PH126 Logic I · Fast Lecture 3
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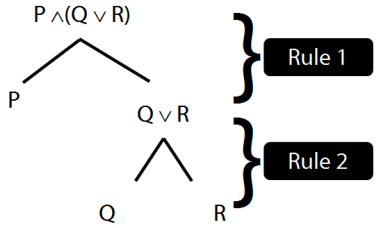
The Syntax of FOL

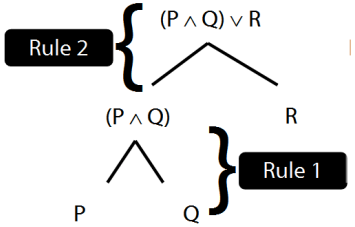
We define what counts as a sentence of FOL using rules. E.g.:

1. If * and # are sentences, then so is(* ∧ #)
2. If * and # are sentences, then so is (* ∨ #)
3. P, Q, R, … are sentences
4. If * is a sentence, then ¬* is a sentence

So:

a. P is a sentence // rule 3
b. ¬P is a sentence // rule 4, a
c. (¬P ∧ Q) is a sentence // rule 1, b, a





Rule of Proof: ∨Elim

∨Elim:

P1 ∨ P2
- P1 … Q
- P2 … Q

… Q

To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

Example proof with ∨Elim

1. R ∨ S
2. R
3. S ∨ R — ∨Intro: 2
4. S
5. S ∨ R — ∨Intro: 4
6. S ∨ R — ∨Elim: 1,2-3,4-5

⊥, ¬

P	¬P	⊥	P∧¬P
T	F	F	F
F	T	F	F

Proof example with ⊥Elim and ⊥Intro

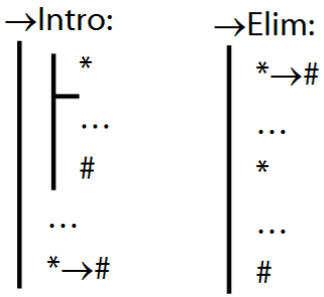
1. R ∧¬R
2. R — ∧Elim: 1
3. ¬R — ∧Elim: 1
4. ⊥ — ⊥Intro: 2,3
5. P — ⊥Elim: 4

Proof example with ¬Intro

1. A ∧ B

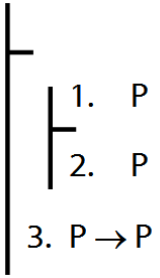
¬(¬A ∨ ¬B)

Rules of Proof for →



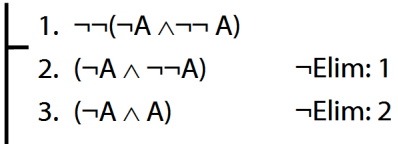
Proof example for →Elim (to complete)

Not all proofs have premises



Scope

Which step of this proof is wrong? Why?



In P∧(Q∨R), the scope of ∧ is P∧(Q∨R)
In P∧(Q∨R), the scope of ∨ is (Q∨R)
In (P∧Q)∨R, the scope of ∧ is (P∧Q)
In (P∧Q)∨R, the scope of ∨ is (P∧Q)∨R
The scope of a connective is the smallest constituent expression which contains that connective.

Exercises 03

For your third seminar
Only for fast groups

7.2, 7.5, 7.6 (tt)
7.9 (truth functions)
8.20–25 (proofs/counterexamples)
9.8–10 (quantifiers)

Exercises 04

For your fourth seminar
Only for fast groups

7.25 (logical equivalence)
8.26–30 (proofs/counterexamples)
9.16.10–15 (translation)
9.17.7–15 (translation)
12.4 (counterexamples)
either
12.5–7 (counterexamples)
or (if possible)
12.8–12 (counterexamples)

NB. DO NOT USE TAUT CON. EVER.