

# MATH 3340 - Scientific Computing Homework 3

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Please note that the deadline will be enforced as per the previous homework. Remember that you are allowed to work in teams of two on this assignment. You are encouraged to prepare your work in  $\text{\LaTeX}$ ; a template will be provided to help you put it all together. If you choose to submit a hard copy, you may submit only one copy for a team, indicating the names of both contributors. Online submission is encouraged, however, in that case both members of a team should submit the PDF file containing their work and showing both their names.

## Instruction

1. Go to <https://www.overleaf.com> and sign in (required).
2. Open [template](#), click *Menu* (up left corner), then *Copy Project*.
3. Go to `LaTeX/meta.tex` (the file `meta.tex` under the folder `LaTeX`) to change the section and your name, e.g.,
  - change title to `\title{MATH 3340-01 Scientific Computing Homework 3}`
  - change author to `\author{Albert Einstein \& Carl F. Gauss}`
4. For Problem 1, 2, 3, you need to write function/script files, store results to output files, and save graphs to figure files. Here are suggested names for function files, script files, output files, and figure files:

Problem	Function File	Script File	Output File	Figure File
1	<code>jacobi.m</code>	<code>hw3_p1.m</code>	<code>hw3_p1.txt</code>	<code>hw3_p1.pdf</code>
2	<code>gaussSeidel.m</code>	<code>hw3_p2.m</code>	<code>hw3_p2.txt</code>	<code>hw3_p2.pdf</code>
3		<code>hw3_p3.m</code>	<code>hw3_p3.txt</code>	<code>hw3_p3.pdf</code>

Once finished, you need to upload these files to the folder `src` on Overleaf. If you have different filenames, please update the filenames in `\lstinputlisting{./src/your_script_name.m}` accordingly. You can code in the provided files in [hw3.zip](#), and use the MATLAB script `save_results.m` to generate the output files and store the graphs to `.pdf` files automatically (the script filenames should be exactly same as listed above).

5. Recompile, download, and submit the generated PDF.
6. You may find  [\$\text{\LaTeX}\$ .Mathematical.Symbols.pdf](#) and the second part of [Lab 01 Slides](#) and [Lab 02 Slides](#) helpful.

# 1 Problem 1

Solve, using a MATLAB code, the following system:

$$\begin{cases} 5x - y + z + w = 9 \\ x + 7y + 2z + 2w = 3 \\ 2x + y + 5z + w = 7 \\ x - y + z + 4w = 6 \end{cases}$$

Use the Jacobi method with a tolerance of  $10^{-5}$  for the norm of the residual. Arrange your results in a table of the form so that you can see how  $x$ ,  $y$ ,  $z$  and  $w$  change with each iteration. You

Table 1: caption

iteration	$x$	$y$	$z$	$w$
0	0	0	0	0
1	1	2	3	4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

can create such a table using `disp` in a loop; you may also use variants of `printf` (in C Language), i.e., `fprintf` or `sprintf` if you are familiar with these functions, but do not use the MATLAB `table` command as it does not do quite what is expected here. Also, as obvious from the table above, start with the zero guess. Moreover, plot the norm of the residual versus the iteration number; use a logarithmic scale on the vertical axis (the residual axis). Turn in your code and the output as described. The code can be organized as:

- (a) a function file that implements Jacobi's method; and
- (b) a script that calls the function with the appropriate inputs and processes the results.

Your plot should have a plot title, axes labels and a legend. Use the `help plot` command and the class notes to investigate the various options available when plotting.

## Solution.

- Function file `jacobi.m`

```

1 function [x, iters, res] = jacobi(A, b, x0, maxIter, tol)
2 %JACOBI: Jacobi Method
3 % Syntax: [x, iters] = jacobi(A, b, x0, maxIter, tol)
4 % Inputs:
5 %   A      = coefficient matrix (matrix)
6 %   b      = right-hand side vector (column vector)
7 %   x0     = solution to the linear system (column vector)
8 %   maxIter = maximum of number of iterations (scalar)
9 %   tol    = tolerance (scalar)
10 % Outputs:
11 %   x      = solution to the linear system (matrix, i-th column is the solution in the i-th
              iteration)
12 %   iters  = number of iterations performed (vector)
13 %   res    = norm of residuals (vector)

```

```

14 %
15 % Author: first_name last_name
16 % Date: mm/dd/yyyy
17
18
19 end

```

- Script file hw3\_p1.m

```

1 % MATH 3340, Semester Year
2 % Homework 3, Problem 1
3 % Author: first_name last_name
4 % Date: mm/dd/yyyy
5
6 clc; clear; figure(1);

```

- Output file hw3\_p1.txt

```

1 iterations      x      y      z      w
2           1    0.0000    0.0000    0.0000    0.0000
3           2    0.0000    0.0000    0.0000    0.0000
4           3    0.0000    0.0000    0.0000    0.0000
5           4    0.0000    0.0000    0.0000    0.0000

```

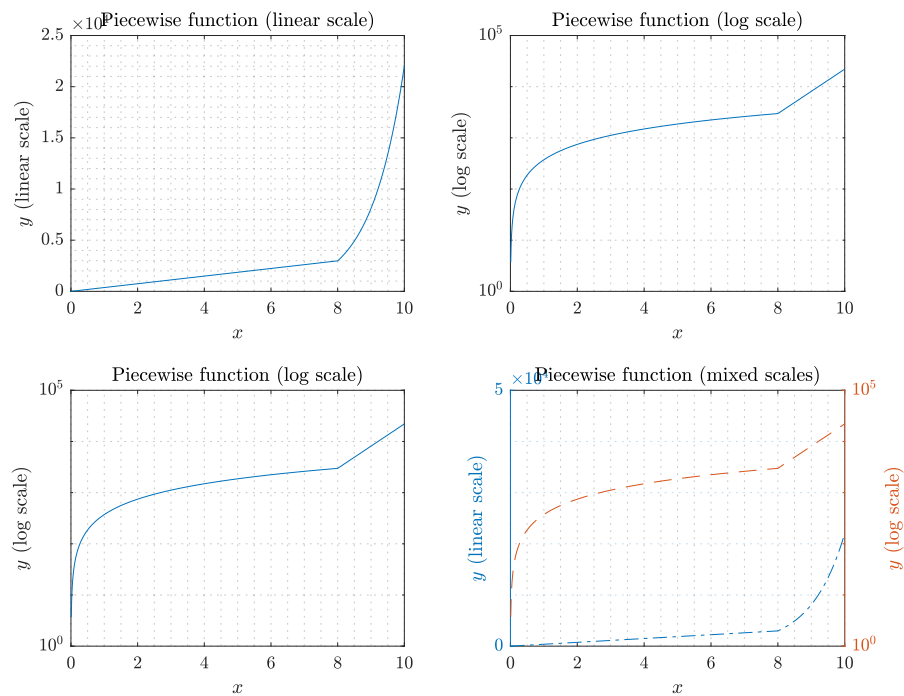


Figure 1

## 2 Problem 2

Solve again the system above, in MATLAB, using the Gauss-Seidel method. Produce the same results as for the previous problem.

**Solution.**

- Function file `gaussSeidel.m`

```

1 function [x, iters, res] = gaussSeidel(A, b, x0, maxIter, tol)
2 %GAUSSSEIDEL: Gauss-Seidel Method
3 % Syntax: [x, iters] = gaussSeidel(A, b, x0, maxIter, tol)
4 % Inputs:
5 %   A       = coefficient matrix (matrix)
6 %   b       = right-hand side vector (column vector)
7 %   x0      = solution to the linear system (column vector)
8 %   maxIter = maximum of number of iterations (scalar)
9 %   tol     = tolerance (scalar)
10 % Outputs:
11 %   x       = solution to the linear system (matrix, i-th column is the solution in the i-th
               iteration)
12 %   iters   = number of iterations performed (vector)
13 %   res     = norm of residuals (vector)
14 %
15 % Author: first_name last_name
16 % Date: mm/dd/yyyy
17
18
19 end

```

- Script file `hw3_p2.m`

```

1 % MATH 3340, Semester Year
2 % Homework 3, Problem 2
3 % Author: first_name last_name
4 % Date: mm/dd/yyyy
5
6 clc; clear; figure(2);

```

- Output file `hw3_p2.txt`

```

1 iterations      x          y          z          w
2           1    0.0000    0.0000    0.0000    0.0000
3           2    0.0000    0.0000    0.0000    0.0000
4           3    0.0000    0.0000    0.0000    0.0000
5           4    0.0000    0.0000    0.0000    0.0000

```

□

## 3 Problem 3

Solve the twenty-one systems of equations:

$$\begin{cases} 7x + y + 2z = 0.01m^2 - 2m \\ x - 5y + 2z = 2 - m \\ 2x + y + 5z = 9 \end{cases}$$

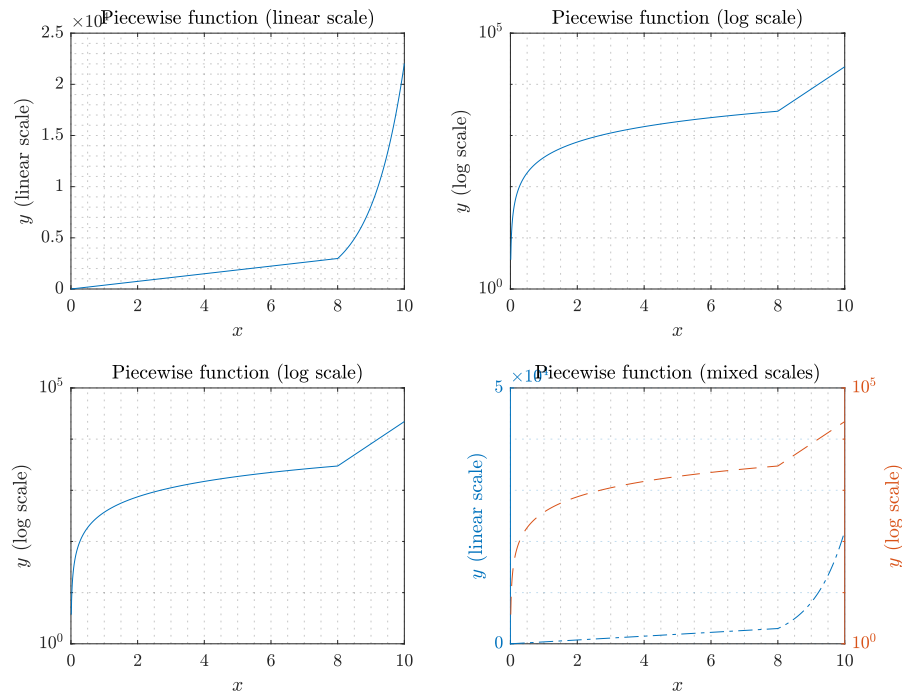


Figure 2

obtained by setting, in turn, the value of the quantity  $m$  on the right-hand side to all integers in between, and including,  $m = 0$  and  $m = 20$ . This time use the LU-decomposition of the system matrix that is implemented in MATLAB by the `lu` function. Perform the decomposition only once, then use the lower- and upper-triangular factors repeatedly to find each successive solution. Turn in the code and a plot of the first and second components of the solution (that is, the values of  $x$  and  $y$ ) as a function of the right-hand side parameter  $m$ .

### Solution.

- Script file `hw3_p3.m`

```

1 % MATH 3340, Semester Year
2 % Homework 3, Problem 3
3 % Author: first_name last_name
4 % Date: mm/dd/yyyy
5
6 clc; clear; figure(3); hold on;
```

- Output file `hw3_p3.txt`

	$m$	$x$	$y$	$z$
1				
2	0	0.0000	0.0000	0.0000
3	1	0.0000	0.0000	0.0000
4	2	0.0000	0.0000	0.0000
5	3	0.0000	0.0000	0.0000
6	4	0.0000	0.0000	0.0000
7	5	0.0000	0.0000	0.0000
8	6	0.0000	0.0000	0.0000

9	7	0.0000	0.0000	0.0000
10	8	0.0000	0.0000	0.0000
11	9	0.0000	0.0000	0.0000
12	10	0.0000	0.0000	0.0000
13	11	0.0000	0.0000	0.0000
14	12	0.0000	0.0000	0.0000
15	13	0.0000	0.0000	0.0000
16	14	0.0000	0.0000	0.0000
17	15	0.0000	0.0000	0.0000
18	16	0.0000	0.0000	0.0000
19	17	0.0000	0.0000	0.0000
20	18	0.0000	0.0000	0.0000
21	19	0.0000	0.0000	0.0000
22	20	0.0000	0.0000	0.0000

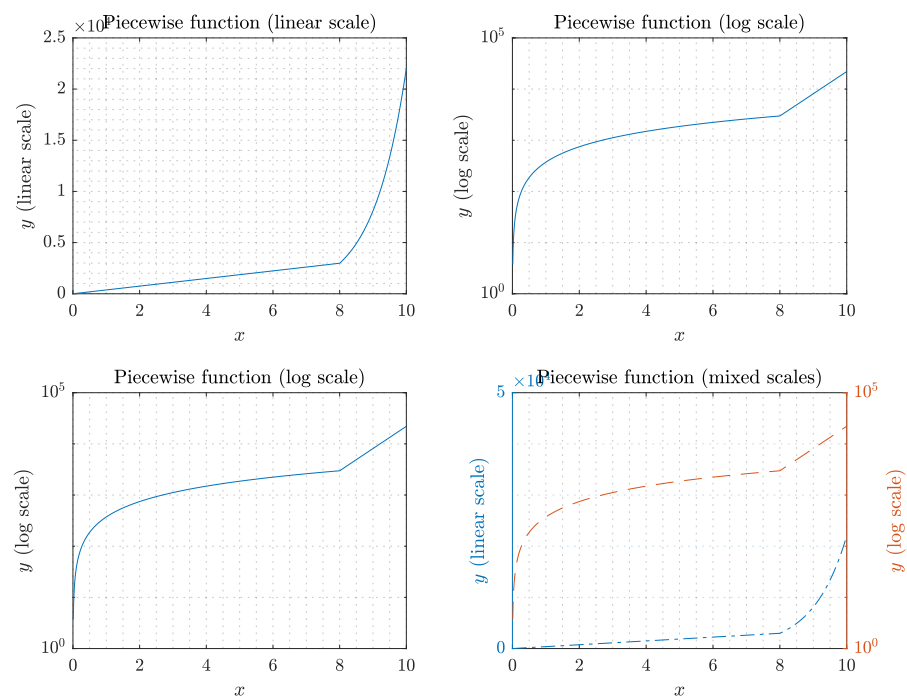


Figure 3

□