

# Logic I: Lecture 13

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

## 1. There Is a Store for Everything

Reading: §11.2, §11.3

There is a store for everything:

$\exists y \forall x \text{ StoreFor}(y, x)$

$\forall y \exists x \text{ StoreFor}(x, y)$

Other sentences to translate:

Wikipedia has an article about everything

Everyone hurts someone they love

Someone hurts everyone she loves

## 2. How Big Is a Truth-Table?

How many truth-functions can be constructed using 2 sentence letters?

P	Q	P % Q
T	T	?
T	F	?
F	T	?
F	F	?

Each of these can be T or F

## 3. Truth-functional completeness

Reading: §7.4

‘A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.’ (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that  $\{\neg, \wedge, \vee\}$  is truth-functionally complete:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$[P \wedge Q] \vee$   
 $[\neg P \wedge Q] \vee$   
 $[\neg P \wedge \neg Q]$

$[P \wedge Q] \vee [\neg P \wedge Q] \vee [\neg P \wedge \neg Q]$

## 4. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. ‘1.1’ refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked ‘\*’ are optional.

10.24–7, \*10.28–9

11.3

11.4, 11.8, 11.9

7.25, 7.26, \*7.28, 7.29

Exercise assuming  $\{\neg, \vee, \wedge\}$  is truth-functionally complete, show that  $\{\neg, \vee\}$  is.