PH126 Logic I Lecture 11
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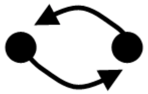
Reflexive, symmetric and transitive

A *reflexive* relation is one that everything bears to itself. (E.g. everything is the SameShape as itself. E.g. of *non*-reflexive: not everything is LeftOf itself).

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y) is symmetric, LeftOf(x,y) is *not* symmetric.)



A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is *not* transitive)

To show that a particular relation lacks one of these properties, we produce and argument and a counterexample.

E.g. Show that NotAdjacent is not transitive …

If NotAdjacent were transitive, the following argument would be logically valid:

1. NotAdjacent(a, b)
2. NotAdjacent(b, c)

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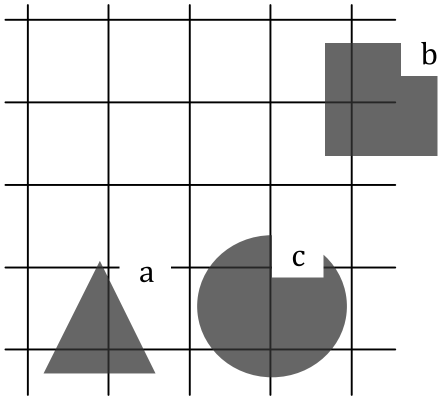
3. NotAdjacent(a, c)

A counterexample to this argument:



Artificial relation examples

EqualToOrLeftOf(x, y) iff
x = y or LeftOf(x, y)

EqualToOrAdjacent(x, y) iff
x=y or Adjacent(x, y)

JohnOrAyesha(x, y) iff
x = John and y = Ayesha
or x = Ayesha and y = John

JohnToAyesha(x, y) iff
x = John and y = Ayesha

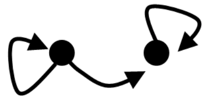
Quantifiers and relations

reflexive $\forall x\, R(x,x)$

symmetric $\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

transitive
$\forall x \forall y \forall z\, (\, [\, R(x,y) \land R(y,z)\,] \rightarrow R(x,z)\,)$

A transitive, reflexive and non-symmetric relation:



That relation is a counterexample to this argument:

$\forall x\, R(x,x)$
$\forall x \forall y \forall z\, (\, [\, R(x,y) \land R(y,z)\,] \rightarrow R(x,z)\,)$

$\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

Formal statement of counterexample:
Domain: {a,b}
R : { <a,a>, <a,b>, <b,b> }

Ex. Is this argument valid?

$\forall x \forall y \forall z\, (\, [\, R(x,y) \land R(y,z)\,] \rightarrow R(x,z)\,)$
$\forall x \forall y\, [\, R(x,y) \rightarrow R(y,x)\,]$

$\forall x\, R(x,x)$

Exercises 06

For your seventh seminar

Not for fast groups

A. From the LPL textbook:

9.15–17, *9.18–19 (trans.)

10.20, 10.22

11.2 (multiple quantifiers)

12.9–10

Short of time? Skip the exercises marked *

B. Each line of the table describes a combination of properties. E.g. line 1 specifies a relation that is reflexive, symmetric and transitive, line 2 specifies a relation that is reflexive, symmetric and not transitive. Complete the table by adding an example relation on each line.

	Example	Reflexive	Symmetric	Transitive
1	= (identity)	Y	Y	Y
2		Y	Y	N
3		Y	N	Y
4		Y	N	N
5		N	Y	Y
6		N	Y	N
7		N	N	Y
8		N	N	N