PH133 Logic Lectures 3 & 4
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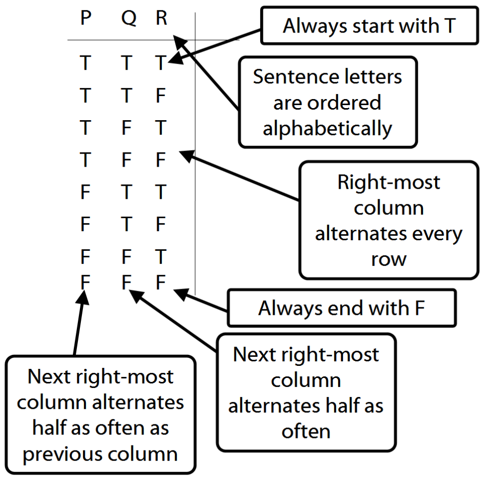
An argument is *logically valid* just if there's no possible situation in which the premises are true and the conclusion false

How to order reference columns
(from lecture 2)



Proofs with ∧Intro and ∨Intro

Example

1. R
2. S
3. R ∨ T — ∨Intro: 1
4. S ∧ (R ∨ T) — ∧Intro: 2,3

¬, ⊥

P	¬P	⊥	P∧¬P
T	F	F	F
F	T	F	F

Proof example

1. R ∧¬R
2. R — ∧Elim: 1
3. ¬R — ∧Elim: 1
4. ⊥ — ⊥Intro: 2,3
5. P — ⊥Elim: 4

Rules of Proof for →

→Intro:

*
…
#
…
*→#

→Elim:

*→#
…
*
…
#

Proof example for →Intro

1. P→Q — //If things move, Zeno is wrong
2. P — //Things move
3. Q — //Zeno is wrong

Proof example for →Elim (to complete)

1. P→Q
2. Q→R

6. P→R

Not all proofs have premises

1. P
2. P

3. P → P

1. $P \lor Q$

2. $\neg Q$

8. P

9. $\neg Q \to P$

A rule of proof for $\lor$

$\lor$Elim:

$P1 \lor P2$

$P1$

…

Q

$P2$

…

Q

…

Q

To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

Example proof

1. $R \lor S$
2. R
3. $S \lor R$ $\lor$Intro: 2
4. S
5. $S \lor R$ $\lor$Intro: 4
6. $S \lor R$ $\lor$Elim: 1,2-3,4-5

Quantifiers -- $\forall$, $\exists$

Everything is broken: $\forall x$ Broken(x)

Something is broken: $\exists x$ Broken(x)

What does $\exists$ mean? We give the meaning of $\exists$ by specifying what it takes for a sentence containing $\exists$ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.