PH126 Logic I Lecture 15
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Quantifiers and identity

Ahura Mazda created everything:
$\forall x\ Created(a,x)$

Ahura Mazda created everything apart from himself: $\forall x\ (\neg(x=a) \to Created(a,x))$

Quantifier proof exercise

1. | $\forall y\ ((\neg y=a) \to Created(a,y))$

10 | $\forall y\ ((\neg\exists x\ Created(x,y)) \to y=a)$

Number

There are at least two squares:
$\exists x\ \exists y\ (Square(x) \wedge Square(y) \wedge \neg x=y)$

At least two squares are broken:
$\exists x\ \exists y\ (Square(x) \wedge Broken(x) \wedge Square(y) \wedge Broken(y) \wedge \neg x=y)$

There are at least three squares:
$\exists x\ \exists y\ \exists z\ (Square(x) \wedge Square(y) \wedge Square(z) \wedge \neg x=y \wedge \neg y=z \wedge \neg x=z)$

There are at most two squares:
¬There are at least three squares
$\neg\exists x\ \exists y\ \exists z\ (Square(x) \wedge Square(y) \wedge Square(z) \wedge \neg x=y \wedge \neg y=z \wedge \neg x=z)$

There are exactly two squares:
There are at most two squares ∧ There are at least two squares

Number: Alternatives

There is at most one square:
$\forall x \forall y((Square(x) \wedge Square(y)) \to x=y)$

There are at most two squares:
$\forall x \forall y \forall z((Square(x) \wedge Square(y) \wedge Square(z)) \to (x=y \vee y=z \vee x=z))$

There is exactly one square:
$\exists x\ (Square(x) \wedge \forall y(Square(y) \to x=y))$

There are exactly two squares:
$\exists x \exists y\ (Square(x) \wedge Square(y) \wedge \neg x=y \wedge \forall z(Square(z) \to (z=x \vee z=y)))$

$\neg\neg P \dashv\vDash P$

$\neg\forall x\ F(x) \dashv\vDash \exists x \neg F(x)$

$\neg(P \to Q) \dashv\vDash P \wedge \neg Q$

Every square is broken
$\forall x\ (Square(x) \to Broken(x))$

$\dashv\vDash\ \neg\neg\forall x\ (Square(x) \to Broken(x))$

$\dashv\vDash\ \neg\exists x\ \neg(Square(x) \to Broken(x))$

$\dashv\vDash\ \neg\exists x\ (Square(x) \wedge \neg Broken(x))$

$\dashv\vDash$ Nothing is square and non-broken
$\neg\exists x\ (Square(x) \wedge \neg Broken(x))$

The

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

There is exactly one square and it is broken

There is at most one square and there is at least one square and it is broken

There is at most one square and there is at least one square and all squares are broken

$\neg\ \exists x\ \exists y\ (Square(x) \wedge Square(y) \wedge \neg x=y)$
$\wedge\ \exists x\ Square(x)$
$\wedge\ \forall x\ (Square(x) \to Broken(x))$

Which shorter sentences are equivalent to this?

Exercises 08

There isn't a seminar for these; you should attempt them over the vacation.

Not for fast groups

11.11, 11.13, *11.10
13.6–7, *13.8–9
13.19, 13.23–27 (*13.28–31)
(*13. 33, 35, 37, 39)
13.43–45, 13.49–50, *13.51–52
14.1, 14.2, 14.3 (*14.4–5) [trans]
*14.10–12, *14.13 [proofs]
*14.26, 14.28 [trans]

Despite what book says, DON'T even THINK about using TAUT CON.

Short of time? Skip the exercises marked *