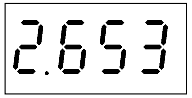
Reporting Measurements

A measurement of any **quantity** f is typically reported like

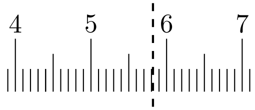
$$f = (3.14 \pm 0.15) \cdot 10^9\ \mathrm{m},$$

a **value** ($3.14 \cdot 10^9\,\mathrm{m}$) and a non-zero positive **uncertainty** ($0.15 \cdot 10^9\,\mathrm{m}$). These have the same **unit** (m) and are **rounded** to the same place (0.01), typically such that the uncertainty has 1 or 2 **significant digits**.

- If f is directly measured once, then this value is reported and the uncertainty is an estimate: If the instrument does not list one, then you may use its resolution, or, for a ruler, half of its resolution.



$f = (2.653 \pm 0.001)\ \mathrm{A}$



$f = (5.80 \pm 0.05)\ \mathrm{cm}$

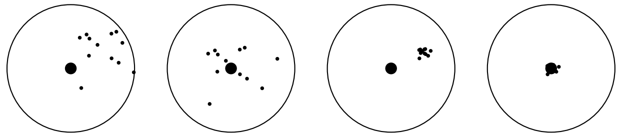
- If f is directly measured $M \gg 1$ times (significantly more than once), then the value is the **estimated mean** and the uncertainty is $1/\sqrt{M}$ times the **estimated standard deviation**.
- If f is calculated from 1 or more independent quantities with values x_i and uncertainties Δx_i, then the value is a function $f(x_i)$ and the uncertainty Δf is calculated by **uncertainty propagation**.

Function	**Gaussian uncertainty propagation**
$f(x_i)$ differentiable	$\Delta f = \sqrt{\sum_j \left(\frac{\partial f(x_i)}{\partial x_j} \Delta x_j\right)^2}$ All cases below follow from this one.
$f = x_1 + x_2$ or $x_1 - x_2$	$\Delta f = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2}$
$f = x_1 x_2$ or x_1/x_2	$\Delta f = \lvert f\rvert \sqrt{(\Delta x_1/x_1)^2 + (\Delta x_2/x_2)^2}$
$f = cx$	$\Delta f = \lvert c\rvert\, \Delta x$
$f = x^c$	$\Delta f = \lvert f\, c\, \Delta x/x\rvert$
$f = \overline{x} = (\sum_{i=1}^{M} x_i)/M$	$\Delta f = \sqrt{\sum_{i=1}^{M} (\Delta x_i)^2}/M \approx \sigma_x/\sqrt{M}$

For deeper insight, you may look into **random variables**, the **Gaussian distribution**, the **central limit theorem** and **confidence intervalls**.

Uncertainty	**Error**
Statistical deviation from true value. Gets smaller with $1/\sqrt{M}$.	**Systematic** deviation from true value. Independent of M.

precise = smaller uncertainty **accurate** = smaller error
imprecise = larger uncertainty **inaccurate** = larger error



imprecise inaccurate | imprecise accurate | precise inaccurate | precise accurate

Beware: Many people use *uncertainty* and *error* interchangeably, or *statistical error* for uncertainty, or *systematic uncertainty* for error.

Mean	**Standard deviation**
Let there be a **sample**, any set of $M \geq 2$ values f_i randomly drawn from a **population**, any set of $N > M$ values g_i.	
Mean of population $\overline{g} = (\sum_{i=1}^{N} g_i)/N$	Standard deviation of population $\sigma_g = \sqrt{\sum_{i=1}^{N} (g_i - \overline{g})^2}/\sqrt{N}$
Estimated mean $\overline{g} \approx (\sum_{i=1}^{M} f_i)/M = \overline{f}$	**Estimated standard deviation** $\sigma_g \approx \sqrt{\sum_{i=1}^{M} \left(f_i - \overline{f}\right)^2}/\sqrt{M-1}$ This underestimates σ_g. Would be unbiased with both sides squared.

Rounding half up		**Gaussian rounding**	
For equally near options, choose the larger one.		For equally near options, choose the one whose last digit is even.	
$1.4999 \approx 1$	$2.4999 \approx 2$	$1.4999 \approx 1$	$2.4999 \approx 2$
$1.5 \approx 2$	$\mathbf{2.5 \approx 3}$	$1.5 \approx 2$	$\mathbf{2.5 \approx 2}$
$1.5001 \approx 2$	$2.5001 \approx 3$	$1.5001 \approx 2$	$2.5001 \approx 3$
Biases towards larger numbers. The mean of rounded uniformly drawn numbers is biased. Often taught in school.		Biases towards even last digits. The mean of rounded uniformly drawn numbers is unbiased. Often preferred by scientists.	