Logic I: Lecture 06

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. ∨Intro and ∨Elim

Reading: §6.2

Disjunction Elimination (∨ Elim)

$$P_1 \vee \ldots \vee P_n$$
$$\vdots$$
$$P_1$$
$$\vdots$$
$$S$$
$$\Downarrow$$
$$P_n$$
$$\vdots$$
$$S$$
$$\vdots$$
$$\triangleright \quad S$$

Disjunction Introduction (∨ Intro)

$$P_i$$
$$\vdots$$
$$\triangleright \quad P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$$

2. ∨Elim and Soundness

Reading: §5.2, §6.2

3. ∨Elim: An Example

To prove a conclusion from a disjunction, prove it from each disjunct.

```
1. R ∨ S
2.
3.
4.
5.
6. S ∨ R
```

4. Not Or

Reading: §3.7

A	B	A ∨ B	¬(A ∨ B)	¬A	¬B	¬A ∨ ¬B
T	T	T	F	F	F	F
T	F	T	F	F	T	T
F	T	T	F	T	F	T
F	F	F	T	T	T	T

5. →Intro: An Example

```
1. P→Q
2. Q→R

6. P→R
```

6. ¬Elim

Reading: §6.3

Negation Elimination (¬ Elim)

$$\neg\neg P$$
$$\vdots$$
$$\triangleright \quad P$$

7. ¬Intro

Reading: §5.3, §6.3

Negation Introduction (¬ Intro)

$$\begin{array}{c|l}
 & P \\
 & \vdots \\
 & \bot \\
\rhd & \neg P
\end{array}$$

The proof with the green X crossed out:

$$\begin{array}{ll}
\text{T} & 1.\ R \vee S \\
 & 2.\quad R \\
 & 3.\quad S \vee R \qquad \vee\text{Intro:2} \\
 & 4.\quad S \\
 & 5.\quad S \vee R \qquad \vee\text{Intro:4} \\
 & 6.\ S \vee R \qquad\quad \vee\text{Elim: 1,2-3,4-5} \\
\text{F} & 7.\ R \wedge S \qquad\quad \wedge\text{Intro: 2,4}
\end{array}$$

R	S	R∨S	R∧S
T	F	T	F

8. Subproofs Are Tricky

What is wrong with the following apparent proof?

$$\begin{array}{ll}
1. & A \wedge B \\
 & \vdots \\
 & \neg(\neg A \vee \neg B)
\end{array}$$

9. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic.*

> 5.1–5.6
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> 6.2–6.6
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> 3.19
>
> 5.18
>
> 6.7–6.10
>
> *6.11–6.12