PH126 Logic I Lecture 8

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Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	A → B
T	T	
T	F	
F	T	
F	F	

'If' and '→' do not match

| ¬A | Marnie will not miss her train |
| A → B | If Marnie misses her train, she will arrive on time. |

'If' and '→' do match

| ¬A ∨ B | America does not exist ∨ Baudrillard is wrong |
| If A, B | If America exists, Baudrillard is wrong |

| If A, B | If you love logic, things will fall into place |
| ¬(A ∧ ¬B) | Not both: you take logic and things don't fall into place |

How to determine the truth of sentences involving ∃

1. Give every object a name.

2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name

3. If ANY OF the new sentences are true, so is the original.

What not to confuse

∃x (Square(x) ∧ Blue(x)) vs.
∃x Square(x) ∧ ∃x Blue(x)

∀x (Square(x) → Blue(x)) vs.
∀x Square(x) → ∀x Blue(x)

¬(P∨Q) vs. ¬P∨¬Q

¬(P∧Q) vs. ¬P∧¬Q

¬ (P→Q) vs. P→¬Q

Scope

In P∧(Q∨R), the scope of ∧ is P∧(Q∨R)

In P∧(Q∨R), the scope of ∨ is (Q∨R)

In (P∧Q)∨R, the scope of ∧ is (P∧Q)

In (P∧Q)∨R, the scope of ∨ is (P∧Q)∨R

The scope of a connective is the smallest constituent expression which contains that connective.

First quantifier rule of proof: ∀Elim

∀Elim

| ∀x S(x)
| …
| S(c)

Proof example: ∀Elim, ∃Intro

1.	∀x (Puf(x) → YelBk(x))	
2.	Puf(a)	
3.	Puf(a) → YelBk(a)	∀Elim:1
4.	YelBk(a)	→Elim: 3,2
5.	∃x YelBk(x)	∃Intro: 4

Fubar rules

∧Fubar:

| *
| …
| * ∧ #

Q1. What would be wrong with adding ∧Fubar to Fitch?

Q2. What would be wrong with having ∧Fubar in any system of proof?

Proof Example using negation-Intro

1. P

5. $\neg\neg P$

More proof examples

1. $A \wedge B$

$\neg(\neg A \vee \neg B)$

1. $\neg(P \vee Q)$

10. $\neg P \wedge \neg Q$

1. $\neg P \wedge \neg Q$

10. $\neg(P \vee Q)$

1. $\neg P \vee R$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \rightarrow R$

Exercises 04

For your fifth seminar

Not for fast groups

(Bit more than usual this week because reading week gives you two weeks to complete these)

3.14–15 (counterexamples)
6.8 (proof)
6.24–7 (proof)
7.1–2, *7.3–6 (truthtables)
8.1 (conditionals; yes/no answers are ok)
8.17–19, *8.20–23
9.1 odd nos only (quantifiers)
9.2 even nos only (quantifiers)
9.4–5, 9.8–10, 9.12 (quantifiers)