

Logic I: Lecture 11

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Readings refer to sections of the course textbook,
Language, Proof and Logic.

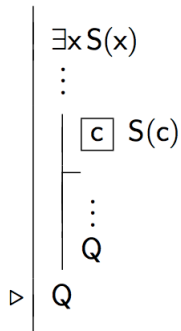
1. Revision: \forall Elim, \exists Intro

Reading: §12.1, §13.1, §13.2

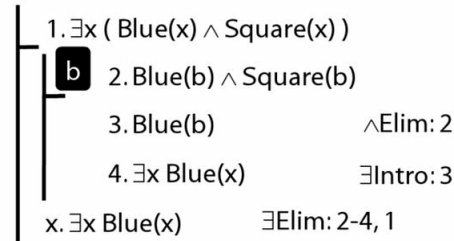
2. \exists Elim

Reading: §12.2, §13.2

Existential Elimination (\exists Elim)

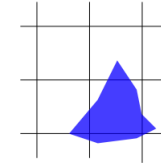


where c does not occur outside the subproof where it is introduced.



3. Don't use \exists with \rightarrow

Is true $\exists x(\text{Square}(x) \rightarrow \text{Broken}(x))$ in this world?



$\exists x(\text{Square}(x) \rightarrow \text{Broken}(x))$

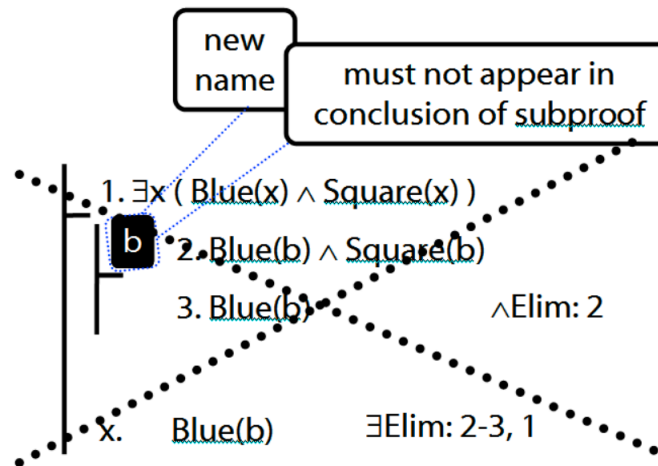
\models

$\exists x(\neg\text{Square}(x) \vee \text{Broken}(x))$

\models

$\exists x(\neg\text{Square}(x)) \vee \exists x(\text{Broken}(x))$

Note this restriction on the use of \exists Elim:



4. Watch Out, Here Come Multiple Quantifiers

Reading: §11.1

5. Something Is Above Something

Reading: §11.1

Something is above something:

$\exists x \exists y \text{Above}(x,y)$

6. Multiple Quantifiers: Everyone Likes Puffins

Reading: §11.1

I like puffins:

$\forall x (\text{Puffin}(x) \rightarrow \text{Likes}(a,x))$

y likes puffins:

$\forall x (\text{Puffin}(x) \rightarrow \text{Likes}(y,x))$

Everyone likes puffins:

$\forall y \forall x (\text{Puffin}(x) \rightarrow \text{Likes}(y,x))$

7. Quantifiers Bind Variables

Reading: §9.3

“If everything is square, everything is blue”

$\forall x \text{ Square}(x) \rightarrow \forall x \text{ Blue}(x)$

This quantifier binds this variable

Typically, a quantifier $\forall x$ or $\exists x$ binds all instances of the variable x in its scope

8. Summary of Quantifier Rules So Far

Reading: §12.1, §12.2, §12.3, §13.1, §13.2

9. \forall Intro

Reading: §12.1, §12.3, §13.1

Universal Introduction (\forall Intro)

$\begin{array}{|l} \boxed{c} \\ \vdots \\ P(c) \\ \hline \triangleright \forall x P(x) \end{array}$

where c does not occur outside the subproof where it is introduced.

$\begin{array}{|l} 1. \forall x (\text{Square}(x) \rightarrow \text{Blue}(x)) \\ \hline 2. \forall x \text{ Square}(x) \\ \hline 3. \boxed{a} \\ 4. \text{Square}(a) \quad \forall\text{Elim: } 2 \\ 5. \text{Square}(a) \rightarrow \text{Blue}(a) \quad \forall\text{Elim: } 1 \\ 6. \text{Blue}(a) \quad \rightarrow\text{Elim: } 4,5 \\ \hline x. \forall x \text{ Blue}(x) \quad \forall\text{Intro: } 3-6 \\ y. \forall x \text{ Square}(x) \rightarrow \forall x \text{ Blue}(x) \quad \rightarrow\text{Intro: } 2-x \end{array}$

Why is this proof incorrect?

$\begin{array}{|l} 1. \forall x \text{ Square}(x) \rightarrow \forall x \text{ Blue}(x) \\ \hline 2. \boxed{b} \\ 3. \text{Square}(b) \rightarrow \text{Blue}(g) \quad \forall\text{Elim: } 1 \\ \hline x. \forall x (\text{Square}(x) \rightarrow \text{Blue}(x)) \quad \forall\text{Intro: } 2-3 \end{array}$

10. \forall Intro: An Incorrect Proof

Reading: §13.1, §13.2

This proof is wrong, but why?:

$\begin{array}{|l} 3. \forall x \text{ Square}(x) \rightarrow \forall x \text{ Blue}(x) \\ \hline 2. \boxed{b} \\ 3. \text{Square}(b) \rightarrow \text{Blue}(b) \quad \forall\text{Elim: } 1 \\ \hline 1. x. \forall x (\text{Square}(x) \rightarrow \text{Blue}(x)) \quad \forall\text{Intro: } 2-3 \end{array}$

There is a counterexample to the argument:

