Logic I: Fast Lecture 02

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Formal Proof: ∧Elim and ∧Intro

Reading: §5.1, §6.1

Conjunction Introduction (∧ Intro)

P_1
⇓
P_n
⋮
▷ $P_1 \land \ldots \land P_n$

Conjunction Elimination (∧ Elim)

$P_1 \land \ldots \land P_i \land \ldots \land P_n$
⋮
▷ P_i

1. P ∧ Q
2. Q ∧ R
3. P — ∧Elim: 1
4. R — ∧Elim: 2
5. P ∧ R — ∧Intro: 3,4

2. ∧Intro and ∨Intro: Compare and Contrast

Reading: §6.1

Disjunction Introduction (∨ Intro)

P_i
⋮
▷ $P_1 \lor \ldots \lor P_i \lor \ldots \lor P_n$

1. R
2. S
3. R ∨ T — ∨Intro: 1
4. S ∧ (R ∨ T) — ∧Intro: 2,3

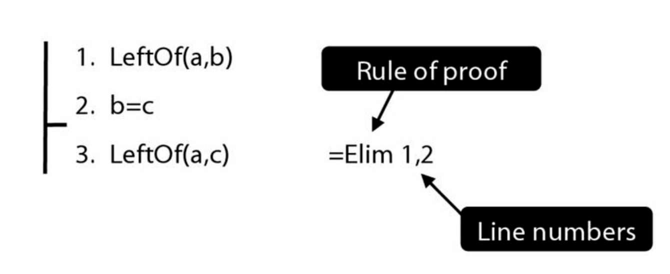
Let us define a new connective with this truth table:

P1	P2	P1 ∨ P2	P1 $\not\leftrightarrow$ P2
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

The following rule is unacceptable. Why?

$\not\leftrightarrow$Intro:
Pi
…
P1 $\not\leftrightarrow$ P2

3. How to Write Proofs



4. Rules of Proof for Identity

Reading: §2.2

Identity Introduction
(= Intro)

$\triangleright \mid n = n$

Identity Elimination
(= Elim)

$\mid P(n)$
$\mid \vdots$
$\mid n = m$
$\mid \vdots$
$\triangleright \mid P(m)$

1. LeftOf(a,b)
2. b=c
3.
4.
5. LeftOf(a,c) =Elim: 1,2

5. DeMorgan: $\neg(A \wedge B) \dashv\vDash \neg A \vee \neg B$

Reading: §3.6

'$\dashv\vDash$' means 'is logically equivalent to', so for now 'has the same truth table as'.

$A \dashv\vDash \neg\neg A$

$\neg(A \wedge B) \dashv\vDash (\neg A \vee \neg B)$

$\neg(A \vee B) \dashv\vDash (\neg A \wedge \neg B)$

$A \rightarrow B \dashv\vDash \neg A \vee B$

$\neg(A \rightarrow B) \dashv\vDash \neg(\neg A \vee B) \dashv\vDash A \wedge \neg B$

6. →Intro, →Elim

Reading: §8.1, §8.2

Conditional Introduction
(→ Intro)

$\mid \mid P$
$\mid \mid \vdots$
$\mid \mid Q$
$\triangleright \mid P \rightarrow Q$

Conditional Elimination
(→ Elim)

$\mid P \rightarrow Q$
$\mid \vdots$
$\mid P$
$\mid \vdots$
$\triangleright \mid Q$

7. →Intro: An Example

1. P→Q
2. Q→R

6. P→R

8. ∨Intro and ∨Elim

Disjunction Introduction
(∨ Intro)

$\mid P_i$
$\mid \vdots$
$\triangleright \mid P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination
(∨ Elim)

$P_1 \vee \ldots \vee P_n$
⋮
  P_1
  ⋮
  S
⇓
  P_n
  ⋮
  S
⋮
▷ S

9. ∨Elim: An Example

To prove a conclusion from a disjunction, prove it from each disjunct.

1. R ∨ S
2.
3.
4.
5.
6. S ∨ R

10. ¬, ⊥

Reading: §6.3

P	¬P	⊥
T	F	F
F	T	F

⊥ Introduction
(⊥ Intro)

P
⋮
¬P
⋮
▷ ⊥

⊥ Elimination
(⊥ Elim)

⊥
⋮
▷ P

11. ¬Elim

Reading: §6.3

Negation Elimination
(¬ Elim)

¬¬P
⋮
▷ P

12. Scope: A Mistaken Application of ¬Elim

What is wrong with this proof?

1. ¬¬(¬A ∧ ¬¬ A)
2. (¬A ∧ ¬¬A) ¬Elim: 1
3. (¬A ∧ A) ¬Elim: 2

13. ¬Intro

Reading: §5.3, §6.3

Negation Introduction
(¬ Intro)

  P
  ⋮
  ⊥
▷ ¬P

14. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*.

5.3–5.6

3.19

4.31

6.7–6.12

6.18–6.20

6.24–6.27

*6.40–6.42