# Logic I: Lecture 12

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

## 1. Relations: Reflexive, Symmetric

*Reading:* §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. everything is the SameShape as itself. E.g. of non-reflexive: not everything is LeftOf itself).

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y) is symmetric, LeftOf(x,y) is not symmetric.)

## 2. Relations: Transitivity

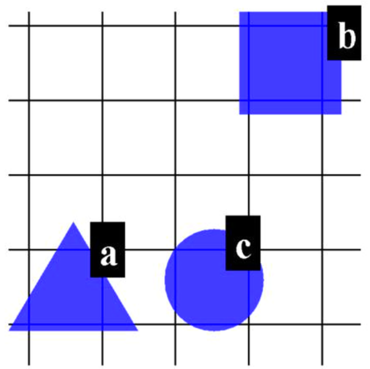
*Reading:* §15.1

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; NotAdjacent is not transitive.)

If NotAdjacent were transitive, the following argument would be logically valid:

1. NotAdjacent(a, b)
2. NotAdjacent(b, c)
3. NotAdjacent(a, c)

A counterexample to this argument:




## 3. Relations: Some Examples

*Reading:* §15.1

| | Reflexive | Symmetric | Transitive |
|---|---|---|---|
| NotEqual | N | Y | N |
| = | Y | Y | Y |
| SameShape | | | |
| DifferentShape | | | |
| LeftOf | | | |
| Adjacent | N | Y | N |

Artificial relations ...

EqualToOrLeftOf(x, y) iff

x = y or LeftOf(x, y)

EqualToOrAdjacent(x, y) iff

x=y or Adjacent(x, y)

JohnOrAyesha(x, y) iff

x = John and y = Ayesha

or x = Ayesha and y = John

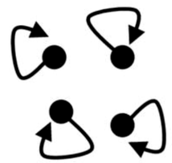
JohnToAyesha(x, y) iff

x = John and y = Ayesha

## 4. Expressing Relations with Quantifiers

*Reading:* §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)
reflexive: ∀x R(x,x)



A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y))
symmetric: ∀x∀y ( R(x,y) → R(y,x) )

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is not transitive)
transitive: $\forall x \forall y \forall z\ (\ (\ R(x,y) \land R(y,z)\ ) \longrightarrow R(x,z)\ )$

## 5. Negating Identity

$\left.\begin{array}{l} \neg a=b \\ \neg(a=b) \end{array}\right\}$ these mean the same thing

## 6. Expressing Counterexamples Formally

*Reading:* §15.1

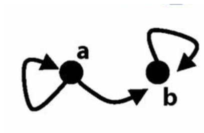
Give a counterexample to this argument:

$\forall x\, R(x,x)$

$\forall x \forall y \forall z\ (\ [\ R(x,y) \land R(y,z)\ ] \rightarrow R(x,z)\ )$

$\forall x \forall y\ [\ R(x,y) \rightarrow R(y,x)\ ]$

Informally:




Formally:

Domain: {a, b}

R: {<a,a>, <a,b>, <b,b>}

## 7. Proof Example: $A \land B$ therefore $\neg(\neg A \lor \neg B)$.

1. $A \land B$

$\neg(\neg A \lor \neg B)$

## 8. Proof Example: P therefore $\neg\neg P$.

1. P

5. $\neg\neg P$

## 9. Proof Example: $S \lor (Q \land R)$ therefore $S \lor Q$.

1. $S \lor (Q \land R)$

$S \lor Q$

## 10. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

9.20

13.1–13.4, 13.10–13.15, *13.16

15.29–15.36