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Coulomb corrections to Bose–Einstein correlations have been greatly exaggerated

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The zero range Gamow factor is not a good representation of the effect of Coulomb corrections on measured two particle correlation functions. Better estimates are provided and it is shown that Coulomb corrections are entangled with others. Procedures currently employed result in a substantial overestimate of Coulomb corrections and hence of the magnitude of the Bose–Einstein enhancement.

1. Introduction

The phenomenon of Bose–Einstein correlation is by now well established and fairly well understood [1]. A better quantitative understanding is dependent on data with excellent statistics in which the correlation function for identical pions can be determined for values of $Q<0.1$ GeV, where Q is the usual variable defined by

$$Q^2=M^2_{\pi\pi}-4m^2_\pi .$$

Such data exist and more are on the way. Very soon our understanding of the phenomenon will become limited by systematic uncertainty in the correction of raw data. The one correction which has so far been made with great confidence is that due to Coulomb repulsion between like sign charged pions. I show here that the standard recipe overestimates the size of the correction to be applied for Coulomb effects and that the Coulomb correction is also dependent on the proportion of pairs in which one pion derives from decay of a particle which travels more than ~ 10 fm (η, ω, η′; weakly decaying particles). The quantity

$$C_2(Q)-1 ,$$

where $C_2(Q)$ is the two particle correlation function, can be over-estimated by more than 15% in the region $0.05<Q<0.2$ GeV.

2. Coulomb corrections

The wave function for two pions interacting through a pure Coulomb field, asymptotic to an incoming plane wave, is [2]

$$\psi_C=\Gamma(1+i\eta)\exp(-\tfrac{1}{2}\eta\pi)\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$$
$$\times F(-i\eta, 1, i[kr-\boldsymbol{k}\cdot\boldsymbol{r}]) , \qquad (1)$$

$$|\psi_C(0)|^2=\frac{2\pi\eta}{\exp(2\pi\eta)-1}=G(\eta) , \qquad (2)$$

where $Q=2k$, $\eta=\pm\alpha m_\pi/Q$ and F is the regular confluent hypergeometric function. The quantity η is positive for Coulomb repulsion and negative for attraction. Below I take η as positive and adopt the approximation $G(-\eta)\simeq G^{-1}(\eta)$.

It has been universally assumed that the appropriate correction, before or after fake pairs have been removed, is given by

$$C_2(Q)=C_2^{\exp}(Q)G^{-1}(\eta(Q)) . \qquad (3)$$

The underlying physical assumption is that the wavefunction over the source can be replaced by its value at the origin, an assumption familiar from nuclear physics.

This assumption is of dubious validity, because the effective source size is ~ 1 fm, yet for $Q=0.1$ GeV the potential energy in a pure Coulomb field balances the kinetic energy at a separation of 0.08 fm, ≪ 1 fm.

I have therefore made a more accurate estimate of the magnitude of this correction by determining the overlap of the square of the Coulomb wave function with a finite source.

The usual Bose–Einstein correlation may be obtained [3] from the expression

$$\int f(r)\,|\,[\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})-\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})]\,|^2 \mathrm{d}^3r\,, \tag{4}$$

where $f(r)$ is the source intensity as a function of the separation, normalised to unity. The commonly employed Gamow correction corresponds to multiplying this form by $G(\eta(Q))$. It is more correct to replace the plane wave functions by Coulomb wave functions asymptotic to outgoing plane waves [4]. The calculation is very simple for a source of form

$$f(r)\propto\exp(-\beta r)\,,$$

and I have therefore compared

$$\int \exp(-\beta r)\ \mathrm{d}^3r\,|\Gamma(1-\mathrm{i}\eta)\exp(-\tfrac{1}{2}\eta\pi)$$
$$\times\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\ F(\mathrm{i}\eta,1,-\mathrm{i}[kr+\boldsymbol{k}\cdot\boldsymbol{r}])\,|^2 \tag{5}$$

with

$$G\int \exp(-\beta r)\ \mathrm{d}^3r\,|\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,|^2\,. \tag{6}$$

(The cross terms in the Coulomb modification of (4) are harder to evaluate but factorisation obtains, approximately.)

The ratio R of (5) to (6) can be expressed as a double series

$$R=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_n^*A_m(\mathrm{i})^n(-\mathrm{i})^m\left(\frac{Q}{\beta}\right)^{m+n}$$
$$\times\frac{(2+m+n)!}{2(m+n+1)}\,, \tag{7}$$

where [2,5]

$$A_0=1\,,$$

$$A_n=\frac{\mathrm{i}\eta(1+\mathrm{i}\eta)\dots(n-1+\mathrm{i}\eta)}{n!n!}\,,\quad n\neq 0\,.$$

In the limit $(Q/\beta)\ll 1$, $R\to 1+3\alpha(m_\pi/\beta)+6\alpha^2(m_\pi/\beta)^2$. Expression (7) is easily evaluated for a given value of β, provided $(Q/\beta)\leqslant 1$; for $Q/\beta>1$ unpleasant oscillations develop. I show in fig. 1 the Coulomb correction factor $C(\beta,Q)^{-1}=(GR)^{-1}$ and for comparison the conventional correction factor G^{-1}. The value of β employed was 0.2 GeV (one inverse fm).

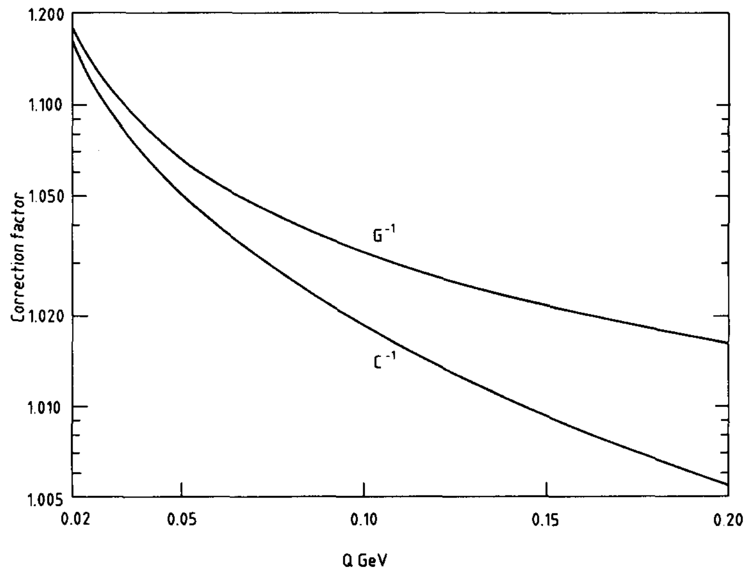

Fig. 1. Coulomb correction factors as a function of Q. The upper curve shows the reciprocal of the Gamow factor G. The lower curve shows the reciprocal of the more appropriate factor C, evaluated for $\beta=0.2$ GeV.

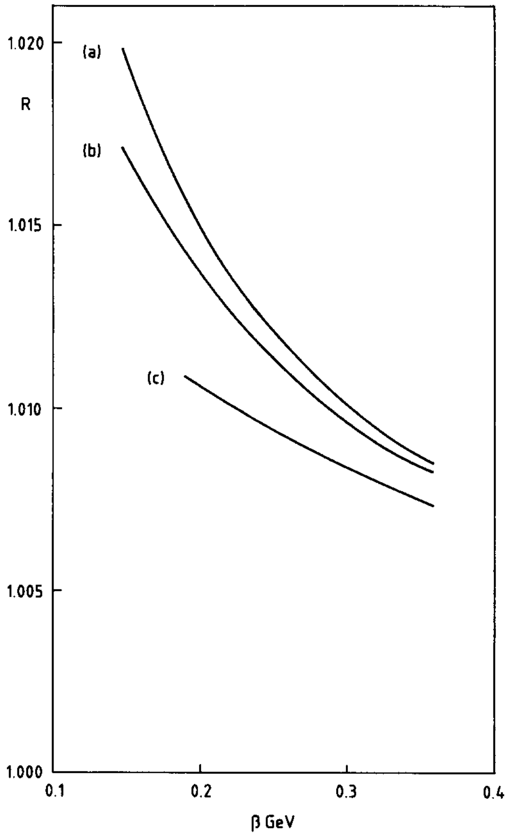

Fig. 2. The quantity R (eq. (7)) as a function of the source size parameter β. Curve (a) is for $Q=0.05$ GeV, (b) is for $Q=0.1$ GeV and (c) for $Q=0.2$ GeV.

(The normalised Fourier transform of $\exp(-\beta r)$ is [4]

$$\tilde{\rho}(Q^2)=\frac{1}{[1+(Q/\beta)^2]^2}, \tag{8}$$

and the corresponding ideal Bose–Einstein correlation is [3]

$$C_2(Q)=1+\tilde{\rho}(Q^2).$$

The Fourier transform $\tilde{\rho}(Q^2)$ falls to one half at $Q=0.643\beta$, to one quarter at $Q=\beta$.)

The variation of R with β is shown in fig. 2. The ratio R is slowly varying with Q and, for $\beta=0.2$ GeV, has a value of 1.015 for $Q=0.01$ GeV ($G^{-1}=1.4$) and falls to 1.01 for $Q=0.2$ GeV ($G^{-1}=1.016$). The finite size of the source results in $(C^{-1}-1)$ being half $(G^{-1}-1)$ at $Q\sim0.13$ GeV.

These effects, of the order of 1% of the Coulomb correction factor, are not without significance. Suppose that for perfect data, with no contributions from fake pairs or long lived decays, a correlation function

$$C_2(Q)=1+\tilde{\rho}(Q^2) \tag{9}$$

applies when all Coulomb effects have been taken out. Then the perfect data have a measured correlation function

$$C_2^{\text{exp}}(Q)=CC_2(Q)=C[1+\tilde{\rho}(Q^2)]. \tag{10}$$

This should be corrected by multiplying with C^{-1} but is, in the conventional treatment, multiplied instead by G^{-1}. This results in an overestimate of $C_2(Q)$ and the value of $\tilde{\rho}(Q^2)$ is spuriously increased by between 3% and 5% (see table 1, mixed pair normalisation). These results obtain if it is assumed that the correlation is normalised with pairs in which pions are drawn from different events; if the initial normalisation is to $\pi^+\pi^-$ pairs then, after trivial corrections, the experimental correlation function should be multiplied by C^{-2} and is in fact multiplied by G^{-2}. The Fourier transform $\tilde{\rho}(Q^2)$ is then overestimated

Table 1
Effect of Gamow correction on ideal data ($\beta=0.2$ GeV).

Q	$C_2(Q)$	Mixed pair normalisation			Unlike sign pair normalisation		
		$C_2^{\text{exp}}(Q)$ $=C_2(Q)C$	$G^{-1}C_2^{\text{exp}}(Q)$	$\frac{\Delta\tilde{\rho}}{\tilde{\rho}}$ (%) [a]	$C_2^{\text{exp}}(Q)$ $=C_2(Q)C^2$	$G^{-2}C_2^{\text{exp}}(Q)$	$\frac{\Delta\tilde{\rho}}{\tilde{\rho}}$ (%) [b]
0.05	1.886	1.794	1.914	3.2	1.707	1.943	6.4
0.10	1.640	1.610	1.662	3.4	1.580	1.685	7.0
0.15	1.410	1.392	1.427	4.1	1.384	1.444	8.3
0.20	1.250	1.230	1.263	5.2	1.236	1.277	10.8

[a] This column gives the fractional overestimate of $C_2(Q)-1$, $\{[G^{-1}C_2^{\text{exp}}(Q)-1]/[C_2(Q)-1]-1\}$.
[b] This column gives the fractional overestimate of $C_2(Q)-1$, $\{[G^{-2}C_2^{\text{exp}}(Q)-1]/[C_2(Q)-1]-1\}$.

by(6–10)% (table 1, unlike sign pair normalisation). Such corrections are already significant. However, Coulomb corrections can be overestimated in another way. This effect is of comparable importance and the two reinforce.

3. Coulomb corrections and long lived states

For $kr \gg 1$ the Coulomb wave functions are asymptotic to plane waves. Widely separated pairs suffer no Coulomb distortion and therefore no Coulomb corrections should be applied to pairs in which one member is a daughter of a long lived state or the two members are daughters of different long lived states. The relevant long lived states are weakly decaying strange particles and the set $\{L\} = \{\eta, \omega, \eta'; c, b\}$. Suppose we have data from which false pairs and decay products of strange particles have been eliminated. Let the true correlation function for all pairs which do not involve a daughter of $\{L\}$ be

$$C_2(Q) = 1 + \tilde{\rho}(Q) , \tag{11}$$

after Coulomb effects have been unfolded. Then the experimentally determined correlation will be, for normalisation with pairs constructed from different events,

$$C_2^{\exp} = xC_2(Q)C + (1-x) , \tag{12}$$

where a fraction $1-x$ of pairs involve a daughter of $\{L\}$. The Coulomb corrected correlation function is

$$xC_2(Q) + (1-x) = 1 + x[C_2(Q) - 1] , \tag{13}$$

whereas the corrected function that has so far been employed is

$$G^{-1}C_2^{\exp}(Q) > 1 + x[C_2(Q) - 1] .$$

The quantity $G^{-1}C_2^{\exp}(Q) - 1$ exceeds $x[C_2(Q) - 1]$ by as much as 10%. The fraction x has been overcorrected and the fraction $1-x$ should not have been corrected at all. Some examples are given in table 2 (mixed pair normalisation).

Things are even worse if $\pi^+\pi^-$ pairs are used as a reference sample. After a trivial correction for the relative number of like and unlike sign pairs, the experimentally determined correlation is

$$C_2^{\exp}(Q) = \frac{xC_2(Q)C + (1-x)}{xC^{-1} + (1-x)} , \tag{14}$$

because close unlike pairs at small Q are enhanced by Coulomb effects. The Coulomb corrected function should again be (13) but invariably the form

$$G^{-2}C_2^{\exp}(Q) > 1 + x[C_2(Q) - 1]$$

is taken as the Coulomb corrected correlation function. In this case $G^{-2}C_2^{\exp}(Q) - 1$ can exceed $x[C_2(Q) - 1]$ by as much as 20% (table 2 (unlike sign pair normalisation)). These numbers may seem surprisingly large: the skeptical reader must perform the calculations himself.

The extent to which these considerations affect the interpretation of any real data set must be left to the proprietors to determine; they may be of relevance to, for example, recent OPAL data [6].

Table 2
Effect of Gamow correction applied to data with a fraction $1-x$ of pairs containing a daughter of $\{L\}$ (β=0.2 GeV), x=0.6.

Q	$C_2(Q)$	$1+x[C_2(Q)-1]$	Mixed pair normalisation			Unlike sign pair normalisation		
			$C_2^{\exp}(Q)$ (eq. (12))	$G^{-1}C_2^{\exp}(Q)$	$\frac{\Delta\tilde{\rho}}{\tilde{\rho}}$ (%) [a]	$C_2^{\exp}(Q)$ (eq. (14))	$G^{-2}C_2^{\exp}(Q)$	$\frac{\Delta\tilde{\rho}}{\tilde{\rho}}$ (%) [b]
0.05	1.886	1.532	1.470	1.568	6.8	1.426	1.623	17.2
0.10	1.640	1.384	1.366	1.410	6.8	1.351	1.440	14.6
0.15	1.410	1.246	1.238	1.265	7.7	1.231	1.289	17.5
0.20	1.250	1.150	1.146	1.164	9.3	1.142	1.179	19.3

[a] This column gives the fractional overestimate of $C_2(Q) - 1$, $\{[G^{-1}C_2^{\exp}(Q) - 1]/x[C_2(Q) - 1] - 1\}$.
[b] This column gives the fractional overestimate of $C_2(Q) - 1$, $\{[G^{-2}C_2^{\exp}(Q) - 1]/x[C_2(Q) - 1] - 1\}$.

4. Multiparticle Coulomb effects

The quantity $C=GR$ has been evaluated for two particles only and a pure Coulomb potential. The Coulomb correction will be closer to unity than $C(\beta, Q)$ because at some level screening due to other charged pions will be effective. In any model, such as the string, in which particles are ordered in space–time, at least one π^- must separate the members of a $\pi^+\pi^+$ pair. The net effect on $\pi^+\pi^-$ is small but $\pi^+\pi^+$ repulsion is turned into attraction via the intervening π^-. For a long range field, it is intuitively the case that this effect will be important only if the π_1^- travels with the $\pi_2^+\pi_3^+$ pair, that is $Q_{12}\sim Q_{13}\sim Q_{23}$. (This limitation does not apply to short range (strong) final state interactions [7].) In the spirit of ref. [7] one might envisage the replacements

$$C(Q_{23})\to C(Q_{23})\langle C^{-1}(Q_{12})C^{-1}(Q_{13})\rangle_{k3}$$

for the $\pi^+\pi^+$ Coulomb correction, and

$$C(Q_{12})\to C(Q_{12})\langle C^{-1}(Q_{23})C(Q_{13})\rangle_{k3}$$

for the $\pi^+\pi^-$ Coulomb correction. The average

$$\langle C^{-1}(Q_{23})C(Q_{13})\rangle_{k3}$$

cannot differ from unity but, because of the rapid variation of $C(Q)$ at small values of Q, the factor

$$\langle C^{-1}(Q_{12})C^{-1}(Q_{13})\rangle_{k3}$$

is very sensitive to the distribution of Q_{12}, Q_{13} associated with a local source of size $\sim\beta^{-1}$. I think it unlikely that this factor exceeds unity by more than 0.005 but I cannot offer any reliable estimate of multiparticle modifications to the Coulomb correction for like sign pairs. The possibility that $|C(Q)-1|$ is in fact an overestimate should be remembered. It is not impossible (for like sign pairs) that $C(Q)-1$ has the wrong sign for Q greater than some characteristic value.

5. A plea to the proprietors of data

There are two corrections to raw Bose–Einstein correlation functions that are highly apparatus dependent and which must be performed by the proprietors of the data. The first is the effect of pairs that are not in fact identical pions. To the extent that particles close in momentum are also produced close in space–time, the Coulomb correction C may be expected to apply to a fraction x of these pairs. (For a false pair, the Coulomb correction C for a given relative momentum Q differs from that for an identical pair, because $\eta=\alpha/v$, where v is the relative velocity.) The second is the effect of pairs in which one member is a daughter of a weakly decaying particle with proper lifetime $\tau>10^{-10}$ s. The number of such pairs is not Coulomb distorted.

After these corrections and no others have been made, the corrected correlation function should be presented before proceeding to further corrections.

There are then corrections which are largely independent of the apparatus but which depend on experimental conditions and so are best made by the proprietors of the data. They fall into two categories.

(a) The effect of pairs in which one particle is a daughter of c or b decay. This correction may depend on the selection of data: cuts are always applied to ensure reliability of data and other selections may be made. For example, a cut on boosted sphericity might be employed in order to reduce the effect of b decay. (Such selections are dangerous and I suggest they be eschewed.) The proportion of pairs containing a daughter of c or b decay is weakly energy dependent and of course depends on the rate at which b and c are produced. The primary quark mix on the Z^0 differs from that produced by virtual photon exchange and hadronic interactions are different again. Simply by counting the number of pairs of like sign particles, I estimate that in e^+e^- annihilation $\sim 30\%$ of like sign pairs contain a single daughter of c, b decay at 35 GeV, falling to $\sim 22\%$ at 60 GeV. On the Z^0, at 90 GeV, I estimate $\sim 18\%$. These figures include pairs in which the members are not identical and pairs in which one member is a daughter of a weakly decaying strange particle. The decay products of c and b are rich in strange particles – some care is necessary to ensure that corrections are not counted twice.

Pairs in which one member is a daughter of b or c decay suffer no Coulomb effects. Pairs in which both members come from the decay of the same b or c likely share both the Bose–Einstein correlation and Coulomb effects.

(b) The effect of pairs in which one member is a daughter of η, ω, η' decay. Such pairs again suffer no

Coulomb effects.

The (two particle) Coulomb correction is trivial in comparison with the above corrections, but entangled with them.

6. Conclusion

If the true Bose–Einstein enhancement, after all corrections have been made, is $C_2(Q)-1$, then experimentally one determines

$$C_2^{\exp}(Q)=[x(1-y)C_2(Q)+xy]C+(1-x) \quad (15)$$

for mixed pair normalisation, or

$$C_2^{\exp}(Q)=\frac{[x(1-y)C_2(Q)+xy]C+(1-x)}{xC^{-1}+(1-x)} \quad (16)$$

for normalisation with pairs of unlike sign. In (15) and (16) above, the fraction of false pairs is y and the fraction of pairs containing a daughter of a long lived state is $1-x$. The Coulomb corrections are entangled with the others and cannot be made multiplicatively, although the factor C can be calculated.

Eqs. (15) and (16) are schematic. An outsider cannot lay down a precise recipe for the analysis of data, but can venture only to suggest the bare bones of an approach. The details must be determined by the proprietors of real data. In the absence of a complete model for generating both Bose–Einstein and Coulomb correlations within any Monte Carlo data set, it is the task of those proprietors to extract from data and tuned Monte Carlo their best estimates of quantities $x(Q)$ and $y(Q)$, analogues of x, y in (15), (16) and hence obtain the quantities of physical interest such as

$$C_2(Q)\,, \quad 1+x_i(C_2(Q)-1)\,,$$

where x_i may represent the effect of a partial selection from $\{L\}$.

Future data will deserve such attention to detail. It is particularly important that the Bose–Einstein enhancement is not overestimated, because if there is a problem in understanding the data it is that the enhancements reported seem too big rather than too small [1].

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