PH126 Logic I Fast Lecture 5

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How to determine the truth of sentences involving ∀

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.

Scope

Underlining shows the scope of the quantifiers

"All squares are blue"

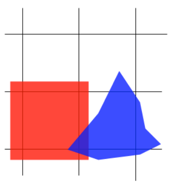
∀x (Square(x) → Blue(x))

"If everything is square, everything is blue"

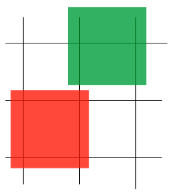
∀x Square(x) → ∀x Blue(x)

Multiple quantifers

1 **∃x ∃y Above(x,y)**
"Something is above something"



1 is FALSE



1 is TRUE

Quantifiers bind variables

Read §9.2 of Barwise & Etchemendy

The Soundness & Completeness Theorems

'**A** ├ B' means there is a proof of B using premises **A**

'├ B' means there is a proof of B using no premises

'**A** ╞ B' means B is a logical consequence of A

'╞ B' means B is a tautology

'**A** $\models_T$ B' means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

Soundness: If **A** ├ B then **A** ╞ B

I.e. if you can prove it in Fitch, it's valid

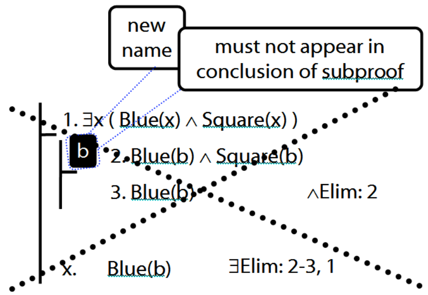
Completeness: If **A** $\models_T$ B then **A** ├ B

I.e. if it's valid just in virtue of the meanings

Proof example: ∃Elim

1.	∃x (Blue(x) ∧ Square(x))	
b 2.	Blue(b) ∧ Square(b)	
3.	Blue(b)	∧Elim: 2
4.	∃x Blue(x)	∃Intro: 3
x.	∃x Blue(x)	∃Elim: 2-4, 1

Invalid use of ∃Elim



Two objects are broken

∃x ∃y (Broken(x)∧Broken(y)∧¬(x=y))

To translate sentences involving number into FOL, use identity.

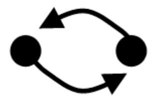
Reflexive, symmetric and transitive

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)

∀x R(x,x)

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y).)

∀x∀y (R(x,y) → R(y,x))



A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is *not* transitive)

∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))

Exercises 06

For your sixth seminar
Only for fast groups

11.2, 11.4

11.8, 11.9, *11.11

13.12, 13.14, 13.16 (quantifier proofs)

13.28

Despite what book says, DON'T even THINK about using TAUT CON.