

Wrong proofs

Step 7 of this proof is wrong. Why?

		R	S	$R \vee S$	$R \wedge S$
T	1. $R \vee S$	T	F	T	F
	2. R				
	3. $S \vee R$			\vee Intro: 2	
	4. S				
	5. $S \vee R$			\vee Intro: 4	
	6. $S \vee R$			\vee Elim: 1,2-3,4-5	
F	7. $R \wedge S$			\wedge Intro: 2,4	

Which step of this proof is wrong? Why?

1. $\neg\neg(\neg A \wedge \neg\neg A)$	
2. $(\neg A \wedge \neg\neg A)$	\neg Elim: 1
3. $(\neg A \wedge A)$	\neg Elim: 2

Scope

In $P \wedge (Q \vee R)$, the scope of \wedge is $P \wedge (Q \vee R)$

In $P \wedge (Q \vee R)$, the scope of \vee is $(Q \vee R)$

In $(P \wedge Q) \vee R$, the scope of \wedge is $(P \wedge Q)$

In $(P \wedge Q) \vee R$, the scope of \vee is $(P \wedge Q) \vee R$

The scope of a connective is the smallest constituent expression which contains that connective.

The Syntax of FOL

We define what counts as a sentence of FOL using rules. E.g.:

3. P, Q, R, ... are sentences

4. If * is a sentence, then $\neg*$ is a sentence

1. If * and # are sentences, then so is $(* \wedge \#)$

2. If * and # are sentences, then so is $(* \vee \#)$

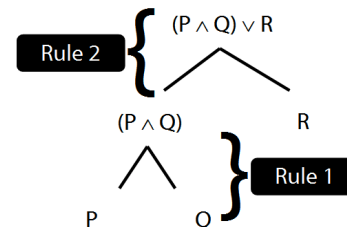
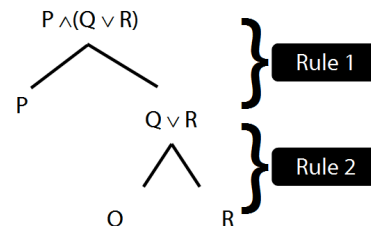
So:

a. P is a sentence // rule 3

b. $\neg P$ is a sentence // rule 4, a

c. $(\neg P \wedge Q)$ is a sentence // rule 1, b, a

Notes: (1) There is no structural ambiguity in FOL because these rules are formulated to ensure that for any FOL sentence, there is exactly one way of constructing it. (2) We used the notion of a sentence in the rules that define what a sentence is.



\neg Intro proof example

1. $A \wedge B$
$\neg(\neg A \vee \neg B)$

Exercises 03

For your fourth seminar

Not for fast groups

4.15–18 (truth tables)

5.1–6 (inference patterns)

*5.18 (counterexample)

6.2–6 (proofs)

6.7, 6.9–10, *6.11–12 (proofs)

Short of time? Skip the exercises marked *