# In-context learning for model-free system identification

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# Standard system identification/supervised machine learning

1. Collect dataset $\mathcal{D} = (u_{1:N}, y_{1:N})$ of input/outputs from system $S$.
2. Apply an algorithm to estimate a model $M(\hat{\theta})$ of $S$:

$$\hat{\theta} = \mathcal{A}(\mathcal{D}) \qquad \text{e.g. } \mathcal{A}(\mathcal{D}) = \arg\min_{\theta \in \Theta} \mathcal{L}(\mathcal{D}, M(\theta))$$

3. Make predictions/simulations using the model on new data:

$$\hat{y}^*_{1:M} = M(u^*_{1:M}; \hat{\theta})$$

Researchers keep on improving learning algorithms and model structures. Can we automate this process? Can we learn the learning algorithm itself?

Meta learning tries to answer this question.


J. Schmidhuber. Evolutionary principles in self-referential learning, or on learning how to learn. *Diploma Thesis, TU Munich, 1987*

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# Our meta learning setting

- We have an infinite stream of datasets from a distribution $p(\mathcal{D})$:

$$\{\mathcal{D}^{(i)} = (u_{1:N}^{(i)}, y_{1:N}^{(i)}),\ i = 1, 2, \dots, \infty\}$$

- $\mathcal{D}^{(i)}$ generated by random system $S^{(i)}$ and input realization $u_{1:N}^{(i)}$
- Different but related to each other. There's a learnable structure!

Can we get better at identifying $S^{(i)}$ as we observe more datasets $\mathcal{D}^{(j)}$?

- $p(\mathcal{D})$ may be a physical simulator where we can change settings
- The learned algorithm could then be applied to real data

Meta learning from a finite collection would also be interesting...

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# In-context learning

Many meta learning strategies around. Here focus on in-context learning.

- Transformers are sufficiently expressive to represent algorithms.
- We train Transformers to behave like an algorithm. We provide:
  - A context, namely an input/output sequence of a system
  - A task, like predicting the next output or simulating for more steps
- The Transformer must learn to identify the system to solve the task!

Context + task may be seen as a prompt to a Large Language Model, which can then continue the word sequence in an optimal way.


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# Two system identification problems

**One-step prediction:**

$$\hat{y}_{k+1} = \mathcal{M}_\phi(u_{1:k}, y_{1:k}).$$

- $(u_1, y_1) \rightarrow \hat{y}_2$
- $(u_{1:2}, y_{1:2}) \rightarrow \hat{y}_3$
- $(u_{1:3}, y_{1:3}) \rightarrow \hat{y}_4$
- $\ldots$
- $(u_{1:N-1}, y_{1:N-1}) \rightarrow \hat{y}_N$

**Multi-step simulation**

$$\hat{y}_{m+1:N} = \mathcal{M}_\phi(u_{1:m}, y_{1:m}, u_{m+1:N})$$

Meta-model receives:

- Full input/output $(u_{1:m}, y_{1:m})$
- Input-only trajectory $u_{m+1:N}$

and generates simulation: $\hat{y}_{m+1:N}$

- If we manage to train a Transformer $\mathcal{M}_\phi$ to solve such problems for a class of systems, it becomes a meta model of that class!
- $\mathcal{M}_\phi$ becomes as powerful as a system identification algorithm!

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# Meta model training

## One-step prediction:

$$\hat{\phi} = \arg\min_{\phi} \mathcal{L}_{\rm pred}(\phi)$$

$$\mathcal{L}_{\rm pred} = \mathbb{E}_{p(\mathcal{D})}\left[\sum_{k=1}^{N-1} \left\|y_{k+1} - \mathcal{M}_{\phi}(y_{1:k}, u_{1:k})\right\|^2\right]$$

$$\mathcal{L}_{\rm pred} \approx \frac{1}{b}\sum_{i=1}^{b}\sum_{k=1}^{N-1} \left\|y^{(i)}_{k+1} - \mathcal{M}_{\phi}(y^{(i)}_{1:k}, u^{(i)}_{1:k})\right\|^2$$

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- Formally, just two boring supervised learning problems.
- The use of powerful architectures and training on a whole class of dynamical systems makes the outcome special.
- If the optimization works out well, the Transformer becomes a meta model of the systems in $p(\mathcal{D})$!
- We have learned a learning algorithm!

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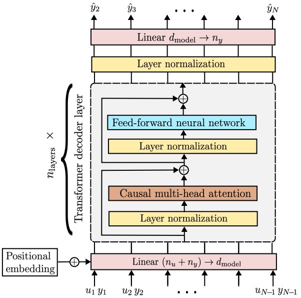
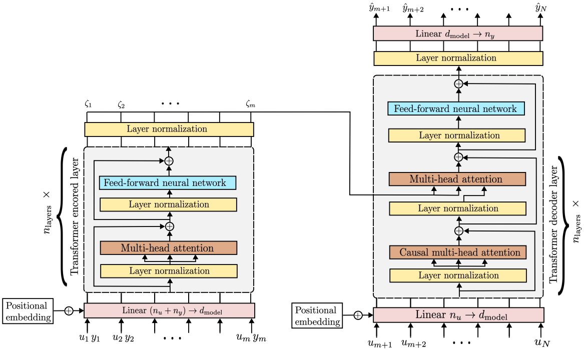
# Transformer architectures

one-step prediction:
decoder-only (∼ GPT-2)




multi-step simulation:
encoder-decoder (∼ language translation)




NLP architectures modified to process real-valued input/output sequences.

# Experiments - System classes

One-step prediction and multi-step simulation on two system classes:

### Linear Time Invariant (LTI):

In state-space form, order $\leq 10$

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k \\
y_{k+1} &= Cx_k
\end{aligned}
$$

- random system matrices
- $A$ constrained to be stable

### Wiener-Hammerstein (WH):

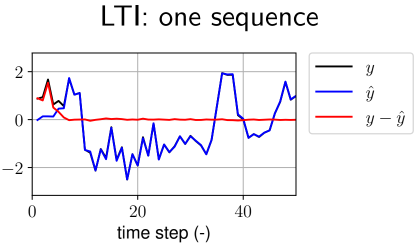
$$\mathbf{u} \rightarrow \boxed{G(z)} \rightarrow \boxed{F(\cdot)} \rightarrow \boxed{G(z)} \rightarrow \mathbf{y}$$

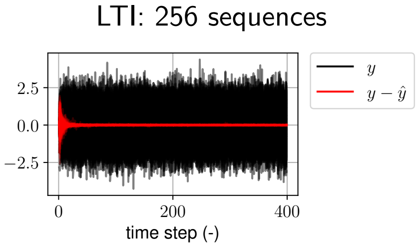
- Sequential LTI $\rightarrow F(\cdot) \rightarrow$ LTI
- Random LTI, order $\leq 5$
- $F(\cdot)$: random feedforward NN.

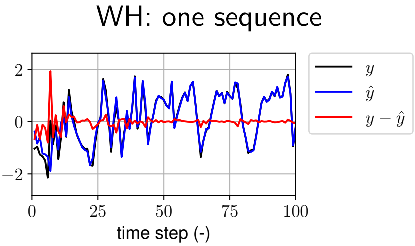
- For both classes, input $u_{1:N}$ is a white Gaussian noise sequence.
- This defines a $p(\mathcal{D})$. We can generate infinite datasets!
- Each dataset from a different input/system realization!

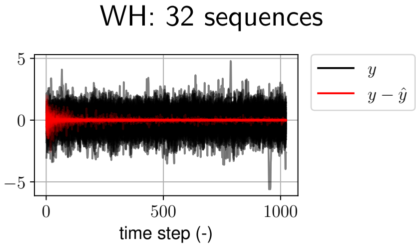
https://github.com/forgi86/sysid-transformers

# Experiments - one-step prediction results

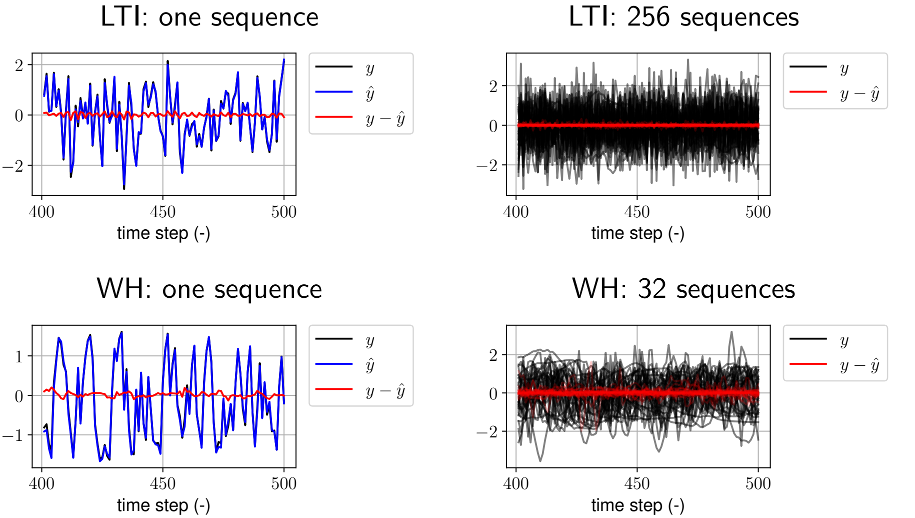


# Experiments - multi-step simulation results



# Conclusions

An in-context learning approach for system identification.

- Model-free, no need to re-train for a specific dataset/system
- Exploits the power of Transformers seen as trainable algorithms
- Seems to work!

Many possible research directions including:

- Transfer learning from one system class to another...
- ... and from simulation to reality.

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Thank you.
Questions?

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