# Logic I: Lecture 07

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

## 1. The Syntax of FOL

*Reading:* §9.3

We define what counts as a sentence of FOL using rules. E.g.:

1. If * and # are sentences, then so is(* ∧ #)

2. If * and # are sentences, then so is (* ∨ #)

3. P, Q, R, … are sentences

4. If * is a sentence, then ¬* is a sentence

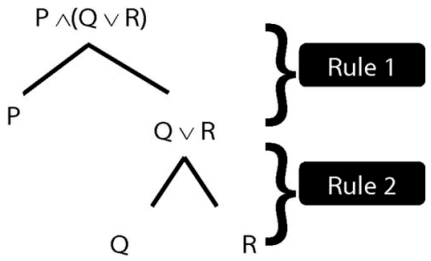
So:

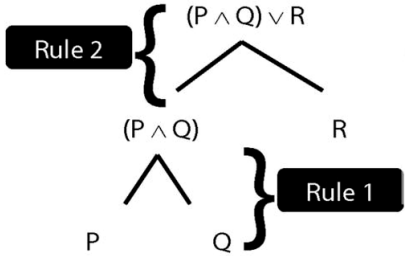
a. P is a sentence // rule 3

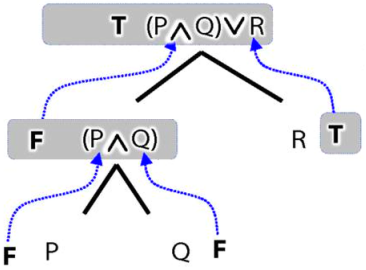
b. ¬P is a sentence // rule 4, a

c. ( ¬P ∧ Q ) is a sentence // rule 1, b, a

There is no structural ambiguity in FOL because these rules are formulated to ensure that for any FOL sentence, there is exactly one way of constructing it.



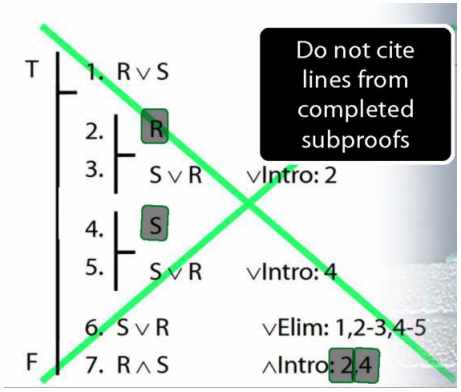




## 2. ¬P ∨ ¬Q compared with ¬(P ∨ Q)

*Reading:* §3.5

## 3. Subproofs Are Tricky: The Answer




## 4. ¬Intro Proof Example

*Reading:* §5.3, §6.3

## 5. DeMorgan: $\neg(A \wedge B) \dashv\models \neg A \vee \neg B$

*Reading:* §3.6, §4.2

'$\dashv\models$' means 'is logically equivalent to', so for now 'has the same truth table as'.

$A \dashv\models \neg\neg A$

$\neg(A \wedge B) \dashv\models (\neg A \vee \neg B)$

$\neg(A \vee B) \dashv\models (\neg A \wedge \neg B)$

$A \longrightarrow B \dashv\models \neg A \vee B$

$\neg(A \longrightarrow B) \dashv\models \neg(\neg A \vee B) \dashv\models A \wedge \neg B$

## 6. Everything Is Broken

*Reading:* §9.1, §9.2

Everything is broken: $\forall x$ Broken(x)

Something is broken: $\exists x$ Broken(x)

## 7. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*.

6.24–6.26

3.19

4.15–18

9.1 odd numbers only

9.2 even numbers only