PH126 Logic I Fast Lecture 7
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"Every time I go to the dentist, somebody dies."
$\forall t\, ((\text{Time}(t) \land \text{ToDentist}(a,t)) \rightarrow \exists x\, (\text{Person}(x) \land \text{DiesAt}(x,t)))$

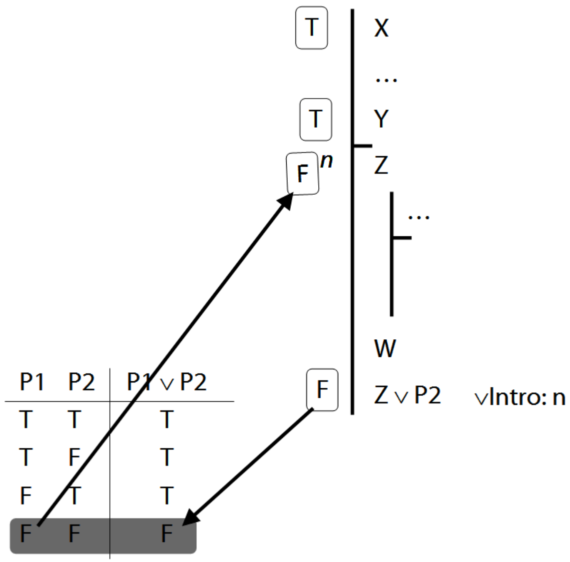
Notation:
A $\vdash$ B means there is a proof of B using premises **A**
$\vdash$ B means there is a proof of B using no premises
A $\vdash$ means the same as A $\vdash \bot$

A $\vDash$ B means B is a logical consequence of A
$\vDash$ B means B is a tautology
A $\vDash$ means the same as **A** $\vDash \bot$
A $\vDash_{TT}$ B means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

Soundness: If **A** $\vdash$ B then **A** $\vDash$ B
I.e. if you can prove it in Fitch, it's valid

Completeness: If **A** $\vDash_{TT}$ B then **A** $\vdash$ B
I.e. if it's valid just in virtue of the meanings of truth functions, you can prove it in Fitch

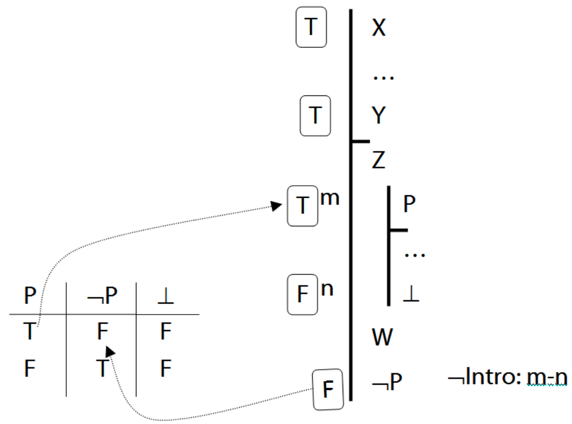
Hint for soundness proof
Take any argument that ends with ∨Intro. Suppose this argument is not valid, i.e. the premises are true and the conclusion false. Then Z must be false. So the argument from the premises to Z (line n) is not a valid argument. So **there is a shorter proof which is not valid**.
Illustration of soundness proof: ∨Intro



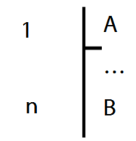
Stipulation: when I say that 'a proof is not valid', I mean that the last step of the proof is not a logical consequence of the premises (including premises of any open subproofs).

How to prove soundness? Outline
Step 1: show that each rule has this property:
- Where the last step in a proof involves that rule, if proof is not valid then there is a shorter proof which is not valid.

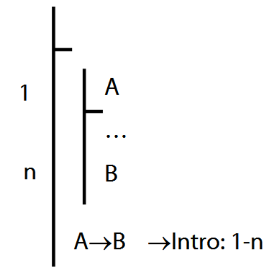
Step 2: Suppose (for a contradiction) that some Fitch proofs are not valid. Select one of the shortest invalid proofs. The last step must involve one of the Fitch rules. Whichever rule it involves, we know that there must be a shorter proof which is not valid. This contradicts the fact that the selected proof is a shortest invalid proof.
Illustration of soundness proof: ¬Intro



Proofs about proofs
1. If A $\vdash$ B then $\vdash$ A→B
Proof Given a proof for A $\vdash$ B …



… we can turn it into a proof for $\vdash$ A→B:



2. If $\vdash A \to B$ then $A \vdash B$

3. If $A \vdash B$ then $A \vdash \neg\neg B$

Proof:

This is the original proof of $A \vdash B$	1	A	
	2	…	
	n	B	
Adding this turns it into a proof of $A \vdash \neg\neg B$	p	$\quad \neg B$	
	q	$\quad \bot$	$\bot$Intro:n,p
		$\neg\neg B$	$\neg$Intro:p-q

4. If $A \vdash C$ then $A \vdash B \to C$

5. If $A \vdash B$ and $A \vdash \neg C$ then $A \vdash \neg(B \to C)$

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Quantifier equivalences

$P \to Q \quad \dashv\vDash \neg Q \to \neg P$

$\forall x\,(\,Square(x) \to Broken(x)\,)$

$\quad \dashv\vDash \forall x\,(\neg Broken(x) \to \neg Square(x)\,)$

$P \to Q \quad \dashv\vDash \neg P \vee Q$

$\forall x\,(\,Square(x) \to Broken(x)\,)$

$\quad \dashv\vDash \forall x\,(\neg Square(x) \vee Broken(x)\,)$

$\neg P \vee Q \dashv\vDash \neg(P \wedge \neg Q)$

$\exists x\, \neg F(x) \dashv\vDash \neg \forall x\, F(x)$

$\forall x\,(\neg Square(x) \vee Broken(x)\,)$

$\quad \dashv\vDash \exists \neg x\,(\,Square(x) \wedge \neg Broken(x)\,)$

$\exists x\, \neg Created(x) \dashv\vDash \neg \forall x\, Created(x)$

Hybrid example

		Every object is either non-square or broken $\forall x\,(\neg Square(x) \vee Broken(x)\,)$
$\boxed{\neg\neg P \dashv\vDash P}$	$\dashv\vDash$	$\neg\neg\forall x\,(\neg Square(x) \vee Broken(x)\,)$
$\boxed{\neg\forall x\, F(x) \dashv\vDash \exists x \neg F(x)}$	$\dashv\vDash$	$\neg\exists x\, \neg(\neg Square(x) \vee Broken(x)\,)$
$\boxed{\neg(P \vee Q) \dashv\vDash \neg P \wedge \neg Q}$	$\dashv\vDash$	$\neg\exists x\,(\neg\neg Square(x) \wedge \neg Broken(x)\,)$
$\boxed{\neg\neg P \dashv\vDash P}$	$\dashv\vDash$	Nothing is square and non-broken $\neg\exists x\,(\,Square(x) \wedge \neg Broken(x)\,)$

Variables

Names : a, b, c, …

Variables : x, y, z, w, …

Variables are for saying several things about one thing even without specifying which thing it is

Someone is loved by everyone

$\exists x \forall y\, Loves(y,x)$

Someone loves everyone

$\exists x \forall y\, Loves(x,y)$

Exercises 08

For your eighth seminar

Only for fast groups

13.51–2

14.4–5

14.13 [proofs]

14.26, 14.28

Despite what book says, DON'T even THINK about using TAUT CON.