Logic (PH133): Lecture 8

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. There Is a Store for Everything

Reading: §11.2, §11.3

There is a store for everything:

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything

Everyone hurts someone they love

Someone hurts everyone she loves

2. There Does Not Exist

Something is not dead:

∃x ¬Dead(x)

Nothing is dead:

¬∃x Dead(x)

Everything is not broken:

∀x ¬Broken(x)

Not everything is broken:

¬∀x Broken(x)

1.		
2.	a=a	=Intro
3.	∃x (x=x)	∃Intro: 2

1.	¬∃x Dead(x)	
2.	Dead(a)	
3.	∃x Dead(x)	∃Intro: 2
4.	⊥	⊥Intro: 1,3
5.	¬Dead(a)	¬Intro: 2-4
6.	∃x ¬Dead(x)	∃Intro: 5

1.	∃x ¬Dead(x)	
2.	¬∃x Dead(x)	

Counterexample:
Domain: {a,b}
Dead : {b}

3. Somebody Is Not Dead

Some person is dead.

∃x(Person(x) ∧ Dead(x))

Some person is not dead.

∃x(Person(x) ∧ ¬Dead(x))

No person is dead.

¬∃x(Person(x) ∧ Dead(x))

Every person is dead.

∀x(Person(x) ⟶ Dead(x))

Every person is not dead.

∀x(Person(x) ⟶ ¬Dead(x))

Not every person is dead.

¬∀x(Person(x) ⟶ Dead(x))

4. Quantifier Equivalences: ¬∀x Created(x) ⫤⊨ ∃x ¬Created(x)

Reading: §10.1, §10.3, §10.4

5. Proof Example: ∃x Dead(x) ⊢ ¬∀x¬ Dead(x).

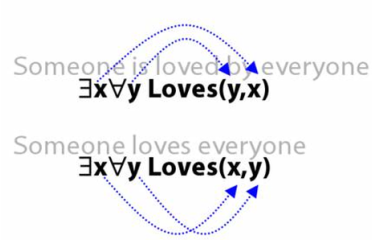
1.	∃x F(x)	
2.	a F(a)	
3.	∀y¬F(y)	
4.	¬F(a)	∀Elim: 3
5.	⊥	⊥Intro: 2,4
6.	¬∀y¬F(y)	¬Intro: 3-5
7.	¬∀y¬F(y)	∃Elim: 2-6

6. Proof Example: ¬∀x Dead(x) ⊢ ∃x¬ Dead(x).

1.	¬∀x Dead(x)	
2.	¬∃x¬Dead(x)	
3.	a	
4.	¬F(a)	
5.	∃x¬Dead(x)	∃Intro: 4
6.	⊥	⊥Intro: 2,5
7.	¬¬F(a)	¬Intro: 4-6
8.	F(a)	¬Elim: 8
9.	∀x Dead(x)	∀Intro: 3-8
10.	⊥	⊥Intro: 1,9
11.	¬¬∃x¬Dead(x)	¬Intro: 2-10
12.	∃x¬Dead(x)	¬Elim: 11

7. Loving and Being Loved

Reading: §11.2, §11.3



8. The End Is Near

Reading: §14.3

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

⫤⊨ There is exactly one square and it is broken

⫤⊨ There is at most one square and there is at least one square and it is broken

⫤⊨ There is at most one square and there is at least one square and all squares are broken

⫤⊨ ¬ ∃x ∃y (Square(x) ∧ Square(y) ∧ ¬x=y)

∧ ∃x Square(x)

∧ ∀x (Square(x) ⟶ Broken(x))

Which shorter sentences are equivalent to this?

∃x (Square(x) ∧ ∀y (Square(y) ⟶ y=x) ∧ Broken(x))

∃x (∀y (Square(y) ↔ y=x) ∧ Broken(x))