PH126 Logic I Lecture 15

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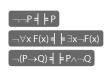
"Every time I go to the dentist, somebody dies."

 $\forall t ((Time(t) \land ToDentist(a,t)) \rightarrow \exists x (Person(x) \land DiesAt(x,t)))$

Quantifier equivalences (continued)

 $\exists x \neg Created(x) = \neg \forall x Created(x)$

Hybrid example



Every square is broken ∀x (Square(x) → Broken(x))

$$= \downarrow \models \neg \neg \forall x \, (\text{Square}(x) \rightarrow \text{Broken}(x) \,)$$

$$= \downarrow \models \neg \exists x \, \neg (\text{Square}(x) \rightarrow \text{Broken}(x) \,)$$

$$= \downarrow \models \neg \exists x \, \neg (\text{Square}(x) \land \neg \text{Broken}(x) \,)$$

Nothing is square and non-broken ¬∃x (Square(x) ∧ ¬Broken(x))

Number

There are at least two squares:

$$\exists x \exists y (Square(x) \land Square(y) \land \neg x=y)$$

At least two squares are broken:

 $\exists x \exists y \ (Square(x) \land Broken(x) \land Square(y) \land Broken(y) \land \neg x=y)$

There are at least three squares:

 $\exists x \exists y \exists z (Square(x) \land Square(y) \land Square(z) \land \neg x=y \land \neg y=z \land \neg x=z)$

There are at most two squares:

¬There are at least three squares ¬ \exists x \exists y \exists z (Square(x) \land Square(y) \land Square(z) \land ¬x=y \land ¬y=z \land ¬x=z)

There are exactly two squares:

There are at most two squares Λ There are at least two squares

Number: Alternatives

There is at most one square:

$$\forall x \forall y ((Square(x) \land Square(y)) \rightarrow x=y)$$

There are at most two squares:

$$\forall x \forall y \forall z ((Square(x) \land Square(y) \land Square(z))$$

There is exactly one square:

$$\exists x (Square(x) \land \forall y (Square(y) \rightarrow x=y))$$

There are exactly two squares:

$$\exists x \exists y (Square(x) \land Square(y) \land \neg x=y \land \forall z (Square(z) \rightarrow (z=x \lor z=y)))$$

Truth-functional completeness (LPL §7.4)

Preliminary: How many truth-functions can be constructed using 2 sentence letters?

Truth-functional completeness: For any truth function, we can easily find a logically equivalent one using only \neg , \lor , \land

Better: 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate L'gic* p. 45)

Illustration of how the proof that $\{\neg, \land, \lor \text{ is truth-functionally complete}\}$:

| Р | Q | $P \rightarrow Q$ | |
|---|---|-------------------|-------------------------|
| T | Т | Т | $[P \land Q] \lor$ |
| T | F | F | |
| F | T | Т | $[\neg P \land Q] \lor$ |
| F | F | Т | [¬P∧ ¬Q] |
| | | | |

$$[P \land Q] \lor [\neg P \land Q] \lor [\neg P \land \neg Q]$$

Exercise assuming $\{\neg, \lor, \land\}$ is truthfunctionally complete, show that $\{\neg, \lor\}$ is?