Logic I: Lecture 15

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Translation from FOL to English

Using the interpretation below, providing English translations of the following sentences of FOL.

Domain: people and actions
D(x) : x is desirable
V(x) : x is virtuous
A(x) : x is an action
P(x,y) : x performed y
a : Ayesha

i. ∀x(D(x) ⟶ V(x))

ii. ∀x((A(x) ∧ D(x)) ⟶ V(x))

iii. ∃x(A(x) ∧ ¬D(x))

iv. ∃x(A(x) ∧ ¬D(x) ∧ V(x))

v. ∃x(A(x) ∧ P(a,x) ∧ ¬V(x))

vi. ¬∃x(

∃y(A(y) ∧ P(x,y) ∧ ¬V(y))

∧

¬∃z(A(z) ∧ P(x,z) ∧ V(z))

)

2. Numerical Quantifiers

Reading: §14.1

There are at least two squares:

∃x ∃y (Square(x) ∧ Square(y) ∧ ¬x=y)

At least two squares are broken:

∃x ∃y (

Square(x) ∧ Broken(x)

∧

Square(y) ∧ Broken(y)

∧

¬x=y

)

There are at least three squares:

∃x ∃y ∃z (

Square(x) ∧ Square(y) ∧ Square(z)

∧

¬x=y ∧ ¬y=z ∧ ¬x=z

)

There are at most two squares:

¬There are at least three squares

¬∃x ∃y ∃z (Square(x) ∧ Square(y) ∧ Square(z) ∧ ¬x=y ∧ ¬y=z ∧ ¬x=z)

There are exactly two squares:

There are at most two squares ∧ There are at least two squares

Number: alternatives

There is at most one square:

∀x ∀y ((Square(x) ∧ Square(y)) ⟶ x=y)

There are at most two squares:

∀x ∀y ∀z (

(Square(x) ∧ Square(y) ∧ Square(z))

⟶

(x=y ∨ y=z ∨ x=z)

)

There is exactly one square:

∃x (Square(x) ∧ ∀y(Square(y) ⟶ x=y))

There are exactly two squares:

∃x∃y (

Square(x) ∧ Square(y) ∧ ¬x=y

∧

∀z(Square(z) ⟶ (z=x ∨ z=y))

)

3. There Is Exactly One

There is one creator (at least one, maybe more).

∃x Creator(x)

Brian is the one and only creator.

Creator(b) ∧ ∀x(Creator(x) ⟶ x=b)

All squares are broken.

$\forall x(\ Sqr(x) \longrightarrow Brkn(x)\ )$

There is one and only one creator.

$\exists y(\ Creator(y) \land \forall x(\ Creator(x) \longrightarrow x{=}y\ )\ )$

or:

$\exists y\ \forall x(\ Creator(x) \leftrightarrow x{=}y\ )$

4. Quantifier Equivalences: $\forall x(Square(x) \longrightarrow Broken(x)) \dashv\vDash \neg\exists x(Square(x) \land \neg Broken(x))$

Reading: §10.3

Every square is broken
$\forall x\ (Square(x) \rightarrow Broken(x)\ )$

$\dashv\vDash\ \neg\neg\forall x\ (Square(x) \rightarrow Broken(x)\ )$

$\dashv\vDash\ \neg\exists x\ \neg(Square(x) \rightarrow Broken(x)\ )$

Nothing is square and non-broken
$\dashv\vDash\ \neg\exists x\ (\ Square(x) \land \neg Broken(x)\ )$

5. Proof Example: $\exists x\ Dead(x) \vdash \neg\forall x\neg\ Dead(x)$.

1. $\exists x\ F(x)$
2. a $F(a)$
3. $\forall y \neg F(y)$
4. $\neg F(a)$ — $\forall$Elim: 3
5. $\perp$ — $\perp$Intro: 2,4
6. $\neg\forall y\neg F(y)$ — $\neg$Intro: 3-5
7. $\neg\forall y\neg F(y)$ — $\exists$Elim: 2-6

6. Proof Example: $\neg\forall x\ Dead(x) \vdash \exists x\neg\ Dead(x)$.

1. $\neg\forall x\ Dead(x)$
2. $\neg\exists x\neg Dead(x)$
3. a
4. $\neg F(a)$
5. $\exists x\neg Dead(x)$ — $\exists$Intro: 4
6. $\perp$ — $\perp$Intro: 2,5
7. $\neg\neg F(a)$ — $\neg$Intro: 4-6
8. $F(a)$ — $\neg$Elim: 8
9. $\forall x\ Dead(x)$ — $\forall$Intro: 3-8
10. $\perp$ — $\perp$Intro: 1,9
11. $\neg\neg\exists x\neg Dead(x)$ — $\neg$Intro: 2-10
12. $\exists x\neg Dead(x)$ — $\neg$Elim: 11

7. Extra Exercises: Proofs

You may not have time to do these exercises involving proofs until after term, but it would be a good idea to complete them at some point.

13.6, 13.7, *13.8, *13.9

13.19, 13.23–13.27, *13.28–13.31

*13.33, 13.35, 13.37, 13.39

13.43–13.45, 13.49–13.50, *13.51–13.52

8. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

14.1–14.3, (*14.4–14.5)

14.10–14.12, *14.13