# CS 132 HW01

Kehan Wang

TOTAL POINTS

## 65 / 70

QUESTION 1

### 1 Problem 2 8 / 10

- **0 pts** Correct
- **2 pts** Incorrect augmented matrix
- **5 pts** Incorrect row reductions
- **3 pts** Incorrect solutions
- **2 Point adjustment**

Everything is correct, except that the student has completed the wrong problem, not the assigned problem. 2-point deduction for now.

QUESTION 2

### 2 Three Planes Intersect? 9 / 10

- **0 pts** Correct
- **2 pts** Incorrect (augmented) matrix
- **6 pts** Incorrect row reductions
- **2 pts** Incorrect conclusion
- **1 Point adjustment**

Conclusion is drawn correctly based on insufficient evidence. Namely, you should also check and make sure that x_1 and x_2 are both solvable, in addition to x_3. 1-point deduction for that.

QUESTION 3

### 3 Problem 20 10 / 10

✓ - **0 pts** **Correct**

- **5 pts** Incorrect row reductions
- **5 pts** Incorrect conclusion

QUESTION 4

### 4 Equation involving g,h, and k 10 / 10

✓ - **0 pts** **Correct**

- **5 pts** Incorrect row reductions
- **5 pts** Incorrect conclusion

QUESTION 5

### 5 Problem 28 8 / 10

- **0 pts** Correct
- **5 pts** Incorrect row reductions
- **5 pts** Incorrect conclusion
- **2 Point adjustment**

Here, you are not allowed to put any restriction on f or g, because the problem states that the system has to be consistent *for all possible values of f and g*. Therefore, the rule of no restriction on f or g is given to you, and the problem is asking you to, then, use this rule and put restriction(s) on a, b, c, d.

QUESTION 6

### 6 Problem 33 10 / 10

✓ - **0 pts** **Correct**

- **2.5 pts** Incorrect 1st equation
- **2.5 pts** Incorrect 2nd equation
- **2.5 pts** Incorrect 3rd equation
- **2.5 pts** Incorrect 4th equation

QUESTION 7

### 7 Problem 34 10 / 10

✓ - **0 pts** **Correct**

- **2 pts** Incorrect augmented matrix
- **6 pts** Incorrect row reductions
- **2 pts** Incorrect solutions

Problem 2.

$x_3 = 0$

$0 \cdot x_1 + 4 \cdot x_2 + (-0) \cdot x_3$

$= 0 + 4x_2 - 0 = 2$

$x_2 = \frac{1}{2}$

$\Rightarrow \begin{bmatrix} 1 & 5 & 2 & -6 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$x_2 = \frac{1}{2}, x_3 = 0$

$x_1 + 5 \cdot x_2 + (2) \cdot x_3 = -6$

$x_1 = -\frac{17}{2}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{17}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Hence, the system is consistent, and the only solution is $x_1 = -\frac{17}{2}, x_2 = \frac{1}{2}, x_3 = 0$

## 1 Problem 2 **8 / 10**

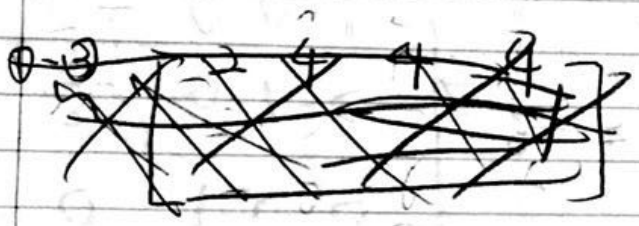
- **0 pts** Correct
- **2 pts** Incorrect augmented matrix
- **5 pts** Incorrect row reductions
- **3 pts** Incorrect solutions
- **2 Point adjustment**

Everything is correct, except that the student has completed the wrong problem, not the assigned problem. 2-point deduction for now.

2.

$$\begin{aligned} 2x_1 + 4x_2 + 4x_3 &= 4 \\ x_2 - 2x_3 &= -2 \\ 2x_1 + 3x_2 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

③ − ① $\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix}$ ②+③ $\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{bmatrix}$



Basing on the above evidence, $x_3 = 1$ the three function have one intersect.

## 2 Three Planes Intersect? 9 / 10

- **0 pts** Correct
- **2 pts** Incorrect (augmented) matrix
- **6 pts** Incorrect row reductions
- **2 pts** Incorrect conclusion
- **1 Point adjustment**

Conclusion is drawn correctly based on insufficient evidence. Namely, you should also check and make sure that x_1 and x_2 are both solvable, in addition to x_3. 1-point deduction for that.

3. Problem 20

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \overset{\textcircled{2}+\textcircled{1}\cdot 2}{\sim} \begin{bmatrix} 2 & 2h & -6 \\ -2 & 4 & 6 \end{bmatrix}$$

$$\textcircled{2}+\textcircled{1} \begin{bmatrix} 1 & h & -3 \\ 0 & 2h+4 & 0 \end{bmatrix}$$

$$0 \cdot x_1 + (2h+4) \cdot x_2 = 0$$

In order to keeping the system consistent, there are two possible cases

In the case of the system have infinte case,

$2h+4=0$, $\boxed{h=-2}$

In the case of the system have unique case,

$2h+4 \neq 0$, but $x_2 = 0 \Rightarrow x_1 = -3$.

Hence, any value of $h$ will let the system consistent. When $h=-2$, the system have infinte case.

### 3 Problem 20 10 / 10

✓ **- 0 pts** **Correct**

**- 5 pts** Incorrect row reductions

**- 5 pts** Incorrect conclusion

3 -9 24 39

4,

$$\begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ -3 & 5 & -9 & k \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix} \quad 3\times① + ③ \rightarrow \quad \begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & -4 & 15 & 3g+k \end{bmatrix}$$

xi ui

$$②+③ \quad \begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & 0 & 0 & 3g+k+h \end{bmatrix}$$

In order to maintain its consistence for any values of $g, k, h$ that $3g+k+h=0$

4 Equation involving g,h, and k **10 / 10**

✓ **- 0 pts** **Correct**

**- 5 pts** Incorrect row reductions

**- 5 pts** Incorrect conclusion

5. Problem 28

$$\begin{aligned} ax_1 + bx_2 &= f \\ cx_1 + dx_2 &= g \end{aligned} \Rightarrow \begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

$$\Rightarrow ② - \frac{c}{a}① \sim \begin{bmatrix} a & b & f \\ 0 & d-\frac{bc}{a} & g-f\frac{c}{a} \end{bmatrix}$$

$$\sim \begin{bmatrix} a & b & f \\ 0 & \frac{ad-bc}{a} & \frac{ag-cf}{a} \end{bmatrix} \overset{②\cdot a}{\sim} \begin{bmatrix} a & b & f \\ 0 & ad-bc & ag-cf \end{bmatrix}$$

In order to let the system consistent,

Case ① $ad-bc=0$ and $ag-cf=0$

$\Rightarrow ad=bc$ and $ag=cf$

In this case, $\begin{bmatrix} a & b & f \\ 0 & 0 & 0 \end{bmatrix}$, it has infinite solutions

Case ② However, when $0\cdot x_1 + \frac{ad-bc}{a}\cdot x_2 = \frac{ag-cf}{a}$ $(ad-bc\neq 0)$

the solution is unique.

Hence, when $ad=bc$, $ag=cf$ the system is consistent and has infinite solution. when $ad-bc\neq 0$, the solution of the system is unique, and the system is consistent as well

## 5 Problem 28 8 / 10

- **0 pts** Correct
- **5 pts** Incorrect row reductions
- **5 pts** Incorrect conclusion

**- 2 Point adjustment**

Here, you are not allowed to put any restriction on f or g, because the problem states that the system has to be consistent *for all possible values of f and g*. Therefore, the rule of no restriction on f or g is given to you, and the problem is asking you to, then, use this rule and put restriction(s) on a, b, c, d.

6. Problem 33

$T_1 = (10+20+T_2+T_4)/4 \Rightarrow 4T_1 - T_2 - T_4 = 30$

$T_2 = (20+40+T_1+T_3)/4 \Rightarrow -T_1 + 4T_2 - T_3 = 60$

$T_3 = (40+30+T_2+T_4)/4 \Rightarrow -T_2 + 4T_3 - T_4 = 70$

$T_4 = (10+30+T_1+T_3)/4 \Rightarrow -T_1 - T_3 + 4T_4 = 40$

## 6 Problem 33 10 / 10

✓ **- 0 pts** **Correct**

**- 2.5 pts** Incorrect 1st equation

**- 2.5 pts** Incorrect 2nd equation

**- 2.5 pts** Incorrect 3rd equation

**- 2.5 pts** Incorrect 4th equation

7. Problem 34

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \quad ① \leftrightarrow ④ \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \times 4$$

$$④ + 4\cdot① \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix} \quad ③ + ④ \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -2 & 0 & 14 & 260 \end{bmatrix}$$

$$0 \quad 1 \quad 0 \quad -1 \quad 5$$

$$-1\cdot① + ② \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -2 & 0 & 14 & 260 \end{bmatrix} \quad ④ + \frac{1}{2}\cdot② \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix}$$

$$③ + \frac{1}{4}② \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix} \quad ③ + \frac{1}{6}④ \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & 0 & 4 & 0 & 120 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix}$$

$$② + \frac{1}{3}④ \quad \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & 0 & 110 \\ 0 & 0 & 4 & 0 & 120 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix} \Rightarrow$$

$$T_1 = 20$$
$$T_2 = \frac{55}{2}$$
$$T_3 = 30$$
$$T_4 = \frac{45}{2}$$

$$① + \frac{1}{4}③ + \left(-\frac{1}{3}\right)④ \quad \begin{bmatrix} -1 & 0 & 0 & 0 & -20 \\ 0 & 4 & 0 & 0 & 110 \\ 0 & 0 & 4 & 0 & 120 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix}$$

## 7 Problem 34 10 / 10

✓ **- 0 pts** **Correct**

**- 2 pts** Incorrect augmented matrix

**- 6 pts** Incorrect row reductions

**- 2 pts** Incorrect solutions