PH126 Logic I Fast Lecture 5
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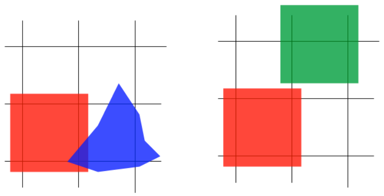
Multiple quantifers

1 **∃x ∃y Above(x,y)**
"Something is above something"



1 is FALSE   1 is TRUE

There is a store for everything

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything
Everyone hurts someone they love
Someone hurts everyone she loves

"Every time I go to the dentist, somebody dies."
∀t ((Time(t) ∧ ToDentist(a,t)) → ∃x (Person(x) ∧ DiesAt(x,t)))

Quantifiers bind variables

Read §9.2 of Barwise & Etchemendy

Two objects are broken

∃x ∃y (Broken(x)∧Broken(y)∧¬(x=y))

To translate sentences involving number into FOL, use identity.

The Soundness & Completeness Theorems

'**A** ⊢ B' means there is a proof of B using premises **A**

' ⊢ B' means there is a proof of B using no premises

'**A** ⊨ B' means B is a logical consequence of A

' ⊨ B' means B is a tautology

'**A** $\models_T$ B' means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

Soundness: If **A** ⊢ B then **A** ⊨ B
I.e. if you can prove it in Fitch, it's valid

Completeness: If **A** $\models_T$ B then **A** ⊢ B
I.e. if it's valid just in virtue of the meanings

Exercise

If the following is valid, give a formal proof. If it is not valid, give a counterexample.

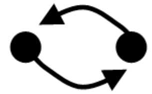
| ∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))
| ∀x∀y [R(x,y) → R(y,x)]
|—
| ∀x R(x,x)

Reflexive, symmetric and transitive

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)

∀x R(x,x)

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y).)



∀x∀y (R(x,y) → R(y,x))

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is *not* transitive)

∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))

Quantifier equivalences

P→Q ⫤⊨ ¬Q→¬P
∀x (Square(x) → Broken(x))
⫤⊨ ∀x (¬Broken(x) → ¬Square(x))

P→Q ⫤⊨ ¬P∨Q
∀x (Square(x) → Broken(x))
⫤⊨ ∀x (¬Square(x) ∨ Broken(x))

¬P∨Q ⫤⊨ ¬(P∧ ¬Q)
∃x ¬F(x) ⫤⊨ ¬∀x F(x)
∀x (¬Square(x) ∨ Broken(x))
⫤⊨ ∃¬x (Square(x) ∧ ¬Broken(x))

∃x ¬Created(x) ⫤⊨ ¬∀x Created(x)

Truth-functional completeness (LPL §7.4)

How many truth-functions can be constructed using 2 sentence letters?

P	Q	P % Q
T	T	?
T	F	?
F	T	?
F	F	?

Each of these can be T or F

Df. 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that $\{\neg, \wedge, \vee\}$ is truth-functionally complete:

P	Q	$P \to Q$	
T	T	T	$[P\wedge Q] \vee$
T	F	F	
F	T	T	$[\neg P\wedge Q] \vee$
F	F	T	$[\neg P\wedge \neg Q]$

$[P\wedge Q] \vee [\neg P\wedge Q] \vee [\neg P\wedge \neg Q]$

Exercise assuming $\{\neg,\vee,\wedge\}$ is truth-functionally complete, show that $\{\neg,\vee\}$ is.

Exercises 06

For your sixth seminar
Only for fast groups

A. From the LPL textbook:

11.2, 11.4
11.8, 11.9, *11.11
13.12, 13.14, 13.16 (quantifier proofs)
13.28

Despite what book says, DON'T even THINK about using TAUT CON.

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain : people and actions
D(x) : x is desirable
V(x) : x is virtuous
A(x) : x is an action
H(x) : x is a person
P(x,y): x performed y

i. $\forall x\ [\ D(x) \to V(x)\ ]$

ii. $\forall x\ [\ [A(x) \wedge D(x)\ ] \to V(x)\ ]\ ]$

iii. $\exists x\ [\ A(x) \wedge \neg[\ D(x) \to V(x)\ ]\ ]$

*iv. $\exists x\ \forall y\ [[[H(x) \wedge A(y)] \wedge P(x,y)] \to V(y)]$

**v. $\neg\exists x\ [\exists y\ [H(x) \wedge P(x,y) \wedge A(y) \wedge \neg V(y)] \wedge \neg\exists z[P(x,z) \wedge A(z) \wedge V(z)]]$