

13.) given

$(\text{begin}(\text{set } x \ 3) \ x) , \quad \rho(x) = 99$

To Prove

The result of evaluation is 3.

Proof:

(Referred to Page 60, 61 & 63)

$D_1 = \text{LITERAL}$

$$\frac{}{\langle \text{LITERAL}(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}$$

$D_2 = \text{FORMALVAR}$

$$\frac{x \in \text{dom } \rho \{x \rightarrow 3\}}{\langle \text{VAR}(x), \xi, \phi, \rho \{x \rightarrow 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho \{x \rightarrow 3\} \rangle}$$

FORMAL ASSIGN

$x \in \text{dom } \rho$

D_1

D_2

$$\langle \text{SET}(x, \text{LITERAL}(3)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \{x \rightarrow 3\} \rangle$$

$$\langle \text{BEGIN}(\text{SET}(x, \text{LITERAL}(3)), \text{VAR}(x)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \{x \rightarrow 3\} \rangle$$

14.) Given

$$\langle \text{IF}(\text{VAR}(x), \text{VAR}(x), \text{LITERAL}(0)), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle v_1, \mathcal{E}', \phi, \rho' \rangle \quad (\text{expression 1})$$

and

$$\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle v_2, \mathcal{E}'', \phi, \rho'' \rangle \quad (\text{expression 2})$$

To Prove

if (expression 1 and expression 2) that implies $v_1 = v_2$

Proof:

We have two cases where $\text{VAR}(x) \in \text{dom } \rho$ (1. if True 2. if False)

We have two cases where $\text{VAR}(x) \in \text{dom } \mathcal{E}$ (1. if True 2. if False)

The differences between formal variable and global variable are subtle and similar enough. So, in this proof we show for $x \in \text{dom } \rho$ and evaluate the cases without loss of generality. The same applies to global var

For expression 2 $\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle v_2, \mathcal{E}'', \phi, \rho'' \rangle$

Using FormalVar,

$$\frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \mathcal{E}', \phi, \rho' \rangle \Downarrow \langle \rho(x), \mathcal{E}'', \phi, \rho'' \rangle}$$

(if TRUE)

$$\frac{\frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle \rho(x), \mathcal{E}', \phi, \rho' \rangle} \quad \frac{\rho(x) \neq 0}{v_1 \neq 0}}{\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle \rho(x), \mathcal{E}', \phi, \rho' \rangle} \quad \frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \mathcal{E}', \phi, \rho' \rangle \Downarrow \langle \underbrace{\rho(x)}_{v_2}, \mathcal{E}'', \phi, \rho'' \rangle}$$

$$\frac{\langle \text{IF}(\underbrace{\text{VAR}(x)}_{e_1}, \underbrace{\text{VAR}(x)}_{e_2}, \underbrace{\text{LITERAL}(0)}_{e_3}), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle \underbrace{\rho(x)}_{v_2}, \mathcal{E}'', \phi, \rho'' \rangle}{\rho(x) = v_2}$$

(if False)

$$\frac{\frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle \rho(x), \mathcal{E}', \phi, \rho' \rangle} \quad \frac{\rho(x) = 0}{v_1 = 0}}{\langle \text{VAR}(x), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle \rho(x), \mathcal{E}', \phi, \rho' \rangle} \quad \frac{\langle \text{LITERAL}(0), \mathcal{E}', \phi, \rho' \rangle \Downarrow \langle 0, \mathcal{E}', \phi, \rho' \rangle}{\langle \text{IF}(\underbrace{\text{VAR}(x)}_{e_1}, \underbrace{\text{VAR}(x)}_{e_2}, \underbrace{\text{LITERAL}(0)}_{e_3}), \mathcal{E}, \phi, \rho \rangle \Downarrow \langle 0, \mathcal{E}'', \phi, \rho'' \rangle}$$

We can observe that in both cases $\text{VAR}(x)$ results in $\rho(x)$. In (if True) case $\rho(x) \neq 0$ which means if-statement is evaluated to $e_2 = \text{VAR}(x)$ which again evaluates to $\rho(x)$. Final evaluation is $\rho(x)$ showing that evaluation of expression 1 = eval. of expression 2

In (if False) case $\rho(x) = 0$ which means if-statement is evaluated to $e_3 = \text{LIT}(0)$

In this case final result is 0 showing that evaluation of expression 1 = eval. of expression 2

Both cases when combined together evaluate to the same thing, thus showing $v_1 = v_2$.

- 21.) Given In ANK, if a variable is not bound to any environment then it implicitly creates a new global variable with value 0.
- In ICON, if a variable is not bound to any environment then it implicitly creates a new local variable with value 0, whose scope is the entire procedure in which assignment appears.

a.) ANK-like semantics for unbound variables.

$$\frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi \{x \rightarrow 0\}, \phi, \rho \rangle} \quad (\text{ANK VAR}) \quad (\xi(x) = 0)$$

$$\frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi' \{x \rightarrow v\}, \phi, \rho' \rangle} \quad (\text{ANK ASSIGN}) \quad (v = 0)$$

b.) ICON-like semantics for unbound variables

$$\frac{x \notin \text{dom } \rho \quad x \notin \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle e'(x), \xi', \phi, \rho' \{x \rightarrow 0\} \rangle} \quad (\text{ICON VAR}) \quad e'(x) = 0$$

$$\frac{x \notin \text{dom } \rho \quad x \notin \text{dom } \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \{x \rightarrow v\} \rangle} \quad (\text{ICON ASSIGN})$$

- c.) I prefer ICON-like semantics. Generally while programming I miss to give valid declarations for counter (ex: i) variables. I would prefer those variables to be in the local scope. ~~as those variables are used to stay in the local scopes.~~ As I keep using the same 'i' variable for wherever I use the counter variables so I would not prefer those messing with my global scope. Thus, placing it in global environment is unnecessary making the ICON-like semantics more apt.

- 20.) To Prove: Impure is deterministic for any e (i.e. global var, literal, formal var, formal assign, global assign, if True, if False, ApplyAdd, ApplyUser) and any environments, there is at most one v such that $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$

Proof:

1.) Literal.

When the last rule used in D_x is LITERAL, Derivation D_x must have the following form:

$$D_x = \frac{}{\langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v_x, \xi_x, \phi, \rho_x \rangle} \quad (\text{LITERAL}) \quad (e = \text{LITERAL}(v))$$

In D_x there is no change in the environments. So, D_y must also end in one of these. So, D_y must have the following form:

$$D_y = \frac{}{\langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v_y, \xi_y, \phi, \rho_y \rangle} \quad (\text{LITERAL})$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $v_x = v_y$, $\xi_x = \xi_y$, $\rho_x = \rho_y$. But because D_{x0} is strictly smaller than D_x , by induction hypothesis we can say, $v_x = v_y$, $\xi_x = \xi_y$, $\rho_x = \rho_y$. So, our obligation is met.

2.) FORMALVAR

When the last rule used in D_x is FORMALVAR, Derivation D_x must have the following form:

$$D_x = \frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho_x(x), \xi_x, \phi, \rho_y \rangle} \quad (\text{FORMALVAR})$$

The form of e is $\text{VAR}(x)$ and it can conclude to be FORMALVAR or GLOBALVAR, so D_y must also end in one of these. But GLOBALVAR requires $x \notin \text{dom } \rho$ & we know that in D_x $x \in \text{dom } \rho$. So D_y can only be FORMALVAR. So, D_y is of the form,

$$D_y = \frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho_y(x), \xi_x, \phi, \rho_y \rangle} \quad (\text{FORMALVAR})$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove $\rho_x(x) = \rho_y(y)$, $\xi_x = \xi_y$ and $\rho_x = \rho_y$. So, using induction hypothesis as there is no change in environments, we can say $\rho_x(x) = \rho_y(y)$, $\xi_x = \xi_y$, $\rho_x = \rho_y$. So, our obligation is met.

3.) GLOBAL VAR

It is similar to FORMAL VAR only that $x \notin \text{dom } \rho$ and $x \in \text{dom } \xi$ is the only difference. So, By induction hypothesis,

$$\xi_x(x) = \xi_y(x), \quad \xi_x = \xi_y, \quad \rho_x = \rho_y.$$

$$D_x = \frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi_x(x), \xi_x, \phi, \rho_x \rangle} \quad (\text{similarly for } D_y)$$

4.) FORMAL ASSIGN (Prof gave it as significant Hint to not repeating)
When the last rule used in D_x is FORMAL ASSIGN, derivation D_x must have following form:

$$D_x = \frac{x \in \text{dom } \rho \quad \frac{D_{x_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x \{x \mapsto v_{0x}\} \rangle}$$

5.) GLOBAL ASSIGN

When the last rule used in D_x is GLOBAL ASSIGN, Derivation D_x must have the following form:

$$\begin{aligned} \text{dom } \xi_x &= \text{dom } \xi'_x \\ \text{dom}(\xi' \{x \mapsto v\}) &= \text{dom } \xi' \cup \{x\} \\ &= \text{dom } \xi \cup \{x\} \\ \text{So, } \text{dom } \xi \cup \{x\} &= \text{dom } \xi \end{aligned}$$

$$D_x = \frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi \quad \frac{D_{x_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x \{x \mapsto v_{0x}\}, \phi, \rho'_x \rangle}$$

The form of e is $\text{SET}(x, e_0)$ and the only rules with $\text{SET}(x, e_0)$ in their conclusions are FORMAL OR GLOBAL ASSIGN, so D_y must be one of these. But FORMAL ASSIGN requires $x \in \text{dom } \rho$ & we know from D_x that $x \notin \text{dom } \rho, x \in \text{dom } \xi$. So, D_y also ends as GLOBAL ASSIGN. Derivation D_y must have the following form

$$D_y = \frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi \quad \frac{D_{y_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0y}, \xi'_y, \phi, \rho'_y \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0y}, \xi'_y \{x \mapsto v_{0y}\}, \phi, \rho'_y \rangle}$$

Our obligation is to prove that induction hypothesis holds for judgement below the line. We must therefore prove $v_{0x} = v_{0y}$, $\xi'_x \{x \mapsto v_{0x}\} = \xi'_y \{x \mapsto v_{0y}\}$ and $\rho'_x = \rho'_y$. But because derivation D_{x_0} is strictly smaller than D_x , we apply induction hypothesis to D_{x_0} . This assumption gives us equations

$$v_{0x} = v_{0y}, \quad \xi'_x = \xi'_y, \quad \rho'_x = \rho'_y$$

& given $v_{0x} = v_{0y}$ and $\xi'_x = \xi'_y$ it follows that $\xi'_x \{x \mapsto v_{0x}\} = \xi'_y \{x \mapsto v_{0y}\}$.
Our obligation is met.

6.) IF TRUE

When the last rule used in D_x is IF TRUE, derivation D_x must have the following form:

$$D_x = \frac{\frac{D_{x_1}}{\langle e_1, \xi, \phi, p \rangle \Downarrow \langle v_{1x}, \xi'_1, \phi, p'_1 \rangle} \quad v_1 \neq 0 \quad \frac{D_{x_2}}{\langle e_2, \xi'_1, \phi, p'_1 \rangle \Downarrow \langle v_{2x}, \xi''_x, \phi, p''_x \rangle}}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, p \rangle \Downarrow \langle v_{2x}, \xi''_x, \phi, p''_x \rangle}$$

The form of e is $\text{IF}(e_1, e_2, e_3)$ and the expression if true and if $v_1 \neq 0$ results in evaluating to if-true statement. So, D_y also evaluates similarly.

$$D_y = \frac{\frac{D_{y_1}}{\langle e_1, \xi, \phi, p \rangle \Downarrow \langle v_{1y}, \xi'_y, \phi, p'_y \rangle} \quad v_1 \neq 0 \quad \frac{D_{y_2}}{\langle e_2, \xi'_y, \phi, p'_y \rangle \Downarrow \langle v_{2y}, \xi''_y, \phi, p''_y \rangle}}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, p \rangle \Downarrow \langle v_{2y}, \xi''_y, \phi, p''_y \rangle}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must therefore prove $v_{2x} = v_{2y}$, $\xi''_x = \xi''_y$, $p''_x = p''_y$, and v_{1x} is the result of evaluation. But D_{x_1} is smaller than D_x we are permitted to assume that the induction hypothesis applies to D_{x_1} & D_{x_2} . This assumption gives us equations

$$v_{1x} = v_{1y}, \quad \xi'_x = \xi'_y, \quad p'_x = p'_y \quad \left(\begin{array}{l} \text{By induction hypothesis, } D_{x_1} \Rightarrow \xi'_1 = \xi'_y \\ \text{As we are using the same environment} \\ \text{variables in } D_{x_1} \text{ as input to } D_{x_2} \end{array} \right)$$

and given $v_{1x} = v_{1y}$, $\xi'_x = \xi'_y$, $p'_x = p'_y$ it follows that if True statement is true and $v_1 \neq 0$ it evaluates to e_2 and $v_{2x} = v_{2y}$, $\xi''_x = \xi''_y$ and $p''_x = p''_y$. Our obligation is met.

7.) IF FALSE (It is totally similar to IF TRUE. So, the difference is that it evaluates to e_3 as if-statement is false and $v_1 = 0$)

$$D_x = \frac{\frac{D_{x_1}}{\langle e_1, \xi, \phi, p \rangle \Downarrow \langle v_{1x}, \xi'_x, \phi, p'_x \rangle} \quad v_1 = 0 \quad \frac{D_{x_2}}{\langle e_3, \xi'_x, \phi, p'_x \rangle \Downarrow \langle v_{3x}, \xi''_x, \phi, p''_x \rangle}}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, p \rangle \Downarrow \langle v_{3x}, \xi''_x, \phi, p''_x \rangle}$$

$$D_y = \frac{\frac{D_{y_1}}{\langle e_1, \xi, \phi, p \rangle \Downarrow \langle v_{1y}, \xi'_y, \phi, p'_y \rangle} \quad v_1 = 0 \quad \frac{D_{y_2}}{\langle e_3, \xi'_y, \phi, p'_y \rangle \Downarrow \langle v_{3y}, \xi''_y, \phi, p''_y \rangle}}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, p \rangle \Downarrow \langle v_{3y}, \xi''_y, \phi, p''_y \rangle}$$

By using induction hypothesis, $v_{1x} = v_{1y}$, $\xi'_x = \xi'_y$, $p'_x = p'_y$ it follows if false stat and $v_1 = 0$ it evaluates to e_3 and so, $v_{3x} = v_{3y}$, $\xi''_x = \xi''_y$ and $p''_x = p''_y$. Our obligation is met.

8.) APPLY ADD

$$\phi(f) = \text{PRIMITIVE}(+)$$

$$D_x = \begin{array}{l} \langle e_1, \mathcal{E}_{0x}, \phi, \rho_{0x} \rangle \Downarrow \langle v_{1x}, \mathcal{E}_{1x}, \phi, \rho_{1x} \rangle \quad - D_1 \\ \langle e_2, \mathcal{E}_{1x}, \phi, \rho_{1x} \rangle \Downarrow \langle v_{2x}, \mathcal{E}_{2x}, \phi, \rho_{2x} \rangle \quad - D_2 \\ \hline \langle \text{APPLY}(f, e_1, e_2), \mathcal{E}_{0x}, \phi, \rho_{0x} \rangle \Downarrow \langle v_{1x} + v_{2x}, \mathcal{E}_{2x}, \phi, \rho_{2x} \rangle \end{array}$$

The form of e is $\text{APPLY}(f, e_1, e_2)$ and this derivation is done sequentially, so, after e_1 is evaluated the environments might change & these new environments \mathcal{E}_{1x} is used for evaluation of e_2 and $\text{PRIMITIVE}(+)$ adds the values of v_{1x} & v_{2x} and stores in new environments $\mathcal{E}_{2x}, \rho_{2x}$. So the Derivation of D_y is of form:

$$\phi(f) = \text{PRIMITIVE}(+)$$

$$D_y = \begin{array}{l} \langle e_1, \mathcal{E}_{0y}, \phi, \rho_{0y} \rangle \Downarrow \langle v_{1y}, \mathcal{E}_{1y}, \phi, \rho_{1y} \rangle \\ \langle e_2, \mathcal{E}_{1y}, \phi, \rho_{1y} \rangle \Downarrow \langle v_{2y}, \mathcal{E}_{2y}, \phi, \rho_{2y} \rangle \\ \hline \langle \text{APPLY}(f, e_1, e_2), \mathcal{E}_{0y}, \phi, \rho_{0y} \rangle \Downarrow \langle v_{1y} + v_{2y}, \mathcal{E}_{2y}, \phi, \rho_{2y} \rangle \end{array}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $v_{1x} = v_{1y}$, $v_{2x} = v_{2y}$ and $\mathcal{E}_{2x} = \mathcal{E}_{2y}$, $\rho_{2x} = \rho_{2y}$. But because the D_1 derivation & D_2 derivations are strictly smaller than D_x by induction hypothesis our assumptions gives us equations,

$$\mathcal{E}_{0x} = \mathcal{E}_{0y}, \mathcal{E}_{1x} = \mathcal{E}_{1y}, v_{1x} = v_{1y}, v_{2x} = v_{2y}, \rho_{1x} = \rho_{1y}, \rho_{0x} = \rho_{0y}$$

and given the above, it follows that $v_{1x} + v_{2x} = v_{1y} + v_{2y}$, $\mathcal{E}_{2x} = \mathcal{E}_{2y}$ and $\rho_{2x} = \rho_{2y}$. Our obligation is met.

9.) APPLY USER

$$\phi(f) = \text{USER}(\langle x_1, x_2 \rangle, e) \quad x_1, x_2 \text{ are all distinct}$$

$$D_x = \begin{array}{l} \langle e_1, \mathcal{E}_{0x}, \phi, \rho_{0x} \rangle \Downarrow \langle v_{1x}, \mathcal{E}_{1x}, \phi, \rho_{1x} \rangle \quad - D_1 \\ \langle e_2, \mathcal{E}_{1x}, \phi, \rho_{1x} \rangle \Downarrow \langle v_{2x}, \mathcal{E}_{2x}, \phi, \rho_{2x} \rangle \quad - D_2 \\ \langle e, \mathcal{E}_{2x}, \phi, \{x_1 \rightarrow v_{1x}, x_2 \rightarrow v_{2x}\} \rangle \Downarrow \langle v_x, \mathcal{E}'_x, \phi, \rho'_x \rangle \\ \hline \langle \text{APPLY}(f, e_1, e_2), \mathcal{E}_{0x}, \phi, \rho_{0x} \rangle \Downarrow \langle v_x, \mathcal{E}'_x, \phi, \rho'_x \rangle \end{array}$$

The form of e is $\text{APPLY}(f, e_1, e_2)$ & this derivation is done one after the other. So, e_1 evaluates in $\mathcal{E}_{0x}, \rho_{0x}$ & results in $\mathcal{E}_{1x}, \rho_{1x}$ & these are used for e_2 & results in $\mathcal{E}_{2x}, \rho_{2x}$. USER evaluates & stores in new environments \mathcal{E}'_x, ρ'_x . So, the Derivation of D_y is of the form:

$$\begin{aligned} \phi(t) &= \text{USER}(\langle x, x \rangle, e) \quad x, x_2 \text{ are all distinct} \\ D_y &= \begin{aligned} &\langle e_1, \xi_{0y}, \phi, \rho_{0y} \rangle \Downarrow \langle v_{1y}, \xi_{1y}, \phi, \rho_{1y} \rangle \\ &\langle e_2, \xi_{1y}, \phi, \rho_{1y} \rangle \Downarrow \langle v_{2y}, \xi_{2y}, \phi, \rho_{2y} \rangle \\ &\langle e, \xi_{2y}, \phi, \{x_1 \rightarrow v_{1y}, x_2 \rightarrow v_{2y}\} \rangle \Downarrow \langle v_y, \xi'_y, \phi, \rho'_y \rangle \end{aligned} \\ &\quad \underline{\langle \text{APPLY}(t, e_1 e_2), \xi_0, \phi, \rho_0 \rangle \Downarrow \langle v_y, \xi'_y, \phi, \rho_{2y} \rangle} \end{aligned}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $v_{1x} = v_{1y}$, $v_{2x} = v_{2y}$, $\xi_{0x} = \xi_{0y}$, $\xi_{1x} = \xi_{1y}$, $\xi_{2x} = \xi_{2y}$, $\rho_{1x} = \rho_{1y}$, $\rho_{2x} = \rho_{2y}$. But because D_1 derivation & D_2 derivations are strictly smaller than D_y by induction hypothesis our assumptions gives us equations,

$$\xi_{0x} = \xi_{0y}, \xi_{1x} = \xi_{1y}, v_{1x} = v_{1y}, v_{2x} = v_{2y}, \rho_{1x} = \rho_{1y} \wedge \rho_{0x} = \rho_{0y}$$

and given above it follows that $v_x = v_y$, $\xi'_{2x} = \xi'_y$, $\rho_{2x} = \rho_{2y}$

Our obligation is met.