PH126 Logic I Fast Lecture 4
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Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	$A \to B$
T	T	
T	F	
F	T	
F	F	

'If' and →

$\neg A$ — There is no cheese in my fridge
$A \to B$ — If there is cheese in my fridge there is chocolate in my pocket.

$\neg A \lor B$ — America does not exist ∨ Baudrillard is wrong
$A \to B$ — If America exists, Baudrillard is wrong

$A \to B$ — If you love logic, things will fall into place
$\neg(A \land \neg B)$ — Not both: you take logic and things don't fall into place

Quantifiers

Everything is broken: $\forall x\ Broken(x)$

Something is broken: $\exists x\ Broken(x)$

What does ∃ mean? We give the meaning of ∃ by specifying what it takes for a sentence containing ∃ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

∃ and ∧ work together

Some object is F and G: $\exists x\ (\ F(x) \land G(x)\ )$

I met a philosopher:

Some object is a philosopher and I met it:

$\exists x\ (\ Philr(x) \land Met(a,x)\ )$

Some square is blue: $\exists x\ (Square(x) \land Blue(x))$

Rooney scored a goal:

$\exists x\ (Goal(x) \land Scored(a,x))$

Obama won an election:

$\exists x\ (Election(x) \land Won(a,x))$

∀ and → work together

All my things are broken:
$\forall x\ (Belongs(a,x) \to Broken(x))$

Everything I do turns bad:
$\forall x\ (\ Does(a,x) \to TurnsBad(x)\ )$

All squares are blue:
$\forall x\ (\ Square(x) \to Blue(x)\ )$

Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

$\varphi \dashv\vDash \psi$ means φ and ψ are logically equivalent.

Terminology: let $\chi[\psi/\varphi]$ be the result of replacing φ with ψ

E.g. $(Q\neg\neg\lor P)\ [P/\neg\neg P] = Q\lor P$

Theorem: If $\varphi \dashv\vDash \psi$ then $\chi \dashv\vDash \chi[\psi/\varphi]$

Exercises 05
For your fifth seminar
Only for fast groups

A. From the LPL textbook:

9.12–13
9.18–19 (trans.)
10.20, *10.24–7
10.28–9
13.2–3, 13.8–9 (quantifier proofs)
13.11, 13.13, 13.15
NEVER USE TAUT CON.

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain : people and actions
$D(x)$: x is desirable
$V(x)$: x is virtuous
$A(x)$: x is an action
$H(x)$: x is a person
$P(x,y)$: x performed y

i. $\forall x\, [\, D(x) \rightarrow V(x)\,]$

ii. $\forall x\, [\, [A(x) \wedge D(x)\,] \rightarrow V(x)\,]\,]$

iii. $\exists x\, [\, A(x) \wedge \neg[\, D(x) \rightarrow V(x)\,]\,]$

*iv. $\exists x\, \forall y\, [[[H(x) \wedge A(y)] \wedge P(x,y)] \rightarrow V(y)]$

**v. $\neg\exists x\, [\exists y\, [H(x) \wedge P(x,y) \wedge A(y) \wedge \neg V(y)] \wedge \neg\exists z[P(x,z) \wedge A(z) \wedge V(z)]]$