### PH126 Logic I Lecture 15

Lecturer: s.butterfill@warwick.ac.uk

## Quantifiers and identity

Ahura Mazda created everything: ∀x Created(a,x)

Ahura Mazda created everything apart from himself:  $\forall x ( \neg(x=a) \rightarrow \text{Created}(a,x) )$ 

## Quantifier proof exercise

1. 
$$\forall y ((\neg y=a) \rightarrow \text{Created}(a,y))$$
  
10  $\forall y ((\neg \exists x \text{Created}(x,y)) \rightarrow y=a)$ 

#### Number

There are at least two squares:  $\exists x \exists y ( Square(x) \land Square(y) \land \neg x=y)$ 

At least two squares are broken:  $\exists x \exists y \ ( Square(x) \land Broken(x) \land Square(y) \land Broken(y) \land \neg x=y)$ 

There are at least three squares:  $\exists x \exists y \exists z ( Square(x) \land Square(y) \land Square(z) \land \neg x=y \land \neg y=z \land \neg x=z)$ 

There are at most two squares:

¬There are at least three squares

¬ $\exists x \exists y \exists z \ ( Square(x) \land Square(y) \land Square(z) \land \neg x=y \land \neg y=z \land \neg x=z)$ 

There are exactly two squares:

There are at most two squares ∧ There are at least two squares

#### **Number: Alternatives**

There is at most one square:  $\forall x \forall y ( (Square(x) \land Square(y)) \rightarrow x=y )$ 

There are at most two squares:  $\forall x \forall y \forall z ( (Square(x) \land Square(y) \land Square(z)) \rightarrow (x=y \lor y=z \lor x=z)$ 

There is exactly one square:  $\exists x \ ( Square(x) \land \forall y ( Square(y) \rightarrow x=y ) )$ 

There are exactly two squares:  $\exists x \exists y (Square(x) \land Square(y) \land \neg x = y \land \forall z (Square(z) \rightarrow (z = x \lor z = y)))$ 

#### The

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken

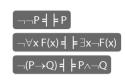
There is exactly one square and it is broken

There is at most one square and there is at least one square and it is broken

There is at most one square and there is at least one square and all squares are broken

 $\neg \exists x \exists y ( Square(x) \land Square(y) \land \neg x=y )$   $\land \exists x Square(x)$  $\land \forall x ( Square(x) \rightarrow Broken(x)$ 

Which shorter sentences are equivalent to this?



# Every square is broken ∀x (Square(x) → Broken(x))

 $\exists \models \neg \exists x (Square(x) \land \neg Broken(x))$ 

#### Exercises 08

There isn't a seminar for these; you should attempt them over the vacation.

Not for fast groups

11.11, 11.13, \*11.10 13.6-7, \*13.8-9 13.19, 13.23-27 (\*13.28-31) (\*13. 33, 35, 37, 39) 13.43-45, 13.49-50, \*13.51-52 14.1, 14.2, 14.3 (\*14.4-5) [trans] \*14.10-12, \*14.13 [proofs] \*14.26, 14.28 [trans]

Despite what book says, DON'T even THINK about using TAUT CON.

Short of time? Skip the exercises marked \*