Basic Forms

(1) $$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1$$

(2) $$\int \frac{1}{x} dx = \ln |x|$$

(3) $$\int u dv = uv - \int v du$$

(4) $$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b|$$

Integrals of Rational Functions

(5) $$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a}$$

(6) $$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$$

(7) $$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$$

(8) $$\int \frac{1}{1+x^2} dx = \tan^{-1} x$$

(9) $$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

(10) $$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2|$$

(11) $$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$$

(12) $$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2|$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a} \ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}}\ dx = 2\sqrt{x \pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a}\ dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \tag{20}$$

$$\int \sqrt{ax+b}\ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2}\ dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x \pm a}}\ dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}}\ dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

(25) $$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right]$$

(26) $$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b}$$

(27) $$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right]$$

(28) $$\int \sqrt{x^3(ax+b)}\,dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|$$

(29) $$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

(30) $$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

(31) $$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2}$$

(32) $$\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|$$

(33) $$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}$$

(34) $$\int \frac{x}{\sqrt{x^2\pm a^2}}\,dx = \sqrt{x^2\pm a^2}$$

(35) $$\int \frac{x}{\sqrt{a^2-x^2}}\,dx = -\sqrt{a^2-x^2}$$

(36) $$\int \frac{x^2}{\sqrt{x^2\pm a^2}}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

(37) $$\int \sqrt{ax^2+bx+c}\,dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right|$$

$$\int x\sqrt{ax^2+bx+c}\,dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\left(-3b^2+2abx+8a(c+ax^2)\right)\right. \tag{38}$$
$$\left.+3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right)$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax\,dx = x\ln ax - x \tag{42}$$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4} \tag{43}$$

$$\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9} \tag{44}$$

$$\int x^n\ln x\,dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n\neq -1 \tag{45}$$

$$\int \frac{\ln ax}{x}\,dx = \frac{1}{2}(\ln ax)^2 \tag{46}$$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{47}$$

$$\int \ln(ax+b)\,dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x, a\neq 0 \tag{48}$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{49}$$

(50) $$\int \ln(x^2 - a^2)\, dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x$$

(51)
$$\int \ln\left(ax^2 + bx + c\right)\, dx = \frac{1}{a}\sqrt{4ac - b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right)$$

(52) $$\int x \ln(ax+b)\, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b)$$

(53) $$\int x \ln\left(a^2 - b^2x^2\right)\, dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right)$$

(54) $$\int (\ln x)^2\, dx = 2x - 2x\ln x + x(\ln x)^2$$

(55) $$\int (\ln x)^3\, dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$$

(56) $$\int x(\ln x)^2\, dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x$$

(57) $$\int x^2(\ln x)^2\, dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x$$

Integrals with Exponentials

(58) $$\int e^{ax}\, dx = \frac{1}{a}e^{ax}$$

(59) $$\int \sqrt{x}e^{ax}\, dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \text{ where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

(60) $$\int xe^x\, dx = (x-1)e^x$$

(61) $$\int xe^{ax}\, dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$

$$\int x^2 e^x \, dx = \left(x^2 - 2x + 2\right) e^x \tag{62}$$

$$\int x^2 e^{ax} \, dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax} \tag{63}$$

$$\int x^3 e^x \, dx = \left(x^3 - 3x^2 + 6x - 6\right) e^x \tag{64}$$

$$\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, \mathrm{d}x \tag{65}$$

$$\int x^n e^{ax} \, dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \text{ where } \Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} \, \mathrm{d}t \tag{66}$$

$$\int e^{ax^2} \, dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right) \tag{67}$$

$$\int e^{-ax^2} \, dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(x\sqrt{a}\right) \tag{68}$$

$$\int x e^{-ax^2} \, dx = -\frac{1}{2a} e^{-ax^2} \tag{69}$$

$$\int x^2 e^{-ax^2} \, dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \tag{70}$$

Integrals with Trigonometric Functions

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax \tag{71}$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{72}$$

$$\int \sin^3 ax \, dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{73}$$

(74) $$\int \sin^n ax \; dx = -\frac{1}{a}\cos ax \;\; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

(75) $$\int \cos ax \; dx = \frac{1}{a}\sin ax$$

(76) $$\int \cos^2 ax \; dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

(77) $$\int \cos^3 axdx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

(78) $$\int \cos^p axdx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$$

(79) $$\int \cos x \sin x \; dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3$$

(80) $$\int \cos ax \sin bx \; dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$$

(81) $$\int \sin^2 ax \cos bx \; dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

(82) $$\int \sin^2 x \cos x \; dx = \frac{1}{3}\sin^3 x$$

(83) $$\int \cos^2 ax \sin bx \; dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

(84) $$\int \cos^2 ax \sin ax \; dx = -\frac{1}{3a}\cos^3 ax$$

(85) $$\int \sin^2 ax \cos^2 bxdx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$

(86) $$\int \sin^2 ax \cos^2 ax \; dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

(87) $$\int \tan ax \ dx = -\frac{1}{a} \ln \cos ax$$

(88) $$\int \tan^2 ax \ dx = -x + \frac{1}{a} \tan ax$$

(89) $$\int \tan^n ax \ dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1 \left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax \right)$$

(90) $$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$$

(91) $$\int \sec x \ dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right)$$

(92) $$\int \sec^2 ax \ dx = \frac{1}{a} \tan ax$$

(93) $$\int \sec^3 x \ dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$$

(94) $$\int \sec x \tan x \ dx = \sec x$$

(95) $$\int \sec^2 x \tan x \ dx = \frac{1}{2} \sec^2 x$$

(96) $$\int \sec^n x \tan x \ dx = \frac{1}{n} \sec^n x, n \neq 0$$

(97) $$\int \csc x \ dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C$$

(98) $$\int \csc^2 ax \ dx = -\frac{1}{a} \cot ax$$

(99) $$\int \csc^3 x \ dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

(100) $$\int \csc^n x \cot x \ dx = -\frac{1}{n} \csc^n x, n \neq 0$$

(101) $$\int \sec x \csc x \ dx = \ln |\tan x|$$

Products of Trigonometric Functions and Monomials

(102) $$\int x\cos x\ dx = \cos x + x\sin x$$

(103) $$\int x\cos ax\ dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

(104) $$\int x^2\cos x\ dx = 2x\cos x + \left(x^2-2\right)\sin x$$

(105) $$\int x^2\cos ax\ dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax$$

(106) $$\int x^n\cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1,-ix) + (-1)^n\Gamma(n+1,ix)\right]$$

(107) $$\int x^n\cos ax\ dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1,-iax) - \Gamma(n+1,ixa)\right]$$

(108) $$\int x\sin x\ dx = -x\cos x + \sin x$$

(109) $$\int x\sin ax\ dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$$

(110) $$\int x^2\sin x\ dx = \left(2-x^2\right)\cos x + 2x\sin x$$

(111) $$\int x^2\sin ax\ dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2}$$

(112) $$\int x^n\sin x\ dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1,-ix) - (-1)^n\Gamma(n+1,-ix)\right]$$

(113) $$\int x\cos^2 x\ dx = \frac{x^2}{4} + \frac{1}{8}\cos 2x + \frac{1}{4}x\sin 2x$$

(114) $$\int x\sin^2 x\ dx = \frac{x^2}{4} - \frac{1}{8}\cos 2x - \frac{1}{4}x\sin 2x$$

(115) $$\int x\tan^2 x\ dx = -\frac{x^2}{2} + \ln\cos x + x\tan x$$

(116) $$\int x\sec^2 x\ dx = \ln\cos x + x\tan x$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \; dx = \frac{1}{2} e^x (\sin x - \cos x) \tag{117}$$

$$\int e^{bx} \sin ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \tag{118}$$

$$\int e^x \cos x \; dx = \frac{1}{2} e^x (\sin x + \cos x) \tag{119}$$

$$\int e^{bx} \cos ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \tag{120}$$

$$\int x e^x \sin x \; dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \tag{121}$$

$$\int x e^x \cos x \; dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \tag{122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \; dx = \frac{1}{a} \sinh ax \tag{123}$$

$$\int e^{ax} \cosh bx \; dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{124}$$

$$\int \sinh ax \; dx = \frac{1}{a} \cosh ax \tag{125}$$

$$\int e^{ax} \sinh bx \; dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{126}$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \tag{127}$$

(128) $$\int e^{ax}\tanh bx\ dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1+\frac{a}{2b},1,2+\frac{a}{2b},-e^{2bx}\right] \\ \qquad -\frac{1}{a}e^{ax}{}_2F_1\left[1,\frac{a}{2b},1+\frac{a}{2b},-e^{2bx}\right] & a\neq b \\ \frac{e^{ax}-2\tan^{-1}[e^{ax}]}{a} & a=b \end{cases}$$

(129) $$\int \cos ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right]$$

(130) $$\int \cos ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right]$$

(131) $$\int \sin ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right]$$

(132) $$\int \sin ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cosh bx\sin ax - a\cos ax\sinh bx\right]$$

(133) $$\int \sinh ax\cosh axdx = \frac{1}{4a}\left[-2ax+\sinh 2ax\right]$$

(134) $$\int \sinh ax\cosh bx\ dx = \frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax - a\cosh ax\sinh bx\right]$$