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CSc-30100

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Assignment 4

1)

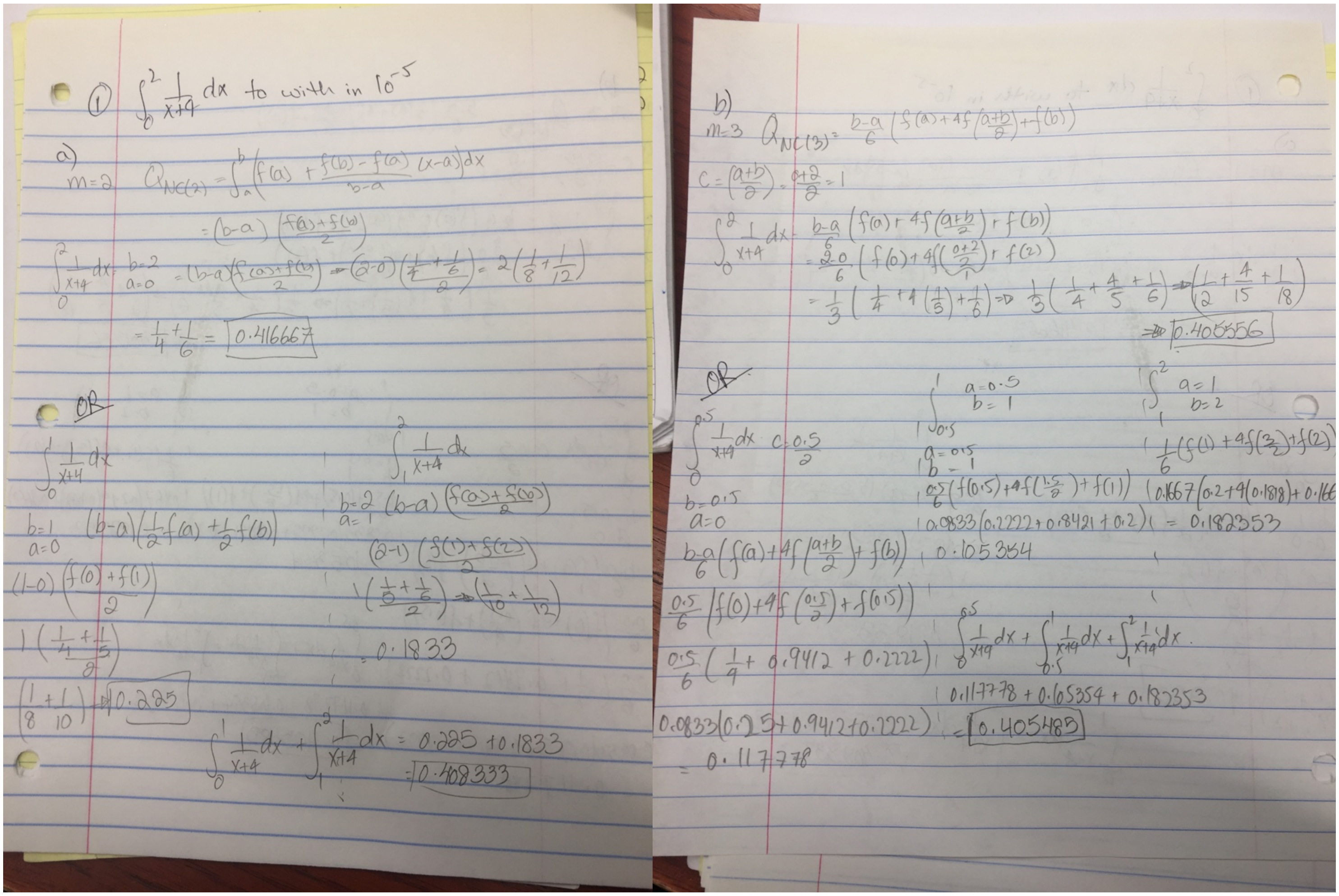
Determine the number of intervals required to approximate

$$\int_0^2 \frac{1}{x+4}\,dx$$

to within 10^{-5} and compute the approximation.

a. Using the Composite Trapezoidal rule;
b. Using the Composite Simpson's rule;
c. Using the Composite Gaussian quadrature rule.

Solution:



method	approximation	expected	error	accept(0/1)	# of n
CompTrapez	0.405471	0.405465	0.0000055	1	46
CompSimpsn	0.405466	0.405465	0.0000013	1	6
CompGauQaud	0.405464	0.405465	0.0000006	1	2

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As you can see in the above figure, the calculations for all methods are approximate similar (Trapezoid, Simson). But the result from the MATLAB code are more precise than doing by hand, since by hand the probability of making errors is higher than by writing a code. As you can see in the result the approximation value vs expected value and percentage of error between 2 values.

Also, we can figure out the number of intervals for all method in the MATLAB code. The Composite Gaussian quadrate rule was little harder to implement, so it was better to write a code and get the results.

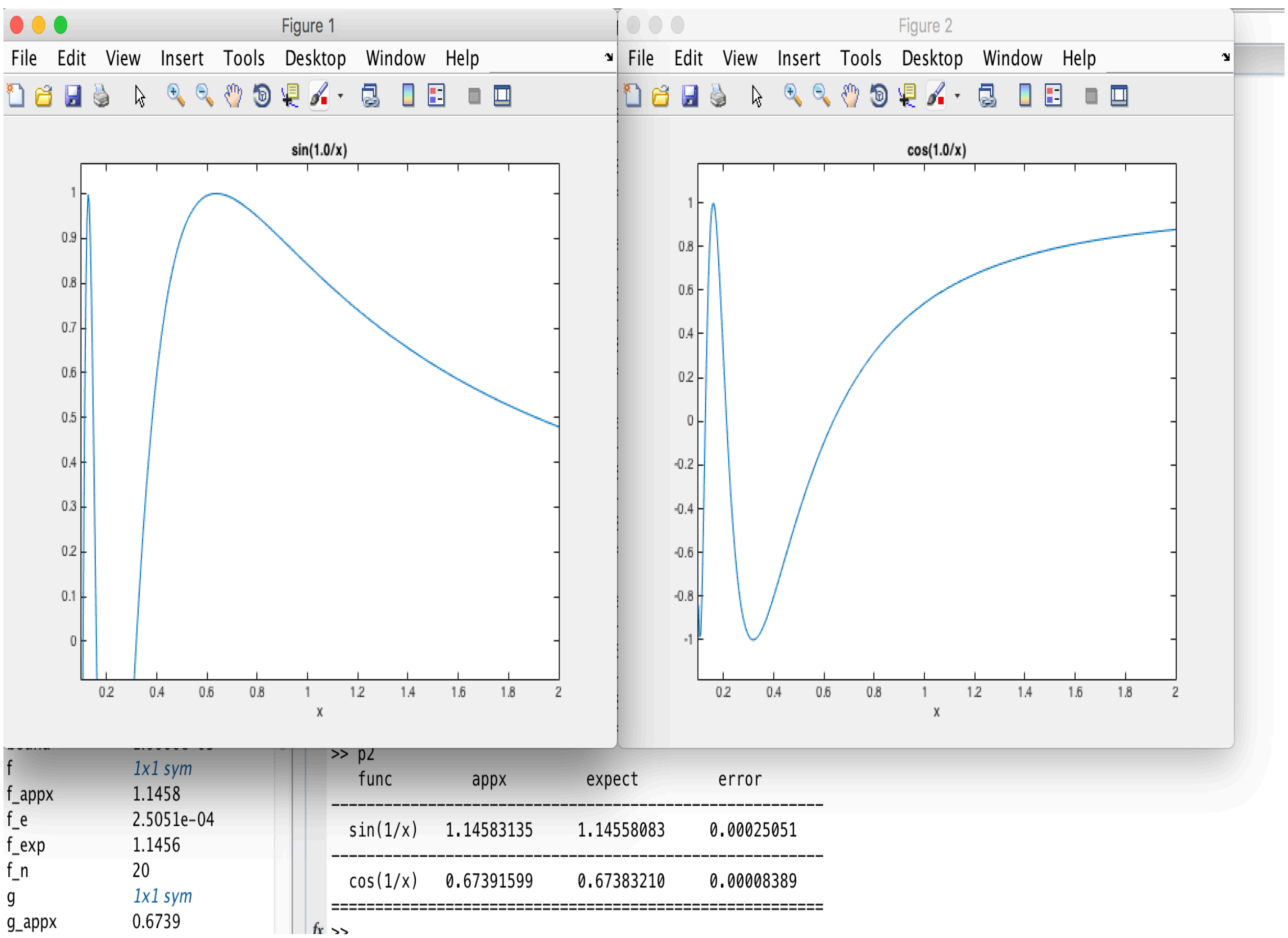
2)

Plot $f(x) = \sin(1/x)$ and $g(x) = \cos(1/x)$ on $[0.1\,, 2]$. Use Adaptive quadrature to approximate the integrals

$$\int_{0.1}^{2} f(x)\,dx \text{ and } \int_{0.1}^{2} g(x)\,dx$$

to within 10^{-3}. Find the number of subintervals used for each function. Are they similar? Explain.

Solution:



As you can see the two graphs sin(1/x) and cos(1/x) on the interval of [0.1,2]. The function and its approximated and expected value is calculated and can be seen in the picture. I have also calculated error by subtracting expected and approximated values. The figures look same at some interval, but after that they look different, because of the functionality of different functions. Sin(1/x) is tend to be have graph that has higher y values than cos(1/x).

3)

Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)\, dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that is exact for all polynomials of degree less then or equal to 3.

Solution:

3)

polynomials $1, x, x^2, x^3, \ldots$

$f(x) = 1 \Rightarrow \int_{-1}^{1} 1\,dx = x\Big|_{-1}^{1} \Rightarrow 2$ $a(1)+b(1)+c(0)+d(0)$

$f(x) = x^1 \Rightarrow \int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} \Rightarrow 0$ $a(-1)+b(1)+c(1)+d(1)$

$f(x) = x^2 \Rightarrow \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} \Rightarrow \frac{2}{3}$ $a(1)+b(1)+c(-2)+d(2)$

$f(x) = x^3 \Rightarrow \int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{1} \Rightarrow 0$ $a(-1)+b(1)+c(3)+3d$

$a+b=2$

$-a+b+c+d=0$

$a+b-2c+2d=\frac{2}{3}$

$-a+b+3c+3d=0$

1st Eq and 3rd Eq

$-(a+b = 2)$

$a+b-2c+2d=\frac{2}{3}$

$\Downarrow$

$-a-b = -2$

$+\ a+b-2c+2d=\frac{2}{3}$

$-2c+2d=\frac{2}{3}-2$

$-2c+2d=\frac{2}{3}-\frac{6}{3}$

$-2c+2d=-\frac{4}{3}$

$2c-2d=\frac{4}{3}$

2nd and 4th Eq

$-a+b+c+d=0$

$-(-a+b+3c+3d=0)$

$-a+b+c+d=0$

$+\ a-b-3c-3d=0$

$-2c-2d=0$

$\frac{-2c-2d}{-2}=\frac{0}{-2}$

$c+d=0$

$\boxed{c=-d}$ $\boxed{c=-(-\frac{1}{3})=\frac{1}{3}}$

$2c-2d=\frac{4}{3}$

$2(-d)-2d=\frac{4}{3}$

$-2d-2d=\frac{4}{3}$

$-4d=\frac{4}{3}$

$\boxed{d=-\frac{1}{3}}$

1^{st} and 2^{nd} EQ

$$a + b = 2$$
$$-a + b + c + d = 0$$

$\rightarrow$ Since $c = \frac{1}{3}$ and $d = \frac{1}{3}$, plug these values.

$$a + b = 2$$
$$-a + b + \frac{1}{3} - \frac{1}{3} = 0$$

$$a + b = 2$$
$$+ \quad -a + b = 0$$

$$2b = 2 \Rightarrow \boxed{b = 1}$$

$$a + b = 2$$
$$a + 1 = 2$$
$$\boxed{a = 1}$$

The constants are $a = 1, b = 1, c = \frac{1}{3}, d = -\frac{1}{3}$

4)

P4.3.6 Let Q_n be the equal spacing composite trapezoidal rule:

$$Q_n = h\left(\frac{1}{2}f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(x_n)\right) \qquad h = \frac{b-a}{n-1},$$

where $x = linspace(a, b, n)$ and we assume that $n \geq 2$. Assume that there is a constant C (independent of n), such that

$$I = \int_a^b f(x)fx = Q_n + Ch^2.$$

(a) Give an expression for $|I - Q_{2n}|$ in terms of $|Q_{2n} - Q_n|$. (b) Write an *efficient* script that computes Q_{2k+1}, where k is the smallest positive integer so that $|I - Q_{2k+1}|$ is smaller than a given positive tolerance `tol`. You may assume that such a k exists. You may assume that the integrand function is available in `f.m` and that it accepts vector arguments.

Solution:

a) I

a) $I = Q_n + ch^2$

(1) this can be written as follows by using the Error composite rule

(2) $I = Q_{2n} + \frac{Ch^2}{4}$

(1-2) The subtracting EQ1 and EQ2.

$|Q_n - Q_{2n}| = \frac{3Ch^2}{4}$

$1 - \frac{1}{4}$ Then solve for Ch^2 we get

$\frac{4}{4} -$ $Ch^2 = \frac{4}{3}|Q_n - Q_{2n}|$

Therefore

$|I - Q_{2n}| = \frac{Ch^2}{4} \Rightarrow \frac{\frac{4}{3}|Q_n - Q_{2n}|}{4}$ $\boxed{\frac{1}{3}|Q_n - Q_{2n}|}$