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The hydrodynamics of hadron matter under a pion interferometric microscope

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Abstract. A method which makes it possible to examine the hydrodynamic motion of hadron matter is proposed. It is based on analysing Bose-Einstein pion correlations. It enables the time, expansion rate and freeze-out temperature of the matter produced in the central region of high-energy hadronic and nuclear collisions to be measured.

Two-particle correlations of identical particles are now widely used in determining the size and shape of the multiparticle generation region in hadronic, nuclear and e^+e^- collisions. The crux of the method is that the width Q_0 of a two-particle correlator, which is considered to be a function of the momentum difference $\Delta\mathbf{P}$, is inversely proportional to the emission region dimension a in a direction parallel to $\Delta\mathbf{P}$ [1–4]. However, this approach does not take into account the possibility of internal relative motion of radiation sources in a system. In this paper, we shall demonstrate how the hydrodynamic motion anticipated for ultrarelativistic nuclei and pp, $\bar{p}p$ collisions [5, 6] can be detected by the pion interferometric method.

1. We base on the pion interferometry theory for a hydrodynamical stage of multiparticle processes, which was developed in our previous papers [7, 8].

The one-particle pion inclusive spectrum in hydrodynamics theory is described by [9]

$$\frac{p^0}{\sigma_T}\frac{d^3\sigma}{d^3p}=p^0 n(p)=\int_{\Sigma_c} d\sigma_\mu\, p^\mu f(pu(x)), \tag{1}$$

where p^μ is the secondary particle 4-momentum, σ_T is the total cross-section, $\Sigma_c: t=t_c(x)$ is the critical hypersurface in space-time where a hadronic fluid reaches the freeze-out temperature T_c and decays into secondary particles, $f=(2\pi)^{-3}\left[\exp\left(\frac{pu}{T_c}\right)-1\right]^{-1}$ and $u^\mu(x)$ is the hydrodynamical 4-velocity of hadronic matter at the decay stage. In this picture, the fluid elements can be considered to be radiation sources. In the general case, these sources do not have identical spectra in their own rest systems [9]; moreover they move with different velocities $u(x)$ and radiate at different times $t=t_c(x)$. The radiation duration time Δt (the decay time of a fluid element) is negligible [5].

The two-particle inclusive $\pi\pi$ cross-section of identical pions in hydrodynamics theory has the form [7]:

$$\frac{p_1^0 p_2^0}{\sigma_T}\frac{d^6\sigma}{d^3p_1\, d^3p_2}=p_1^0 p_2^0\, n(p_1)\, n(p_2)[1+R(p_1,p_2)], \tag{2}$$

where the correlator $R(p_1,p_2)$ arises from interference of identical pions

$$\begin{aligned}&4p_1^0 p_2^0\, n(p_1)\, n(p_2)\, R(p_1,p_2)\\&=\iint_{\Sigma_c} d\sigma_\mu(x_1)\, d\sigma_\nu(x_2)(p_1^\mu+p_2^\mu)(p_1^\nu+p_2^\nu)\\&\quad\times f(p_1u(x_1))\, f(p_2u(x_2))\cos(p_1-p_2)(x_1-x_2).\end{aligned} \tag{3}$$

It is appropriate to mention here that the interferometry method for moving sources was also studied in papers [10] for the special case of a scaling expansion. But the model of pion emission by currents used in these papers is not suitable for the hydrodynamics theory of multiparticle production. This causes discrepancies in the initial expression of (3) and, of course, in numerical results. The hydrodynamic approach deals with real local thermodynamic equilibrium for a pion gas at the decay hypersurface Σ_c [5, 9]. This initial suggestion gives correlator (3) unambiguously [7]. The explicit calculations of correlator

(3) are easily performed in terms of the auxiliary function [8]

$$J(p_1, p_2) = \tfrac{1}{2} \int_{\Sigma_c} d\sigma_\mu(x)(p_1^\mu + p_2^\mu)$$
$$\times f(p_1 u(x)) \exp[-i(p_1 - p_2) x] \tag{4}$$

so that

$$p^0 n(p) \quad = J(p, p)$$
$$R(p_1, p_2) = \mathrm{Re}\,[J(p_1, p_2)\, J(p_2, p_1)/J(p_1, p_1)\, J(p_2, p_2)]. \tag{5}$$

2. In the hydrodynamics theory of multiparticle production, the one-dimensional expansion of matter along the collision axis $x^{\|} = x$ is assumed to be an initial approximation. So the speedograph transformation may be used:

$$T_0 x = e^{-z}\left(\frac{\partial \chi}{\partial z} \operatorname{sh} y - \frac{\partial \chi}{\partial y} \operatorname{ch} y\right)$$
$$T_0 x = e^{-z}\left(\frac{\partial \chi}{\partial z} \operatorname{ch} y - \frac{\partial \chi}{\partial y} \operatorname{sh} y\right), \tag{6}$$

where $z = \ln(T/T_0)$, $y = \operatorname{arth} v(t, x)$ is a hydrodynamic rapidity of the fluid element moving along the x axis, T_0 is some reference point of temperature, and $\chi(z, y)$ is the hydrodynamic potential. The explicit forms of the potentials are well known for Landau and scaling models [5, 11]. The decay region Σ_c is limited by longitudinal coordinates $X_{\max} = -X_{\min} = \frac{a}{2}$, or rapidities $y_{\max} = -y_{\min} = Y$ in c.m.s.

Let us now deal with the rapidity variables of the secondary pions π_1, π_2

$$p_i^0 = m_{i\perp} \operatorname{ch} \theta_i,\quad p_i^{\|} = m_{i\perp} \operatorname{sh} \theta_i;$$
$$m_\perp^2 = m_\pi^2 + p_\perp^2;\qquad Q = \frac{p_1^{\|} - p_2^{\|}}{2}$$
$$P = \frac{p_1^{\|} - p_2^{\|}}{2};$$
$$\alpha = \frac{\theta_1 - \theta_2}{2} \approx \frac{Q}{\sqrt{P^2 + m_\perp^2}},\qquad \theta = \frac{\theta_1 + \theta_2}{2} \approx \operatorname{arch}(P/m_\perp). \tag{7}$$

Here $p_i^{\|}$ is the momentum projection on the collision axis, $\mathbf{p}_{i\perp}$ are the transverse components. When we analyse the two-particle correlator, we assume $\mathbf{p}_{1\perp} = \mathbf{p}_{2\perp}$. Using variables (6, 7), we can then write

$$J(p_1, p_2) = \frac{S_\perp m_\perp \operatorname{ch} \alpha}{(2\pi)^3 T_c} \int_{-Y}^{Y} dy [\Phi(y) \operatorname{ch}(\theta - y)$$
$$- \Psi(y) \operatorname{sh}(\theta - y)]$$
$$\times f\left(\frac{m_\perp}{T_c} \operatorname{ch}(y - \theta - \alpha)\right)$$
$$\cdot \exp[2i \operatorname{sh} \alpha (\chi_z \operatorname{sh}(y - \theta) - \chi_y \operatorname{ch}(y - \theta))], \tag{8}$$

where $\Phi(y)$ and $\Psi(y)$ are the known functions [9]

$$\Phi(y) = \chi_z - \chi_{yy},\ \Psi(y) = \chi_y - \chi_{zy};$$
$$\chi_z = \frac{\partial \chi}{\partial z}, \ldots. \tag{9}$$

Here $S_\perp = \pi r_\perp^2$ is the transversal area of the hydrodynamic tube at the final stage.

If only particles with $m_\perp \gg T_c$ are selected, the Bose-Einstein factor f in (8) can be replaced by a Boltzman exponent. Under the above assumption, the function $J(p_1, p_2)$ can be evaluated by the saddle-point method in the central rapidity region $|\theta_i| \ll Y$. The equation which defines the saddle point is

$$\operatorname{th}(y_0 - \theta) = \operatorname{th} \alpha \frac{1 + 2i\,\Phi(y_0)}{1 + 2i \operatorname{th} \alpha\, \Psi(y_0)}. \tag{10}$$

If $\alpha = 0$, the obvious solution of (10) allows us to evaluate the one-particle spectra (1), (5) and to find the pion rapidity distribution

$$H(\theta) = \frac{dN}{d\theta} = \tau(\theta)\, S_\perp\, N_0 \left(\frac{T_c}{m_\pi}\right);$$
$$\tau(\theta) = \frac{\Phi(\theta)}{T_c} \cong \left(\frac{du^{\|}}{dx}\right)^{-1}. \tag{11}$$

Here $N_0 = (g\, T_c^3/2\pi^2)\, F(m_\pi/T_c)$ [5], $g = 3$ for the pion triplet, and $\frac{du^{\|}}{dx}$ is the gradient of the 4-velocity longitudinal component of the fluid element, which moves with rapidity θ.

The solution of (10) for $\alpha \neq 0$ can be obtained by the successive approximation detailed in [8]. If we measure correlations only in the region $\operatorname{th}^2 \alpha \ll 1$ and neglect the terms $\approx \operatorname{th}^4 \alpha$ in the pion correlator, the hydrodynamic correlator in the central rapidity region for pions with equal transversal masses $m_{1\perp} = m_{2\perp} (|\mathbf{p}_{1\perp}| = |\mathbf{p}_{2\perp}|)$ has the form:

$$R(\alpha, \theta) = \lambda \exp[-4 m_\perp T_c \tau^2(\theta) \operatorname{th}^2 \alpha] \cos[(4 m_\perp$$
$$+ 6 T_c)\, \tau(\theta) \operatorname{th}^2 \alpha]. \tag{12}$$

The factor $\lambda = 1$ at $\mathbf{p}_{1\perp} = \mathbf{p}_{1\perp}$. The factor $\lambda < 1$ in the general case when the directions of momenta $\mathbf{p}_{1\perp}$ and $\mathbf{p}_{2\perp}$ are not fixed to be exactly parallel, and when we take into account that the interference is not complete for various reasons.

Contrary to the standard model with motionless sources [1–4], the correlator (12) does not depend on a size of the system. If one determines it formally using the usual interferometric method via the correlator width $Q_0 : a_{\text{eff}} \approx 1/Q_0$ [connected with the rapidity width α_0 according to (7)], there are then two effective lengths, depending on the hydrodynamic regime. If we deal with the moderate velocity gradients of a hadron fluid, $\frac{du^{\parallel}}{dx} = \frac{1}{\tau} \ll T_c$, the effective length $a_{\text{eff}} = a_T = \tau \sqrt{\frac{T_c}{m_\perp}}$ is the longitudinal size of a fluid element forming the density of a one-particle spectrum at the points $p_1, p_2 \approx p/2$ [8]. In this case, the correlator (12) shows exponential behaviour. If the correlator behaviour is oscillatory, the velocity gradient is large, $\frac{du^{\parallel}}{dx} > T_c$, and the effective length $a_{\text{eff}} = a_H = \sqrt{\frac{\tau}{m_\perp}}$ is the distance between the fluid elements which contribute to the one-particle spectrum densities at points p_1 and p_2. The distance a_H exceeds the sizes a_T of the elements themselves: $a_H > a_T$. The typical regimes of correlator $R(Q, P)$ behaviour for $\bar{p}p$ collisions ($T_c \cong m_\pi, r_\perp \cong 1/m_\pi$) in the Landau model [5] and scaling model [6, 11, 12] are plotted in Fig. 1 for $P=0$. The factor $\lambda = 1$. The effective dimensions depending upon the hydrodynamic velocity gradient and the heat broadening of the hydrodynamic spectrum are of the order $a_{\text{eff}} \approx 1 \div 5$ fm, while the whole lengths of the decaying system are within $15 \div 80$ fm. Therefore the energy density ε calculated using $\varepsilon = E/S_\perp a_{\text{eff}}$ would be an overestimation if a system possessing developed hydrodynamic motion is mistaken for a system without internal motion.

3. A qualitative experimental test indicates what variant of matter evolution takes place in high-energy hadronic or nuclear collisions.

a) The formation of an intermediate massive cluster (fireball), i.e. the absence of developed hydrodynamic motion.

If the rest spherical cluster of radius r decays, the correlator is described by [1, 8]

$$R(\Delta \mathbf{p}) = \frac{(p_1^0 + p_2^0)^2}{4 p_1^0 p_2^0} \left[3 \frac{j_1(|\Delta \mathbf{p}| r)}{|\Delta \mathbf{p}| r} \right]^2, \tag{13}$$

where $j_1(x)$ is the spherical Bessel function. The first zero at $|\Delta \mathbf{p}| = \Delta \mathbf{p}_0$ in (13) enables one to determine the cluster size: $r = 4.49/\Delta \mathbf{p}_0$.

If the prolate shape rest system radiates, the correlator is approximated by a 3-dimensional Gaussian form [4]. Its widths determine the longitudinal a and the transverse $r_\perp$ effective sizes of the emitting object:

$$R(\Delta \mathbf{p}) = \exp[-r_\perp^2 |\Delta \mathbf{p}_\perp|^2] \exp[-a^2 (\Delta \mathbf{p}^{\parallel})^2]. \tag{14}$$

At present, interferometric analysis of the dimensions and shape of an emission region is based on models of this type. The common property of models (13, 14) is that correlator $R(Q, P)$ in momentum space depends only on the momentum difference $2Q$ in the direction of interest, but not on the momentum sum $2P$. The width of the same correlator $R(\alpha, \theta)$ expressed in rapidity variables decreases according to (7) when the rapidity sum 2θ increases (see Fig. 2).

b) The emergence of a global hydrodynamic regime for matter evolution in events with high multiplicity fluctuations of $\bar{p}p$ collisions and ultra relativistic heavy ion collisions.

In accordance with the basic results of the hydrodynamic approach [5, 6, 11, 12], the function $\tau(\theta)$ connected with the one-particle rapidity spectrum via (11) is a constant in the scaling model, or slowly de-

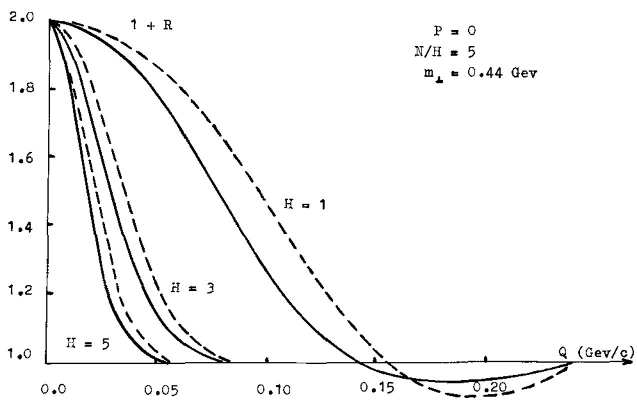

Fig. 1. The Q dependences of correlator $R(Q, P)$ at $P=0$ for the scaling model S (continuous line), and the Landau model L (dashes), at different plateau heights $H(0)$ in $p\bar{p}$ collisions. When $H=1$, $a_s = 17$ fm, $a_L = 14$ fm, $a_{\text{eff}} = 1$ fm; when $H=3$ $a_S = 50$ fm, $a_L = 42$ fm, $a_{\text{eff}} = 2.5$ fm; when $H=5$ $a_S = 80$ fm, $a_L = 70$ fm, $a_{\text{eff}} = 5$ fm

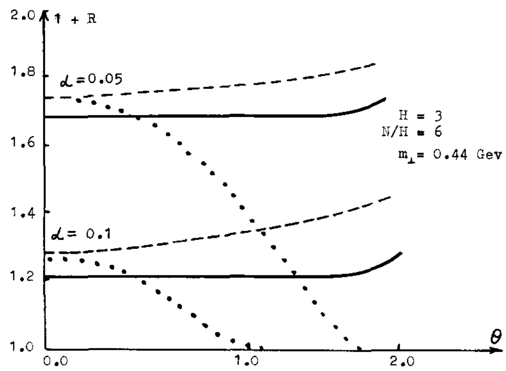

Fig. 2. The θ dependence of correlator $R(\alpha, \theta)$ at different α for hydrodynamic S, L models and the rest cluster (C) model (dots). The effect of hydrodynamic boundaries [8] begins manifesting itself near $\theta = 2$

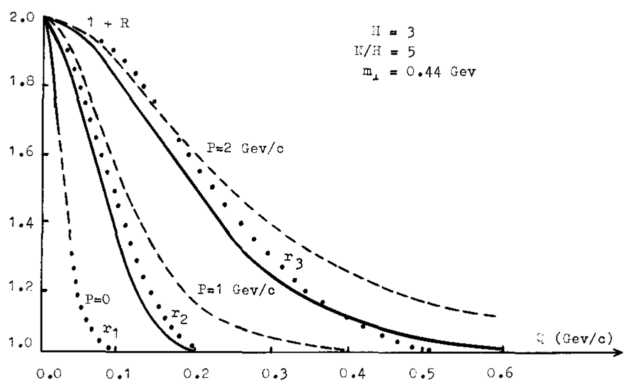

Fig. 3. Correlator $R(Q, P)$ at different values P for S, L and C models. The hydrodynamic S, L correlators at different values P are approximated by the cluster model with different cluster radii r: $r_1 = 4.3$ fm, $r_2 = 1.8$ fm, $r_3 = 0.8$ fm. At $P=0$ the curves of the S, L, C models are indistinguishable in the figure scale

creases, as in the Landau model, when θ increses (in c.m.s.). If a plateau in the central region of the rapidity distribution is observed, the width α_0 of the hydrodynamic correlator $R(\alpha, \theta)$ in the rapidity variables does not change when the detected-particles rapidity sum 2θ increases in c.m.s. (see Fig. 2). The same correlator $R(Q, P)$ in momentum variables broadens with increasing momentum sum $2P$, due to (7) (see Fig. 3):

$$Q_0(P \neq 0) = Q_0(P=0)\sqrt{1 + P^2/m_\perp^2}. \tag{15}$$

This gives rise to an impression of decreasing source size $a_{\text{eff}} \sim 1/Q_0(P)$ according to (15) when the momentum sum increases. Thus the rapidity difference is a natural variable for hydrodynamic correlators, and the momentum difference is a natural variable for the radiation interference from a rest media. If the rapidity plateau is absent (the Landau model leads to a Gaussian-type falling of a rapidity distribution), the hydrodynamic correlator gains a broadening additional to (15) (see Figs. 2, 3). Note also that the effective hydrodynamic length has a specific dependence on the transverse mass of detecting particles, $a_{\text{eff}} \sim 1/\sqrt{m_\perp}$, for all hydrodynamic models.

c) The break-up of hot quark-gluon matter into drops because of large density fluctuations during phase $qg - h$ transition [12].

If the global hydrodynamic regime breaks down and a fluid decays into drops, the typical rapidity distance between drops is unity [11]. As a result, the contribution to the correlator comes from the radiation of a single drop only (the typical rapidity width α_0 of the correlator $\alpha_0 \ll 1$). Owing to homogeneity in the rapidity distribution of drops [12], the correlator does not depend on the rapidity sum 2θ in a longitudinal direction, as does the scaling-hydrodynamics correlator: $R_{\text{drop}}(\alpha, \theta) = R_{\text{drop}}(\alpha)$. Moreover, because of the spherical symmetry of drop decay, its correlation characteristics must be the same for any decay mechanism:

$$R_{\text{drop}}(\mathbf{p}_1 - \mathbf{p}_2,\ \mathbf{p}_1 + \mathbf{p}_2 = 0) = R(|\mathbf{p}_1 - \mathbf{p}_2|).$$

4. If the tests indicate the existence of a global hydrodynamic regime, one can analyse the regime in detail using correlator (12). We shall now demonstrate how to analyse the scaling regime of matter evolution in nucleus-nucleus collisions [6, 12]. For the scaling model, the decay isotherm has the form $\tau^2 = t^2 - x^2$, and $\tau = \left(\dfrac{du^{\|}}{dx}\right)^{-1}$ means the proper time of system expansion. The longitudinal velocity distribution has the form $v_{\text{hydr}} = \dfrac{x}{t}$.

In hadronic and nuclear collisions, the parameter of hydrodynamic flow change from one collision event to another, so it is necessary to express the parameters in terms of observable quantities and to select events with the same parameters. The plateau height in the central region $H = H(0)$ (in c.m.s.) is taken as one of the main observables quantifying the physical picture. The height of the plateau H and the total pion multiplicity N in the central region are connected with the hydrodynamic parameters of the scaling model [8, 12] (for the Landau model, see [8]):

$$\tau = \frac{H}{S_\perp N_0(T_c)}; \qquad Y = \frac{N}{2H}; \qquad a_S = 2\tau \operatorname{sh} Y_S;$$
$$s_0 = \frac{3.7 N_0(T_c)}{\tau_0}\tau, \tag{16}$$

where s_0 is the initial entropy density, $\tau_0 \approx (0.5 \div 1)$ fm is the initial time when the hydrodynamic stage is formed. Note that measuring only the plateau height H in ultrarelativistic nuclear collisions does not enable τ to be reliable determined, since only 40% of the variations in transverse radius $r_\perp$ and the temperature T_c at the final stage of matter evolution lead to a change in τ by an order of magnitude, due to the presence of the factor $S_\perp N_0(T_c)$ in (11). The proper time of the expansion τ and the freeze-out temperature T_c can be determined directly from the correlation data.

The correlator behaviour at chosen $m_\perp \gg T_c$ (or $m_\perp \gtrsim \bar{m}_\perp$) depends on the two parameters τ and T_c (and the common normalizing multiplier λ). It can be determined by fitting the correlation data with (12) when the plateau height $H(0)$ is fixed. Remember that if the plateau is absent in events with a fixed value of $H(0)$, then $\tau(\theta)$ is the inverse gradient of the longitudinal component of the hydrodynamical 4-velocity $u^{\|}(x)$ at the final stage. If the values τ and T_c are determined at fixed $H(0)$, one can find the transverse area $S_\perp$ according to (16): $S_\perp = H/\tau N_0(T_c)$.

In conclusion, we shall demonstrate the possibilities of correlation analysis in elucidating the character of hadronic matter evolution in ultrarelativistic nucleus-nucleus collisions. Here we are concerned with the transition of a one-dimensional hydrodynamic regime into a three-dimensional one, and the occurence of a mixed $qg-h$ phase as a manifestation of the first-order phase transition.

If $\tau \simeq r_\perp$ in central collisions, intense transverse motion has no time to develop and the transverse area at the moment of decay is equal to the nuclear area $S_\perp \simeq S_A$. The oscillation factor dominates in correlator (12) because τ is small. If at the same time $T_c \lesssim 140$ MeV, this would indicate standard hydrodynamics [5]. The QCD degrees of freedom do not manifest themselves. If $T_c > 140$ MeV, this can indicate the specific mechanism of concentration freezing [13]. The mechanism appears because the relaxation processes which guarantee the settlement of local thermodynamic equilibrium have no time to compensate the changes connected with the fast rate of system expansion. The picture corresponds to the hard equation of state at all phases of hydrodynamic evolution.

If the mixed phase is realized, the proper time τ becomes long [12]. The greater part of the matter moves by inertia during this phase, because there is no pressure gradient. If the initial energy density does not exceed the energy density in a mixed phase, the transverse expansion of the greater part of the matter is negligible. The system decays, in fact, just after the hadronic phase is formed (during time $\Delta\tau \simeq r_\perp$), because all the parameters of the system are close to its decay values. In this case, the correlation analysis must lead to $\tau \gg r_\perp$ and $S_\perp \simeq S_A$. If $\tau \gg r_\perp$ at $S_\perp \gg S_A$, this would be evidence that the active hydrodynamic regime already forms in the quark-gluon phase. The transverse motion developed is preserved by inertia in the mixed phase (simple estimations give $S_\perp \approx (3 \div 4)\, S_A$).

The method described for determining the freeze-out temperature and the transverse area from the correlation data on longitudinal momenta enables us to separate the contributions to the transverse momentum spectrum from heat radiation and transverse hydrodynamic motion.

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