

# MATH 3341 — Fall 2021

## Lab 06: LU Decomposition

If you haven't downloaded and unzipped [Math.3341.zip](#). Download and unzip it under H: (H Drive if you are working on the Remote Lab). Change the current working directory by typing `cd H:\Math.3341\Math.3341.Lab.06` in the Command Window, and type `edit lab_06_script` in the Command Window to edit `lab_06_script.m`.

### 1 SOLVE A SYSTEM WITH LU DECOMPOSITION

- (a) Define matrix **A** and vector **b** as (1.1).

$$\underbrace{\begin{bmatrix} 7 & -26 & 45 & -47 \\ 1 & 2 & 3 & 4 \\ 2 & -11 & -12 & -13 \\ 4 & -17 & 30 & 35 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} -98 \\ 30 \\ -108 \\ 200 \end{bmatrix}}_{\mathbf{b}} \quad (1.1)$$

- (b) Calculate the LU decomposition **L**, **U** of the matrix **A**.

- (c) Solve the following system (1.2) and store the solution to **z**.

$$L\mathbf{z} = \mathbf{b}. \quad (1.2)$$

- (d) Then solve the following system (1.3) and store the solution to **x**.

$$U\mathbf{x} = \mathbf{z}. \quad (1.3)$$

- (e) Check your solution by calculating the norm of the residual  $\|A\mathbf{x} - \mathbf{b}\|_2$  and store the result to **res**.

### 2 VARYING THE VECTOR **b**

Suppose we want to solve the system for each integer value of  $m$  in between  $m = 0$  and  $m = 20$ . This time use the LU decomposition of the system matrix; perform the decomposition only once and use the lower and upper triangular factors repeatedly to find each successive solution. Then generate a table (Table 1) and a plot (Figure 1) of the solution versus the integer  $m$ .

$$\begin{cases} 3x + y + z = m \\ x - 5y + 2z = 5 \\ 2x + y + 5z = 10 \end{cases} \quad (2.1)$$

To do this you'll follow the steps below:

- (a) Define coefficient matrix **A** given in (2.1), and get the LU decomposition **L**, **U** of the matrix **A**.
- (b) Define a vector **m** which ranges from 0 to 20 with step size 1.

- (c) Then create a for-loop, of which the loop iterator  $i$  starts from 1 to `length(m)`. In the body of the loop, define a column vector  $\mathbf{b}$  as the right-hand side of (2.1), where  $m$  should be the  $i$ th component of  $\mathbf{m}$ . Then repeat (c) and (d) in Part 1. Store the solution  $\mathbf{x}$  to the  $i$ th row of  $\mathbf{X}$ .
- (d) Format the output of  $\mathbf{m}$  and  $\mathbf{X}$  to a file called `solution.tex` as you did in Part 3 of Lab 05:
- Use `fprintf` to print out the setup for the *table* and *tabular* environments. The first column of the table is centered while the rest three columns are right-justified in  $\text{\LaTeX}$ .
  - Between `\toprule` and `\midrule`, use `fprintf` to print out the heading of the table. The column widths are 4, 11, 11, 11, respectively.
  - Between `\midrule` and `\bottomrule`, use a for-loop to print each row of the table. Note that the  $i$ th row of the table consists of the  $i$ th component of  $\mathbf{m}$  and the  $i$ th row of matrix  $\mathbf{X}$ . The column widths are 2, 9, 9, 9, respectively. For floating point numbers, output 6 digits after the decimal point.
  - Call `type('solution.tex')` to print the content of `solution.tex`.
- (e) Plot the solution versus  $m$  using a for-loop as you did in Part 4 of Lab 05:
- Get the size of  $\mathbf{X}$  and assign it to `XSize`. Define a cell array `styles`, of which the entries are dashed line with hexagram, dotted line with pentagram, solid line with diamond.
  - Use a for-loop to plot each column of  $\mathbf{X}$  versus  $\mathbf{m}$  in the same figure window with the above styles.
  - Add labels, title, grid, legend as shown in Figure 1.
  - Save the plot to a file named `lab_06_plot.pdf`.

Type `diary('lab_06_output.txt')` in the Command Window, run the script file `lab_06_script.m`, and type `diary off` in the Command Window. Upload `lab_06_output.txt`, `lab_06_script.m`, `solution.tex`, and `lab_06_plot.pdf` to the folder `src` on Overleaf.

On Overleaf, open `body.tex` under the folder `LaTeX`. In the last section of the report, you will reproduce Section 3 using  $\text{\LaTeX}$ . You may find the following helpful:

- You may use environments such as `equation`, `cases`, `figure`, and `table`.
- You may use `\includegraphics[width=amount unit]{/path/to/figure.pdf}` to specify the width of a figure. In our case, the width of the figure is `0.85\textwidth`.
- You may use `\ref{labelName}` to refer to figures, tables; use `\eqref{labelName}` to refer to equations.
- For special symbols, you may look them up in  [\$\text{\LaTeX}\$ .Mathematics.Symbols.pdf](#).
- You may use `\input{/path/to/solution.tex}` to include the table you got from MATLAB.

Recompile and submit the PDF file generated by Overleaf to WyoCourses.

### 3 BASICS OF L<sup>A</sup>T<sub>E</sub>X

#### 3.1 LU DECOMPOSITION

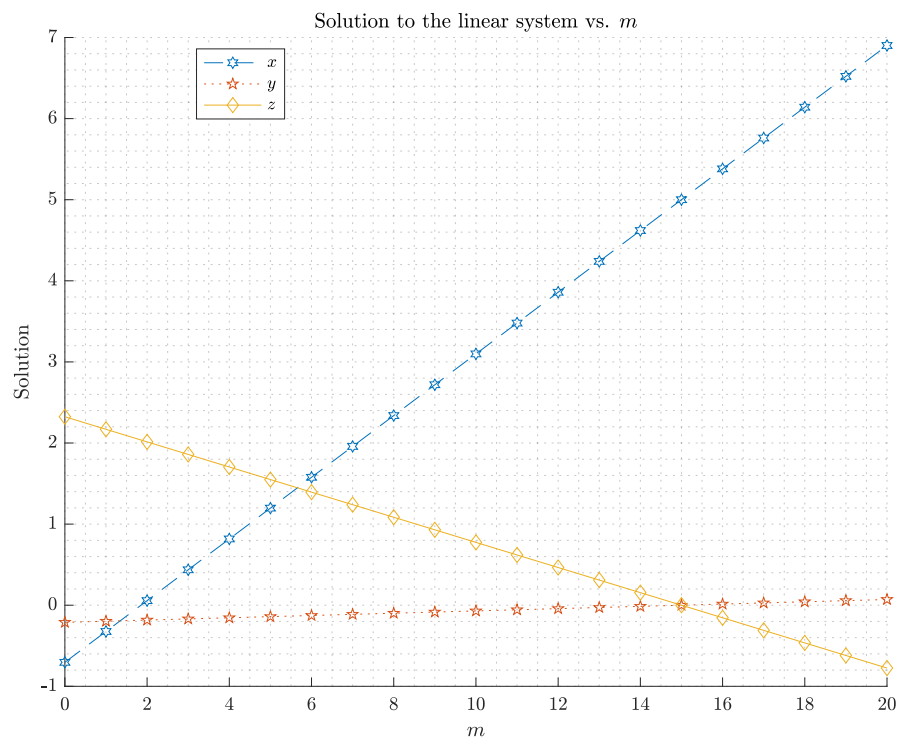
Given the linear system (3.1)

$$\begin{cases} 3x + y + z = m \\ x - 5y + 2z = 5 \\ 2x + y + 5z = 10 \end{cases} \quad (3.1)$$

where  $m = 0, 1, 2, \dots, 20$ . Using LU Decomposition we can obtain the solution to the linear system (3.1) for corresponding  $m$  (see Table 1 and Figure 1).

Table 1: Solution to the linear system

$m$	$x$	$y$	$z$
0	-0.704225	-0.211268	2.323944
1	-0.323944	-0.197183	2.169014
2	0.056338	-0.183099	2.014085
3	0.436620	-0.169014	1.859155
4	0.816901	-0.154930	1.704225
5	1.197183	-0.140845	1.549296
6	1.577465	-0.126761	1.394366
7	1.957746	-0.112676	1.239437
8	2.338028	-0.098592	1.084507
9	2.718310	-0.084507	0.929577
10	3.098592	-0.070423	0.774648
11	3.478873	-0.056338	0.619718
12	3.859155	-0.042254	0.464789
13	4.239437	-0.028169	0.309859
14	4.619718	-0.014085	0.154930
15	5.000000	-0.000000	0.000000
16	5.380282	0.014085	-0.154930
17	5.760563	0.028169	-0.309859
18	6.140845	0.042254	-0.464789
19	6.521127	0.056338	-0.619718
20	6.901408	0.070423	-0.774648

Figure 1: Solution to the linear system vs.  $m$