

MATH 3341 — Fall 2021

Lab 12: Romberg Integration

If you haven't downloaded and unzipped **Math.3341.zip**. Download and unzip it under H: (H Drive if you are working on the Remote Lab). Change the current working directory by typing `cd H:\Math.3341\Math.3341.Lab.12` in the Command Window, and type `edit lab_12_script` in the Command Window to edit `lab_12_script.m`.

1. Download the file `Math.3341.Lab.12.zip`, un-zip it.
2. The algorithm for Romberg integration is given below (see Algorithm 1). Please implement the algorithm in MATLAB in the provided function file `lab_12_romberg.m`.

Algorithm 1: Romberg integration: approximates $I = \int_a^b f(x) dx$ using n intervals.

Function `romberg(f, a, b, n):`
Input : f is the integrand, a is the lower bound, b is the upper bound, n is the number of subintervals.
Output: The integral of $f(x)$ over the interval $[a, b]$ using $1, 2, 3, \dots, n$ subintervals.
 $h \leftarrow b - a;$
 $R_{1,1} \leftarrow [f(a) + f(b)] \cdot h/2;$
for $k \leftarrow 2$ **to** n **do**
 $R_{k,1} \leftarrow \frac{1}{2} \left[R_{k-1,1} + h \sum_{j=1}^{2^{k-2}} f(a + (2j-1) \cdot h/2) \right];$
 for $j \leftarrow 2$ **to** k **do**
 $R_{k,j} \leftarrow R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1};$
 end
 $h \leftarrow h/2;$
end
return $[R_{1,1}, R_{2,2}, R_{3,3}, \dots, R_{n,n}];$
end

3. Run the script file `lab_12_script.m` to verify your function is working.
4. Uncomment line 18 through line 52 in the script file `lab_12_script.m`, and add more test functions to `lab_12_script.m`:

(a) $\int_0^\pi x^3 \sin x \, dx.$

(b) $\int_1^5 x^3 (\ln x)^2 \, dx.$

(c) $\int_{e^e}^{e^4} \ln \ln \ln x \, dx.$

5. Add plots for Romberg integration error of test functions $g(x)$, $h(x)$, $p(x)$ against n , which are also indicated in the comments (around line 65 through line 69).

6. Run `diary('lab_12_output.txt')`, run the script file `lab_12_script.m`, then call `diary off` to save the output to the specified text file.
7. Upload `lab_12_output.txt`, `lab_12_figure.pdf`, `lab_12_script.m`, and `lab_12_romberg.m` to Overleaf, recompile and submit the .pdf report to WyoCourses.