PH126 Logic I Lecture 10
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Scope

Underlining shows the scope of the quantifiers

"All squares are blue"
$\forall$**x (Square(x) $\rightarrow$ Blue(x))**

"If everything is square, everything is blue"
$\forall$**x Square(x)** $\rightarrow$ $\forall$**x Blue(x)**

Quantifiers bind variables

Read §9.2 of Barwise & Etchemendy

Quantifiers and number

To translate statements involving number into FOL, use identity

E.g. Two objects are broken:

$\exists x \exists y$ (Broken(x) $\wedge$ Broken(y) $\wedge \neg$(x=y))

Ex. Translate ∀Three objects are broken into FOL∀

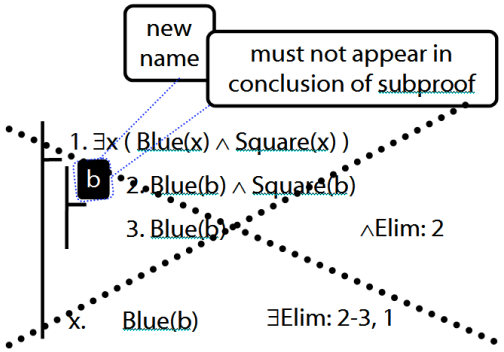
Some object is broken.
$\exists$x Broken(x)

Two objects are broken
~~$\exists$x Broken(x) $\wedge$ $\exists$y Broken(y)~~
~~$\exists x \exists$y (Broken(x) $\wedge$ Broken(y))~~
$\exists x \exists$y (Broken(x) $\wedge$ Broken(y) $\wedge \neg$(x=y))

Proof example: ∃Elim

1. $\exists$x (Blue(x) $\wedge$ Square(x))
   - b
     - 2. Blue(b) $\wedge$ Square(b)
     - 3. Blue(b) $\wedge$Elim: 2
     - 4. $\exists$x Blue(x) $\exists$Intro: 3
   - x. $\exists$x Blue(x) $\exists$Elim: 2-4, 1

Invalid use of ∃Elim



Tonk

*Intro	*Elim
P_i	P_1*P_2
…	…
P_1*P_2	P_i

Exercises 05

(Same as from handout for lecture 09)

For your sixth seminar. Not for fast groups

A. From the LPL textbook:

7.9 (truth functions)
6.17–20 (proof)
6.33, 6.40
DO NOT USE TAUT CON. EVER.
8.24–25 (proofs/counterexamples)
12.4–5, *12.6–7 (counterexamples)

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain : people and actions
D(x) : x is desirable
V(x) : x is virtuous
A(x) : x is an action
H(x) : x is a person
P(x,y) : x performed y

i. $\forall x$ [D(x) $\rightarrow$ V(x)]

ii. $\forall x$ [[A(x)$\wedge$ D(x)] $\rightarrow$ V(x)]]

iii. $\exists x$ [A(x) $\wedge \neg$[D(x) $\rightarrow$ V(x)]]

*iv. $\exists x \forall y$ [[[H(x) $\wedge$ A(y)] $\wedge$ P(x,y)] $\rightarrow$ V(y)]

**v. $\neg\exists x$ [$\exists y$ [H(x) $\wedge$ P(x,y) $\wedge$ A(y) $\wedge \neg$V(y)]
$\wedge \neg\exists z$[P(x,z) $\wedge$ A(z) $\wedge$ V(z)]]