Logic (PH133): Lecture 7

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. ∃Intro

Reading: §13.2

Disjunction Introduction (∨ Intro)

$$\begin{array}{l} \quad\vdots\ P_i \\ \quad\vdots \\ \triangleright\ P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \end{array}$$

4. YelBk(a)
5. ∃x YelBk(x)

2. ∃Elim

Reading: §12.2, §13.2

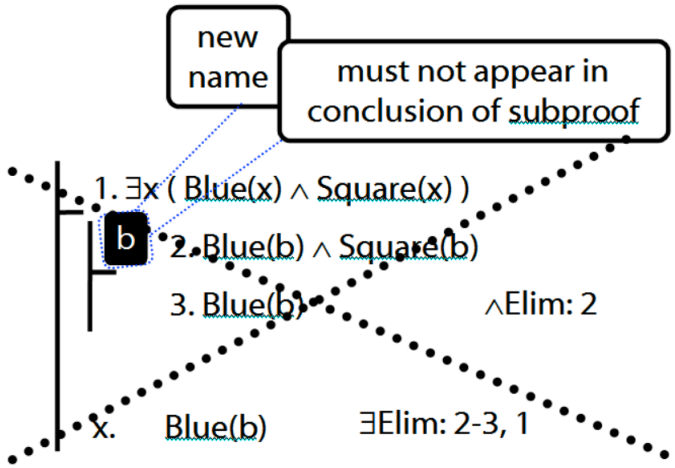
Existential Elimination (∃ Elim)

$$\begin{array}{l} \exists x\, S(x) \\ \quad\vdots \\ \quad\boxed{c}\ S(c) \\ \quad\vdots \\ \quad Q \\ \triangleright\ Q \end{array}$$

where c does not occur outside the subproof where it is introduced.

1. ∃x (Blue(x) ∧ Square(x))
   - b
     - 2. Blue(b) ∧ Square(b)
     - 3. Blue(b) ∧Elim: 2
     - 4. ∃x Blue(x) ∃Intro: 3
   - x. ∃x Blue(x) ∃Elim: 2-4, 1

Note this restriction on the use of ∃Elim:



3. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:
∀x(Dscrdn(x) ⟶ Wps(x))

All **French** discordians weep:
∀x((**Frnch(x)** ∧ Dscrdn(x)) ⟶ Wps(x))

All French discordians weep **and wail**:
∀x((Frnch(x) ∧ Dscrdn(x)) ⟶ (Wps(x) ∧ **Wls(x)**))

All French discordians weep and wail **except Gillian Deleude**:
∀x((Frnch(x) ∧ Dscrdn(x) ∧ ¬**(x=a)**) ⟶ (Wps(x) ∧ Wls(x)))

4. Scope and Quantifiers

Reading: §9.5, §9.6

Underlining shows the scope of the quantifiers:

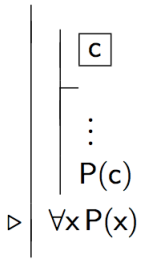
"All squares are blue"
$\forall x\,(\,\underline{\text{Square}(x) \rightarrow \text{Blue}(x)}\,)$

"If everything is square, everything is blue"
$\underline{\forall x\,\text{Square}(x)} \rightarrow \underline{\forall x\,\text{Blue}(x)}$

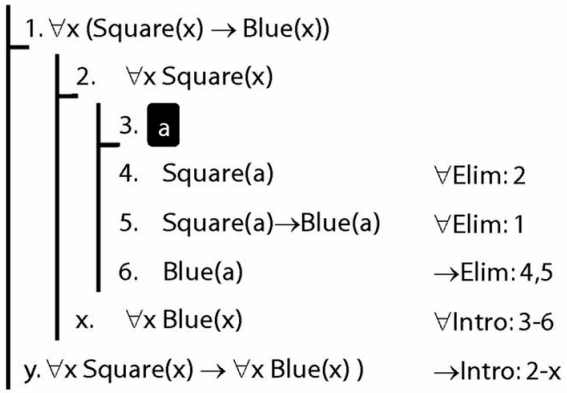
5. ∀Intro

Reading: §12.1, §12.3, §13.1

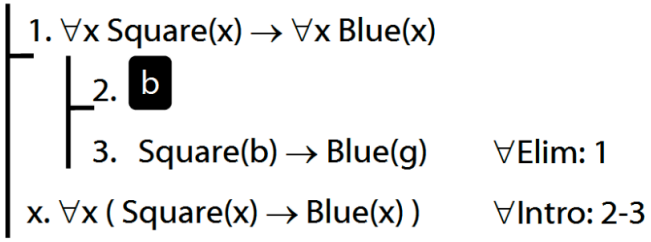
Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.



Why is this proof incorrect?



6. Summary of Quantifier Rules

Reading: §13.1, §13.2

∀Elim
If it's true of everything it's true of Baudrillard

∃Intro
If it's true of Baudrillard it's true of something

∃Elim
If it's true of something and Q follows no matter which something it is, then Q

∀Intro
If it's true of an arbitrary thing, then it's true of everything.

7. Something Is Above Something

Reading: §11.1

Something is above something:

$\exists x\,\exists y\,\text{Above}(x,y)$

8. There Is Exactly One

There is one creator (at least one, maybe more).

$\exists x\,\text{Creator}(x)$

Ahura Mazda is the one and only creator.

$\text{Creator}(a) \wedge \forall x(\,\text{Creator}(x) \longrightarrow x{=}a\,)$

All squares are broken.

$\forall x(\,\text{Sqr}(x) \longrightarrow \text{Brkn}(x)\,)$

There is one and only one creator.

$\exists y(\,\text{Creator}(y) \wedge \forall x(\,\text{Creator}(x) \longrightarrow x{=}y\,)\,)$

or:

$\exists y\,\forall x(\,\text{Creator}(x) \leftrightarrow x{=}y\,)$