

1.

$$f(1) = -1$$

$$-1 = a + b + c$$

$$f(2) = 3$$

$$3 = a + 2b + 4c$$

$$f(3) = 13$$

$$13 = a + 3b + 9c$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 13 \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1}} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 8 & 14 \end{bmatrix}$$

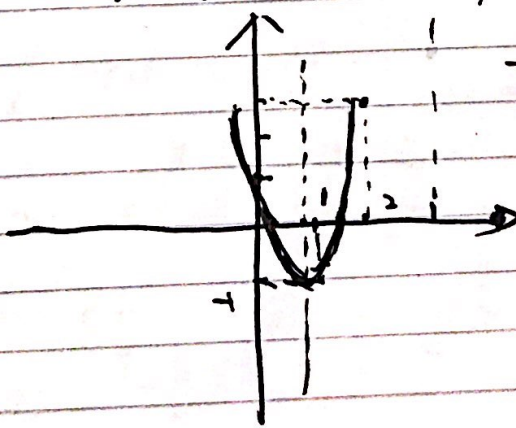
$$R_3 = R_3 - 2R_2 \quad \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Hence,  ~~$a=3, b=4, c=2$~~   $a=1, b=-5, c=3$

~~$$f(t) = 3 + 4t + 2t^2$$~~

$$f(t) = 1 - 5t + 3t^2$$



$$\text{B/L } f(t) = 0 \quad t = \frac{5 \pm \sqrt{13}}{6}$$

$$t=0, f(t)=1$$

$$\frac{b}{-2a} = \frac{5}{6}$$



$$16 \quad 3b \quad 5c$$

$$30 + 4$$

2.

$$x=5, y=5$$

$$x=4, y=6$$

$$x=6, y=2$$

$$a + 5b + 5c + 50 = 0$$

$$a + 4b + 6c + 52 = 0$$

$$a + 6b + 2c + 40 = 0$$

$$\begin{bmatrix} 1 & 5 & 5 & -50 \\ 1 & 4 & 6 & -52 \\ 1 & 6 & 2 & -40 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & 5 & -50 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & -3 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 5 & -50 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & 5 & -50 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{bmatrix} \Rightarrow a = -20, b = -2, c = -4$$

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

$$x^2 - 2x + y^2 - 4y = 20$$

$$(x-1)^2 + (y-2)^2 = 25$$

$\therefore$  center of the circle is  $(1, 2)$

Radius is  $\sqrt{25} = 5$

3. a) 
$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is a reduced echelon form, because all nonzero rows are above any rows of all zeros, all entries in a column below a leading entry are zeros, the leading entries are 1 and go from top-left to bottom-right, also 1 is the only nonzero entry in its column.

b) 
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

This is an echelon form, because all entries in a column below a leading entry are zeros, and each leading entry of a row is in a column to the right of the leading entry of the row above it. However, in Row 2, there is a 1 above the entry.

c) 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

This is not an echelon form/reduced echelon form because the entry of Row 2 is not at the right side of Row 1's entry.

d) 
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is an echelon form because all nonzero rows are above any rows of all zeros. And the entry is from top-left to bottom-right and all entries in a column below a leading entry are zero. However, the entry in Row 2 is 2, not 1.

4. 
$$\begin{bmatrix} \boxed{1} & 3 & 5 & 7 \\ 3 & \boxed{2} & 7 & 9 \\ 5 & 7 & 9 & \boxed{1} \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - 3R_1 \\ R_3 = R_3 - 5R_1}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} \boxed{1} & 3 & 5 & 7 \\ 0 & \boxed{2} & 3 & 3 \\ 0 & -8 & -16 & -34 \end{bmatrix}$$

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$$R_3 + 8R_2 \rightarrow \begin{bmatrix} \boxed{1} & 3 & 5 & 7 \\ 0 & \boxed{2} & 3 & 3 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

Pivot cols 1, 2, 4.

5.

$$8. \begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \xrightarrow{2-R_1} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & 4 \end{bmatrix}$$

$$\begin{cases} x_1 = -9 \\ x_2 = 4 \\ x_3 \text{ is free} \end{cases}$$

6.

Ex 2

$$u = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$u+v = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$u-2v = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

7. Ex 10

$$x_1 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}$$

8. Ex 12

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = b$$

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \begin{array}{l} R_2 = R_2 + 2R_1 \\ R_3 = R_3 - 2R_1 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{bmatrix}$$

$$\checkmark R_3 = R_3 - R_2 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \Rightarrow \text{no solutions}$$

Hence, this is not a linear combination.



9. Ex 14  $\begin{matrix} x_1 & x_2 & x_3 \\ \uparrow & \uparrow & \uparrow \end{matrix}$

$$A = \begin{bmatrix} 1 & -2 & 6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$[a_1 \ a_2 \ a_3 \ b] \sim \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}$$

$$R_3 = R_3 - R_1 \quad \sim \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 1 & -\frac{2}{11} \end{bmatrix} \quad R_2 = R_2 - 7R_3 \quad \sim \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 1 & 0 & -\frac{41}{33} \\ 0 & 0 & 1 & -\frac{2}{11} \end{bmatrix}$$

Hence the solution is  $x = \frac{245}{33}$ ,  $y = -\frac{41}{33}$ ,  $z = -\frac{2}{11}$

10 Ex 28

a.  $27.6 X_1 + 30.2 X_2$  million Btu

b. 
$$\begin{bmatrix} \text{heat} \\ \text{sulfur dioxide} \\ \text{particulate} \end{bmatrix} = X_1 \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix} + X_2 \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$$

c. 
$$\begin{bmatrix} 162 \\ 23610 \\ 1623 \end{bmatrix} = X_1 \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix} + X_2 \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$$

$$\begin{bmatrix} 27.6 & 30.2 & 162 \\ 3100 & 6400 & 23610 \\ 250 & 360 & 1623 \end{bmatrix}$$



1.

```
a = [[1, 2, 3, -16], [5, 4, 6, -41], [10, 9, 8, -50]]
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[]: False
```

```
backsubstitution(AEchelon)
```

```
Out[]: [[1.0, 0.0, 0.0, -3.0], [0.0, 1.0, -0.0, 4.0], [0.0, 0.0, 1.0, -7.0]]
```

2.

```
a = np.loadtxt('h2m0.txt')
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[]: False
```

```
backsubstitution(AEchelon)
```

```
Out[]:
```

```
array([[ 1.,  0.,  0., -3.],  
       [ 0.,  1., -0.,  4.],  
       [ 0.,  0.,  1., -7.]])
```

3.

```
a = np.loadtxt('h2m1.txt')
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[85]: False
```

```
backsubstitution(AEchelon)
```

```
Out[86]:
```

```
array([[ 1.,  0.,  0.,  0.,  0.,  0.,  1.],  
       [ 0.,  1.,  0.,  0.,  0.,  0.,  5.],  
       [ 0.,  0.,  1., -0., -0., -0.,  3.],  
       [ 0.,  0.,  0.,  1., -0., -0.,  9.],  
       [ 0.,  0.,  0.,  0.,  1.,  0.,  7.],  
       [ 0.,  0.,  0.,  0.,  0.,  1.,  8.]])
```

4.

```
a = np.loadtxt('h2m2.txt')
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[89]: True
```

```
backsubstitution(AEchelon)
```

```
Out[90]:
```

```
array([[ 1.        ,  0.        ,  0.        ,  0.        ,  0.        ,  
        1.61000062,  0.        ],  
       [ 0.        ,  1.        ,  0.        ,  0.        ,  0.        ,  
       -0.7864653 ,  0.        ],  
       [ 0.        ,  0.        ,  1.        , -0.        , -0.        ,  
        1.85848134, -0.        ],  
       [ 0.        ,  0.        ,  0.        ,  1.        , -0.        ,  
        0.20696884, -0.        ],  
       [ 0.        ,  0.        ,  0.        ,  0.        ,  1.        ,  
       -1.15401597, -0.        ],  
       [ 0.        ,  0.        ,  0.        ,  0.        ,  0.        ,  
        0.        ,  1.        ]])
```

5.

```
A = np.loadtxt('h2m3.txt')
```

```
AEchelon = forwardElimination(A)
```

```
inconsistentSystem(AEchelon)
```

```
Out[234]: False
```

```
backsubstitution(AEchelon)
```

```
Out[235]:
```

```
array([[ 1.,  0.,  0.,  0.,  0.,  2.],  
       [ 0.,  1.,  0.,  0.,  0., -2.],  
       [ 0.,  0.,  1.,  0.,  0.,  4.],  
       [ 0.,  0.,  0.,  1.,  0., -4.],  
       [ 0.,  0.,  0.,  0.,  1.,  3.],  
       [ 0.,  0.,  0.,  0.,  0.,  0.]])
```

6.

```
a = np.loadtxt('h2m4.txt')
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[153]: False
```

```
backsubstitution(AEchelon)
```

```
Out[154]:
```

```
array([[ 1.      ,  0.      ,  0.      , -1.04878049, -2.41463415,
         2.      ],
       [ 0.      ,  1.      ,  0.      ,  2.07317073,  4.12195122,
        -2.      ],
       [ 0.      ,  0.      ,  1.      , -1.90243902, -4.17073171,
         4.      ]])
```

```
7.
```

```
A = np.loadtxt('h2m5.txt')
```

```
AEchelon = forwardElimination(A)
```

```
inconsistentSystem(AEchelon)
```

```
Out[239]: False
```

```
backsubstitution(AEchelon)
```

```
Out[240]:
```

```
array([[ 1.,  0.,  0., -15.],
       [ 0.,  1.,  0.,  22.],
       [ 0.,  0.,  1., -81.],
       [ 0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.]])
```

```
8.
```

```
a = np.loadtxt('h2m6.txt')
```

```
AEchelon = forwardElimination(a)
```

```
inconsistentSystem(AEchelon)
```

```
Out[158]: False
```

```
backsubstitution(AEchelon)
```

```
Out[159]:
```

```
array([[ 1.      ,  0.      ,  0.      ,  0.06065384,  0.87918104,  
        3.51623543],  
       [ 0.      ,  1.      , -0.      ,  0.26277451,  0.21397584,  
       -0.88362151],  
       [ 0.      ,  0.      ,  1.      ,  1.20489538, -0.25831521,  
       -1.18473639]])
```



