Logic (PH133): Lecture 7

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. Something Is Above Something

Reading: §11.1

Something is above something:

∃x ∃y Above(x,y)

2. There Is Exactly One

There is one creator (at least one, maybe more).

∃x Creator(x)

Ahura Mazda is the one and only creator.

Creator(a) ∧ ∀x(Creator(x) ⟶ x=a)

All squares are broken.

∀x(Sqr(x) ⟶ Brkn(x))

There is one and only one creator.

∃y(Creator(y) ∧ ∀x(Creator(x) ⟶ x=y))

or:

∃y ∀x(Creator(x) ↔ x=y)

3. ∃Intro

Reading: §13.2

Disjunction Introduction (∨ Intro)

P_i
⋮
▷ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

4. ∃Elim

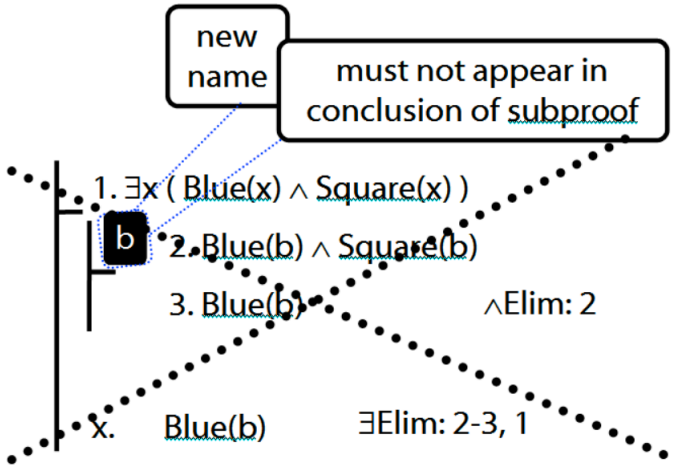
Reading: §12.2, §13.2

Existential Elimination (∃ Elim)

∃x S(x)
⋮
[c] S(c)
⋮
Q
▷ Q

where c does not occur outside the subproof where it is introduced.

Note this restriction on the use of ∃Elim:



5. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:
∀x(Dscrdn(x) ⟶ Wps(x))

All **French** discordians weep:
∀x((**Frnch(x)** ∧ Dscrdn(x)) ⟶ Wps(x))

All French discordians weep **and wail**:
$\forall x((Frnch(x) \wedge Dscrdn(x)) \rightarrow (Wps(x) \wedge \mathbf{Wls(x)}))$

All French discordians weep and wail **except Gillian Deleude**:
$\forall x((Frnch(x) \wedge Dscrdn(x) \wedge \neg\mathbf{(x=a)}) \rightarrow (Wps(x) \wedge Wls(x)))$

6. Scope and Quantifiers

Reading: §9.5, §9.6

Underlining shows the scope of the quantifiers:

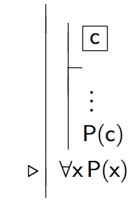
"All squares are blue"
$\forall\mathbf{x (Square(x) \rightarrow Blue(x))}$

"If everything is square, everything is blue"
$\forall\mathbf{x\ Square(x)} \rightarrow \forall\mathbf{x\ Blue(x)}$

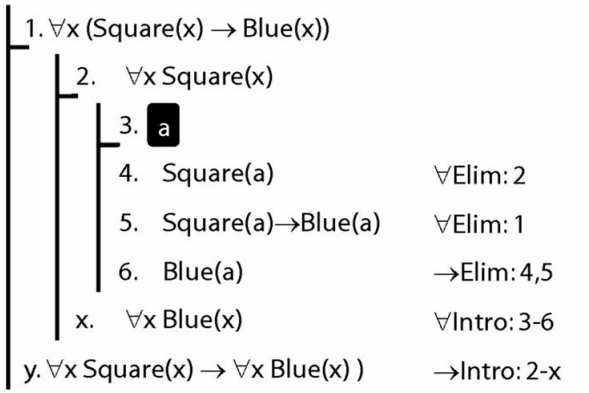
7. ∀Intro

Reading: §12.1, §12.3, §13.1

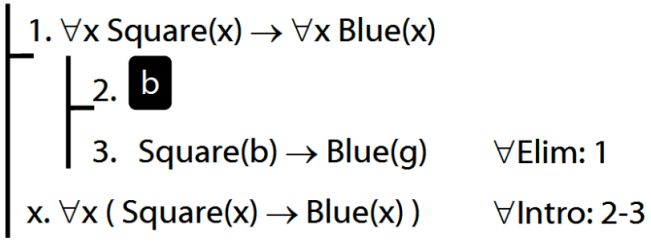
Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.



Why is this proof incorrect?



8. Summary of Quantifier Rules

Reading: §13.1, §13.2

∀Elim
If it's true of everything it's true of Baudrillard

∃Intro
If it's true of Baudrillard it's true of something

∃Elim
If it's true of something and Q follows no matter which something it is, then Q

∀Intro
If it's true of an arbitrary thing, then it's true of everything.

9. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

'Two things are broken' might be translated as:

$\exists x \exists y (Broken(x) \wedge Broken(y) \wedge \neg(x=y))$