STA 531: Final Project
Predicting Pitches in Baseball

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1 Introduction

Sabermetrics, or the statistical analysis of baseball, has grown immensely in popularity over the past 15 years and is widely used to analyze the performance of baseball players and baseball teams. One of the main focuses of Sabermetricians has been comparing the performance of individual players. While the statistical analysis of baseball has been around for many years, there are still areas of research that have been less explored that would be beneficial for players and coaches. One such area of research that has been explored but not with any great depth is pitch prediction. The motivation behind this project is that it would be of great value for a batter to know the upcoming pitch. We conduct our analysis from the outlook of a batter and attempt to predict upcoming pitches given a sequence of pitches by a particular pitcher and a set of covariates about the state of the game at the time of each pitch. The pitcher we have chosen to analyze is Los Angeles Dodgers pitcher Clayton Kershaw, who is widely considered the best pitcher in Major League Baseball.

2 Data

We conduct our analysis using MLB.com's PITCHf/x data that is available publicly. The data set contains pitch-by-pitch data on all MLB teams from 2013 to 2015 and includes information regarding the state of the game at the time of the pitch (number of outs, count, number of baserunners, pitcher, batter, etc.) as well as information about the trajectory of the pitch (pitch type, velocity, acceleration, break, etc.). There are 2,114,497 observations of pitches in the data set and 39 different initial variables. We focus our analysis on the game state variables and the pitch type variable for Clayton Kershaw.

3 Methods

In this paper, we apply various statistical methods aimed at predicting pitch sequences of Clayton Kershaw. We then compare the results of the various methods. The two main statistical techniques that we use are Markov Models and Multinomial Logistic Regression to predict the sequence of pitches. We elaborate on the methods used in the following sections. To compare the different methods, we use cross-validation with a Monte Carlo estimate of percent accuracy in the predictions as our comparison metric.

3.1 Sampling from Probability Vectors

The first, and most naive, method that we implement as a baseline is using the sample probability vectors for each of Clayton Kershaw's pitches to predict future pitches. This method does not consider any of the game state covariates from our data. We obtain the sample probability vector by getting the counts for each pitch thrown by Clayton Kershaw and dividing by the total number of pitches that Clayton Kershaw has

thrown. We then sample pitches from this probability vector and compare this sampled sequence to an actual sequence of 100 pitches thrown by Clayton Kershaw to obtain the percent accuracy of our predictions. We repeat this procedure 100 times and use the mean of the accuracy scores as our metric as there in inherent randomness in the sampling.

3.2 Markov Model

Next, we use a simple Markov model. This Markov model only uses an initial probability vector and a transition matrix to predict the next pitch in a sequence. The transition matrix was created from the entire sequence of Clayton Kershaw's pitches. We use this transition matrix to predict the next pitch in a sequence of Clayton Kershaw's pitches. Cross validation is also applied with this method by training, or creating the transition matrix, on a portion of the data, and then testing this Markov model on the held out set. The mean percent accuracy in predictions of 100 Monte Carlo samples was once again used as the comparison metric.

3.3 Hidden Markov Model

After applying a simple Markov model, a more complex hidden Markov model is implemented on Clayton Kershaw's pitch sequence. Given a sequence of his pitches, to obtain the optimal parameters for the model, we use the Baum-Welch algorithm. Once the algorithm determined the parameters, we obtain the probabilities of the upcoming pitch given the previous pitches, $p(x_{n+1}|x_{1:n})$, using the Forward Algorithm. Both algorithms are described in detail below.

3.3.1 Baum-Welch

The Baum-Welch algorithm applies expectation maximization to hidden Markov models to obtain parameters for the Hidden Markov Model. These parameters are the initial probability vector, the transition matrix from state to state, and the emission matrix. The Baum-Welch algorithm is an iterative algorithm that uses the forward-backward algorithm at each iteration to estimate these parameters.

The forward-backward algorithm is as follows:

In the forward algorithm, we sum over $z_1, z_2, \ldots, z_n$ in that order, and derive a recursion for computing $p(x_{1:j}, z_j)$ for each $z_j = 1, \ldots, m$ and each $j = 1, \ldots, n$. In the backward algorithm, we sum over $z_n, z_{n-1}, \ldots, z_1$, and derive a recursion for computing $p(x_{j+1:n}|z_j)$ for each $z_j = 1, \ldots, m$ and each $j = 1, \ldots, n$.

The forward-backward algorithm was implemented as shown below. The log of the probabilities were taken in order to deal with arithmetic underflow/overflow and R's inability to store such a low probability. In addition, the log-sum-exp trick was used to deal with a similar problem.

Forward algorithm:
1. For each $z_1, \ldots, m$, compute $g_1(z_1) = \log p(z_1) + \log p(x_1|z_1)$
2. For each $j = 2, \ldots, n$ for each $z_j = 1, \ldots, m$, compute:

$$\log s_j(z_j) = g_j(z_j) = \log \sum_{z_{j-1}} \exp[g_{j-1}(z_{j-1}) + \log p(z_j|z_{j-1}) + \log p(x_j|z_j)]$$

3. $\log p(x_{1:n}) = \log \sum_{z_n} \exp(g_n(z_n))$

And $g_j(z_j) = \log p(x_{1:j}, z_j)$

Backward algorithm:
1. For each $z_n = 1, \ldots, m$, define $r_n(z_n) = 0$
2. For each $j = n-1, n-2, \ldots, 1$, for each $z_j = 1, \ldots, m$ compute:

$$r_j(z_j) = \log \sum_{z_{j+1}} \exp(\log p(z_{j+1}|z_j) + \log p(x_{j+1}|z_{j+1}) + r_{j+1}(z_{j+1}))$$

And $r_j(z_j) = \log p(x_{j+1:n}|z_j)$

The Baum-Welch algorithm is implemented as follows:
Using the following formula:

$$\gamma_{ti} = P(Z_t = i|x)$$
$$\beta_{tij} = P(Z_{t-1} = i, Z_t = j|x)$$
$$\pi_j = \frac{\gamma_{1i}}{\sum_{j=1}^{m} \gamma_{1j}}$$
$$T_{ij} = \frac{\sum_{t=2}^{n} \beta_{tij}}{\sum_{t=1}^{n-1} \gamma_{ti}}$$

The algorithm is:
1. Randomly initialize π, T, and $\phi = (\phi_1, \ldots, \phi_m)$
2. Iteratively repeat the following two steps, until convergence:
(a) E-step: Compute the γ and β using the forward-backward algorithm.
(b) M-step: Update π, T, and ϕ using the formulas above.

This algorithm was implemented in Python, and the results from the Baum-Welch were used in the Forward algorithm to do probabilistic inference, which is described below.

3.3.2 Forward Algorithm

Once the parameters are estimated for the transition matrix, the emission matrix, and the initial probability matrix, we can combine these parameters with results from the forward algorithm to get probabilities for the next observation given the previous observation.

We can predict x_{j+1} and $x_{1:j}$ using:

$$p(x_{j+1}|x_{1:j}) \propto p(x_{1:j}, x_{j+1}) = \sum_{z_j, z_{j+1}} p(x_{1:j}, x_{j+1}, z_j, z_{j+1})$$
$$= \sum_{z_j, z_j+1} p(x_{1:j}, z_j) p(z_{j+1}|z_j) p(x_{j+1}|z_{j+1})$$

Cross validation was also applied in this setting. The Baum-Welch algorithm was applied to the training set. Then as we ran through a sequence of pitches in the testing set, we use the forward algorithm and the estimated parameters to get probabilities of the next pitch. These predictions are compared to the true sequence of pitches in order to obtain an estimated percent accuracy in predictions.

3.4 Multinomial Logistic Regression

Next, we explore multinomial logistic regression models to predict the upcoming pitch given a set of covariates regarding the state of the game at the time of the pitch. Multinomial logistic regression is a classification scheme that generalizes logistic regression to a multivariate scheme. Because a pitcher can throw multiple types of pitches at a given observation, we have a vector of possible pitches.

Given the multinomial data with J categories and the p-dimensional predictor variables, we can forecast in which j category a future data point y^* at the predictor x^* will be.

Multinomial logistic regression can be understood as a set of independent binary regressions. If we have J possible outcomes, we can imagine running $J-1$ binary regression regression models, which are compared against the one pivot outcome.

$$\frac{\Pr(Y_i = 1)}{\Pr(Y_i = K)} = \beta_1 \cdot X_i$$
$$\frac{\Pr(Y_i = 2)}{\Pr(Y_i = K)} = \beta_2 \cdot X_i$$
$$\cdots\cdots$$
$$\frac{\Pr(Y_i = K-1)}{\Pr(Y_i = K)} = \beta_{K-1} \cdot X_i$$

For each possible outcome, there are separate vectors of regression coefficients. Therefore, we can calculate the probability of observing a category j after at a time occurrence as:

$$P(y^* = j | x^*, \beta, n^* = 1) = e^{x^* \beta_j} / \sum_{k=1}^{J} e^{x^* \beta_k}.$$

Once we calculate these probabilities for a set of covariates, we can predict what the pitch will be for the set of covariates.

For this dataset, there were specific variables that we decided to use in our models because they have been traditionally known to influence pitching strategy. We also used cross validation of accuracy scores to select the best set of variables for our model by choosing variables that created the highest percent accuracy when implemented on the testing set.

3.5 Cross Validation

Throughout our analysis we implement 5-fold Cross Validation. Cross validation is a model validation technique to assess how statistical analysis will generalize to another independent data set. In cross validation, we divide the dataset into a training set and a test set. The training set's covariates and pitches are used to run and train the model, and the covariates of the test set are used to predict the pitches. The predicted pitches are then compared to the true pitches in the test set. The 5 folds refers to the number of times we divide the data into training and test sets. We average the results across the 5 different results. Cross validation helps problems such as over-fitting a dataset because it tests the model against another dataset.

4 Results

4.1 Exploratory Data Analysis

To begin our analysis, we perform some exploratory data analysis to explore potential covariates for our models that influence the type of pitch thrown by Clayton Kershaw. Below are some of the results from the

exploratory data analysis of proportions of the different pitches across covariates. The more the proportion varies across the columns, the more evidence that the proportion of pitches change across the covariates.

	CH	CU	FF	SL
0-0	0.012	0.016	0.810	0.161
0-1	0.035	0.287	0.431	0.247
0-2	0.000	0.281	0.424	0.294
1-0	0.032	0.000	0.614	0.354
1-1	0.026	0.185	0.425	0.364
1-2	0.000	0.388	0.328	0.284
2-0	0.004	0.000	0.909	0.087

Table 1: Proportion Table for Count

	CH	CU	FF	SL
0	0.013	0.137	0.581	0.269
1	0.014	0.150	0.562	0.274
2	0.016	0.166	0.566	0.253

Table 2: Proportion Table for Pre-Outs

4.2 Predicting and Cross Validation

All of the methods were implemented for the pitcher Clayton Kershaw. He uses four different types of pitches, four-seam fastballs (FF), sliders (SL), curveballs (CU), and change-ups (CH)and he has about 9,400 pitch observations in the dataset. These methods include sampling from the sample probability vector, a simple Markov chain, a hidden Markov model, and various multinomial logistic regression models. The total percent accuracy was calculated for each method. The only issue came with implementing the Baum-Welch algorithm. After many attempts, the product of the algorithm did not produce viable results, and therefore, the results are not included below. For the logistic regression, a simple forward variable selection process was used to iteratively add variables to maximize the percent accuracy. The variable "count" (as is balls, strikes) was found to make the biggest difference in percent accuracy followed by "inning" and "previous pitch". After those variables, adding or changing other variables did not make much of a difference. Therefore, we have shown results for three models with the following variables used:

Model #	Variables
LR1	count
LR2	count, pre outs, inning
LR3	pre outs, count, pitch number, runners count, pitch count top inning, bat side, inning, prev pitch type

The results of these from testing within sample is shown on the next page. This table shows percent accuracy for each true pitch type along with total percent accuracy calculated in the final column.

The results from testing using 5-fold Cross Validation is shown on the next page. Once again, this table shows percent accuracy for each true pitch type along with total percent accuracy calculated in the final column for the various methods.

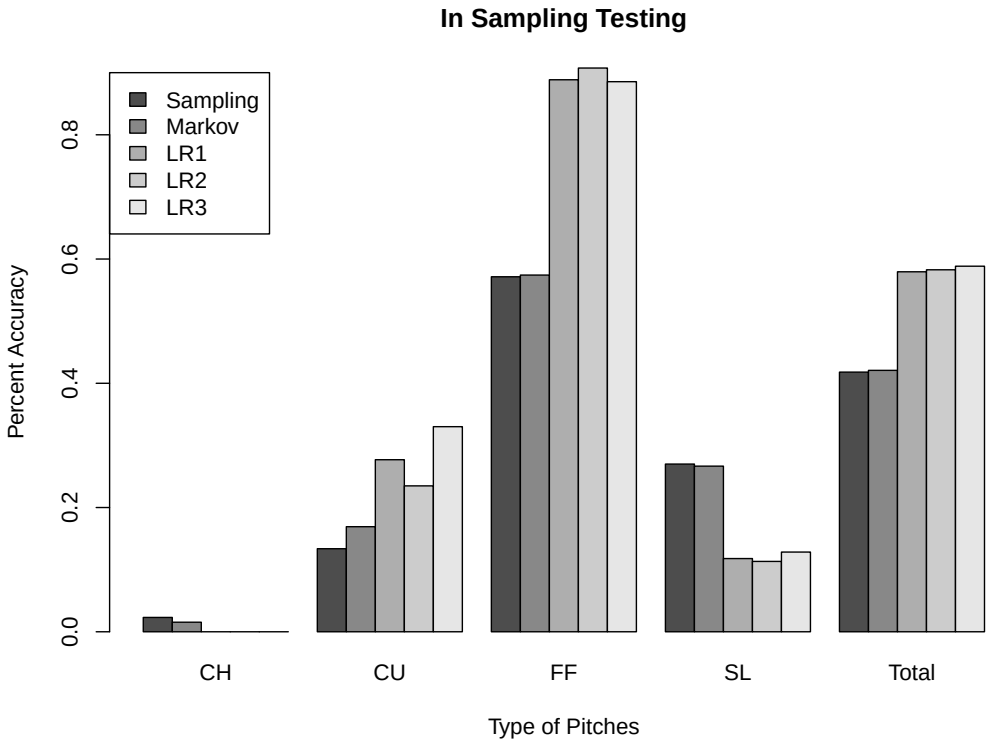
These tables are also shown below as bar plots.

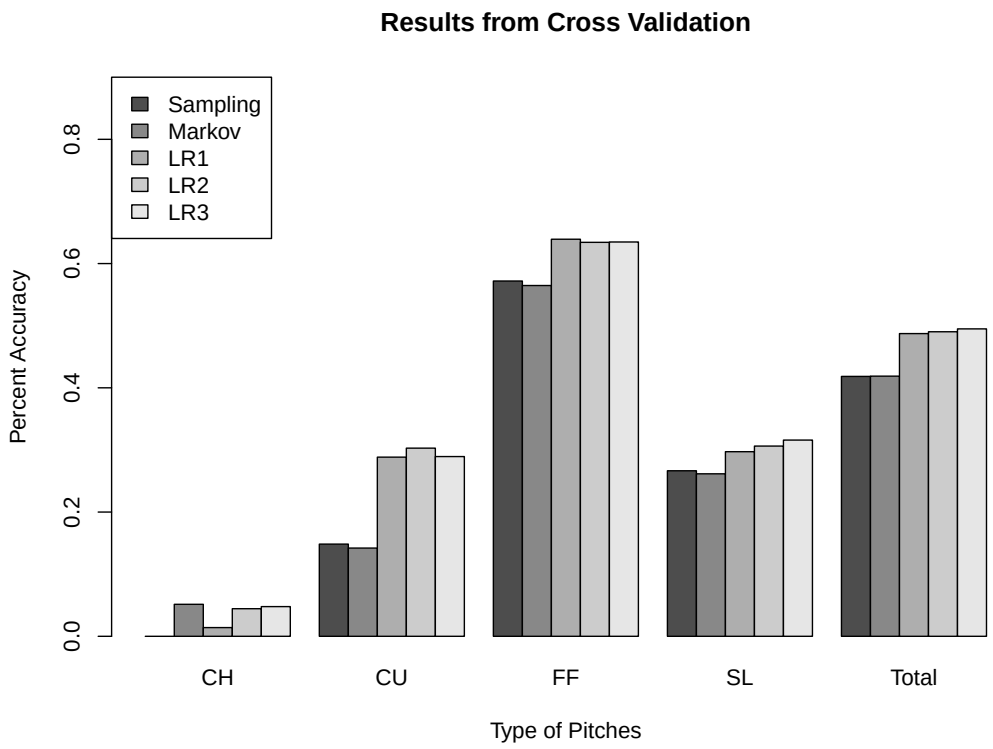
	CH	CU	FF	SL	Total
Sampling	0.0231	0.1337	0.5715	0.2700	0.4180
Markov	0.0154	0.1691	0.5742	0.2666	0.4208
LR1	0.0000	0.2770	0.8886	0.1178	0.5795
LR2	0.0000	0.2349	0.9074	0.1132	0.5827
LR3	0.0000	0.3301	0.8855	0.1283	0.5885

Table 3: Percent Accuracies from In-Sample Testing

	CH	CU	FF	SL	Total
Sampling	0.0000	0.1484	0.5719	0.2665	0.4183
Markov	0.0515	0.1420	0.5647	0.2616	0.4188
LR1	0.0140	0.2884	0.6392	0.2972	0.4873
LR2	0.0444	0.3029	0.6342	0.3062	0.4902
LR3	0.0478	0.2893	0.6347	0.3159	0.4949

Table 4: Percent Accuracies from Cross Validation





As we can see from the tables and the bar plots, the best performing models are the multinomial logistic regression models. The Markov model does not have a much better performance than simply sampling from the probability vector. We can also see that predicting fast balls seems to be more accurate than other pitches. The cross validation technique also shows that it is necessary in order to accurately measure the predictive accuracy of our model on out-of-sample sets.

5 Conclusion

In this paper, we have presented the predictive accuracy of various statistical methods in predicting baseball pitches thrown by Clayton Kershaw. The applications of various Markov models and multinomial logistic regressions have shown to be useful as well as the success of cross validation to check the results of a model. However, there is still much room for improvement in predicting baseball pitches. To our surprise, none of our methods performed better than simply predicting a fastball every single time. One area of further investigation would be to see if this is true for all pitchers or if Clayton Kershaw's success is partially due to his inherent unpredictability. Another shortcoming of our methodology is that we focused exclusively on one particular pitcher. This project can be further expanded by examining multiple pitchers and the differences in predicting pitches for different pitchers.