

# MATH 3341 — Fall 2021

## Lab 09: Ill-conditioned Matrices and Finite Precision Arithmetic

If you haven't downloaded and unzipped [Math.3341.zip](#). Download and unzip it under H: (H Drive if you are working on the Remote Lab). Change the current working directory by typing `cd H:\Math.3341\Math.3341.Lab.09` in the Command Window, and type `edit lab_09_script` in the Command Window to edit `lab_09_script.m`.

### 1 ILL-CONDITIONED SYSTEMS

- (a) Open the function file `lab_09_function.m`.
- (b) Generate an  $n \times n$  Hilbert matrix `A`.
- (c) Create an  $n \times 1$  all-one vector `b`.
- (d) Now solve the ill-conditioned linear system  $Ax = b$  for  $x$  by means of  $x = A^{-1}b$ :
  - Use `invhilb` to find  $A^{-1}$ , and store the exact solution  $x$  to `x_exact`.
  - Use `\` to solve the linear system and store the approximate solution  $\hat{x}$  to `x_backslash`.
  - Use `inv` to find  $A^{-1}$ , and store the approximate solution  $\hat{x}$  to `x_inv`.
- (e) Calculate the infinity norm of relative error of `x_backslash` and `x_inv` using the formula below, and store each relative error to `error_backslash` and `error_inv`, respectively.

$$\text{error}_{\text{relative}} = \frac{\|\hat{x} - x\|_{\infty}}{\|x\|_{\infty}}.$$

- (f) Calculate the 1-norm condition number of matrix `A` using `cond` and store the result to `cond_A`.
- (g) Next, go to `lab_09_script.m`, create a vector named `n`: `n = [9, 11, 13, 15]`. Using a for-loop to call `lab_09_function` by passing each entry of `n` as the input argument. What do you notice about the results?

### 2 FINITE PRECISION ARITHMETIC

- (a) Use either a for-loop or `sum` to calculate

$$s = \sum_{i=1}^{10} 0.1 = 0.1 + 0.1 + \cdots + 0.1.$$

Then calculate the error `error_1 = abs(s - 1)`.

- (b) Calculate

$$b = 2 - 3 \left( \frac{4}{3} - 1 \right).$$

Then calculate the error `error_2 = abs(b - 1)`.

- (c) Calculate each side of the following equation in the variables `lhs` and `rhs`:

$$1 + a + a^2 + a^3 + a^4 + a^5 = \frac{1 - a^6}{1 - a},$$

for the value  $a = 0.3$ . Then calculate the error `error_3 = abs(lhs - rhs)`. Hint: you can use either a for-loop or `polyval` to calculate `lhs`.

- (d) Calculate the following and compare their output.

```
1 x = 1e16 + 1 - 1e16;  
2 y = 1e16 - 1e16 + 1;  
3 z = 1e16 - (1e16 - 1);
```

- (e) Calculate the following and compare their output.

```
4 u = 1 + 0.1 - 1;  
5 v = 1 - 1 + 0.1;  
6 w = 1 - (1 - 0.1);
```

Once you finish, call `diary('lab_09_output.txt')`, then run the script file `lab_09_script.m`, and call `diary off` to save the resulting output. Then you need to upload files `lab_09_script.m`, `lab_09_function.m`, and `lab_09_output.txt` to Overleaf, and answer the corresponding questions in `body.tex`.