

# Lab 06: LU Decomposition

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## 1 OUTPUT: lab\_06\_output.txt

```
1 lab_06_script
2 A =
3     7   -26   45  -47
4     1    2    3    4
5     2   -11  -12  -13
6     4   -17   30   35
7 b =
8   -98
9    30
10  -108
11   200
12 L =
13   1.0000    0    0    0
14   0.1429    1.0000    0    0
15   0.2857  -0.6250    1.0000    0
16   0.5714  -0.3750  -0.1111    1.0000
17 U =
18   7.0000 -26.0000  45.0000 -47.0000
19    0    5.7143  -3.4286  10.7143
20    0    0 -27.0000   7.1250
21    0    0    0  66.6667
22 z =
23  -98.0000
24   44.0000
25  -52.5000
26  266.6667
27 x =
28   1.0000
29   2.0000
30   3.0000
31   4.0000
32 res =
33   3.5527e-15
34
35 \begin{table}[!hbt]
```

```

36 \centering
37 \caption{Solution to the linear system}
38 \label{tab:solution}
39 \begin{tabular}{crrr}
40 \toprule
41 $m$ & $x$ & $y$ & $z$ \\
42 \midrule
43 $ 0$ & $-0.704225$ & $-0.211268$ & $ 2.323944$ \\
44 $ 1$ & $-0.323944$ & $-0.197183$ & $ 2.169014$ \\
45 $ 2$ & $ 0.056338$ & $-0.183099$ & $ 2.014085$ \\
46 $ 3$ & $ 0.436620$ & $-0.169014$ & $ 1.859155$ \\
47 $ 4$ & $ 0.816901$ & $-0.154930$ & $ 1.704225$ \\
48 $ 5$ & $ 1.197183$ & $-0.140845$ & $ 1.549296$ \\
49 $ 6$ & $ 1.577465$ & $-0.126761$ & $ 1.394366$ \\
50 $ 7$ & $ 1.957746$ & $-0.112676$ & $ 1.239437$ \\
51 $ 8$ & $ 2.338028$ & $-0.098592$ & $ 1.084507$ \\
52 $ 9$ & $ 2.718310$ & $-0.084507$ & $ 0.929577$ \\
53 $10$ & $ 3.098592$ & $-0.070423$ & $ 0.774648$ \\
54 $11$ & $ 3.478873$ & $-0.056338$ & $ 0.619718$ \\
55 $12$ & $ 3.859155$ & $-0.042254$ & $ 0.464789$ \\
56 $13$ & $ 4.239437$ & $-0.028169$ & $ 0.309859$ \\
57 $14$ & $ 4.619718$ & $-0.014085$ & $ 0.154930$ \\
58 $15$ & $ 5.000000$ & $-0.000000$ & $ 0.000000$ \\
59 $16$ & $ 5.380282$ & $ 0.014085$ & $-0.154930$ \\
60 $17$ & $ 5.760563$ & $ 0.028169$ & $-0.309859$ \\
61 $18$ & $ 6.140845$ & $ 0.042254$ & $-0.464789$ \\
62 $19$ & $ 6.521127$ & $ 0.056338$ & $-0.619718$ \\
63 $20$ & $ 6.901408$ & $ 0.070423$ & $-0.774648$ \\
64 \bottomrule
65 \end{tabular}
66 \end{table}
67 diary off

```

## 2 SCRIPT: lab\_06\_script.m

```

1 % Math 3341, Fall 2021
2 % Lab 06: LU Decomposition
3 % Author: Melissa Butler
4 % Date: 09/27/2021
5
6 clear; close all; clc;
7 % Change default text interpreter to LaTeX
8 set(groot, 'defaultTextInterpreter','latex');
9 set(groot, 'defaultAxesTickLabelInterpreter','latex');
10 set(groot, 'defaultLegendInterpreter','latex')
11 format compact
12
13 %% 1 Solve a System with LU Decomposition
14 % 1(a)
15 A = [7 -26 45 -47; 1 2 3 4; 2 -11 -12 -13; 4 -17 30 35]
16 b = [-98; 30; -108; 200;]
17 % 1(b)
18 [L U] = lu(A)
19 % 1(c)
20 z = L \ b
21 % 1(d)
22 x = U \ z
23 % 1(e)
24 res = norm(A * x - b, 2)
25
26 %% 2 Varying the Vector b
27 % 2(a)
28 A = [3 1 1; 1 -5 2; 2 1 5];
29 [L U] = lu(A);
30 % 2(b)
31 m = 0:20;
32 % 2(c)
33 for i = 1:length(m)
34     b = [m(i); 5; 10];
35     z = L \ b;
36     x = U \ z;
37     X(i, :) = x;
38 end
39 % 2(d)
40 file_handle = fopen('solution.tex', 'w');
41 fprintf(file_handle, '\\begin{table}[!hbt] \n');
42 fprintf(file_handle, '\\centering \n');
43 fprintf(file_handle, '\\caption{Solution to the linear system} \n');
44 fprintf(file_handle, '\\label{tab:solution} \n');
45 fprintf(file_handle, '\\begin{tabular}{cccc} \n');
46 fprintf(file_handle, '\\toprule \n');
47 fprintf(file_handle, '%4s & %11s & %11s & %11s \\\\ \n', '$m$', '$x$', '$y$', '$z$');
48 fprintf(file_handle, '\\midrule \n');
49 for i = 1:length(m)
50     fprintf(file_handle, '%2d$ & %9.6f$ & %9.6f$ & %9.6f$ \\\\ \n', m(i), X(i, 1), X(i, 2), X(i,
    3));

```

```
51 end
52 fprintf(file_handle, '\\bottomrule\\n');
53 fprintf(file_handle, '\\end{tabular}\\n');
54 fprintf(file_handle, '\\end{table}\\n');
55 fclose(file_handle);
56 type('solution.tex');
57 % 2(e)
58 figure(1); hold on;
59 X_size = size(X);
60 styles = {'h--', 'p:', 'd-'};
61 for j = 1:X_size(2)
62     plot(m, X(:, j), styles{j})
63 end
64 xlabel('$m$');
65 ylabel('Solution');
66 title('Solution to the linear system vs. $m$');
67 grid minor;
68 legend({'$x$', '$y$', '$z$'}, 'Location', 'best');
69 name = 'lab_06_plot';
70 fig = figure(1); % Set figure i as current figure window
71 set(fig, 'PaperPositionMode', 'auto'); % Set paper position mode to 'auto'
72 pos = get(fig, 'PaperPosition'); % Get figure window paper position
73 set(fig, 'PaperSize', [pos(3) pos(4)]); % Set figure paper size
74 print(fig, '-dpdf', name); % Save figure
```

### 3 BASICS OF L<sup>A</sup>T<sub>E</sub>X

`\subsection{LU Decomposition}`

Given the linear system `\eqref{eq:varyRHS}`

`\begin{equation}`

`\label{eq:varyRHS}`

`\begin{cases}`

`3x + y + z = m \\`

`x - 5y + 2z = 5 \\`

`2x + y + 5z = 10`

`\end{cases}`

`\end{equation}`

where  $m = 0, 1, 2, \ldots, 20$ . Using LU Decomposition we can obtain the solution to the linear system `\eqref{eq:varyRHS}` for corresponding  $m$  (see Table `\ref{tab:solution}` and Figure `\ref{fig:solution}`).

`\input{../src/solution.tex}`

`\begin{figure}[!hbt]`

`\centering`

`\includegraphics[width=0.85\textwidth]{../src/lab_06_plot.pdf}`

`\caption{Solution to the linear system vs.  $m$ }`

`\label{fig:solution}`

`\end{figure}`