PH126 Logic I Fast Lecture 4
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Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	A → B
T	T	
T	F	
F	T	
F	F	

'If' and →

| ¬A | There is no cheese in my fridge
| A → B | If there is cheese in my fridge there is chocolate in my pocket.

| ¬A ∨ B | America does not exist ∨ Baudrillard is wrong
| A → B | If America exists, Baudrillard is wrong

| A → B | If you love logic, things will fall into place
| ¬(A ∧ ¬B) | Not both: you take logic and things don't fall into place

What does ∃ mean?

We give the meaning of ∃ by specifying what it takes for a sentence containing ∃ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

∃ and ∧ work together

Some object is F and G: ∃x (F(x) ∧ G(x))

Some square is blue: ∃x (Square(x) ∧ Blue(x))

∀ and → work together

All my things are broken:
∀x (Belongs(a,x) → Broken(x))

All squares are blue:
∀x (Square(x) → Blue(x))

Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

φ ⫤ ⊨ ψ means φ and ψ are logically equivalent.

Terminology: let χ[ψ/φ] be the result of replacing φ with ψ

E.g. (Q¬¬∨P) [P/¬¬P] = Q∨P

Theorem: If φ ⫤ ⊨ ψ then χ ⫤ ⊨ χ[ψ/φ]

How to determine the truth of sentences involving ∀

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.

Scope

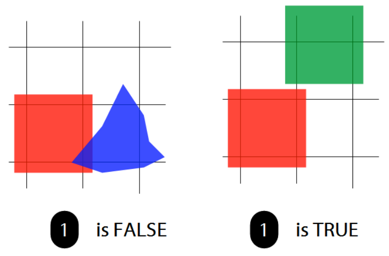
Underlining shows the scope of the quantifiers

"All squares are blue"
∀x (Square(x) → Blue(x))

"If everything is square, everything is blue"
∀x Square(x) → ∀x Blue(x)

Multiple quantifers

1 **∃x ∃y Above(x,y)**
"Something is above something"



Quantifiers bind variables

Read §9.2 of Barwise & Etchemendy

Proof example: ∃Elim

1. ∃x (Blue(x) ∧ Square(x))

  $\boxed{b}$ 2. Blue(b) ∧ Square(b)

  3. Blue(b)   ∧Elim: 2

  4. ∃x Blue(x)   ∃Intro: 3

x. ∃x Blue(x)   ∃Elim: 2-4, 1

∀Intro

  $\boxed{c}$

  …

  S(c)

∀x S(x)

Existential Elimination (∃ Elim)

∃x S(x)

⋮

  $\boxed{c}$ S(c)

  ⋮

  Q

▷ Q

where c does not occur outside the subproof where it is introduced.

Invalid use of ∃Elim

new name

must not appear in conclusion of subproof

1. ∃x (Blue(x) ∧ Square(x))

  $\boxed{b}$ 2. Blue(b) ∧ Square(b)

  3. Blue(b)   ∧Elim: 2

x. Blue(b)   ∃Elim: 2-3, 1

∀Intro *Example proof*

1. ∀x (Square(x) → Blue(x))

  2. ∀x Square(x)

    3. $\boxed{a}$

    4. Square(a)   ∀Elim: 2

    5. Square(a)→Blue(a)   ∀Elim: 1

    6. Blue(a)   →Elim: 4,5

  x. ∀x Blue(x)   ∀Intro: 3-6

y. ∀x Square(x) → ∀x Blue(x))   →Intro: 2-x

Intuitve summary of quantifier rules

∀Elim
If it's true of everything it's true of Baudrillard

∃Intro
If it's true of Baudrillard it's true of something

∃Elim
If it's true of something and Q follows no matter which something it is, then Q

∀Intro
If it's true of an arbitrary thing, then it's true of everything.

Two objects are broken

∃x ∃y (Broken(x)∧Broken(y)∧¬(x=y))

To translate sentences involving number into FOL, use identity.

Why is this proof incorrect?

1. ∀x Square(x) → ∀x Blue(x)

  2. $\boxed{b}$

  3. Square(b) → Blue(g)   ∀Elim: 1

x. ∀x (Square(x) → Blue(x))   ∀Intro: 2-3

Exercises 05

For your fifth seminar
Only for fast groups

9.12–13
9.18–19 (trans.)
10.20, *10.24–7
10.28–9
13.2–3, 13.8–9 (quantifier proofs)
13.11, 13.13, 13.15
NEVER USE TAUT CON.