# Logic (PH133): Lecture 5

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#### 1. Not Or

Reading: §3.7

Α	В	$A \vee B$	¬(A ∨ B)	¬A	¬B	¬A ∨ ¬B
Т	Т	Т	F	F	F	F
Т	F	Т	F	F	Т	Т
F	Т	Т	F	Т	F	Т
F	F	F	Т	Т	Т	Т

# 2. I Met a Philosopher

Reading: §9.2, §9.3, §9.5

# 3. All Squares Are Blue

Reading: §9.2, §9.3, §9.5

# 4. Vegetarians Are Evil

Reading: §9.2, §9.3, §9.5

$$\forall x ( Evil(x) \rightarrow HatesMeat(x) )$$

$$\forall x ( HatesMeat(x) \rightarrow Vegetarian(x) )$$

$$\forall x ( Vegetarian(x) \rightarrow Evil(x) )$$

# 5. Counterexamples with Quantifiers

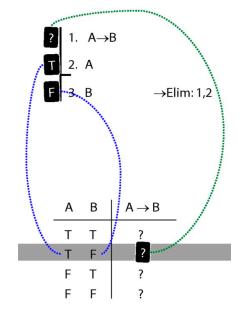
	Evil(x)	HatesMeat(x)	Vegetarian(x)
Ayesha	no	no	yes

# 6. What does ' $\rightarrow$ ' mean?

Reading: §7.1

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth table for  $\rightarrow$  is fixed if we accept  $\rightarrow$ Elim and  $\rightarrow$ Intro.

How do the rules of proof for  $\rightarrow$  fix its truth table?



#### 7. Not If

If she has seen it, I am dead.

 $A \rightarrow B$ 

That's not true.

**□**(A → B)

If she has seen it, I am not dead.

A → ¬B

Α	В	A → B	$\neg(A \rightarrow B)$	$A \rightarrow \neg B$
Τ	Τ	Т	F	F
Τ	F	F	Т	Т
F	Т	Т	F	Т
F	F	Т	F	Т

#### 8. $\leftrightarrow$ : truth tables and rules

Α	В	$A \leftrightarrow B$
Т	Т	Т
Τ	F	F
F	Т	F
F	F	Т

### **Biconditional Elimination** $(\leftrightarrow Elim)$

$$\begin{array}{|c|c|} \hline P \leftrightarrow Q \ (\mathrm{or} \ Q \leftrightarrow P) \\ \vdots \\ P \\ \vdots \\ \hline P \\ Q \end{array}$$

#### **Biconditional Introduction** $(\leftrightarrow Intro)$

$$\begin{array}{c|c} & P \\ & \vdots \\ & Q \\ & Q \\ & \vdots \\ & P \\ & P \leftrightarrow Q \end{array}$$

#### What does ∀ mean?

Reading: §9.4

We give the meaning of ∀ by specifying what it takes for a sentence containing ∀ to be true:

1. Give every object a name.

- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.

### 10. Does 'if' mean what ' $\rightarrow$ ' means?

Reading: §7.3

These two arguments are valid: does that mean that 'if' means what ' $\rightarrow$ ' means?

### 11. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:

 $\forall x ( Dscrdn(x) \rightarrow Wps(x) )$ 

All **French** discordians weep:

 $\forall x ( (Frnch(x) \land Dscrdn(x)) \rightarrow Wps(x) )$ 

All French discordians weep and wail:  $\forall x ((Frnch(x) \land Dscrdn(x)) \rightarrow (Wps(x) \land Wls(x))$ 

All French discordians weep and wail except Gillian Deleude:

$$\forall x ( (Frnch(x) \land Dscrdn(x) \land \neg(x=a)) \rightarrow (Wps(x) \land Wls(x)) )$$

The English argument isn't valid; the FOL argument is valid; therefore 'if' can't mean what ' $\rightarrow$ ' means?