**PH126 Logic I** · Lecture 3
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### Proofs with ∧

Rules of proof are given near the end of the textbook.

*example*

| | | |
|---|---|---|
| 1. | P ∧ Q | |
| 2. | Q ∧ R | |
| 3. | P | ∧Elim: 1 |
| 4. | R | ∧Elim: 2 |
| 5. | P ∧ R | ∧Intro: 3,4 |

### Proofs with identity

*example* (with missing justification)

| | |
|---|---|
| 1. | LeftOf(a,b) |
| 2. | b=c |
| 3. | |
| 4. | |
| 5. | LeftOf(a,c) |

### Truth-functional connectives

A *connective* joins one or more sentences to make a new sentence. E.g. ∧, ¬, because

A sentence joined by a connective is a *constituent sentence*. E.g. P in "P because Q"

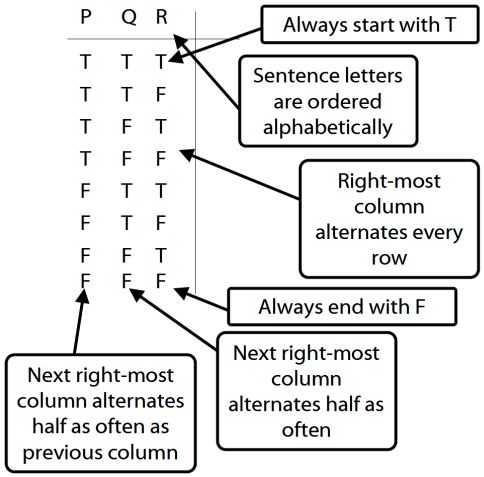
A *truth functional* connective produces a new sentence whose truth value depends only on the truth values of its constituent sentences.

When P and Q are both true, "P because Q" is sometimes true and sometimes false. Therefore, 'because' is not a truth-functional connective.

### Complex truth table example

| P | Q | R | (P ∧ Q) ∨ R |
|---|---|---|---|
| T | T | T | |
| T | T | F | |
| T | F | T | |
| T | F | F | |
| F | T | T | |
| F | T | F | |
| F | F | T | |
| F | F | F | |

### How to order reference columns




### Logical Validity

Argument 3

| | | |
|---|---|---|
| 1. | ┐ | (P ∧ Q) ∨ R |
| 2. | ┘ | P ∨ ¬P |

Argument 3b

| | | |
|---|---|---|
| 1. | ┐ | |
| 2. | ┘ | P ∨ ¬P |

Argument 4

| | | |
|---|---|---|
| 1. | ┐ | P ∧ ¬P |
| 2. | ┘ | (P ∧ Q) ∨ R |

P ∨ ¬P is a *logical truth* (see p. 568)

P ∧ ¬P is a *contradiction* (see p. 564)

### Exercises 01

*For your second seminar. Not for fast groups*

A. From the textbook:

2.5–6 (informal proofs with identity)

2.8, 2.10, 2.12, 2.21 (counterexamples)

3.1–2 (negation)

3.5, 3.7 (conjunction)

4.1–2 (truth tables)

B. Explicate each of the following in one or two sentences.

- logical consequence
- logically sound argument
- FOL
- Fitch format