Abu Butt May 18, 2017
CSc 301 Prof. Gladkova

Assignment 5

1)

a)

1) $L_1 \dots\rightarrow$ corresponds to length of 2 points.

a) $L_1 = (u_1 - u_2)^2 + (v_1 - v_2)^2$ $L_2 = (u_1 - u_3)^2 + (v_1 - v_3)^2$ $L_3 = (-1 - u_1)^2 + (0 + v_1)^2$

$L_4 = (.5 - u_2)^2 + (1 - v_2)^2$ $L_5 = (0 - u_3)^2 + (-1 - v_3)^2$ $L_6 = (1 - u_3)^2 + (.5 - v_3)^2$

$L_7 = (1 - u_2)^2 + (.5 - v_2)^2$ minimize $\|Ax - b\|^2$

$$A = \begin{array}{c} \begin{array}{cccccc} a_1 & a_2 & a_3 & v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ .5 \\ 1 \\ 0 \\ -1 \\ 1 \\ .5 \\ 1 \\ .5 \end{pmatrix}$$

$$x = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

b)

1(b)

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \bar{A}$$

$$\Rightarrow \quad \|\bar{A}u - b\|^2$$

$$\bar{b} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ .5 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

For $\|\hat{A}v - \hat{b}\|$

$$\hat{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \hat{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ .5 \\ .5 \end{pmatrix}$$

Matrix A in part a is composte of Us and Vs components
But in part B $\bar{A}$ and $\hat{A}$ are the matrix of those components,
Since both have the same results.

2)

2)

For point y, which will minimizes the line C_i, we divide C_i and solve.

$$A = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \qquad b = \begin{pmatrix} P_{0u} + t_0 q_{0u} \\ \vdots \\ P_n u + t_n q_n u \end{pmatrix}$$

For a point to minimize the distance, we need to use least Square.

$$\hat{A} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \qquad \hat{b} = \begin{pmatrix} P_0 V + t_0 q_0 V \\ \vdots \\ P_n V + t_n q_n V \end{pmatrix} \qquad = \underset{\hat{x}}{\operatorname{argmin}} \left\| \hat{A}\hat{x} - \hat{b} \right\|^2$$

We keep doing it to find the point on V, Now we have (u, v) which will determine the distance is minimized from the lines.

3)

3) $p(t) = x_1 + x_2 t + x_3 t^2 + x_4 t^3$

after taking derivative of $p(t)$

$p'(t) = x_2 + 2x_3 t + 3x_4 t^2$

Since we are given 4 points and interval $[0,1]$ and plug the points into $p(t)$. and $p(t_i) \approx y_i$, $i = 1, \ldots, 4$.

For $i=1$ $\quad p(t_1) = x_1 + x_2 t_1 + x_3 t_1^2 + x_4 t_1^3 \approx y_1$

$i=2$ $\quad p(t_2) = x_1 + x_2 t_2 + x_3 t_2^2 + x_4 t_2^3 \approx y_2$

$i=3$ $\quad p(t_3) = x_1 + x_2 t_3 + x_3 t_3^2 + x_4 t_3^3 \approx y_3$

$i=4$ $\quad p(t_4) = x_1 + x_2 t_4 + x_3 t_4^2 + x_4 t_4^4 \approx y_4$.

Now we plug the interval values into the derivative of $p(t)$ and $p'(0) \approx 0$, $p'(1) \approx 0$

$p'(t) = x_2 + 2x_3 t + 3x_4 t^2$.

$t = 0, 1$

$p'(0) = x_2 + 2x_3(0) + 3x_4(0)^2 = x_2 \approx 0$

$p'(1) = x_2 + 2x_3(1) + 3x_4(1)^2 = x_2 + 2x_3 + 3x_4 \approx 0$

Now we do the average

$\int_0^1 p(t)\,dt \approx p(\tfrac{1}{2})$

$\int_0^1 p(t) - p(\tfrac{1}{2}) =$ after doing integral we get

$\frac{x_3}{12} + \frac{x_4}{64} \approx 0$

Now we need to coefficients of all the Equations/variables to form a matrix.

$$A = \underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ 1 & t_3 & t_3^2 & t_3^3 \\ 1 & t_4 & t_4^2 & t_4^2 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \frac{1}{12} & \frac{1}{64} \end{bmatrix}}_{4} \quad b = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{1} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Matrix A dimension 7×4

Matrix B dimension 7×1

Matrix A consists of the points t_i from the equations Coefficients.

Matrix B consists of the points by solving the Equation at point t_i.