

PH133 Logic Lecture 5

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Truth table for \rightarrow

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for \rightarrow is fixed if we accept \rightarrow Elim and \rightarrow Intro.

How do the rules of proof for \rightarrow fix its truth table?

A	B	$A \rightarrow B$
T	T	
T	F	
F	T	
F	F	

'If' and ' \rightarrow ' do not match

	$\neg A$	Marnie will not miss her train
	$A \rightarrow B$	If Marnie misses her train, she will arrive on time.

'If' and ' \rightarrow ' do match

	$\neg A \vee B$	America does not exist \vee Baudrillard is wrong
	If A, B	If America exists, Baudrillard is wrong
	If A, B	If you love logic, things will fall into place
	$\neg(A \wedge \neg B)$	Not both: you take logic and things don't fall into place

Quantifiers

Everything is broken: $\forall x \text{ Broken}(x)$

Something is broken: $\exists x \text{ Broken}(x)$

What does \exists mean? We give the meaning of \exists by specifying what it takes for a sentence containing \exists to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY of the new sentences are true, so is the original.

Translations

Some persuasive and useful arguments are not valid.

$\exists x(\text{Persuasive}(x) \wedge \text{Useful}(x) \wedge \text{Argument}(x) \wedge \neg \text{Valid}(x))$

All discordians weep.

$\forall x(\text{Dscrdn}(x) \rightarrow \text{Wps}(x))$

All French discordians weep.

$\forall x((\text{Frnch}(x) \wedge \text{Dscrdn}(x)) \rightarrow \text{Wps}(x))$

All French discordians weep and wail.

$\forall x((\text{Frnch}(x) \wedge \text{Dscrdn}(x)) \rightarrow (\text{Wps}(x) \wedge \text{Wls}(x)))$

All French discordians weep and wail except Gillian Deleude.

$\forall x((\text{Frnch}(x) \wedge \text{Dscrdn}(x) \wedge \neg x=a) \rightarrow (\text{Wps}(x) \wedge \text{Wls}(x)))$

Fubar rules*

\wedge Fubar:

| *
| ...
| * \wedge #

Q1. What would be wrong with adding \wedge Fubar to Fitch?

Q2. What would be wrong with having \wedge Fubar in any system of proof?

Tonk

*Intro

*Elim

	P_i		$P_1 * P_2$

	$P_1 * P_2$		P_i

[*I made up the Fubar rules. You don't need to know them, and you won't find them in the textbook. They're only there as an illustration.]