PH126 Logic I Lecture 11
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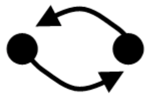
Reflexive, symmetric and transitive

A *reflexive* relation is one that everything bears to itself. (E.g. everything is the SameShape as itself. E.g. of *non*-reflexive: not everything is LeftOf itself).

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y) is symmetric, LeftOf(x,y) is *not* symmetric.)



A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is *not* transitive)

To show that a particular relation lacks one of these properties, we produce and argument and a counterexample.

E.g. Show that NotAdjacent is not transitive …

If NotAdjacent were transitive, the following argument would be logically valid:

1. NotAdjacent(a, b)
2. NotAdjacent(b, c)

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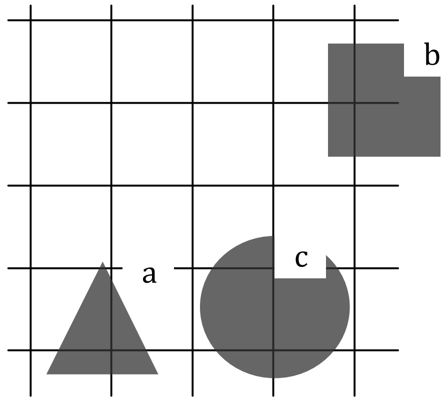
3. NotAdjacent(a, c)

A counterexample to this argument:



Artificial relation examples

EqualToOrLeftOf(x, y) iff
x = y or LeftOf(x, y)

EqualToOrAdjacent(x, y) iff
x=y or Adjacent(x, y)

JohnOrAyesha(x, y) iff
x = John and y = Ayesha
or x = Ayesha and y = John

JohnToAyesha(x, y) iff
x = John and y = Ayesha

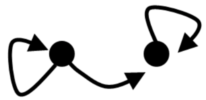
Quantifiers and relations

reflexive $\forall x\, R(x,x)$

symmetric $\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

transitive
$\forall x \forall y \forall z\, (\, [\, R(x,y) \wedge R(y,z)\,] \rightarrow R(x,z)\,)$

A transitive, reflexive and non-symmetric relation:



That relation is a counterexample to this argument:

$\forall x\, R(x,x)$
$\forall x \forall y \forall z\, (\, [\, R(x,y) \wedge R(y,z)\,] \rightarrow R(x,z)\,)$

$\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

Formal statement of counterexample:
Domain: {a,b}
R : { <a,a>, <a,b>, <b,b> }

Ex. Is this argument valid?

$\forall x \forall y \forall z\, (\, [\, R(x,y) \wedge R(y,z)\,] \rightarrow R(x,z)\,)$
$\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

$\forall x\, R(x,x)$