PH126 Logic I · Lecture 5
Lecturer: s.butterfill@warwick.ac.uk

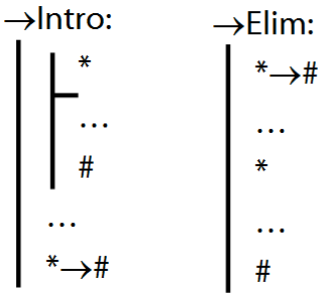
⊥, ¬

P	¬P	⊥	P∧¬P
T	F	F	F
F	T	F	F

Proof example

1.	R∧¬R	
2.	R	∧Elim: 1
3.	¬R	∧Elim: 1
4.	⊥	⊥Intro: 2,3
5.	P	⊥Elim: 4

Rules of Proof for →



Proof example for →Intro

1.	P→Q	//If things move, Zeno is wrong
2.	P	//Things move
3.	Q	//Zeno is wrong

Proof example for →Elim (to complete)

1. P→Q
2. Q→R

6. P→R

Not all proofs have premises

1. P
2. P

3. P → P

1. P ∨ Q
2. ¬Q

8. P

9. ¬Q → P

Rules of Proof for ∨

∨Elim:

P1 ∨ P2

  P1

  …

  Q

  P2

  …

  Q

…

Q

To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

Example proof

1. R ∨ S
2. R
3. S ∨ R ∨Intro: 2
4. S
5. S ∨ R ∨Intro: 4
6. S ∨ R ∨Elim: 1,2-3,4-5

The Syntax of FOL

We define what counts as a sentence of FOL using rules. E.g.:

3. P, Q, R, … are sentences
4. If * is a sentence, then ¬* is a sentence
1. If * and # are sentences, then so is(* ∧ #)
2. If * and # are sentences, then so is (* ∨ #)

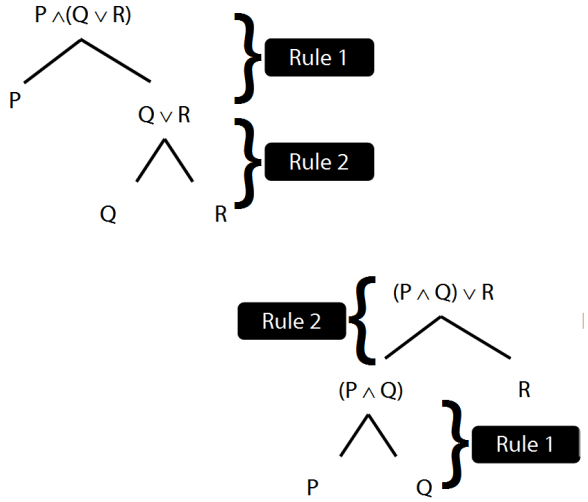
So:

a. P is a sentence // rule 3

b. ¬P is a sentence // rule 4, a

c. (¬P ∧ Q) is a sentence // rule 1, b, a

Notes: (1) There is no structural ambiguity in FOL because these rules are formulated to ensure that for any FOL sentence, there is exactly one way of constructing it. (2) We used the notion of a sentence in the rules that define what a sentence is.



Exercises 02

For your third seminar

Not for fast groups

3.8, 3.12–13 (∨ and ∧)

3.20, 3.21, 3.22 (trans.)

4.4–7 (truth tables)

4.12–4.14 (truth tables)

5.1–4 (validity)

6.1 (proofs)