HO CHI MINH UNIVERSITY OF TECHNOLOGY

GRADUATE THESIS

Nonlinear finite element approach for contact problems in hyper-elastic models

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"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

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Abstract

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Rubber, plastic, their similar materials and many other polymer materials are hyper elastic materials. We can see the use of these materials increasingly common in engineering with familiar products such as tires, car front-end covers, products made of plastic and rubber, etc. So solving the problems of hyper-elastic materials is essential in engineering. Solving these problems is a difficult engineering task. Since hyper-elastic materials have a stress-and-strain relationship that is nonlinear. Therefore, a suitable and effective method is required. And one of the most commonly used methods in engineering is the finite element method (FEM). . This paper discusses solving the contact problem between two elastic materials. In addition to using FEM, we also use high-order elements to deal with advantages such as: less element usage, more accurate results, geometrical flexibility....The contribution of the paper is that Matlab programs can calculate and simulate the specific contact problem between two hyper-elastic materials. To increase reliability, the obtained results are verified with the solutions given by FEA program. In summary, this paper said that it is high feasibility to use high-order elements in computational programming. With its advantages, the high-order elements are used in many contact problems requiring high accuracy.

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Contents

List of Figures

List of Tables

List of Abbreviations

LAH	**L**ist **A**bbreviations **H**ere
WSF	**W**hat (it) **S**tands **F**or

Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^{8}\ \mathrm{m\,s^{-1}}$ (exact)

List of Symbols

$\{u\}$	Displacement vector
$\{\varepsilon\}$	Strain vector
$[D]$	Differentiation operation matrix
$\{\sigma\}$	Stress vector
$[E]$	Elasticity matrix
λ, μ	Lame constants
v	Poisson's ratio
Π	Total potential
$\{p^v\}$	Body force vector
$\{p^s\}$	Surface force vector
$\{q\}$	Nodal displacement vector
$[N]$	Shape function matrix
$[B]$	Displacement differentiation matrix
$[K]$	Element stiffness matrix
$\{f\}$	Load vector
$\{p\}$	Actual forces vector
$\{h\}$	Thermal vector

For my Family…

Chapter 1

Introduction

1.1 Introduction of Contact problems

The contact problems are very importance in industrial applications in mechanical and civil engineering. The range of application are profusely such as metal forming processes, drilling problems, bearings or crash analysis of cars. Other applications are related to biomechanics where human joints, implant or teeth are considered. Due to this variety contact problems are today combined either with large elastic or inelastic deformations including time dependent responses. Thermal coupling might have to be considered, see the cooling of electronic devices, the heat removal within nuclear power plant vessels or thermal insulation of astronautic vehicles. Even stability behavior has to be linked tocontact, like wrinkling arising in metal forming problems. [1] Due to this technical importance a great number of researchers have investigated contact problems. Starting with the classical analytical work of Hertz (1882) on the elastic contact of two spheres the deformation of the bodies being in contact has been taken into account. However only very few problems involving contact can be solved analytically. Thus for most industrial applications numerical methods have to be applied when the contacting bodies have complex geometries . Due to that the solution of contact problems with finite element methods has a relatively long history. [1] The following introductory remarks are related to the steps which have to be followed when treating contact problems within the finite element method. [1]

1.2 Introduction of Hyper-elastic materials

Hyper-elastic materials are designed for modeling rubber or rubber-like materials in which the elastic deformation can be extremely large.

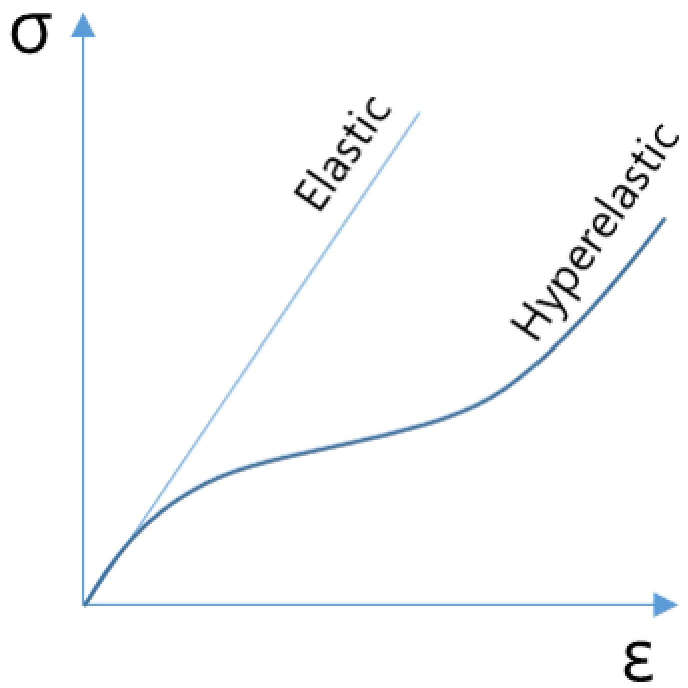

FIGURE 1.1: Introduction of hyper-elastic materials.

Hyper-elastic materials use something called a strain energy density function to derive the relationship between stress and strain. This allows them to model the relationship between stress and strain accurately even when the strain is between 100 % to 700 %, depending on the exact hyper-elastic model that is used. Various hyper-elastic models are accurate over different range of strains. You must choose the model to use depending on the expected range of strains, the computational expense of the formulation and the amount of data that you have to define in the stress-strain relationship.

1.3 Study objective

In this study, contact problems with hyper-elastic materials will be considered in cases of frictionless sliding and frictionless compressing and the obtained results will be verified with the solution given by commercial program. From the theories and algorithms about contact problems and hyper – elastics materials based on finite element method, some programs have to be made to resolve the problem of contact for hyper – elastic materials. These example will be programmed by using MATLAB and Python programming languages, then the results which will be obtained from these programs will be compared to the results of commercial program (ANSYS).

Chapter 2

Finite Element Method - Theoretical foundations and applications

2.1 Elasticity theory and applications

Chapter 3

Finite element algorithms for contact problems

3.1 Finite element method for solid mechanics problems [2][3]

3.1.1 Problem statement

The displacements along coordinate axes x, y and z are defined by the displacement vector $\{u\}$

$$\{u\} = \{\begin{array}{ccc} u & v & w \end{array}\} \tag{3.1}$$

Six different strain components are able to place in the strain vector $\{\varepsilon\}$:

$$\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}\} \tag{3.2}$$

Which are related to strains for elastic body by the Hook's law:

$$\begin{array}{l} \{\sigma\} = [E]\{\varepsilon^e\} = [E]\left(\{\varepsilon\} - \{\varepsilon^t\}\right) \\ \{\varepsilon^t\} = \{\alpha T \quad \alpha T \quad \alpha T \quad 0 \quad 0 \quad 0\} \end{array} \tag{3.3}$$

Here $\{\varepsilon^e\}$ is the elastic part of strains; $\{\varepsilon^t\}$ is the thermal part of strains; α is the coefficient of thermal expansion; T is temperature. The elasticity matrix [E] has the following appearance:

$$[E] = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} N \tag{3.4}$$

Where λ and μ are elastic Lame constants which can be expressed through the Young's modulus E and Poisson's ratio ν :

$$\begin{array}{l} \lambda = \dfrac{vE}{(1+v)(1-2v)} \\ \mu = \dfrac{E}{2(1+v)} \end{array} \tag{3.5}$$

The purpose of finite element solution of elastic problem is used to find such displacement field which provides minimum to the functional of total potential energy

Π:

$$\Pi = \int_V \frac{1}{2} \{\varepsilon^e\}^T \{\sigma\} dV - \int_V \{u\}^T \{p^v\} \, dV - \int_S \{u\}^T \{p^s\} \, dS \tag{3.6}$$

Appendix A

Frequently Asked Questions

A.1 How do I change the colors of links?

The color of links can be changed to your liking using:
`\hypersetup{urlcolor=red}`, or
`\hypersetup{citecolor=green}`, or
`\hypersetup{allcolor=blue}`.
If you want to completely hide the links, you can use:
`\hypersetup{allcolors=.}`, or even better:
`\hypersetup{hidelinks}`.
If you want to have obvious links in the PDF but not the printed text, use:
`\hypersetup{colorlinks=false}`.

Bibliography

[1] P. Wriggers, *Finite Element Algorithms for Contact Problems* **1995**.

[2] G. P. Nikishkov, Aizu-Wakamatsu, *Introduction to the Finite Element Method* **2004**.

[3] A. F. Bower, B. Raton, *Applied Mechanics of Solids* **2010**.