Section 1.4

1. problem 2, 4, 6, 8 ··· 20

(2). $\begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

the product is undefined because A_x is defined only if the number of columns of A equals the number of entries in X.

(4) $\begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 8\cdot 1 + 3\cdot 1 + (-4)\cdot 1 \\ 5\cdot 1 + 1\cdot 1 + 2\cdot 1 \end{bmatrix}$

$= \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

(6) $\begin{bmatrix} 7 & -3 \\ 2 & 1 \\ 9 & -6 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

First, according to the definition of Ax,

$A_x = [a_1 \; a_2 \; \cdots \; a_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 a_1 + \cdots x_n a_n$

in the given matrix, the columns of A equasl the number of entry

$a_1 = \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix}$ $a_2 = \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix}$ $x_1 = -2$ $x_2 = -5$

$Ax = x_1 a_1 + x_2 a_2 = (-2)\cdot \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} + (-5)\cdot \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix}$

Therefore, $-2 \cdot \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} + (-5) \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

⑧ $Ax = [a_1\ a_2 \cdots a_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$a_1 = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $a_2 = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ $a_3 = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$ $a_4 \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$x_1 = z_1$ $x_2 = z_2$ $x_3 = z_3$ $x_4 = z_4$

$\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$, the column of A equals the number of entries X.

$$\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

$$4 \cdot z_1 - 4z_2 - 5z_3 + 3z_4 = 4$$
$$-2 \cdot z_1 + 5z_2 + 4z_3 + 0 \cdot z_4 = 13$$

⑩ $8x_1 - x_2 = 4$
$5x_1 + 4x_2 = 1$
$x_1 - 3x_2 = 2$

$$x_1\begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -1 & 4 \\ 5 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

⑫
$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{3}{5} \\ 0 & 1 & 0 & -\frac{4}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = \frac{3}{5}$, $x_2 = -\frac{4}{5}$, $x_3 = 1$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 0 \end{bmatrix}$$

(14) $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$

If u in the subset of R^3 spanned by the columns of A the system reflects the agumented matrix as

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & \frac{7}{5} & -\frac{7}{5} & \frac{8}{5} \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & \frac{29}{5} \end{bmatrix}$$

$0 \neq \frac{29}{5}$, this is inconsistent, and that was impossible

(10) $$\begin{bmatrix} 1 & -3 & -4 & b_1 \\ -3 & 2 & 6 & b_2 \\ 5 & -1 & -8 & b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1+b_2 \\ 0 & 14 & 12 & b_3-5b_1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1+b_2 \\ 0 & 0 & 0 & b_1+2b_2+b_3 \end{bmatrix}$$

It requires $b_1+2b_2+b_3=0$ in order to have solution for $Ax=b$

If $b_1+2b_2+b_3 \neq 0$, for all possible b, $Ax=b$ doesn't have a solution.

(18) $B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 3 & -2 & 2 \\ 0 & \boxed{1} & 1 & -5 \\ 0 & 0 & 0 & \boxed{-7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basing on Theorem 4, it must has a position in every row. After row reduction of B, it only has leading entry in row 1, 2, 3, not 4. So The column of B not span R^4, at same time, $Bx=y$ doesn't have a solution for each y in R^4 (Theorem 4 as well)

(20) According to Row Reduction in 18, we know B does not have a pivot position in every row, applying Theorem 4, every vector in R^4 can not be written as a linear combination of the columns of the matrix B above, and the columns of B does not span R^3, since each column of B is in R^4.

Section 1.6

2. Problem 12

a. $$\begin{cases} x_1 + x_2 = 200 \\ x_1 = 40 + x_3 + x_4 \\ x_2 + x_3 = 100 + x_5 \\ 60 = x_4 + x_5 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 200 \\ 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & -1 & -1 & -1 & 0 & -160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & -1 & -1 & -1 & 0 & -160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & -1 & -1 & 0 & 1 & -100 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & -1 & -1 & -1 & 0 & -160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free} \end{cases}$$

b. when x_4 is closed, $x_4 = 0$

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_5 = 60 \end{cases}$$

c). The minimum value of x_1 is 40, (when $x_3 = 0$).

3. Problem 14.

$$X_1 = 100 + X_2 \qquad X_1 - X_2 = 100$$
$$X_2 + 50 = X_3 \qquad X_2 - X_3 = -50$$
$$X_3 = 120 + X_4 \qquad X_3 - X_4 = 120$$
$$X_4 + 150 = X_5 \quad \sim \quad X_4 - X_5 = -150$$
$$X_5 = 80 + X_6 \qquad X_5 - X_6 = 80$$
$$X_6 + 100 = X_1 \qquad -X_1 + X_6 = -100$$

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{array}\right] \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$R_5 = R_6 + R_3 \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & -1 & 0 & 1 & 70 \end{array}\right] \begin{array}{c} R_5 = R_5 + R_3 \\ R_6 = R_6 + R_5 \end{array} \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow X_2, X_6$ is free and $X_2 = X_6$. (Row 5, Row 6)

R_1: $X_1 = 100 + X_6$

R_2: $X_3 = 100 + X_6 - 50 = 50 + X_6$

R_3: $X_4 = X_6 - 70$

R_4: $X_5 = X_6 + 80$

Basing on this situation, on negative $(X_1 \dots X_6)$ will cause the change the direction of map, hence $X_4 \geq 0 \Rightarrow X_6 - 70 \geq 0 \Rightarrow X_6 \geq 70$

so smallest $\boxed{X_6 = 70}$

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Section 1.7

4. Problem 2

This vector is

$v_1 = \begin{bmatrix} 8 \\ 0 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}$ $v_3 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$

when $x_1v_1 + x_2v_2 + x_3v_3 = 0$ ($x_1, x_2, x_3 \ldots x_p$ are zero) it will be conisidered linearly independent.

$$\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_1} \begin{bmatrix} \boxed{2} & -8 & 1 & 0 \\ 0 & \boxed{5} & 4 & 0 \\ 0 & 0 & \boxed{-3} & 0 \end{bmatrix}$$

In this matrix, it only has one trivial solution, since it doesn't have free variable, then the vectors are linearly independent.

5. problem 4.

$v_1 \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, $v_2 \begin{bmatrix} -2 \\ -8 \end{bmatrix}$

A set of two vectors $\{v_1, v_2\}$ is linearly dependent if and only if one of the vectors is a multiple of the other. However, $2 \cdot v_1 = 2 \cdot \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix} \neq \begin{bmatrix} -2 \\ -8 \end{bmatrix}$ (v_2). Hence the vectors are linearly independent.

$p<1$ $s<4$

4 3

6. problem 6

$$A=\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

$Ax=0$

$$\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ -4 & -3 & 0 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \xrightarrow[R_4=R_4-5R_1]{R_3=R_3+4R_1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & -3 & 12 & 0 \\ 0 & 4 & -9 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3=R_2+R_3} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & -4 & 16 & 0 \\ 0 & 4 & -9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 0 & 3 & 0 \\ 0 & \boxed{-1} & 4 & 0 \\ 0 & 0 & \boxed{7} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$ only has one trival solution, this is linearly independent.

7. problem 8 $\quad A=\begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$ $\quad Ax=0$

$$\begin{bmatrix} 1 & -3 & 3 & -2 & 0 \\ -3 & 7 & -1 & 2 & 0 \\ 0 & 1 & -4 & 3 & 0 \end{bmatrix} \xrightarrow{R_2=R_2+3R_1} \begin{bmatrix} 1 & -3 & 3 & -2 & 0 \\ 0 & -2 & 8 & -4 & 0 \\ 0 & 1 & -4 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & -3 & 3 & -2 & 0 \\ 0 & \boxed{-1} & 4 & -2 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \end{bmatrix}$$

This is linearly dependent, because x_3 is free variable, also by Theorom 8, it has more columns than rows.

11, 13
h=0 All h

8. Problem 10

a. If v_3 in Span (v_1, v_2)

$$v_1 x_1 + v_2 x_2 = v_3$$

$$x_1 \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ 0 & 0 & h+6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & h+6 \end{bmatrix}$$

The system is not consistent, since $0 \neq -1$

Hence for no h v_3 is not Span (v_1, v_2)

b. $v_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}$ $v_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}$

we see $v_2 = -2v_1$

Hence the vectors is linearly dependent for any values of h because no matter what the value of h is, v_2 are multiple of another vector.

9. Problem 12

$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix}$

If linearly dependent $Ax=b$

$\underset{-12}{}$ 16

$\begin{bmatrix} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 4 & 0 \\ -4 & 7 & h & 0 \\ 2 & -6 & 8 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & -5 & h+16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

x_3 is free, so for all value of h it is linearly dependent.

10. problem 14

$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$

$\begin{bmatrix} 1 & -5 & 1 & 0 \\ -1 & 7 & 1 & 0 \\ 3 & 8 & h & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 23 & h-3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 23 & h-3 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & h-26 & 0 \end{bmatrix}$

If linearly dependent, $h-26=0$ in order to make x_3 free variable. Hence when $\boxed{h=26}$, the set of vectors are linearly dependent

11. Problem 24

Leading entry $\blacksquare$

the starred entries $*$ (any value)

0

2x2 matrix with linearly dependent columns

For all 2x2 matrixs, it could have 2 pivot position or 1 or none.

① When it has 2 pivots, the matrix has only one trivial solution, then the set of vectors is independent so ignore this case.

② when it has 1 pivot, the matrix can be

$\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

③ when it doesn't have pivot, the matrix should be $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, all possible 2x2 matrix with linearly dependent columns are $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

12. Problem 26

4x3 matrix, $A = [a_1 \ a_2 \ a_3]$

$\{a_1, a_2\}$ linearly independent // $x_1 a_1 + x_2 a_2 = 0$ has only trivial solution

a_3 not Span (a_1, a_2) $\Rightarrow$ $x_1 a_1 + x_2 a_2 = a_3$ we can not say

According to all of above. $x_1 = x_2 = 0$

Assume $\{a_1, a_2 \ a_3\}$ is linearly dependent,

then $x_1 a_1 + x_2 a_2 + x_3 a_3 = 0$

$\Rightarrow x_1 a_1 + x_2 a_2 = -x_3 a_3$

$-\frac{x_1}{x_3} a_1 - \frac{x_2}{x_3} a_2 = a_3$

which reflect a_3 is in Span$\{a_1, a_2\}$

However, in contradiction, a_3 not Span (a_1, a_2)

$\Rightarrow \{a_1, a_2, a_3\}$ is linearly independent.

$\Rightarrow Ax = 0$ only has trivial solution

$\Rightarrow$ In a 4x3 matrix, when $Ax = 0$ only has trivial solution, it must have pivot in every column.

$$\Rightarrow \begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$$

13. Problem 28

5×7 matrix   span R^5

$$\begin{array}{cc} & 7 \\ 5 & \begin{bmatrix} \quad & \quad & \quad \\ & & \end{bmatrix} \end{array}$$

In order to span R^5, according to Theorem 4, it must have a pivot position in each row. Since it has 5 rows, it requires to have $\boxed{5}$ pivot position.

1.

```
A = np.loadtxt('h3A1.txt')

x = np.loadtxt('h3x1.txt')

AxIP(A,x)
Out[13]:
array([ 1.70000000e+01,  1.30000000e+01,  1.50000000e+01, -3.55271368e-15,
        2.00000000e+00,  4.00000000e+00])

AxVS(A,x)
Out[14]:
array([ 1.70000000e+01,  1.30000000e+01,  1.50000000e+01, -3.55271368e-15,
        2.00000000e+00,  4.00000000e+00])
```

2.

```
A = np.loadtxt('h3A2.txt')

x = np.loadtxt('h3x2.txt')

AxIP(A,x)
Out[45]: array([10.,  9.,  8.,  7.,  6.,  5.])

AxVS(A,x)
Out[46]: array([10.,  9.,  8.,  7.,  6.,  5.])
```