PH126 Logic I · Lecture 7
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Proof example with →

1. $P \to Q$
2. $\neg Q$

6. $\neg P$

How to determine the truth of sentences involving ∀

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL of the new sentences are true, so is the original.

Quantifiers

Everything is broken: $\forall x$ Broken(x)

Something is broken: $\exists x$ Broken(x)

∀Elim

$\forall x\ S(x)$

…

$S(c)$

∃Intro

$S(a)$

…

$\exists x\ S(x)$

First quantifier rule of proof: ∀Elim

∀Elim

$\forall x\ S(x)$

…

$S(c)$

Proof example: ∀Elim, ∃Intro

1.	$\forall x\ (\text{Puf}(x) \to \text{YelBk}(x))$	
2.	Puf(a)	
3.	$\text{Puf}(a) \to \text{YelBk}(a)$	∀Elim:1
4.	YelBk(a)	→Elim: 3,2
5.	$\exists x$ YelBk(x)	∃Intro: 4

Proof example

1. $\neg P \vee R$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \to R$

Conjunction Introduction
($\wedge$ Intro)

> P_1
> $\Downarrow$
> P_n
> $\vdots$
> $\triangleright$ $P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination
($\wedge$ Elim)

> $P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$
> $\vdots$
> $\triangleright$ P_i

Disjunction Introduction
($\vee$ Intro)

> P_i
> $\vdots$
> $\triangleright$ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination
($\vee$ Elim)

> $P_1 \vee \ldots \vee P_n$
> $\vdots$
> > P_1
> > $\vdots$
> > S
>
> $\Downarrow$
> > P_n
> > $\vdots$
> > S
>
> $\vdots$
> $\triangleright$ S

Negation Introduction
($\neg$ Intro)

> > P
> > $\vdots$
> > $\perp$
>
> $\triangleright$ $\neg P$

Negation Elimination
($\neg$ Elim)

> $\neg\neg P$
> $\vdots$
> $\triangleright$ P

$\perp$ Introduction
($\perp$ Intro)

> P
> $\vdots$
> $\neg P$
> $\vdots$
> $\triangleright$ $\perp$

$\perp$ Elimination
($\perp$ Elim)

> $\perp$
> $\vdots$
> $\triangleright$ P

Conditional Introduction
($\rightarrow$ Intro)

> > P
> > $\vdots$
> > Q
>
> $\triangleright$ $P \rightarrow Q$

Conditional Elimination
($\rightarrow$ Elim)

> $P \rightarrow Q$
> $\vdots$
> P
> $\vdots$
> $\triangleright$ Q

Biconditional Introduction
($\leftrightarrow$ Intro)

> > P
> > $\vdots$
> > Q
>
> > Q
> > $\vdots$
> > P
>
> $\triangleright$ $P \leftrightarrow Q$

Biconditional Elimination
($\leftrightarrow$ Elim)

> $P \leftrightarrow Q$ (or $Q \leftrightarrow P$)
> $\vdots$
> P
> $\vdots$
> $\triangleright$ Q

Reiteration
(Reit)

> P
> $\vdots$
> $\triangleright$ P

Identity Introduction
($=$ Intro)

> $\triangleright$ $n = n$

Identity Elimination
($=$ Elim)

> $P(n)$
> $\vdots$
> $n = m$
> $\vdots$
> $\triangleright$ $P(m)$

Universal Elimination
($\forall$ Elim)

> $\forall x\, S(x)$
> $\vdots$
> $\triangleright$ $S(c)$

Universal Introduction
($\forall$ Intro)

> > $\boxed{c}$
> > $\vdots$
> > $P(c)$
>
> $\triangleright$ $\forall x\, P(x)$

where c does not occur outside the subproof where it is introduced.

Existential Introduction
($\exists$ Intro)

> $S(c)$
> $\vdots$
> $\triangleright$ $\exists x\, S(x)$

Existential Elimination
($\exists$ Elim)

> $\exists x\, S(x)$
> $\vdots$
> > $\boxed{c}$ $S(c)$
> > $\vdots$
> > Q
>
> $\triangleright$ Q

where c does not occur outside the subproof where it is introduced.