PH126 Logic I Fast Lecture 8
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¬¬P ⫤ ⊨ P

¬∀x F(x) ⫤ ⊨ ∃x¬F(x)

¬(P→Q) ⫤ ⊨ P∧¬Q

Every square is broken
∀x (Square(x) → Broken(x))

⫤ ⊨ ¬¬∀x (Square(x) → Broken(x))

⫤ ⊨ ¬∃x ¬(Square(x) → Broken(x))

⫤ ⊨ ¬∃x (Square(x) ∧ ¬Broken(x))

⫤ ⊨ Nothing is square and non-broken
¬∃x (Square(x) ∧ ¬Broken(x))

Number

There are at least two squares:
∃x ∃y (Square(x) ∧ Square(y) ∧ ¬x=y)

At least two squares are broken:
∃x ∃y (Square(x) ∧ Broken(x) ∧ Square(y) ∧ Broken(y) ∧ ¬x=y)

There are at least three squares:
∃x ∃y ∃z (Square(x) ∧ Square(y) ∧ Square(z) ∧ ¬x=y ∧ ¬y=z ∧ ¬x=z)

There are at most two squares:
¬There are at least three squares
¬∃x ∃y ∃z (Square(x) ∧ Square(y) ∧ Square(z) ∧ ¬x=y ∧ ¬y=z ∧ ¬x=z)

There are exactly two squares:
There are at most two squares ∧ There are at least two squares

Number: Alternatives

There is at most one square:
∀x∀y((Square(x) ∧ Square(y)) → x=y)

There are at most two squares:
∀x∀y∀z((Square(x)∧Square(y)∧Square(z))
→
(x=y∨y=z∨x=z)
)

There is exactly one square:
∃x (Square(x) ∧ ∀y(Square(y) → x=y))

There are exactly two squares:
∃x∃y (Square(x) ∧ Square(y) ∧ ¬x=y ∧
∀z(Square(z) → (z=x ∨ z=y)))

The

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

⇔There is exactly one square and it is broken

∃x (Square(x) ∧
∀y(Square(y) → x=y) ∧
Broken(x)
)