

Quantifier Recap

Everything is broken
 $\forall x \text{ Broken}(x)$

All my things are broken
 $\forall x (\text{BelongsToMe}(x) \rightarrow \text{Broken}(x))$

Something is broken
 $\exists x \text{ Broken}(x)$

Something of mine is broken
 $\exists x (\text{BelongsToMe}(x) \wedge \text{Broken}(x))$

How to determine truth of a sentence with an existential quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

How to determine truth of a sentence with a universal quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL OF the new sentences are true, so is the original.

Multiple quantifiers

There is a store for everything

$\exists y \forall x \text{ StoreFor}(y, x)$

$\forall y \exists x \text{ StoreFor}(x, y)$

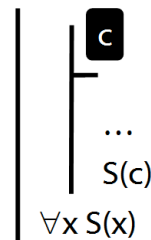
Other sentences to translate:

Wikipedia has an article about everything

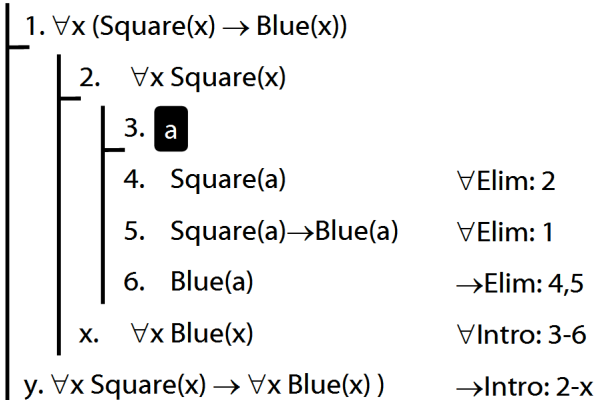
Everyone hurts someone they love

Someone hurts everyone she loves

\forall Intro



\forall Intro Example proof



Exercise

If the following is valid, give a formal proof. If it is not valid, give a counterexample.

$\forall x \forall y \forall z ([R(x, y) \wedge R(y, z)] \rightarrow R(x, z))$

$\forall x \forall y [R(x, y) \rightarrow R(y, x)]$

$\forall x R(x, x)$

"Every time I go to the dentist, somebody dies."

$\forall t ((\text{Time}(t) \wedge \text{ToDentist}(a, t)) \rightarrow \exists x (\text{Person}(x) \wedge \text{DiesAt}(x, t)))$

Truth-functional completeness (LPL §7.4)

How many truth-functions can be constructed using 2 sentence letters?

P	Q	P % Q
T	T	?
T	F	?
F	T	?
F	F	?

Each of these can be T or F

Df. 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that $\{\neg, \wedge, \vee\}$ is truth-functionally complete:

P	Q	$P \rightarrow Q$	
T	T	T	$[P \wedge Q] \vee$
T	F	F	
F	T	T	$[\neg P \wedge Q] \vee$
F	F	T	$[\neg P \wedge \neg Q]$

$$[P \wedge Q] \vee [\neg P \wedge Q] \vee [\neg P \wedge \neg Q]$$

Exercise assuming $\{\neg, \vee, \wedge\}$ is truth-functionally complete, show that $\{\neg, \vee\}$ is.

Quantifier equivalences

$$P \rightarrow Q \quad \equiv \quad \vdash \neg Q \rightarrow \neg P$$

$$\forall x (\text{Square}(x) \rightarrow \text{Broken}(x))$$

$$\equiv \quad \vdash \forall x (\neg \text{Broken}(x) \rightarrow \neg \text{Square}(x))$$

$$P \rightarrow Q \quad \equiv \quad \vdash \neg P \vee Q$$

$$\forall x (\text{Square}(x) \rightarrow \text{Broken}(x))$$

$$\equiv \quad \vdash \forall x (\neg \text{Square}(x) \vee \text{Broken}(x))$$

$$\neg P \vee Q \equiv \vdash \neg(P \wedge \neg Q)$$

$$\exists x \neg F(x) \equiv \vdash \neg \forall x F(x)$$

$$\forall x (\neg \text{Square}(x) \vee \text{Broken}(x))$$

$$\equiv \quad \vdash \exists \neg x (\text{Square}(x) \wedge \neg \text{Broken}(x))$$

$$\exists x \neg \text{Created}(x) \equiv \vdash \neg \forall x \text{Created}(x)$$

Exercises 07

For your seventh seminar

Only for fast groups

11.10 (nb. especially no. 20)

11.13

13.43–45

13.49–50

14.2–3

14.10–11

Despite what book says, DON'T even THINK about using TAUT CON.