PH126 Logic I Lecture 9

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Proof example

1. $\neg P \lor R$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \to R$

The Soundness & Completeness Theorems

'**A** $\vdash$ B' means there is a proof of B using premises **A**

'$\vdash$ B' means there is a proof of B using no premises

'**A** $\models$ B' means B is a logical consequence of A

'$\models$ B' means B is a tautology

'**A** $\models_T$ B' means B is a logical consequence of A *just in virtue of the meanings of truth-functions* (the textbook LPL calls this 'tautological consequence')

Soundness: If **A** $\vdash$ B then **A** $\models$ B

I.e. if you can prove it in Fitch, it's valid

Completeness: If **A** $\models_T$ B then **A** $\vdash$ B

I.e. if it's valid just in virtue of the meanings of truth functions, you can prove it in Fitch

Proof Example using negation-Intro

1. P

5. $\neg\neg P$

'If' and '$\to$' do not match

$\neg A$	Marnie will not miss her train
$A \to B$	If Marnie misses her train, she will arrive on time.

'If' and '$\to$' do match

$\neg A \lor B$	America does not exist $\lor$ Baudrillard is wrong
If A, B	If America exists, Baudrillard is wrong

If A, B	If you love logic, things will fall into place
$\neg(A \land \neg B)$	Not both: you take logic and things don't fall into place

Proof example

1. $A \land B$

$\neg(\neg A \lor \neg B)$

Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

If * and # are logically equivalent, then you take a complex formula and replace * with # without changing the truth table of the complex formula.

Example. Because $\neg(R \lor \neg P)$ is logically equivalent to $\neg R \land P$, it follows that:

$P \lor \neg(Q \land \neg(R \lor \neg P))$ is logically equivalent to $P \lor \neg(Q \land (\neg R \land P))$

And because $\neg(Q \land (\neg R \land P))$ is logically equivalent to $\neg Q \lor (R \lor \neg P)$, it follows that $P \lor \neg(Q \land (\neg R \land P))$ is logically equivalent to $P \lor (\neg Q \lor (R \lor \neg P))$.

Fubar rules

$\wedge$Fubar:

| *
| …
| * $\wedge$ #

Q1. What would be wrong with adding ∧Fubar to Fitch?

Q2. What would be wrong with having ∧Fubar in any *system of proof?*

Scope

In $P\wedge(Q\vee R)$, the scope of $\wedge$ is $P\wedge(Q\vee R)$

In $P\wedge(Q\vee R)$, the scope of $\vee$ is $(Q\vee R)$

In $(P\wedge Q)\vee R$, the scope of $\wedge$ is $(P\wedge Q)$

In $(P\wedge Q)\vee R$, the scope of $\vee$ is $(P\wedge Q)\vee R$

The scope of a connective is the smallest constituent expression which contains that connective.

Exercises 05

For your sixth seminar

Not for fast groups

A. From the LPL textbook:

7.9 (truth functions)
6.17–20 (proof)
6.33, 6.40
DO NOT USE TAUT CON. EVER.
8.24–25 (proofs/counterexamples)
12.4–5, *12.6–7 (counterexamples)

B. For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain : people and actions
D(x) : x is desirable
V(x) : x is virtuous
A(x) : x is an action
H(x) : x is a person
P(x,y): x performed y

i. $\forall x\ [\ D(x) \rightarrow V(x)\ ]$

ii. $\forall x\ [\ [A(x)\wedge D(x)\ ] \rightarrow V(x)\ ]\ ]$

iii. $\exists x\ [\ A(x) \wedge \neg[\ D(x) \rightarrow V(x)\ ]\ ]$

*iv. $\exists x\ \forall y\ [[[H(x) \wedge A(y)] \wedge P(x,y)] \rightarrow V(y)]]$

**v. $\neg\exists x\ [\exists y\ [H(x) \wedge P(x,y) \wedge A(y) \wedge \neg V(y)]$
$\wedge \neg\exists z[P(x,z) \wedge A(z) \wedge V(z)]]$