

Proof example with \rightarrow

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|--|----------------------|
| | 1. $P \rightarrow Q$ |
| | 2. $\neg Q$ |
| | |
| | 6. $\neg P$ |

How to determine the truth of sentences involving \forall

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.

Quantifiers

Everything is broken: $\forall x \text{ Broken}(x)$

Something is broken: $\exists x \text{ Broken}(x)$

What does \exists mean? We give the meaning of \exists by specifying what it takes for a sentence containing \exists to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

| \forall Elim | \exists Intro |
|------------------|------------------|
| $\forall x S(x)$ | $S(a)$ |
| ... | ... |
| $S(c)$ | $\exists x S(x)$ |

First quantifier rule of proof: \forall Elim

| \forall Elim |
|------------------|
| $\forall x S(x)$ |
| ... |
| $S(c)$ |

Proof example: \forall Elim, \exists Intro

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|--|--|-------------------------|
| | 1. $\forall x (Puf(x) \rightarrow YelBk(x))$ | |
| | 2. $Puf(a)$ | |
| | 3. $Puf(a) \rightarrow YelBk(a)$ | \forall Elim:1 |
| | 4. $YelBk(a)$ | \rightarrow Elim: 3,2 |
| | 5. $\exists x YelBk(x)$ | \exists Intro: 4 |

Exercises 04

For your fifth seminar

Not for fast groups

(Bit more than usual this week because reading week gives you two weeks to complete these)

3.14–15 (counterexamples)

6.8 (proof)

6.24–7 (proof)

7.1–2, *7.3–6 (truthtables)

8.1 (conditionals; yes/no answers are ok)

8.17–19, *8.20–23

9.1 odd nos only (quantifiers)

9.2 even nos only (quantifiers)

9.4–5, 9.8–10, 9.12 (quantifiers)