PH126 Logic I Lecture 8

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Truth table for \rightarrow

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for \rightarrow is fixed if we accept \rightarrow Elim and \rightarrow Intro.

How do the rules of proof for \rightarrow fix its truth table?

Α	В	$A \rightarrow B$
Т	Т	
Т	F	
F	T	
F	F	

'If' and '→' do not match

'If' and '→' do match

F	- ¬A∨B If A, B	America does not exist \vee Baudrillard is wrong
	If A, B	If America exists, Baudrillard is wrong
\vdash	If A, B	If you love logic, things will fall into place Not both: you take logic and things don't fall into place
	¬(A ∧ ¬B)	Not both: you take logic and things don't fall into place

How to determine the truth of sentences involving \exists

- 1. Give every object a name.
- 2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
- 3. If ANY OF the new sentences are true, so is the original.

What not to confuse

 $\exists x \ (Square(x) \land Blue(x)) \ vs.$ $\exists x \ Square(x) \land \exists x \ Blue(x)$

 $\forall x \ (Square(x) \rightarrow Blue(x)) \ vs.$ $\forall x \ Square(x) \rightarrow \forall x \ Blue(x)$

 $\neg (P \lor Q) \text{ vs. } \neg P \lor \neg Q$

 $\neg (P \land Q) \text{ vs. } \neg P \land \neg Q$

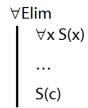
 $\neg (P \rightarrow Q) \text{ vs. } P \rightarrow \neg Q$

Scope

In $P\Lambda(QVR)$, the scope of Λ is $P\Lambda(QVR)$ In $P\Lambda(QVR)$, the scope of V is (QVR)In $(P\Lambda Q)VR$, the scope of Λ is $(P\Lambda Q)$ In $(P\Lambda Q)VR$, the scope of V is $(P\Lambda Q)VR$

The scope of a connective is the smallest constituent expression which contains that connective.

First quantifier rule of proof: ∀Elim



Proof example: ∀Elim, ∃Intro

∃x YelBk(x)

1.
$$\forall x (Puf(x) \rightarrow YelBk(x))$$

2. $Puf(a)$
3. $Puf(a) \rightarrow YelBk(a) \quad \forall Elim: 1$
4. $YelBk(a) \rightarrow Elim: 3,2$

∃Intro: 4

Fubar rules



- Q1. What would be wrong with adding Λ Fubar to Fitch?
- Q2. What would be wrong with having Λ Fubar in any system of proof?

Proof Example using negation-Intro



More proof examples

Exercises 04

For your fifth seminar

Not for fast groups

(Bit more than usual this week because reading week gives you two weeks to complete these)

- 3.14–15 (counterexamples)
- 6.8 (proof)
- 6.24–7 (proof)
- 7.1–2, *7.3–6 (truthtables)
- 8.1 (conditionals; yes/no answers are ok)
- 8.17-19, *8.20-23
- 9.1 odd nos only (quantifiers)
- 9.2 even nos only (quantifiers)
- 9.4–5, 9.8–10, 9.12 (quantifiers)