PH126 Logic I Lecture 15
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“Every time I go to the dentist, somebody dies.”
∀t ((Time(t) ∧ ToDentist(a,t)) → ∃x (Person(x) ∧ DiesAt(x,t)))

Quantifier equivalences (continued)

∃x ¬Created(x) ⫤⊨ ¬∀x Created(x)

Hybrid example

¬¬P ⫤⊨ P
¬∀x F(x) ⫤⊨ ∃x¬F(x)
¬(P→Q) ⫤⊨ P∧¬Q

Every square is broken
∀x (Square(x) → Broken(x))
⫤⊨ ¬¬∀x (Square(x) → Broken(x))
⫤⊨ ¬∃x ¬(Square(x) → Broken(x))
⫤⊨ ¬∃x (Square(x) ∧ ¬Broken(x))
⫤⊨ Nothing is square and non-broken
¬∃x (Square(x) ∧ ¬Broken(x))

Number

There are at least two squares:
∃x ∃y (Square(x) ∧ Square(y) ∧ ¬x=y)

At least two squares are broken:
∃x ∃y (Square(x) ∧ Broken(x) ∧ Square(y) ∧ Broken(y) ∧ ¬x=y)

There are at least three squares:
∃x ∃y ∃z (Square(x) ∧ Square(y) ∧ Square(z) ∧ ¬x=y ∧ ¬y=z ∧ ¬x=z)

There are at most two squares:
¬There are at least three squares
¬∃x ∃y ∃z (Square(x) ∧ Square(y) ∧ Square(z) ∧ ¬x=y ∧ ¬y=z ∧ ¬x=z)

There are exactly two squares:
There are at most two squares ∧ There are at least two squares

Number: Alternatives

There is at most one square:
∀x∀y((Square(x) ∧ Square(y)) → x=y)

There are at most two squares:
∀x∀y∀z((Square(x)∧Square(y)∧Square(z))
→
(x=y∨y=z∨x=z)
)

There is exactly one square:
∃x (Square(x) ∧ ∀y(Square(y) → x=y))

There are exactly two squares:
∃x∃y (Square(x) ∧ Square(y) ∧ ¬x=y ∧
∀z(Square(z) → (z=x ∨ z=y)))

Truth-functional completeness (LPL §7.4)

Preliminary: How many truth-functions can be constructed using 2 sentence letters?

P	Q	P % Q
T	T	?
T	F	?
F	T	?
F	F	?

Each of these can be T or F

Truth-functional completeness: For any truth function, we can easily find a logically equivalent one using only ¬, ∨, ∧

Better: ‘A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.’ (Bostock, *Intermediate L’gic* p. 45)

Illustration of how the proof that {¬, ∧, ∨ is truth-functionally complete}:

P	Q	P → Q	
T	T	T	[P∧Q] ∨
T	F	F	
F	T	T	[¬P∧Q] ∨
F	F	T	[¬P∧ ¬Q]

[P∧Q] ∨ [¬P∧Q] ∨ [¬P∧ ¬Q]

Exercise assuming {¬,∨,∧} is truth-functionally complete, show that {¬,∨} is?