Logic I: Lecture 11

s.butterfill@warwick.ac.uk

Readings refer to sections of the course textbook, *Language, Proof and Logic.*

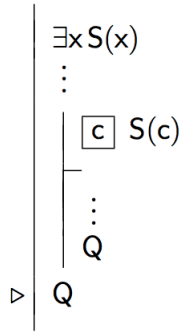
1. Revision: ∀Elim, ∃Intro

Reading: §12.1, §13.1, §13.2

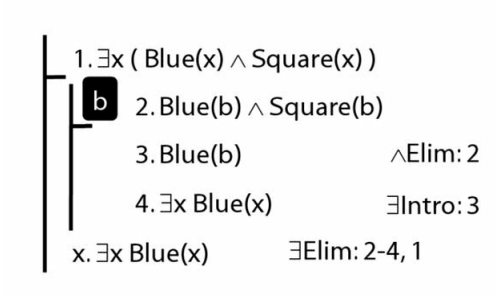
2. ∃Elim

Reading: §12.2, §13.2

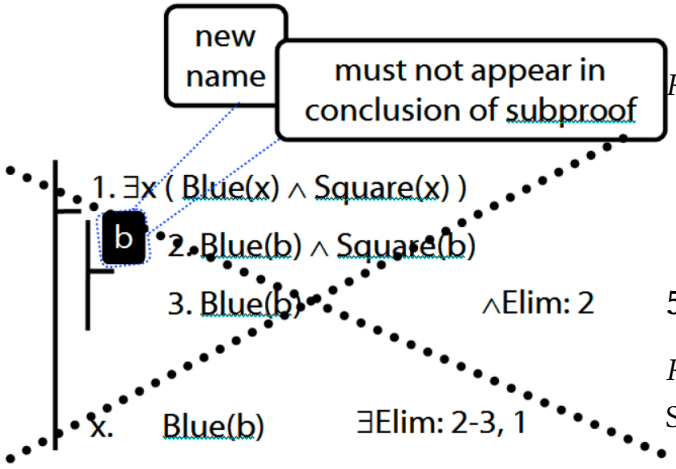
Existential Elimination (∃ Elim)



where c does not occur outside the subproof where it is introduced.

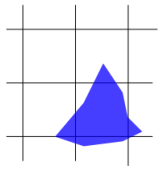


Note this restriction on the use of ∃Elim:



3. Don't use ∃ with →

Is true ∃x(Square(x) → Broken(x)) in this world?



∃x(Square(x) → Broken(x))

⫤⊨

∃x(¬Square(x) ∨ Broken(x))

⫤⊨

∃x(¬Square(x)) ∨ ∃x(Broken(x))

4. Watch Out, Here Come Multiple Quantifiers

Reading: §11.1

5. Something Is Above Something

Reading: §11.1

Something is above something:
∃x ∃y Above(x,y)

6. Multiple Quantifiers: Everyone Likes Puffins

Reading: §11.1

I like puffins:

$\forall x$ (Puffin(x) $\longrightarrow$ Likes(a,x))

y likes puffins:

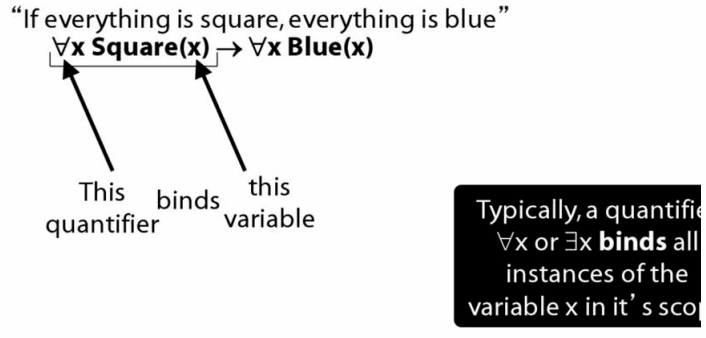
$\forall x$ (Puffin(x) $\longrightarrow$ Likes(y,x))

Everyone likes puffins:

$\forall y \forall x$ (Puffin(x) $\longrightarrow$ Likes(y,x))

7. Quantifiers Bind Variables

Reading: §9.3



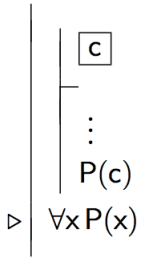
8. Summary of Quantifier Rules So Far

Reading: §12.1, §12.2, §12.3, §13.1, §13.2

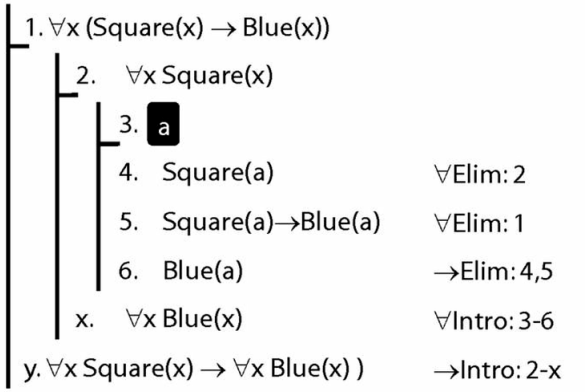
9. ∀Intro

Reading: §12.1, §12.3, §13.1

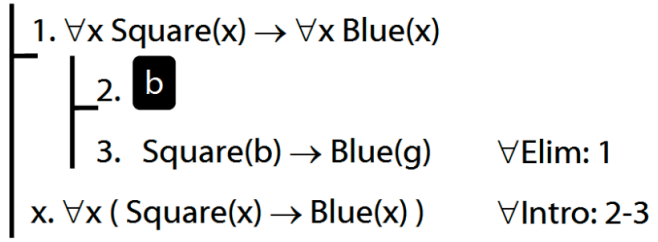
Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.



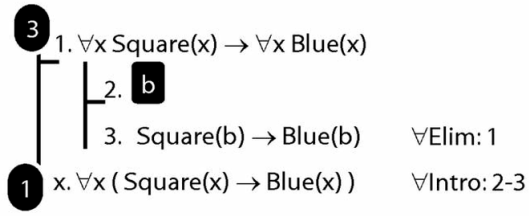
Why is this proof incorrect?



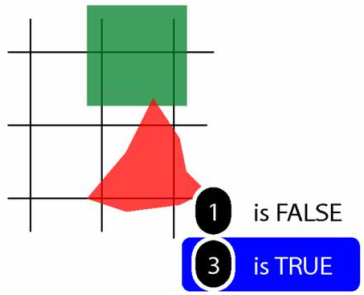
10. ∀Intro: An Incorrect Proof

Reading: §13.1, §13.2

This proof is wrong, but why?:



There is a counterexample to the argument:



11. Exercises

These exercises will be discussed in seminars the week after this lecture. The numbers below refer to the numbered exercises in the course textbook, e.g. '1.1' refers to exercise 1.1. on page 39 of the second edition of *Language, Proof and Logic*. Exercises marked '*' are optional.

6.17–6.20

6.33, 6.40

8.24–8.25

9.10

9.15–9.17

*9.18–9

11.2

12.4–12.5

*12.6–12.7

12.9–12.10