Logic (PH133): Lecture 7

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. Watch Out, Here Come Multiple Quantifiers

Reading: §11.1

2. Something Is Above Something

Reading: §11.1

Something is above something:

∃x ∃y Above(x,y)

3. There Is Exactly One

There is one creator (at least one, maybe more).

∃x Creator(x)

Ahura Mazda is the one and only creator.

Creator(a) ∧ ∀x(Creator(x) ⟶ x=a)

All squares are broken.

∀x(Sqr(x) ⟶ Brkn(x))

There is one and only one creator.

∃y(Creator(y) ∧ ∀x(Creator(x) ⟶ x=y))

or:

∃y ∀x(Creator(x) ↔ x=y)

4. ∃Intro

Reading: §13.2

Disjunction Introduction (∨ Intro)

$$\begin{array}{l} \quad P_i \\ \quad \vdots \\ \triangleright \quad P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \end{array}$$

5. ∃Elim

Reading: §12.2, §13.2

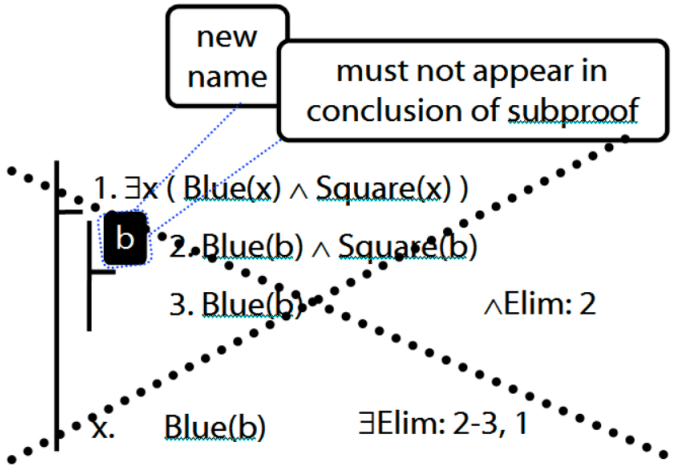
Existential Elimination (∃ Elim)

$$\begin{array}{l} \exists x\, S(x) \\ \vdots \\ \quad \boxed{c}\ S(c) \\ \quad \vdots \\ \quad Q \\ \triangleright \quad Q \end{array}$$

where c does not occur outside the subproof where it is introduced.

1. ∃x (Blue(x) ∧ Square(x))
   b
   2. Blue(b) ∧ Square(b)
   3. Blue(b) ∧Elim: 2
   4. ∃x Blue(x) ∃Intro: 3
x. ∃x Blue(x) ∃Elim: 2-4, 1

Note this restriction on the use of ∃Elim:



6. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:
∀x(Dscrdn(x) ⟶ Wps(x))

All **French** discordians weep:
∀x((**Frnch(x)** ∧ Dscrdn(x)) ⟶ Wps(x))

All French discordians weep **and wail**:
$\forall x((\text{Frnch}(x) \wedge \text{Dscrdn}(x)) \rightarrow (\text{Wps}(x) \wedge \mathbf{\text{Wls}(x)}))$

All French discordians weep and wail **except Gillian Deleude**:
$\forall x((\text{Frnch}(x) \wedge \text{Dscrdn}(x) \wedge \mathbf{\neg(x=a)}) \rightarrow (\text{Wps}(x) \wedge \text{Wls}(x)))$

7. Scope and Quantifiers

Reading: §9.5, §9.6

Underlining shows the scope of the quantifiers:

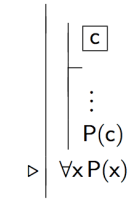
"All squares are blue"
$\forall \mathbf{x (Square(x) \rightarrow Blue(x))}$

"If everything is square, everything is blue"
$\forall \mathbf{x\ Square(x)} \rightarrow \forall \mathbf{x\ Blue(x)}$

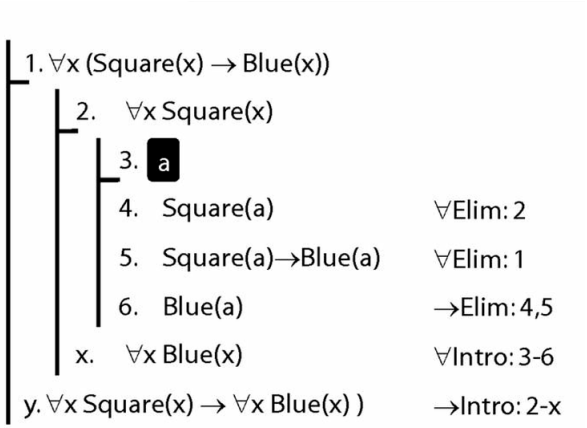
8. ∀Intro

Reading: §12.1, §12.3, §13.1

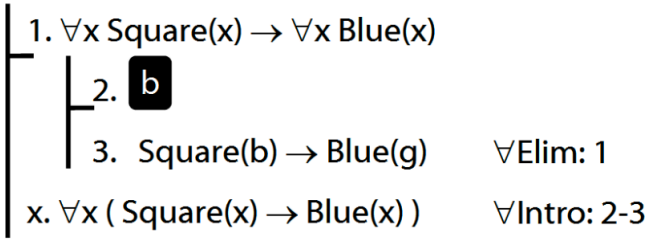
Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.



Why is this proof incorrect?



9. Summary of Quantifier Rules

Reading: §13.1, §13.2

∀Elim
If it's true of everything it's true of Baudrillard

∃Intro
If it's true of Baudrillard it's true of something

∃Elim
If it's true of something and Q follows no matter which something it is, then Q

∀Intro
If it's true of an arbitrary thing, then it's true of everything.

10. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

'Two things are broken' might be translated as:

$\exists x \exists y (\text{Broken}(x) \wedge \text{Broken}(y) \wedge \neg(x=y))$