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Coulomb corrections for interferometry analysis of expanding hadron systems

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Abstract

The problem of the Coulomb corrections to the two-boson correlation functions for the systems formed in ultra-relativistic heavy ion collisions is considered for large effective volumes predicted in the realistic evolution scenarios taking into account the collective flows. A simple modification of the standard zero-distance correction (so called Gamow or Coulomb factor) has been proposed for such a kind of systems. For $\pi^+\pi^+$ and K^+K^+ correlation functions this approximate analytical approach is compared with the exact numerical results and a good agreement is found for typical conditions at SPS, RHIC and even LHC energies.

One of the main goals of the experiments with ultrarelativistic heavy ions is a possibility to study the properties of the hadronic matter at very high temperatures and energy densities. There is a direct connection between these properties and space-time evolution of the systems created in nuclear collisions. The important features of this evolution can be accessed using the boson interferometry (correlation) method [1,2]. The space-time structure of the source is usually represented in terms of the interferometric radii R_j. They result from a Gaussian fit of the correlation function $C(p_1,p_2)$ defined as a ratio of the two-particle spectrum to the product of the single-particle ones. The interferometric radii squared are just the slopes of the correlation function in the squares of the components of the relative 4-momentum $q = p_1 - p_2$ in different regions of the mean 4-momentum $p = (p_1 + p_2)/2$. Due to azimuthal symmetry, the radii are usually sampled in terms of rapidity y and transverse momentum p_T: $R_j = R_j(p_T, y)$.

The real experimental situation with the interferometry analysis is complicated because of the effects of the strong and Coulomb final state interactions (FSI). In heavy ion collisions the effect of the short-range strong interaction is usually small and can be neglected. On the contrary, the effect of the Coulomb FSI dominates the behavior of the spectra of two charged particles at very small relative momenta. It is often taken into account with the help of the simple Coulomb factor representing the modulus squared of the non-relativistic Coulomb wave func-

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tion at zero distance. This procedure, sometimes called Gamow correction, is well founded if the effective size of the emitting source is much smaller than the pair Bohr radius $|a|$ and smaller or compatible with the inverse relative momentum. At the same time, the experimental behavior of the effective sizes in $A+A$ collisions still well follows the $R \sim (dN/dy)^{1/3}$ tendency [3]. It means that for future experiments at RHIC and LHC the average distances r^* between any two emitted particles in their c.m.s. will be larger, Coulomb interaction will be weaker and so the simple Gamow correction will be too strong and will have to be modified. This modification is more important for energetic particles since the spatial separation of the particles in their c.m.s. increases with the pair momentum simply due to the Lorentz transformation. This increase is usually attenuated due to a strong correlation between momentum and position of the emitted particles generated by various dynamical mechanisms, like resonance production, string fragmentation or hydrodynamic expansion. At the same time, at large transverse momenta these mechanisms (the flows in particular) can lead to a strong anisotropy of the effective emission volume in the pair rest frame.

As an example of the 3-dimensional hydrodynamic expansion, relevant for ultra-relativistic heavy ion collisions, we will consider [4] a cylindrically symmetric radiating system of a Gaussian transverse radius $\overline{R}_T$, expanding, in the final stage, in the boost-invariant manner in the longitudinal direction [5,6]: $r_L/t = \tanh y_L$, and linearly (at small r_T) in the transversal rapidity y_T in the radial direction: $r_T = \overline{R}_v y_T$. Its decay is assumed at a freeze-out temperature T on the freeze-out hypersurface: $\tau = \sqrt{t^2 - r_L^2} \cong$ const. For such a kind of system the correlation function of non-interacting bosons at sufficiently large transverse mass $m_T = (m^2 + p_T^2)^{1/2} \gg T$ is well approximated by a Gaussian and takes especially simple form in so called longitudinally co-moving system (LCMS) in which the boson pair is emitted transverse to the collision axis ($p_L = 0$):

$$C(p_1, p_2) = 1 + \exp\left(-q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2\right), \tag{1}$$

where the components of the vector $\boldsymbol{q}$ are chosen parallel to the collision axis (z = Longit.), parallel to the vector $\boldsymbol{p}_T$ (x = Outward) and perpendicular to the production plane (z,x) of the pair (y = Sideward) [1]. The expressions of the interferometric radii R_j through the parameters τ, $\overline{R}_T$, $\overline{R}_v$ and T can be found in [4]. In the considered simple model (infinite rapidity plateau) these radii are functions of the transverse momentum p_T only. In particular, in the case of one-dimensional expansion in longitudinal direction (when $\overline{R}_v \gg \overline{R}_T$ and the transverse flows are absent) the large-m_T results [9,10] are recovered: $R_z = \tau\sqrt{T/m_T}$ and $R_x = R_y = \overline{R}_T$.

It should be noted that the direct application of this model to pion correlations is complicated by a non-Gaussian shape of the correlation function [11] which is related with a small pion mass and a large resonance contribution increasing with the decreasing p_T. Nevertheless, just for demonstration aims, we have applied the asymptotic (large-m_T) formulae of Ref. [4] to the NA49 data on pion interferometric radii in Pb + Pb collisions measured at midrapidity in different p_T regions [12]. A good agreement is found for $T = m_\pi$ and $\tau = \overline{R}_T = \overline{R}_v = 8.6$ fm. Below we will use these parameters of the system evolution in a study of the Coulomb corrections to the kaon correlations as well. For LHC energies we will scale this parameters according to the law $R \sim (dN/dy)^{1/3}$. Taking into account that the midrapidity density of negative pions per unit rapidity for SPS Pb + Pb collisions is about 180 while for LHC energy it is expected in the range 1200–4000 [13], we will assume the LHC effective sizes by a factor of $10^{1/3} \doteq 2.2$ larger than those at SPS.

Regarding the effect of the final state interaction on particle correlations, practically it can be calculated on the conditions of sufficient smoothness of the single-particle spectrum as compared with a sharp q-dependence of the two-particle one due to the effect of quantum statistics and FSI and – of a small phase-space density of the emitted particles. Then the two particles with a small relative momentum can be considered in isolation of the rest of the system and the effect of FSI reduces to the substitution of the free plane wave amplitudes by the non-

[1] Near $q = 0$ this parametrization has rather general meaning, up to a possible cross term $\propto q_x q_z$ – the only one allowed on the requirement of azimuthal symmetry [7,8].

symmetrized Bethe–Salpeter amplitudes in the continuous spectrum of the two-particle states [14,15]: $e^{i\tilde{q}x} \to \psi_{\tilde{q}}(x)$, where $\tilde{q} = q$ for equal-mass particles, otherwise $\tilde{q} = q - p(qp)/p^2$. For non-identical bosons we have [15]:

$$C(p_1,p_2) = \Lambda \left\langle \left| \psi_{\tilde{q}}(x) \right|^2 \right\rangle + (1-\Lambda), \tag{2}$$

where the averaging is done over the relative 4-coordinates $x = x_1 - x_2$ of those bosons which can give an observable correlation effect. The corresponding fraction of such bosons is taken into account by a momentum dependent suppression parameter $\Lambda \leq 1$ [2]. For identical bosons the Bethe-Salpeter amplitude has to be properly symmetrized. In the considered case of spinless bosons:

$$\psi_{\tilde{q}}(x) \to \frac{1}{\sqrt{2}} \left[\psi_q(x) + \psi_{-q}(x) \right]. \tag{3}$$

In particular, for non-interacting identical spinless bosons Eqs. (2) and (3) recover the well known result [1]: $C(p_1,p_2) = 1 + \Lambda\langle\cos(qx)\rangle$. For non-identical particles, in the case of no FSI, $C(p_1,p_2) = 1$.

The Bethe–Salpeter amplitude $\psi_{\tilde{q}}(x)$ is usually approximated by its value at equal emission times in the two-particle c.m.s. ($t^* \equiv t_1^* - t_2^* = 0$) which coincides with a stationary solution of the scattering problem $\psi_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)$; in the two-particle c.m.s. $\boldsymbol{p}_1^* = -\boldsymbol{p}_2^* = \boldsymbol{k}^*$, $\tilde{\boldsymbol{q}}^* = 2\boldsymbol{k}^*$, $\tilde{q}_0^* = 0$. Note that the substitution $\psi_{\tilde{q}}(x) \to \psi_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)$ is possible on condition [15] $|t^*| \ll mr^{*2}$ which is usually satisfied for heavy particles like kaons or nucleons. But even for pions emitted in typical conditions in heavy ion collisions, the $t^* = 0$ approximation merely leads to a slight overestimation ($< 10\%$) of the strong FSI effect and, it doesn't influence the leading zero-distance ($r^* \ll |a|$) effect of the Coulomb FSI.

In further analytical formulae we will neglect the effect of the short range interaction which is known to be small for the pairs of like charged bosons produced in heavy ion collisions. We will control the corresponding distortions using the ''switch on'' and ''switch off'' strong FSI options in the code of Ref. [15]. For charged particles we then have [15,16]

$$\psi_{-\boldsymbol{k}^*}(\boldsymbol{r}^*) \doteq \psi^c_{-\boldsymbol{k}^*}(\boldsymbol{r}^*) \equiv e^{i\delta_c}\sqrt{A_c(\eta)}\, e^{-i\boldsymbol{k}^*\boldsymbol{r}^*} F(-i\eta,1,i\xi), \tag{4}$$

where $\xi = \boldsymbol{k}^*\boldsymbol{r}^* + k^*r^* \equiv k^*r^*(1+\cos\theta^*)$, $\eta = 1/(k^*a)$, a is the Bohr radius ($a = \pm 388$ fm and ± 111 fm for like/unlike charged pions and kaons, respectively), δ_c is the Coulomb s-wave phase shift, $A_c(\eta) \equiv |\psi^c_{-\boldsymbol{k}^*}(0)|^2 = 2\pi\eta[\exp(2\pi\eta) - 1]^{-1}$ is the usual Coulomb factor, $F(\alpha,1,z) = 1 + \alpha z/1!^2 + \alpha(\alpha+1)z^2/2!^2 + \cdots$ is the confluent hypergeometric function.

The correlation function for unlike charged bosons is then determined by the finite-size Coulomb factor $\tilde{A}_c = \langle |\psi^c_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)|^2 \rangle$:

$$C_{+-}(p_1,p_2) = \Lambda_{+-}\tilde{A}_c^{+-} + (1-\Lambda_{+-}). \tag{5}$$

For identical bosons, due to the effect of symmetrization, there appears another Coulomb factor $\tilde{B}_c = \left\langle \mathrm{Re}\left[\psi^c_{-\boldsymbol{k}^*}(\mathbf{r}^*)\, \psi^{c*}_{\boldsymbol{k}^*}(\boldsymbol{r}^*) \right] \right\rangle / \langle\cos(qx)\rangle$:

$$C_{++}(p_1,p_2) = \Lambda_{++}\left[\tilde{A}_c^{++} + \tilde{B}_c\langle\cos(qx)\rangle \right] + (1-\Lambda_{++}), \tag{6}$$

On the condition $\langle r^* \rangle / |a| \ll 1$, the Coulomb factors $\tilde{A}_c$ and $\tilde{B}_c$ coincide at $k^*\langle r^* \rangle \ll 1$. In the limit $\langle r^* \rangle / |a| \to 0$ they are both equal to the factor $A_c(\eta)$ and the simple size-independent Coulomb (Gamow) correction is recovered. Since, at SPS and even at LHC energies, the relative difference between the factors $\tilde{A}_c$ and $\tilde{B}_c$ at $k^*\langle r^* \rangle \ll 1$ is rather small ($\tilde{B}_c/\tilde{A}_c \approx 1 - \frac{2}{3}\langle r^{*2} \rangle / a^2$) [3] and since the behavior of the factor $\tilde{B}_c$ at large $k^*\langle r^* \rangle \gg 1$ is not important (due to a rapid vanishing of the interference term $\tilde{B}_c\langle\cos(qx)\rangle \to 0$), we will adopt here a simple ansatz $\tilde{B}_c = \tilde{A}_c$. Eq. (6) then leads to

[2] The suppression parameter Λ is not to be mixed with the so called chaoticity parameter related to the coherent sources.

[3] For pions in SPS Pb+Pb-collisions at $p_T =$ 0.15, 0.4, 0.8 and 1.6 GeV/c, we find in the considered hydrodynamical model that, at small $k^* \ll 1/\langle r^* \rangle$, the ratio $\tilde{B}_c / \tilde{A}_c$ is lower than unity by 0.3%, 1.2%, 3.5% and 10%, respectively. For kaons the deviation of this ratio from unity is about the same except for the lowest p_T-value of 0.15 GeV/c when it is about twice as large. At LHC we expect the increase of these deviations by a factor of $\sim 10^{2/3} \approx 5$.

the following approximation for the correlation function of two identical charged bosons:

$$C_{++}(p_1,p_2) = \Lambda_{++}\tilde{A}_c^{++}C_{QS}(p_1,p_2) + (1-\Lambda_{++}), \tag{7}$$

where $C_{QS} = 1 + \langle\cos(qx)\rangle$ is the pure QS correlation function.

Regarding the factor $\tilde{A}_c$, at $k^*|a| \ll 1$ it can be approximated by a truncated series resulting from the expansion of the confluent hypergeometric function $F(-i\eta,1,i\xi)$ in powers of $\eta\xi \equiv r^*(1+\cos\theta^*)/a$: $\tilde{A}_c \cong A_c(\eta)\langle 1 + 2\eta\xi + 3(\eta\xi)^2/2 + \cdots\rangle$, while at $k^*\langle r^*\rangle \gg 1$, it is simply given by a Jacobian of the transformation to the Coulomb shifted particle momenta (see, e.g. [17,18]), i.e., ignoring the angular dependence: $\tilde{A}_c \cong \left\langle\left[1 - 2/(r^* a k^{*2})\right]^{1/2}\right\rangle_{k^*}$ [4].

Compared with the large-k^* behavior of the zero-distance Coulomb factor: $A_c \to 1 - \pi/(ak^*)$, the true size-dependent Coulomb factor approaches 1 much faster: $\tilde{A}_c \to 1 - \langle(r^* a k^{*2})^{-1}\rangle_{k^*}$. The long k^*-tail of the factor A_c, in fact, revealed itself in the corresponding false tail observed in the ''Gamow corrected'' correlation functions in a number of experiments (see, e.g., [19]). It should be stressed that the fail of the simple Gamow correction has nothing to do with the large particle multiplicities and/or with a rapid expansion of the source in ultra-relativistic nuclear collisions, as suggested in [18,19]. In fact, the NA35 correlation data for both like and unlike charge pion pairs appears to be in a good agreement with the standard two-body FSI calculations accounting for the finite size Coulomb effect according to Eq. (6).

In Fig. 1 we demonstrate the fail of the simple Gamow correction for the $\pi^+\pi^+$ and K^+K^+ correlation functions using the RQMD v1.08 event generator [20] to simulate Pb + Pb collisions at 158 GeV/nucleon. The correlation functions are evaluated by weighting the simulated boson pairs with the modulus squared of the properly symmetrized two-particle wave function calculated with the FSI code [15], taking into account both the Coulomb as well as the strong interaction in the final state.

Tailing the behavior of the Coulomb factor $\tilde{A}_c(k^*)$ in the quantum region $k^*\langle r^*\rangle \ll 1$ and in the quasi-classical one $k^*\langle r^*\rangle \gg 1$, we arrive, neglecting its angular dependence, at a simple ansatz:

$$\tilde{A}_c \cong A_c(\eta)\left[1 + 2\frac{\langle r^*\rangle}{a}\left(1 + d_2\frac{\langle r^*\rangle}{a}\right)\right]\theta(\tilde{k} - k^*) + \left(1 - \frac{d}{a\langle r^*\rangle k^{*2}}\right)\theta(k^* - \tilde{k}), \tag{8}$$

where $\theta(x) = 1$ for $x \geq 0$, otherwise $\theta(x) = 0$. Fixing further the second reduced moment $d_2 = \langle r^{*2}\rangle/\langle r^*\rangle^2$ at the isotropic Gaussian value $d_2 = \tilde{d}_2 = 3\pi/8$, this ansatz contains three parameters $\langle r^*\rangle$, $\tilde{k}$ and d, out of them one ($\langle r^*\rangle$) can be now considered as free and the two others can be fixed at the values:

$$\tilde{k}\langle r^*\rangle \cong \frac{\pi}{4}\left[1 + 2\frac{\langle r^*\rangle}{a}\left(1 + d_2\frac{\langle r^*\rangle}{a}\right)\right], \tag{9}$$

$$d = a\langle r^*\rangle\tilde{k}^2 \times\left\{1 - A_c(\eta)\left[1 + 2\frac{\langle r^*\rangle}{a}\left(1 + d_2\frac{\langle r^*\rangle}{a}\right)\right]\right\}, \tag{10}$$

which follow from the continuity requirements on the function (8) and its derivative at the tailing point $\tilde{k}$ and approximately correspond to the best fit of the factor $\tilde{A}_c$. Note that though the tailing value of the parameter d in Eq. (10) is the maximal possible one, it is still slightly lower than the value $d \doteq d_{-1} \equiv \langle r^*\rangle\langle r^{*-1}\rangle$ corresponding to the quasi-classical large-k^* limit. This underestimation represents few per cent for the isotropic Gaussian $\boldsymbol{r}^*$-distribution ($d_{-1} = 4/\pi$); it increases for a superposition of Gaussians or - when introducing an anisotropy, both leading to an increase of d_{-1} and d_2. For example, at $R_y \approx R_z \approx R$ and $R/R_x^* =$ 0.5, 0.2 and 0, we have $d_{-1}/\tilde{d}_{-1} =$ 1.05, 1.29 and ∞, $d_2/\tilde{d}_2 =$ 3.2, 5.5 and 6.7, respectively. Unfortunately, the account of the large anisotropies is not reduced to the substitution of the corresponding d_2-values in Eq. (8). Their full account would require a more sophisti-

[4] The averaging over the relative emission coordinates $\boldsymbol{r}^*$ here depends on k^* since it is applicable only for $r^* > 2/(|a|k^{*2})$, corresponding to the momenta k^* larger than the classical boundary momentum $k_b = (|a|r^*/2)^{-1/2}$. This dependence is important only for k^* close or smaller than the mean boundary momentum $\langle k_b\rangle$.

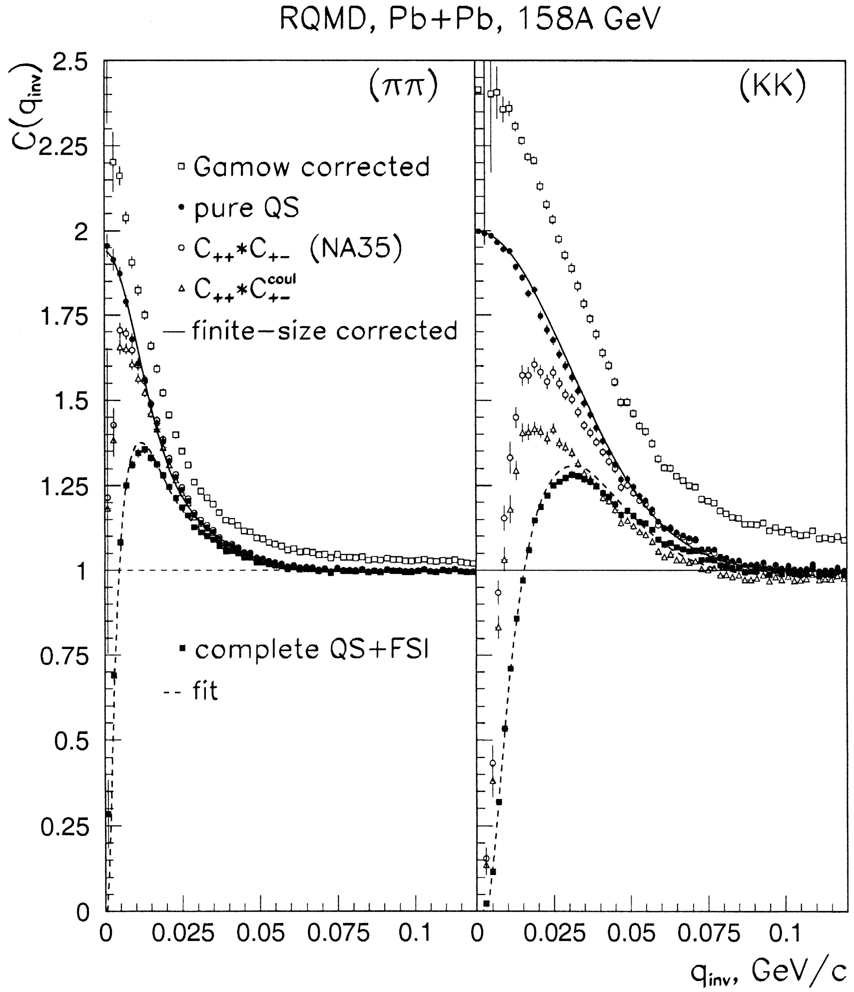

Fig. 1. The comparison of the Coulomb corrected two-boson correlation functions with the pure QS and the complete (QS + FSI) ones simulated with the RQMD code for Pb + Pb collisions at SPS energy. The curves correspond to the fits of the complete correlation functions according to Eqs. (7) and (8) using the two-Gaussian $(\pi\pi)$ or single-Gaussian (KK) parametrizations for the pure QS correlation function.

cated treatment of the finite-size corrections, accounting also for the neglected violation ($\sim \langle r^{*2}\rangle/a^2$) of the ansatz $\tilde{B}_c/\tilde{A}_c = 1$, for the k^*-dependence of the factors $\tilde{A}_c$ and $\tilde{B}_c$ at $k^* < \tilde{k}$ and for their directional dependence [5] and, eventually, for the higher than quadratic in $\langle r^*\rangle/a$ terms. Another possibility is the choice $d_2 = \tilde{d}_2 = 3\pi/8$ in Eq. (8), which appears to be an optimal one, effectively accounting for a partial compensation of the above mentioned factors and allowing to use the considered simple ansatz even at relatively large anisotropies.

It should be mentioned that for certain functional forms of the isotropic $\boldsymbol{r}^*$-distribution there exist analytical expressions for the finite-size Coulomb factors $\tilde{A}_c$ and $\tilde{B}_c$ obtained in the approximation linear in η [21]. These formulae, representing infinite series for a given r^*-distribution, are much more complicated than the simple tailing ansatz (8). Since

[5] For example, the term $2d_2\langle r^{*2}/a^2\rangle$ in Eq. (8) generally has to be substituted by $(3/2)\langle r^{*2}/a^2 + (\hat{\boldsymbol{k}}^*\boldsymbol{r}^*)^2/a^2\rangle$. For an anisotropic $\boldsymbol{r}^*$-distribution the directional dependence arises, even at $k^* \to 0$, from the term $\langle(\hat{\boldsymbol{k}}^*\boldsymbol{r}^*)^2/a^2\rangle = 2(\hat{k}_x^{*2}R_x^{*2} + \hat{k}_y^{*2}R_y^{*2} + k_z^{*2}R_z^{*2})/a^2$. Here $\hat{k}_j^* = k_j^*/k^*$ and $R_j^* = (\langle r_j^{*2}\rangle/2)^{1/2}$ are equal to the interferometric LCMS radii, except for $R_x^* = \frac{m_T}{m}R_x$.

the main error introduced by these approximations, as well as by the ansatz in Eqs. (7) and (8), is due to the neglect of the directional dependence and the contribution of the strong FSI (see below), we think that our simple ansatz is sufficient for a fast analysis of the correlation functions of charged pions and kaons and a preliminary estimate of the interferometric radii.

The quantity $\langle r^* \rangle$ can be, in principle, determined by fitting the correlation function of unlike charge bosons according to Eqs. (5) and (8). For kaons this is however practically impossible due to substantial contribution of the strong FSI. Another possibility is not to consider $\langle r^* \rangle$ as an additional free parameter but rather express it through the parameters $\{\Lambda_i, R_i\}$ describing the pure QS correlation function C_{QS}: $\langle r^* \rangle = \langle r^* \rangle_{\{\Lambda_i, R_i\}}$ (see, e.g., Eq. (13)) and fit the measured correlation function of like charge bosons according to Eqs. (7) and (8). The results of such fits of the $\pi^+\pi^+$ and K^+K^+ correlation functions simulated using RQMD at SPS energy are also shown in Fig. 1. To achieve a good agreement, for kaons it was sufficient to parametrize the k^*-dependence of the pure QS correlation function by a single Gaussian: $R = 4.7$ fm; $\langle r^* \rangle = 4R/\sqrt{\pi} =$ 10.5 fm. A two Gaussian parametrization was necessary for pions: the fitted fractions $\Lambda_i =$ 0.61, 0.33 and corresponding radii $R_i =$ 13.5, 6 fm; $\langle r^* \rangle = (4/\sqrt{\pi})(\Lambda_1 R_1 + \Lambda_2 R_2)/(\Lambda_1 + \Lambda_2) =$ 24.5 fm.

Since the two-pion correlation function at small p_T cannot be simply approximated by a single Gaussian, it may be useful to try to extract the pure QS correlation function without using its explicit form. This can be done at sufficiently small $\langle r^* \rangle / |a| \ll 1$ when the ratio $\tilde{A}_c(k^*)/A_c(\eta)$ depends on the nature of the two-particle system mainly through the linear combination of the inverse pair Bohr ''radius'' a (in particular, when the quadratic in $\langle r^* \rangle / a$ term in Eq. (8) can be neglected). Since $a = \pm|a|$ for the pairs of like and unlike charge bosons, respectively, we can then relate, on certain conditions (see below), the correlation functions C_{++} and C_{+-}. Thus, assuming that there is no difference in the production of the positive and negative bosons of the same type and neglecting the effect of the strong FSI, we can use Eqs. (5), (7) and the linear in η expansion of the function $F(-i\eta, 1, i\xi)$ in Eq. (4) to get in the absence of the long-lived sources ($\Lambda = 1$) and the sources of the ''coherent'' bosons, the following expression for the pure QS correlation function:

$$C_{QS}(p_1, p_2) \cong \frac{C_{++}(p_1, p_2)}{A_c(\eta_{++})} \times \left[2 - \frac{C_{+-}(p_1, p_2)}{A_c(\eta_{+-})} \right]^{-1}. \qquad (11)$$

With the considered accuracy, accounting for the terms linear in a^{-1}, we can also write, just by multiplying Eqs. (5) and (6) at $\Lambda = 1$, an alternative expression for C_{QS} valid on the same conditions as Eq. (11):

$$C_{QS}(p_1, p_2) \cong C_{++}(p_1, p_2) C_{+-}(p_1, p_2) \times \left[A_c(\eta_{++}) A_c(\eta_{+-}) \right]^{-1}. \qquad (12)$$

The latter expression can be compared with the intuitive prescription recently suggested by the NA35 collaboration [19]. This prescription coincides with Eq. (12) up to the inverse product of the Coulomb factors and thus it explodes at $k^* < 2\pi/|a|$. This explosion is demonstrated in Fig. 1, where we compare the NA35 prescription with the pure QS correlation functions simulated with the RQMD code. We can see that this prescription is applicable for pions at $q_{inv} > 10$ MeV while for kaons, even in the absence of the strong interaction, only at $q_{inv} > 35$ MeV. In Fig. 1 we also demonstrate the effect of the strong FSI. It may be seen that this effect is much more important for kaons, namely for K^+K^- system, thus preventing to use, in this case, not only the NA35 prescription, but also those in Eqs. (11) or (12). Note that the fitting procedure discussed above is practically not affected by the strong FSI since it deals only with K^+K^+ pairs.

In Fig. 2 we demonstrate the validity of Eq. (11) for the same pion pairs as in Fig. 1, simulated using the RQMD model for Pb + Pb collisions at SPS. To follow as closely as possible the applicability conditions of this equation, we have excluded the pions from the K_s^0- and Λ-decays and also, in Fig. 2a, switched off the strong FSI. The maximal deviation is about 2%. For the NA35 prescription the same is

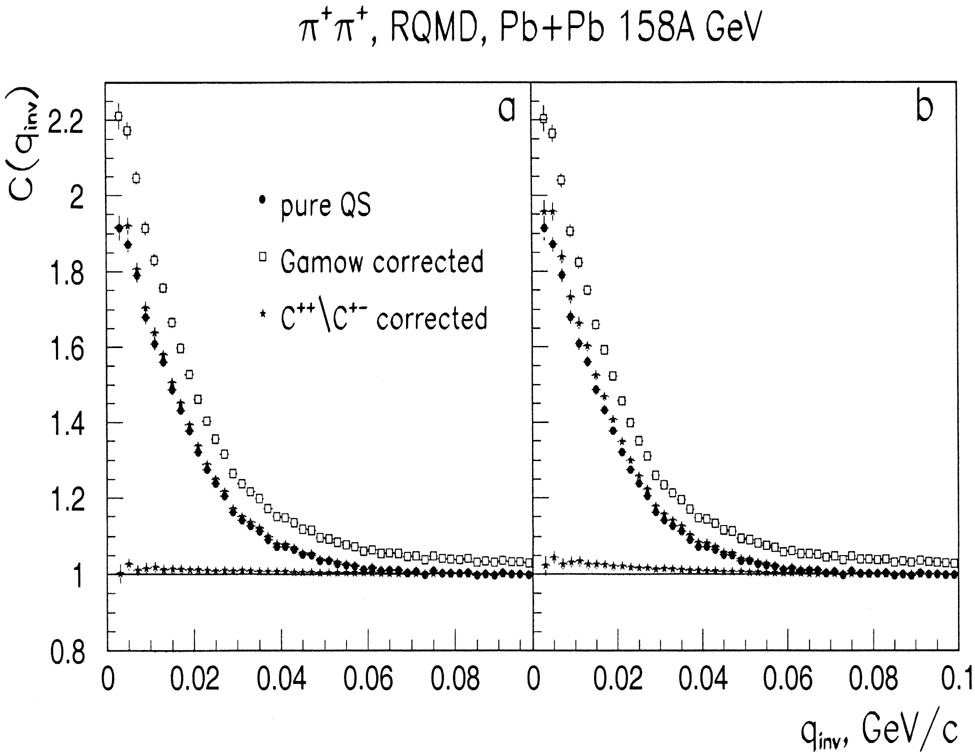

Fig. 2. The comparison of the Coulomb corrected two-pion correlation functions with the pure QS one simulated with the RQMD code for Pb + Pb collisions at SPS energy with the strong FSI switched off (a) and switched on (b). The $C^{++}\backslash C^{+-}$-correction is calculated according to Eq. (11). The points near the unity correspond to the ratio of the reconstructed pure QS correlation function to the simulated one.

true only at $q_{\mathrm{inv}} \equiv 2k^* > 10$ MeV. When the strong FSI is switched on (Fig. 2b) the maximal discrepancy is increased to about 4%. At LHC the effect of the strong FSI will be by a factor of about $10^{1/3}$ smaller due to the increased effective size.

Since the correlation function is expected to be strongly directional dependent (see Table 1) a more informative procedure than the 1-dimensional fit as in Fig. 1 would be to make the fit at a fixed pair momentum or at a fixed p_T, using for C_{QS} the

Table 1
The effective system sizes in the c.m.s. of the pion and kaon pairs calculated for various p_T-values in the 3-dimensional hydrodynamical model [4] for SPS Pb + Pb collisions. The model parameters are fixed based on the NA49 data (see text). To be closer to the data, the radius R_x^* is increased by 8%; $\langle r^* \rangle$ is calculated using Eq. (13) with $R^2 = (R_y^2 + R_z^2)/2$

p_T, GeV/c	0.15		0.4		0.8		1.6	
system	$\pi^+\pi^+$	K^+K^+	$\pi^+\pi^+$	K^+K^+	$\pi^+\pi^+$	K^+K^+	$\pi^+\pi^+$	K^+K^+
R_x^*, fm	9.0	4.4	16.7	5.5	28.1	7.7	47.7	12.8
R_y, fm	5.5	4.0	4.5	3.7	3.5	3.3	2.6	2.6
R_z, fm	6.7	4.5	4.1	3.7	2.8	2.7	1.6	1.7
$\langle r^* \rangle$, fm	16.2	9.7	21.5	9.8	32.9	11.0	54.2	15.5

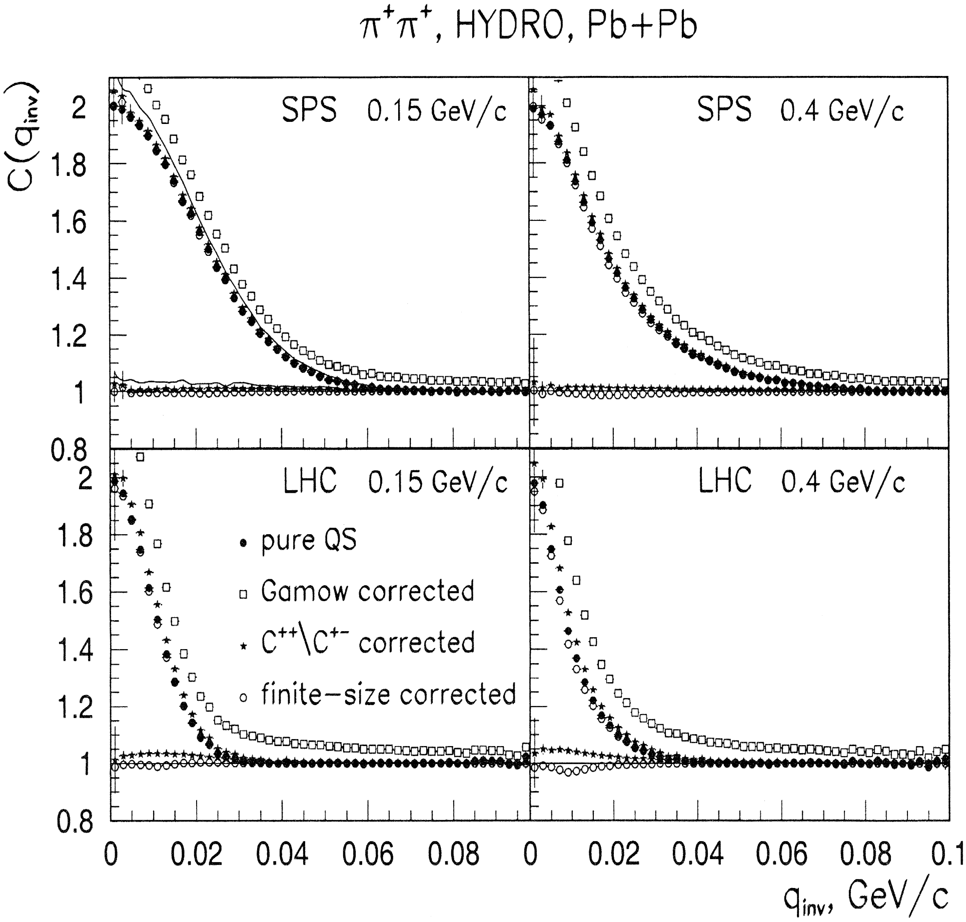

Fig. 3. The same as in Fig. 2 for the hydrodynamical calculations [4] of the two-pion correlation functions at SPS and LHC energies for $p_T = 0.15$ and 0.4 GeV/c. The broken lines in the left upper figure show the effect of the strong FSI. Also shown are the results of the finite-size correction similar to those in Fig. 1, using however the 3-dimensional Gaussian parametrization (1) for the pure QS correlation function; the corresponding radii for SPS energy are given in Table 1, those for LHC energy are taken by a factor of $10^{1/3}$ larger.

parametrization in 3-dimensional $\boldsymbol{k}^*$-space [6] similar to that in Eq. (1). For the anisotropic Gaussian, $\langle r^* \rangle$ can be expressed analytically through the interferometric radii in the case of practical interest, when $R_y \cong R_z \cong R$ and $R_x^* = \frac{m_T}{m} R_x \geq R$. Then

$$\langle r^* \rangle = \frac{2}{\sqrt{\pi}} R_x^* \left[1 + (1-\epsilon^2) \frac{1}{2\epsilon} \ln \frac{1+\epsilon}{1-\epsilon} \right], \quad (13)$$

where $\epsilon = \left(1 - \left(R/R_x^*\right)^2\right)_{1/2}$.

[6] Recall that in LCMS $q_j = 2k_j^*$, except for $q_x = 2\frac{m_T}{m}k_x^*$, and that $k^* = (\boldsymbol{q}^2 - q_0^2)^{1/2}/2$. In the simple hydrodynamic model [4] we could use the known analytical dependence of the interferometric radii on p_T and perform the fit even in the 4-dimensional $\{\boldsymbol{k}^*, p_T\}$-space.

In Figs. 3 and 4 we present the results of such 3-dimensional fits of the $\pi^+\pi^+$ and K^+K^+ correlation functions at SPS and LHC energies calculated using the hydrodynamical model of Ref. [4]. The corresponding effective sizes for SPS Pb + Pb collisions are given in Table 1. The effective sizes at LHC are assumed by a factor of $10^{1/3}$ larger. Also shown are the results of the Gamow correction and, in Fig. 3, of the parametrization independent $C^{++} \backslash C^{+-}$-correction. Except for the SPS ($p_T = 0.15$ GeV/c) case in Fig. 3 we neglect here the effect of the strong FSI which is of minor importance for the Gamow and finite-size corrections involving only the like charged bosons.

It can be seen from Fig. 3 that, in the absence of

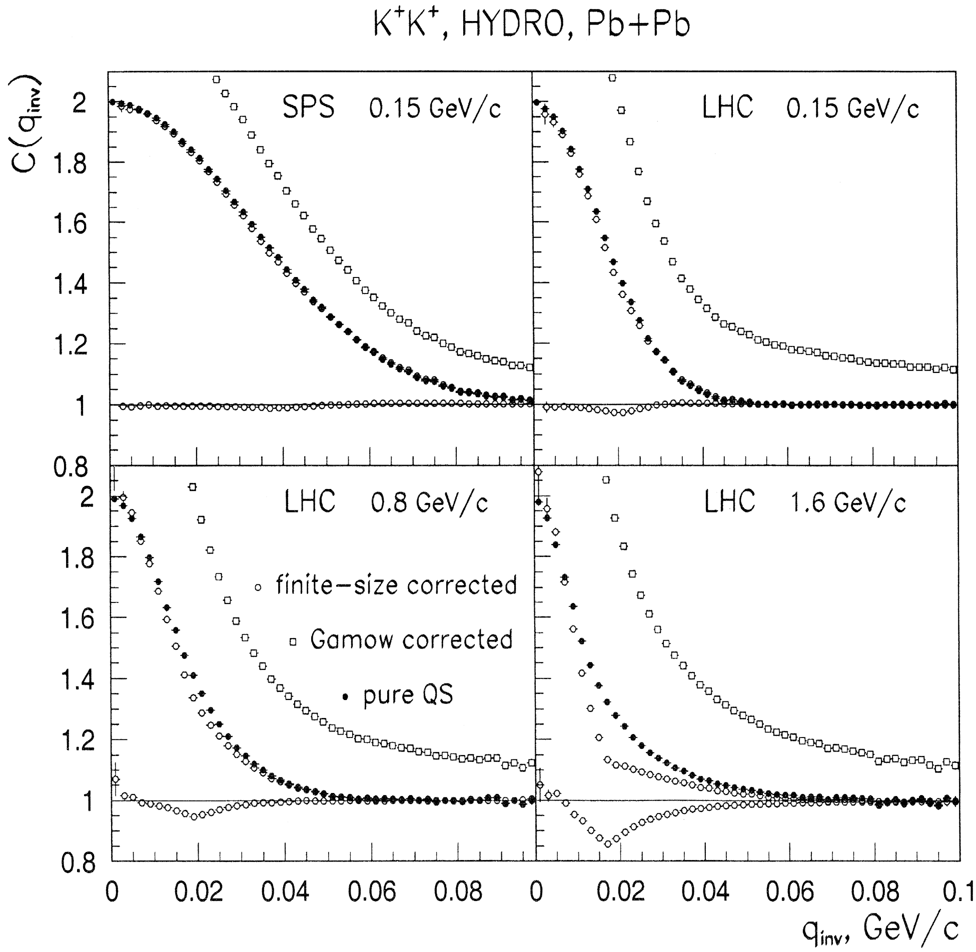

Fig. 4. The same as in Fig. 3 (except for the $C^{++}\backslash C^{+-}$-correction) for the two-kaon correlation functions at SPS and LHC energies for $p_T = 0.15$, 0.8 and 1.6 GeV/c.

the strong FSI, the maximal deviation of both our analytical prescriptions from the input QS two-pion correlation functions is 1.-1.5% for typical SPS conditions. For rather anisotropic LHC source at $p_T = 0.4$ GeV/c (see Table 1) it reaches 3% (finite-size correction) and 5% ($C^{++}\backslash C^{+-}$-correction). The influence of the strong FSI on the $C^{++}\backslash C^{+-}$-correction is demonstrated in Fig. 3 for the SPS ($p_T = 0.15$ GeV/c) case. It can be seen that the maximal discrepancy is increased up to 4-5%. This discrepancy remains practically the same up to LHC energy where it is, however, mainly related to the violation of the linearity condition $\langle r^* \rangle / |a| \ll 1$.

For kaons the violation of the latter condition is substantially stronger thus making necessary to account for the quadratic in $\langle r^* \rangle / a$ terms in the ansatz (8) for the finite-size Coulomb correction factor. Then, for $p_T < 0.5$ GeV/c the calculated pure QS correlation function coincides with the exact one within few per cent even at LHC energy (upper part of Fig. 4). At larger values of p_T the agreement becomes less good due to the increasing anisotropy and the effective size of the source (see Table 1). In bottom part of Fig. 4 we show the results for LHC energy at $p_T = 0.8$ and 1.6 GeV/c, the maximal deviation from the input correlation function averaged over the direction of vector $\boldsymbol{k}^*$ reaching 7% and 16%, respectively. For the $\boldsymbol{k}^*$-direction parallel and transverse to the outward x-direction, the deviations differ from the average one by a factor of about

1.6 and 0.7, respectively. For pions at these large values of p_T the errors introduced by the ansatz (8) are about the same as for kaons due to similar values of the ratio $\langle r^* \rangle / |a|$ (see Table 1).

Let us summarize the results. In this paper, instead of the usual zero-distance Coulomb factor, we suggest a simple analytical approximation of the finite-size Coulomb correction allowing one to extract the pure QS correlation functions of two identical charged pions or kaons in the case of relatively large effective emission volumes. This analytical correction, containing in principle no additional parameters to those describing the pure QS correlation function (e.g., in a hydrodynamic model), allows for a fast interferometry analysis and reasonable preliminary estimate of the model parameters even at RHIC and LHC energies.

A parametrization independent procedure for the extraction of the pure QS two-pion correlation function is suggested based on the measured $\pi^+\pi^+$ and $\pi^+\pi^-$ ones. Contrary to a similar NA35 procedure, it is applicable also in the region of very small relative momenta. This procedure is not applicable for kaons due to the non-negligible contribution of the strong K^+K^- interaction in final state and the failure, at very high energies, of the linear in $\eta = (k^* a)^{-1}$ approximation. Both the analytical approaches fail at very large transverse momenta due to a rapid increase of the anisotropy and the effective size of the source.

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