PH133 Logic Lecture 8

Lecturer: s.butterfill@warwick.ac.uk

Quantifier equivalences

$P \to Q \quad \dashv\vDash \neg Q \to \neg P$

$\forall x\ (\ Square(x) \to Broken(x)\ )$

$\quad \dashv\vDash \forall x\ (\neg Broken(x) \to \neg Square(x)\ )$

$P \to Q \quad \dashv\vDash \neg P \lor Q$

$\forall x\ (\ Square(x) \to Broken(x)\ )$

$\quad \dashv\vDash \forall x\ (\neg Square(x) \lor Broken(x)\ )$

$\neg P \lor Q \dashv\vDash \neg(P \land \neg Q)$

$\exists x\ \neg F(x) \dashv\vDash \neg \forall x\ F(x)$

$\forall x\ (\neg Square(x) \lor Broken(x)\ )$

$\quad \dashv\vDash \exists x \neg(\ Square(x) \land \neg Broken(x)\ )$

$\exists x\ \neg Created(x) \dashv\vDash \neg \forall x\ Created(x)$

Someone is loved by everyone

$\exists x \forall y$ Loves(y,x)

Someone loves everyone

$\exists x \forall y$ Loves(x,y)

Quantifiers and identity

Ahura Mazda created everything:

$\quad \forall x\ Created(a,x)$

Ahura Mazda created everything apart from himself: $\forall x\ (\ \neg(x=a) \to Created(a,x)\ )$

Quantifiers and negation

Some person is dead.

$\quad \exists x(Person(x) \land Dead(x))$

Some person is not dead.

$\quad \exists x(Person(x) \land \neg Dead(x))$

No person is dead.

$\quad \neg \exists x(Person(x) \land Dead(x))$

Every person is dead.

$\quad \forall x(Person(x) \to Dead(x))$

Every person is not dead.

$\quad \forall x(Person(x) \to \neg Dead(x))$

Not every person is dead.

$\quad \neg \forall x(Person(x) \to Dead(x))$

Multiple quantifiers

'There is a store for everything'

$\quad \exists y \forall x\ StoreFor(y,x)$

$\quad \forall y \exists x\ StoreFor(x,y)$

Other sentences to translate:

'Wikipedia has an article about everything'

'Everyone hurts someone they love'

'Someone hurts everyone she loves'

Intuitve summary of quantifier rules

$\forall$Elim

If it's true of everything it's true of Baudrillard

$\exists$Intro

If it's true of Baudrillard it's true of something

$\exists$Elim

If it's true of something and Q follows no matter which something it is, then Q

$\forall$Intro

If it's true of an arbitrary thing, then it's true of everything.

Number

There are at least two squares:

$\quad \exists x\ \exists y\ (\ Square(x) \land Square(y) \land \neg x=y)$

At least two squares are broken:

$\quad \exists x\ \exists y\ (\ Square(x) \land Broken(x) \land Square(y) \land Broken(y) \land \neg x=y)$

There are at least three squares:

$\quad \exists x\ \exists y\ \exists z\ (\ Square(x) \land Square(y) \land Square(z) \land \neg x=y \land \neg y=z \land \neg x=z)$

There are at most two squares:

$\neg$There are at least three squares

$\neg\exists x\ \exists y\ \exists z\ (\ Square(x) \land Square(y) \land Square(z) \land \neg x=y \land \neg y=z \land \neg x=z)$

There are exactly two squares:

There are at most two squares $\land$
There are at least two squares

Number: Alternatives

There is at most one square:

$\forall x \forall y(\ (Square(x) \land Square(y)) \rightarrow x=y\ )$

There are at most two squares:

$$\forall x \forall y \forall z(\ (Square(x) \land Square(y) \land Square(z)) \rightarrow (x=y \lor y=z \lor x=z)\ )$$

There is exactly one square:

$\exists x\ (\ Square(x) \land \forall y(\ Square(y) \rightarrow x=y\ ))$

There are exactly two squares:

$\exists x \exists y\ (Square(x) \land Square(y) \land \neg x=y \land \forall z(\ Square(z) \rightarrow (z=x \lor z=y)\ ))$

The

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

There is exactly one square and it is broken

There is at most one square and there is at least one square and it is broken

There is at most one square and there is at least one square and all squares are broken

$\neg\ \exists x\ \exists y\ (\ Square(x) \land Square(y) \land \neg x=y\ )$
$\land\ \exists x\ Square(x)$
$\land\ \forall x\ (\ Square(x) \rightarrow Broken(x))$