Logic I: Fast Lecture 02

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Formal Proof: $\wedge$Elim and $\wedge$Intro

Reading: §5.1, §6.1

Conjunction Introduction ($\wedge$ Intro)

P_1
$\Downarrow$
P_n
$\vdots$
$\triangleright$ $P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination ($\wedge$ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$
$\vdots$
$\triangleright$ P_i

1.	$P \wedge Q$	
2.	$Q \wedge R$	
3.	P	$\wedge$Elim: 1
4.	R	$\wedge$Elim: 2
5.	$P \wedge R$	$\wedge$Intro: 3,4

2. $\wedge$Intro and $\vee$Intro: Compare and Contrast

Reading: §6.1

Disjunction Introduction ($\vee$ Intro)

P_i
$\vdots$
$\triangleright$ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

1.	R	
2.	S	
3.	$R \vee T$	$\vee$Intro: 1
4.	$S \wedge (R \vee T)$	$\wedge$Intro: 2,3

Let us define a new connective with this truth table:

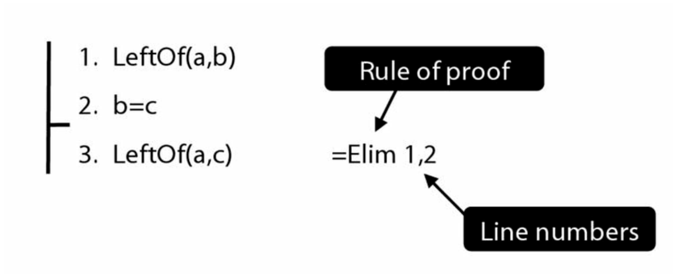
P1	P2	P1 $\vee$ P2	P1 $\not\leftrightarrow$ P2
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

The following rule is unacceptable. Why?

$\not\leftrightarrow$Intro:

Pi
...
P1 $\not\leftrightarrow$ P2

3. How to Write Proofs



4. Rules of Proof for Identity

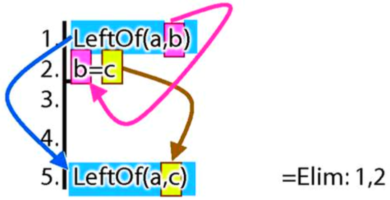
Reading: §2.2

Identity Introduction
(= Intro)

$\triangleright$ | n = n

Identity Elimination
(= Elim)

| P(n)
| ⋮
| n = m
| ⋮
$\triangleright$ | P(m)



5. DeMorgan: ¬(A ∧ B) ⫤⊨ ¬A ∨ ¬B

Reading: §3.6

'⫤⊨' means 'is logically equivalent to', so for now 'has the same truth table as'.

A ⫤⊨ ¬¬A

¬(A ∧ B) ⫤⊨ (¬A ∨ ¬B)

¬(A ∨ B) ⫤⊨ (¬A ∧ ¬B)

A ⟶ B ⫤⊨ ¬A ∨ B

¬(A ⟶ B) ⫤⊨ ¬(¬A ∨ B) ⫤⊨ A ∧ ¬B

6. ⟶Intro, ⟶Elim

Reading: §8.1, §8.2

Conditional Introduction
(→ Intro)

| | P
| |—
| | ⋮
| | Q
$\triangleright$ | P → Q

Conditional Elimination
(→ Elim)

| P → Q
| ⋮
| P
| ⋮
$\triangleright$ | Q

7. ⟶Intro: An Example

| 1. P→Q
| 2. Q→R
|—
|
| 6. P→R

8. ∨Intro and ∨Elim

Disjunction Introduction
(∨ Intro)

| P_i
| ⋮
$\triangleright$ | $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination
(∨ Elim)

$P_1 \vee \ldots \vee P_n$
$\vdots$
P_1
$\vdots$
S
$\Downarrow$
P_n
$\vdots$
S
$\vdots$
$\triangleright$ S

9. ∨Elim: An Example

To prove a conclusion from a disjunction, prove it from each disjunct.

1. $R \vee S$
2.
3.
4.
5.
6. $S \vee R$

10. ¬, ⊥

Reading: §6.3

P	¬P	⊥
T	F	F
F	T	F

⊥ Introduction
(⊥ Intro)

P
$\vdots$
$\neg P$
$\vdots$
$\triangleright$ $\perp$

⊥ Elimination
(⊥ Elim)

$\perp$
$\vdots$
$\triangleright$ P

11. ¬Elim

Reading: §6.3

Negation Elimination
(¬ Elim)

$\neg\neg P$
$\vdots$
$\triangleright$ P

12. Scope: A Mistaken Application of ¬Elim

What is wrong with this proof?

1.	$\neg\neg(\neg A \wedge \neg\neg A)$	
2.	$(\neg A \wedge \neg\neg A)$	¬Elim: 1
3.	$(\neg A \wedge A)$	¬Elim: 2

13. ¬Intro

Reading: §5.3, §6.3

Negation Introduction
(¬ Intro)

P
$\vdots$
$\perp$
$\triangleright$ $\neg P$