Rules of Thumb for Logic 1

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There are exceptions to these rules of thumb. But they are often useful.

1. Proofs

1.1. Starting

First ask, 'Which *Elim* rule can apply to this premise?' for each premise. Apply any *Elim* rules you can first (except $\forall$*Elim*—see below).

Then ask, 'Which *Intro* rule would get me to this conclusion?'

If you still can't get to the conclusion, try using $\neg$*Intro*. (You can do use $\neg$*Intro* even if the conclusion isn't a negated sentence. For example, if the conclusion is $A \lor B$, create a subproof with $\neg(A \lor B)$ as premise, derive a contradiction, use $\neg$*Intro* to get $\neg\neg(A \lor B)$ then use $\neg$*Elim*.)

1.2. $\forall$*Elim*

Use $\forall$*Elim* as late as possible in your proof.

Only apply $\forall$*Elim* using names that already occur in your proof.

1.3. $\bot$

Don't use $\bot$*Elim*: you need $\neg$*Intro*.

When using $\lor$*Elim*, if you are struggling to get two subproofs with matching conclusions try using $\bot$*Elim* or $\lor$*Intro*.

1.4. What to do with $\neg$

Having sentences that start with negation ($\neg$) as premises is awkward. Learning some standard proofs will help you.

If you have $\neg(A \to B)$, you can get A like this:

1	$\neg(A \to B)$	
2	$\quad \neg A$	
3	$\qquad A$	
4	$\qquad \bot$	$\bot$*Intro*: 2,3
5	$\qquad B$	$\bot$*Elim*: 4
6	$\quad A \to B$	$\to$ *Intro*: 3–5
7	$\quad \bot$	$\bot$*Intro*: 1, 7
8	$\neg\neg A$	$\neg$*Intro*: 2–7
9	A	$\neg$*Elim*: 8

If you have $\neg(A \to B)$, you can get $\neg B$ like this:

1	$\neg(A \to B)$	
2	$\quad B$	
3	$\qquad A$	
4	$\qquad B$	*Reit*: 2
5	$\quad A \to B$	$\to$ *Intro*: 3–4
6	$\quad \bot$	$\bot$*Intro*: 1, 7
7	$\neg B$	$\neg$*Intro*: 2–6

If you have $\neg(A \lor B)$, you can get $\neg A$ like this:

1	$\neg(A \lor B)$	
2	$\quad A$	
3	$\quad (A \lor B)$	$\lor$*Intro*: 2
4	$\quad \bot$	$\bot$*Intro*: 1, 3
5	$\neg A$	$\neg$*Intro*: 2–4

You can use $\neg\exists x Blue(x)$ almost as if it were $\forall x \neg Blue(x)$: you can get $\neg Blue(b)$ like this:

1	$\neg\exists x Blue(x)$	
2	$\quad Blue(b)$	
3	$\quad \exists x Blue(x)$	$\exists$*Intro*: 2
4	$\quad \bot$	$\bot$*Intro*: 1, 3
5	$\neg Blue(b)$	$\neg$*Intro*: 2–4

2. Translation

Use $\forall$ with $\to$, e.g.

$$\forall x(Square(x) \to Broken(x))$$

means all squares are broken.

Use $\exists$ with $\land$, e.g.

$$\exists x(Square(x) \land Broken(x))$$

means some square is broken.

English sentences with mixed quantifiers are ambiguous (e.g. 'There is a store for everything.').

Conjunction Introduction
($\wedge$ Intro)

P_1
$\Downarrow$
P_n
$\vdots$
$\triangleright\ P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination
($\wedge$ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$
$\vdots$
$\triangleright\ P_i$

$\perp$ Introduction
($\perp$ Intro)

P
$\vdots$
$\neg P$
$\vdots$
$\triangleright\ \perp$

$\perp$ Elimination
($\perp$ Elim)

$\perp$
$\vdots$
$\triangleright\ P$

Identity Introduction
($=$ Intro)

$\triangleright\ n = n$

Identity Elimination
($=$ Elim)

$P(n)$
$\vdots$
$n = m$
$\vdots$
$\triangleright\ P(m)$

Disjunction Introduction
($\vee$ Intro)

P_i
$\vdots$
$\triangleright\ P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination
($\vee$ Elim)

$P_1 \vee \ldots \vee P_n$
$\vdots$
P_1
$\vdots$
S
$\Downarrow$
P_n
$\vdots$
S
$\vdots$
$\triangleright\ S$

Conditional Introduction
($\rightarrow$ Intro)

P
$\vdots$
Q
$\triangleright\ P \rightarrow Q$

Conditional Elimination
($\rightarrow$ Elim)

$P \rightarrow Q$
$\vdots$
P
$\vdots$
$\triangleright\ Q$

Biconditional Introduction
($\leftrightarrow$ Intro)

P
$\vdots$
Q
Q
$\vdots$
P
$\triangleright\ P \leftrightarrow Q$

Biconditional Elimination
($\leftrightarrow$ Elim)

$P \leftrightarrow Q$ (or $Q \leftrightarrow P$)
$\vdots$
P
$\vdots$
$\triangleright\ Q$

Universal Elimination
($\forall$ Elim)

$\forall x\, S(x)$
$\vdots$
$\triangleright\ S(c)$

Universal Introduction
($\forall$ Intro)

$\boxed{c}$
$\vdots$
$P(c)$
$\triangleright\ \forall x\, P(x)$

where c does not occur outside the subproof where it is introduced.

Negation Introduction
($\neg$ Intro)

P
$\vdots$
$\perp$
$\triangleright\ \neg P$

Negation Elimination
($\neg$ Elim)

$\neg\neg P$
$\vdots$
$\triangleright\ P$

Reiteration
(Reit)

P
$\vdots$
$\triangleright\ P$

Existential Introduction
($\exists$ Intro)

$S(c)$
$\vdots$
$\triangleright\ \exists x\, S(x)$

Existential Elimination
($\exists$ Elim)

$\exists x\, S(x)$
$\vdots$
$\boxed{c}\ S(c)$
$\vdots$
Q
$\triangleright\ Q$

where c does not occur outside the subproof where it is introduced.