PH126 Logic I Lecture 12

Lecturer: s.butterfill@warwick.ac.uk

Quantifier Recap

Everything is broken ∀x Broken(x)

All my things are broken $\forall x \text{ (BelongsToMe}(x) \rightarrow \text{Broken}(x) \text{)}$

Something is broken ∃x Broken(x)

Something of mine is broken ∃x (BelongsToMe(x) ∧ Broken(x))

How to determine truth of a sentence with an existential quantifier as the main connective

- 1. Give every object a name.
- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
- 3. If ANY OF the new sentences are true, so is the original.

How to determine truth of a sentence with a universal quantifier as the main connective

- 1. Give every object a name.
- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
- 3. If ALL OF the new sentences are true, so is the original.

Multiple quantifiers

'There is a store for everything'

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

'Wikipedia has an article about everything'

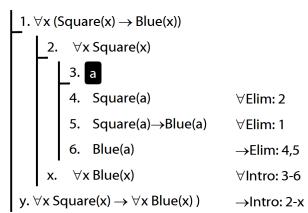
'Everyone hurts someone they love'

'Someone hurts everyone she loves'

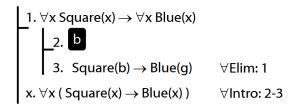
∀Intro



∀Intro Example proof



Why is this proof incorrect?



Intuitve summary of quantifier rules $\forall Elim$

If it's true of everything it's true of Baudrillard

∃Intro

If it's true of Baudrillard it's true of something

∃Elim

If it's true of something and Q follows no matter which something it is, then Q

∀Intro

If it's true of an arbitrary thing, then it's true of everything.

Notation:

A | B means there is a proof of B using premises **A**

B means there is a proof of B using no premises

 $A \mid$ means the same as $A \mid \bot$

A ⊨ B means B is a logical consequence of A ⊨ B means B is a tautology

 $\mathbf{A} \models \text{means the same as } \mathbf{A} \models \bot$

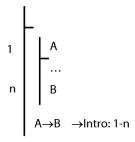
 $A \models_{TT} B$ means B is a logical consequence of A just in virtue of the meanings of truth-functions (the textbook LPL calls this 'tautological consequence')

Proofs about proofs

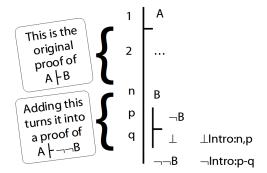
1. If $A \vdash B$ then $\vdash A \rightarrow B$ Proof Given a proof for $A \vdash B$...



... we can turn it into a proof for $\vdash A \rightarrow B$:



- 2. If $\mid A \rightarrow B$ then $A \mid B$
- 3. If A \vdash B then A $\vdash \neg \neg B$ *Proof*:



- 4. If A \vdash C then A \vdash B \rightarrow C
- 5. If A \vdash B and A \vdash \neg C then A \vdash \neg (B \rightarrow C)

Exercises 06

For your seventh seminar Not for fast groups

A. From the LPL textbook: 9.15–17, *9.18–19 (trans.) 10.20, 10.22 11.2 (multiple quantifiers) 12.9–10

Short of time? Skip the exercises marked *

B. Each line of the table describes a combination of properties. E.g. line 1 specifies a relation that is reflexive, symmetric and transitive, line 2 specifies a relation that is reflexive, symmetric and not transitive. Complete the table by adding an example relation on each line.

	Example	Reflexive	Symmetric	Transitive
1	=	Y	Y	Y
	(identity)			
2		Y	Y	N
3		Y	N	Y
4		Y	N	N
5		N	Y	Y
6		N	Y	N
7		N	N	Y
8		N	N	N