Logic (PH133): Lecture 6

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. DeMorgan: ¬(A ∧ B) ⫤⊨ ¬A ∨ ¬B

Reading: §3.6, §4.2

'⫤⊨' means 'is logically equivalent to', so for now 'has the same truth table as'.

A ⫤⊨ ¬¬A

¬(A ∧ B) ⫤⊨ (¬A ∨ ¬B)

¬(A ∨ B) ⫤⊨ (¬A ∧ ¬B)

A ⟶ B ⫤⊨ ¬A ∨ B

¬(A ⟶ B) ⫤⊨ ¬(¬A ∨ B) ⫤⊨ A ∧ ¬B

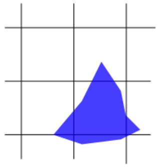
2. Negation and the arrow: A ⟶ ¬B ⊭ ¬(A ⟶ B)

Reading: §3.6

A	B	A → ¬B	A → B	¬(A → B)
T	T	F	T	F
T	F	T	F	T
F	T	T	T	F
F	F	T	T	F

3. Don't use ∃ with ⟶

Is true ∃x(Square(x) ⟶ Broken(x)) in this world?



∃x(Square(x) ⟶ Broken(x))

⫤⊨

∃x(¬Square(x) ∨ Broken(x))

⫤⊨

∃x(¬Square(x)) ∨ ∃x(Broken(x))

4. ¬Intro

Reading: §5.3, §6.3

Negation Introduction (¬ Intro)

P

⋮

⊥

▷ ¬P

1. A ∧ B

¬(¬A ∨ ¬B)

5. ¬Intro Proof Example

Reading: §5.3, §6.3

1. P→Q
2. ¬Q

6. ¬P

6. Subproofs Are Tricky

What is wrong with the following apparent proof?

R	S	$R \lor S$	$R \land S$
T	F	T	F

T	1.	$R \lor S$	
	2.	R	
	3.	$S \lor R$	$\lor$Intro: 2
	4.	S	
	5.	$S \lor R$	$\lor$Intro: 4
	6.	$S \lor R$	$\lor$Elim: 1,2-3,4-5
F	7.	$R \land S$	$\land$Intro: 2,4

7. ∀Elim

Reading: §13.1

Universal Elimination
(∀ Elim)

$$\begin{array}{l|l} & \forall x\, S(x) \\ & \vdots \\ \triangleright & S(c) \end{array}$$