PH126 Logic I Fast Lecture 6
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Quantifier equivalences

P→Q ⫤⊨ ¬Q→¬P
∀x (Square(x) → Broken(x))
⫤⊨ ∀x (¬Broken(x) → ¬Square(x))

P→Q ⫤⊨ ¬P∨Q
∀x (Square(x) → Broken(x))
⫤⊨ ∀x (¬Square(x) ∨ Broken(x))

¬P∨Q ⫤⊨ ¬(P∧ ¬Q)
∃x ¬F(x) ⫤⊨ ¬∀x F(x)
∀x (¬Square(x) ∨ Broken(x))
⫤⊨ ∃¬x (Square(x) ∧ ¬Broken(x))

∃x ¬Created(x) ⫤⊨ ¬∀x Created(x)

Hybrid example

Every object is either non-square or broken
∀x (¬Square(x) ∨ Broken(x))

¬¬P ⫤⊨ P	⫤⊨ ¬¬∀x (¬Square(x) ∨ Broken(x))
¬∀x F(x) ⫤⊨ ∃x¬F(x)	⫤⊨ ¬∃x ¬(¬Square(x) ∨ Broken(x))
¬(P∨Q) ⫤⊨ ¬P∧¬Q	⫤⊨ ¬∃x (¬¬Square(x) ∧ ¬Broken(x))
¬¬P ⫤⊨ P	⫤⊨ Nothing is square and non-broken **¬∃x (Square(x) ∧ ¬Broken(x))**

Truth-functional completeness (LPL §7.4)

How many truth-functions can be constructed using 2 sentence letters?

P	Q	P % Q
T	T	?
T	F	?
F	T	?
F	F	?

Each of these can be T or F

Df. 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that {¬, ∧, ∨} is truth-functionally complete:

P	Q	P → Q	
T	T	T	[P∧Q] ∨
T	F	F	
F	T	T	[¬P∧Q] ∨
F	F	T	[¬P∧ ¬Q]

[P∧Q] ∨ [¬P∧Q] ∨ [¬P∧ ¬Q]

Exercise assuming {¬,∨,∧} is truth-functionally complete, show that {¬,∨} is.

Variables

Names : a, b, c, …

Variables : x, y, z, w, …

Variables are for saying several things about one thing even without specifying which thing it is

Notation:
A ⊢ B means there is a proof of B using premises **A**
⊢ B means there is a proof of B using no premises
A ⊢ means the same as A ⊢ ⊥

A ⊨ B means B is a logical consequence of A
⊨ B means B is a tautology
A ⊨ means the same as **A** ⊨ ⊥
A $\models_{TT}$ B means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

Soundness: If **A** ⊢ B then **A** ⊨ B
I.e. if you can prove it in Fitch, it's valid

Completeness: If **A** $\models_{TT}$ B then **A** ⊢ B
I.e. if it's valid just in virtue of the meanings of truth functions, you can prove it in Fitch

Proofs about proofs

1. If $A \vdash B$ then $\vdash A \to B$

Proof Given a proof for $A \vdash B$ …

1	A
	…
n	B

… we can turn it into a proof for $\vdash A \to B$:

1	$\mid A$	
	$\mid$ …	
n	$\mid B$	
	$A \to B$	$\to$Intro: 1-n

2. If $\vdash A \to B$ then $A \vdash B$

3. If $A \vdash B$ then $A \vdash \neg\neg B$

Proof:

This is the original proof of $A \vdash B$	1	A	
	2	…	
	n	B	
Adding this turns it into a proof of $A \vdash \neg\neg B$	p	$\mid \neg B$	
	q	$\mid \bot$	$\bot$Intro:n,p
		$\neg\neg B$	$\neg$Intro:p-q

4. If $A \vdash C$ then $A \vdash B \to C$

5. If $A \vdash B$ and $A \vdash \neg C$ then $A \vdash \neg(B \to C)$

Someone is loved by everyone

$\exists x \forall y\, \mathrm{Loves}(y,x)$

Someone loves everyone

$\exists x \forall y\, \mathrm{Loves}(x,y)$

"Every time I go to the dentist, somebody dies."

$\forall t\, ((\mathrm{Time}(t) \wedge \mathrm{ToDentist}(a,t)) \to \exists x\, (\mathrm{Person}(x) \wedge \mathrm{DiesAt}(x,t)))$

Exercises 07

For your seventh seminar

Only for fast groups

11.10 (nb. especially no. 20)
11.13
13.43–45
13.49–50
14.2–3
14.10–11

Despite what book says, DON'T even THINK about using TAUT CON.