

# MATH 3340 - Scientific Computing Homework 8

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The deadline will be strictly enforced. If you do not submit in time there will be a 20% penalty for each day you're late. If you do not submit in time there will be a 20% penalty upfront plus another 20% for each day you're late. Remember that you are allowed to work in teams of two on this assignment. You are encouraged to prepare your work in  $\text{\LaTeX}$ ; a template will be provided to help you put it all together. If you choose to submit a hard copy, you may submit only one copy for a team, indicating the names of both contributors. Online submission is encouraged, however, in that case both members of a team should submit the PDF file containing their work and showing both their names.

*All plots generated in this homework should have a title, legend, and labeled  $x$  and  $y$ -axes.*

## Instruction

1. Go to <https://www.overleaf.com> and sign in (required).
2. Open [template](#), click *Menu* (up left corner), then *Copy Project*.
3. Go to `LaTeX/meta.tex` (the file `meta.tex` under the folder `LaTeX`) to change the section and your name, e.g.,
  - change title to `\title{MATH 3340-01 Scientific Computing Homework 8}`
  - change author to `\author{Albert Einstein \& Carl F. Gauss}`
4. For Problem 1 and 3, you are encouraged to type solutions in  $\text{\LaTeX}$ . But if you want to write it on the printout, make sure your scanned work is *clear* enough, and compile all solutions *in order*, i.e., 1, 2, 3, in a single PDF (failure to do so will lead to points deduction).
5. For Problem 2 and 3, you need to write function/script files, store results to output files, and save graphs to figure files. Here are suggested names for function files, script files, output files, and figure files:

Problem	Function File	Script File	Output File	Figure File
2	monteCarlo.m	hw8_p2.m	hw8_p2.txt	
3	goldenSection.m	hw8_p3.m	hw8_p3.txt	hw8_p3.pdf

Once finished, you need to upload these files to the folder `src` on Overleaf. If you have different filenames, please update the filenames in `\lstinputlisting{./src/your_script_name.m}` accordingly. You can code in the provided files in [hw8.zip](#), and use the MATLAB script `save_results.m` to generate the output files and store the graphs to `.pdf` files automatically (the script filenames should be exactly same as listed above).

6. Recompile, download and upload the generated PDF to WyoCourses.
7. You may find  [\$\text{\LaTeX}\$ .Mathematical.Symbols.pdf](#) and the second part of [Lab 01 Slides](#) and [Lab 02 Slides](#) helpful.

**Problem 1.** This computation should be done by hand. Use Romberg integration to compute the  $R_{3,3}$  approximation for

$$I = \int_1^3 (x^3 - 1)e^{-x^2} dx. \quad (1.1)$$

Perform all calculations by rounding off to four decimal places.

**Solution.**

- $R$  obtained by Romberg integration:

$$\begin{bmatrix} R_{11} \\ R_{21} & R_{22} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} =$$

- Approximation of (1.1) :

$$R_{3,3} =$$

- All the calculations step by step:



**Problem 2.** Write a code that integrates the function  $f(x, y, z) = 0.7(x^2 + y^2 + z^2)$  over the unit sphere  $S = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$  using the Monte-Carlo method in three dimensions. Run the code with a sample of  $M = 10^6$  points; do this ten times and compute the average of the ten results. Then compare this average result with the exact value of the integral, which can be easily calculated analytically. Print the error: the absolute value of the difference between the exact value and the average you got from your ten runs.

**Solution.**

- Output file hw8\_p2.txt:

```

1 Monte Carlo integration for f(x, y, z) = x^3 + y^2 + z over the ball x^2 + y^2 + z^2 <= 4: (REPLACE
  THIS WITH YOUR OWN OUTPUT)
2      I_monte_carlo      I_exact      error
3      26.775929      26.808257      0.032329

```

- Function file monteCarlo.m:

```

1 function I = monteCarlo(f, checker, xmin, xmax, ymin, ymax, zmin, zmax, N)
2 %MONTECARLO: Monte Carlo Integration of f(x, y)
3 % Input:
4 %     f: function handle, the integrand of the integral
5 % checker: function handle, set the function value to zero if the points (x, y, z) is
   outside the region
6 %     xmin: scalar value, lower bound of x coordinate of the bounding box for the region
7 %     xmax: scalar value, upper bound of x coordinate of the bounding box for the region
8 %     ymin: scalar value, lower bound of y coordinate of the bounding box for the region
9 %     ymax: scalar value, upper bound of y coordinate of the bounding box for the region
10 %     zmin: scalar value, lower bound of z coordinate of the bounding box for the region
11 %     zmax: scalar value, upper bound of z coordinate of the bounding box for the region
12 %     N: scalar value, number of samples
13 % Output:
14 %     I: the integral of f(x, y, z) over the specific region
15 % Author: first_name last_name
16 % Date: mm/dd/yyyy
17
18 % PUT YOUR CODE HERE
19
20 end

```

- Script file hw8\_p2.m:

```

1 % MATH 3340, Semester Year
2 % Homework 8, Problem 2
3 % Author: first_name last_name
4 % Date: mm/dd/yyyy
5
6 clear; close all; clc;
7 rng(1204);
8 % PUT YOUR CODE HERE

```



**Problem 3.** Write a MATLAB function that implements the golden search method. Use this function to find the minimum of the function  $f(x) = \cos(x) - \sin(x)$  on the interval  $[1, 3]$  with a tolerance  $T = 10^{-7}$ . You should first plot the function and check that it is unimodal on this interval. Also, find the number of iterations needed to locate the value of the minimum with a tolerance of at least  $T = 0.1$ . Do the latter calculation by hand.

**Solution.**

- The number of iterations needed to locate the value of the minimum with a tolerance of at least  $T = 0.1$ :

- Output file `hw8_p3.txt`:

```
1 f(x) = x^2 + 4*x + 3 reaches the minimum -1.00000000 at x = -2.00000003. (REPLACE THIS WITH YOUR OWN OUTPUT)
```

- Figure file `hw8_p3.pdf`:

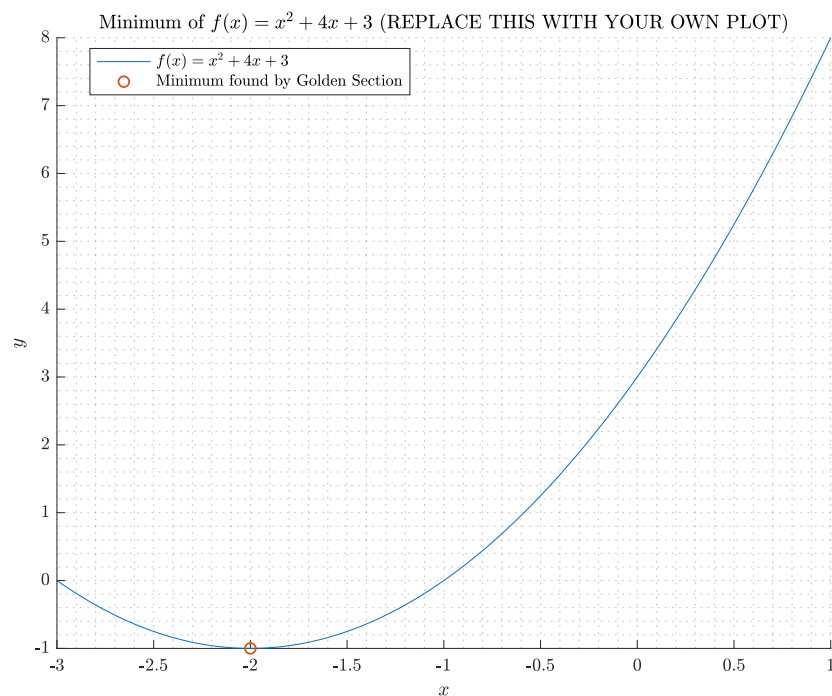


Figure 1: Minimum of  $f(x)$

- Function file `goldenSection.m`:

```
1 function [m, fm] = goldenSection(f, a, b, tol)
2
3 %GOLDENSECTION
4 % Syntax: [m] = goldenSection(f, a, b, tol)
5 % Inputs:
6 %   f   = function of which the minimum is desired, function handle
7 %   a   = left endpoint of the interval
8 %   b   = right endpoint of the interval
9 %   tol = tolerance
10 % Outputs:
11 %   m   = the minimizer at which the minimum of f(x) can be obtained
12 %   fm  = the minimum of the function
13 % Author: first_name last_name
14 % Date: mm/dd/yyyy
15
16 % PUT YOUR CODE HERE
17
18 end
```

- Script file `hw8_p3.m`:

```
1 % MATH 3340, Semester Year
2 % Homework 8, Problem 3
3 % Author: first_name last_name
4 % Date: mm/dd/yyyy
5
6 clear; close all; clc;
7 set(groot, 'defaulttextinterpreter', 'latex');
8 set(groot, 'defaultAxesTickLabelInterpreter', 'latex');
9 set(groot, 'defaultLegendInterpreter', 'latex');
10 figure(1); hold on;
11
12 % PUT YOUR CODE HERE
```

