**PH126 Logic I** Fast Lecture 6
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### Quantifier Recap

Everything is broken
**$\forall x$ Broken(x)**

All my things are broken
**$\forall x$ ( BelongsToMe(x) $\rightarrow$ Broken(x) )**

Something is broken
**$\exists x$ Broken(x)**

Something of mine is broken
**$\exists x$ ( BelongsToMe(x) $\wedge$ Broken(x) )**

### How to determine truth of a sentence with an existential quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

### How to determine truth of a sentence with a universal quantifier as the main connective

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ALL OF the new sentences are true, so is the original.

### Multiple quantifiers

There is a store for everything

$\exists y \forall x$ StoreFor(y,x)

$\forall y \exists x$ StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything
Everyone hurts someone they love
Someone hurts everyone she loves

$\forall$Intro

```
|  | c
|  |----
|  | ...
|  | S(c)
| ∀x S(x)
```

**$\forall$Intro** *Example proof*

```
| 1. ∀x (Square(x) → Blue(x))
|----
|  | 2.  ∀x Square(x)
|  |----
|  |  | 3. a
|  |  |----
|  |  | 4.  Square(a)              ∀Elim: 2
|  |  | 5.  Square(a)→Blue(a)      ∀Elim: 1
|  |  | 6.  Blue(a)                →Elim: 4,5
|  | x.  ∀x Blue(x)                ∀Intro: 3-6
| y. ∀x Square(x) → ∀x Blue(x) )  →Intro: 2-x
```

### Exercise

If the following is valid, give a formal proof. If it is not valid, give a counterexample.

```
| ∀x∀y∀z ( [ R(x,y) ∧ R(y,z) ] → R(x,z) )
| ∀x∀y [ R(x,y) → R(y,x) ]
|----
| ∀x R(x,x)
```

**"Every time I go to the dentist, somebody dies."**
$\forall t$ ((Time(t) $\wedge$ ToDentist(a,t) ) $\rightarrow$ $\exists x$ ( Person(x) $\wedge$ DiesAt(x,t) ) )

### Truth-functional completeness (LPL §7.4)

How many truth-functions can be constructed using 2 sentence letters?

| P | Q | P % Q |
|---|---|---|
| T | T | ? |
| T | F | ? |
| F | T | ? |
| F | F | ? |

Each of these can be T or F

Df. 'A set of truth-functors is said to be *expressively adequate* (or sometimes *functionally complete*) iff, for every truth-function whatever, there is a formula containing only those truth-functors which express that truth-function, i.e. which has as its truth-table the truth-table specifying that function.' (Bostock, *Intermediate Logic* p. 45)

Illustration of the proof that $\{\neg, \wedge, \vee\}$ is truth-functionally complete:

| P | Q | $P \rightarrow Q$ | |
|---|---|---|---|
| T | T | T | $[P \wedge Q] \vee$ |
| T | F | F | |
| F | T | T | $[\neg P \wedge Q] \vee$ |
| F | F | T | $[\neg P \wedge \neg Q]$ |

$[P \wedge Q] \vee [\neg P \wedge Q] \vee [\neg P \wedge \neg Q]$

*Exercise* assuming $\{\neg,\vee,\wedge\}$ is truth-functionally complete, show that $\{\neg,\vee\}$ is.

**Quantifier equivalences**

$P \rightarrow Q \quad \Dashv\,\models \neg Q \rightarrow \neg P$

$\forall x\ (\ Square(x) \rightarrow Broken(x)\ )$

$\quad \Dashv\,\models \forall x\ (\neg Broken(x) \rightarrow \neg Square(x)\ )$

$P \rightarrow Q \quad \Dashv\,\models \neg P \vee Q$

$\forall x\ (\ Square(x) \rightarrow Broken(x)\ )$

$\quad \Dashv\,\models \forall x\ (\neg Square(x) \vee Broken(x)\ )$

$\neg P \vee Q \Dashv\,\models \neg(P \wedge \neg Q)$

$\exists x\ \neg F(x) \Dashv\,\models \neg \forall x\ F(x)$

$\forall x\ (\neg Square(x) \vee Broken(x)\ )$

$\quad \Dashv\,\models \exists \neg x\ (\ Square(x) \wedge \neg Broken(x)\ )$

$\exists x\ \neg Created(x) \Dashv\,\models \neg \forall x\ Created(x)$

**Exercises 07**

*For your seventh seminar*

*Only for fast groups*

11.10 (nb. especially no. 20)
11.13
13.43–45
13.49–50
14.2–3
14.10–11

*Despite what book says, DON'T even THINK about using TAUT CON.*