Rules of Thumb for Logic 1

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There are exceptions to these rules of thumb. But they are often useful.

1. Proofs

1.1. Starting

First ask, 'Which Elim rule can apply to this premise?' for each premise. Apply any Elim rules you can first (except $\forall Elim$ —see below).

Then ask, 'Which *Intro* rule would get me to this conclusion?'

If you still can't get to the conclusion, try using $\neg Intro$. (You can do use $\neg Intro$ even if the conclusion isn't a negated sentence. For example, if the conclusion is $A \lor B$, create a subproof with $\neg (A \lor B)$ as premise, derive a contradiction, use $\neg Intro$ to get $\neg \neg (A \lor B)$ then use $\neg Elim$.)

1.2. ∀*Elim*

Use $\forall Elim$ as late as possible in your proof.

Only apply $\forall Elim$ using names that already occur in your proof.

1.3. ⊥

Don't use $\perp Elim$: you need $\neg Intro$.

When using $\forall Elim$, if you are struggling to get two subproofs with matching conclusions try using $\bot Elim$ or $\lor Intro$.

1.4. What to do with \neg

Having sentences that start with negation (\neg) as premises is awkward. Learning some standard proofs will help you.

If you have $\neg(A \rightarrow B)$, you can get A like this:

If you have $\neg(A \rightarrow B)$, you can get $\neg B$ like this:

$$\begin{array}{c|cccc}
1 & \neg(A \rightarrow B) \\
2 & B \\
3 & A \\
4 & B & Reit: 2 \\
5 & A \rightarrow B & \rightarrow Intro: 3-4 \\
6 & \bot & \bot Intro: 1, 7 \\
7 & \neg B & \neg Intro: 2-6
\end{array}$$

If you have $\neg (A \lor B)$, you can get $\neg A$ like this:

$$\begin{array}{c|cccc}
1 & \neg(A \lor B) \\
2 & A \\
3 & (A \lor B) & \lor Intro: 2 \\
4 & \bot & \bot Intro: 1, 3 \\
5 & \neg A & \neg Intro: 2-4
\end{array}$$

You can use $\neg \exists x Blue(x)$ almost as if it were $\forall x \neg Blue(x)$: you can get $\neg Blue(b)$ like this:

$$\begin{array}{c|cccc}
1 & \neg \exists xBlue(x) \\
2 & Blue(b) \\
3 & \exists xBlue(x) & \exists Intro: 2 \\
4 & \bot & \bot Intro: 1, 3 \\
5 & \neg Blue(b) & \neg Intro: 2-4
\end{array}$$

2. Translation

Use \forall with \rightarrow , e.g.

$$\forall x(Square(x) \rightarrow Broken(x))$$

means all squares are broken.

Use \exists with \land , e.g.

$$\exists x(Square(x) \land Broken(x))$$

means some square is broken.

English sentences with mixed quantifiers are ambiguous (e.g. 'There is a store for everything.').

Conjunction Introduction (∧ Intro)

Conjunction Elimination $(\land Elim)$

$$| P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_i \rangle$$

$$| P_i \rangle$$

$$\begin{vmatrix} P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_i \\ \vdots \\ P_i \end{vmatrix}$$

\perp Introduction $(\perp Intro)$

\perp Elimination

Identity Introduction (= Intro)

$$\triangleright$$
 $\mathbf{n} = \mathbf{n}$

Identity Elimination (= Elim)

$$\begin{array}{c} P(n) \\ \vdots \\ n=m \\ \vdots \\ P(m) \end{array}$$

Disjunction Introduction (∨ Intro)

$$P_i \\ \vdots \\ P_1 \lor \ldots \lor P_i \lor \ldots \lor P_n$$

Disjunction Elimination (∨ Elim)





 $(\leftrightarrow Intro)$

 $(\rightarrow Intro)$

Conditional Introduction

Biconditional Introduction

Conditional Elimination $(\rightarrow Elim)$

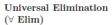
Biconditional Elimination

 $P \leftrightarrow Q \ (\mathrm{or} \ Q \leftrightarrow P)$



 $(\leftrightarrow Elim)$

Universal Introduction (∀ Intro)



where c does not occur outside the subproof where it is introduced.

Negation Introduction (¬ Intro)



(¬ Elim)



Negation Elimination



(∃ Intro)

Existential Introduction

Existential Elimination (∃ Elim)



where c does not occur outside the subproof where it is introduced.