

# Logic I: Lecture 03

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

## 1. Logical Validity and Truth Tables

Reading: §4.3

Truth tables can be used to show that an argument is valid. To illustrate ...

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

$\wedge$                        $\wedge$                        $\wedge$   
 premise                      premise                      conclusion

  

T	$P \vee Q$
F	$\neg P$
	Q

To establish that an argument is valid:

1. Create truth tables for each premise and the conclusion.
2. Check whether there is a row of the truth table where all premises are true and the conclusion is false.
3. If not, the argument is valid.

## 2. Translating a Simple Argument

Reading: §3.2

## 3. Formal Proof: $\wedge$ Elim and $\wedge$ Intro

Reading: §5.1, §6.1

### Conjunction Introduction ( $\wedge$ Intro)

$P_1$
$\Downarrow$
$P_n$
$\vdots$
$\triangleright P_1 \wedge \dots \wedge P_n$

### Conjunction Elimination ( $\wedge$ Elim)

$P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n$
$\vdots$
$\triangleright P_i$

1. $P \wedge Q$	
2. $Q \wedge R$	
3. $P$	$\wedge$ Elim: 1
4. $R$	$\wedge$ Elim: 2
5. $P \wedge R$	$\wedge$ Intro: 3,4

## 4. Rules of Proof for Identity

Reading: §2.2

### Identity Introduction (= Intro)

$\triangleright n = n$
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## Identity Elimination (= Elim)

$P(n)$   
 $\vdots$   
 $n = m$   
 $\vdots$   
 $\triangleright P(m)$

## 5. How to Write Proofs

$\vdash$ 

1. LeftOf(a,b)
2.  $b=c$
3. LeftOf(a,c)

Rule of proof

=Elim 1,2

Line numbers

## 6. Complex Truth Tables

Reading: §3.3, §3.5

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Always start with T  
 Sentence letters are ordered alphabetically  
 Right-most column alternates every row  
 Always end with F  
 Next right-most column alternates half as often as previous column  
 Next right-most column alternates half as often

Complex truth table example:

P	Q	R	$(P \wedge Q) \vee R$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

## 7. Logic Makes Me Die Inside

Reading: §2.1

## 8. Tautologies and Contradictions

Reading: §4.1, §4.2

Argument 3

$\vdash$ 

1.  $(P \wedge Q) \vee R$
2.  $P \vee \neg P$

Argument 3b

$\vdash$ 

1.  $P \vee \neg P$

Argument 4

$\vdash$ 

1.  $P \wedge \neg P$
2.  $(P \wedge Q) \vee R$

$P \vee \neg P$  is a *logical truth*

logical truth defined p. 568

$P \wedge \neg P$  is a *contradiction*

contradiction defined p. 564

## 9. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*.

5.1-5.4

6.1

3.12, 3.13

4.4-4.7

4.12-14

4.1, 4.2