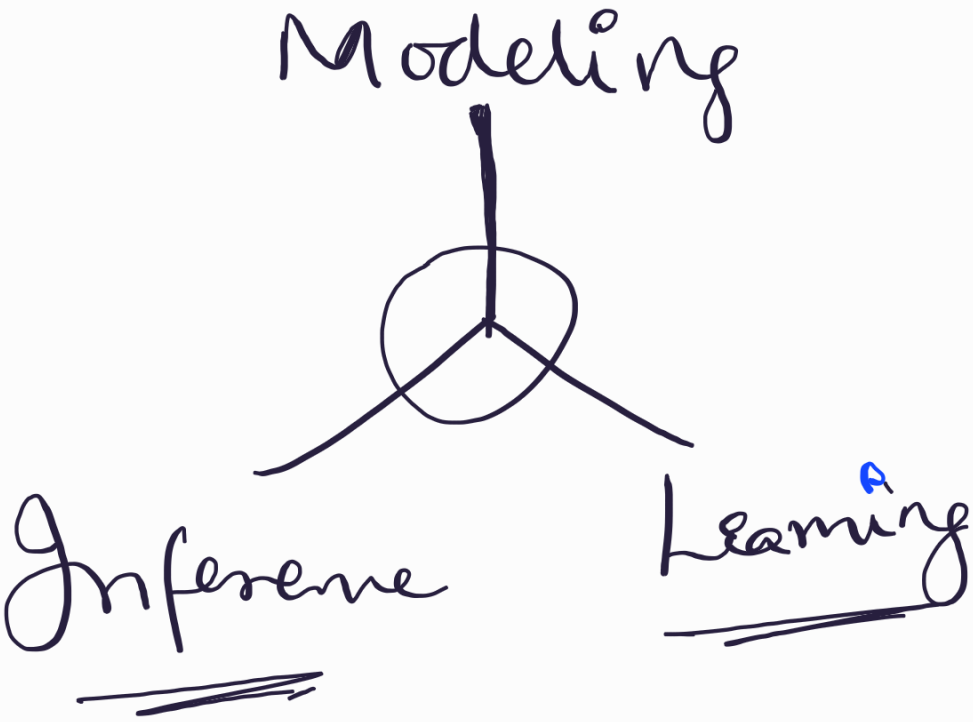
# AI Theory

Question: Find two neighbouring countries, one that speaks Hungarian and the other begins with "A".

Constraint Satisfaction Problems

Chapter 5 from Book

Paradigm:



# State based models – Takeaway

Model local interaction

Inference: Global optimal sol.

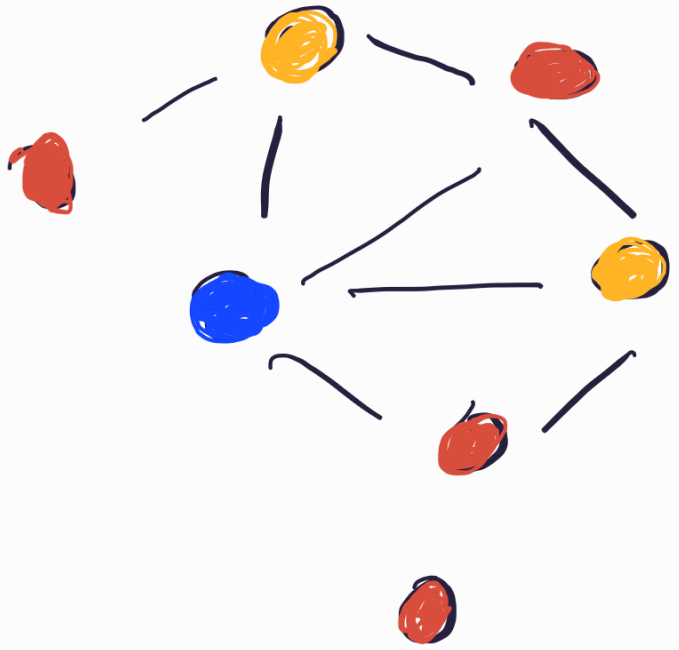
→ Review of state through colors examples:

⇒ Graph - coloring examples

7 nodes - 3 opt maximum

states count = $3^7$

failure | success states



many configurations like these.

challenge: explore and find such states (configuration)

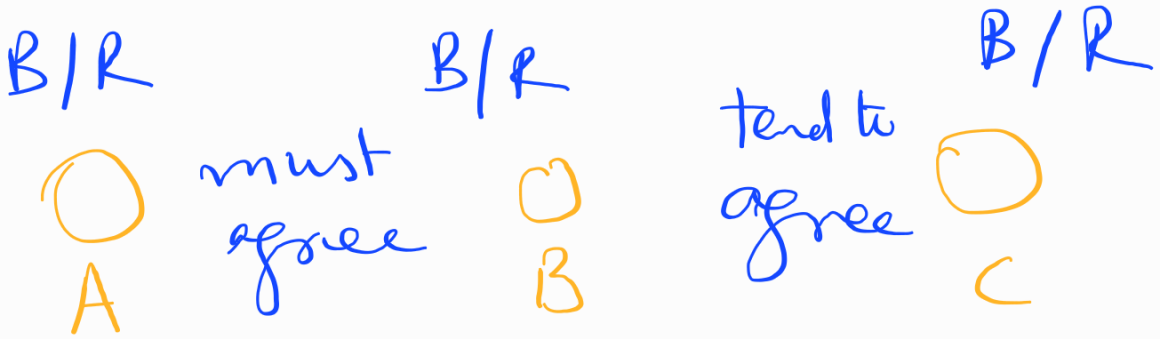
State: Partial assignment of color

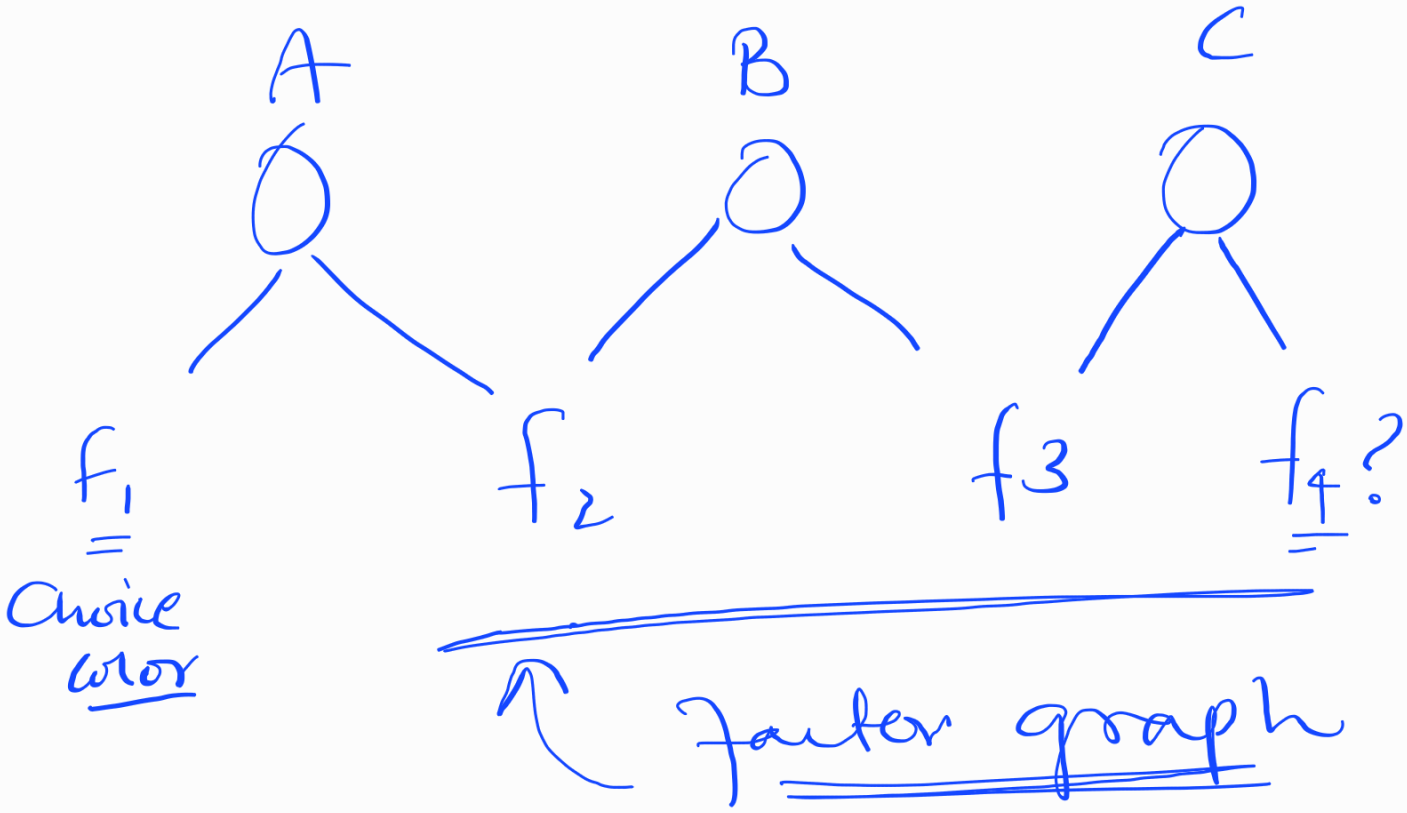
Action: Assign uncolored node a suitable color

Order of color matters.

Although solvable by search, solve it in high-level variable approach.

# Factor Graphs:



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| $x_1$ | $f_1(x_1)$ |
|---|---|
| R | 0 |
| B | 1 |

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| R | R | 1 |
| R | B | 0 |
| B | R | 0 |
| B | B | 1 |

| $x_2$ | $x_3$ | $f(x_2, x_3)$ |
|---|---|---|
| R | R | 3 |
| R | B | 2 |
| B | R | 2 |
| B | B | 3 |

| $x_3$ | $f(x_3)$ |
|---|---|
| R | 2 |
| B | 1 |

one choice, not compulsion

$f_2(x_1, x_2) = [x_1 = x_2]$

$f(x_2, x_3) = [x_2 = x_3] + 2$

choose those weights that maximize the rewards.

## Defining factor graph:

**Variables:**

$$X_1, X_2, \dots, X_n \mid \boxed{X_i \in \text{Domain}_i}$$

$\Rightarrow$ each variable might have a sp. domain.

**Factors:**

$$f_1, \dots, f_m, \quad \underline{f_j(X) \geq 0}$$

factor can't have value in negative

$\Rightarrow$ If constraints conflict, we compare the importance.

Example : map coloring

Variables :

$X =$ nodes

Domain $= \{R, G, B\}$

Factors :

(Provinces)

$f_1(X) = [WA \neq NT]$

$f_2(X)$ . . . .

⋮

## Defining Some Terms :

Scope of Factor :

Set of variable it depends on.

## Arity of Factor:

Count of variables in scope.

## In the factor graph examples

$2^3$ combos

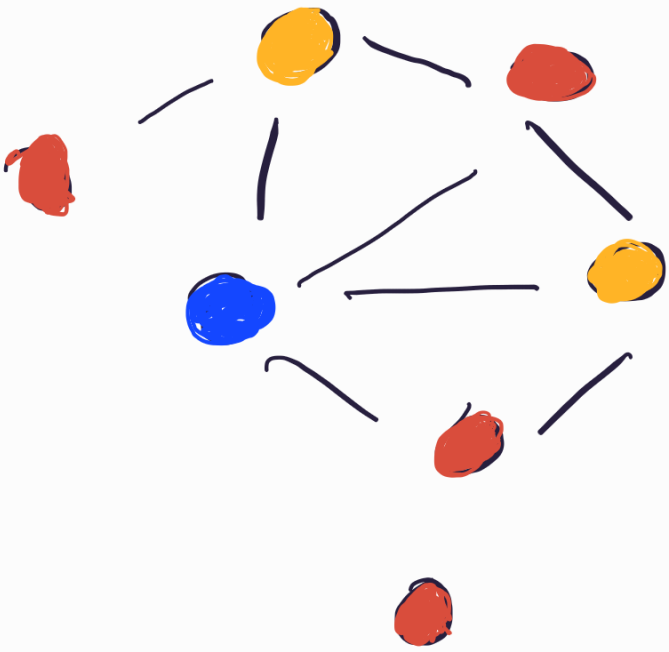
| $X_1$ | $X_2$ | $X_3$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $w$ |
|---|---|---|---|---|---|---|---|
| R | R | R | 0 | 1 | 3 | 2 | 0 |
| R | R | B | 0 | 1 | 2 | 1 | 0 |
| R | B | R | 0 | 0 | 2 | 2 | 0 |
| R | B | B | 0 | 0 | 3 | 1 | 0 |
| B | R | R | 1 | 0 | 3 | 2 | 0 |
| B | R | B | 1 | 0 | 2 | 1 | 0 |
| B | B | R | 1 | 1 | 2 | 2 | 4 |
| B | B | B | 1 | 1 | 3 | 1 | 3 |

assignment weights

$$w = \prod_{i=1}^{m} f_i$$

find max $(w)$

Example → map colors



Calculate Assignment Weights :

if neighbour color same: 0

else = 1

## Constraint Satisfaction Problem (CSP):

A CSP is a factor graph

where

$$f_j(x) \in \{0, 1\} \text{ for all } j = 1 \dots m$$

The constraint is satisfied iff $f_j = 1$

**Consistent Assignment:** If all weights are 1.

# Dynamic Ordering:

↳ How to reach consistent assignment?

↳ How to move towards your answer?