# Logic I: Lecture 15

s.butterfill@warwick.ac.uk

Readings refer to sections of the course textbook, Language, Proof and Logic.

### Translation from FOL to English

Using the interpretation below, providing English translations of the following sentences of FOL.

```
Domain: people and actions
         D(x): x is desirable
          V(x) : x \text{ is virtuous}
          A(x): x is an action
         P(x,y): x performed y
          a : Ayesha
i. \forall x (D(x) \rightarrow V(x))
ii. \forall x ((A(x) \land D(x)) \rightarrow V(x))
iii. \exists x (A(x) \land \neg D(x))
iv. \exists x (A(x) \land \neg D(x) \land V(x))
v. \exists x (A(x) \land P(a,x) \land \neg V(x))
vi. ¬∃x(
     \exists y (A(y) \land P(x,y) \land \neg V(y))
     Λ
     \neg \exists z (A(z) \land P(x,z) \land V(z))
```

# 2. Numerical Quantifiers

```
Reading: §14.1
There are at least two squares:
    \exists x \exists y (Square(x) \land Square(y) \land \neg x=y)
At least two squares are broken:
    ∃x ∃y (
        Square(x) \wedge Broken(x)
        Λ
        Square(y) \wedge Broken(y)
        \neg x = y
There are at least three squares:
    ∃x ∃y ∃z (
        Square(x) \land Square(y) \land Square(z)
        Λ
        \neg x = y \land \neg y = z \land \neg x = z
There are at most two squares:
    ¬There are at least three squares
    \neg \exists x \exists y \exists z (Square(x) \land Square(y) \land Square(z))
\land \neg x = y \land \neg y = z \land \neg x = z)
There are exactly two squares:
    There are at most two squares ∧ There are at
```

least two squares

#### Number: alternatives

```
There is at most one square:
    \forall x \ \forall y \ ( (Square(x) \land Square(y)) \rightarrow x=y )
There are at most two squares:
    ∀x ∀y ∀z (
        (Square(x) \land Square(y) \land Square(z))
        (x=y \lor y=z \lor x=z)
There is exactly one square:
    \exists x ( Square(x) \land \forall y ( Square(y) \rightarrow x=y ) )
There are exactly two squares:
    ∃х∃у (
        Square(x) \land Square(y) \land \neg x = y
        \forall z ( Square(z) \rightarrow (z=x \lor z=y) )
```

## There Is Exactly One

```
There is one creator (at least one, maybe more).
   ∃x Creator(x)
Brian is the one and only creator.
  Creator(b) \land \forall x (Creator(x) \rightarrow x=b)
```

All squares are broken.

$$\forall x ( Sqr(x) \rightarrow Brkn(x) )$$

There is one and only one creator.

 $\exists y \ \forall x (\ Creator(x) \leftrightarrow x = y)$ 

$$\exists y ( Creator(y) \land \forall x ( Creator(x) \longrightarrow x=y ) )$$
 or:

4. Quantifier Equivalences:  $\forall x (Square(x) \rightarrow Broken(x)) = \exists x (Square(x) \land \neg Broken(x))$ 

Reading: §10.3

Every square is broken  $\forall x \text{ (Square(x)} \rightarrow \text{Broken(x)} \text{ )}$   $| \downarrow | \vdash \neg \neg \forall x \text{ (Square(x)} \rightarrow \text{Broken(x)} \text{ )}$   $| \downarrow | \vdash \neg \exists x \neg (\text{Square(x)} \rightarrow \text{Broken(x)} \text{ )}$   $| \downarrow | \vdash \neg \exists x \neg (\text{Square(x)} \rightarrow \text{Broken(x)} \text{ )}$   $| \downarrow | \vdash \neg \exists x \text{ (Square(x)} \land \neg \text{Broken(x)} \text{ )}$ 

5. Proof Example:  $\exists x \ Dead(x) \vdash \neg \forall x \neg Dead(x)$ .

1. 
$$\exists x F(x)$$

2.  $a F(a)$ 

3.  $\forall y \neg F(y)$ 

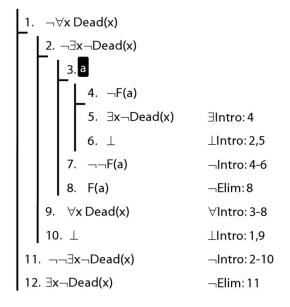
4.  $\neg F(a)$   $\forall Elim: 3$ 

5.  $\bot$   $\bot Intro: 2,4$ 

6.  $\neg \forall y \neg F(y)$   $\neg Intro: 3-5$ 

7.  $\neg \forall y \neg F(y)$   $\exists Elim: 2-6$ 

6. Proof Example:  $\neg \forall x \text{ Dead}(x) \vdash \exists x \neg \text{ Dead}(x)$ .



### 7. Extra Exercises: Proofs

You may not have time to do these exercises involving proofs until after term, but it would be a good idea to complete them at some point.

### 8. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language*, *Proof and Logic*. Exercises marked '\*' are optional.