PH126 Logic I Lecture 9
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Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	A → B
T	T	
T	F	
F	T	
F	F	

‘If’ and ‘→’ do not match

| ¬A | Marnie will not miss her train |
| A → B | If Marnie misses her train, she will arrive on time. |

‘If’ and ‘→’ do match

| ¬A ∨ B | America does not exist ∨ Baudrillard is wrong |
| If A, B | If America exists, Baudrillard is wrong |

| If A, B | If you love logic, things will fall into place |
| ¬(A ∧ ¬B) | Not both: you take logic and things don’t fall into place |

Multiple quantifiers: simple examples

“Something is above something”
∃x ∃y Above(x,y)

“Everyone likes puffins”
∀y ∀x (Puffin(x) → Likes(y,x))

Puffin(x) : x is a puffin

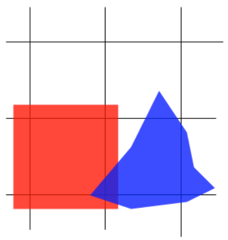
b : Steve

Likes(x,y) : x likes y

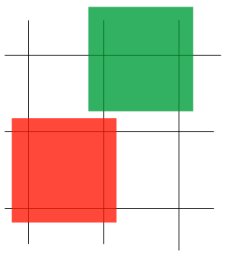
“Something makes someone want to die inside”
∃x ∃y WantToDieInside(x,y)

WantToDieInside(x,y) : x makes y want to die inside

1 **∃x ∃y Above(x,y)**
“Something is above something”



1 is FALSE



1 is TRUE

The Soundness & Completeness Theorems

‘**A** ⊢ B’ means there is a proof of B using premises **A**

‘ ⊢ B’ means there is a proof of B using no premises

‘**A** ⊨ B’ means B is a logical consequence of A

‘ ⊨ B’ means B is a tautology

‘**A** $\models_T$ B’ means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this ‘tautological consequence’)

Soundness: If **A** ⊢ B then **A** ⊨ B

I.e. if you can prove it in Fitch, it’s valid

Completeness: If **A** $\models_T$ B then **A** ⊢ B

I.e. if it’s valid just in virtue of the meanings of truth functions, you can prove it in Fitch

Proof Example using negation-Intro

1. P

5. ¬¬P

Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

If * and # are logically equivalent, then you take a complex formula and replace * with # without changing the truth table of the complex formula.

Example. Because $\neg(R \vee \neg P)$ is logically equivalent to $\neg R \wedge P$, it follows that:

$P \vee \neg(Q \wedge \neg(R \vee \neg P))$ is logically equivalent to $P \vee \neg(Q \wedge (\neg R \wedge P))$

And because $\neg(Q \wedge (\neg R \wedge P))$ is logically equivalent to $\neg Q \vee (R \vee \neg P)$, it follows that $P \vee \neg(Q \wedge (\neg R \wedge P))$ is logically equivalent to $P \vee (\neg Q \vee (R \vee \neg P))$.

Example proof

1. $S \vee (Q \wedge R)$

$S \vee Q$

Fubar rules

$\wedge$Fubar:

*

…

$* \wedge \#$

Q1. What would be wrong with adding ∧Fubar to Fitch?

Q2. What would be wrong with having ∧Fubar in any *system of proof?*

Tonk

*Intro

P_i

…

$P_1 * P_2$

*Elim

$P_1 * P_2$

…

P_i

Exercises 05

For your sixth seminar

Not for fast groups

A. From the LPL textbook:

- 7.9 (truth functions)
- 6.17–20 (proof)
- 6.33, 6.40
- DO NOT USE TAUT CON. EVER.
- 8.24–25 (proofs/counterexamples)
- 12.4–5, *12.6–7 (counterexamples)

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain : people and actions
D(x) : x is desirable
V(x) : x is virtuous
A(x) : x is an action
H(x) : x is a person
P(x,y): x performed y

i. $\forall x\ [D(x) \rightarrow V(x)]$

ii. $\forall x\ [[A(x) \wedge D(x)] \rightarrow V(x)]]$

iii. $\exists x\ [A(x) \wedge \neg[D(x) \rightarrow V(x)]]$

*iv. $\exists x\ \forall y\ [[[H(x) \wedge A(y)] \wedge P(x,y)] \rightarrow V(y)]$

**v. $\neg\exists x\ [\exists y\ [H(x) \wedge P(x,y) \wedge A(y) \wedge \neg V(y)] \wedge \neg\exists z[P(x,z) \wedge A(z) \wedge V(z)]]$