Logic I: Lecture 16

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. Every Time I Go to the Dentist Someone Dies

Reading: §11.2

∀t (

(Time(t) ∧ ToDentist(a,t))

→

∃x (Person(x) ∧ TimeOfDeath(x,t))

)

2. There Is Exactly One

There is one creator (at least one, maybe more).

∃x Creator(x)

Brian is the one and only creator.

Creator(b) ∧ ∀x(Creator(x) → x=b)

All squares are broken.

∀x(Sqr(x) → Brkn(x))

There is one and only one creator.

∃y(Creator(y) ∧ ∀x(Creator(x) → x=y))

or:

∃y ∀x(Creator(x) ↔ x=y)

3. Could There Be Nothing?

Reading: §13.2

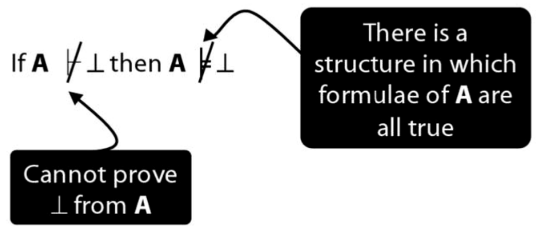
1.	a=a	=Intro
2.	∃x (x=x)	∃ Intro: 1

1.	...	
...		
m.	∀x (Train(x) ∨ ¬Train(x))	...
n.	Train(a) ∨ ¬Train(a)	∀Elim: m
o.	a=a	=Intro
p.	∃x (x=x)	∃ Intro: 1

1.	¬(a=a)	
2.	a=a	=Intro
3.	⊥	⊥Intro: 1,3
4.	¬¬(a=a)	¬Intro: 1-3
5.	a=a	¬Elim: 5
6.	∃x (x=x)	∃ Intro: 5

4. The Essence of the Completeness Theorem

Reading: §8.3



Arrange the sentence letters in a series: P1, P2, P3, ...

Define a structure, *h*, as follows.

Take each sentence letter, Pi, in turn.

- If **A** ⊢ Pi then *h*(Pi) = True

- If **A** ⊢ ¬Pi then *h*(Pi) = False

- Otherwise *h*(Pi) = True

Every sentence of **A** is true in the structure *h*

Therefore **A** ⊭⊥

5. Proofs about Proofs

If A ⊢ B then ⊢ A⟶B

Proof Given a proof for A ⊢ B …

1	A	
	…	
n	B	

… we can turn it into a proof for ⊢ A⟶B:

1	A	
	…	
n	B	
	A→B	→Intro: 1-n

If ⊢ A⟶B then A ⊢ B

If A ⊢ B then A ⊢ ¬¬B

Proof:

This is the original proof of A ⊢ B

1	A	
2	…	
n	B	

Adding this turns it into a proof of A ⊢ ¬¬B

p	¬B	
q	⊥	⊥Intro:n,p
	¬¬B	¬Intro:p-q

If A ⊢ C then A ⊢ B⟶C

If A ⊢ B and A ⊢ ¬C then A ⊢ ¬(B⟶C)

6. Does ‘if’ mean what ‘⟶’ means?

Reading: §7.3

These two arguments are valid: does that mean that ‘if’ means what ‘⟶’ means?

¬A ∨ B	America does not exist ∨ Baudrillard is wrong
If A, B	If America exists, Baudrillard is wrong

If A, B	If you love logic, things will fall into place
¬(A ∧ ¬B)	Not both: you take logic and things don't fall into pla…

The English argument isn't valid; the FOL argument is valid; therefore ‘if’ can't mean what ‘⟶’ means?

¬A	Marnie will not miss her train
A → B	If Marnie misses her train, she will arrive on time.

7. Proof Example: ∃x Dead(x) ⊢ ¬∀x¬Dead(x).

1.	∃x F(x)	
2.	$\boxed{a}$ F(a)	
3.	∀y¬F(y)	
4.	¬F(a)	∀Elim: 3
5.	⊥	⊥Intro: 2,4
6.	¬∀y¬F(y)	¬Intro: 3-5
7.	¬∀y¬F(y)	∃Elim: 2-6

8. Proof Example: ¬∀x Dead(x) ⊢ ∃x¬ Dead(x).

1.	¬∀x Dead(x)	
2.	¬∃x¬Dead(x)	
3.	$\boxed{a}$	
4.	¬F(a)	
5.	∃x¬Dead(x)	∃Intro: 4
6.	⊥	⊥Intro: 2,5
7.	¬¬F(a)	¬Intro: 4-6
8.	F(a)	¬Elim: 8
9.	∀x Dead(x)	∀Intro: 3-8
10.	⊥	⊥Intro: 1,9
11.	¬¬∃x¬Dead(x)	¬Intro: 2-10
12.	∃x¬Dead(x)	¬Elim: 11

9. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

14.10–14.12, *14.13