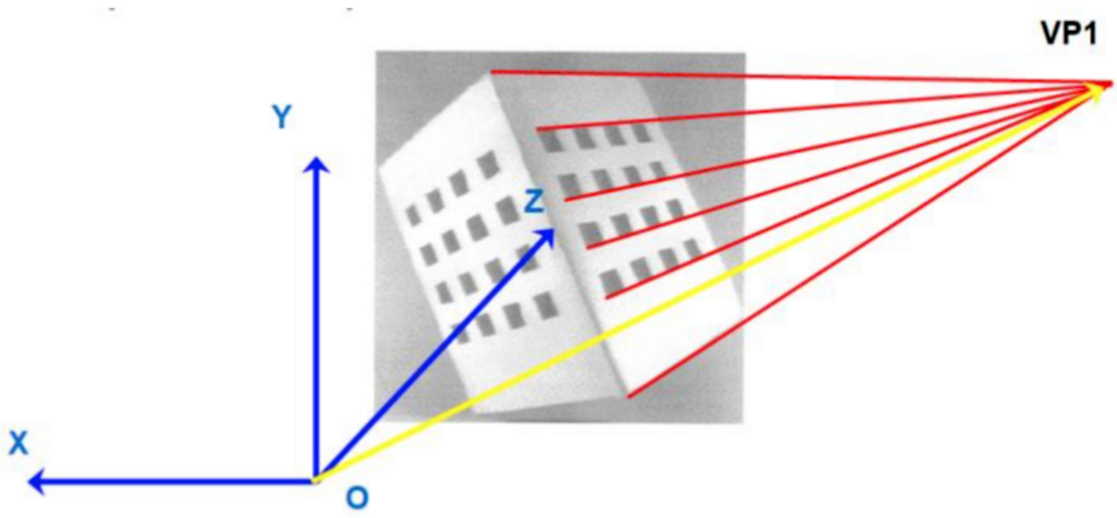
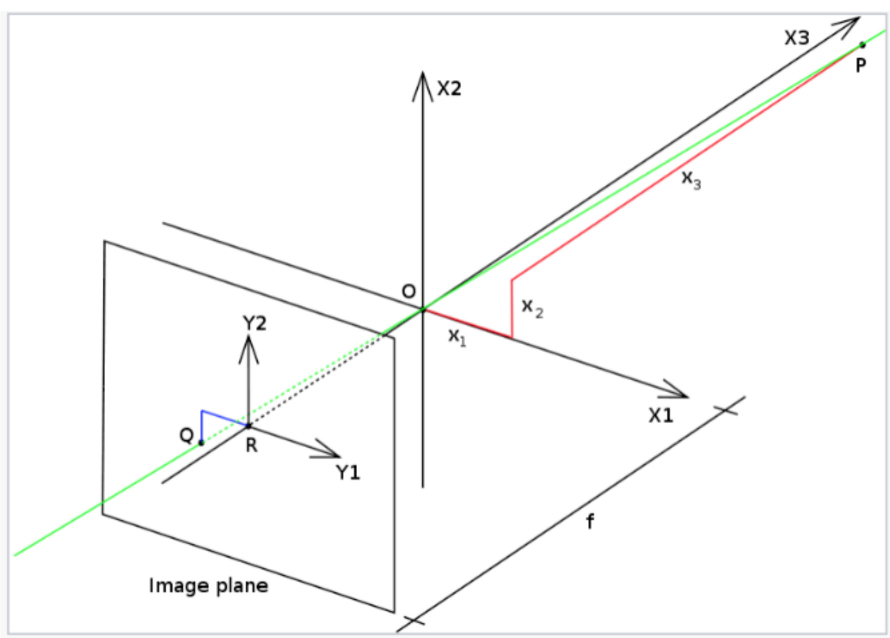
Abu Butt-3097 October 28, 2017
CSc-47100 Prof. Zhigang Zhu

Assignment 3

1)

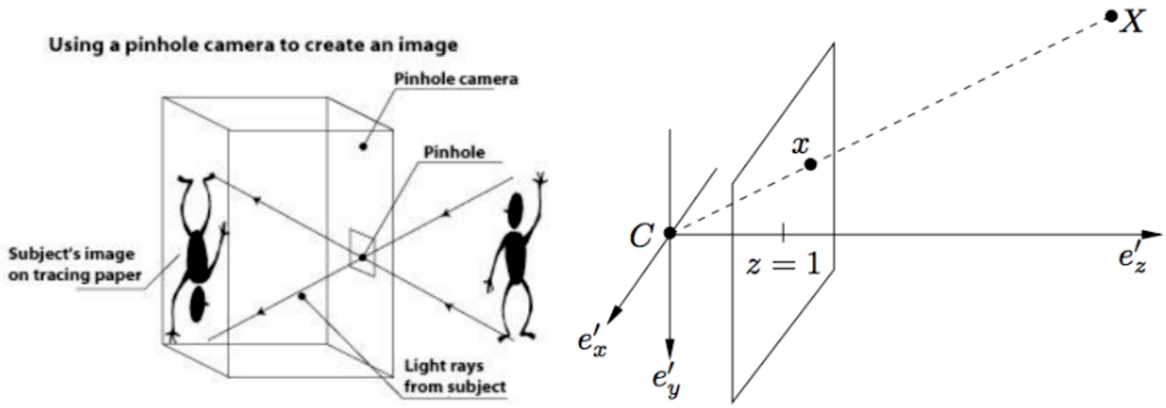


One of the important property of the camera model is Vector OV, which is formed from the center if the projection to vanishing point to the parallel lines.

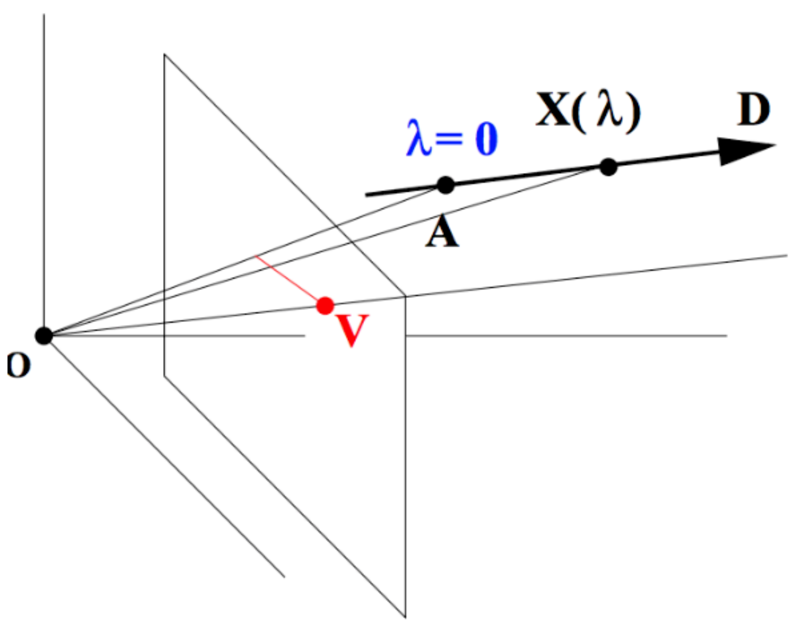


Above image is a geometry of a pinhole camera. As you can see, a 3D orthogonal coordinate system with its origin at O. This is also the position where the camera is located. The 3 axes are

X1, X2, X3. Axis X3 is pointing in the viewing direction of the camera and is called the optical axis. The 3D plane which intersect with axes X1 and X2 is the front side of the camera.



The mathematical image on the right side, whereas pinhole camera image is on the left side.



A line which consists of 3D points is represented as:

$$\mathbf{X}(\lambda) = \mathbf{A} + \lambda \mathbf{D}$$

if we take the derivative we will get:

$$\mathbf{v} = \lim_{\lambda \to \pm\infty} \mathbf{x}(\lambda) = f \frac{\mathbf{A} + \lambda \mathbf{D}}{A_Z + \lambda D_Z}$$

$$= f \frac{\mathbf{D}}{D_Z} = f \begin{pmatrix} D_X / D_Z \\ D_Y / D_Z \\ 1 \end{pmatrix}$$

v only depends on the direction D. All the parallel lines meet at same vanishing point.

2)

With the plane equation, we would add one more constraint for all the points on the plane.

$$\mathbf{N_x X_w + n_y Y_w + n_z Z_w = d}$$

This equation can be written as

$$\mathbf{n^T P_w = d}$$

where n is the parameter of the plane and P_w is a 3D point on the plane.
Therefore we Pw = (Xw, Yw, 0, 1)^T. The 3D points (Xw, Yw, Zw) becomes a 2D point(Xw, Yw) and we get the following matrix:

This is equation Q1:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -fr_{11} & -fr_{12} & -fT_x \\ -fr_{21} & -fr_{22} & -fT_y \\ r_{31} & r_{32} & T_z \end{bmatrix} \begin{pmatrix} X_w \\ Y_w \\ 1 \end{pmatrix}$$

If we assume nz to be equal to 1, the plane equation can be written as :

$$\mathbf{Z_w = d - n_x X_w - n_y Y_w}$$

Then we can easily transform the 3x3 matrix for the more general matrix by plugging the above equation into the equation Q1 with the matrix M

$$\mathbf{M} = \begin{bmatrix} -fr_{11} & -fr_{12} & -fr_{13} & -fT_x \\ -fr_{21} & -fr_{22} & -fr_{23} & -fT_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

then is equation Q2:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -f(r_{11}-n_x r_{13}) & -f(r_{12}-n_y r_{13}) & -f(dr_{13}+T_x) \\ -f(r_{21}-n_x r_{23}) & -f(r_{22}-n_y r_{23}) & -f(dr_{23}+T_y) \\ (r_{31}-n_x r_{33}) & (r_{32}-n_y r_{33}) & (dr_{33}+T_z) \end{bmatrix} \begin{pmatrix} X_w \\ Y_w \\ 1 \end{pmatrix}$$

For any point on the plane, we can calculate the 3D coordinates in the real world coordinates.

The we need to find out Mext for 3x4 matrix that only include extrinsic parameters.

$$\mathbf{M}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^T & T_x \\ \mathbf{R}_2^T & T_y \\ \mathbf{R}_3^T & T_z \end{bmatrix}$$

whereas Mint is a 3x3 matrix that only includes the intrinsic parematers:

$$\mathbf{M}_{\text{int}} = \begin{bmatrix} -f_x & 0 & o_x \\ 0 & -f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

If we combine both Mext and Mint then subitute in eq. Q1, we will have a linera representation of the perspective projection transformation:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \mathbf{M}_{\text{int}}\mathbf{M}_{\text{ext}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

then dividing both sides by X3, that we defined (x1, x2, x3)T:

$$\begin{pmatrix} x_{im} \\ y_{im} \end{pmatrix} = \begin{pmatrix} x_1 / x_3 \\ x_2 / x_3 \end{pmatrix}$$

3)

The center of the projection of the camera is O. Also, there are 2 mutual orthogonal set of parallel lines, L1, L2 and L3. Then assume, there is a triangle formed by these lines, triangle V1 V2 V3 and these are also the vanishing points (V1, V2, V3).

We must prove that the vector OVi from the pinhole to a vanishing point Vi is parallel to the mutually orthogonal parallel lines in 3D spaces in the first question.

This means that OVi is perpendicular to OVj as long as I is not equal to j, therefore OV1 is perpendicular to V2V3 and OV2 is perpendicular to V1V3, and so on. In addition, Vihi is the distance form Vi, therefore V1h1 is also perpendicular to V2V3, and so on. Let's define the center of the image as o, so line Oo is also perpendicular to the image plane. Also, the point o is also the projection of point O in the image plane, which lies on all 3 Vihi.

When using the Orthocenter theorem, we do not need any information about the camera parameters to find the vanishing points. Also, we do not need any focal length information for the Orthocenter theorem. But instead we only need to know the orthogonal relations of the altitude and aspect ratio of the image.

There are 3 generated vanishing points from the sets of parallel lines that are mutually orthogonal to each other. Having a 3 sets of orthogonal parallel line sets does not guarantee that a set of parallel lines will generate vanishing points. But the camera view angle matter. As an example, if the camera is perpendicular to the plane which has the set of the parallel lines, then the images of the parallel would still be parallel. The orthocenter theorem only works on images without lens distortions.