PH126 Logic I Fast Lecture 6

Lecturer: s.butterfill@warwick.ac.uk

Quantifier equivalences

P→Q = |
$$\models \neg Q \rightarrow \neg P$$

 $\forall x (Square(x) \rightarrow Broken(x))$
= | $\models \forall x (\neg Broken(x) \rightarrow \neg Square(x))$

$$P \rightarrow Q = | \neg P \lor Q$$

 $\forall x (Square(x) \rightarrow Broken(x))$
 $= | \neg F \lor X (\neg Square(x) \lor Broken(x))$

$$\neg P \lor Q \neq \models \neg (P \land \neg Q)$$

$$\exists x \neg F(x) \neq \models \neg \forall x F(x)$$

$$\forall x (\neg Square(x) \lor Broken(x))$$

$$\neq \models \exists \neg x (Square(x) \land \neg Broken(x))$$

$$\exists x \neg Created(x) \nmid \neg \forall x Created(x)$$

Hybrid example

Every object is either non-square or broken ∀x (¬Square(x) ∨ Broken(x))

$$\neg P = | P | \qquad \qquad | P = | P | \qquad \qquad | P = | P | \qquad | P = | P$$

Truth-functional completeness (LPL §7.4)

How many truth-functions can be constructed using 2 sentence letters?

Р	Q	P % Q
T	T	? 📉
T	F	? Each of these
F	T	? can be T or F
F	F	? *

Df. 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that $\{\neg, \land, \lor\}$ is truthfunctionally complete:

Q P-	→ Q
- -	
: 1	F
- -	Γ [¬P∧Q] ∨
: -	Γ [¬P∧ ¬Q]

$$[P \land Q] \lor [\neg P \land Q] \lor [\neg P \land \neg Q]$$

Exercise assuming $\{\neg, \lor, \land\}$ is truth-functionally complete, show that $\{\neg, \lor\}$ is.

Variables

Names: a, b, c, ...

Variables: x, y, z, w, ...

Variables are for saying several things about one thing even without specifying which thing it is

Notation:

A ├ B means there is a proof of B using premises **A**

B means there is a proof of B using no premises

 $A \vdash$ means the same as $A \vdash \bot$

A ⊨ B means B is a logical consequence of A ⊨ B means B is a tautology A ⊨ means the same as A ⊨ ⊥ A ⊨ TT B means B is a logical consequence of A just in virtue of the meanings of truth-functions (the textbook LPL calls this 'tautological consequence')

Soundness: If $A \models B$ then $A \models B$ I.e. if you can prove it in Fitch, it's valid

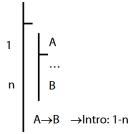
Completeness: If $\mathbf{A} \models_{TT} \mathbf{B}$ then $\mathbf{A} \models \mathbf{B}$ I.e. if it's valid just in virtue of the meanings of truth functions, you can prove it in Fitch

Proofs about proofs

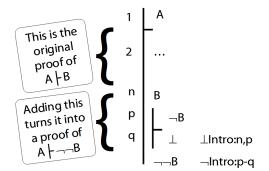
1. If $A \vdash B$ then $\vdash A \rightarrow B$ Proof Given a proof for $A \vdash B$...



... we can turn it into a proof for $\vdash A \rightarrow B$:



- 2. If $\mid A \rightarrow B$ then $A \mid B$
- 3. If A \vdash B then A $\vdash \neg \neg B$ *Proof*:



- 4. If A \vdash C then A \vdash B \rightarrow C
- 5. If A \vdash B and A \vdash ¬C then A \vdash ¬(B \rightarrow C)





"Every time I go to the dentist, somebody dies."

 $\forall t \ ((Time(t) \ \land ToDentist(a,t) \) \quad \rightarrow \quad \exists x \ (\ Person(x) \land DiesAt(x,t) \) \)$

Exercises 07

For your seventh seminar Only for fast groups

11.10 (nb. especially no. 20)

11.13

13.43-45

13.49-50

14.2 - 3

14.10-11

Despite what book says, DON'T even THINK about using TAUT CON.