

Quantifier equivalences

$$P \rightarrow Q \quad \models \quad \neg Q \rightarrow \neg P$$

$$\forall x (\text{Square}(x) \rightarrow \text{Broken}(x)) \\ \models \quad \models \quad \forall x (\neg \text{Broken}(x) \rightarrow \neg \text{Square}(x))$$

$$P \rightarrow Q \quad \models \quad \neg P \vee Q$$

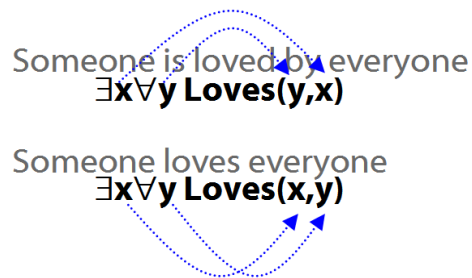
$$\forall x (\text{Square}(x) \rightarrow \text{Broken}(x)) \\ \models \quad \models \quad \forall x (\neg \text{Square}(x) \vee \text{Broken}(x))$$

$$\neg P \vee Q \quad \models \quad \neg (P \wedge \neg Q)$$

$$\exists x \neg F(x) \quad \models \quad \neg \forall x F(x)$$

$$\forall x (\neg \text{Square}(x) \vee \text{Broken}(x)) \\ \models \quad \models \quad \exists x \neg (\text{Square}(x) \wedge \neg \text{Broken}(x))$$

$$\exists x \neg \text{Created}(x) \quad \models \quad \neg \forall x \text{Created}(x)$$



Quantifiers and identity

Ahura Mazda created everything:
 $\forall x \text{Created}(a, x)$

Ahura Mazda created everything apart from himself: $\forall x (\neg (x=a) \rightarrow \text{Created}(a, x))$

Quantifiers and negation

Some person is dead.
 $\exists x (\text{Person}(x) \wedge \text{Dead}(x))$

Some person is not dead.
 $\exists x (\text{Person}(x) \wedge \neg \text{Dead}(x))$

No person is dead.
 $\neg \exists x (\text{Person}(x) \wedge \text{Dead}(x))$

Every person is dead.
 $\forall x (\text{Person}(x) \rightarrow \text{Dead}(x))$

Every person is not dead.
 $\forall x (\text{Person}(x) \rightarrow \neg \text{Dead}(x))$

Not every person is dead.
 $\neg \forall x (\text{Person}(x) \rightarrow \text{Dead}(x))$

Multiple quantifiers

'There is a store for everything'

$$\exists y \forall x \text{StoreFor}(y, x)$$

$$\forall y \exists x \text{StoreFor}(x, y)$$

Other sentences to translate:

'Wikipedia has an article about everything'

'Everyone hurts someone they love'

'Someone hurts everyone she loves'

Intuitive summary of quantifier rules

\forall Elim

If it's true of everything it's true of Baudrillard

\exists Intro

If it's true of Baudrillard it's true of something

\exists Elim

If it's true of something and Q follows no matter which something it is, then Q

\forall Intro

If it's true of an arbitrary thing, then it's true of everything.

Number

There are at least two squares:

$$\exists x \exists y (\text{Square}(x) \wedge \text{Square}(y) \wedge \neg x=y)$$

At least two squares are broken:

$$\exists x \exists y (\text{Square}(x) \wedge \text{Broken}(x) \wedge \text{Square}(y) \wedge \text{Broken}(y) \wedge \neg x=y)$$

There are at least three squares:

$$\exists x \exists y \exists z (\text{Square}(x) \wedge \text{Square}(y) \wedge \text{Square}(z) \wedge \neg x=y \wedge \neg y=z \wedge \neg x=z)$$

There are at most two squares:

\neg There are at least three squares
 $\neg \exists x \exists y \exists z (\text{Square}(x) \wedge \text{Square}(y) \wedge \text{Square}(z) \wedge \neg x=y \wedge \neg y=z \wedge \neg x=z)$

There are exactly two squares:

There are at most two squares \wedge
There are at least two squares

Number: Alternatives

There is at most one square:

$\forall x \forall y ((\text{Square}(x) \wedge \text{Square}(y)) \rightarrow x=y)$

There are at most two squares:

$\forall x \forall y \forall z ($
 $(\text{Square}(x) \wedge \text{Square}(y) \wedge \text{Square}(z))$
 \rightarrow
 $(x=y \vee y=z \vee x=z)$
 $)$

There is exactly one square:

$\exists x (\text{Square}(x) \wedge \forall y (\text{Square}(y) \rightarrow x=y))$

There are exactly two squares:

$\exists x \exists y (\text{Square}(x) \wedge \text{Square}(y) \wedge \neg x=y \wedge$
 $\forall z (\text{Square}(z) \rightarrow (z=x \vee z=y)))$

The

‘The’ can be a quantifier, e.g. ‘the square is broken’. How to formalise it?

The square is broken

There is exactly one square and it is
broken

There is at most one square and there is at
least one square and it is broken

There is at most one square and there is at
least one square and all squares are
broken

$\neg \exists x \exists y (\text{Square}(x) \wedge \text{Square}(y) \wedge \neg x=y)$
 $\wedge \exists x \text{Square}(x)$
 $\wedge \forall x (\text{Square}(x) \rightarrow \text{Broken}(x))$