PH133 Logic Lecture 6
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Rule of proof: ∀Elim

∀Elim

| $\forall x\, S(x)$
| …
| $S(c)$

DeMorgan and other equivalences

$\neg(A \wedge B) \dashv\vDash (\neg A \vee \neg B)$
$\neg(A \vee B) \dashv\vDash (\neg A \wedge \neg B)$
$A \rightarrow B \dashv\vDash \neg A \vee B$
$\neg(A \rightarrow B) \dashv\vDash \neg(\neg A \vee B) \dashv\vDash A \wedge \neg B$

Scope

In $P \wedge (Q \vee R)$, the scope of $\wedge$ is $P \wedge (Q \vee R)$
In $P \wedge (Q \vee R)$, the scope of $\vee$ is $(Q \vee R)$
In $(P \wedge Q) \vee R$, the scope of $\wedge$ is $(P \wedge Q)$
In $(P \wedge Q) \vee R$, the scope of $\vee$ is $(P \wedge Q) \vee R$

The scope of a connective is the smallest constituent expression which contains that connective.

Subproofs are tricky

Step 7 of this proof is wrong. Why?

R	S	R ∨ S	R ∧ S
T	F	T	F

T | 1. $R \vee S$
| | 2. | R
| | 3. | $S \vee R$ ∨Intro: 2
| | 4. | S
| | 5. | $S \vee R$ ∨Intro: 4
| 6. $S \vee R$ ∨Elim: 1,2-3,4-5
F | 7. $R \wedge S$ ∧Intro: 2,4

Which step of this proof is wrong? Why?

| 1. $\neg\neg(\neg A \wedge \neg\neg A)$
| 2. $(\neg A \wedge \neg\neg A)$ ¬Elim: 1
| 3. $(\neg A \wedge A)$ ¬Elim: 2

What not to confuse

$\exists x\, (\text{Square}(x) \wedge \text{Blue}(x))$ vs.
$\exists x\, \text{Square}(x) \wedge \exists x\, \text{Blue}(x)$
$\neg(P \vee Q)$ vs. $\neg P \vee \neg Q$
$\neg(P \wedge Q)$ vs. $\neg P \wedge \neg Q$
$\neg(P \rightarrow Q)$ vs. $P \rightarrow \neg Q$

Proof example with →

| 1. $P \rightarrow Q$
| 2. $\neg Q$
| 6. $\neg P$

¬Intro proof example

| 1. $A \wedge B$
| $\neg(\neg A \vee \neg B)$