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How to measure which sort of particles was emitted earlier and which later

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Abstract

A method allowing to directly measure delays in the emission of particles of different types at time scales as short as 10^{-23}–10^{-22} s is suggested.

Since the pioneering papers of Kopylov and Podgoretsky (see the review [1]) it is well-known that a study of the directional dependence of the correlations of two identical particles at small relative velocities can be used to extract information on the emission times and the form of the emission region. The distribution of the relative space-time coordinates of the emission points of two identical particles is always symmetric. In contrast, this may not be the case for nonidentical particles. We show here that, due to the effect of final state interactions (FSI), the correlations of two nonidentical particles are sensitive not only to the anisotropy of the distribution of the relative space-time coordinates of the emission points, but also to its asymmetry. We show how to use this effect to directly measure the delays in the emission of particles of different types.

We start with the usual assumption of a sufficiently small density of the produced multi-particle system in momentum space, such that the correlation of two nonidentical particles with a small relative velocity is influenced by the effect of their mutual FSI only. We define the correlation function $R(p_1, p_2)$ of the two particles as the ratio of their differential production cross section to that which would be observed in the absence of the effects of FSI. Following Kopylov and Podgoretsky we introduce the normalized probability $W_s(x_1, p_1; x_2\ p_2)$ for the emission of two non-interacting particles with total spin S and 4-momenta p_1 and p_2 by one-particle sources. These are treated classically and thus are described by the parameters $x_i\{t_i,\ \boldsymbol{r}_i\}$ representing the coordinates of the source centers at the times of their decay. Assuming the momentum dependence of the emission probability to be negligible when varying the 4-momenta p_1 and p_2 by the amount characteristic for the correlation due to FSI, and taking into account that FSI leads to the substitution of the plane wave $e^{ip_1x_1+ip_2x_2}$ by the nonsymmetrized Bethe–Salpeter amplitudes in the continuous spectrum of the two-particle states $\psi^{S(+)}_{p_1p_2}(x_1, x_2)$, we get [2]

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$$R(p_1,p_2) = \sum_S \int d^4x_1 d^4x_2 W_S(x_1,p_1;x_2,p_2) \times |\psi^{S(+)}_{p_1p_2}(x_1,x_2)|^2 = \sum_S \rho_S(p_1,p_2)\langle|\psi^{S(+)}_{p_1p_2}(x)|^2\rangle_S , \tag{1}$$

where the mean $\langle f\rangle_S = \int d^4x_1 d^4x_2 W_S f/(\int d^4x_1 d^4x_2 \times W_S)$ is a function of p_1 and p_2, and

$$\rho_S(p_1,p_2) = \int d^4x_1 d^4x_2 W_S(x_1,p_1;x_2,p_2), \quad \sum_S \rho_S = 1 \tag{2}$$

describes the population of the two-particle states with total spin S. The amplitude $\psi^{S(+)}_{p_1p_2}(x)$, depending only on the relative 4-coordinate $x \equiv \{t, \boldsymbol{r}\} = x_1 - x_2$, is obtained from the Bethe–Salpeter amplitude after separation of the two-particle c.m.s. motion: $\psi^{S(+)}_{p_1p_2}(x_1,x_2) = e^{iPX}\psi^{S(+)}_{p_1p_2}(x)$, where $X = [(p_1P)x_1 + (p_2P)x_2]/P^2$ is the c.m.s. 4-coordinate and $P \equiv 2p = p_1 + p_2$. At equal emission times $t^* = t_1^* - t_2^* = 0$ in the two-particle c.m.s. it coincides with a stationary solution of the scattering problem $\psi^{S(+)}_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)$, $\boldsymbol{k}^* = \boldsymbol{p}_1^* = -\boldsymbol{p}_2^*$, having at large r^* the asymptotic form of a superposition of the plane and outgoing spherical waves. It can be shown [2] that the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ can usually be substituted by this solution (*equal time* approximation).

Let us now demonstrate the sensitivity of the correlation function of two nonidentical particles to the sign of the difference of the emission times $t = t_1 - t_2$. Let us consider sufficiently small momenta k^* of the particles in their c.m.s. so that their interaction is dominated by s-waves. The wave function $\psi^{S(+)}_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)$ in the absence of Coulomb interaction then takes on the form

$$\psi^{S(+)}_{-\boldsymbol{k}^*}(\boldsymbol{r}^*) = e^{-i\boldsymbol{k}^*\boldsymbol{r}^*} + \phi^S_{k^*}(r^*) , \tag{3}$$

where the scattered wave $\phi^S_{k^*}(r^*)$ is independent of the directions of the vectors $\boldsymbol{k}^*$ and $\boldsymbol{r}^*$. Neglecting the interaction of the two particles with the residual system and assuming that the conditions of *equal time* approximation are fulfilled, we can write the correlation function in the form[3]

$$R(p_1,p_2) = \sum_S \rho_S\langle|\psi^{S(+)}_{-\boldsymbol{k}^*}(\boldsymbol{r}^*)|^2\rangle_S = 1 + \sum_S \rho_S\langle|\phi^S_{k^*}(r^*)|^2 + 2\,\mathrm{Re}\,\phi^S_{k^*}(r^*)\cos\boldsymbol{k}^*\boldsymbol{r}^* - 2\,\mathrm{Im}\,\phi^S_{k^*}(r^*)\sin\boldsymbol{k}^*\boldsymbol{r}^*\rangle_S . \tag{4}$$

Consider now the behaviour of the vector $\boldsymbol{r}^*$ in the limit $|vt| \gg r$. Making the Lorentz transformation from the rest frame of the source to the c.m.s. of the two particles: $r_L^* = \gamma(r_L - vt)$, $r_T^* = r_T$, we see that, in the considered limit, the vector $\boldsymbol{r}^*$ is only slightly affected by averaging over the spatial distance $r \ll |vt|$ of the emission points in the rest frame of the source: $\boldsymbol{r}^* \doteq -\gamma\boldsymbol{v}t$ is nearly parallel or antiparallel to the velocity vector $\boldsymbol{v}$ of the pair, depending on the sign of the time difference t. Therefore, the correlation function is sensitive to sign (t) due to the odd term $\sim \sin\boldsymbol{k}^*\boldsymbol{r}^*$[4].

For charged particles there arise additional odd terms due to the confluent hypergeometrical function $F(\alpha,1,z) = 1 + \alpha z + \alpha(\alpha+1)(z/2!)^2 + \ldots$ modifying the plane wave in Eq. (3):

$$\psi^{S(+)}_{-\boldsymbol{k}^*}(\boldsymbol{r}^*) = e^{i\delta}\sqrt{A_C(k^*)}\left\{e^{-i\boldsymbol{k}^*\boldsymbol{r}^*} \times F\left[\frac{-i}{k^*a}, 1, i(\boldsymbol{k}^*\boldsymbol{r}^* + k^*r^*)\right] + \phi^S_{ck^*}(r^*)\right\}, \tag{5}$$

where $\delta = \arg\Gamma[1 + i/(k^*a)]$ is the Coulomb s-wave phase shift,

$$A_c(k^*) = \frac{2\pi}{k^*a}\left[\exp\left(\frac{2\pi}{k^*a}\right) - 1\right]^{-1} \tag{6}$$

is the modulus squared of the Coulomb wave function at zero distance and a is the Bohr radius of the two-particle system. Clearly, at a given distance r^*, the effect of the odd component in the Coulomb wave function is of increasing importance with a decreasing Bohr radius of the particle pair, i.e. for particles of greater masses or electric charges.

Thus the sign of the mean time difference $\langle t\rangle$ can be determined provided the sign of the scalar product $\boldsymbol{k}^*\boldsymbol{v}$ is fixed. A straightforward way to determine $\langle t\rangle$ is to measure the correlation functions $R_+(\boldsymbol{k}^*\boldsymbol{v} \geq 0)$ and $R_-(\boldsymbol{k}^*\boldsymbol{v} < 0)$. Depending on $\langle t\rangle$, their ratio R_+/R_-

[3] The correlation function of two nonidentical particles was originally introduced in Ref. [2] in a symmetrized form, so that the odd term in the variable $\boldsymbol{k}^*\boldsymbol{r}^*$ was omitted.

[4] The odd term in the correlation function could be enhanced in the case of a p-wave resonance near the threshold. However there are no such resonances for particle pairs of practical interest.

should show a peak or a dip in the region of small k^* and approach 1 both at $k^* \to 0$ and $k^* \to \infty$.

As the sign of the scalar product $\boldsymbol{k}^*\boldsymbol{v}$ is practically equal to that of the difference of particle velocities $v_1 - v_2$ (this equality is always valid for particles of equal masses), the sensitivity of the correlation functions R_+ and R_- to the sign of the difference of the particle emission times has a simple explanation in terms of the classical trajectory approach (see, e.g., [3]). Clearly, the interaction between the particles in the case of an earlier transmission of the faster particle will be different compared with the case of its later emission (the interaction time being longer in the latter case, leading to a stronger correlation).

For quantitative estimates we assume that a compound nucleus isotropically emits unpolarized particles with their energies distributed according to Maxwellian law with a temperature $T = 4$ MeV. The corresponding mean velocity of pp, pd or np pairs at small values of k^* is $\langle v \rangle \sim 0.12$. The distribution of the 4-coordinates of the particle sources is approximated by a Gauss function

$$W_S(x_1,p_1;x_2,p_2) \propto \rho_S \exp\left(-\frac{{r_1}^2}{2r_0^2} - \frac{(t_1-\langle t\rangle)^2}{2\tau_0^2}\right) \times \exp\left(\frac{{r_2}^2}{2r_0^2} - \frac{t_2^2}{2\tau_0^2}\right). \tag{7}$$

In the considered case of unpolarized particles with spins s_1 and s_2, the population probability of the spin-S states is $\rho_S = (2S+1)/[(2s_1+1)(2s_2+1)]$. We assume $r_0 = 3.5$ fm in correspondence with the mass number of the emitting nucleus $A = 120$. Requiring the same dispersion of the difference $t = t_1 - t_2$ of the emission times for the Gaussian and the exponential decay laws, the parameter τ_0 can be identified with the emitter lifetime τ. In the case of evaporation processes τ is typically several hundreds fm/c (leading to $\langle r^* \rangle \simeq v\tau$ of several tens fm). The above parameters roughly describe particle emission in the reaction $^{40}\mathrm{Ar} + ^{108}\mathrm{Ag}$ at 44 MeV/nucleon [3]. Instead of the 6-dimensional correlation function $R(p_1,p_2)$ we calculate the 1-dimensional one integrating over the one-particle spectra:

$$R(k^*) = \frac{1}{N(k^*)}\sum_{i=1}^{N(k^*)}\sum_S \rho_S|\psi^{S(+)}_{p_{1_i},p_{2_i}}(x_{1_i},x_{2_i})|^2, \tag{8}$$

where $N(k^*)$ is the number of generated particle pairs in a given k^* bin.

Our calculations show that the correlation functions R_+, R_- and their ratio are quite sensitive to $\langle t \rangle$ provided that $\langle t \rangle^2$ is comparable to or higher than the dispersions $\langle t_i^2 - \langle t_i \rangle^2 \rangle$. In Figs. 1 and 2 we present the results for pd and np systems assuming $\tau_0 = 50$ fm/c and $\langle t \rangle = -100$, 50 and 100 fm/c. We see that the ratios R_+/R_- strongly deviate from unity in the region of small k^* achieving their extremal values at ~ 15 and 30 MeV/c for pd and np systems, respectively. Figs. 1 and 2 also demonstrate that, in accordance with the discussion following Eq. (4), the deviation of the ratio R_+/R_- from 1 is correlated with the mean difference of emission times of the two particles $\langle t \rangle$ (of course, $R_+ = R_-$ at $\langle t \rangle = 0$).

We have also studied how the correlation functions R_+ and R_- are influenced by the Coulomb field of the residual nucleus and the accepted angular range. Taking $Z = 51$ for the charge of the residual nucleus leads to an about twice as large mean kinetic energy of the emitted protons or deuterons as compared with that of neutrons, increasing the mean velocity of the considered pairs to $\langle v \rangle \sim 0.15$. The corresponding increase in the characteristic distance $\langle r^* \rangle \simeq \langle v \rangle \tau$ leads to a noticeable suppression of the correlation. Further suppression arises due to 3-body final state interactions between the two particles and the residual nucleus [4]. It appears that these effects only slightly affect the ratio R_+/R_-, somewhat suppressing mainly the tails of the maxima and minima displayed in Figs. 1 and 2. These maxima and minima become wider when the accepted angular range is narrowed. The maximal effect would be achieved at zero width of the accepted angular interval, when the vectors $\boldsymbol{v}$ and $\boldsymbol{k}^*$ can only be parallel or antiparallel.

It should be stressed that the delays in the emission of the particles of various types are quite sensitive to the production mechanism. Thus, in the framework of the coalescence model, we expect the mean emission time of a deuteron to be half that of a proton assuming that those of the proton and neutron are approximately equal and sufficiently large. Note that the former condition is expected to be valid except for the very low energy reactions when the proton emission should be delayed due to the Coulomb barrier. Concerning particle production at high energies, it is usually assumed that, for example, kaons are pro-

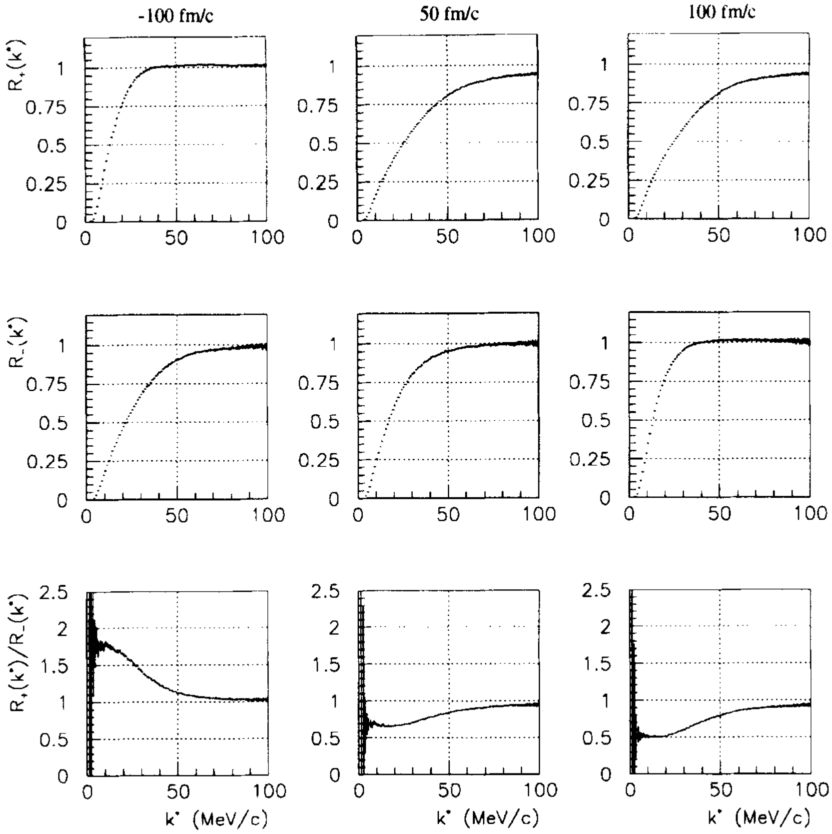

Fig. 1. The pd correlation functions $R_+(vk^* \geq 0)$, $R_-(vk^* < 0)$ and their ratio calculated for unpolarized particles emitted isotropically according to Maxwellian law with a temperature of 4 MeV. The threshold of 5 MeV in the kinetic energy of the charged particles and the accepted range of 60°–120° in polar and azimuthal angles are assumed in the source rest frame. The distribution of the space-time coordinates of the particle sources is approximated by a product of Gauss functions with $\langle \frac{1}{3} r_i^2 \rangle^{1/2} \equiv r_0 = 3.5$ fm, $\langle t_i^2 - \langle t_i \rangle^2 \rangle^{1/2} \equiv \tau_0 = 50$ fm/c and $\langle t_p - t_d \rangle = -100$, 50 and 100 fm/c.

duced earlier than pions due to the smaller interaction cross-sections. Another example is a delay expected between kaon and antikaon emission in the case of the strangeness distillation from the mixed hadronic and quark-gluon plasma phase.

We summarize our results. We have shown that correlations between two nonidentical particles allow to measure directly the difference of their mean emission times including its sign. Thus a new possibility is open to determine which sort of particles (e.g., protons or deuterons, pions or kaons) was produced earlier and which later. For this we suggest to study the correlation functions of two nonidentical particles separately for the angles between the relative velocity $\boldsymbol{k}^*/\mu$ and the total pair velocity $\boldsymbol{v}$ less and greater than 90°.

It should be stressed that our method can be used not only to determine the differences of the mean emission times of various particles but, in general, it yields information on the asymmetry of the distribution of the relative space-time coordinates of the particle emission points.

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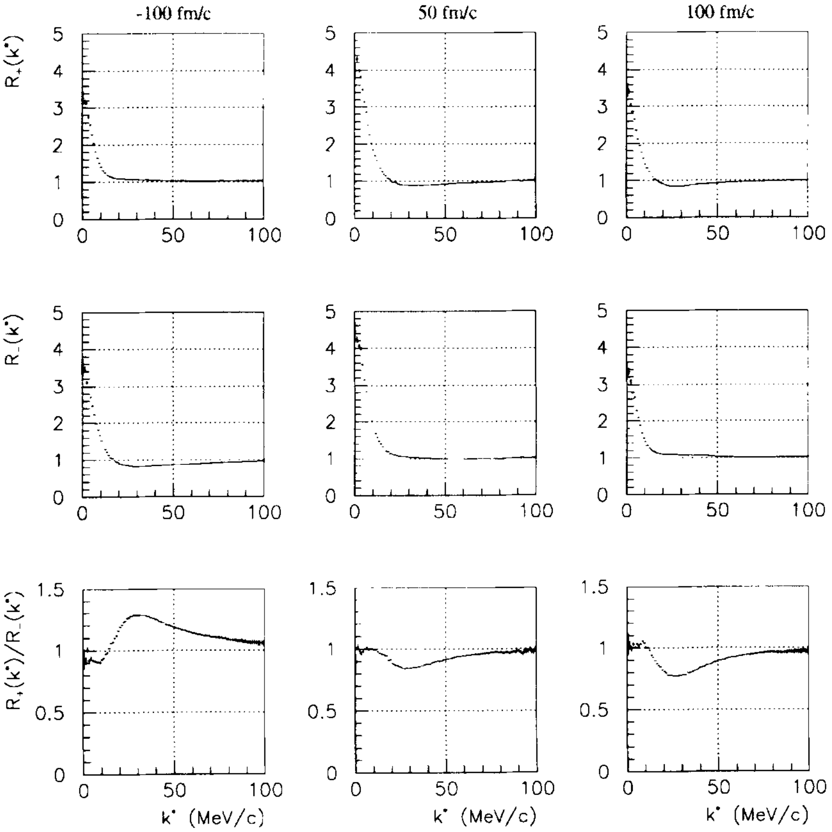

Fig. 2. The same as in Fig. 1 for the *np* system.

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