

**Conjunction Introduction**  
( $\wedge$  Intro)

$$\begin{array}{|l} P_1 \\ \vdots \\ P_n \\ \hline \triangleright P_1 \wedge \dots \wedge P_n \end{array}$$

**Conjunction Elimination**  
( $\wedge$  Elim)

$$\begin{array}{|l} P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n \\ \hline \triangleright P_i \end{array}$$

**$\perp$  Introduction**  
( $\perp$  Intro)

$$\begin{array}{|l} P \\ \vdots \\ \neg P \\ \hline \triangleright \perp \end{array}$$

**$\perp$  Elimination**  
( $\perp$  Elim)

$$\begin{array}{|l} \perp \\ \vdots \\ P \\ \hline \triangleright P \end{array}$$

**Identity Introduction**  
(= Intro)

$$\triangleright n = n$$

**Identity Elimination**  
(= Elim)

$$\begin{array}{|l} P(n) \\ \vdots \\ n = m \\ \vdots \\ P(m) \\ \hline \triangleright P(m) \end{array}$$

**Disjunction Introduction**  
( $\vee$  Intro)

$$\begin{array}{|l} P_i \\ \vdots \\ \hline \triangleright P_1 \vee \dots \vee P_i \vee \dots \vee P_n \end{array}$$

**Disjunction Elimination**  
( $\vee$  Elim)

$$\begin{array}{|l} P_1 \vee \dots \vee P_n \\ \vdots \\ \hline \begin{array}{|l} P_1 \\ \vdots \\ S \end{array} \\ \hline \downarrow \\ \begin{array}{|l} P_n \\ \vdots \\ S \end{array} \\ \hline \triangleright S \end{array}$$

**Conditional Introduction**  
( $\rightarrow$  Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ Q \end{array} \\ \hline \triangleright P \rightarrow Q \end{array}$$

**Conditional Elimination**  
( $\rightarrow$  Elim)

$$\begin{array}{|l} P \rightarrow Q \\ \vdots \\ P \\ \hline \triangleright Q \end{array}$$

**Biconditional Introduction**  
( $\leftrightarrow$  Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ Q \end{array} \\ \hline \begin{array}{|l} Q \\ \vdots \\ P \end{array} \\ \hline \triangleright P \leftrightarrow Q \end{array}$$

**Biconditional Elimination**  
( $\leftrightarrow$  Elim)

$$\begin{array}{|l} P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ \vdots \\ P \\ \hline \triangleright Q \end{array}$$

**Universal Elimination**  
( $\forall$  Elim)

$$\begin{array}{|l} \forall x S(x) \\ \vdots \\ \hline \triangleright S(c) \end{array}$$

**Universal Introduction**  
( $\forall$  Intro)

$$\begin{array}{|l} \boxed{c} \\ \vdots \\ P(c) \\ \hline \triangleright \forall x P(x) \end{array}$$

where  $c$  does not occur outside the subproof where it is introduced.

**Negation Introduction**  
( $\neg$  Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ \perp \end{array} \\ \hline \triangleright \neg P \end{array}$$

**Negation Elimination**  
( $\neg$  Elim)

$$\begin{array}{|l} \neg \neg P \\ \vdots \\ P \\ \hline \triangleright P \end{array}$$

**Reiteration**  
(Reit)

$$\begin{array}{|l} P \\ \vdots \\ \hline \triangleright P \end{array}$$

**Existential Introduction**  
( $\exists$  Intro)

$$\begin{array}{|l} S(c) \\ \vdots \\ \hline \triangleright \exists x S(x) \end{array}$$

**Existential Elimination**  
( $\exists$  Elim)

$$\begin{array}{|l} \exists x S(x) \\ \vdots \\ \hline \begin{array}{|l} \boxed{c} S(c) \\ \vdots \\ Q \end{array} \\ \hline \triangleright Q \end{array}$$

where  $c$  does not occur outside the subproof where it is introduced.