**PH126 Logic I** Lecture 12

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### Quantifier Recap

Everything is broken

$\forall x\ \mathbf{Broken(x)}$

All my things are broken

$\forall x\ (\mathbf{BelongsToMe(x)} \rightarrow \mathbf{Broken(x)})$

Something is broken

$\exists x\ \mathbf{Broken(x)}$

Something of mine is broken

$\exists x\ (\mathbf{BelongsToMe(x)} \land \mathbf{Broken(x)})$

### How to determine truth of a sentence with an existential quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

### How to determine truth of a sentence with a universal quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL OF the new sentences are true, so is the original.

### Multiple quantifiers

'There is a store for everything'

$\exists y \forall x\ StoreFor(y,x)$

$\forall y \exists x\ StoreFor(x,y)$

Other sentences to translate:

'Wikipedia has an article about everything'

'Everyone hurts someone they love'

'Someone hurts everyone she loves'

$\forall$Intro

| **c**
| ...
| S(c)
$\forall x\ S(x)$

### $\forall$Intro *Example proof*

| | | |
|---|---|---|
| 1. | $\forall x\ (Square(x) \rightarrow Blue(x))$ | |
| 2. | $\forall x\ Square(x)$ | |
| 3. | **a** | |
| 4. | Square(a) | $\forall$Elim: 2 |
| 5. | $Square(a) \rightarrow Blue(a)$ | $\forall$Elim: 1 |
| 6. | Blue(a) | $\rightarrow$Elim: 4,5 |
| x. | $\forall x\ Blue(x)$ | $\forall$Intro: 3-6 |
| y. | $\forall x\ Square(x) \rightarrow \forall x\ Blue(x)$ | $\rightarrow$Intro: 2-x |

*Why is this proof incorrect?*

| | | |
|---|---|---|
| 1. | $\forall x\ Square(x) \rightarrow \forall x\ Blue(x)$ | |
| 2. | **b** | |
| 3. | $Square(b) \rightarrow Blue(g)$ | $\forall$Elim: 1 |
| x. | $\forall x\ (Square(x) \rightarrow Blue(x))$ | $\forall$Intro: 2-3 |

### Intuitve summary of quantifier rules

**$\forall$Elim**
If it's true of everything it's true of Baudrillard

**$\exists$Intro**
If it's true of Baudrillard it's true of something

**$\exists$Elim**
If it's true of something and Q follows no matter which something it is, then Q

**$\forall$Intro**
If it's true of an arbitrary thing, then it's true of everything.

Notation:

$\mathbf{A} \vdash B$ means there is a proof of B using premises **A**

$\vdash B$ means there is a proof of B using no premises

$\mathbf{A} \vdash$ means the same as $A \vdash \bot$

$\mathbf{A} \models B$ means B is a logical consequence of A

$\models B$ means B is a tautology

$\mathbf{A} \models$ means the same as $\mathbf{A} \models \bot$

$\mathbf{A} \models_{TT} B$ means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

**Proofs about proofs**

1. If A $\vdash$ B then $\vdash$ A$\to$B

*Proof* Given a proof for A $\vdash$ B …

| | | |
|---|---|---|
| 1 | A | |
| | … | |
| n | B | |

… we can turn it into a proof for $\vdash$ A$\to$B:

| | | |
|---|---|---|
| 1 | A | |
| | … | |
| n | B | |
| | A$\to$B | $\to$Intro: 1-n |

2. If $\vdash$ A$\to$B then A $\vdash$ B

3. If A $\vdash$ B then A $\vdash$ ¬¬B

*Proof*:

| | | | |
|---|---|---|---|
| This is the original proof of A $\vdash$ B | 1 | A | |
| | 2 | … | |
| | n | B | |
| Adding this turns it into a proof of A $\vdash$ ¬¬B | p | ¬B | |
| | q | $\perp$ | $\perp$Intro:n,p |
| | | ¬¬B | ¬Intro:p-q |

4. If A $\vdash$ C then A $\vdash$ B$\to$C

5. If A $\vdash$ B and A $\vdash$ ¬C then A $\vdash$ ¬(B$\to$C)

**Exercises 06**

*For your seventh seminar*

*Not for fast groups*

A. From the LPL textbook:

9.15–17, *9.18–19 (trans.)

10.20, 10.22

11.2 (multiple quantifiers)

12.9–10

Short of time? Skip the exercises marked *

B. Each line of the table describes a combination of properties. E.g. line 1 specifies a relation that is reflexive, symmetric and transitive, line 2 specifies a relation that is reflexive, symmetric and not transitive. Complete the table by adding an example relation on each line.

| | Example | Reflexive | Symmetric | Transitive |
|---|---|---|---|---|
| 1 | = (identity) | Y | Y | Y |
| 2 | | Y | Y | N |
| 3 | | Y | N | Y |
| 4 | | Y | N | N |
| 5 | | N | Y | Y |
| 6 | | N | Y | N |
| 7 | | N | N | Y |
| 8 | | N | N | N |