Continuing from my previous essay, in this piece I explain the limitation of Gödel's theorem from the very structure of its proof.

Regarding the proof, I refer to the demonstration of Gödel's theorem by Natalie Wolchover (editor of Quantamagazine.org) as well as Professor Phạm Việt Hưng on YouTube channel Nhận thức mới.

1. First, the metamathematical statement “It is impossible to prove the proposition with Gödel number Sub(y,y,17)” leads to two limitations:

- First, this statement itself creates the boundary of the logical system (i.e., we have drawn a circle). When the proposition with Gödel number Sub(y,y,17) cannot be proven, it means the system does not yet contain sufficient information to prove it. To do so requires something outside the system, i.e., outside the circle. This statement sets the limit of the system, but in fact the logical system is open.

Within a logical system there are propositions that cannot be proven simply because the system itself has limits. When humans discover new elements and extend the boundary of the system, those propositions become provable. The logical system is open, and as its boundaries are extended, the system reaches a higher level.

To illustrate, consider Euclid's axiomatic system: it consists of the five basic postulates for planar geometry introduced by Euclid. These five postulates form the system's limit. In truth, Euclid's axiomatic system is not closed — it is limited by the five postulates Euclid chose, and we named the system after him. In fact, those five postulates are themselves theorems within a system of greater scope (where they can be proven).

- Second, when we say “cannot be proven,” we are still raising the problem of proof. In reality, when the logical system reaches what I call the state “Me,” the system itself is the sole existence. Outside the system there is nothing, and thus no need for proof. In this state there is no issue of true or false within the system; rather, what is true or false is decided by the system itself. Truth or falsity are simply states that the system assigns to its own elements.

2. The next step in the proof involves substitution (Sub) of n (the Gödel number of the initial metamathematical statement) for y (i.e., Sub(n,n,17) replacing Sub(y,y,17)). This substitution yields the Gödel number Sub(n,n,17), since the substitution has just taken place.

Thus, Sub(n,n,17) = It is impossible to prove Sub(n,n,17).

Two cases arise here:

(1) We cannot treat the final substitution (Gödel number Sub(n,n,17)) as identical to the earlier Sub(y,y,17) in the previous proposition. The two Subs are different: the earlier substitution we call Sub 1, while the final substitution is Sub 2, representing the step where n replaces y.

(2) Suppose Sub(y,y,17) in the original metamathematical statement is indeed the same substitution as that occurring within the Gödel number during the proof steps (i.e., the Sub in the statement is the same as the Sub used to replace the Gödel number in the statement). Then the result is:

Sub = It is impossible to prove Sub.

In other words: Substitution = It is impossible to prove Substitution. Here we encounter a looping error — a circularity — which leads to the result that Substitution is unprovable (Substitution). The parenthetical repetition of “Substitution” is not a second proposition but simply a restatement.

Thus, through these steps (a circular loop), we arrive again at the result that “Sub is unprovable.” This is in fact the original metamathematical statement, not a new proposition with two separate clauses. Therefore, we return to step 1, where the statement already introduced the two limitations I described earlier.

3. In Gödel’s proof, substitution is the core. In reality, substitution cannot serve as representation. Substitution in itself is neither true nor false; its validity requires supplementary meaning.

To summarize, the proof of Gödel's theorem reveals several aspects:

First, the original metamathematical statement already imposed the limit (drew the circle), in the form of: There exists a proposition in the system that cannot be proven.

Second, the subsequent steps amount to nothing more than a circular error, a logical loop, producing not a new proposition but merely the original statement.

Third, substitution cannot function as representation.

To make it easier to visualize and systematize, I will take the very example of the liar paradox that the mathematician Gödel began with when presenting his discovery.

First, the very act of raising the issue of lying already sets a boundary when someone says: “I am lying.”

Next, let us analyze the statement "I am lying." There are two possibilities:

The notion of lying in this statement may differ from the notion in "He is lying."

+ Suppose that "He is lying" refers to the very same statement (the self-reference that many take to be paradoxical). In fact, there is no paradox at all. There is only one outcome: he is lying when he says "I am lying." By a looping error — a logical slip — we artificially separate this into two states and call it a paradox. In truth, what we face is a logical error born of linguistic confusion or fixation on the word "lying."

These are just a few of my personal perspectives that I wish to share, drawn from my own contemplations. Thank you for reading these lines.