Lab 12: Romberg Integration

Melissa Butler

November 15, 2021

1 Figure and Output

1.1 Figure file: `lab_12_figure.pdf`

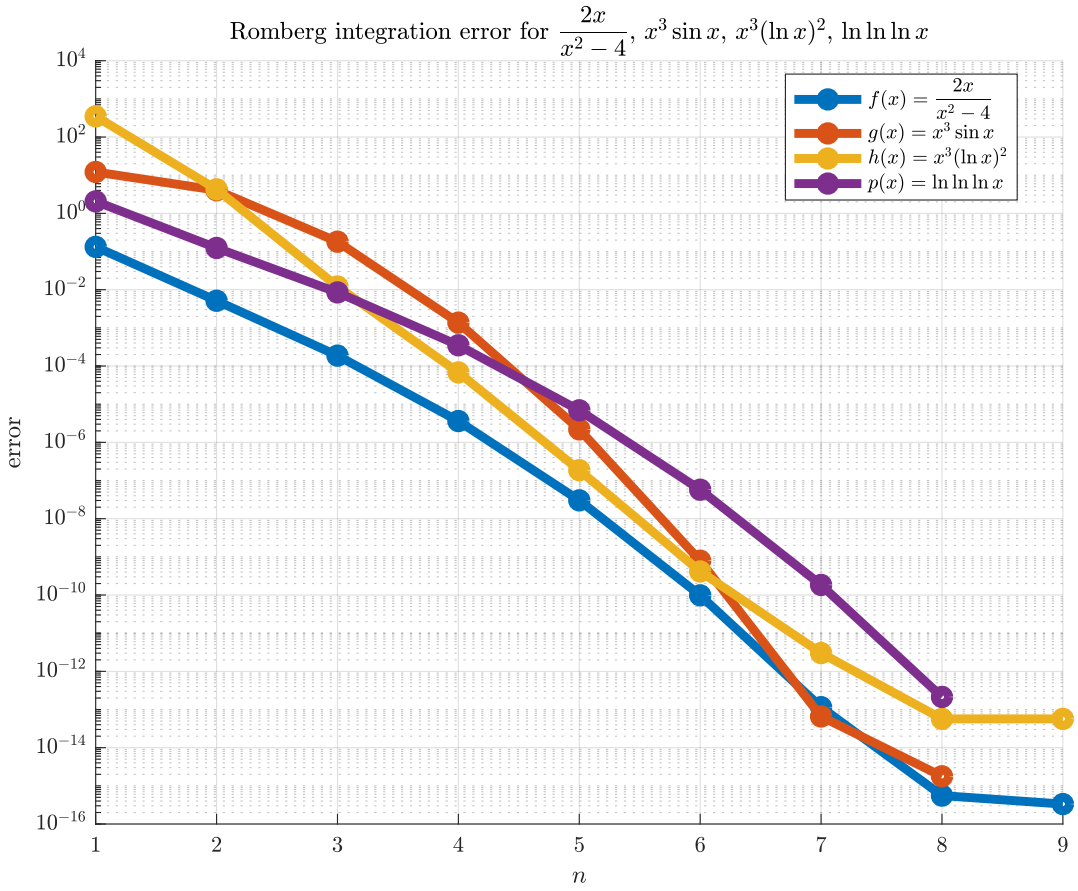

Figure 1: Romberg Integration error vs. n

1.2 Output file: lab_12_output.txt

```
lab_12_script
---------------------- Test function 1 ----------------------
Exact solution: -0.733969175080200320
Using integral: -0.733969175080200542, Error:  2.22045e-16
Romberg integration:
N =  1, I = -0.866666666666666696, Error: 1.32697e-01
N =  2, I = -0.739105339105339154, Error: 5.13616e-03
N =  3, I = -0.734156668376617527, Error: 1.87493e-04
N =  4, I = -0.733972793808240320, Error: 3.61873e-06
N =  5, I = -0.733969205320933460, Error: 3.02407e-08
N =  6, I = -0.733969175178207478, Error: 9.80072e-11
N =  7, I = -0.733969175080315561, Error: 1.15241e-13
N =  8, I = -0.733969175080200875, Error: 5.55112e-16
N =  9, I = -0.733969175080200653, Error: 3.33067e-16

---------------------- Test function 2 ----------------------
Exact solution: 12.156720758761061063
Using integral: 12.156720758761061063, Error:  0.00000e+00
Romberg integration:
N =  1, I =  0.000000000000005965, Error: 1.21567e+01
N =  2, I =  8.117424252833536968, Error: 4.03930e+00
N =  3, I = 12.337938222467283111, Error: 1.81217e-01
N =  4, I = 12.155349307627609790, Error: 1.37145e-03
N =  5, I = 12.156722948791598427, Error: 2.19003e-06
N =  6, I = 12.156720757968916047, Error: 7.92145e-10
N =  7, I = 12.156720758761126788, Error: 6.57252e-14
N =  8, I = 12.156720758761062839, Error: 1.77636e-15
N =  9, I = 12.156720758761061063, Error: 0.00000e+00

---------------------- Test function 3 ----------------------
Exact solution: 298.495537150497682433
Using integral: 298.495537150497568746, Error:  1.13687e-13
Romberg integration:
N =  1, I = 647.572598495058628032, Error: 3.49077e+02
N =  2, I = 302.757858010192023812, Error: 4.26232e+00
N =  3, I = 298.507671982975352876, Error: 1.21348e-02
N =  4, I = 298.495605132144476102, Error: 6.79816e-05
N =  5, I = 298.495537336960126140, Error: 1.86462e-07
N =  6, I = 298.495537150083464439, Error: 4.14218e-10
N =  7, I = 298.495537150494669731, Error: 3.01270e-12
N =  8, I = 298.495537150497625589, Error: 5.68434e-14
N =  9, I = 298.495537150497625589, Error: 5.68434e-14

---------------------- Test function 4 ----------------------
Exact solution:  8.503961262414033939
Using integral:  8.503961262414033939, Error:  0.00000e+00
```

```
Romberg integration:
N =  1, I =  6.441862549748430311, Error: 2.06210e+00
N =  2, I =  8.379727967291962898, Error: 1.24233e-01
N =  3, I =  8.495468707104780748, Error: 8.49256e-03
N =  4, I =  8.503611316913707086, Error: 3.49946e-04
N =  5, I =  8.503954342599357119, Error: 6.91981e-06
N =  6, I =  8.503961204935768237, Error: 5.74783e-08
N =  7, I =  8.503961262230388840, Error: 1.83645e-10
N =  8, I =  8.503961262413820776, Error: 2.13163e-13
N =  9, I =  8.503961262414033939, Error: 0.00000e+00
diary off
```

2 Function and Script

2.1 Function file: lab_12_romberg.m

```
function val = lab_12_romberg(f, a, b, n)
%LAB_12_ROMBERG Approximates the integral using Romberg technique
% up to order O(h^(2n)) accuracy
% INPUT:
%   f:  f(x), a function handle
%   a: lower bound, a scalar
%   b: upper bound, a scalar
%   n: number of subintervals, a scalar
% OUTPUT:
%   val: diagonal elements of table, a vector of length n
%        to be more exact, val = [R(1, 1), R(2, 2), ..., R(n, n)];

h = b - a;
R = zeros(n);
R(1, 1) = (f(a) + f(b)) * h / 2;
for k = 2:n
    R(k, 1) = (R(k - 1, 1) + h * sum(f(a + (2 * (1:2^(k-2)) - 1) * h / 2))) / 2;
    for j = 2:k
        R(k, j) = R(k, j - 1) + (R(k, j - 1) - R(k - 1, j - 1)) / (4^(j - 1) - 1);
    end
    h = h / 2;
end
val = diag(R);
end
```

2.2 Script file: lab_12_script.m

```
% Math 3341, Fall 2021
% Lab 12: Romberg Integration
% Author: Melissa Butler
% Date: 11/15/2021

clear; clc;
n = 9;

%% Test function 1: f(x) = 2 * x / (x^2 - 4).
fprintf('---------------------- Test function 1 ----------------------\n');
% lower bound and upper bound for f
f_bound = [1, 1.6];
% define symbolic function
syms x
f_symbolic = 2 * x / (x ^ 2 - 4);
% define anynomous function
f_handle = @(x) 2 * x ./ (x .^ 2 - 4);
If_romberg = lab_12_romberg(f_handle, f_bound(1), f_bound(2), n);
error_f = lab_12_compare_integral(f_symbolic, x, f_handle, f_bound, If_romberg, n);

%% 4(a) Test function 2: g(x) = x^3 * sin(x).
fprintf('\n---------------------- Test function 2 ----------------------\n');
% lower bound and upper bound for g
g_bound = [0, pi];
% define symbolic function
syms x
g_symbolic = x^3 * sin(x);
% define anynomous function
g_handle = @(x) x.^3 .* sin(x);
Ig_romberg = lab_12_romberg(g_handle, g_bound(1), g_bound(2), n);
error_g = lab_12_compare_integral(g_symbolic, x, g_handle, g_bound, Ig_romberg, n);

%% 4(b) Test function 3: h(x) = x^3 * (log(x))^2.
fprintf('\n---------------------- Test function 3 ----------------------\n');
% lower bound and upper bound for h
h_bound = [1, 5];
% define symbolic function
syms x
h_symbolic = x^3 * (log(x))^2;
% define anynomous function
h_handle = @(x) x.^3 .* (log(x)).^2;
Ih_romberg = lab_12_romberg(h_handle, h_bound(1), h_bound(2), n);
error_h = lab_12_compare_integral(h_symbolic, x, h_handle, h_bound, Ih_romberg, n);

%% 4(c) Test function 4: p(x) = log(log(log(x))).
fprintf('\n---------------------- Test function 4 ----------------------\n');
% lower bound and upper bound for p
p_bound = [exp(1)^exp(1), exp(1)^4];
% define symbolic function
syms x
p_symbolic = log(log(log(x)));
% define anynomous function
```

```
p_handle = @(x) log(log(log(x)));
Ip_romberg = lab_12_romberg(p_handle, p_bound(1), p_bound(2), n);
error_p = lab_12_compare_integral(p_symbolic, x, p_handle, p_bound, Ip_romberg, n);

%% Visualize the error
% Change default text interpreter to LaTeX
set(groot,'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

fig = figure(1);
hold on;
% plot error of f(x) against n
plot(error_f, 'o-', 'LineWidth', 4);
% plot error of g(x) against n
plot(error_g, 'o-', 'LineWidth', 4);
% plot error of h(x) against n
plot(error_h, 'o-', 'LineWidth', 4);
% plot error of p(x) against n
plot(error_p, 'o-', 'LineWidth', 4);
% Formatting graph
grid on;
legend({
    '$\displaystyle f(x) = \frac{2x}{x^2 - 4}$',...
    '$g(x) = x^3 \sin{x}$',...
    '$h(x) = x^3 (\ln{x})^2$',...
    '$p(x) = \ln{\ln{\ln{x}}}$'}, 'Location', 'best');
set(gca, 'YScale', 'log');
xlabel('$n$');
ylabel('error');
title('Romberg integration error for $\displaystyle \frac{2x}{x^2 - 4}$, $x^3 \sin{x}$, $x^3 (\ln{x
    })^2$, $\ln{\ln{\ln{x}}}$');

% save plots
fig.PaperPositionMode = 'auto';
f1_pos = fig.PaperPosition;
fig.PaperSize = [f1_pos(3) f1_pos(4)];
print(fig, '-dpdf', 'lab_12_figure.pdf')
```