A branch-and-cut-and-price algorithm for service network design considering heterogeneous fleets and service capacity decision

Zujian Wang
Department of Industrial Engineering
Tsinghua University, Beijing 100084, China
wang-zj14@mails.tsinghua.edu.cn

Abstract

Service network design addresses decisions related to selecting transportation services and distributing origin-to-destination commodity flow. In this paper, we consider the usage of heterogeneous fleets to provide transportation services. The amounts of fleets employed by each service decide its capacity. We propose both arc-based and cycle-path based models to formulate the problem. A branch-and-cut-and-price algorithm is presented to solve the problem. The method includes pricing and cutting techniques, as well as a local search algorithm to obtain upper bound. The computation study indicates the efficiency of the proposed algorithm.

Keywords: a heterogeneous fleet; column generation;

1 Introduction

Ha ha Lazić et al. (2010)

2 Literature review

3 Model formulation

In this section, we introduce two kinds of model formulations to describe the problem.

3.1 Arc-based formulation

A set of commodities K represents different products that require to be transited from particular origin to destination terminals. Each commodity $k \in K$ requires a certain quantity demand d_k. The fixed cost of selecting and operating transportation service $(i, j) \in A$ by vehicle type $f \in F$ is denoted h_{ij}^f. The unit flow cost on each arc (i, j) with respect to commodity $k \in K$ is denoted c_{ij}^k. The capacity of vehicle type f is denoted by u^f. We define $N_i^+ : \{j \in N : (i, j) \in A\}, N_i^- : \{j \in N : (j, i) \in A\}$. For each commodity k and node i, define

$$\omega_i^k = \begin{cases} d_k & if \quad i = o(k) \\ -d_k & if \quad i = d(k) \\ 0 & otherwise \end{cases}$$

We define two sets of decision variables:

- Continuous variable: x_{ij}^k denotes the amount of commodity $k \in K$ that flows on arc $(i,j) \in A$;
- Integer variable: y_{ij}^f indicates the amount of fleet type $f \in F$ that provide services on arc $(i,j) \in A$.

The model is formulated as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f \tag{1}$$

s.t.

$$\sum_{j \in N_i^+} x_{ij}^k - \sum_{j \in N_i^-} x_{ji}^k = w_i^k \quad \forall i \in N, \forall k \in K, \tag{2}$$

$$\sum_{j \in N_i^+} y_{ij}^f - \sum_{j \in N_i^-} y_{ji}^f = 0 \quad \forall i \in N, \forall f \in F, \tag{3}$$

$$\sum_{k \in K} x_{ij}^k \leq \sum_{f \in F} u^f y_{ij}^f \quad \forall (i,j) \in A, \tag{4}$$

$$x_{ij}^k \geq 0 \quad \forall (i,j) \in A, \forall k \in K, \tag{5}$$

$$y_{ij}^f \in Z_0^+ \quad \forall (i,j) \in A, \forall f \in F, \tag{6}$$

The objective function (1) minimizes the total cost calculated as the sum of the flow costs on arcs plus the fixed costs for selecting and operating transportation service with respect to different types of vehicles. Constrains (2) give the flow conversation relations for each commodity and each node. Equations (3) are the design-balance constraints, which indicate that the total number of each type of vehicle entering a node equals the total number of each type of vehicle leaving that node. Through constrains (4), we enforce service capacity restrictions on each arc. Finally, Constrains (5) and (6) give variable-type restrictions.

3.2 Cycle-path formulation

Cycle-path based service network design formulation, presented in Andersen et al. (2011) has been proved outperforming other form of formulation. We define heterogeneous vehicle design-cycle $q \in Q^f$

consisting of a sequence of paths $p \in P^k$ which transit commodities $k \in K$ by heterogeneous fleets $f \in F$. The fixed cost of cycle q provided transportation services by vehicle type $f \in F$ is denoted h_q^f. The unit flow cost of path p with respect to vehicle type $f \in F$ is denoted c_p^k.

We define two sets of decision variables:

- Cycle integer variables: y_q^f denotes the amount of vehicles used by each cycle;
- Path continuous variables: x_p^k denotes the volume of specific commodity on path p with respect to vehicle type v.

$$\min \sum_{f \in F} \sum_{q \in Q^f} h_q^f y_q^f + \sum_{k \in K} \sum_{p \in P^k} c_p^k x_p^k \tag{7}$$

s.t.

$$\sum_{k \in K} \sum_{p \in P^k} \delta_{ij}^p x_p^k \leq \sum_{f \in F} \sum_{q \in Q^f} u^f \alpha_{ij}^q y_q^f \quad \forall (i,j) \in A \tag{8}$$

$$\sum_{p \in P^k} x_p^k = d^k \quad \forall k \in K \tag{9}$$

$$x_p^k \geq 0 \quad \forall p \in P^k, \forall k \in K \tag{10}$$

$$y_q^f \in Z_0^+ \quad \forall q \in Q^f, \forall f \in F \tag{11}$$

The objective function (7) minimizes the total cost calculated as the sum of the flow costs on paths plus the fixed costs for selected cycles with respect to specific type of vehicle.Through constrains (8), we enforce service capacity restrictions on each arc. Constrains (9) ensure demand satisfaction corresponding to the flow conservation equations for each commodity. Finally, Constrains (10) and (11) gives variable-type restrictions.

4 Solution method

Since the service network design problem is NP-hard, the exact algorithm is difficult to solve the large scale instances optimally. Heuristics and metaheuristics are the common choices in real-life application. But the drawback of heuristics algorithm is that the results cannot be evaluated by lower bound. Therefore we propose a branch-and-cut-and-price method combining the exact and the heuristics algorithm to solve the problem. The exact part mainly contains column generation and cutting plane method to obtain lower bound. The heuristic method uses local search and tabu search techniques to find feasible solutions at nodes on search tree.

4.1 Column generation

Andersen et al. (2011) proposed a branch-and-price algorithm for service network design with asset management constrains. However heterogeneous fleets are not taken into consideration in the literature. This paper aims to be devoted to filling the gap.The linear relaxation of the cycle-path formulation constitutes the master problem(MP) which is solved by the column generation approach. The number of variables in the MP is exponentially increasing thus it is necessary to concentrate on a restricted master problem(RMP) which contain only a subset of variables in the MP. The column generation approach generates columns dynamically until no additional column can be found.

4.1.1 Restricted master problem

The restricted master problem(RMP) works on subsets of cycles and paths. Dual variables η_{ij} and ω_k are associated with constrains (8) and (9) respectively.To search for columns whose reduced cost is negative but are not included in the RMP, we solve the linear relaxation of RMP by use CPLEX as the LP-solver. The dual variable values in the solution of the RMP are past on to the subproblem for generating new columns. We initialize the RMP with a limited set of cycle-path variables. To avoid demand constraints violation, the RMP includes path-vehicle flow decision variables entirely.

4.1.2 Cycle subproblem

A cycle satisfying design-balance requirements consists of paths providing services $s \in \mathscr{S}$ by the same type of vehicle. Thus the objective of the cycle-selection subproblem is to find cycle-vehicle variables y_{qv} with negative reduced cost which is given by:

$$RC_{fq} = h_q^f + \sum_{(i,j)\in A} u^f \alpha_{ij}^q \eta_{ij} = \sum_{(i,j)\in q} (h_{ij}^f + u^f \eta_{ij}) \tag{12}$$

We initialize the network with a subset of cycles to avoid constraints violation and ensure RMP being feasible. Each time we add a certain number of cycle-vehicle variables with least reduced cost to the RMP.

4.1.3 Path subproblem

$$RP_{kp} = c_p^k - \sum_{(i,j)\in A} \delta_{ij}^p \eta_{ij} - \sigma_k = \sum_{(i,j)\in p} (c_{ij}^k - \eta_{ij}) - \sigma_k \tag{13}$$

4.2 Cutting plane

The generation of valid inequalities can help strength the lower bound obtained by column generation. We introduce two sets of valid inequalities including strong inequalities and cutset inequalities.

4.2.1 Strong inequalities(SI)

SI have been widely used to improve the quality of the LP lower bound. SI added to the model in our paper are defined as

$$x_{ij}^k \leq d^k \sum_{f\in F} y_{ij}^f \quad \forall (i,j) \in A, \forall k \in K. \tag{14}$$

4.2.2 Cutset inequalities(CI)

CI have been used by Chouman and Crainic (2015) to strength the lower bound. As for heterogeneous fleet, the CI have been presented by Kim et al. (1999) which are defined as

$$\sum_{f\in F} u^f Y_{S,\bar{S}}^f \geq D_{S,\bar{S}} \tag{15}$$

where we define cutset $(S, \bar{S})$ by partitioning the node set N into any nonempty subset S and its complement $\bar{S} = N\backslash S$. An arc (i,j) that connects node i in S to node j in $\bar{S}$ belongs to the cutset $(S, \bar{S})$. Let $Y_{S,\bar{S}}^f$ denote the total amount of type f vehicles used on the cutset $(S, \bar{S})$ arcs, i.e., $\sum_{(i,j)\in(S,\bar{S})} y_{ij}^f$ for the arc-based formulation and $\sum_{q\in Q^f}\sum_{(i,j)\in(S,\bar{S})} \alpha_{ij}^q y_q^f$ for the cycle-path formulation. Let $D_{S,\bar{S}}$ denote the aggregate demand of all commodities with their origin in$S$ and destination in $\bar{S}$. CI ensure the total demand that must flow from S to $\bar{S}$ will be satisfied by enough capacity on the arcs of $(S, \bar{S})$.

In general, the LP solution will not violate CI. Therefore, we strength CI by using a integer rounding procedure to produce *Chvátal-Gomory* (C-G) cuts.

$$\sum_{f\in F} \left(\left\lceil \frac{u_f}{u_l} \right\rceil Y_{S,\bar{S}}^f \right) \geq \left\lceil \frac{D_{S,\bar{S}}}{u_l} \right\rceil \quad \forall l \in F \tag{16}$$

4.3 Local search

Since the application of SND employing heterogeneous fleets by logistics enterprises will deal with large scale of data, it is necessary to present efficient algorithms to find high-quality feasible solutions efficiently. To satisfy the need of solving large scale of instances, we propose a two-stage heuristics algorithm to find feasible solutions.

At each node on the searching tree, the column generation method will provide the value of x_{ij}^k and y_{ij}^f. The arc subset A_1 includes arcs with nonzero flow on it. Another arc subset A_2 includes arcs with nonzero fleets on it. It is easy to find that $A_1 \subseteq A_2$ because of the capacity constraints. The values of $\sum_{(i,j)\in A_2} u^f y_{ij}^f$ decide the arc capacity u_{ij} in A_1.

4.3.1 Stage 1: capacitated multi-commodity minimum cost flow problem (CMCF)

Without considering the design-balance constraints, the capacitated multi-commodity minimum cost flow problem (CMCF) is obtained for given arc capacity. The solutions of the CMCF determine the flow distribution of each commodity on each arc $(i,j) \in A_1$.

$$\min \sum_{(i,j)\in A_1} \sum_{k\in K} \bar{c}_{ij}^k x_{ij}^k \tag{17}$$

s.t.

$$\sum_{j\in N_i^+} x_{ij}^k - \sum_{j\in N_i^-} x_{ji}^k = w_i^k \quad \forall i \in N, \forall k \in K, \tag{18}$$

$$\sum_{k\in K} x_{ij}^k \leq u_{ij} \quad \forall (i,j) \in A_1, \tag{19}$$

$$x_{ij}^k \geq 0 \quad \forall (i,j) \in A_1, \forall k \in K. \tag{20}$$

After solving the CMCF and redistributing the flow on each arc, the arcs without flow are removed from A_1.

4.3.2 Stage 2: fleet assignment problem (FAP)

After the flow distribution of origin-to-destination commodities has been completed, we need to assign heterogeneous fleets to provide transportation services. Let Ω_{ij} be the total flow of all commodities distributed on arc $(i,j) \in A_2$, which equals $\sum_{k\in K} \bar{x}_{ij}^k$. The mathematical formulation of Stage 2 defined on arc set A_2 is as follow:

$$\min \sum_{f\in F} \sum_{(i,j)\in A_2} h_{ij}^f y_{ij}^f \tag{21}$$

s.t.

$$\sum_{j\in N_i^+} y_{ij}^f - \sum_{j\in N_i^-} y_{ji}^f = 0 \quad \forall i \in N, \forall f \in F \tag{22}$$

$$\sum_{f\in F} u^f y_{ij}^f \geq \Omega_{ij} \quad \forall (i,j) \in A_2 \tag{23}$$

$$y_{ij}^f \in Z_0^+ \quad \forall (i,j) \in A_2, \forall f \in F \tag{24}$$

Constraints (23) ensure each service arc will provide sufficient vehicle capacity for the total flow.

4.3.3 Cycle-based neighborhoods

To improve the upper bound, we introduce cycle-based neighborhoods to search for better feasible solutions. The cycle-based neighborhoods was first proposed by Crainic to solve fixed-charge capacity multicommodity network design.

5 Numerical experiments and analyses

The computation study is implemented in C++, using CPLEX 12.63 as the MIP-solver. All experiments are performed using a computer with four 64-bit 2.4 GHz Intel Core processors and 4 GB of RAM, running Window 10.

6 Conclusion

Acknowledgments

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