PH126 Logic I Lecture 9

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Proof example

1.	¬P∨R
2.	-
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	P→R

The Soundness & Completeness Theorems

- 'A | B' means there is a proof of B using premises A
- ' \vdash B' means there is a proof of B using no premises
- 'A | B' means B is a logical consequence of A
- ' ⊨ B' means B is a tautology
- 'A \models_T B' means B is a logical consequence of A just in virtue of the meanings of truth-functions (the textbook LPL calls this 'tautological consequence')

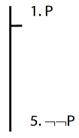
Soundness: If $A \models B$ then $A \models B$

I.e. if you can prove it in Fitch, it's valid

Completeness: If $\mathbf{A} \models_T \mathbf{B}$ then $\mathbf{A} \models \mathbf{B}$

I.e. if it's valid just in virtue of the meanings of truth functions, you can prove it in Fitch

Proof Example using negation-Intro



'If' and '→' do not match

 $\begin{array}{|c|c|c|c|c|}\hline \neg A & \text{Marnie will not miss her train} \\ A \rightarrow B & \text{If Marnie misses her train, she will arrive on time.} \end{array}$

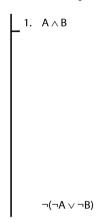
'If' and '→' do match

¬A∨B America does not exist ∨ Baudrillard is wrong

If A, B If America exists, Baudrillard is wrong

If A, B If you love logic, things will fall into place $\neg (A \land \neg B) \text{ Not both: you take logic and things don't fall into place}$

Proof example



Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

If * and # are logically equivalent, then you take a complex formula and replace * with # without changing the truth table of the complex formula.

Example. Because \neg (R $\lor \neg$ P) is logically equivalent to \neg R \land P, it follows that:

 $P \lor \neg (Q \land \neg (R \lor \neg P))$ is logically equivalent to $P \lor \neg (Q \land (\neg R \land P))$

And because \neg ($Q \land (\neg R \land P)$) is logically equivalent to $\neg Q \lor (R \lor \neg P)$, it follows that $P \lor \neg (Q \land (\neg R \land P))$ is logically equivalent to $P \lor (\neg Q \lor (R \lor \neg P))$.

Fubar rules

∧Fubar:

* ... *^#

Q1. What would be wrong with adding ∧Fubar to Fitch?

Q2. What would be wrong with having Λ Fubar in any system of proof?

Scope

In $P \land (Q \lor R)$, the scope of \land is $P \land (Q \lor R)$ In $P \land (Q \lor R)$, the scope of \lor is $(Q \lor R)$ In $(P \land Q) \lor R$, the scope of \land is $(P \land Q)$ In $(P \land Q) \lor R$, the scope of \lor is $(P \land Q) \lor R$

The scope of a connective is the smallest constituent expression which contains that connective.

Exercises 05

For your sixth seminar
Not for fast groups

A. From the LPL textbook:

7.9 (truth functions)

6.17-20 (proof)

6.33, 6.40

DO NOT USE TAUT CON. EVER.

8.24-25 (proofs/counterexamples)

12.4–5, *12.6–7 (counterexamples)

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain: people and actions

D(x) : x is desirable

V(x) : x is virtuous

A(x): x is an action

H(x) : x is a person

P(x,y): x performed y

i. $\forall x [D(x) \rightarrow V(x)]$

ii. $\forall x [[A(x) \land D(x)] \rightarrow V(x)]]$

iii. $\exists x [A(x) \land \neg [D(x) \rightarrow V(x)]]$

*iv. $\exists x \ \forall y \ [[[H(x) \land A(y)] \land P(x,y)] \rightarrow V(y)]$

**v. $\neg \exists x [\exists y [H(x) \land P(x,y) \land A(y) \land \neg V(y)] \land \neg \exists z [P(x,z) \land A(z) \land V(z)]]$