New analysis of the KLOE data on the $\phi\to\eta\pi^0\gamma$ decay

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In this paper we present an analysis of the recent high-statistical KLOE data on the $\phi\to\eta\pi^0\gamma$ decay. This decay mainly goes through the $a_0\gamma$ intermediate state. The results obtained differ from those of previous fits: the data prefer a high a_0 mass and a quite large a_0 coupling to the $K\bar{K}$.

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I. INTRODUCTION

The lightest scalar mesons $a_0(980)$ and $f_0(980)$, discovered more than 30 years ago, became a hard problem for the naive quark-antiquark $(q\bar{q})$ model from the outset. On the one hand the almost exact degeneracy of the masses of the isovector $a_0(980)$ and isoscalar $f_0(980)$ states seemingly revealed a structure similar to the structure of the vector ρ and ω mesons, and on the other hand the strong coupling of $f_0(980)$ with the $K\bar{K}$ channel pointed unambiguously to a considerable part of the strange quark pair $s\bar{s}$ in the wave function of $f_0(980)$. It was noted late in the 1970s that in the MIT bag model there are light four-quark scalar states and it was suggested that $a_0(980)$ and $f_0(980)$ might be these states [1]. From that time the $a_0(980)$ and $f_0(980)$ resonances came to be the beloved children of light quark spectroscopy (see, for example, Refs. [2–4]).

Ten years later it was proposed in Ref. [5] to study radiative ϕ decays $\phi\to a_0\gamma\to\eta\pi^0\gamma$ and $\phi\to f_0\gamma\to\pi^0\pi^0\gamma$ to solve the puzzle of the lightest scalar mesons. Over the next ten years before the experiments of 1998, this question was examined from different points of view [6–10].

Now these decays have been studied not only theoretically[1] but also experimentally. The first measurements were reported by the SND [13–16] and CMD-2 [17] Collaborations, which obtain the following branching ratios:

$$\mathrm{Br}(\phi\to\gamma\pi^0\eta)=(8.8\pm1.4\pm0.9)\times10^{-5}\quad[15],$$

$$\mathrm{Br}(\phi\to\gamma\pi^0\pi^0)=(12.21\pm0.98\pm0.61)\times10^{-5}\quad[16],$$

$$\mathrm{Br}(\phi\to\gamma\pi^0\eta)=(9.0\pm2.4\pm1.0)\times10^{-5},$$

$$\mathrm{Br}(\phi\to\gamma\pi^0\pi^0)=(9.2\pm0.8\pm0.6)\times10^{-5}\quad[17].$$

More recently, the KLOE Collaboration has measured [18,19]

$$\mathrm{Br}(\phi\to\gamma\pi^0\eta)=(8.51\pm0.51\pm0.57)\times10^{-5}\quad\text{in}\quad\eta\to\gamma\gamma\quad[18],$$

$$\mathrm{Br}(\phi\to\gamma\pi^0\eta)=(7.96\pm0.60\pm0.40)\times10^{-5}\quad\text{in}\quad\eta\to\pi^+\pi^-\pi^0\quad[18],$$

$$\mathrm{Br}(\phi\to\gamma\pi^0\pi^0)=(10.9\pm0.3\pm0.5)\times10^{-5}\quad[19],$$

in agreement with the Novosibirsk data [15–17] but with a considerably smaller error.

In this work we present a new analysis of the recent KLOE data on the $\phi\to\eta\pi^0\gamma$ decay [18,20]. In contradistinction to [18], we (1) treat the a_0 mass m_{a_0} as a free parameter of the fit, (2) fit the phase δ of the interference between $\phi\to a_0\gamma\to\eta\pi^0\gamma$ (signal) and $\phi\to\rho^0\pi^0\to\eta\pi^0\gamma$ (background) reactions, and (3) use new more precise experimental values of the input parameters.

All formulas for the $\phi\to(a_0\gamma+\rho^0\pi^0)\to\eta\pi^0\gamma$ reaction taking the background into account are shown in Sec. II and the Appendix. The results of the four different fits are presented in Sec. III. A brief summary is given in Sec. IV.

II. THE FORMALISM OF THE $\phi\to a_0\gamma\to\eta\pi^0\gamma$ AND $\phi\to\rho^0\pi^0\to\eta\pi^0\gamma$ REACTIONS

In Ref. [11] it was shown that the process $\phi\to a_0\gamma\to\eta\pi^0\gamma$ dominates in $\phi\to\eta\pi^0\gamma$ decay (see also [5,7], where it was predicted in the four-quark model). This was confirmed in [18,20]. Nevertheless, the main background process $\phi\to\rho\pi^0\to\eta\pi^0\gamma$ should be taken into account also (see [11,18]).

The amplitude of the background process $\phi(p)\to\pi^0\rho^0\to\gamma(q)\pi^0(k_1)\eta(k_2)$ is [11]

$$M_B=\frac{g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho(p-k_1)}\phi_\alpha k_{1\mu}p_\nu\epsilon_\delta(p-k_1)_\omega q_\epsilon\epsilon_{\alpha\beta\mu\nu}\epsilon_{\beta\delta\omega\epsilon}. \tag{1}$$

According to the one-loop mechanism of the decay $\phi\to K^+K^-\to\gamma a_0$, suggested in Ref. [5], the amplitude of the signal $\phi\to\gamma a_0\to\gamma\pi^0\eta$ has the form

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[1]of the theoretical work appearing after the first experiments, note [11,12], where mass spectra are considered.

$$M_a = g(m)\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\left((\phi\epsilon) - \frac{(\phi q)(\epsilon p)}{(pq)}\right), \qquad (2)$$

where $m^2=(k_1+k_2)^2$, ϕ_α and ϵ_μ are the polarization vectors of the ϕ meson and photon, and the function $g(m)$ is given in the Appendix.

The mass spectrum is

$$\frac{d\Gamma(\phi\to\gamma\pi^0\eta,m)}{dm} = \frac{d\Gamma_{a_0}(m)}{dm} + \frac{d\Gamma_{back}(m)}{dm} + \frac{d\Gamma_{int}(m)}{dm}, \qquad (3)$$

where the mass spectrum for the signal is

$$\frac{d\Gamma_{a_0}(m)}{dm} = \frac{2}{\pi}\frac{m^2\Gamma(\phi\to\gamma a_0,m)\Gamma(a_0\to\pi^0\eta,m)}{|D_{a_0}(m)|^2} = \frac{2|g(m)|^2 p_{\eta\pi}(m_\phi^2-m^2)}{3(4\pi)^3 m_\phi^3}\left|\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\right|^2. \qquad (4)$$

The mass spectrum for the background process $\phi\to\pi^0\rho\to\gamma\pi^0\eta$ is [11]

$$\frac{d\Gamma_{back}(m)}{dm} = \frac{(m_\phi^2-m^2)p_{\pi\eta}}{128\pi^3 m_\phi^3}\int_{-1}^{1} dx A_{back}(m,x), \qquad (5)$$

where

$$A_{back}(m,x) = \frac{1}{3}\sum |M_B|^2$$
$$= \frac{1}{24}(m_\eta^4 m_\pi^4 + 2m^2 m_\eta^2 m_\pi^2 \tilde m_\rho^2 - 2m_\eta^4 m_\pi^2 \tilde m_\rho^2 - 2m_\eta^2 m_\pi^4 \tilde m_\rho^2 + 2m^4\tilde m_\rho^4 - 2m^2 m_\eta^2 \tilde m_\rho^4 + m_\eta^4 \tilde m_\rho^4 - 2m^2 m_\pi^2 \tilde m_\rho^4$$
$$+ 4m_\eta^2 m_\pi^2 \tilde m_\rho^4 + m_\pi^4 \tilde m_\rho^4 + 2m^2\tilde m_\rho^6 - 2m_\eta^2\tilde m_\rho^6 - 2m_\pi^2\tilde m_\rho^6 + \tilde m_\rho^8 - 2m_\eta^4 m_\pi^2 m_\phi^2 - 2m^2 m_\eta^2 m_\phi^2 \tilde m_\rho^2 + 2m_\eta^2 m_\pi^2 m_\phi^2 \tilde m_\rho^2$$
$$- 2m^2 m_\phi^2 \tilde m_\rho^4 + 2m_\eta^2 m_\phi^2 \tilde m_\rho^4 - 2m_\phi^2 \tilde m_\rho^6 + m_\eta^4 m_\phi^4 + m_\phi^4 \tilde m_\rho^4)\left|\frac{g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho(\tilde m_\rho)}\right|^2, \qquad (6)$$

and

$$\tilde m_\rho^2 = m_\eta^2 + \frac{(m^2+m_\eta^2-m_\pi^2)(m_\phi^2-m^2)}{2m^2} - \frac{(m_\phi^2-m^2)x}{m}p_{\pi\eta},$$

$$p_{\pi\eta} = \frac{\sqrt{(m^2-(m_\eta-m_\pi)^2)(m^2-(m_\eta+m_\pi)^2)}}{2m}. \qquad (7)$$

Note that there is a misprint in Eq. (6) of Ref. [11], which describes $A_{back}(m,x)$: the seventh term in the brackets $+2m_\eta^4\tilde m_\rho^4$ should be replaced by $+m_\eta^4\tilde m_\rho^4$, as above in Eq. (6) of this paper. We emphasize that all evaluations in Ref. [11] were done with the correct formula.

The term of the interference between the signal and the background processes is written in the following way:

$$\frac{d\Gamma_{int}(m)}{dm} = \frac{(m_\phi^2-m^2)p_{\pi\eta}}{128\pi^3 m_\phi^3}\int_{-1}^{1} dx A_{int}(m,x), \qquad (8)$$

where

$$A_{int}(m,x) = \frac{2}{3}\mathrm{Re}\sum M_a M_B^* = \frac{1}{3}\left((m^2-m_\phi^2)\tilde m_\rho^2 + \frac{m_\phi^2(\tilde m_\rho^2-m_\eta^2)^2}{m_\phi^2-m^2}\right) \times \mathrm{Re}\left\{\frac{e^{i\delta}g(m)g_{a_0K^+K^-}g_{a_0\pi\eta}g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho^*(\tilde m_\rho)D_{a_0}(m)}\right\}. \qquad (9)$$

Note that the phase δ is not taken into account in [11]. The inverse propagator of the scalar meson R (a_0 in our case), is presented in Refs. [5,7,21,22]:

$$D_R(m) = m_R^2 - m^2 + \sum_{ab}[\mathrm{Re}\Pi_R^{ab}(m_R^2) - \Pi_R^{ab}(m^2)], \qquad (10)$$

where $\sum_{ab}[\mathrm{Re}\Pi_R^{ab}(m_R^2)-\Pi_R^{ab}(m^2)] = \mathrm{Re}\Pi_R(m_R^2)-\Pi_R(m^2)$ takes into account the finite width corrections of the resonance, which are the one-loop contributions to the self-energy of the R resonance from the two-particle intermediate ab states. For the pseudoscalar ab mesons, $\Pi_R^{ab}(m^2)$ are expressed in the Appendix.

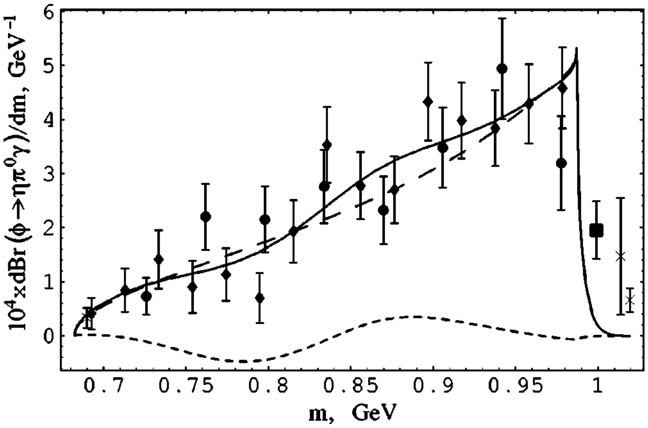

FIG. 1. The comparison of fit 1 (the solid line) with the KLOE data (points). The circles correspond to the $\phi \to \eta\pi^0\gamma$, $\eta \to \pi^+\pi^-\pi^0$ data, the diamond and square to the $\phi \to \eta\pi^0\gamma$, $\eta \to 2\gamma$ ones. The signal contribution and the interference term are shown with the dashed and dotted lines. The cross points are omitted in fitting. The square (0.999 GeV) is omitted in fits 3 and 4.

In our case we take into account the intermediate states $ab = \eta\pi^0$, $K\bar{K}$, and $\eta'\pi^0$:

$$\Pi_{a_0} = \Pi_{a_0}^{\eta\pi^0} + \Pi_{a_0}^{K^+K^-} + \Pi_{a_0}^{K^0\bar{K}^0} + \Pi_{a_0}^{\eta'\pi^0}, \quad (11)$$

$g_{a_0K^+K^-} = -g_{a_0K^0\bar{K}^0}$. Note that the $\eta'\pi^0$ contribution is of small importance due to the high threshold. Even fitting with $|g_{a_0\eta'\pi^0}| = 0$ changes the results by less than 10% of their errors. We set $|g_{a_0\eta'\pi^0}| = |1.13 g_{a_0K^+K^-}|$ according to the four-quark model (see [5]), but this is almost the same as the two-quark model prediction $|g_{a_0\eta'\pi^0}| = |1.2\, g_{a_0K^+K^-}|$ (see [5]).

The inverse propagator of the ρ meson has the following expression:

$$D_\rho(m) = m_\rho^2 - m^2 - i m^2 \frac{g_{\rho\pi\pi}^2}{48\pi}\left(1 - \frac{4m_\pi^2}{m^2}\right)^{3/2}. \quad (12)$$

The coupling constants $g_{\phi K^+K^-} = 4.376 \pm 0.074$ and $g_{\phi\rho\pi} = 0.814 \pm 0.018$ GeV^{-1} are taken from the new most precise measurement Ref. [23].[2] Note that in Refs. [11,18] the value $g_{\phi K^+K^-} = 4.59$ was obtained using the [24] data. The coupling constant $g_{\rho\eta\gamma} = 0.56 \pm 0.05$ GeV^{-1} is obtained from the data of Ref. [25] with the help of the expression

TABLE I. Results of various fits. DF indicates degree of freedom.

Fit	m_{a_0} (MeV)	$\frac{g_{a_0K^+K^-}^2}{4\pi}$ (GeV)^2	$g_{a_0\eta\pi}/g_{a_0K^+K^-}$	δ (deg)	χ^2/DF
1	1003^{+32}_{-13}	$0.82^{+0.81}_{-0.27}$	$1.06^{+0.20}_{-0.27}$	27 ± 29	24.2/20
2	995^{+22}_{-8}	$0.65^{+0.42}_{-0.18}$	$1.17^{+0.17}_{-0.24}$	0	25.2/21
3	994^{+22}_{-8}	$0.62^{+0.4}_{-0.17}$	$1.21^{+0.17}_{-0.24}$	21 ± 30	16.3/19
4	992^{+14}_{-7}	$0.55^{+0.27}_{-0.13}$	$1.26^{+0.16}_{-0.2}$	0	16.9/20

$$\Gamma(\rho \to \eta\gamma) = \frac{g_{\rho\eta\gamma}^2}{96\pi m_\rho^3}(m_\rho^2 - m_\eta^2)^3. \quad (13)$$

III. RESULTS

The KLOE data on the $\phi \to \eta\pi^0\gamma$ decay may be found in Table 5 of Ref. [20] (see also Fig. 1 in the given paper). The data are separated into two samples: points 1–10 of this table correspond to the sample of $\phi \to \eta\pi^0\gamma$, $\eta \to \pi^+\pi^-\pi^0$ events, while points 11–27 correspond to the sample of $\phi \to \eta\pi^0\gamma$, $\eta \to 2\gamma$ events. Note that as in Refs. [18,20], we do not fit first, tenth, and 27th points of this table (cross points in Fig. 1). We emphasize that the tenth (1.014 GeV) and 27th (1.019 GeV) points are obvious artifacts because the mass spectrum behavior on the right slope of the resonance has the form (photon energy)^3 according to gauge invariance.

In the experiment the whole mass region $(m_\eta + m_{\pi^0}, m_\phi)$ is divided into some number of bins. Experimenters measure the average value $\bar{B}_i$ (i is the number of the bin) of $d\mathrm{Br}(\phi \to \eta\pi^0\gamma)/dm$ around each ith bin:[3]

$$\bar{B}_i = \frac{1}{m_{i+1} - m_i}\int_{m_i}^{m_{i+1}} d\mathrm{Br}(\phi \to \eta\pi^0\gamma)/dm. \quad (14)$$

In this case one should define the χ^2 function as

$$\chi^2 = \sum_i \frac{(\bar{B}_i^{th} - \bar{B}_i^{exp})^2}{\sigma_i^2}, \quad (15)$$

where $\bar{B}_i^{exp}$ are the experimental results, σ_i are the experimental errors, and

$$\bar{B}_i^{th} = \frac{1}{m_{i+1} - m_i}\int_{m_i}^{m_{i+1}} d\mathrm{Br}^{th}(\phi \to \eta\pi^0\gamma)/dm,$$

where $d\mathrm{Br}^{th}(\phi \to \eta\pi^0\gamma)/dm$ is the theoretical curve.

[2]We consider the SND measurement of $g_{\phi K^+K^-}$ as the most reliable one because all ϕ decay modes were analyzed simultaneously in this experiment. Note that using Ref. [25]'s constant has almost no effect on the results.

[3]We emphasize that all three omitted points correspond to mass bins that lie partially outside the permitted mass region $(m_\eta + m_{\pi^0}, m_\phi)$. So these points (i.e., mass bins) cannot be analyzed.

TABLE II. Results for the branching ratio, the signal contribution, and R.

Fit	$\Gamma_{a_0\eta\pi^0}$ (MeV)	R	$10^5\times\mathrm{Br}(\phi\to(a_0\gamma+\rho\pi^0)\to\eta\pi^0\gamma)$	$10^5\times\mathrm{Br}(\phi\to a_0\gamma\to\eta\pi^0\gamma)$
1	153^{+22}_{-17}	3.4 ± 1.7	7.6 ± 0.4	7.3 ± 0.4
2	148^{+17}_{-15}	4.3 ± 1.7	7.6 ± 0.4	7.1 ± 0.4
3	149^{+19}_{-16}	4.5 ± 1.7	7.6 ± 0.4	7.2 ± 0.4
4	146^{+17}_{-15}	5.0 ± 1.6	7.6 ± 0.4	7.1 ± 0.4

The free parameters of the fit are $m_{a_0}, g^2_{a_0K^+K^-}/4\pi$, the phase δ (we assume it is constant), and the ratio $g_{a_0\eta\pi}/g_{a_0K^+K^-}$. The results are shown in Table I (fit 1).[4] The quality of the fit is good. The phase δ is consistent with zero, so we make a fit with $\delta=0$ (fit 2 in Table I).

To check the correctness of treating the phase δ as a constant, we have done a fit with δ taken in the form $\delta(m)=bp_{\eta\pi}(m)$ (the phase of the elastic background in $\eta\pi^0$ scattering may have such behavior), and found that the constant $b=2.8\pm3.2\ \mathrm{GeV}^{-1}$ is also consistent with zero. Changes of the other values are not important.

Since the discrepancy between the fits and the experimental point number 26 (0.999 GeV) in Table 5 of Ref. [20] (the square in Fig. 1) is about three standard deviations (i.e., this point may be an artifact also), we make another fit without this point (fit 3). The phase δ is again consistent with zero, so we make a fit without it (fit 4).

In Table II we present the results for the total branching ratio $\mathrm{Br}(\phi\to(a_0\gamma+\rho\pi^0)\to\eta\pi^0\gamma)$, the signal contribution $\mathrm{Br}(\phi\to a_0\gamma\to\eta\pi^0\gamma)$, $\Gamma_{a_0\eta\pi^0}\equiv\Gamma(a_0\to\eta\pi^0,m_{a_0})=g^2_{a_0\eta\pi}\rho_{\eta\pi^0}(m_{a_0})/(16\pi m_{a_0})$, and the ratio $R=g^2_{f_0K^+K^-}/g^2_{a_0K^+K^-}$. The last is obtained using the Ref. [19] value $g^2_{f_0K^+K^-}/(4\pi)=2.79\pm0.12\ \mathrm{GeV}^2$. The branching ratio of the background $\mathrm{Br}(\phi\to\rho\pi^0\to\eta\pi^0\gamma)$ accounts for $(0.5\pm0.1)\times10^{-5}$.

IV. CONCLUSION

Note that the obtained value of the ratio $g_{a_0\eta\pi}/g_{a_0K^+K^-}$ does not contradict the first predictions based on the four-quark model of the a_0: $g_{a_0\eta\pi}/g_{a_0K^+K^-}\approx0.85$ [5].[5] But even if $g_{a_0\eta\pi}/g_{a_0K^+K^-}$ deviates from 0.85, there is no problem, because this variant of the four-quark model is rather rough; it is considered as a guide.

For all fits the obtained value of R differs from the value $R=7.0\pm0.7$ from [18]. So the conclusion that the constant $g^2_{a_0K^+K^-}/(4\pi)$ is small, obtained in [18,20] [$g^2_{a_0K^+K^-}/(4\pi)=0.4\pm0.04\ \mathrm{GeV}^2$], is the result of the parameter restrictions, especially fixing m_{a_0} at the Particle Data Group 2000 value 984.8 MeV. Note that a high a_0 mass is also needed to describe the $\gamma\gamma\to\eta\pi^0$ experiment (see [27]).

There should be no confusion due to the large a_0 width. In the peripheral production of the a_0 (for example, in the reaction $\pi^-p\to\eta\pi^0n$) the mass spectrum is given by the relation

$$\frac{dN_{\eta\pi^0}}{dm}\sim S_{per}(m)=\frac{2m^2}{\pi}\frac{\Gamma(a_0\to\eta\pi^0,m)}{|D_{a_0}(m)|^2}. \qquad (16)$$

The effective (visible) width of this distribution is much less then the nominal width $\Gamma_{a_0\eta\pi^0}$. For example, for the fit 1 results (Table I) the effective width is ~50 MeV (see Fig. 2).

As noted in Ref. [28], there is no problem with the relation between the branching ratios of a_0 and f_0 production in ϕ radiative decays. The early predictions [5] are based on the one-loop mechanism $\phi\to K^+K^-\to a_0\gamma\to\eta\pi^0\gamma$ and $\phi\to K^+K^-\to f_0\gamma\to\pi\pi\gamma$ at $m_{a_0}=980$ MeV, $m_{f_0}=975$ MeV, and $g_{a_0K^+K^-}=g_{f_0K^+K^-}$,[6] which leads to $\mathrm{Br}(\phi\to a_0\gamma\to\eta\pi^0\gamma)\approx\mathrm{Br}(\phi\to f_0\gamma\to\pi\pi\gamma)$. But it is shown in Ref. [7] that the relation between the branching ratios of a_0 and f_0 production in ϕ radiative decays essentially depends on the a_0-f_0 mass splitting. This strong mass dependence is

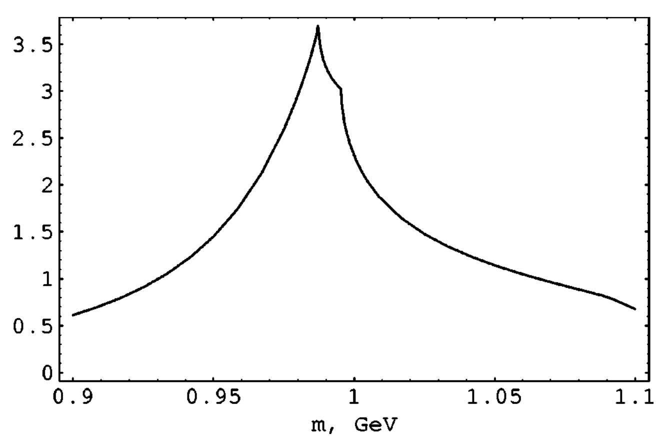

FIG. 2. Plot of the function $S_{per}(m)(\mathrm{GeV}^{-1})$, for fit 1 results.

[4] Note that fitting without averaging the theoretical curve [changing $\overline{B}^{th}_i\to d\mathrm{Br}^{th}(\phi\to\eta\pi^0\gamma)/dm|_{m=(m_{i+1}+m_i)/2}$] results in a worse χ^2 per degree of freedom $=28.8/20$. The results in this case are consistent within errors with those obtained by averaging the theoretical curve.

[5] Note that the prediction $g_{a_0\eta\pi}/g_{a_0K^+K^-}\approx0.93$, made in [1], was corrected in [26].

[6] We emphasize that isotopic invariance does not require $g_{a_0K^+K^-}=g_{f_0K^+K^-}$.

the result of gauge invariance, the (photon energy)^3 law on the right slope of the resonance. Our present analysis confirms this conclusion. Note that a noticeable deviation from the naive four-quark model equality $g_{a_0K^+K^-}=g_{f_0K^+K^-}$ is not crucial. What is more important is the mechanism of production of the a_0 and f_0 through the charged kaon loop, i.e., the four-quark transition. As is shown in Ref. [4], this gives strong evidence in favor of the four-quark model of the a_0 (f_0).

Note that the constant $g^2_{f_0K^+K^-}/(4\pi)$ can also differ a lot from those obtained in [19]. The point is that the extraction of this constant is very model dependent. For example, a fit taking into account the mixing of the resonances can decrease the value of $g^2_{f_0K^+K^-}/(4\pi)$ considerably. For instance, by fitting the data of [16] without mixing, one has $g^2_{f_0K^+K^-}/(4\pi)=2.47^{+0.37}_{-0.51}$ GeV^2 [16], while fitting and taking the mixing into account gives $g^2_{f_0K^+K^-}/(4\pi)=1.29\pm0.017$ GeV^2 [11]. Note also that in [19] the phase δ_B of the background is taken from [11], where it is obtained by simultaneous fitting of the $m_{\pi^0\pi^0}$ spectrum and the phase δ_0 of the $\pi\pi$ scattering, taking into account the mixing of the resonances. In [19] the mixing is not taken into account, so the additional phase dealing with it is omitted.

We emphasize once more that the KLOE data completely confirm the K^+K^--loop mechanism of the $\phi\to a_0\gamma$ decay, suggested in Ref. [5].

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APPENDIX

The forms of $g_R(m)$ and $g(m)=g_R(m)/g_{RK^+K^-}$ everywhere over the m region are in Refs. [5] and [29], respectively.

For $m<2m_{K^+}$,

$$g(m)=\frac{e}{2(2\pi)^2}g_{\phi K^+K^-}\Bigg(1+\frac{1-\rho^2(m^2)}{\rho^2(m_\phi^2)-\rho^2(m^2)}\times\Bigg\{2|\rho(m^2)|\arctan\frac{1}{|\rho(m^2)|}-\rho(m_\phi^2)\lambda(m_\phi^2)+i\pi\rho(m_\phi^2)-[1-\rho^2(m_\phi^2)]\Bigg[\frac{1}{4}[\pi+i\lambda(m_\phi^2)]^2-\left(\arctan\frac{1}{|\rho(m^2)|}\right)^2\Bigg]\Bigg\}\Bigg), \tag{A1}$$

where

$$\rho(m^2)=\sqrt{1-\frac{4m_{K^+}^2}{m^2}},\quad \lambda(m^2)=\ln\frac{1+\rho(m^2)}{1-\rho(m^2)},$$

$$\frac{e^2}{4\pi}=\alpha=\frac{1}{137}. \tag{A2}$$

For $m\geqslant 2m_{K^+}$,

$$g(m)=\frac{e}{2(2\pi)^2}g_{\phi K^+K^-}\Bigg\{1+\frac{1-\rho^2(m^2)}{\rho^2(m_\phi^2)-\rho^2(m^2)}\Bigg[\rho(m^2)\times[\lambda(m^2)-i\pi]-\rho(m_\phi^2)[\lambda(m_\phi^2)-i\pi]-\frac{1}{4}[1-\rho^2(m_\phi^2)]([\pi+i\lambda(m_\phi^2)]^2-[\pi+i\lambda(m^2)]^2)\Bigg]\Bigg\}. \tag{A3}$$

For $\Pi_R^{ab}(m^2)$ in the case of pseudoscalar ab mesons and $m_a\geqslant m_b$ one has [3,9,21,22,30] the following.[7] For $m\geqslant m_+$,

$$\Pi_R^{ab}(m^2)=\frac{g_{Rab}^2}{16\pi}\Bigg[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a}+\rho_{ab}\left(i+\frac{1}{\pi}\ln\frac{\sqrt{m^2-m_-^2}-\sqrt{m^2-m_+^2}}{\sqrt{m^2-m_-^2}+\sqrt{m^2-m_+^2}}\right)\Bigg]. \tag{A4}$$

For $m_-\leqslant m<m_+$,

$$\Pi_R^{ab}(m^2)=\frac{g_{Rab}^2}{16\pi}\Bigg[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a}-|\rho_{ab}(m)|+\frac{2}{\pi}|\rho_{ab}(m)|\arctan\frac{\sqrt{m_+^2-m^2}}{\sqrt{m^2-m_-^2}}\Bigg]. \tag{A5}$$

For $m<m_-$,

$$\Pi_R^{ab}(m^2)=\frac{g_{Rab}^2}{16\pi}\Bigg[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a}-\frac{1}{\pi}\rho_{ab}(m)\ln\frac{\sqrt{m_+^2-m^2}-\sqrt{m_-^2-m^2}}{\sqrt{m_+^2-m^2}+\sqrt{m_-^2-m^2}}\Bigg], \tag{A6}$$

[7] Note that in Ref. [21] $\Pi_R^{ab}(m^2)$ differs by a real constant from those determined in the other listed works in the case of $m_a\neq m_b$, but obviously this has no effect on Eq. (10).

and

$$\rho_{ab}(m)=\sqrt{\left(1-\frac{m_+^2}{m^2}\right)\left(1-\frac{m_-^2}{m^2}\right)},\quad m_\pm=m_a\pm m_b. \tag{A7}$$

The constants g_{Rab} are related to the width

$$\Gamma(R\to ab,m)=\frac{g_{Rab}^2}{16\pi m}\rho_{ab}(m). \tag{A8}$$

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