

## Proofs with $\wedge$

Rules of proof are given near the end of the textbook.

example

1.	$P \wedge Q$	
2.	$Q \wedge R$	
3.	$P$	$\wedge\text{Elim: } 1$
4.	$R$	$\wedge\text{Elim: } 2$
5.	$P \wedge R$	$\wedge\text{Intro: } 3,4$

## Proofs with identity

example (with missing justification)

1.	LeftOf(a,b)
2.	$b=c$
3.	
4.	
5.	LeftOf(a,c)

## Proofs with $\wedge\text{Intro}$ and $\vee\text{Intro}$

Example

1.	$R$	
2.	$S$	
3.	$R \vee T$	$\vee\text{Intro: } 1$
4.	$S \wedge (R \vee T)$	$\wedge\text{Intro: } 2,3$

After the premises, each line in a proof is guaranteed to be a logical consequence of the lines above.

P1	P2	$P1 \leftrightarrow P2$
T	T	F
T	F	T
F	T	T
F	F	F

The following rule is unacceptable. Why?

$\leftrightarrow\text{Intro:}$
$P_i$
...
$P1 \leftrightarrow P2$

## Truth-functional connectives

A *connective* joins one or more sentences to make a new sentence. E.g.  $\wedge$ ,  $\neg$ , because

A sentence joined by a connective is a *constituent sentence*. E.g.  $P$  in “ $P$  because  $Q$ ”

A *truth functional* connective produces a new sentence whose truth value depends only on the truth values of its constituent sentences.

When  $P$  and  $Q$  are both true, “ $P$  because  $Q$ ” is sometimes true and sometimes false.

Therefore, ‘because’ is not a truth-functional connective

## How to order reference columns

[not in lecture]

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Annotations:

- Always start with T
- Sentence letters are ordered alphabetically
- Right-most column alternates every row
- Always end with F
- Next right-most column alternates half as often as previous column
- Next right-most column alternates half as often

## $A \wedge B \vee C$

A	B	C	$(A \wedge B) \vee C$	$A \wedge (B \vee C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	T	F
F	F	F	F	F

## Ambiguity

Lexical ambiguity, e.g. "give me a note"

Structural ambiguity, e.g. "Two puffins ate six fish", "I shot an elephant in my pyjamas"

What is the source of structural ambiguity in natural languages?

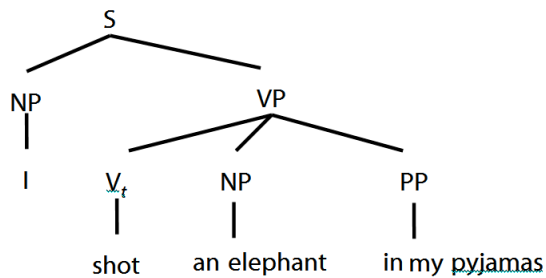
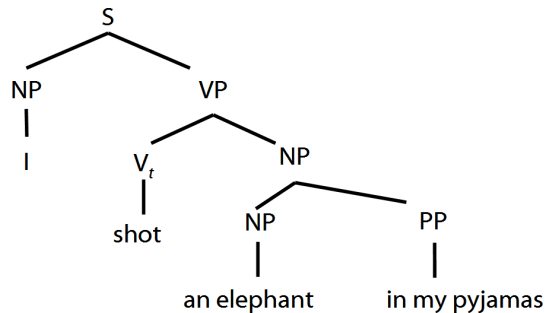
Rule 1: a NP followed by a VP is a S

Rule 2: a  $V_t$  followed by a NP is a VP

Rule 3: a NP followed by a PP is a S

Rule 4: A  $V_t$  followed by a NP then a PP is a VP

Two derivations of Groucho Marx' claim, "I shot an elephant in my pyjamas"



- Not every sequence of words is a sentence.

- Which sequences of words are sentences is determined by rules.

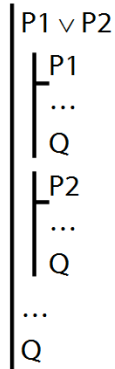
- The rules specify how to construct sentences from fragments.

- The rules impose a tree-like structure on sentences.

- Structural ambiguity occurs when the same linear sequence of words can be derived from different rules.

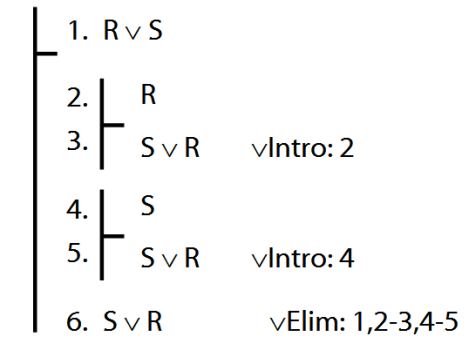
## Rule of Proof: $\vee$ Elim

$\vee$ Elim:



To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

Example proof with  $\vee$ Elim



## Exercises 02

For your second seminar  
Only for fast groups

3.19 (translation)

4.31 (equivalences)

5.3–6 (validity)

6.7–12 (proofs)

6.18–6.20 (proofs with subproofs)

either:

6.24–6.27

or, if possible:

6.40–6.42