Logic I: Fast Lecture 05

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Vegetarians Are Evil

Reading: §9.2, §9.3, §9.5

∀x (Evil(x) → HatesMeat(x))
∀x (HatesMeat(x) → Vegetarian(x))

∀x (Vegetarian(x) → Evil(x))

2. Counterexamples with Quantifiers

	Evil(x)	HatesMeat(x)	Vegetarian(x)
Ayesha	no	no	yes

3. Something Is Above Something

Reading: §11.1

Something is above something:
∃x ∃y Above(x,y)

4. Multiple Quantifiers: Everyone Likes Puffins

Reading: §11.1

I like puffins:

∀x (Puffin(x) → Likes(a,x))

y likes puffins:

∀x (Puffin(x) → Likes(y,x))

Everyone likes puffins:

∀y ∀x (Puffin(x) → Likes(y,x))

5. Relations: Transitivity

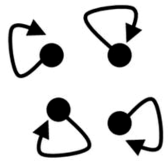
Reading: §15.1

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; NotAdjacent is not transitive.)

6. Expressing Relations with Quantifiers

Reading: §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)
reflexive: ∀x R(x,x)



A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y))
symmetric: ∀x∀y (R(x,y) → R(y,x))

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is not transitive)
transitive: ∀x∀y∀z ((R(x,y) ∧ R(y,z)) → R(x,z))

7. Expressing Counterexamples Formally

Reading: §15.1

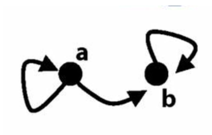
Give a counterexample to this argument:

∀x R(x,x)

∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))

∀x∀y [R(x,y) → R(y,x)]

Informally:



Formally:

Domain: {a,b}

R : { <a,a>, <a,b>, <b,b> }

Domain: {a, b}

R:<a,a>, <a,b>, <b,b>

8. There Is a Store for Everything

Reading: §11.2, §11.3

There is a store for everything:

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything

Everyone hurts someone they love

Someone hurts everyone she loves

9. ∀Intro: An Incorrect Proof

Reading: §13.1, §13.2

This proof is wrong, but why?:

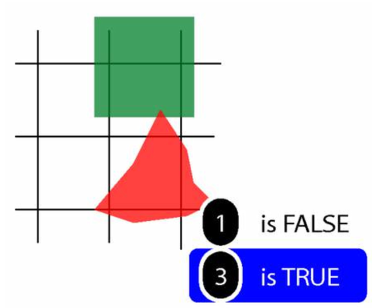
3 1. ∀x Square(x) → ∀x Blue(x)

2. b

3. Square(b) → Blue(b) ∀Elim: 1

1 x. ∀x (Square(x) → Blue(x)) ∀Intro: 2-3

There is a counterexample to the argument:



10. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

'Two things are broken' might be translated as:

∃x ∃y (Broken(x) ∧ Broken(y) ∧ ¬(x=y))

11. Quantifier Equivalences: ∀x(Square(x) → Broken(x)) ⫤⊨ ∀x(¬Broken(x) → ¬Square(x))

Reading: §10.3

P	Q	P→Q	¬Q→¬P
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

12. Quantifier Equivalences: ∀x Created(x) ⫤⊨ ¬∃x Created(x)

Reading: §10.3, §10.4

13. Soundness and Completeness: Statement of the Theorems

Reading: §8.3, §13.4

'A ⊢ B' means there is a proof of B using premises A

'⊢ B' means there is a proof of B using no premises

'A ⊨ B' means B is a logical consequence of A

'⊨ B' means B is a tautology

'A $\vDash_{TT}$ B' means B is a logical consequence of A just in virtue of the meanings of truth-functions (the textbook LPL calls this 'tautological consequence')

Soundness: If A $\vdash$ B then A $\vDash$ B

i.e. if you can prove it in Fitch, it's valid

Completeness: If A $\vDash_{TT}$ B then A $\vdash$ B

i.e. if it's valid just in virtue of the meanings of the truth-functional connectives, then you can prove it in Fitch.

14. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

13.28–13.31

11.2, 11.4

15.29

15.33

15.34

15.35

15.29

15.33

15.34

15.35

11.8, 11.9, *11.11