PH126 Logic I Lecture 8
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Proofs of the Day

1. $\neg(P \lor Q)$

10. $\neg P \land \neg Q$

1. $\neg P \land \neg Q$

10. $\neg(P \lor Q)$

Proof example

1. $\neg P \lor R$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \to R$

Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	A → B
T	T	
T	F	
F	T	
F	F	

How to determine the truth of sentences involving ∀

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.

What not to confuse

$\exists x\,(\text{Square}(x) \land \text{Blue}(x))$ vs.
$\exists x\,\text{Square}(x) \land \exists x\,\text{Blue}(x)$

$\forall x\,(\text{Square}(x) \to \text{Blue}(x))$ vs.
$\forall x\,\text{Square}(x) \to \forall x\,\text{Blue}(x)$

$\neg(P \lor Q)$ vs. $\neg P \lor \neg Q$

$\neg(P \land Q)$ vs. $\neg P \land \neg Q$

$\neg(P \to Q)$ vs. $P \to \neg Q$

Disjunction Introduction
(∨ Intro)

$$\begin{array}{r|l} & P_i \\ & \vdots \\ \triangleright & P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \end{array}$$

Disjunction Elimination
(∨ Elim)

$$\begin{array}{r|l} & P_1 \vee \ldots \vee P_n \\ & \vdots \\ & \begin{array}{|l} P_1 \\ \hline \vdots \\ S \end{array} \\ & \Downarrow \\ & \begin{array}{|l} P_n \\ \hline \vdots \\ S \end{array} \\ & \vdots \\ \triangleright & S \end{array}$$

Conditional Introduction
(→ Intro)

$$\begin{array}{r|l} & \begin{array}{|l} P \\ \hline \vdots \\ Q \end{array} \\ \triangleright & P \to Q \end{array}$$

Conditional Elimination
(→ Elim)

$$\begin{array}{r|l} & P \to Q \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Biconditional Introduction
(↔ Intro)

$$\begin{array}{r|l} & \begin{array}{|l} P \\ \hline \vdots \\ Q \end{array} \\ & \begin{array}{|l} Q \\ \hline \vdots \\ P \end{array} \\ \triangleright & P \leftrightarrow Q \end{array}$$

Biconditional Elimination
(↔ Elim)

$$\begin{array}{r|l} & P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Universal Elimination
(∀ Elim)

$$\begin{array}{r|l} & \forall x\, S(x) \\ & \vdots \\ \triangleright & S(c) \end{array}$$

Universal Introduction
(∀ Intro)

$$\begin{array}{r|l} & \begin{array}{|l} \boxed{c} \\ \hline \vdots \\ P(c) \end{array} \\ \triangleright & \forall x\, P(x) \end{array}$$

where c does not occur outside the subproof where it is introduced.

Negation Introduction
(¬ Intro)

$$\begin{array}{r|l} & \begin{array}{|l} P \\ \hline \vdots \\ \bot \end{array} \\ \triangleright & \neg P \end{array}$$

Negation Elimination
(¬ Elim)

$$\begin{array}{r|l} & \neg\neg P \\ & \vdots \\ \triangleright & P \end{array}$$

Reiteration
(Reit)

$$\begin{array}{r|l} & P \\ & \vdots \\ \triangleright & P \end{array}$$

Existential Introduction
(∃ Intro)

$$\begin{array}{r|l} & S(c) \\ & \vdots \\ \triangleright & \exists x\, S(x) \end{array}$$

Existential Elimination
(∃ Elim)

$$\begin{array}{r|l} & \exists x\, S(x) \\ & \vdots \\ & \begin{array}{|l} \boxed{c}\ S(c) \\ \hline \vdots \\ Q \end{array} \\ \triangleright & Q \end{array}$$

where c does not occur outside the subproof where it is introduced.

⊥ Introduction
(⊥ Intro)

$$\begin{array}{r|l} & P \\ & \vdots \\ & \neg P \\ & \vdots \\ \triangleright & \bot \end{array}$$

⊥ Elimination
(⊥ Elim)

$$\begin{array}{r|l} & \bot \\ & \vdots \\ \triangleright & P \end{array}$$

Identity Introduction
(= Intro)

$$\begin{array}{r|l} \triangleright & n = n \end{array}$$

Identity Elimination
(= Elim)

$$\begin{array}{r|l} & P(n) \\ & \vdots \\ & n = m \\ & \vdots \\ \triangleright & P(m) \end{array}$$