

Logic I: Lecture 05

s.butterfill@warwick.ac.uk

Readings refer to sections of the course textbook,
Language, Proof and Logic.

1. \neg , \perp

Reading: §6.3

| P | $\neg P$ | \perp |
|---|----------|---------|
| T | F | F |
| F | T | F |

\perp Introduction (\perp Intro)

| | |
|----------|--|
| \vdash | $\begin{array}{c} P \\ \vdots \\ \neg P \end{array}$ |
| \vdash | \perp |

\perp Elimination (\perp Elim)

| | |
|----------|---|
| \vdash | $\begin{array}{c} \perp \\ \vdots \\ P \end{array}$ |
|----------|---|

2. \rightarrow Intro, \rightarrow Elim

Reading: §8.1, §8.2

Conditional Introduction (\rightarrow Intro)

| | |
|----------|--|
| \vdash | $\begin{array}{c} \vdash P \\ \vdots \\ Q \end{array}$ |
| \vdash | $P \rightarrow Q$ |

Conditional Elimination (\rightarrow Elim)

| | |
|----------|---|
| \vdash | $\begin{array}{c} P \rightarrow Q \\ \vdots \\ P \end{array}$ |
| \vdash | Q |

3. \rightarrow Intro: An Example

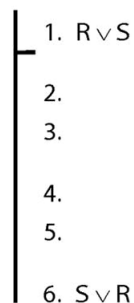
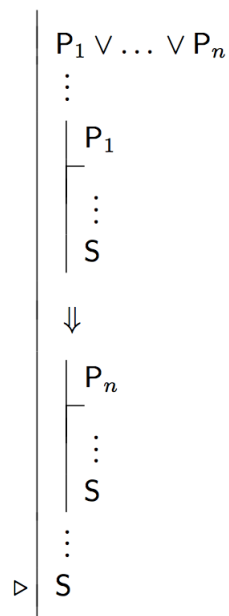
| | |
|----------|---|
| \vdash | $\begin{array}{c} 1. P \rightarrow Q \\ 2. Q \rightarrow R \end{array}$ |
| \vdash | $6. P \rightarrow R$ |

4. \vee Intro and \vee Elim

Disjunction Introduction (\vee Intro)

| | |
|----------|---|
| \vdash | $\begin{array}{c} P_i \\ \vdots \end{array}$ |
| \vdash | $P_1 \vee \dots \vee P_i \vee \dots \vee P_n$ |

Disjunction Elimination (\vee Elim)



7. Not Or

Reading: §3.7

| A | B | $A \vee B$ | $\neg(A \vee B)$ | $\neg A$ | $\neg B$ | $\neg A \vee \neg B$ |
|---|---|------------|------------------|----------|----------|----------------------|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | T |
| F | T | T | F | T | F | T |
| F | F | F | T | T | T | T |

8. DeMorgan: $\neg(A \wedge B) \models \neg A \vee \neg B$

Reading: §3.6, §4.2

‘ \models ’ means ‘is logically equivalent to’, so for now ‘has the same truth table as’.

$$A \models \neg\neg A$$

$$\neg(A \wedge B) \models (\neg A \vee \neg B)$$

$$\neg(A \vee B) \models (\neg A \wedge \neg B)$$

$$A \rightarrow B \models \neg A \vee B$$

$$\neg(A \rightarrow B) \models \neg(\neg A \vee B) \models A \wedge \neg B$$

9. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. ‘1.1’ refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*.

6.2–6.6

3.19

4.15–18

5. \vee Elim and Soundness

Reading: §5.2, §6.2

6. \vee Elim: An Example

To prove a conclusion from a disjunction, prove it from each disjunct.