

Conjunction Introduction
(\wedge Intro)

$$\begin{array}{|l} P_1 \\ \vdots \\ P_n \\ \hline \triangleright P_1 \wedge \dots \wedge P_n \end{array}$$

Conjunction Elimination
(\wedge Elim)

$$\begin{array}{|l} P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n \\ \hline \triangleright P_i \end{array}$$

\perp Introduction
(\perp Intro)

$$\begin{array}{|l} P \\ \vdots \\ \neg P \\ \hline \triangleright \perp \end{array}$$

\perp Elimination
(\perp Elim)

$$\begin{array}{|l} \perp \\ \vdots \\ P \\ \hline \triangleright P \end{array}$$

Identity Introduction
(= Intro)

$$\triangleright n = n$$

Identity Elimination
(= Elim)

$$\begin{array}{|l} P(n) \\ \vdots \\ n = m \\ \vdots \\ P(m) \\ \hline \triangleright P(m) \end{array}$$

Disjunction Introduction
(\vee Intro)

$$\begin{array}{|l} P_i \\ \vdots \\ \hline \triangleright P_1 \vee \dots \vee P_i \vee \dots \vee P_n \end{array}$$

Disjunction Elimination
(\vee Elim)

$$\begin{array}{|l} P_1 \vee \dots \vee P_n \\ \vdots \\ \hline \begin{array}{|l} P_1 \\ \vdots \\ S \end{array} \\ \vdots \\ \hline \begin{array}{|l} P_n \\ \vdots \\ S \end{array} \\ \hline \triangleright S \end{array}$$

Conditional Introduction
(\rightarrow Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ Q \end{array} \\ \hline \triangleright P \rightarrow Q \end{array}$$

Conditional Elimination
(\rightarrow Elim)

$$\begin{array}{|l} P \rightarrow Q \\ \vdots \\ P \\ \vdots \\ Q \\ \hline \triangleright Q \end{array}$$

Biconditional Introduction
(\leftrightarrow Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ Q \end{array} \\ \hline \begin{array}{|l} Q \\ \vdots \\ P \end{array} \\ \hline \triangleright P \leftrightarrow Q \end{array}$$

Biconditional Elimination
(\leftrightarrow Elim)

$$\begin{array}{|l} P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ \vdots \\ P \\ \vdots \\ Q \\ \hline \triangleright Q \end{array}$$

Universal Elimination
(\forall Elim)

$$\begin{array}{|l} \forall x S(x) \\ \vdots \\ S(c) \\ \hline \triangleright S(c) \end{array}$$

Universal Introduction
(\forall Intro)

$$\begin{array}{|l} \boxed{c} \\ \vdots \\ P(c) \\ \hline \triangleright \forall x P(x) \end{array}$$

where c does not occur outside the subproof where it is introduced.

Negation Introduction
(\neg Intro)

$$\begin{array}{|l} \begin{array}{|l} P \\ \vdots \\ \perp \end{array} \\ \hline \triangleright \neg P \end{array}$$

Negation Elimination
(\neg Elim)

$$\begin{array}{|l} \neg \neg P \\ \vdots \\ P \\ \hline \triangleright P \end{array}$$

Reiteration
(Reit)

$$\begin{array}{|l} P \\ \vdots \\ P \\ \hline \triangleright P \end{array}$$

Existential Introduction
(\exists Intro)

$$\begin{array}{|l} S(c) \\ \vdots \\ \hline \triangleright \exists x S(x) \end{array}$$

Existential Elimination
(\exists Elim)

$$\begin{array}{|l} \exists x S(x) \\ \vdots \\ \hline \begin{array}{|l} \boxed{c} S(c) \\ \vdots \\ Q \end{array} \\ \hline \triangleright Q \end{array}$$

where c does not occur outside the subproof where it is introduced.