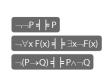
## PH126 Logic I Fast Lecture 8

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## Number

There are at least two squares:  $\exists x \exists y ( Square(x) \land Square(y) \land \neg x=y)$ 

At least two squares are broken:  $\exists x \exists y \ ( Square(x) \land Broken(x) \land Square(y) \land Broken(y) \land \neg x=y)$ 

There are at least three squares:  $\exists x \exists y \exists z ( Square(x) \land Square(y) \land Square(z) \land \neg x = y \land \neg y = z \land \neg x = z)$ 

There are at most two squares:

¬There are at least three squares

¬ $\exists x \exists y \exists z \text{ ( Square(x) } \land \text{ Square(y) } \land \text{ Square(z) } \land \neg x=y \land \neg y=z \land \neg x=z)$ 

There are exactly two squares:

There are at most two squares ∧ There are at least two squares

## **Number: Alternatives**

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There is at most one square: \forall x \forall y ( (Square(x) \land Square(y)) \rightarrow x=y )

There are at most two squares: \forall x \forall y \forall z ( (Square(x) \land Square(y) \land Square(z)) \rightarrow (x=y \lor y=z \lor x=z) )

There is exactly one square: \exists x ( Square(x) \land \forall y ( Square(y) \rightarrow x=y ))

There are exactly two squares: \exists x \exists y ( Square(x) \land Square(y) \land \neg x=y \land y=y )
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 $\forall z ( Square(z) \rightarrow (z=x \lor z=y) ) )$ 

## The

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

⇔There is exactly one square and it is broken

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\exists x ( Square(x) \land \forall y ( Square(y) \rightarrow x=y ) \land Broken(x) )
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