# PH126 Logic I · Lecture 7

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Proof example with  $\rightarrow$ 

6. ¬P

# How to determine the truth of sentences involving $\forall$

- 1. Give every object a name.
- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
- 3. If ALL of the new sentences are true, so is the original.

## Quantifiers

Everything is broken: ∀x Broken(x) Something is broken: ∃x Broken(x)

∀Elim	∃Intro
$\forall x S(x)$	S(a)
S(c)	∃x S(x)

# First quantifier rule of proof: ∀Elim

# **Proof example:** ∀Elim, ∃Intro

1. 
$$\forall x ( Puf(x) \rightarrow YelBk(x) )$$

2. Puf(a)

3. 
$$Puf(a) \rightarrow YelBk(a) \forall Elim:1$$

4. YelBk(a) 
$$\rightarrow$$
Elim: 3,2

5. 
$$\exists x \, YelBk(x)$$
  $\exists Intro: 4$ 

# **Proof example**

#### Conjunction Introduction (∧ Intro)

### Conjunction Elimination $(\land Elim)$

$$| P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_i \rangle$$

$$| P_i \rangle$$

$$\begin{vmatrix} P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_i \\ \vdots \\ P_i \end{vmatrix}$$

#### $\perp$ Introduction $(\perp Intro)$

#### $\perp$ Elimination

#### Identity Introduction (= Intro)

$$\triangleright$$
  $\mathbf{n} = \mathbf{n}$ 

#### Identity Elimination (= Elim)

$$\begin{array}{c} P(n) \\ \vdots \\ n=m \\ \vdots \\ P(m) \end{array}$$

### Disjunction Introduction (∨ Intro)

$$P_i \\ \vdots \\ P_1 \lor \ldots \lor P_i \lor \ldots \lor P_n$$

### Disjunction Elimination (∨ Elim)





 $(\leftrightarrow Intro)$ 

 $(\rightarrow Intro)$ 

Conditional Introduction

Biconditional Introduction

#### Conditional Elimination $(\rightarrow Elim)$

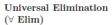
Biconditional Elimination

 $P \leftrightarrow Q \ (\mathrm{or} \ Q \leftrightarrow P)$ 



 $(\leftrightarrow Elim)$ 

### Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.

#### Negation Introduction (¬ Intro)



# (¬ Elim)



# Negation Elimination



(∃ Intro)

**Existential Introduction** 

#### **Existential Elimination** (∃ Elim)



where c does not occur outside the subproof where it is introduced.