Logic I: Lecture 08

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Everything Is Broken

Reading: §9.1, §9.2

Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)

2. All Squares Are Blue

Reading: §9.2, §9.3, §9.5

3. What does ∀ mean?

Reading: §9.4

We give the meaning of ∀ by specifying what it takes for a sentence containing ∀ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.
3. If ALL of the new sentences are true, so is the original sentence.

4. Vegetarians Are Evil

Reading: §9.2, §9.3, §9.5

∀x (Evil(x) → HatesMeat(x))

∀x (HatesMeat(x) → Vegetarian(x))

∀x (Vegetarian(x) → Evil(x))

5. Not If

If she has seen it, I am dead.

A → B

That's not true.

¬(A → B)

If she has seen it, I am not dead.

A → ¬B

A	B	A → B	¬(A → B)	A → ¬B
T	T	T	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

6. Scope: A Mistaken Application of ¬Elim

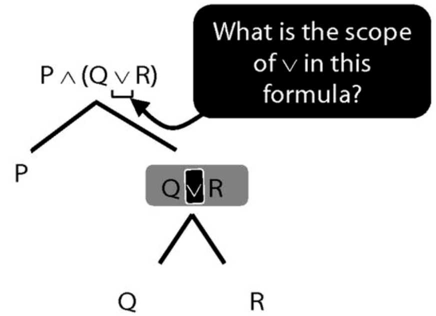
What is wrong with this proof?

1.	¬¬(¬A ∧¬¬ A)	
2.	(¬A ∧ ¬¬A)	¬Elim: 1
3.	(¬A ∧ A)	¬Elim: 2

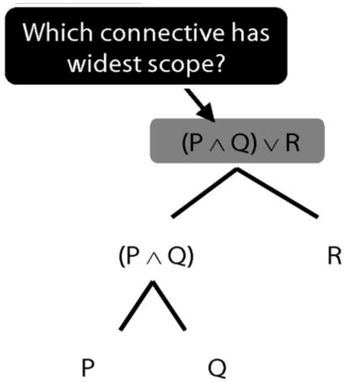
7. Scope

Reading: §3.5

The *scope* of a connective (token) is the sentence containing it lowest in the tree.



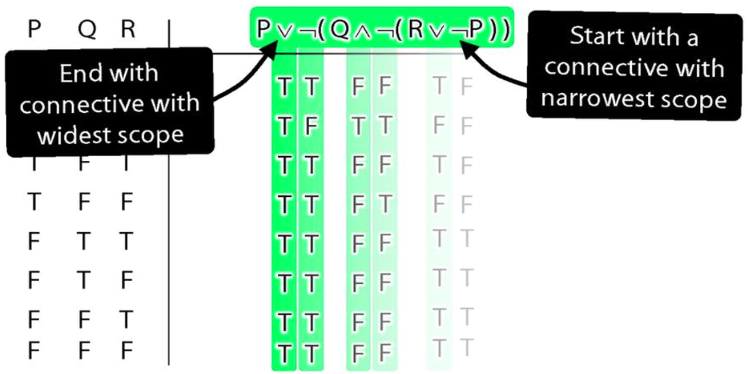
The connective with *widest scope* is the one whose scope is the whole sentence.



A rule of proof can only be applied to the connective with widest scope.

1. $\neg\neg(\neg R \wedge \neg\neg R)$
2. $(\neg R \wedge \neg\neg R)$ ¬Elim: 1
3. ~~$(\neg R \wedge R)$ ¬Elim: 2~~

When we do truth tables, the order we do the columns in is determined by scope.



8. Proof Example: $\neg(P \vee Q)$ therefore $\neg P \wedge \neg Q$

1. $\neg(P \vee Q)$

10. $\neg P \wedge \neg Q$

1. $\neg P \wedge \neg Q$

10. $\neg(P \vee Q)$

9. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

7.1–7.2, *7.3–7.6

9.1 odd numbers only

9.2 even numbers only

9.4–9.5

9.8–9.10

6.8

3.14

3.15

6.24–6.27

Conjunction Introduction
($\wedge$ Intro)

$$\begin{array}{r|l} & P_1 \\ & \Downarrow \\ & P_n \\ & \vdots \\ \triangleright & P_1 \wedge \ldots \wedge P_n \end{array}$$

Conjunction Elimination
($\wedge$ Elim)

$$\begin{array}{r|l} & P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n \\ & \vdots \\ \triangleright & P_i \end{array}$$

Disjunction Introduction
($\vee$ Intro)

$$\begin{array}{r|l} & P_i \\ & \vdots \\ \triangleright & P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \end{array}$$

Disjunction Elimination
($\vee$ Elim)

$$\begin{array}{r|l} & P_1 \vee \ldots \vee P_n \\ & \vdots \\ & \left|\begin{array}{l} P_1 \\ \hline \vdots \\ S \end{array}\right. \\ & \Downarrow \\ & \left|\begin{array}{l} P_n \\ \hline \vdots \\ S \end{array}\right. \\ & \vdots \\ \triangleright & S \end{array}$$

Negation Introduction
($\neg$ Intro)

$$\begin{array}{r|l} & \left|\begin{array}{l} P \\ \hline \vdots \\ \bot \end{array}\right. \\ \triangleright & \neg P \end{array}$$

Negation Elimination
($\neg$ Elim)

$$\begin{array}{r|l} & \neg\neg P \\ & \vdots \\ \triangleright & P \end{array}$$

$\bot$ Introduction
($\bot$ Intro)

$$\begin{array}{r|l} & P \\ & \vdots \\ & \neg P \\ & \vdots \\ \triangleright & \bot \end{array}$$

$\bot$ Elimination
($\bot$ Elim)

$$\begin{array}{r|l} & \bot \\ & \vdots \\ \triangleright & P \end{array}$$

Conditional Introduction
($\rightarrow$ Intro)

$$\begin{array}{r|l} & \left|\begin{array}{l} P \\ \hline \vdots \\ Q \end{array}\right. \\ \triangleright & P \rightarrow Q \end{array}$$

Conditional Elimination
($\rightarrow$ Elim)

$$\begin{array}{r|l} & P \rightarrow Q \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Biconditional Introduction
($\leftrightarrow$ Intro)

$$\begin{array}{r|l} & \left|\begin{array}{l} P \\ \hline \vdots \\ Q \end{array}\right. \\ & \left|\begin{array}{l} Q \\ \hline \vdots \\ P \end{array}\right. \\ \triangleright & P \leftrightarrow Q \end{array}$$

Biconditional Elimination
($\leftrightarrow$ Elim)

$$\begin{array}{r|l} & P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Reiteration
(Reit)

$$\begin{array}{r|l} & P \\ & \vdots \\ \triangleright & P \end{array}$$

Identity Introduction
($=$ Intro)

$$\begin{array}{r|l} \triangleright & n = n \end{array}$$

Identity Elimination
($=$ Elim)

$$\begin{array}{r|l} & P(n) \\ & \vdots \\ & n = m \\ & \vdots \\ \triangleright & P(m) \end{array}$$

Universal Elimination
($\forall$ Elim)

$$\begin{array}{r|l} & \forall x\, S(x) \\ & \vdots \\ \triangleright & S(c) \end{array}$$

Universal Introduction
($\forall$ Intro)

$$\begin{array}{r|l} & \left|\begin{array}{l} \boxed{c} \\ \hline \vdots \\ P(c) \end{array}\right. \\ \triangleright & \forall x\, P(x) \end{array}$$

where c does not occur outside the subproof where it is introduced.

Existential Introduction
($\exists$ Intro)

$$\begin{array}{r|l} & S(c) \\ & \vdots \\ \triangleright & \exists x\, S(x) \end{array}$$

Existential Elimination
($\exists$ Elim)

$$\begin{array}{r|l} & \exists x\, S(x) \\ & \vdots \\ & \left|\begin{array}{l} \boxed{c}\ S(c) \\ \hline \vdots \\ Q \end{array}\right. \\ \triangleright & Q \end{array}$$

where c does not occur outside the subproof where it is introduced.