PROTON PICTURES OF HIGH-ENERGY NUCLEAR COLLISIONS

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Correlations between protons emitted with nearly equal momenta are shown to be sensitive to the space-time structure of high-energy heavy-ion collisions. A quantal estimate indicates that final-state interactions and the exclusion principle result in a rich, experimentally accessible correlation structure for relative proton-proton momenta $\lesssim 50$ MeV/c which can be used to determine the size, velocity, and lifetime of the collision volume

Relativistic nuclear collisions offer an exciting possibility for the study of the behavior of hadronic matter at extreme temperatures and densities. However, the proton inclusive spectra from these collisions [1] can be understood qualitatively through a variety of methods [1–4]. These range from nuclear fireballs and relativistic hydrodynamics to classical trajectory, intra-nuclear cascade, and knock-out models. It is therefore probable that a determination of the correct reaction mechanism will require more exclusive characterizations of the collision products. The simplest of these is the two-proton (p-p) correlation function. We have discussed previously [4] how large angle (back-to-back) azimuthal correlations indicate unambiguously the degree of equilibration achieved in relativistic collisions. In this note, we show that small-angle p-p correlations can be used to probe the space-time structure of the collision, in effect providing snapshots of the reacting system.

It is well known that the correlation between two particles is sensitive to the character of their source. The Brown–Twiss interferometer [5] uses photon-photon correlations to determine the size of stars. Similarly, following a suggestion by Goldhaber et al. [6] and elaboration by Kopylov et al. [7], π-π correlations have been used recently to measure the hadronic fireball produced in high-energy proton-proton collisions [8]. In both of these methods interactions between the emitted particles may be neglected so that the correlation between quanta with nearly equal momenta is due primarily to their boson nature. In contrast, anti-symmetry of the fermion wavefunction results in an anti-correlation between nearly parallel protons. Furthermore, the nuclear and coulomb p-p interactions cannot be neglected. Instead of obscuring the effect however, these interactions enhance the sensitivity of the proton technique, as we show below.

It is simple to estimate the magnitude of the small-angle p-p correlation expected in relativistic heavy-ion collisions [9]. Consider two protons which undergo simultaneous independent final scatterings with two other hadrons before leaving the system. Let the two scattering sites be separated by a distance $\boldsymbol{r}$ and let the momenta of the protons just after scattering be equal, $\boldsymbol{p}$, and orthogonal to $\boldsymbol{r}$. In the most probable spintriplet state, the anti-symmetry of the p-p spatial wavefunction induces an anti-correlation over a range of relative p-p momenta $\Delta p \approx \hbar/r$ and hence an angular anti-correlation over an interval $\Delta\phi = \Delta p/p \approx \hbar/rp$. An additional anti-correlation is due to the protons' mutual coulomb repulsion, which causes them to follow divergent trajectories as they leave the system and ultimately imparts to each a momentum orthogonal to $\boldsymbol{p}$ of magnitude $\sqrt{me^2/r}$. Here, m and e are respectively the proton mass and charge. This contribution to the angular anti-correlation then has a width $\Delta\phi \approx 2\sqrt{me^2/r}/p$. For $r = 3$ fm and $p = 531$ MeV/c (corresponding non-relativistically to a kinetic energy of 150 MeV), $\Delta\phi = 7.1^\circ$ and 4.6° respectively for the effect of statistics and the coulomb repulsion. From these estimates we may infer the existence of a measurable anti-correlation between nearly parallel protons which is sensitive to the size of the system. We have ignored the essentially geometrical complications of non-simultaneity of the final scatterings and non-orthogonality of $\boldsymbol{r}$ and $\boldsymbol{p}$. There

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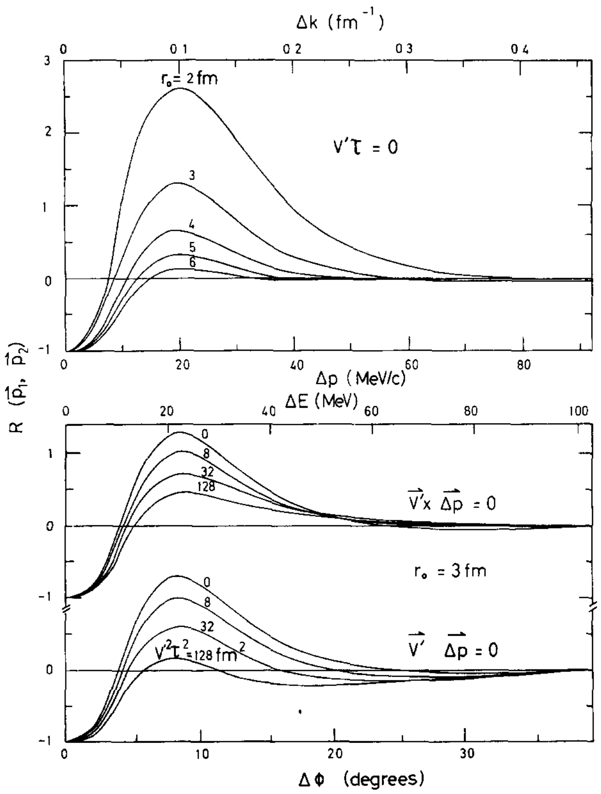

Fig. 1. Upper portion: The two-proton correlation function for various values of r_0 when $V'\tau = 0$. The horizontal axes are labeled by the p-p relative momentum Δp and wave number, $\Delta k = \Delta p/\hbar$. Lower portion: The correlation function for various values of $V'\tau$ when $r_0 = 3$ fm. The horizontal axes are labeled by kinetic energy and azimuthal angle differences for experiments detecting 150 MeV protons at $\theta = 30°$, as described in the text.

is also a distribution in the values of r for any given collision. These effects are expected to diminish the anti-correlation. More importantly, we have omitted the nuclear p-p interaction. Its contribution to the correlation scales as $(a/r)^3$, i.e. proportional to the fractional volume of the system within the range a of the nuclear force from a given point. Since the interaction is attractive at low relative p-p momenta, it is expected to cause a positive small-angle correlation. Although all of these effects can be estimated classically, they are treated more simply and accurately in the following quantal calculation.

A more precise estimate of the expected small-angle p-p correlations can be obtained within the context of nonrelativistic quantum mechanics. Let $D(\boldsymbol{r}t, \boldsymbol{p})$ be the impact parameter-averaged space-time distribution of final scatterings which produce a proton of momentum p during a heavy-ion collision. Since it is impossible to specify simultaneously and precisely both $\boldsymbol{r}$ and $\boldsymbol{p}$, D must be interpreted in terms of wave packets centered about these mean values. It is convenient to normalize D so that its integral over all space-time is $\sigma^{-1}\,\mathrm{d}\sigma/\mathrm{d}\boldsymbol{p}$, where $\mathrm{d}\sigma/\mathrm{d}\boldsymbol{p}$ and $\sigma = \int \mathrm{d}\boldsymbol{p}(\mathrm{d}\sigma/\mathrm{d}\boldsymbol{p})$ are respectively the differential and total proton inclusive cross-sections. Let two protons be emitted independently with equal momenta $\boldsymbol{p}$ from space-time points $(\boldsymbol{r}_1 t_1)$, $(\boldsymbol{r}_2 t_2)$, where $t_2 \geqslant t_1$. The joint probability of observing protons with momenta $\boldsymbol{p}_1, \boldsymbol{p}_2$ (both approximately equal to $\boldsymbol{p}$) is then given by the square of the overlap between the single-particle wave packets centered at $\boldsymbol{r}_1' = \boldsymbol{r}_1 + \boldsymbol{V}(t_2 - t_1), \boldsymbol{r}_2$ and the final state wavefunction. Here, $\boldsymbol{V} = \boldsymbol{p}/m = (\boldsymbol{p}_1 + \boldsymbol{p}_2)/2m$ is the laboratory velocity of the p-p center-of-mass. After similarly considering the case $t_1 > t_2$, the double differential two-proton inclusive cross-section may be approximated as

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{p}_1\mathrm{d}\boldsymbol{p}_2} = \int_{-\infty}^{\infty}\mathrm{d}t_1\,\mathrm{d}t_2 \int \mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2\, D(\boldsymbol{r}_1 t_1, \boldsymbol{p}) D(\boldsymbol{r}_2 t_2, \boldsymbol{p})$$

$$\times \{\tfrac{1}{4}|{}^1\Psi_{\boldsymbol{p}_1\boldsymbol{p}_2}(\boldsymbol{r}_1', \boldsymbol{r}_2)|^2 + \tfrac{3}{4}|{}^3\Psi_{\boldsymbol{p}_1\boldsymbol{p}_2}(\boldsymbol{r}_1', \boldsymbol{r}_2)|^2\}\,. \quad (1)$$

The singlet and triplet p-p scattering wavefunctions for protons of momenta $\boldsymbol{p}_1, \boldsymbol{p}_2$ are respectively ${}^1\Psi$ and ${}^3\Psi$. They are respectively symmetric and anti-symmetric under the interchange of their spatial or momentum arguments and satisfy the two-body p-p Schroedinger equation containing nuclear and coulomb potentials. In formulating eq. (1), we have neglected the influence of the nuclear mean field on the single-particle trajectories and the final state wavefunction. As a result, Ψ is the product of a plane wave in the p-p center-of-mass coordinate and ψ, the wavefunction for relative p-p motion. This latter function depends only upon the relative p-p momentum, $\Delta\boldsymbol{p} = (\boldsymbol{p}_1 - \boldsymbol{p}_2)/2$ and separation, $\boldsymbol{r}$. Furthermore, since we show below that the correlation mechanism is effective only over a very small range of Δp, we have neglected the spatial variation in ψ over the width of the single-particle wave packets and have assumed that both protons have the same momentum, $\boldsymbol{p}$, immediately after their final scatterings. We have also taken the proton spin distribution to be statistical.

In order to explore the dependence of the two-proton correlation upon the space-time distribution of final scatterings, it is convenient to parametrize D as

$$D(\boldsymbol{r}t,\boldsymbol{p}) = \frac{1}{\sigma}\frac{d\sigma}{d\boldsymbol{p}}\left(\frac{1}{\pi^{3/2}r_0^3}\,e^{-(\boldsymbol{r}-\boldsymbol{V}_0 t)^2/r_0^2}\right)\left(\frac{1}{\pi^{1/2}\tau}\,e^{-t^2/\tau^2}\right) \quad (2)$$

The parameters r_0 and τ are measures of the spatial and temporal extent of the region producing protons with momentum $\boldsymbol{p}$, while $\boldsymbol{V}_0$ is the laboratory velocity of this region, taken to lie along the beam direction. These three parameters generally depend on $\boldsymbol{p}$ through the details of the collision. While this is an admittedly crude parametrization, it is doubtful that a more detailed description of D could be safely determined from experiment. When eq. (2) is inserted into (1), after some elementary manipulations the double differential cross-section may be written in terms of the two-proton correlation function $R(\boldsymbol{p}_1,\boldsymbol{p}_2)$ as:

$$\frac{1}{\sigma}\frac{d\sigma}{d\boldsymbol{p}_1 d\boldsymbol{p}_2} = \left(\frac{1}{\sigma}\frac{d\sigma}{d\boldsymbol{p}}\right)^2 (R(\boldsymbol{p}_1,\boldsymbol{p}_2)+1)\,, \quad (3\text{a})$$

$$R(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{1}{(2\pi)^{3/2}r_0^2\rho}\int d\boldsymbol{r}\exp\{-[r^2-(\boldsymbol{r}\cdot\boldsymbol{V}'\tau/\rho)^2]/2r_0^2\} \times\{\tfrac{1}{4}|{}^1\psi_{\Delta\boldsymbol{p}}(\boldsymbol{r})|^2+\tfrac{3}{4}|{}^3\psi_{\Delta\boldsymbol{p}}(\boldsymbol{r})|^2-1\}\,. \quad (3\text{b})$$

Here $\boldsymbol{V}'=\boldsymbol{V}-\boldsymbol{V}_0$ and the distance ρ is $\sqrt{r_0^2+(V'\tau)^2}$. To evaluate eq. (3b) numerically, the exponential involving $(\boldsymbol{r}\cdot\boldsymbol{V}'\tau/\rho)^2$ may be expanded in a Taylor series to give R in terms of one-dimensional integrals involving the radial wavefunctions of a partial wave expansion of ψ. In the presence of a nuclear tensor or spin-orbit interaction, $|{}^3\psi|^2$ must be taken as the appropriate incoherent average over initial and final spin projections.

We have evaluated eq. (3b) using the coulomb and Reid soft-core, central, spin-orbit,,and diagonal tensor potentials [10] in all p-p channels with $\ell \leqslant 2$. Because of the small values of Δp over which R is appreciably different from zero, the tensor coupling of the ${}^3P_2$ and ${}^3F_2$ channels has been safely neglected and only the coulomb interaction has been used in channels with $\ell \geqslant 3$. In fact, the results we present below are essentially unchanged if the nuclear potential is negelcted in all channels except ${}^1S_0$.

In the upper portion of fig. 1, we show $R(\boldsymbol{p}_1,\boldsymbol{p}_2)$ for various r_0 when $\tau = 0$ (neglegibly small collision time). In this case, as can be seen from eq. (3b), R is a function of Δp only. These curves thus describe both the correlation between protons emitted in different directions with the same energy and the correlation between protons emitted in the same direction with different energies. The most prominent feature is the large positive correlation due to the attractive S-wave nuclear interaction. This enhancement peaks near $\Delta p = 20$ MeV/c and decreases with increasing r_0, since the short-range potential is not effective in correlating protons originating at widely separated points in space. For small Δp ($\lesssim 10$ MeV/c), the long-range repulsive coulomb interaction prevents two protons from ever having precisely equal final momenta, independent of how far apart in space (or time) they originate, so that $R(\Delta p = 0) = -1$ for all r_0. Note that for physically plausible values of r_0, the width of the coulomb dip near $\Delta p = 0$ increases with increasing r_0, in contrast to the simple estimate presented above. This is because the nuclear peak decreases more rapidly ($\sim 1/r_0^3$) than does the coulomb repulsion ($\sim 1/\sqrt{r_0}$). For large Δp ($\gtrsim 30$ MeV/c), channels with $\ell > 0$ become important. Both the repulsive coulomb force in these partial waves and the smaller relative importance for the S-wave enhancement then cause a decrease and eventually a slight negative minimum in R. For larger values of Δp, R approaches zero.

When the collision time τ is non-vanishing, the correlation given by eq. (3) depends upon both Δp and the angle between $\Delta\boldsymbol{p}$ and $\boldsymbol{V}'$. To explore this dependence, it is useful to consider two limiting cases: $\Delta\boldsymbol{p}$ perpendicular to $\boldsymbol{V}'$ ($\Delta\boldsymbol{p}\cdot\boldsymbol{V}' = 0$) and $\Delta\boldsymbol{p}$ parallel to $\boldsymbol{V}'$ ($\Delta\boldsymbol{p}\times\boldsymbol{V}' = 0$). The first case may be realized experimentally by detecting two protons with equal kinetic energies and equal angles θ relative to the beam direction, but separated by an azimuthal angle $\Delta\phi$, where $\Delta p = p\sin\theta\sin(\Delta\phi/2)$. The second case can be isolated experimentally by a combination of two measurements. The first determines the correlation between protons emitted in the same direction with kinetic energies differing by $\Delta E = 2p\Delta p/m$, while the second measures correlations between protons with the same kinetic energy and ϕ, but separated by an angle $\Delta\theta$, where $\Delta p = p\sin(\Delta\theta/2)$. To the extend that the correlation as a function of Δp is different for $\Delta\boldsymbol{p}\cdot\boldsymbol{V}' = 0$ and $\Delta\boldsymbol{p}\times\boldsymbol{V}' = 0$, the combination of ΔE and $\Delta\theta$ measurements can be used to determine the direction of $\boldsymbol{V}'$ in the plane containing $\boldsymbol{V}_0$ and $\boldsymbol{V}$, so that V_0, r_0, and τ can all be determined. Alternatively, if V_0 is known by other methods (from the fireball model [1] or equal to half the beam velocity in a collision between ions with equal mass) a measurement of either ΔE or $\Delta\theta$ correlations in conjunction with a $\Delta\phi$ measurement would both determine r_0 and τ.

In the lower portion of fig. 1, we show $R(\boldsymbol{p}_1, \boldsymbol{p}_2)$ for various values of $V'\tau$ when $r_0 = 3$ fm. These curves are generally functions of Δp only, as given by the scale in the upper portion of the figure. However, for the purposes of experimental orientation we have labeled the axes with values of $\Delta\phi$ and ΔE which correspond to experiments detecting 150 MeV protons at $\theta = 30°$. The appropriate values for $\Delta\theta$ are approximately one-half of those given for $\Delta\phi$. Note that if we assume $V_0 \ll V$, so that $V' \sim V$, for 150 MeV protons a value of $V'^2\tau^2 = 128\ \mathrm{fm}^2$ corresponds to $\tau \approx 7 \times 10^{-23}$ sec. Increasing values of $V'\tau$ have much the same effect as increasing values of r_0, separating the protons in space-time and therefore decreasing the nuclear correlation peaks. This effect is more pronounced when $\Delta\boldsymbol{p} \cdot \boldsymbol{V}' = 0$, where for $\Delta p \approx 40$ MeV/c the rapid decrease of nuclear attraction relative to coulomb and statistics effects causes a strong negative minimum.

The results presented above indicate the existence of detectable correlations between protons with nearly equal momenta which depends sensitively upon the spatial and temporal extend of the collision process. While more refined calculations of the effect are not warranted until experimental confirmation of its existence, the extraction of quantitative information from such correlations may require a more detailed consideration of several uncertainties in the present calculation. We have neglected the width of the single-particle wave packets relative to the spatial variation of ψ. In the first (knock-out) scatterings of the collision [4], this width is comparable to that estimated from the initial momentum uncertainty for nucleons within the nucleus, i.e. the Fermi momentum, $p_{\mathrm{F}} = 268$ MeV/c. The corresponding distance $\approx 1/\hbar p_{\mathrm{F}} = 0.7$ fm is safely small in comparison to the wavelength for relative p-p motion, $2\pi\hbar/\Delta p$, in the region of interest. However, wave packets for nucleons emitted in the later stages of the collision may be broadened by many binary scatterings, resulting in a weaker correlation. Although modification of eq. (1) to assume gaussian wave packets (of unspecified width) is possible, an estimate of the effect can be made by replacing r_0 by $\sqrt{r_0^2 + (\Delta r)^2}$, where Δr is the width of the packets. We have also neglected the effect of the proton-nucleus mean field, which should be less important for experiments with higher energy protons. The nuclear contribution to this potential is a sensitive functional of the shape of the system and involves the largely unknown optical potential in these situations. The contribution of the proton-nucleus coulomb interaction to the p-p correlation can be estimated as in ref. [9] to have a range $\Delta\phi \approx mZe^2/p^2r_0$, where Z is the nuclear atomic number. For $Z = 50$, $r_0 = 3$ fm and $p = 531$ MeV/c, $\Delta\phi = 5°$, which is comparable to the effect of the coulomb p-p interaction. It should be noted that the simultaneous treatment of any proton-nucleus mean field and the p-p interaction entails complicated quantal 3-body calculations.

The mechanism for small angle proton-proton correlations we have presented is basically model independent. The only essential assumption is the independence of the final scatterings which generate the correlated protons. Although collective flow patterns or the emission of "chunks" of nuclear matter during the collision could cause correlations of purely dynamical nature, these would be expected to extend over a far wider range of phase space than does the effect discussed here. Small-angle correlations could then be an opportunity to test various microscopic models of heavy-ion collisions. Because of the equilibration processes which occur in relativsitic collisions, there is a rough correlation between the proton energy and the time of emission [11]. The determination of r_0, τ, and V_0 as functions of p could then give information about the size of the system at various times. For example, r_0 for high-energy knock-out protons emitted in the forward direction [4] is a measure of the size of the overlap region (impact parameter) in the early stage of the collision. Measurements of this parameter in coincidence with multiplicity might be used to establish a definite link between multiplicity and central collisions. The values of r_0, τ, and V_0 for lower energy "thermalized" protons could similarly contain information about the dynamics of the nuclear fireball [1]. Finally, although in this work we have assumed an isotropic parametrization of the collision volume [2], the angular dependence of the two-proton correlation function can also yield information about the shape of the collision volume [8,9]. The extension of our calculations to allow for such anisotropy is straightforward.

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