13.) <u>Given</u>

( begin ( set x 3) x ) , $\rho(x) = 99$

<u>To Prove</u>

The result of evaluation is 3.

<u>Proof:</u> (Referred to Page 60, 61 & 63)

$D_1$ = LITERAL

$$\frac{}{\langle \text{LITERAL}(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}$$

$D_2$ = FORMALVAR

$$\frac{x \in dom\ \rho\{x \to 3\}}{\langle \text{VAR}(x), \xi, \phi, \rho\{x \to 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \to 3\} \rangle}$$

FORMAL ASSIGN

$$\frac{\dfrac{x \in dom\ \rho \qquad D_1}{\langle \text{SET}(x, \text{LITERAL}(3)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \to 3\} \rangle} \qquad D_2}{\langle \text{BEGIN}(\text{SET}(x, \text{LITERAL}(3)), \text{VAR}(x)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \to 3\} \rangle}$$

14.) Given

$\langle IF(VAR(x), VAR(x), LITERAL(0)), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle$ (expression1)

and

$\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi'', \phi, \rho'' \rangle$ (expression2)

To Prove

If (expression1 and expression 2) that implies $V_1 = V_2$

Proof:

We have two cases where $VAR(x) \in dom\ \rho$ (1. If True 2. If False)

We have two cases where $VAR(x) \in dom\ \xi$ (1. If True 2. If False)

The differences between formal variable and global variable are subtle and similar enough. So, In this proof we show for $x \in dom\ \rho$ and evaluate the cases without loss of generality. The same applies to global var

For expression 2 $\langle VAR(x), \xi, \phi, \rho \rangle$

Using FormalVar, $\dfrac{x \in dom\ \rho}{\langle VAR(x), \xi', \phi, \rho' \rangle \Downarrow \langle \rho(x), \xi'', \phi, \rho'' \rangle}$

(If TRUE)

$$\dfrac{\dfrac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi', \phi, \rho' \rangle} \quad \begin{matrix}\rho(x) \neq 0 \\ v_1 \neq 0\end{matrix} \quad \dfrac{x \in dom\ \rho}{\langle VAR(x), \xi', \phi, \rho' \rangle \Downarrow \langle \underbrace{\rho(x)}_{v_2}, \xi'', \phi, \rho'' \rangle}}{\langle IF(\underbrace{VAR(x)}_{e_1}, \underbrace{VAR(x)}_{e_2}, \underbrace{LITERAL(0)}_{e_3}), \xi, \phi, \rho \rangle \Downarrow \langle \underbrace{\rho(x)}_{V_2}, \xi'', \phi, \rho'' \rangle}$$

$\rho(x) = V_2$

(If False)

$$\dfrac{\dfrac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi', \phi, \rho' \rangle} \quad \begin{matrix}\rho(x) = 0 \\ v_1 = 0\end{matrix} \quad \overline{\langle LITERAL(0), \xi', \phi, \rho' \rangle \Downarrow \langle 0, \xi', \phi, \rho' \rangle}}{\langle IF(\underbrace{VAR(x)}_{e_1}, \underbrace{VAR(x)}_{e_2}, \underbrace{LITERAL(0)}_{e_3}), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi'', \phi, \rho'' \rangle}$$

We can observe that in both cases $VAR(x)$ results in $\rho(x)$. In (If True) case $\rho(x) \neq 0$ which means if-statement is evaluated to $e_2 = VAR(x)$ which again evaluates to $\rho(x)$. Final evaluation is $\rho(x)$ showing that evaluation of expression1 = eval. of expression2

In (If False) case $\rho(x) = 0$ which means if-statement is evaluated to $e_3 = LIT(0)$. In this case final result is 0 showing that evaluation of expression 1 = eval. of expression 2

Both cases when combined together evaluate to the same thing, thus showing $V_1 = V_2$.

21.) <u>Given</u> In ANK, if a variable is not bound to any environment then it implicitly creates a new global variable with value 0.

In ICON, if a variable is not bound to any environment then it implicitly creates a new local variable with value 0, whose scope is the entire procedure in which assignment appears.

a.) ANK-like semantics for unbound variables.

$$\frac{x \notin \text{dom}\,\rho \qquad x \in \text{dom}\,\xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi\{x \to 0\}, \phi, \rho \rangle} \;(\text{ANKVAR}) \quad (\xi(x) = 0)$$

$$\frac{x \notin \text{dom}\,\rho \qquad x \in \text{dom}\,\xi \qquad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \to v\}, \phi, \rho' \rangle} \;(\text{ANK ASSIGN}) \quad (v = 0)$$

b.) ICON-like semantics for unbound variables

$$\frac{x \notin \text{dom}\,\rho \qquad x \notin \text{dom}\,\xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho'(x), \xi', \phi, \rho'\{x \to 0\} \rangle} \;(\text{ICON VAR}) \quad \rho'(x) = 0$$

$$\frac{x \notin \text{dom}\,\rho \qquad x \notin \text{dom}\,\xi \qquad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \to v\} \rangle} \;(\text{ICON ASSIGN})$$

c.) I prefer ICON-like semantics. Generally while programming I miss to give valid declarations for counter (Ex: i) variables. I would prefer those variables to be in the local scope. ~~as those variables are used to stay in the local scopes~~. As I keep using the same 'i' variable for wherever I use the counter variables. so I would not prefer those messing with my global scope. Thus, placing it in global environment is unnecessary making the ICON-like semantics more apt.

20.) To Prove: Impcore is deterministic for any e (i.e globalvar, literal, formal var, formal assign, global assign, if True, if False, ApplyAdd, ApplyUser) and any environments, there is atmost one v such that $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$

Proof:

1.) Literal

When the last rule used in $D_x$ is LITERAL, Derivation $D_x$ must have the following form:

$$D_x = \frac{}{\langle LITERAL(v), \xi, \phi, \rho \rangle \Downarrow \langle v_x, \xi_x, \phi, \rho_x \rangle} \ (LITERAL) \quad (e = LITERAL(v))$$

In $D_x$ there is no change in the environments. So, $D_y$ must also end in one of these. So, $D_y$ must have the following form:

$$D_y = \frac{}{\langle LITERAL(v), \xi, \phi, \rho \rangle \Downarrow \langle v_y, \xi_y, \phi, \rho_y \rangle} \ (LITERAL)$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $v_x = v_y$, $\xi_x = \xi_y$, $\rho_x = \rho_y$

~~But because $D_{x_0}$ is strictly smaller than $D_x$~~, By induction hypothesis we can say, $v_x = v_y$, $\xi_x = \xi_y$, $\rho_x = \rho_y$. So, our obligation is met

2.) FORMALVAR

When the last rule used in $D_x$ is FORMALVAR, Derivation $D_x$ must have the following form:

$$D_x = \frac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho_x(x), \xi_x, \phi, \rho_x \rangle} \ (FORMALVAR)$$

The form of e is VAR(x) and it can conclude to be FORMALVAR or GLOBALVAR, so $D_y$ must also end in one of these. But GLOBALVAR requires $x \notin dom\ \rho$ & we know that in $D_x$ $x \in dom\ \rho$. So $D_y$ can only be FORMALVAR. So, $D_y$ is of the form,

$$D_y = \frac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho_y(x), \xi_y, \phi, \rho_y \rangle} \ (FORMALVAR)$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove $\rho_x(x) = \rho_y(x)$, $\xi_x = \xi_y$ and $\rho_x = \rho_y$. So, using induction hypothesis as there is no change in environments, we can say $\rho_x(x) = \rho_y(x)$, $\xi_x = \xi_y$, $\rho_x = \rho_y$. So, our obligation is met.

3.) GLOBAL VAR

It is similar to FORMALVAR only that $x \notin \text{dom}\,\rho$ and $x \in \text{dom}\,\xi$ is the only difference. So, By induction hypothesis,

$$\xi_x(x) = \xi_y(x), \quad \xi_x = \xi_y, \quad \rho_x = \rho_y.$$

$$D_x = \frac{x \notin \text{dom}\,\rho \quad x \in \text{dom}\,\xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi_x(x), \xi_x, \phi, \rho_x \rangle} \qquad \text{(similarly for } D_y\text{)}$$

4.) FORMAL ASSIGN (Prof gave it as significant Hint so not repeating)

When the last rule used in $D_x$ is FORMALASSIGN, derivation $D_x$ must have following form:

$$D_x = \frac{x \in \text{dom}\,\rho \qquad \dfrac{D_{x_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x\{x \mapsto v_{0x}\} \rangle}$$

5.) GLOBAL ASSIGN

When the last rule used in $D_x$ is GLOBAL ASSIGN, Derivation $D_x$ must have the following form:

$\text{dom}\,\xi_x = \text{dom}\,\xi'_x$
$\text{dom}(\xi'\{x \mapsto v\}) = \text{dom}\,\xi' \cup \{x\}$
$= \text{dom}\,\xi \cup \{x\}$
So, $\text{dom}\,\xi \cup \{x\} = \text{dom}\,\xi$

$$D_x = \frac{x \notin \text{dom}\,\rho \quad x \in \text{dom}\,\xi \qquad \dfrac{D_{x_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x, \phi, \rho'_x \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0x}, \xi'_x\{x \to v_{0x}\}, \phi, \rho'_x \rangle}$$

The form of $e$ is $\text{SET}(x, e_0)$ and the only rules with $\text{SET}(x, e_0)$ in their conclusions are FORMAL OR GLOBAL ASSIGN, so $D_y$ must be one of these. But FORMALASSIGN requires $x \in \text{dom}\,\rho$ & we know from $D_x$ that $x \notin \text{dom}\,\rho$, $x \in \text{dom}\,\xi$. So, $D_y$ also ends as GLOBALASSIGN. Derivation $D_y$ must have the following form

$$D_y = \frac{x \notin \text{dom}\,\rho \quad x \in \text{dom}\,\xi \qquad \dfrac{D_{y_0}}{\langle e_0, \xi, \phi, \rho \rangle \Downarrow \langle v_{0y}, \xi'_y, \phi, \rho'_y \rangle}}{\langle \text{SET}(x, e_0), \xi, \phi, \rho \rangle \Downarrow \langle v_{0y}, \xi'_y\{x \to v_{0y}\}, \phi, \rho'_y \rangle}$$

Our obligation is to prove that induction hypothesis holds for judgement below the line. We must therefore prove $v_{0x} = v_{0y}$, $\xi'_x\{x \to v_{0x}\} = \xi'_y\{x \to v_{0y}\}$ and $\rho'_x = \rho'_y$. But because derivation $D_{x_0}$ is strictly smaller than $D_x$, we apply induction hypothesis to $D_{x_0}$. This assumption gives us equations

$$v_{0x} = v_{0y}, \quad \xi'_x = \xi'_y, \quad \rho'_x = \rho'_y$$

& given $v_{0x} = v_{0y}$ and $\xi'_x = \xi'_y$ it follows that $\xi'_x\{x \to v_{0x}\} = \xi'_y\{x \to v_{0y}\}$

Our obligation is met.

6.) IF TRUE

When the last rule used in $D_x$ is IF TRUE, derivation $D_x$ must have the following form :

$$D_x = \frac{\dfrac{D_{x_1}}{\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_{1x}, \xi'_x, \phi, \rho'_x \rangle} \quad v_1 \neq 0 \quad \dfrac{D_{x_2}}{\langle e_2, \xi'_x, \phi, \rho'_x \rangle \Downarrow \langle v_{2x}, \xi''_x, \phi, \rho''_x \rangle}}{\langle IF(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_{2x}, \xi''_x, \phi, \rho''_x \rangle}$$

The form of $e$ is $IF(e_1, e_2, e_3)$ and the expression if true and if $v_1 \neq 0$ results in evaluating to if-true statement. So, $D_y$ also evaluates similarly.

$$D_y = \frac{\dfrac{D_{y_1}}{\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_{1y}, \xi'_y, \phi, \rho'_y \rangle} \quad v_1 \neq 0 \quad \dfrac{D_{y_2}}{\langle e_2, \xi'_y, \phi, \rho'_y \rangle \Downarrow \langle v_{2y}, \xi''_y, \phi, \rho''_y \rangle}}{\langle IF(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_{2y}, \xi''_y, \phi, \rho''_y \rangle}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must therefore prove $V_{2x} = V_{2y}$, $\xi''_x = \xi''_y$, $\rho''_x = \rho''_y$ and while $e_2$ is the result of evaluation. But $D_{x_1}$ is smaller than $D_x$ we are permitted to assume that the induction hypothesis applies to $D_{x_1}$ & $D_{x_2}$. This assumption gives us equations

By induction hypothesis, ($D_{x_1} \Rightarrow \xi = \xi'$, $\xi' = \xi'_x$) (As we are using the same environment variables in $D_{x_1}$ as input to $D_{x_2}$)

$$v_{1x} = v_{1y}, \quad \xi'_x = \xi'_y, \quad \rho'_x = \rho'_y$$

and given $v_{1x} = v_{1y}$, $\xi'_x = \xi'_y$, $\rho'_x = \rho'_y$ it follows that if True statement is true and $v_1 \neq 0$ it evaluates to $e_2$ and $V_{2x} = V_{2y}$, $\xi''_x = \xi''_y$ and $\rho''_x = \rho''_y$. Our obligation is met.

7.) IF FALSE (It is totally similar to IF TRUE So, the difference is that it evaluates to $e_3$ as if-statement is false and $v_1 = 0$)

$$D_x = \frac{\dfrac{D_{x_1}}{\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_{1x}, \xi'_x, \phi, \rho'_x \rangle} \quad v_1 = 0 \quad \dfrac{D_{x_2}}{\langle e_3, \xi'_x, \phi, \rho'_x \rangle \Downarrow \langle v_{3x}, \xi''_x, \phi, \rho''_x \rangle}}{\langle IF(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_{3x}, \xi''_x, \phi, \rho''_x \rangle}$$

$$D_y = \frac{\dfrac{D_{y_1}}{\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_{1y}, \xi'_y, \phi, \rho'_y \rangle} \quad v_1 = 0 \quad \dfrac{D_{y_2}}{\langle e_3, \xi'_y, \phi, \rho'_y \rangle \Downarrow \langle v_{3y}, \xi''_y, \phi, \rho''_y \rangle}}{\langle IF(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_{3y}, \xi''_y, \phi, \rho''_y \rangle}$$

By using induction hypothesis, $v_{1x} = v_{1y}$, $\xi'_x = \xi'_y$, $\rho'_x = \rho'_y$ it follows if false stmt and $v_1 = 0$ it evaluates to $e_3$ and so, $V_{3x} = V_{3y}$, $\xi''_x = \xi''_y$ and $\rho''_x = \rho''_y$.
Our obligation is met.

8.) APPLY ADD

$$D_x = \frac{\begin{array}{ll} \phi(f) = \text{PRIMITIVE}(+) & \\ \langle e_1, \xi_{0_x}, \phi, \rho_{0_x}\rangle \Downarrow \langle V_{1_x}, \xi_{1_x}, \phi, \rho_{1_x}\rangle & — D_1 \\ \langle e_2, \xi_{1_x}, \phi, \rho_{1_x}\rangle \Downarrow \langle V_{2_x}, \xi_{2_x}, \phi, \rho_{2_x}\rangle & — D_2 \end{array}}{\langle \text{APPLY}(f, e_1, e_2), \xi_{0_x}, \phi, \rho_{0_x}\rangle \Downarrow \langle V_{1_x} + V_{2_x}, \xi_{2_x}, \phi, \rho_{2_x}\rangle}$$

The form of e is APPLY$(f, e_1, e_2)$ and this derivation is done sequentially, So, after $e_1$ is evaluated the environments might change & those new environments $\xi_{1_x}$ is used for evaluation of $e_2$ and primitive (+) adds the values of $V_{1_x}$ & $V_{2_x}$ and stores in new environments $\xi_{2_x}$, $\rho_{2_x}$. So the Derivation of $D_y$ is of form:

$$D_y = \frac{\begin{array}{l} \phi(f) = \text{PRIMITIVE}(+) \\ \langle e_1, \xi_{0_y}, \phi, \rho_{0_y}\rangle \Downarrow \langle V_{1_y}, \xi_{1_y}, \phi, \rho_{1_y}\rangle \\ \langle e_2, \xi_{1_y}, \phi, \rho_{1_y}\rangle \Downarrow \langle V_{2_y}, \xi_{2_y}, \phi, \rho_{2_y}\rangle \end{array}}{\langle \text{APPLY}(f, e_1, e_2), \xi_{0_y}, \phi, \rho_{0_y}\rangle \Downarrow \langle V_{1_y} + V_{2_y}, \xi_{2_y}, \phi, \rho_{2_y}\rangle}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $V_{1_x} = V_{1_y}$, $V_{2_x} = V_{2_y}$ and $\xi_{2_x} = \xi_{2_y}$, $\rho_{2_x} = \rho_{2_y}$. But because the $D_1$ derivation & $D_2$ derivations are strictly smaller than $D_x$ by induction hypothesis our assumptions gives us equations,

$\xi_{0_x} = \xi_{0_y}$, $\xi_{1_x} = \xi_{1_y}$, $V_{1_x} = V_{1_y}$, $V_{2_x} = V_{2_y}$, $\rho_{1_x} = \rho_{1_y}$, $\rho_{0_x} = \rho_{0_y}$

and given the above, it follows that $V_{1_x} + V_{2_x} = V_{1_y} + V_{2_y}$, $\xi_{2_x} = \xi_{2_y}$ and $\rho_{2_x} = \rho_{2_y}$. Our obligation is met.

9.) APPLY USER

$$D_x = \frac{\begin{array}{ll} \phi(f) = \text{USER}(\langle x_1, x_2\rangle, e) \quad x_1, x_2 \text{ are all distinct} & \\ \langle e_1, \xi_{0_x}, \phi, \rho_{0_x}\rangle \Downarrow \langle V_{1_x}, \xi_{1_x}, \phi, \rho_{1_x}\rangle & — D_1 \\ \langle e_2, \xi_{1_x}, \phi, \rho_{1_x}\rangle \Downarrow \langle V_{2_x}, \xi_{2_x}, \phi, \rho_{2_x}\rangle & — D_2 \\ \langle e, \xi_{2_x}, \phi, \{x_1 \to V_{1_x}, x_2 \to V_{2_x}\}\rangle \Downarrow \langle V_x, \xi'_x, \phi, \rho'_x\rangle & \end{array}}{\langle \text{APPLY}(f, e_1, e_2), \xi_{0_x}, \phi, \rho_{0_x}\rangle \Downarrow \langle V_x, \xi'_x, \phi, \rho_{2_x}\rangle}$$

The form of e is APPLY$(f, e_1, e_2)$ & this derivation is done one after the other. So, $e_1$ evaluates in $\xi_{0_x}$, $\rho_{0_x}$ & results in $\xi_{1_x}$, $\rho_{1_x}$ & these are used for $e_2$ & results in $\xi_{2_x}$, $\rho_{2_x}$. USER evaluates & stores in new environments $\xi'_x$ & $\rho_{2_x}$. So, the Derivation of $D_y$ is of the form:

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$$\phi(f) = \mathrm{USER}(\langle x_1, x_2 \rangle, e) \qquad x_1, x_2 \text{ are all distinct}$$

$$D_y = \frac{\begin{array}{c} \langle e_1, \xi_{0_y}, \phi, \rho_{0_y} \rangle \Downarrow \langle v_{1_y}, \xi_{1_y}, \phi, \rho_{1_y} \rangle \\ \langle e_2, \xi_{1_y}, \phi, \rho_{1_y} \rangle \Downarrow \langle v_{2_y}, \xi_{2_y}, \phi, \rho_{2_y} \rangle \\ \langle e, \xi_{2_y}, \phi, \{x_1 \to v_{1_y}, x_2 \to v_{2_y}\} \rangle \Downarrow \langle v_y, \xi'_y, \phi, \rho'_y \rangle \end{array}}{\langle \mathrm{APPLY}(f, e_1, e_2), \xi_0, \phi, \rho_0 \rangle \Downarrow \langle v_y, \xi'_y, \phi, \rho_{2_y} \rangle}$$

Our obligation is to prove that induction hypothesis holds for the judgement below the line. We must prove that $v_{1_x} = v_{1_y}$, $v_{2x} = v_{2y}$, $\xi_{0x} = \xi_{0y}$, $\xi_{1_x} = \xi_{1_y}$, $\xi_{2x} = \xi_{2y}$, $\rho_{1x} = \rho_{1y}$, $\rho_{2x} = \rho_{2y}$. But because $D_1$ derivation & $D_2$ derivations are strictly smaller than $D_y$ by induction hypothesis our assumptions gives us equations,

$\xi_{0x} = \xi_{0y}$, $\xi_{1x} = \xi_{1y}$, $v_{1x} = v_{1y}$, $v_{2x} = v_{2y}$, $\rho_{1x} = \rho_{1y}$ & $\rho_{0x} = \rho_{0y}$

and given above it follows that $v_x = v_y$, $\xi'_{2x} = \xi'_y$, $\rho_{2x} = \rho_{2y}$

Our obligation is met.