**PH126 Logic I** Lecture 12
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## Quantifier Recap

Everything is broken
$\forall x\ \mathbf{Broken(x)}$

All my things are broken
$\forall x\ (\mathbf{BelongsToMe(x)} \rightarrow \mathbf{Broken(x)})$

Something is broken
$\exists x\ \mathbf{Broken(x)}$

Something of mine is broken
$\exists x\ (\mathbf{BelongsToMe(x)} \wedge \mathbf{Broken(x)})$

## How to determine truth of a sentence with an existential quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

## How to determine truth of a sentence with a universal quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL OF the new sentences are true, so is the original.

## Multiple quantifiers

'There is a store for everything'

$\exists y \forall x$ StoreFor(y,x)

$\forall y \exists x$ StoreFor(x,y)

Other sentences to translate:
'Wikipedia has an article about everything'
'Everyone hurts someone they love'
'Someone hurts everyone she loves'

∀Intro

| | |
|---|---|
| | c |
| | … |
| | S(c) |
| ∀x S(x) | |

**∀Intro** *Example proof*

| | | |
|---|---|---|
| 1. ∀x (Square(x) → Blue(x)) | | |
| | 2. ∀x Square(x) | |
| | | 3. a |
| | | 4. Square(a) ∀Elim: 2 |
| | | 5. Square(a)→Blue(a) ∀Elim: 1 |
| | | 6. Blue(a) →Elim: 4,5 |
| | x. ∀x Blue(x) | ∀Intro: 3-6 |
| y. ∀x Square(x) → ∀x Blue(x) ) | | →Intro: 2-x |

*Why is this proof incorrect?*

| | | |
|---|---|---|
| 1. ∀x Square(x) → ∀x Blue(x) | | |
| | 2. b | |
| | 3. Square(b) → Blue(g) | ∀Elim: 1 |
| x. ∀x ( Square(x) → Blue(x) ) | | ∀Intro: 2-3 |

## Intuitve summary of quantifier rules

**∀Elim**
If it's true of everything it's true of Baudrillard

**∃Intro**
If it's true of Baudrillard it's true of something

**∃Elim**
If it's true of something and Q follows no matter which something it is, then Q

**∀Intro**
If it's true of an arbitrary thing, then it's true of everything.