Detailed predictions for two-pion correlations in ultrarelativistic heavy-ion collisions

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We calculate two-pion correlation functions from a dynamical string model situation and from a Bjorken-scaling solution. The effects of hadronic decays and final-state Coulomb and strong interactions are included. Correlation functions are shown to depend strongly on both the total transverse momenta and the direction of the relative momentum of the pairs.

I. INTRODUCTION

At sufficiently high energy density, the density of mesons or baryons would become so large that they overlap and the usual hadronic degrees of freedom become inappropriate. One then enters the domain of the quark-gluon plasma. Lattice-gauge calculations have suggested that this phase transition is first order with a tremendous latent heat, perhaps in the neighborhood of 10 GeV/fm^3. For a large range of energy densities, corresponding to the mixed phase, the temperature and pressure are constant.

Fortunately, ultrarelativistic heavy-ion collisions should provide the means to create such an energetic system over several hundred cubic fermi. Unfortunately, we cannot measure the pressure and temperatures of these systems by inserting thermomenters and barometers. Instead we must rely on signatures from the products of the collisions. The temperature can be inferred either from the slope of the transverse momentum spectra or from the mean transverse momentum of the emitted particles. However, without knowing the size of the system one cannot determine the energy density. Determining the size of the system would allow one to extract an energy density which could be plotted against the temperature to yield an equation of state. One could gain insight into the pressure of the system by knowing the lifetime of the expansion. A low pressure manifests itself by allowing the system to remain together for a much longer time than would be otherwise possible. This extension of the lifetime will occur for any theoretical scenario which includes the latent heat.[1] Two such models are hydrodynamic expansion by shock waves[2] and slowly evaporating globs of plasma.[3] Thus a measurement of the reaction time yields insight into the pressure.

Correlation measurements can yield extremely detailed information about the spatial and temporal nature of the reaction. These measurements are crucial if one is to infer the equation of state of the matter or understand the essential dynamics of the reaction. The correlation function is the ratio of the two-particle probability to the product of the single-particle probabilities:

$$C(\mathbf{K},\mathbf{k})=\frac{P(\mathbf{p}_1,\mathbf{p}_2)}{P(\mathbf{p}_1)P(\mathbf{p}_1)}\ , \qquad \mathbf{K}=\mathbf{p}_1+\mathbf{p}_2,\quad \mathbf{k}=(\mathbf{p}_1-\mathbf{p}_2)\ . \tag{1.1}$$

There is structure in the correlation function arising from the Bose-Einstein nature of the particles, and final-state interactions between the pions. Small and short-lived sources have stronger correlations which allows one to determine the lifetime and size of the system. This six-dimensional function depends on all aspects of the final Wigner distribution; therefore it explores manifestations of plasma formation and the latent heat. Even if there is no phase transition, correlation measurements provide an extremely stringent test of any picture of an ultrarelativistic reaction.

Predictions for correlation functions have been performed for a variety of dynamical models. A hydrodynamic expansion of a spherically shaped region was investigated in Ref. 2. The phase transition manifested itself very clearly for the case where there was no initial collective expansion. The case of a Bjorken tube thermally dissolving at a specific time has been investigated by several authors.[1,4,5] Bertsch and collaborators[2] considered a cascade model of a pionic gas along with globs of plasma which slowly emit particles according to time-reversal arguments. They showed that even for small globs (1 fm) and large capture radii (2 fm which means a more rapid decay) the globs still manifest themselves in the correlation function through longer apparent lifetimes. Recently, two-pion correlations were calculated[6] from the SPACER[7] version of FRITIOF[8] where the production of the various hadrons comes from the LUND[9] string model and the mesons do not interact after their formation. This is considered a good description for a system if there is no collective behavior and no plasma is formed.

The effects of long-lived hadronic resonances was first estimated in Ref. 5. Reference 6 contains a more thorough calculation of the effects of the resonances by generating the complete space-time points of all the decay products along with their momenta. These decays extend the apparent lifetime and alter the shape of the correlation function substantially. It has been pointed out that kaon interferometry should be less dependent on the extension of the lifetime due to decays.[10] In this paper we also include the effects of the Coulomb repulsion and the hadronic attraction between the two negative pions. In Sec. II the formalism for including these interactions is developed. The effects of the interactions are examined by looking at correlations from Gaussian sources. Section III presents results from a thermalized Bjorken model where the population of various resonances is assumed to be thermal and the dissolution happens at a specific proper time. Correlations from SPACER are presented in Sec. IV. For both models we show results for two different transverse momenta, all at midrapidity. For each momentum cut we perform three directional cuts for the direction of the relative momenta. This is necessary for independently discerning the transverse size, the velocity gradient or proper time τ, and the lifetime of the emission. The status of experiment is briefly discussed in Sec. V and the prospects for future study are discussed in the conclusion.

II. DERIVATION OF FORMALISM

By definition, dynamical models provide the phase-space distribution of particles during a collision. Usually, the output of these models is confined to the single-particle Wigner distribution, $f(\mathbf{p},\mathbf{r},t)$ which denotes the classical probability of viewing a particle of momentum $\mathbf{p}$ at point $\mathbf{x}$ and time t. Even nonrelativistic quantum-mechanical models such as the time-dependent Hartree-Fock provide only single-particle wave functions $\psi(x,t)$ from which the Wigner distribution can be obtained:

$$f(\mathbf{p},\mathbf{r},t)=\int d^3\delta r e^{i\mathbf{p}\cdot\delta\mathbf{r}}\psi^*(\mathbf{r}+\delta\mathbf{r}/2,t)\psi(\mathbf{r}-\delta\mathbf{r}/2,t)\ . \tag{2.1}$$

We wish to calculate the two-particle probabilities and therefore the correlation functions. In principle, this requires complete quantum-mechanical knowledge of the two-particle wave function which is not available in any current models. Fortunately, we can approximate the two-particle probability from the single-particle probabilities and knowledge of how the two particles interact with one another. In this section we write down the quantum-mechanical expressions then explain what assumptions must be made to make the problem tractable, that is to predict the correlation functions from single-particle emission probabilities and knowledge of the two-particle relative wave function.

The single-particle emission probability depends on the transition matrix element into a state with a pion with momentum $\mathbf{p}$. We define the matrix element as $M_F(x)$ which leaves the remaining particles in a state denoted by F. Using the evolution operator U we write the single-particle probability as

$$P(\mathbf{p})=\sum_F \left|\int d^4x\, M_F(x)U(x,\mathbf{p},t=\infty)\right|^2\ , \tag{2.2}$$

By squaring the matrix element we can write this as

$$P(\mathbf{p})=\int d^4q\, d^4x\, S(q,x)W_p(q,x)\ , \tag{2.3}$$

where,

$$S(q,x)\equiv\sum_F \int d^4\delta x M_F(x+\delta x/2)M_F(x-\delta x/2)e^{iq\cdot x}\ , \tag{2.4}$$

$$W_p(q,x)\equiv\int d^4\delta x U^\dagger(x+\delta x/2,\mathbf{p})U(x-\delta x/2,\mathbf{p})e^{iq\cdot x}$$
$$=\delta^4(q-p),\quad p_0=E_p\ .$$

Thus $S(q,x)$ can be thought of as the classical probability for creating a particle of momentum $\mathbf{q}$ at space-time point x when q_0 is E_q. This function can therefore be obtained from a host of dynamical models. The two-particle probability can be obtained in a similar manner:

$$P(\mathbf{p}_1,\mathbf{p}_2)=\sum_F \left|\int d^4x_1 d^4x_2 M_{F1}(x_1)M_{F2}(x_2)U(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2,t=\infty)\right|^2$$
$$=\int d^4q_1 d^4x_1 d^4q_2 d^4x_2 S_1(q_1,x_1)S_2(q_2,x_2)W_{p1,p2}(q_1,x_1,q_2,x_2)\ . \tag{2.5}$$

We have made an assumption that the matrix element can be factorized, otherwise the emission is not independent even for nonidentical noninteracting particles. The two-particle evolution matrix for identical but noninteracting particles is

$$U(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)=\frac{1}{\sqrt{2}}[\exp(ip_1\cdot x_1+ip_2\cdot x_2)\pm\exp(ip_1\cdot x_2+ip_2\cdot x_1)]\ . \tag{2.6}$$

This leads to the following expression for W

$$W_{p1,p2}(q_1,x_1,q_2,x_2)=\tfrac{1}{2}\delta^4(p_1-q_1)\delta^4(p_2-q_2)+\tfrac{1}{2}\delta^4(p_1-q_2)\delta^4(p_2-q_1)$$
$$\pm\delta^4(K/2-q_1)\delta^4(K/2-q_2)\cos[k\cdot(x_1-x_2)]\ , \tag{2.7}$$

where K is the total momenta, p_1+p_2, and k is the relative momenta (p_1-p_2). Inserting the expression for W into the expression for the probability one obtains

$$P(\mathbf{p}_1,\mathbf{p}_2)=\int d^4x_1 d^4x_2\{S(p_1,x_1)S(p_2,x_2)\pm S(K/2,x_1)S(K/2,x_2)\cos[k\cdot(x_1-x_2)]\}\ . \tag{2.8}$$

In the latter term the zeroth component of $K/2$ is not equal to $E_{K/2}$ but is instead equal to $(E_{p1}+E_{p2})/2$. This prohibits associating the term with the single-particle emission probabilities. Unless we have knowledge of the off-shell behavior of the emission functions $S(p,x)$ we must make an approximation to get a tractable formalism. Since the relative momentum of interest is usually small, we can approximate the two-particle probability and therefore the correlation function as

$$C(\mathbf{p}_1,\mathbf{p}_2)=\frac{\int d^4x_1 d^4x_2\{S(K/2,x_1)S(K/2,x_2)\pm S(K/2,x_1)S(K/2,x_2)\cos[k\cdot(x_1-x_2)]\}}{\int d^4x_1 d^4x_2 S(K/2,x_1)S(K/2,x_2)} \tag{2.9}$$

with the zeroth component of $K/2$ set equal to $E_{K/2}$. This varies slightly from some of the other formalisms presented in the literature. For instance, in some derivation the product of source terms in the exchange term is replaced by the square root of the product of four source functions evaluated at $(\mathbf{p}_1,x_1)$, $(\mathbf{p}_2,x_1)$, $(\mathbf{p}_1,x_2)$, and $(\mathbf{p}_2,x_2)$. These derivations are all identical when the relative momentum is small. For large relative momenta one should closely examine the complete quantum result shown in Eq. (2.8) and decide if any approximation is warranted.

For small $\mathbf{k}$, k_0 can be written as $\mathbf{k}\cdot v_{K/2}$ which allows the argument in the cosine to be written as $(\mathbf{k}[x_1-x_2-v_{K/2}(t_2-t_1)]$. Thus it appears to be the distance between the two particles after both have been emitted that determines the correlation function. When viewed in the rest frame of the two-particle pair $v_{K/2}=0$ and the relative distance is all that remains. In this frame the relative energy is also zero as the momenta are back to back. Thus if the relative momentum is calculated in the rest frame of the two-particle pair, it has no zeroth component and the magnitude of the relative momentum is the Lorentz-invariant relative momentum. For all calculations presented in this paper we consider the relative momentum in the rest frame of the two-particle pair. The difference between this choice and the rest frame of the source (assuming it is known) is a contraction factor γ when the relative momentum is parallel to the total momentum. The relative momentum along the direction of the total momentum becomes smaller by a factor of γ when one boosts to the frame of the two-particle pair, while the other two components of the relative momentum are unchanged. We also need a convention for specifying the direction of the relative momentum. The beam axis and the pair's total momentum define the reaction plane. The out-of-plane direction is the one perpendicular to these two and is one of the three directions. Next we boost along the beam axis to a frame where the component of the total momentum along the beam axis is zero. The other two directions are then the beam direction and that parallel to the total momentum in this Lorentz frame. Since we are considering the Lorentz frame where the total momentum is transverse to the beam direction, these last two directions are orthogonal. We believe that the sources of the pions will be distributed over a wide variety of rapidity but with relatively small transverse velocities. Therefore the dimension parallel to the total momentum will correspond closely to that coming straight from the source in the rest frame of the source. It is this dimension of the Wigner distribution which would be most extended by long-lived sources.

We can write these three components of the relative momentum in terms of the relative and total momentum components in the laboratory frame. In the laboratory the components of the total and relative momentum are defined as

$$\begin{aligned} K_{\rm lab}&=(K^0_{\rm lab},K^{\rm beam}_{\rm lab},K^{\rm perp}_{\rm lab},0)\ ,\\ k_{\rm lab}&=(k^0_{\rm lab},k^{\rm beam}_{\rm lab},k^{\rm perp}_{\rm lab},k^{\rm out}_{\rm lab})\ . \end{aligned} \tag{2.10}$$

In the frame described above the new components of the total and relative momentum are

$$\begin{aligned} &(K^0)^2=K^2_{\rm lab},\quad \mathbf{K}=0,\quad k^0=0,\quad k^{\rm out}=k^{\rm out}_{\rm lab},\\ &k^{\rm beam}=\frac{K^0_{\rm lab}k^{\rm beam}_{\rm lab}-K^{\rm beam}_{\rm lab}k^0_{\rm lab}}{\sqrt{(K^0_{\rm lab})^2-(K^{\rm lab}_{\rm beam})^2}}\ ,\\ &(k^{{\rm along}K})^2=k^2-(k^{\rm beam})^2-(k^{\rm out})^2\ . \end{aligned} \tag{2.11}$$

Equation (2.9) can also be extended to include correlations due to strong and Coulomb interactions between the pions. The nonrelativistic result is then,

$$\begin{aligned} C(\mathbf{p}_1,\mathbf{p}_2)&=\int d^3r\,F_K(\mathbf{r})|\phi(\mathbf{k},\mathbf{r})|^2\ ,\\ F_K(\mathbf{r})&=\frac{\int d^3R\,dt_1dt_2S(K/2,t_1,\mathbf{R}+\mathbf{r}/2-v_{K/2}(t_2-t_1))S(K/2,t_2,\mathbf{R}-\mathbf{r}/2)}{\int d^4x\,S(\frac{1}{2}\mathbf{K},x)|^2}\ . \end{aligned} \tag{2.12}$$

Since we are considering the frame where the total momentum is zero, the nonrelativistic assumption is exact. We find the wave function for the pions by numerically solving the Schrödinger equation for pions interacting via the Coulomb and strong interactions. An expression for the Coulomb wave function can be found in the appendix of Messiah's text book.[11] The correction to the wave function due to strong interactions is confined to

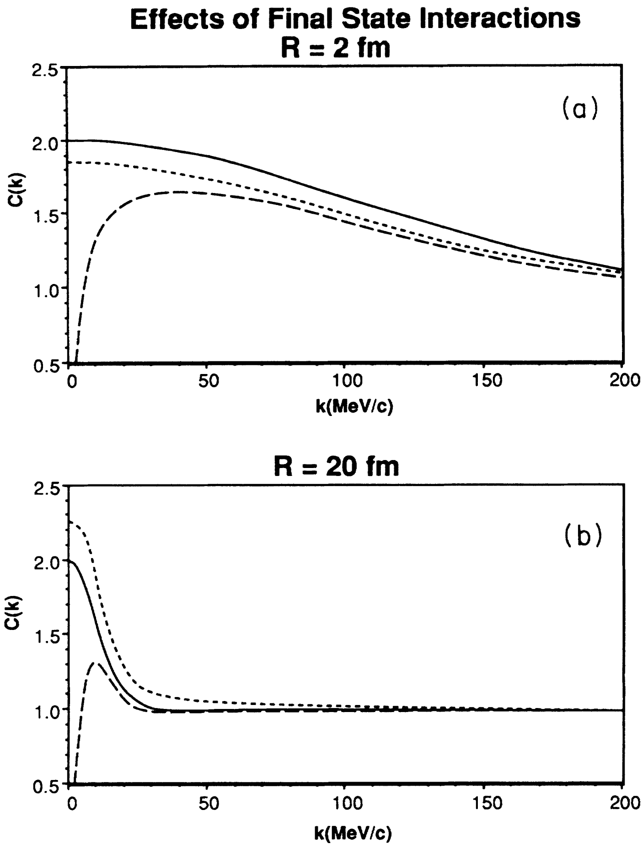

FIG. 1. Correlation functions are shown for Gaussian sources of size (a) 2 and (b) 20 fm. The solid lines are the results neglecting Coulomb and strong interactions. The dashed lines are the correlation functions as they are measured including the final-state interactions. The dotted lines are the Gamow corrected versions of the true correlation functions. The variations of the dotted lines and the solid lines are due to the strong interaction for the small source (a) and due to the inclusion of the complete Coulomb interaction for the large source (b).

the first partial wave. The form of the potential is Yukawa, with the mass of the exchanged particle being that of the ρ meson:

$$V=\frac{V_0}{m_\rho r}\exp(-m_\rho r),\quad V_0=2.6\text{ GeV} . \tag{2.13}$$

The strength of the coupling was chosen to match the behavior of the phase shifts. Any potential which matches the phase shifts should give extremely similar results.

Coulomb interactions play an important role in shaping the correlation function. If the separation of the like charged pions is much less than the two-pion Bohr radius of 390 fm, the correction due to Coulomb interactions is the Gamow factor $G(k)$ which does not depend on the relative position. The wave function $\phi_{\rm Coul}$ for $r \ll 390$ fm in terms of the wave function without Coulomb ϕ_0 is

$$|\phi_{\rm Coul}(\mathbf{k},\mathbf{r})|^2 \approx |\phi_0(\mathbf{k},\mathbf{r})|^2 G(k)^2,$$
$$G(k)=\frac{2\pi\eta}{e^{2\pi\eta}-1},\quad \eta=\frac{m_\pi e^2}{2k} \tag{2.14}$$

Thus we can get the correlation function if Coulomb interaction were not present by dividing the experimental correlation functions by the square of the Gamow factor. These are referred to as Gamow-corrected functions. The Gamow factor goes to zero at small momentum which means that the correction factor goes to infinity. This makes the correction dangerous, for if there are pions coming from very-long-lived resonances or if there is some particle misidentification, a spurious peak will be present in the correlation function.

Figure 1 demonstrates the corrections to the correlation function for Gaussian sources of 2 and 20 fm. The solid line shows the correlation function for the case with no interactions and only identical particle interference. The dashed lines show the full correlation functions with strong and Coulomb interactions included. The dotted line shows the Gamow corrected result. For the 2-fm source the strong-interaction correction is important, yielding perhaps as much as a 10% reduction in the correlation function while the Coulomb corrections beyond Gamow are only about 1%. For the 20-fm source the correction due to the strong interaction is negligible but the complete Coulomb corrections change the Gamow result by about 10%. In our calculations we present the Gamow corrected correlation functions, where the strong and Coulomb interactions have been included. These should then be compared to Gamow corrected experimental results.

III. RESULTS FOR THE BJORKEN TUBE

The Bjorken model[12] describes the distribution of collective velocities along the beam direction. The velocities and the local times at which those velocities apply are chosen such that all reference frames related by boosts along the beam direction are equivalent. Originally this formalism was justified by the fact that the distribution of particles per rapidity if fairly flat. The properties of the matter such as temperature and pressure are assumed to depend only on the proper time $\tau=\sqrt{t^2-z^2}$, and the collective velocity along the beam direction is given by:

$$\gamma v=\frac{z}{\tau} , \tag{3.1}$$

Thus for any element of the expanding matter, one can boost to its rest frame and find that the velocity gradient $1/\tau$ as well as all other local properties of the matter depending only on τ. This parametrization remains valid as the matter expands regardless of the conditions of the expansion.

We present results for only the most simple picture, thermal emission at one proper time. In the local frame the matter instantly dissolves into a thermal mixture of the various hadrons. Both the kinetic energies and the chemical makeup of the hadrons are from thermal equilibrium. We assume zero baryon chemical potential and we neglect hadronic states of mass greater than 1 GeV. There are then two sources of pions, direct thermal pions and secondary ones from resonances like the ρ or the ω. The emission point of the secondary pions is chosen randomly to correspond to the appropriate exponential decays.

In Fig. 2 we show results for an instantaneously dissolving Bjorken tube. The Bjorken time was chosen at 3 fm. The transverse distribution of the primary fragments in Gaussian with a radius of $R_T=3$ fm, $\rho(x)=\exp(-x^2/R^2)$. The temperature is 175 MeV. Figure 2(a) shows the correlation function for pion pairs whose average momentum is 200 MeV. Figure 2(b) shows the correlation function for pairs with an average momentum of 500 MeV. For each case the correlation function is shown for the relative momentum being in three different directions, along the beam axis, out of the reaction plane, and parallel to the total momentum. As mentioned earlier the longitudinal reference frame we are considering is where the total momentum of the pair is purely transverse to the beam axis for defining the three directions.

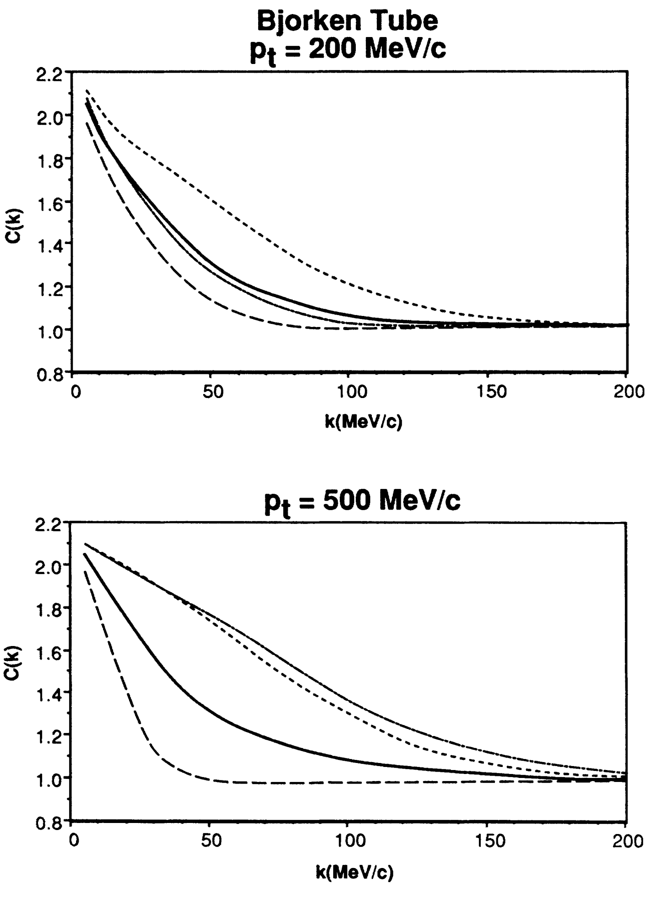

FIG. 2. Correlation functions are shown for a Bjorken tube which thermally dissolves at a specific proper time $\tau=3$ fm/c. The transverse radius is 3 fm and the temperature is 175 MeV. Results are shown for two transverse momenta, $pt=200$ MeV/c and $pt=500$ MeV/c. The filled lines show the prediction for the correlation function where all directions of the relative momentum are averaged over. Also shown are the three directional cuts for the direction of the relative momentum k. The dashed, dotted, and dot-dashed lines represent, respectively, the cuts with the relative momentum parallel to the total momentum, perpendicular to both the total momentum and the beam axis, and parallel to the beam axis.

The out-of-plane cuts are not so strongly dependent on the transverse energy as this cut mainly gives information about the transverse spatial size. The cuts for the relative momentum being parallel to the total momentum of the pair exhibit a greater sensitivity to the total transverse momentum. Pions with higher transverse momentum are less likely to come from a long-lived resonance like an omega. If the Lorentz effect of boosting to the pair's rest frame is taken into account, the correlation function for the higher transverse momentum cut corresponds to a noticeably smaller effective source size. The cuts for the relative momentum being along the beam axis demonstrate an enormous sensitivity to the total transverse momentum. The effective size along the beam axis becomes very small for higher transverse momentum cuts because pions of higher transverse momentum are more directly correlated to the collective flow of the matter.[2] This is for the same reason that particles of greater mass are more correlated to the collective flow. Particles of a given velocity, v_z along the beam axis, are emitted from a region where the collective velocities differ from v_z by no more than the Boltzmann velocities. For heavy particles the Boltzmann velocities are less therefore they are emitted from sources more locally correlated to the part of the source moving with that velocity. Therefore they yield smaller effective sources.

If one believes in instantaneous thermal emission, the correlation functions can yield an excellent measurement of the velocity gradient which is $1/\tau$ in this model. By knowing the velocity gradient dv/dz as well as the energy per unit rapidity dE/dy, one can determine the energy density dE/dz. This would then allow one to determine the equation of state. The important assumption in this determination is that the particles were emitted at one proper time. Otherwise the correlation functions depend on many more parameters.

The shape of the correlation function is decidedly non-Gaussian. This is due to the many resonances that produce pions. The biggest culprit is the omega which has a lifetime of near 20 fm/c. Since about 20% of the pions come from the omegas only 64% of the pairs consist of both pions emitted from other more short-lived sources. Since there are a variety of resonances with a variety of lifetimes, the correlation function appears somewhat like the superposition of several Gaussians. We recommend against showing results only in terms of a simple parametrization as the entire shape of the correlation function is extremely important.

IV. SPACER RESULTS

Here we present predictions from the SPACER version of FRITIOF for 200A GeV sulphur collisions on a gold target. Results were presented in Ref. 6 for oxygen-induced collisions. Due to the different technique implied here the statistics are several orders of magnitude better than the first paper. We have sufficient statistics to predict the correlation functions for any reasonable cut on the momenta. For the results shown here 2×10^3

event were simulated.

Figure 3(a) shows the correlation functions for pion pairs at midrapidity with a transverse average momentum of 200 MeV/c. Figure 3(b) shows the correlation functions for pion pairs with an average transverse momentum of 500 MeV/c. For each example the correlation function is shown for the three specific angular directions as well as for a cut summed over all angles.

The SPACER results are quite similar to the Bjorken results. For the various directional cuts the behavior of the correlation function as the transverse momentum is varied is qualitatively the same as the results from the Bjorken case discussed in the preceding section. Note that for the case when all directions of the relative momentum are averaged over, the correlation functions for the two transverse momentum cuts look similar. This kind of cut disguises nearly all the strong behavior in the transverse momentum cut. The physical reasons are the same except that the argument about the velocity gradient is no longer founded. The dependence of the cut of the relative momentum along the beam axis shows an even stronger sensitivity to the transverse energy. This sensitivity could yield insight into some strongly debated details of how mesons are created from strings. Also, like the Bjorken tube results, the shapes are much more exponential than Gaussian. The full shape is again very rich and matching single parameter fits can destroy much of the available information.

These results are only a fraction of the available predictions from SPACER. Here we only show results for two transverse momenta and one rapidity. When the data eventually improve many more cuts and plots will be warranted. Due to the Monte Carlo nature of the calculations it is easy to extend these programs to mimic any experimental cuts that may be performed in the future.

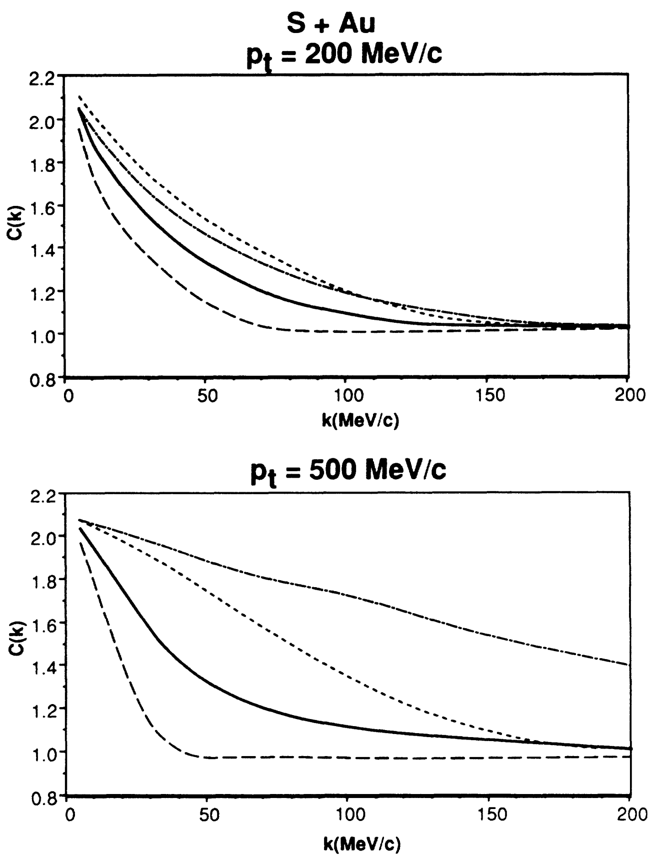

FIG. 3. Correlation functions as predicted by the SPACER version of FRITIOF are depicted. Results are shown for two transverse momenta, $pt = 200$ MeV/c and $pt = 500$ MeV/c. The solid lines show the predictions for the correlation functions where all directions of the relative momentum are averaged over. Also shown are the three directional cuts for the direction of the relative momentum **k**. The dashed, dotted, and dot-dashed lines represent, respectively, the cuts with the relative momentum parallel to the total momentum, perpendicular to both the total momentum and the beam axis, and parallel to the beam axis.

V. COMPARISON TO EXPERIMENT

At this point the most exciting experimental data at 200 nucleons GeV comes from streamer chamber data taken by the NA35 group at CERN for both oxygen[13] and sulphur induced reactions. Unfortunately the statistics are not yet sufficient to determine correlation functions accurately for all the momentum and directional cuts necessary to get a very complete picture of the collision. The one clear conclusion we can draw from the results is that the shape of the correlation function is exponential and not Gaussian. This suggests emission from a variety of time scales in agreement with the results of the SPACER model. However, for the midrapidity data the overall size is much larger than the results of SPACER. Both the theoretical models and the data need to be improved. The effects of subsequent scattering of produced hadrons need to be included in the theoretical descriptions. Recently, dynamical models have been able to accommodate some features of rescattering.[14] This may lead to either broader or more narrow peaks depending on whether the cross sections for the long-lived resonances like the omegas are changed and whether the lifetime of the reaction is significantly extended.

The WA80 group[15] has made some rather surprising measurements of the π^0-π^0 correlation functions at high transverse energy by reconstructing the neutral pions from the observed photons. The width of the correlation functions are several hundred MeV, contradicting the results of Fig. 2(b). Even if the corresponding size along the beam axis was zero for these energy cuts and if the lifetimes were zero, the width of the correlation function should be much more narrow due to the transverse size of the reaction region. This is especially true since the reaction region should appear larger by the Lorentz factor for these high-transverse-momentum cuts.

From experience with proton-proton correlations from intermediate-energy collisions we know that the statistics necessary for making many momentum and directional cuts are of the order of tens of 10^6 of pairs per unit of rapidity. Current streamer-chamber experiments only look at a few hundred events. It should be possible for a dedicated experimental apparatus to obtain the necessary statistics to make the detailed comparisons proposed here.

VI. CONCLUSIONS

By including the effects of final-state Coulomb and strong interactions we have improved the theoretical apparatus for predicting correlation functions. We have also presented more detailed predictions on the effects of various cuts on both energy and directions of the relative momentum than were given in Ref. 6. These detailed predictions of the shape of the correlation functions for the various proposed cuts unveils a rich structure to the measurements. For midrapidity data there are four degrees of freedom to explore, the magnitude of the pair's transverse momentum and all three components of the pion's relative momentum. The correlation function shows a strong dependence on all four of these degrees of freedom. Forcing theoretical models to eventually fit four dimensions of experimental results should yield great insight into many of the most important aspects of the reaction's dynamics.

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