PH133 Logic Lecture 7
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Proof example: ∀Elim, ∃Intro

1.	∀x (Puf(x) → YelBk(x))	
2.	Puf(a)	
3.	Puf(a) → YelBk(a)	∀Elim:1
4.	YelBk(a)	→Elim: 3,2
5.	∃x YelBk(x)	∃Intro: 4

∀Elim

∀x S(x)

…

S(c)

∃Intro

S(a)

…

∃x S(x)

'If' and '→' do match

¬A ∨ B	America does not exist ∨ Baudrillard is wrong
If A, B	If America exists, Baudrillard is wrong

If A, B	If you love logic, things will fall into place
¬(A ∧ ¬B)	Not both: you take logic and things don't fall into place

Multiple quantifiers: simple examples

"Something is above something"
∃x ∃y Above(x,y)

"Everyone likes puffins"
∀y ∀x (Puffin(x) → Likes(y,x))

Puffin(x) : x is a puffin

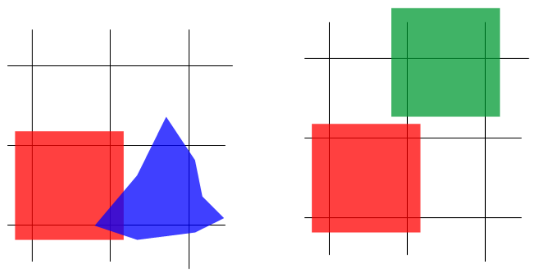
b : Steve

Likes(x,y) : x likes y

"Something makes someone want to die inside"
∃x ∃y WantToDieInside(x,y)

WantToDieInside(x,y) : x makes y want to die inside

(1) **∃x ∃y Above(x,y)**
"Something is above something"



(1) is FALSE (1) is TRUE

Scope

Underlining shows the scope of the quantifiers

"All squares are blue"
∀x (Square(x) → Blue(x))

"If everything is square, everything is blue"
∀x Square(x) → ∀x Blue(x)

Quantifiers bind variables

Read §9.2 of Barwise & Etchemendy

Proof example: ∃Elim

	1. ∃x (Blue(x) ∧ Square(x))	
b	2. Blue(b) ∧ Square(b)	
	3. Blue(b)	∧Elim: 2
	4. ∃x Blue(x)	∃Intro: 3
	x. ∃x Blue(x)	∃Elim: 2-4, 1

Invalid use of ∃Elim

new name

must not appear in conclusion of subproof

	1. ∃x (Blue(x) ∧ Square(x))	
b	2. Blue(b) ∧ Square(b)	
	3. Blue(b)	∧Elim: 2
	x. Blue(b)	∃Elim: 2-3, 1

Conjunction Introduction
($\wedge$ Intro)

P_1
$\Downarrow$
P_n
$\vdots$
$\triangleright$ $P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination
($\wedge$ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$
$\vdots$
$\triangleright$ P_i

Disjunction Introduction
($\vee$ Intro)

P_i
$\vdots$
$\triangleright$ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination
($\vee$ Elim)

$P_1 \vee \ldots \vee P_n$
$\vdots$
> P_1
> $\vdots$
> S

$\Downarrow$
> P_n
> $\vdots$
> S

$\vdots$
$\triangleright$ S

Negation Introduction
($\neg$ Intro)

> P
> $\vdots$
> $\perp$

$\triangleright$ $\neg P$

Negation Elimination
($\neg$ Elim)

$\neg\neg P$
$\vdots$
$\triangleright$ P

$\perp$ Introduction
($\perp$ Intro)

P
$\vdots$
$\neg P$
$\vdots$
$\triangleright$ $\perp$

$\perp$ Elimination
($\perp$ Elim)

$\perp$
$\vdots$
$\triangleright$ P

Conditional Introduction
($\rightarrow$ Intro)

> P
> $\vdots$
> Q

$\triangleright$ $P \rightarrow Q$

Conditional Elimination
($\rightarrow$ Elim)

$P \rightarrow Q$
$\vdots$
P
$\vdots$
$\triangleright$ Q

Biconditional Introduction
($\leftrightarrow$ Intro)

> P
> $\vdots$
> Q

> Q
> $\vdots$
> P

$\triangleright$ $P \leftrightarrow Q$

Biconditional Elimination
($\leftrightarrow$ Elim)

$P \leftrightarrow Q$ (or $Q \leftrightarrow P$)
$\vdots$
P
$\vdots$
$\triangleright$ Q

Reiteration
(Reit)

P
$\vdots$
$\triangleright$ P

Identity Introduction
($=$ Intro)

$\triangleright$ $n = n$

Identity Elimination
($=$ Elim)

$P(n)$
$\vdots$
$n = m$
$\vdots$
$\triangleright$ $P(m)$

Universal Elimination
($\forall$ Elim)

$\forall x\, S(x)$
$\vdots$
$\triangleright$ $S(c)$

Universal Introduction
($\forall$ Intro)

> $\boxed{c}$
> $\vdots$
> $P(c)$

$\triangleright$ $\forall x\, P(x)$

where c does not occur outside the subproof where it is introduced.

Existential Introduction
($\exists$ Intro)

$S(c)$
$\vdots$
$\triangleright$ $\exists x\, S(x)$

Existential Elimination
($\exists$ Elim)

$\exists x\, S(x)$
$\vdots$
> $\boxed{c}$ $S(c)$
> $\vdots$
> Q

$\triangleright$ Q

where c does not occur outside the subproof where it is introduced.