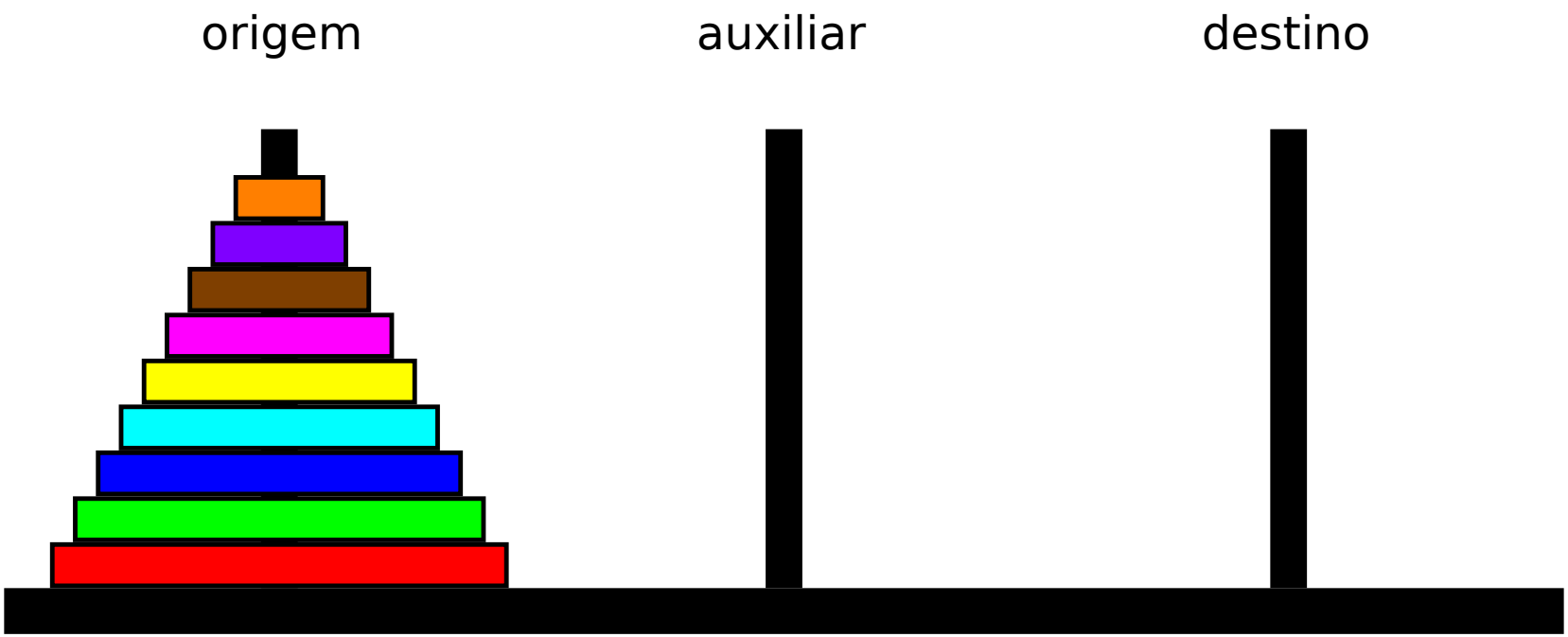
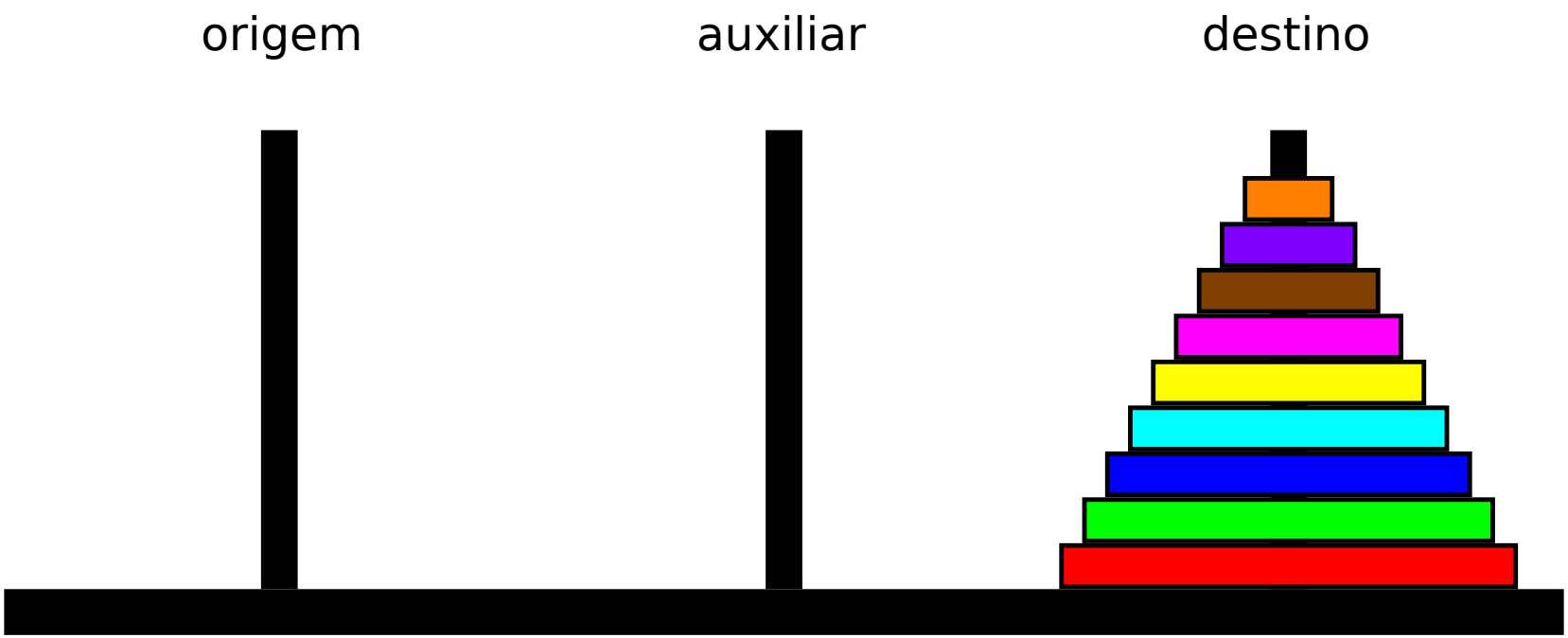
- 3 torres e 64 discos de tamanhos diferentes.
- Todos os discos começam na primeira torre. (origem)
- O objetivo é movê-los para a terceira torre. (destino)
- A segunda torre pode ser utilizada para ajudar. (auxiliar)

Torres de Hanói



Torres de Hanói



Torres de Hanói

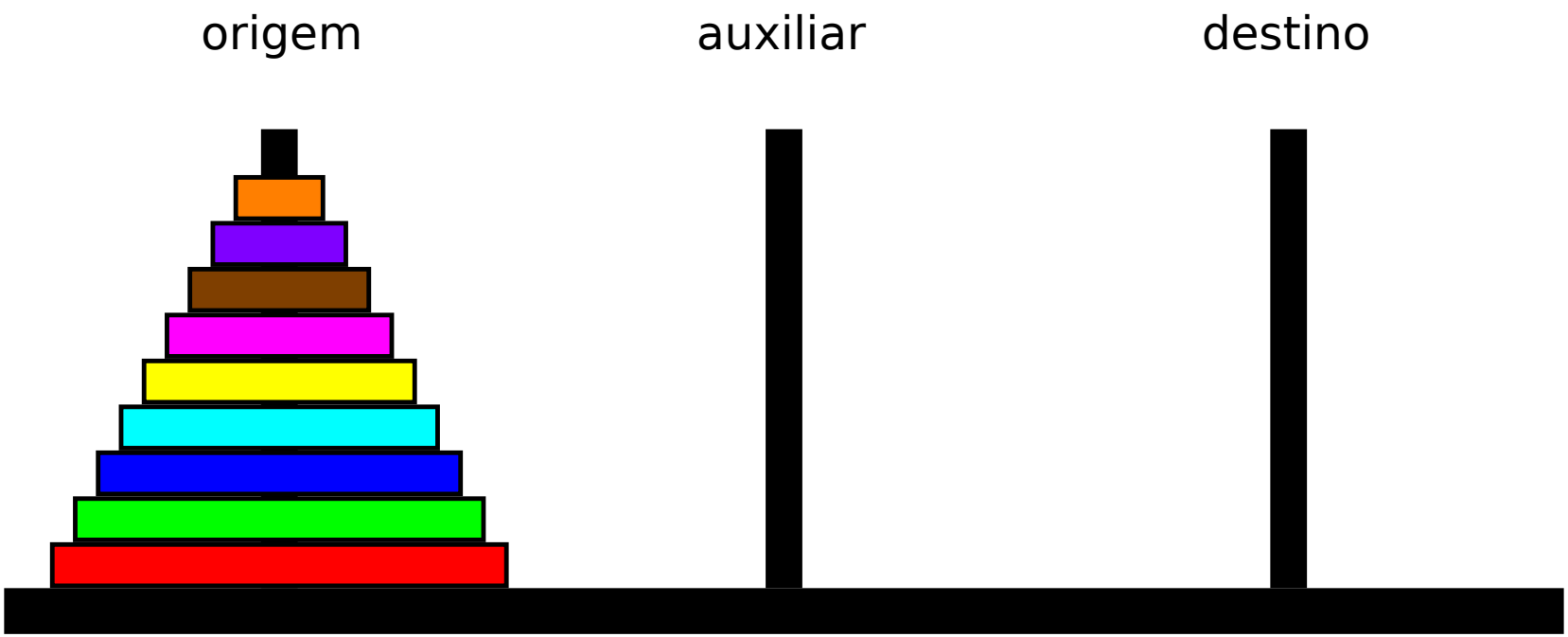
- Apenas um disco pode ser movido de cada vez.

Torres de Hanói

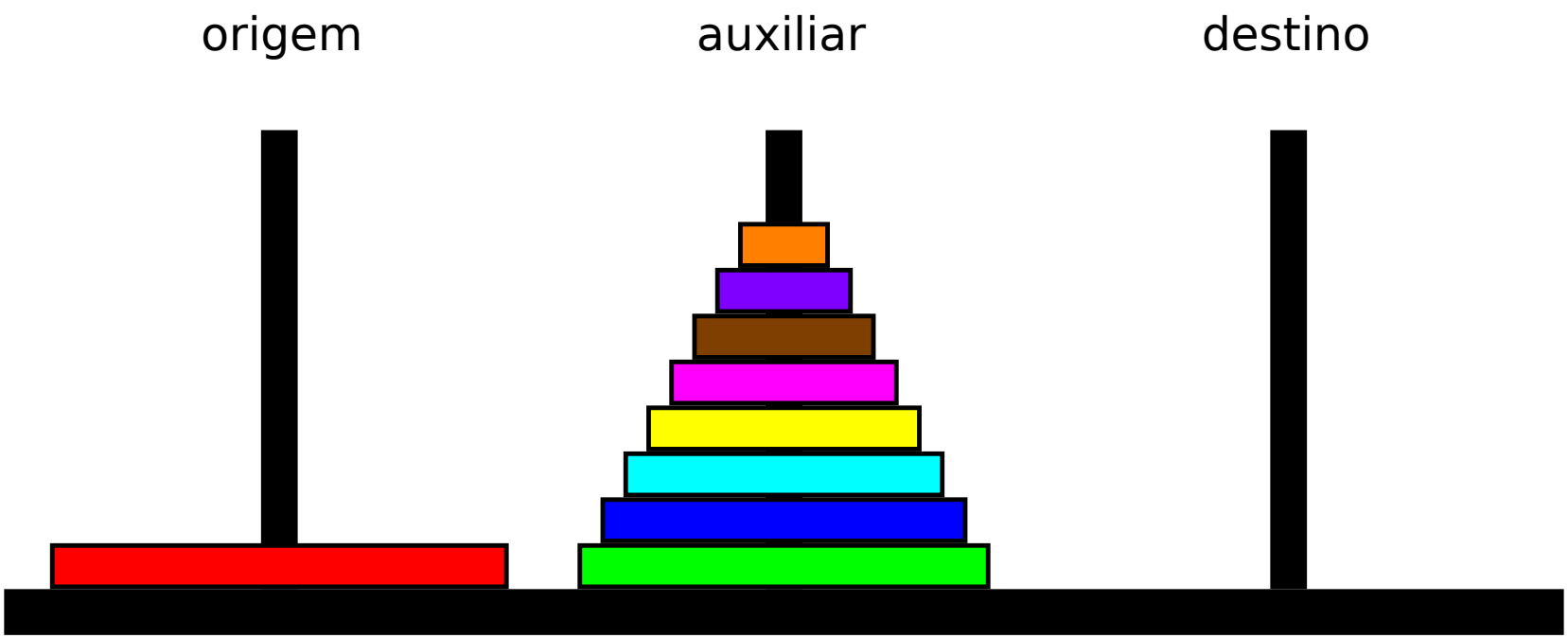
- Apenas um disco pode ser movido de cada vez.

- Um disco maior nunca pode ficar em cima de um menor.

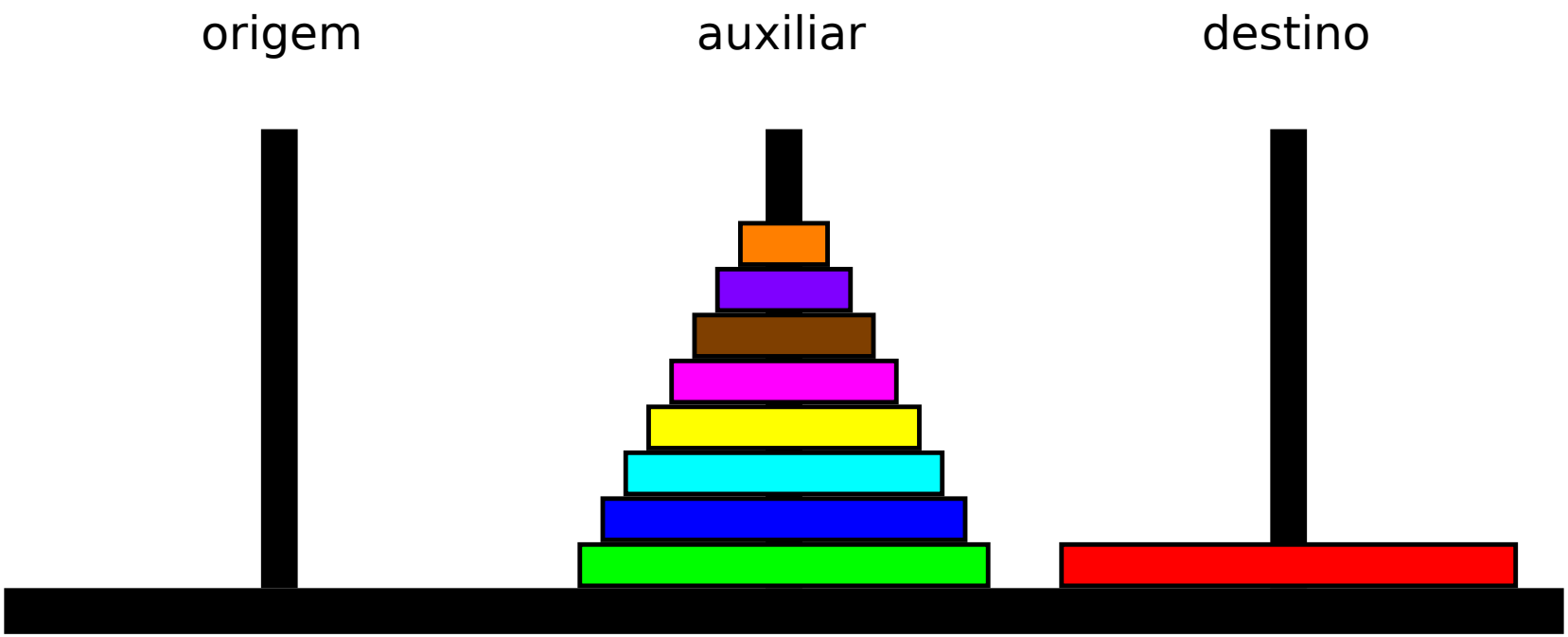
Torres de Hanói



Torres de Hanói



Torres de Hanói



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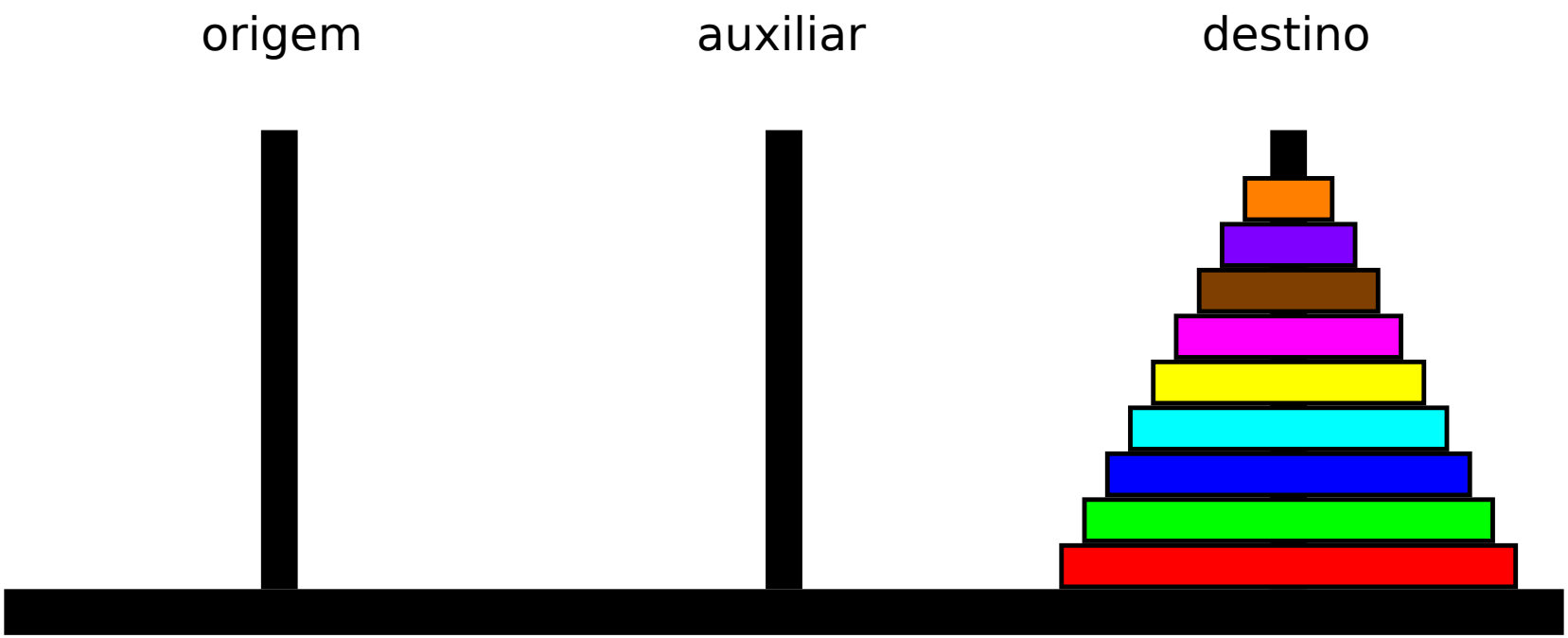

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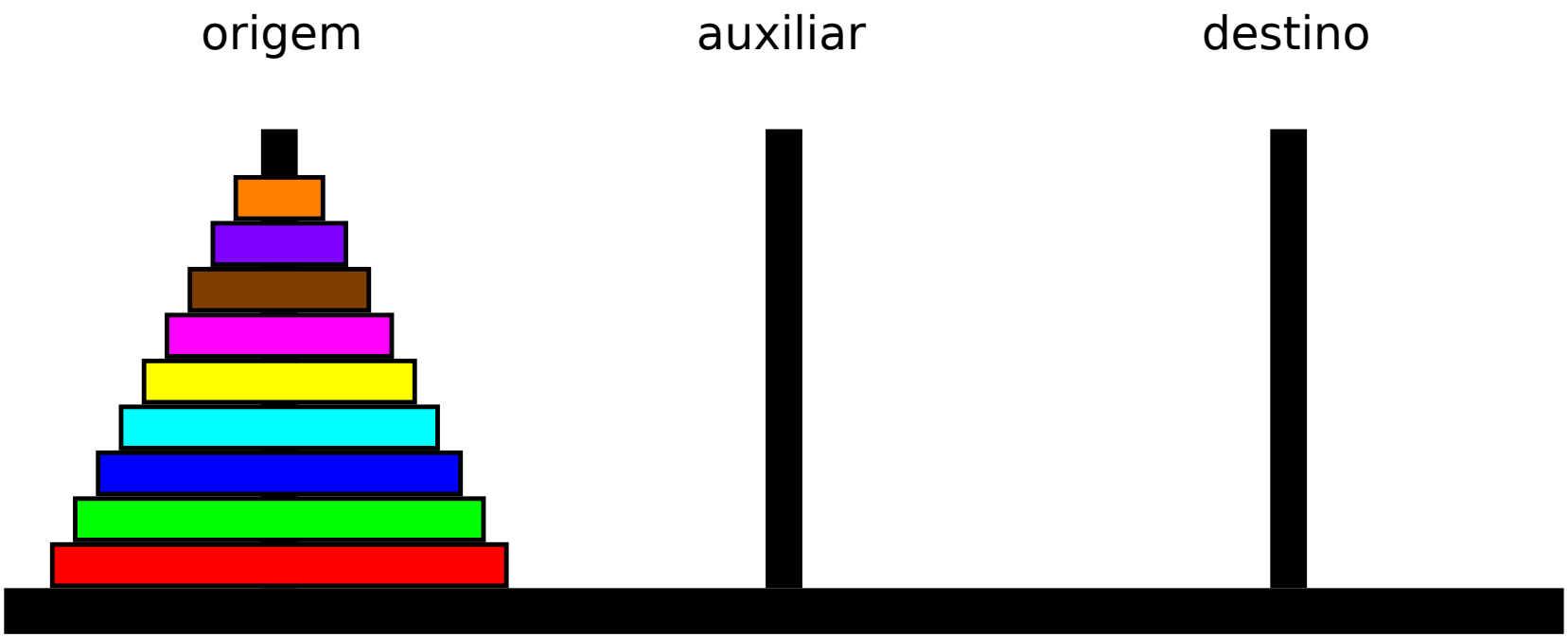

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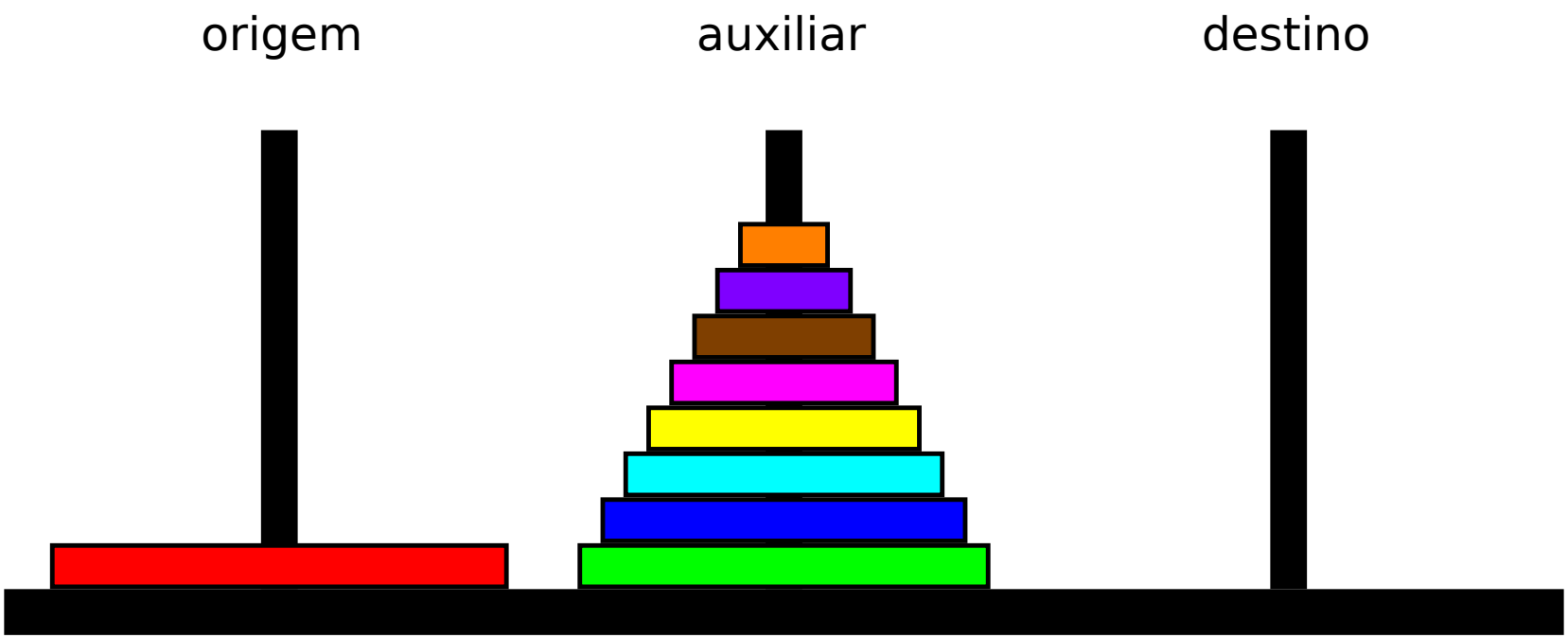

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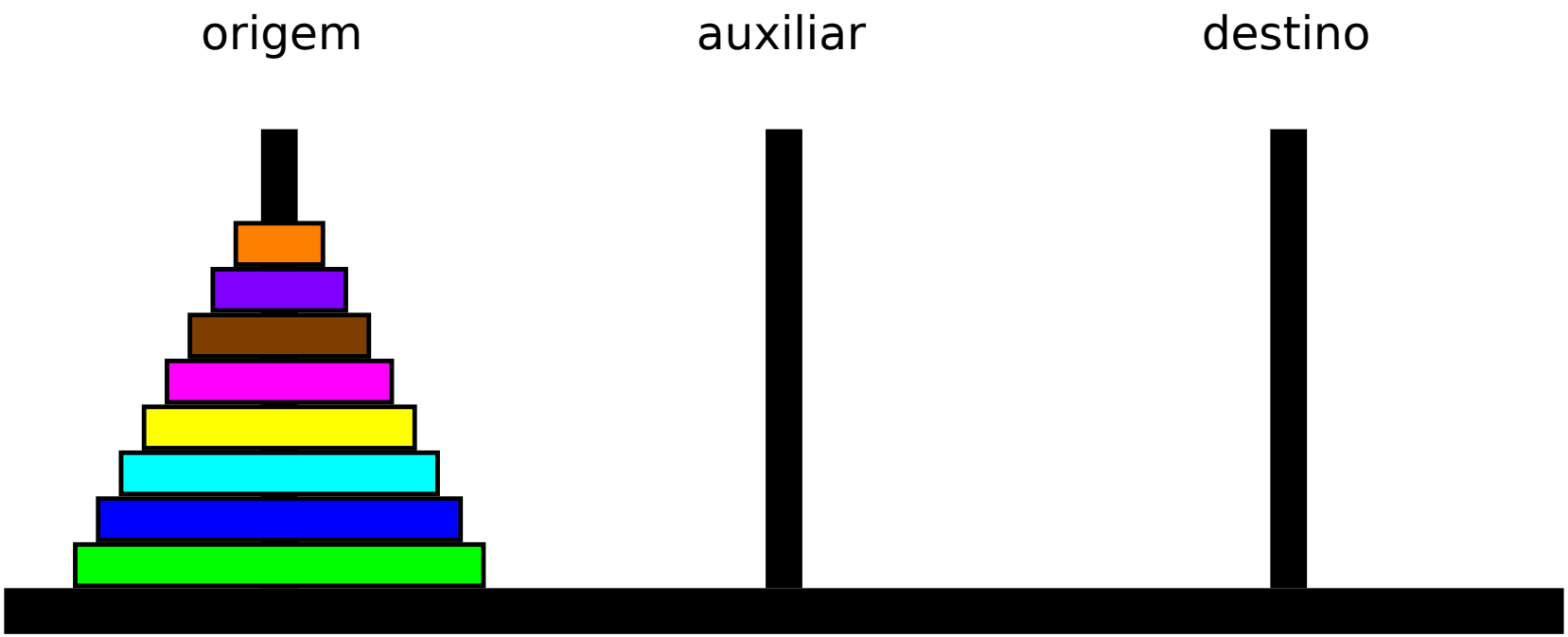

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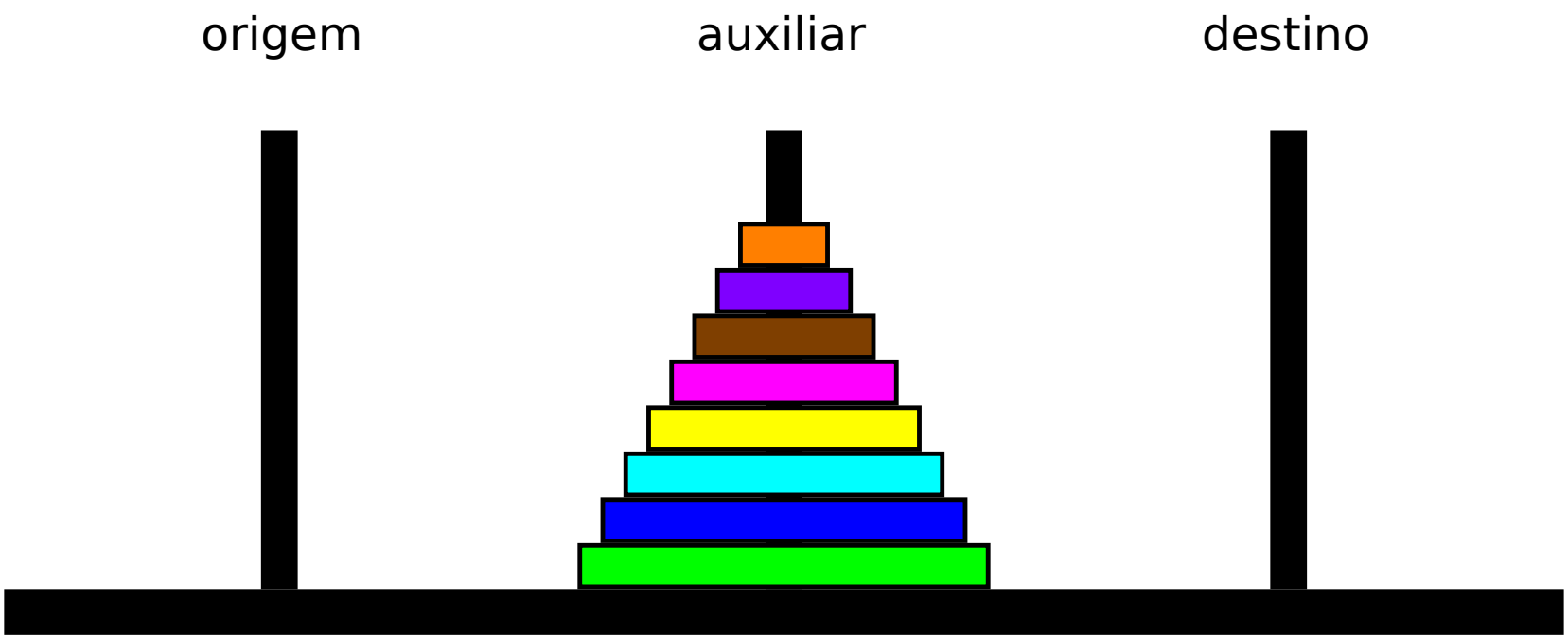

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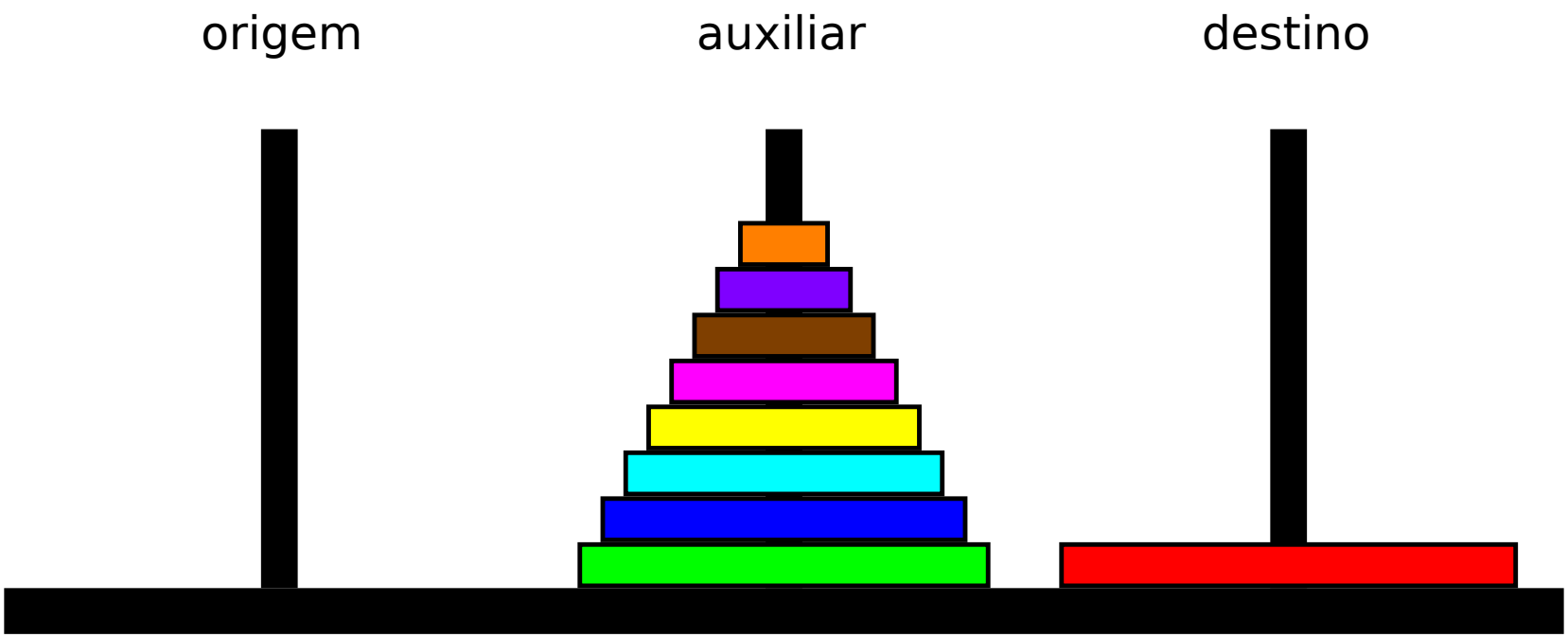

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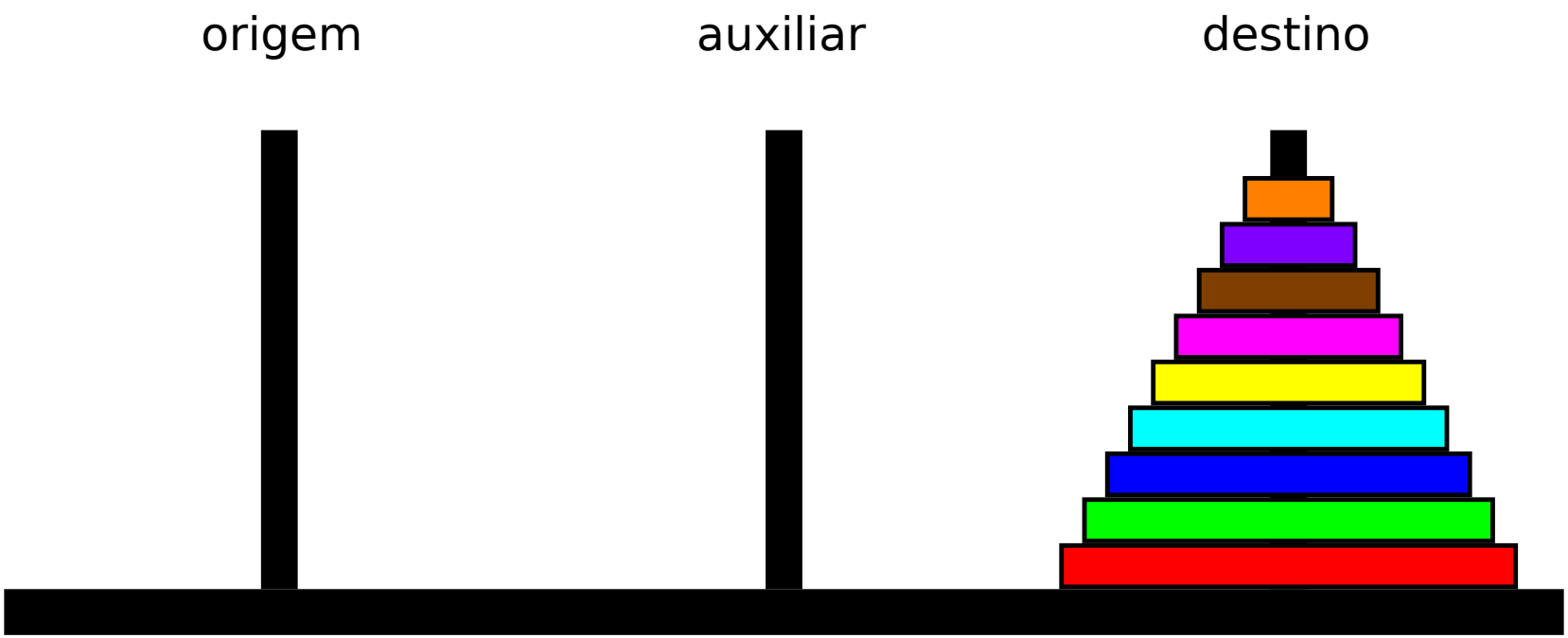

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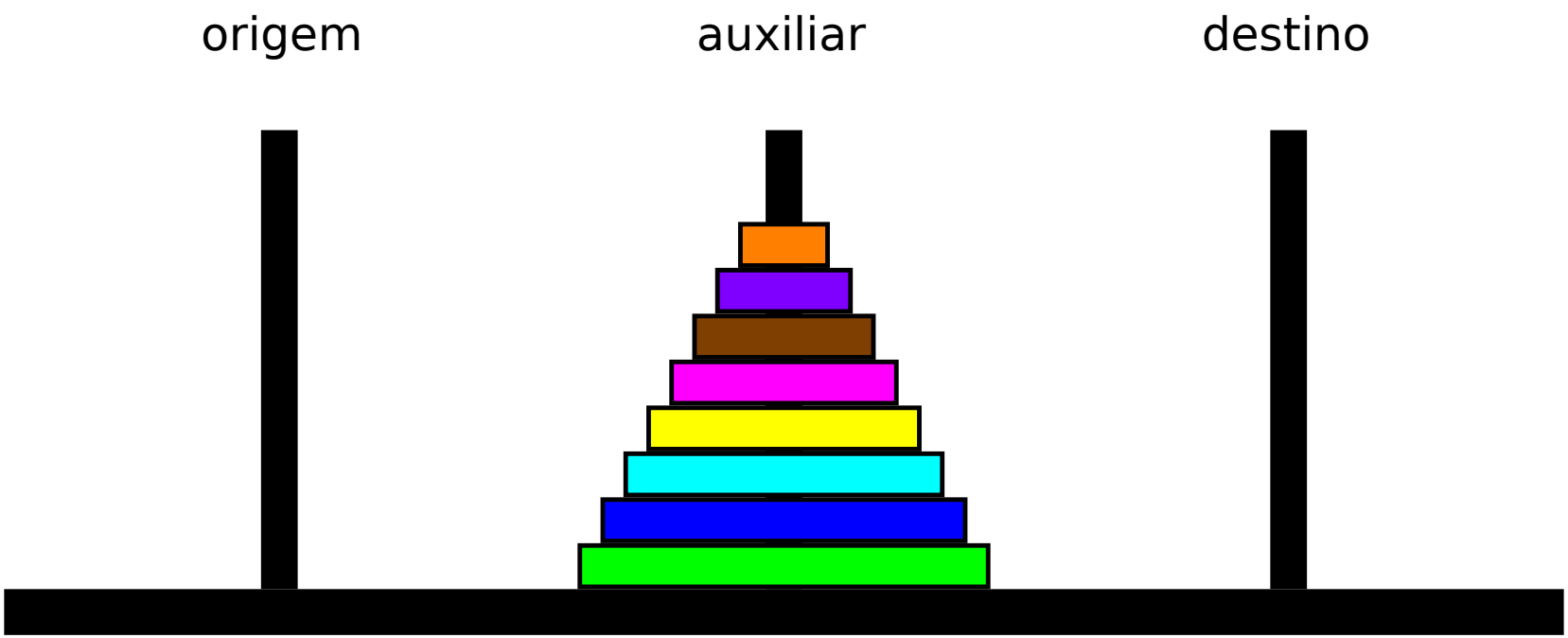

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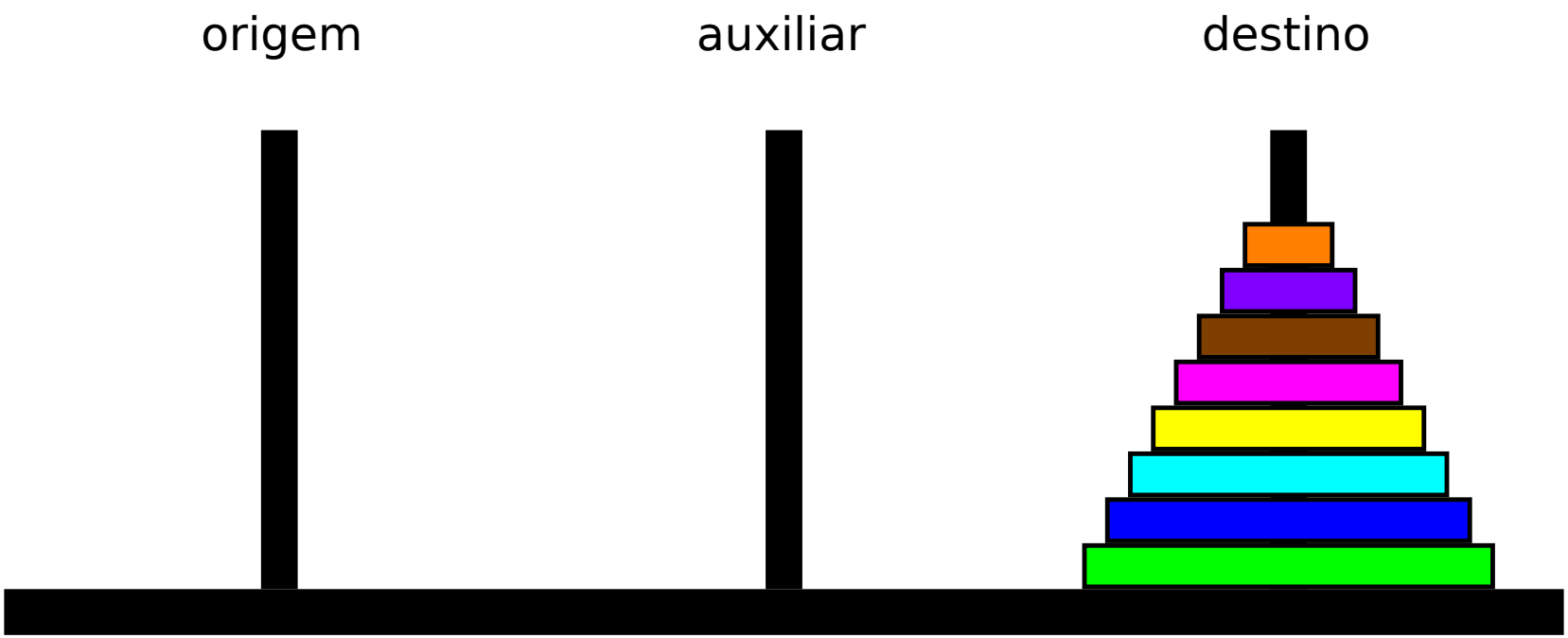
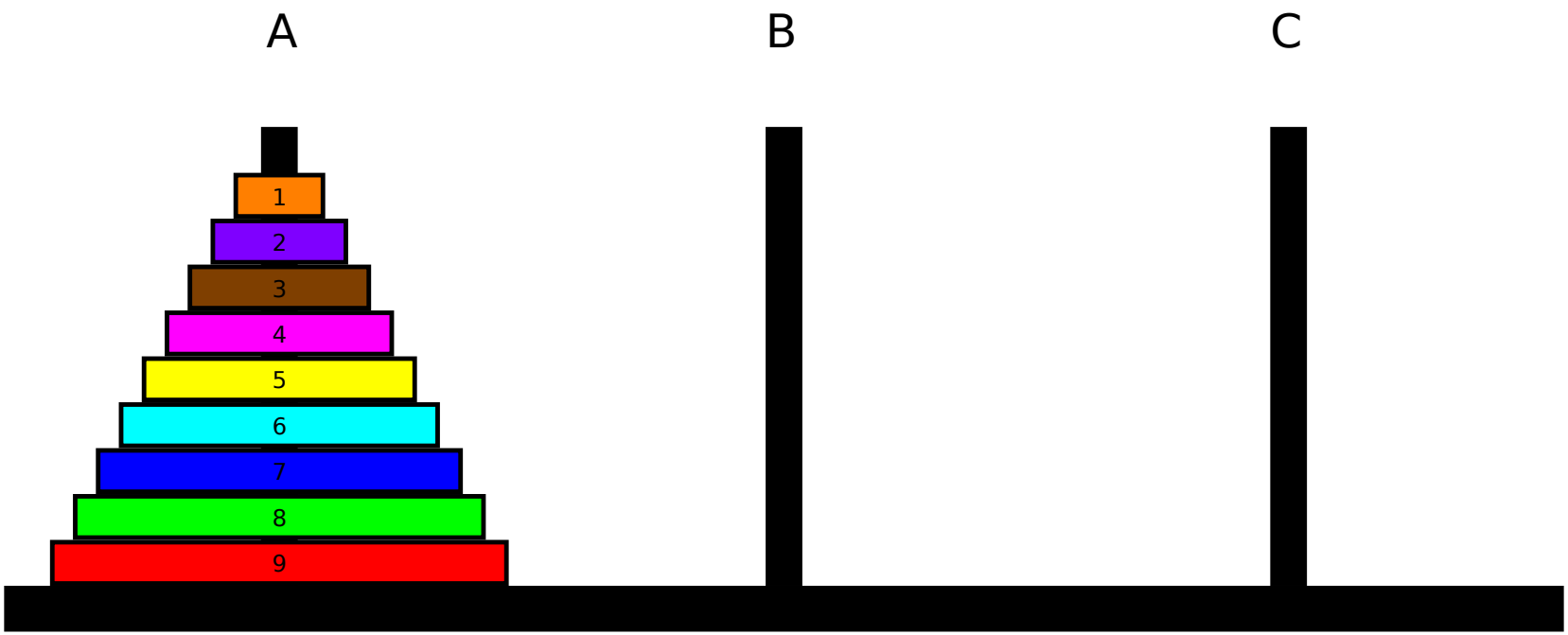


Torres de Hanói

Hanoi(n, *origem*, *auxiliar*, *destino*)

Torres de Hanói

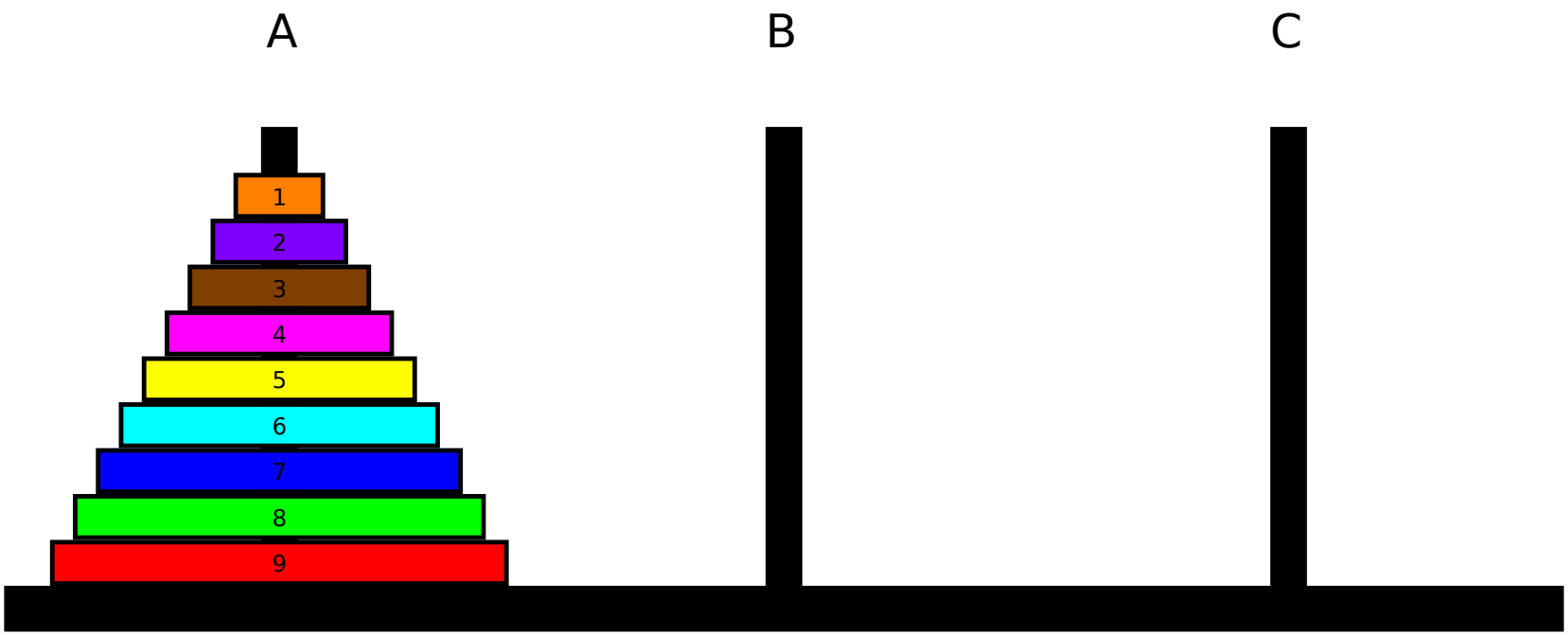
Hanoi(*n*, *origem*, *auxiliar*, *destino*)



Torres de Hanói

Hanoi(*n*, *origem*, *auxiliar*, *destino*)

OBJETIVO: Hanoi(9, A, B, C)



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Hanoi(n, *origem*, *auxiliar*, *destino*)

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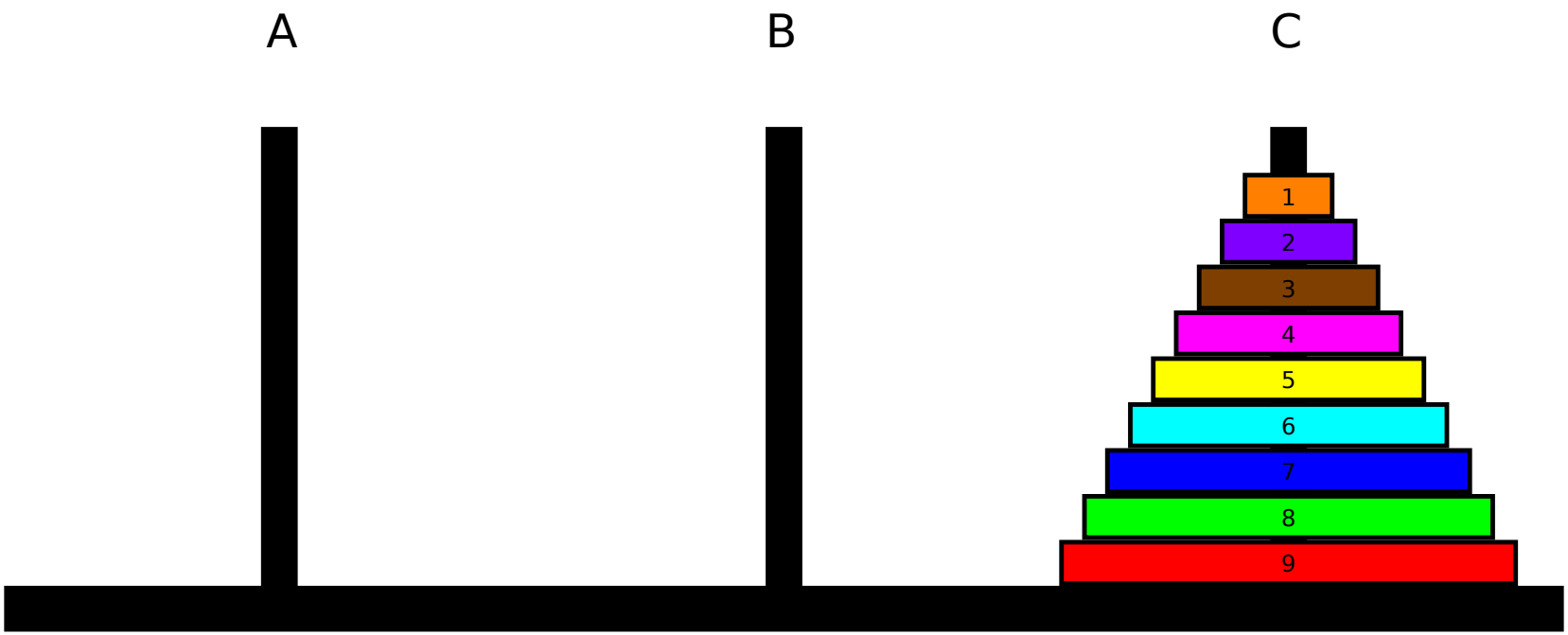
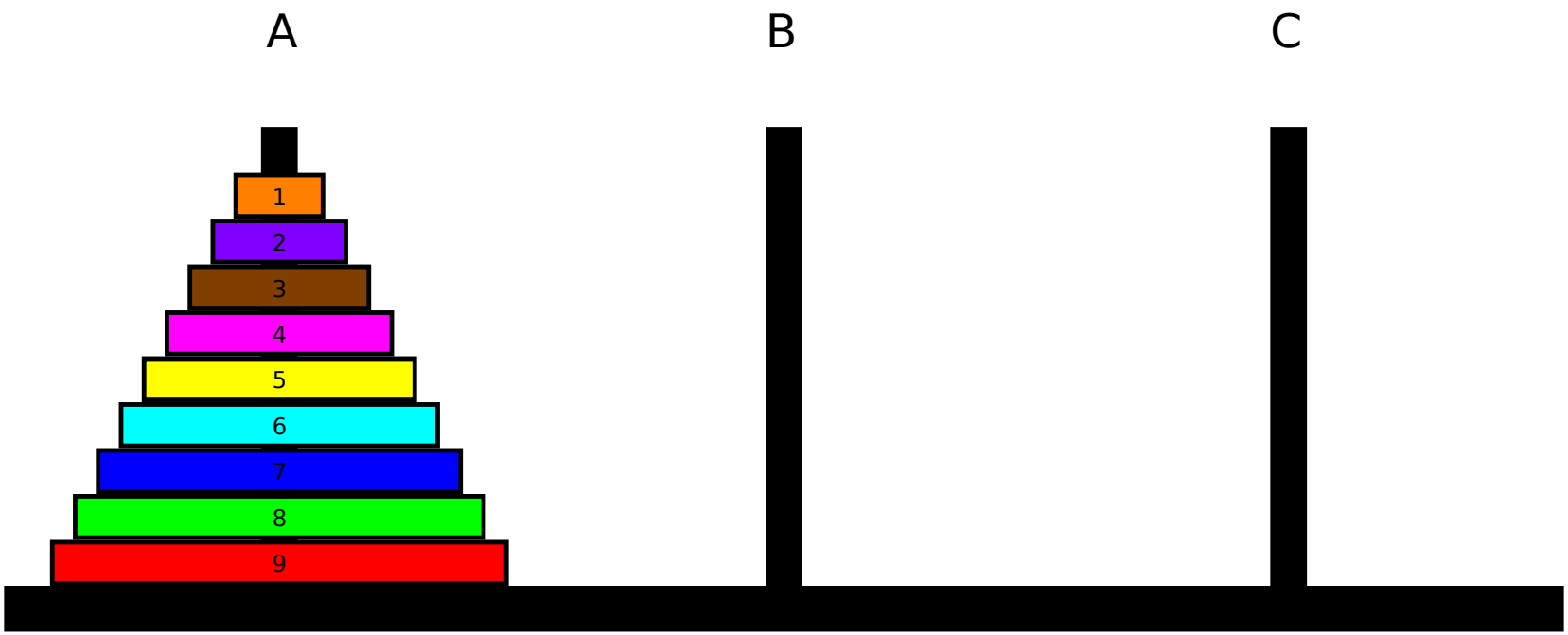


Torres de Hanói

Hanoi(*n*, *origem*, *auxiliar*, *destino*)

PASSO 1: chamar Hanoi(8, A, C, B)



Torres de Hanói

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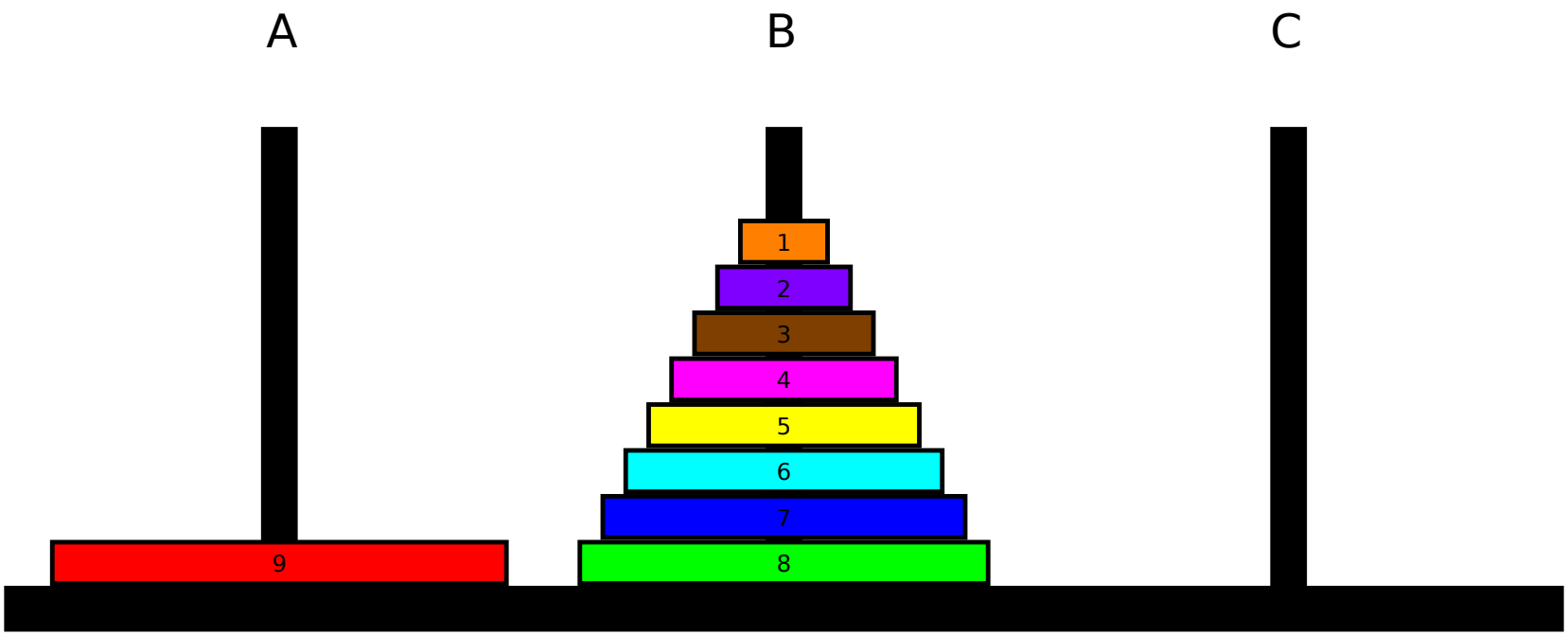
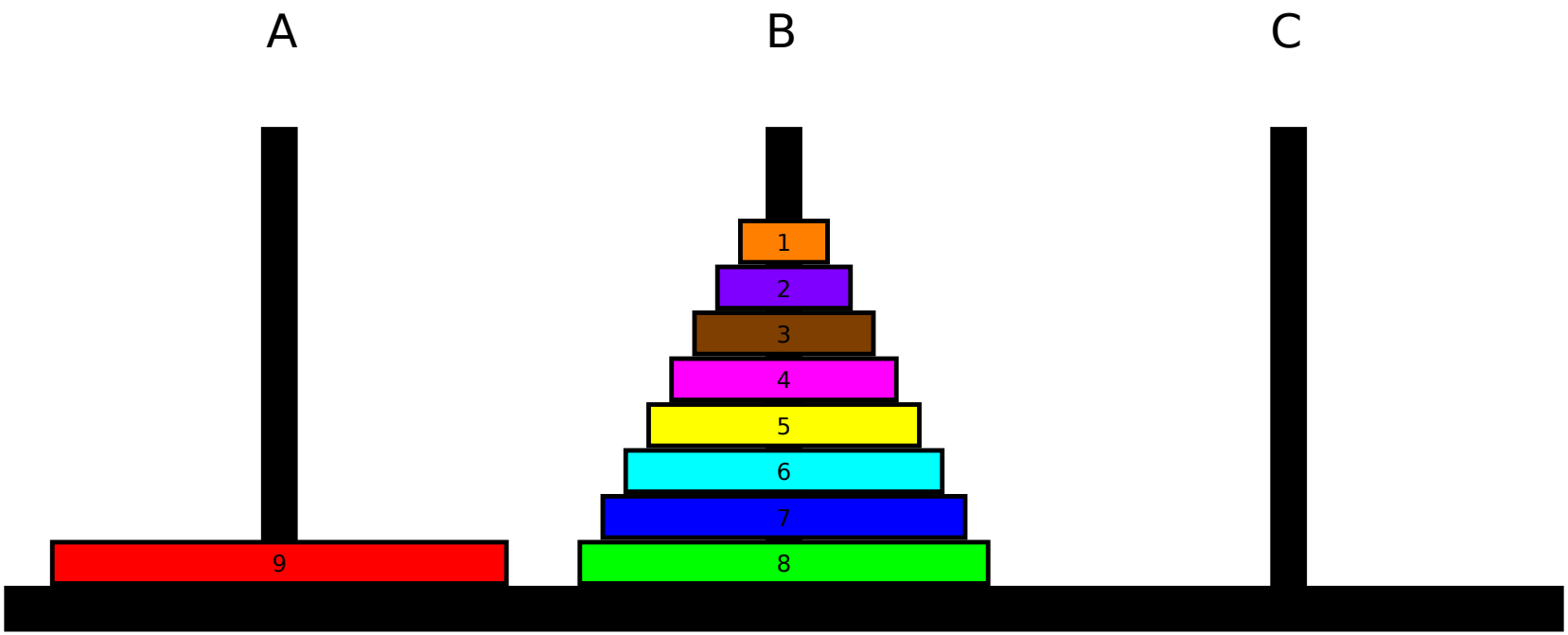
PASSO 1: chamar Hanoi(8, A, C, B)



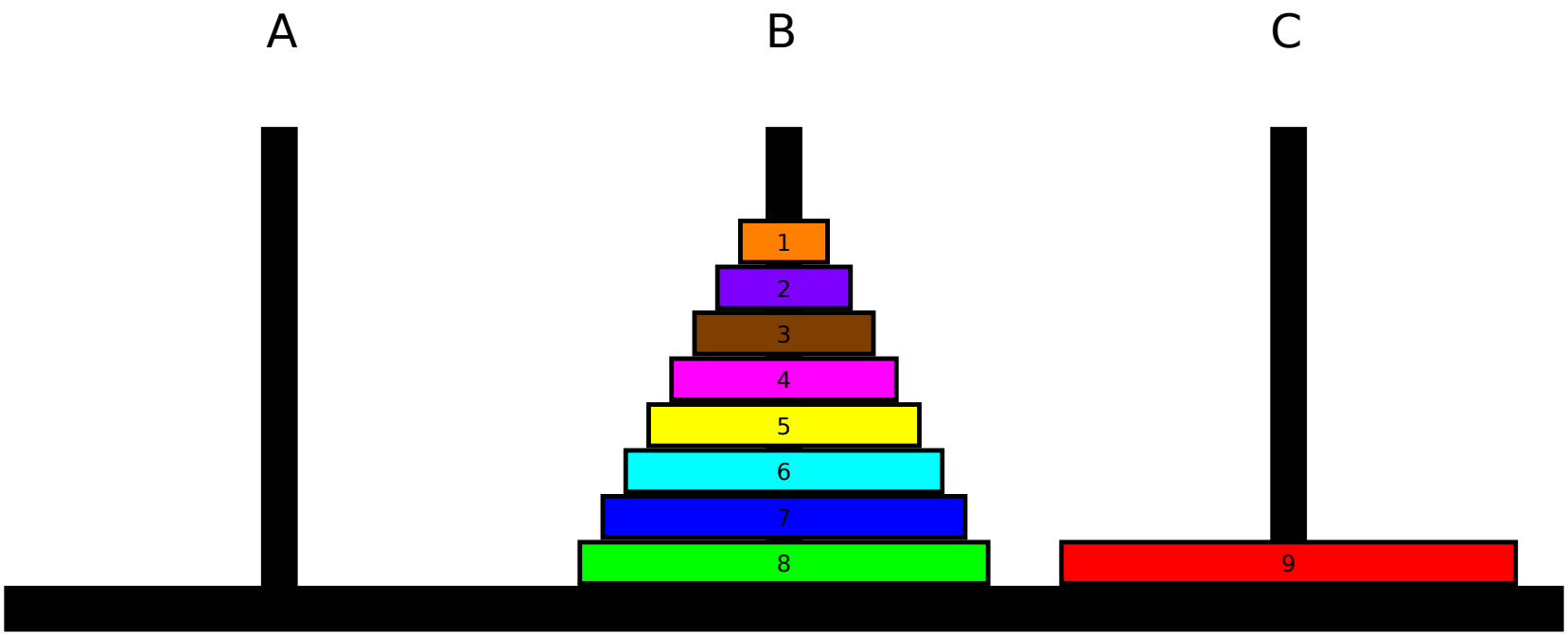
Hanoi(*n*, *origem*, *auxiliar*, *destino*)

PASSO 2: mover disco 9 de A a C



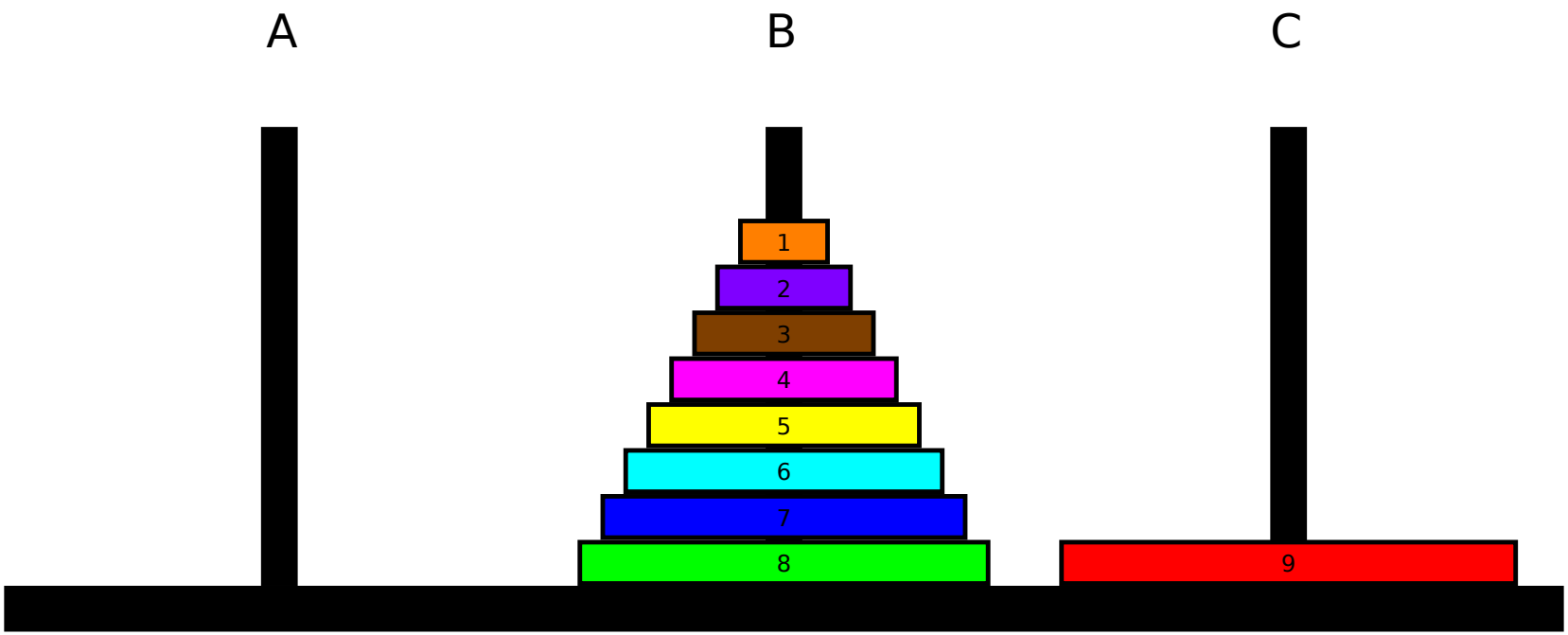
Hanoi(*n*, *origem*, *auxiliar*, *destino*)

PASSO 2: mover disco 9 de A a C



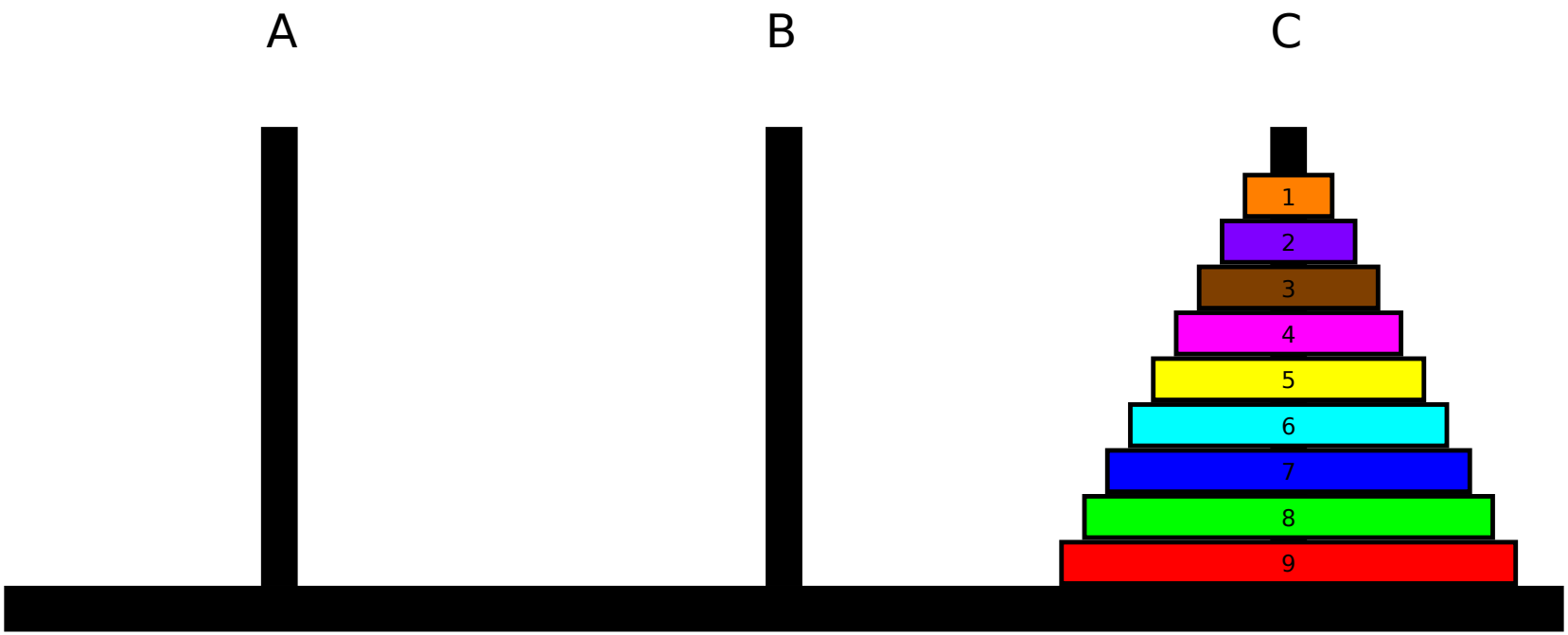
Hanoi(*n*, *origem*, *auxiliar*, *destino*)

PASSO 3: chamar Hanoi(8, B, A, C)



Hanoi(*n*, *origem*, *auxiliar*, *destino*)

PASSO 3: chamar Hanoi(8, B, A, C)



Torres de Hanói

Hanoi(9, A, B, C)

chamar Hanoi(8, A, C, B)

mover disco 9 de A a C

chamar Hanoi(8, B, A, C)

Torres de Hanói

Hanoi(*n*, *origem*, *auxiliar*, *destino*)

chamar Hanoi($n-1$, *origem*, *destino*, *auxiliar*)

mover disco n de *origem* a *destino*

chamar Hanoi($n-1$, *auxiliar*, *origem*, *destino*)

Torres de Hanói

Hanoi(n, *origem*, *auxiliar*, *destino*)

se $n > 1$
  chamar Hanoi($n - 1$, *origem*, *destino*, *auxiliar*)

mover disco n de *origem* a *destino*

se $n > 1$
  chamar Hanoi($n - 1$, *auxiliar*, *origem*, *destino*)

Exemplo

Escreva uma função recursiva que calcula x^y, onde x, y são inteiros e $y \geq 0$.

Exemplo

x^y

Exemplo

$$x^y = x \cdot x^{y-1}$$

Exemplo

$$x^y = x \cdot x^{y-1}$$
$$x^0 = 1$$

Exemplo

Potencia(x, y)

se $y = 0$
devolve 1

devolve $x \cdot$Potencia(x, $y - 1$)

Exercício

Escreva uma função recursiva que calcula $n!$, onde $n \geq 0$.

Exercício

$n!$

Exercício

$$n! = n \cdot (n-1)!$$

Exercício

$$n! = n \cdot (n-1)!$$
$$0! = 1$$

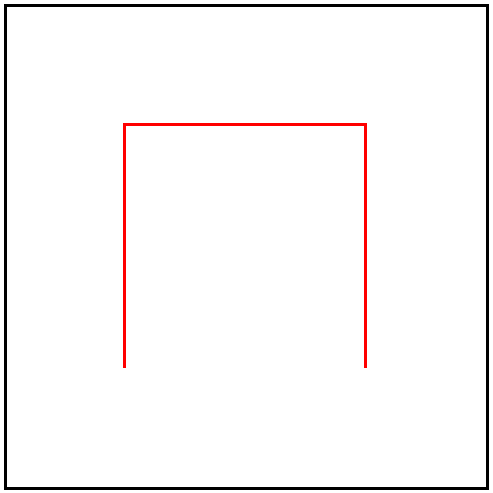
Exemplo

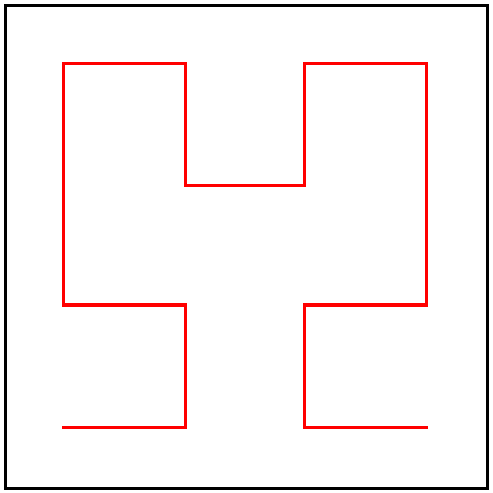
Fatorial(n)

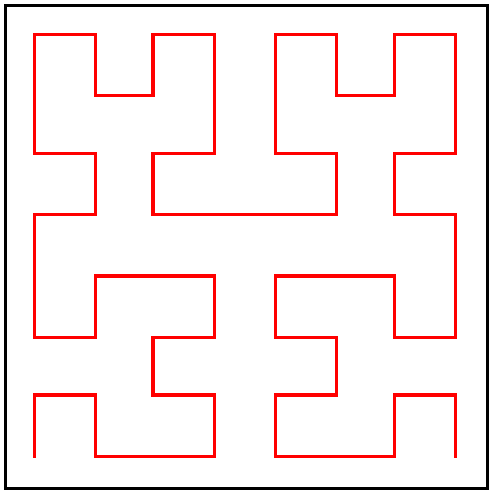
se $n = 0$
  devolve 1

devolve $n \cdot$Fatorial$(n - 1)$

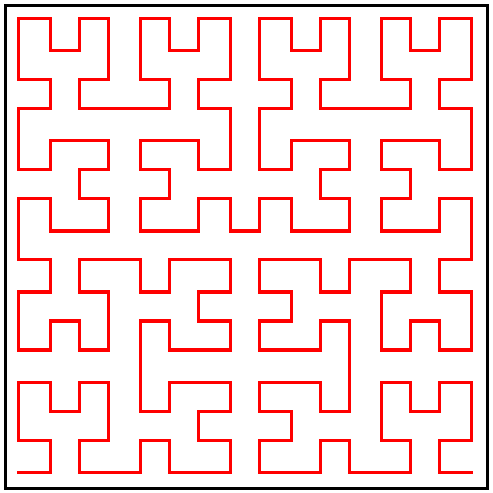
Desenhos recursivos

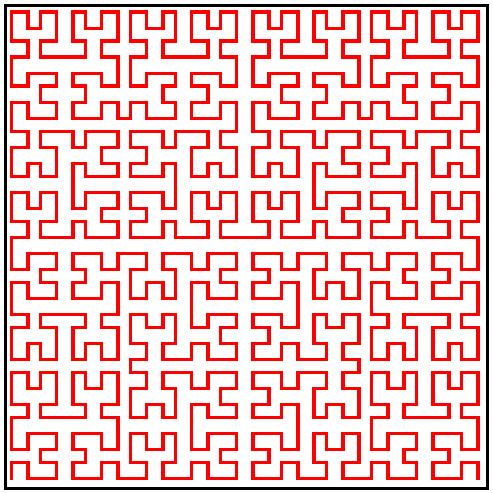


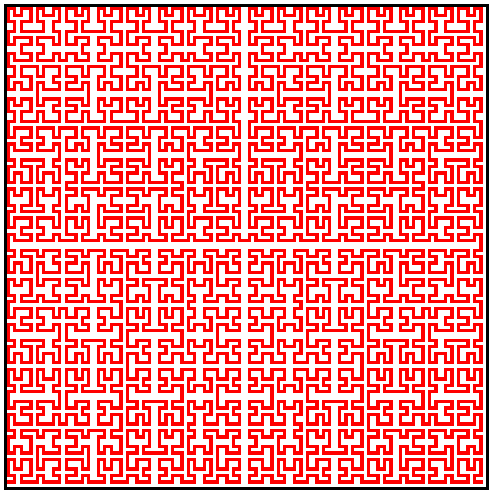




Desenhos recursivos







Torres de Hanói

Hanoi(n, *origem*, *auxiliar*, *destino*)

se $n > 1$
  chamar Hanoi($n - 1$, *origem*, *destino*, *auxiliar*)

mover disco n de *origem* a *destino*

se $n > 1$
  chamar Hanoi($n - 1$, *auxiliar*, *origem*, *destino*)

Torres de Hanói

Número de movimentos: $2^n - 1$

Torres de Hanói

Número de movimentos: $2^{64} - 1$

Torres de Hanói

Número de movimentos: 18,446,744,073,709,551,615

Supondo um movimento por segundo,
leva quase 585 bilhões de anos.