# Rules of Thumb

Logic 1: PH126

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## 1. About

There are exceptions to these rules of thumb. But they are often useful.

## 2. Proofs

### 2.1. Starting

First ask, 'Which $Elim$ rule can apply to this premise?' for each premise. Apply any $Elim$ rules you can first (except $\forall Elim$—see below).

Then ask, 'Which $Intro$ rule would get me to this conclusion?'

If you still can't get to the conclusion, try using $\neg Intro$. (You can do use $\neg Intro$ even if the conclusion isn't a negated sentence. For example, if the conclusion is $A \vee B$, create a subproof with $\neg(A \vee B)$ as premise, derive a contradiction, use $\neg Intro$ to get $\neg\neg(A \vee B)$ then use $\neg Elim$.)

### 2.2. $\forall Elim$

Use $\forall Elim$ as late as possible in your proof.

Only apply $\forall Elim$ using names that already occur in your proof.

### 2.3. $\perp$

Don't use $\perp Elim$: you need $\neg Intro$.

When using $\vee Elim$, if you are struggling to get two subproofs with matching conclusions try using $\perp Elim$ or $\vee Intro$.

### 2.4. What to do when ...

Having sentences that start with negation ($\neg$) as premises is awkward. Learning some standard proofs will help you.

If you have $\neg(A \vee B)$, you can get $\neg A$ like this:

| | | |
|---|---|---|
| 1 | $\neg(A \vee B)$ | |
| 2 |   $A$ | |
| 3 |   $(A \vee B)$ | $\vee Intro$, 2 |
| 4 |   $\perp$ | $\perp Intro$, 1, 3 |
| 5 | $\neg A$ | $\neg Intro$ 2–4 |

If you have $\neg(A \vee B)$, you can also get $\neg B$ using a proof just like the one above.

If you have $\neg(A \to B)$, you can get $A$ like this:

| | | |
|---|---|---|
| 1 | $\neg(A \to B)$ | |
| 2 |   $\neg A$ | |
| 3 |     $A$ | |
| 4 |     $\perp$ | $\perp Intro$, 2,3 |
| 5 |     $B$ | $\perp Elim$, 4 |
| 6 |   $A \to B$ | $\to Intro$, 3–5 |
| 7 |   $\perp$ | $\perp Intro$, 1, 7 |
| 8 | $\neg\neg A$ | $\neg Intro$ 2–7 |
| 9 | $A$ | $\neg Elim$ 8 |

If you have $\neg(A \to B)$, you can get $\neg B$ like this:

| | | |
|---|---|---|
| 1 | $\neg(A \to B)$ | |
| 2 |   $B$ | |
| 3 |     $A$ | |
| 4 |     $B$ | $Reit$, 2 |
| 5 |   $A \to B$ | $\to Intro$, 3–4 |
| 6 |   $\perp$ | $\perp Intro$, 1, 7 |
| 7 | $\neg B$ | $\neg Intro$ 2–6 |

## 3. Translation

Use $\forall$ with $\to$, e.g.

$$\forall x(Square(x) \to Broken(x))$$

means all squares are broken.

Use $\exists$ with $\wedge$, e.g.

$$\exists x(Square(x) \wedge Broken(x))$$

means some square is broken.

English sentences with mixed quantifiers are ambiguous (e.g. 'There is a store for everything.').