

B.J. Brenton

BUAD 502b

Module 7

25.2 Use Monte Carlo simulation to estimate the probability that all six faces appear exactly once in six tosses of a fair die.

Answer should be $\frac{6!}{6^6} \cong 0.015$

```
> # 25.2
> # random throw of 6 6-sided die
> # iterate nrep times
> # use unique function to test x
> # if length of unique values is 6 (all appear exactly once)
> # then iterate the count
> nrep = 100000
> count = 0
> for (i in 1:nrep) {
+   x = sample(1:6, 6, replace = TRUE)
+   if (length(unique(x)) == 6) count = count + 1
+ }
> print (count/nrep)
[1] 0.0157
```

25.3 A waiting line consists of 40 men and 10 women arranged in random order. Use Monte Carlo simulation to estimate the probability that no two women in line are adjacent to one another.

```
> #25.3
> # create vector with random placement of 40 NA's and 10 1's
> # iterate loop nrep times
> # use the diff function, adjacent 1's will deliver 0 not NA
> # sum number of NA's in result, if all NA's (49)
> # then no two women in line are adjacent
> # iterate the count when this occurs
> nrep = 100000
> count = 0
> for (i in 1:nrep) {
+   x = sample(c(rep(NA, 40), rep(1, 10))) #women are 1
+   if (sum(is.na(diff(x))) == 49 ) count = count + 1
+ }
> print (count/nrep)
[1] 0.10857
```

25.4 You roll a fair die five times. Write R code that conducts a Monte Carlo simulation experiment with 500,000 replications that estimates the probability that you will see a string of three or more consecutive ones.

```
> # 25.4
> # random throw of 5 6-sided dice
> # iterate nrep times
> # three repeating ones will be in position 123, 234, or 345
> # sum these positions, if equal to three they are all 1's
> # then iterate the count
> nrep = 500000
> count = 0
> for (i in 1:nrep) {
+   x = sample(1:6, 5, replace = TRUE)
+   if ( sum(x[1:3]) == 3 ) count = count + 1
+   if ( sum(x[2:4]) == 3 ) count = count + 1
+   if ( sum(x[3:5]) == 3 ) count = count + 1
+ }
> print(count/nrep)
[1] 0.0137
```

25.10 Write a sequence that describes the quantity that is being estimated in the R monte Carlo simulation code given below.

```
> nrep = 100000
> count = 0
> for (i in 1:nrep) {
+   x = sample(6, 5, replace = TRUE)
+   if (min(x) >= 2) count = count + 1
+ }
> print(count/nrep)
```

Use a Monte Carlo simulation experiment with 100,000 replications that estimates the probability that when you roll 5 fair dice you do not see any ones.

Check: solution: $\frac{5^5}{6} \cong .4019$

```
> #25.10
> nrep = 100000
> count = 0
> for (i in 1:nrep) {
+   x = sample(6, 5, replace = TRUE)
+   if (min(x) >= 2) count = count + 1
+ }
> print(count/nrep)
[1] 0.4019
```

26.2 Give a 90% confidence interval for the mean annual rainfall in the United States using the precip data set.

```
> #26.2
> n      = length(precip)      # n is the sample size
> xbar   = mean(precip)        # sample mean
> sdev   = sd(precip)          # sample standard deviation
> alph   = 0.1                 # alpha = 0.1 for 90% conf interval
> crit   = qt(1 - alph / 2, n - 1) # quantile for the t-distribution
> half   = crit * sdev / sqrt(n) # interval halfwidth
> xbar + c(-1, 1) * half       # 90% confidence interval
[1] 32.15435 37.61708
> print(paste("Confidence Interval: (", round(xbar - half, 3),
+            ", ", round(xbar + half, 3), ")", sep = ""))
[1] "Confidence Interval: (32.154, 37.617)"
```

27.3 Let A be the 2×3 matrix $\begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 4 \end{bmatrix}$. Calculate the value of AA^T by hand, then execute R commands to verify your result.

$$A = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} -1 & 0 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A \cdot A^T = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 14 & 20 \end{bmatrix}$$

$$C_{11} = (-1) \cdot (-1) + (1) \cdot (1) + (3) \cdot (3) = 11$$

$$C_{12} = (-1) \cdot (0) + (1) \cdot (2) + (3) \cdot (4) = 14$$

$$C_{21} = (0) \cdot (-1) + (2) \cdot (1) + (4) \cdot (3) = 14$$

$$C_{22} = (0) \cdot (0) + (2) \cdot (2) + (4) \cdot (4) = 20$$

```
> A <- matrix(-1:4, nrow = 2, ncol = 3)
> AT <- t(A)
> print (A %*% AT)
      [,1] [,2]
[1,]   11   14
[2,]   14   20
```