PH126 Logic I Lecture 8

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Proofs of the Day

Proof example

Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for \rightarrow is fixed if we accept \rightarrow Elim and \rightarrow Intro.

How do the rules of proof for \rightarrow fix its truth table?

Α	В	$A \rightarrow B$
Т	Т	
Т	F	
F	Т	
F	F	

How to determine the truth of sentences involving \forall

- 1. Give every object a name.
- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
- 3. If ALL of the new sentences are true, so is the original.

What not to confuse

$$\exists x \ (Square(x) \land Blue(x)) \ vs.$$

 $\exists x \ Square(x) \land \exists x \ Blue(x)$

$$\forall x \ (\text{Square}(x) \rightarrow \text{Blue}(x)) \text{ vs.}$$

 $\forall x \ \text{Square}(x) \rightarrow \forall x \ \text{Blue}(x)$

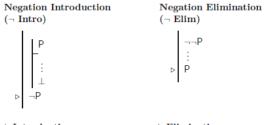
$$\neg (P \lor Q) \text{ vs. } \neg P \lor \neg Q$$

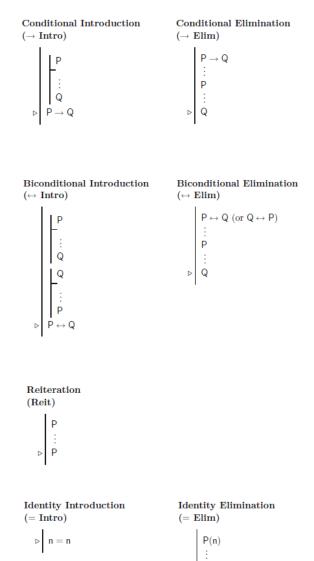
$$\neg (P \land Q) \text{ vs. } \neg P \land \neg Q$$

$$\neg (P \rightarrow Q) \text{ vs. } P \rightarrow \neg Q$$

(∨ Intro) (¬ Intro)

$$\begin{array}{c|c} \textbf{Disjunction Introduction} & \textbf{Disjunction Elimination} \\ (\vee \textbf{ Intro}) & (\vee \textbf{ Elim}) \\ \hline \\ P_i \\ \vdots \\ P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \\ \hline \\ \vdots \\ P_1 \\ \hline \\ \vdots \\ S \\ \hline \\ \downarrow \\ \end{array}$$





n = m

P(m)

