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Analysis of the nature of the $\phi\to\gamma\pi\eta$ and $\phi\to\gamma\pi^0\pi^0$ decays

N. N. Achasov* and V. V. Gubin†
Laboratory of Theoretical Physics, Sobolev Institute for Mathematics, 630090, Novosibirsk-90, Russia
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We study interference patterns in the $\phi\to(\gamma a_0+\pi^0\rho)\to\gamma\pi\eta$ and $\phi\to(\gamma f_0+\pi^0\rho)\to\gamma\pi^0\pi^0$ reactions. Taking into account the interference, we fit the experimental data and show that the background reaction does not distort the $\pi^0\eta$ spectrum in the decay $\phi\to\gamma\pi\eta$ everywhere over the energy region and does not distort the $\pi^0\pi^0$ spectrum in the decay $\phi\to\gamma\pi^0\pi^0$ in the wide region of the $\pi^0\pi^0$ system invariant mass, $m_{\pi\pi}>670$ MeV, or when the photon energy is less than 300 MeV. We discuss the details of the scalar meson production in the radiative decays and note that there are reasonable arguments in favor of the one-loop mechanism $\phi\to K^+K^-\to\gamma a_0$ and $\phi\to K^+K^-\to\gamma f_0$. We discuss also distinctions between the four-quark, molecular, and two-quark models and argue that the Novosibirsk data give evidence in favor of the four-quark nature of the scalar $a_0(980)$ and $f_0(980)$ mesons.

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I. INTRODUCTION

As was shown in a number of papers, see Refs. [1–6], and references therein, the study of the radiative decays $\phi\to\gamma a_0\to\gamma\pi\eta$ and $\phi\to\gamma f_0\to\gamma\pi\pi$ can shed light on the problem of the scalar $a_0(980)$ and $f_0(980)$ mesons. These decays have been studied not only theoretically but also experimentally. Present time data have already been obtained from Novosibirsk with the detectors SND [7–10] and CMD-2 [11], which give the following branching ratios: $\mathrm{BR}(\phi\to\gamma\pi\eta)=(0.88\pm0.14\pm0.09)\times10^{-4}$ [9], $\mathrm{BR}(\phi\to\gamma\pi^0\pi^0)=(1.221\pm0.098\pm0.061)\times10^{-4}$ [10] and $\mathrm{BR}(\phi\to\gamma\pi\eta)=(0.9\pm0.24\pm0.1)\times10^{-4}$, $\mathrm{BR}(\phi\to\gamma\pi^0\pi^0)=(0.92\pm0.08\pm0.06)\times10^{-4}$ [11].

These data give evidence in favor of the four-quark $(q^2\overline{q}^2)$ [1,12–16] nature of the scalar $a_0(980)$ and $f_0(980)$ mesons. Note that the isovector $a_0(980)$ meson is produced in the radiative ϕ meson decay as intensively as the well-studied η' meson involving essentially strange quarks $s\overline{s}$ ($\approx66\%$), responsible for the decay.

As shown in Refs. [1,3,17], the background situation for studying the radiative decays $\phi\to\gamma a_0\to\gamma\pi^0\eta$ and $\phi\to\gamma f_0\to\gamma\pi^0\pi^0$ is very good. For example, in the case of the decay $\phi\to\gamma a_0\to\gamma\pi^0\eta$, the process $\phi\to\pi^0\rho\to\gamma\pi^0\eta$ is the dominant background. The estimation for the soft, by strong interaction standard, photon energy, $\omega<100$ MeV, gives $\mathrm{BR}(\phi\to\pi^0\rho^0\to\gamma\pi^0\eta,\omega<100\ \mathrm{MeV})\approx1.5\times10^{-6}$. The influence of the background process is negligible, provided $\mathrm{BR}(\phi\to\gamma a_0\to\gamma\pi^0\eta,\omega<100\ \mathrm{MeV})\geqslant10^{-5}$. In this paper, in Sec. II, we calculate the expression for the $\phi\to\gamma\pi^0\eta$ decay amplitude taking into account the interference between the $\phi\to\gamma a_0\to\gamma\pi^0\eta$ and $\phi\to\pi^0\rho\to\gamma\pi^0\eta$ processes. We show that for the obtained experimental data the influence of the background processes is negligible everywhere over the photon energy region.

The situation with $\phi\to\gamma f_0\to\gamma\pi^0\pi^0$ decay is not much different. As was shown in [1,3,17] the dominant background is the $\phi\to\pi^0\rho^0\to\gamma\pi^0\pi^0$ process with $\mathrm{BR}(\phi\to\pi^0\rho^0\to\gamma\pi^0\pi^0,\omega<100\ \mathrm{MeV})\approx6.4\times10^{-7}$. The influence of this background process is negligible, provided $\mathrm{BR}(\phi\to\gamma f_0\to\gamma\pi^0\pi^0,\omega<100\ \mathrm{MeV})\geqslant5\times10^{-6}$.

The exact calculation of the interference patterns between the decays $\phi\to\gamma f_0\to\gamma\pi^0\pi^0$ and $\phi\to\rho^0\pi\to\gamma\pi^0\pi^0$, which we present in this paper in Sec. III, shows that the influence of the background in the decay $\phi\to\gamma\pi^0\pi^0$ for the obtained experimental data is negligible in the wide region of the $\pi^0\pi^0$ invariant mass, $m_{\pi\pi}>670$ MeV, or in the photon energy region $\omega<300$ MeV.

In Sec. IV we discuss the mechanism of the scalar meson production in the radiative decays and show that experimental data obtained in Novosibirsk give the reasonable arguments in favor of the one-loop mechanism $\phi\to K^+K^-\to\gamma a_0$ and $\phi\to K^+K^-\to\gamma f_0$ of these decays . In the same place we discuss also distinctions between the four-quark, molecular, and two-quark models and explain why these data give evidence in favor of the four-quark nature of the scalar $a_0(980)$ and $f_0(980)$ mesons.

II. INTERFERENCE BETWEEN THE REACTIONS $\phi\to\gamma a_0\to\gamma\pi^0\eta$ AND $\phi\to\pi^0\rho^0\to\gamma\pi^0\eta$

As was shown in Refs. [1,3] the background process $e^+e^-\to\phi\to\pi^0\rho^0\to\gamma\pi^0\eta$ is dominant. The amplitudes of the processes $e^+e^-\to\rho^0(\omega)\to\eta\rho^0(\omega)\to\gamma\pi^0\eta$ are much less than the amplitudes of the $e^+e^-\to\rho^0(\omega)\to\pi^0\omega(\rho^0)\to\gamma\pi^0\eta$ processes. In its turn, the amplitudes of the $e^+e^-\to\rho^0(\omega)\to\pi^0\omega(\rho^0)\to\gamma\pi^0\eta$ processes are much less than the amplitudes of the $e^+e^-\to\phi\to\pi^0\rho^0\to\gamma\pi^0\eta$ processes. The amplitude of the $e^+e^-\to\phi\to\eta\phi\to\gamma\pi^0\eta$ process is also much less than the amplitude of $e^+e^-\to\phi\to\pi^0\rho^0\to\gamma\pi^0\eta$ process.

The amplitude of the background process $\phi(p)\to\pi^0\rho^0\to\gamma(q)\pi^0(k_1)\eta(k_2)$ is

$$M_B=\frac{g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho(p-k_1)}\phi_\alpha k_{1\mu}p_\nu\epsilon_\delta(p-k_1)_\omega q_\epsilon\epsilon_{\alpha\beta\mu\nu}\epsilon_{\beta\delta\omega\epsilon}. \tag{1}$$

*Email address: achasov@math.nsc.ru
†Email address: gubin@math.nsc.ru

For the amplitude of the signal $\phi\to\gamma a_0\to\gamma\pi^0\eta$ we use the model suggested in Ref. [1], in which the one-loop mechanism of the decay $\phi\to K^+K^-\to\gamma a_0$ is considered:

$$M_a=g(m)\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\left((\phi\epsilon)-\frac{(\phi q)(\epsilon p)}{(pq)}\right), \qquad (2)$$

where $m^2=(k_1+k_2)^2$, ϕ_α, and ϵ_μ are the polarization vectors of ϕ meson and photon, the function $g(m)$ was determined in Refs. [1,3]. The mass spectrum is

$$\frac{\Gamma(\phi\to\gamma\pi\eta)}{dm}=\frac{d\Gamma_{a_0}}{dm}+\frac{d\Gamma_{\rm back}(m)}{dm}\pm\frac{d\Gamma_{\rm int}(m)}{dm}, \qquad (3)$$

where the mass spectrum for the signal is

$$\frac{d\Gamma_{a_0}}{dm}=\frac{2}{\pi}\frac{m^2\Gamma[\phi\to\gamma a_0(m)]\Gamma[a_0(m)\to\pi\eta]}{|D_{a_0}(m)|^2}$$
$$=\frac{2|g(m)|^2p_{\eta\pi}(m_\phi^2-m^2)}{3(4\pi)^3m_\phi^3}\left|\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\right|^2. \qquad (4)$$

Accordingly, the mass spectrum for the background process $e^+e^-\to\phi\to\pi^0\rho\to\gamma\pi^0\eta$ is

$$\frac{d\Gamma_{\rm back}(m)}{dm}=\frac{(m_\phi^2-m^2)p_{\pi\eta}}{128\pi^3m_\phi^3}\int_{-1}^{1}dxA_{\rm back}(m,x), \qquad (5)$$

where

$$A_{\rm back}(m,x)=\frac{1}{3}\sum|M_B|^2$$
$$=\frac{1}{24}(m_\eta^4m_\pi^4+2m^2m_\eta^2m_\pi^2\tilde m_\rho{}^2-2m_\eta^4m_\pi^2\tilde m_\rho{}^2$$
$$-2m_\eta^2m_\pi^4\tilde m_\rho{}^2+2m^4\tilde m_\rho{}^4-2m^2m_\eta^2\tilde m_\rho{}^4$$
$$+2m_\eta^4\tilde m_\rho{}^4-2m^2m_\pi^2\tilde m_\rho{}^4+4m_\eta^2m_\pi^2\tilde m_\rho{}^4$$
$$+m_\pi^4\tilde m_\rho{}^4+2m^2\tilde m_\rho{}^6-2m_\eta^2\tilde m_\rho{}^6-2m_\pi^2\tilde m_\rho{}^6$$
$$+\tilde m_\rho{}^8-2m_\eta^4m_\pi^2m_\phi^2-2m^2m_\eta^2m_\phi^2\tilde m_\rho{}^2$$
$$+2m_\eta^2m_\pi^2m_\phi^2\tilde m_\rho{}^2-2m^2m_\phi^2\tilde m_\rho{}^4+2m_\eta^2m_\phi^2\tilde m_\rho{}^4$$
$$-2m_\phi^2\tilde m_\rho{}^6+m_\eta^4m_\phi^4+m_\phi^4\tilde m_\rho{}^4)\left|\frac{g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho(\tilde m_\rho)}\right|^2 \qquad (6)$$

and

$$\tilde m_\rho{}^2=m_\eta^2+\frac{(m^2+m_\eta^2-m_\pi^2)(m_\phi^2-m^2)}{2m^2}-\frac{(m_\phi^2-m^2)x}{m}p_{\pi\eta}$$
$$p_{\pi\eta}=\frac{\sqrt{[m^2-(m_\eta-m_\pi)^2][m^2-(m_\eta+m_\pi)^2]}}{2m}. \qquad (7)$$

The interference between the background process amplitude and the signal amplitude is written in the following way:

$$\frac{d\Gamma_{\rm int}(m)}{dm}=\frac{(m_\phi^2-m^2)p_{\pi\eta}}{128\pi^3m_\phi^3}\int_{-1}^{1}dxA_{\rm int}(m,x), \qquad (8)$$

where

$$A_{\rm int}(m,x)=\frac{2}{3}{\rm Re}\sum M_aM_B^*$$
$$=\frac{1}{3}\left((m^2-m_\phi^2)\tilde m_\rho{}^2+\frac{m_\phi^2(\tilde m_\rho{}^2-m_\eta^2)^2}{m_\phi^2-m^2}\right)$$
$$\times{\rm Re}\left\{\frac{g(m)g_{a_0K^+K^-}g_{a_0\pi\eta}g_{\phi\rho\pi}g_{\rho\eta\gamma}}{D_\rho^*(\tilde m_\rho)D_{a_0}(m)}\right\}. \qquad (9)$$

The inverse propagator of a_0 meson, $D_{a_0}(m)$, is presented in Refs. [1,3]. The inverse propagator of ρ meson has the following expression:

$$D_\rho(m)=m_\rho^2-m^2-im^2\frac{g_{\rho\pi\pi}^2}{48\pi}\left(1-\frac{4m_\pi^2}{m^2}\right)^{3/2}. \qquad (10)$$

We use the coupling constant $g_{\phi K^+K^-}=4.68\pm0.05$ obtained form the decay $\phi\to K^+K^-$ [18], and the coupling constant $g_{\rho\eta\gamma}=0.572\pm0.08$ GeV^{-1} obtained from the decay $\rho\to\eta\gamma$ [19], with the help of the following expressions:

$$\Gamma(\phi\to K^+K^-)=\frac{g_{\phi K^+K^-}^2}{48\pi}m_\phi\left(1-\frac{4m_K^2}{m_\phi^2}\right)^{3/2},$$

$$\Gamma(\rho\to\eta\gamma)=\frac{g_{\rho\eta\gamma}^2}{96\pi m_\rho^3}(m_\rho^2-m_\eta^2)^3. \qquad (11)$$

The coupling constant $g_{\phi\rho\pi}=0.811\pm0.081$ GeV^{-1} is obtained using the data on the decay $\phi\to\rho\pi\to\pi^+\pi^-\pi^0$ [18] with the help of the formulas from Ref. [20].

The fit of the experimental data from the SND detector [9], taking into account the relation $g_{a_0\pi\eta}=0.85g_{a_0K^+K^-}$ resulting from the $q^2\overline{q}^2$ model [1], chooses the constructive interference and gives

$$m_{a_0}=985.51\pm0.8 \quad \text{MeV},$$
$$g_{a_0K^+K^-}=2.747\pm0.428 \quad \text{GeV},$$
$$\frac{g_{a_0K^+K^-}^2}{4\pi}=0.6\pm0.015 \quad \text{GeV}^2,$$
$$\chi^2/N_{\rm DF}=3.1/6. \qquad (12)$$

The total branching ratio, taking into account the interference, is ${\rm BR}(\phi\to(\gamma a_0+\pi^0\rho)\to\gamma\pi\eta)=(0.79\pm0.2)\times10^{-4}$, the branching ratio of the signal is $\mathrm{BR}(\phi\to\gamma a_0\to\gamma\pi\eta)=(0.75\pm0.2)\times10^{-4}$ and the branching ratio of the background is $\mathrm{BR}(\phi\to\rho^0\pi^0\to\gamma\pi^0\eta)=3.43\times10^{-6}$. So, the integral part of the interference is negligible. The influence of the interference on the mass spectrum of the $\pi\eta$ system is also negligible, see Fig. 1.

The difference of the obtained parameters (12) from the parameters found in Ref. [9], which are $m_{a_0}=994\pm^{33}_{8}$ MeV, $g^2_{a_0K^+K^-}/4\pi=1.05\pm^{0.36}_{0.25}\ \mathrm{GeV}^2$, is due to the fact that in Ref. [9] a more refined fitting was performed considering the event distribution inside of the each bin. Notice that this difference is less than two standard deviations.

Let us specially emphasize that the value $g^2_{a_0K^+K^-}/4\pi=0.6\pm0.015\ \mathrm{GeV}^2$ obtained by us on no account points to the possibility of the $K\bar{K}$ molecule description [2] of the a_0 meson. In the $K\bar{K}$ molecule model, the imaginary part of the K^+K^- loop is dominant because the real part of the K^+K^- loop is suppressed by the wave function of the molecule [4], see also Sec. IV. Due to this fact, we have $\mathrm{BR}(\phi\to\gamma a_0\to\gamma\pi\eta)\approx1.5\times10^{-5}$ [4] in the $K\bar{K}$ molecule model at the same coupling constant and $m_{a_0}=985$ MeV, which is almost by six times less than the experimental value $\mathrm{BR}(\phi\to\gamma\pi\eta)=(0.88\pm0.14\pm0.09)\times10^{-4}$ [9]. The divergence is by five standard deviations. In addition, in the case of molecule, the bump in the spectrum of the $\pi\eta$ system is much narrower than the experimentally observed [4], see Sec. IV.

III. INTERFERENCE BETWEEN THE $e^+e^-\to\gamma f_0\to\gamma\pi^0\pi^0$ AND $e^+e^-\to\phi\to\pi^0\rho\to\gamma\pi^0\pi^0$ REACTIONS

When analyzing the $\phi\to\gamma f_0\to\gamma\pi^0\pi^0$ decay, one should take into account the mixing of the f_0 meson with the isosinglet scalar states. The whole formalism of the mixing of two scalar f_0 and σ mesons was considered in Ref. [3]. In this paper, we consider only expressions in regard to the interference with the background reactions.

As was shown in Refs. [1,3], the dominant background is the $e^+e^-\to\phi\to\pi^0\rho\to\gamma\pi^0\pi^0$ reaction. The amplitude of the $e^+e^-\to\rho\to\pi^0\omega\to\gamma\pi^0\pi^0$ reaction is much less than the amplitude of the $e^+e^-\to\phi\to\pi^0\rho\to\gamma\pi^0\pi^0$ reaction. In its turn, the amplitude of the $e^+e^-\to\omega\to\pi^0\rho\to\gamma\pi^0\pi^0$ reaction is much less than the amplitude of the $e^+e^-\to\rho\to\pi^0\omega\to\gamma\pi^0\pi^0$ reaction.

The amplitude of the background decay $\phi(p)\to\pi^0\rho\to\gamma(q)\pi^0(k_1)\pi^0(k_2)$ is written in the following way:

$$M_{\mathrm{back}}=g_{\rho\pi^0\phi}g_{\rho\pi^0\gamma}\phi_\alpha p_\nu\epsilon_\delta q_\epsilon\epsilon_{\alpha\beta\mu\nu}\epsilon_{\beta\delta\omega\epsilon}\times\left(\frac{k_{1\mu}k_{2\omega}}{D_\rho(q+k_2)}+\frac{k_{2\mu}k_{1\omega}}{D_\rho(q+k_1)}\right). \tag{13}$$

The amplitude of the signal $\phi\to\gamma(f_0+\sigma)\to\gamma\pi^0\pi^0$ takes into account the mixing of f_0 and σ mesons, see Ref. [3],

$$M_{f_0}=g(m)e^{i\delta_B}\left((\phi\epsilon)-\frac{(\phi q)(\epsilon p)}{(pq)}\right)\times\left(\sum_{R,R'}g_{RK^+K^-}G^{-1}_{RR'}g_{R'\pi^0\pi^0}\right), \tag{14}$$

where $R,R'=f_0,\sigma$. The matrix of propagators is defined in Ref. [3]. The phase of the signal amplitude is formed by the phase of the triangle diagram ($\phi\to K^+K^-\to\gamma R$) and by the phase of $\pi\pi$ scattering which in its turn is defined by the phase of the $f_0-\sigma$ complex, and by the phase of the elastic background of $\pi\pi$ scattering δ_B see details in Refs. [6,3,13].

The mass spectrum of the process is

$$\frac{\Gamma(\phi\to\gamma\pi^0\pi^0)}{dm}=\frac{d\Gamma_{f_0}}{dm}+\frac{d\Gamma_{\mathrm{back}}(m)}{dm}\pm\frac{d\Gamma_{\mathrm{int}}(m)}{dm}, \tag{15}$$

where the mass specrum of the signal has the form

$$\frac{d\Gamma_{f_0}}{dm}=\frac{|g(m)|^2\sqrt{m^2-4m_\pi^2}(m_\phi^2-m^2)}{3(4\pi)^3m_\phi^3}\times\left|\sum_{R,R'}g_{RK^+K^-}G^{-1}_{RR'}g_{R'\pi^0\pi^0}\right|^2. \tag{16}$$

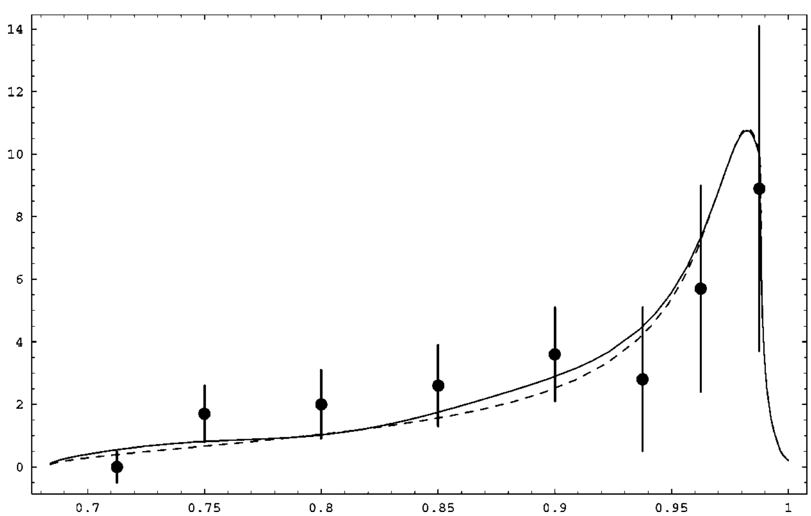

FIG. 1. Fitting of $d\mathrm{BR}(\phi\to\gamma\pi\eta)/dm$ with the background is shown with the solid line, the signal contribution is shown with the dashed line.

The mass spectrum for the background process $e^+e^-\to\phi\to\pi^0\rho\to\gamma\pi^0\pi^0$ is

$$\frac{d\Gamma_{\rm back}(m)}{dm}=\frac{1}{2}\,\frac{(m_\phi^2-m^2)\sqrt{m^2-4m_\pi^2}}{256\pi^3 m_\phi^3}\int_{-1}^{1}dxA_{\rm back}(m,x),\tag{17}$$

where

$$\begin{aligned}A_{\rm back}(m,x)&=\frac{1}{3}\sum|M_{\rm back}|^2\\
&=\frac{1}{24}g^2_{\phi\rho\pi}g^2_{\rho\pi\gamma}\Big\{(m_\pi^8+2m^2m_\pi^4\tilde m_\rho^2-4m_\pi^6\tilde m_\rho^2+2m^4\tilde m_\rho^4-4m^2m_\pi^2\tilde m_\rho^4+6m_\pi^4\tilde m_\rho^4+2m^2\tilde m_\rho^6-4m_\pi^2\tilde m_\rho^6+\tilde m_\rho^8-2m_\pi^6m_\phi^2\\
&\quad-2m^2m_\pi^2\tilde m_\rho^2m_\phi^2+2m_\pi^4\tilde m_\rho^2m_\phi^2-2m^2\tilde m_\rho^4m_\phi^2+2m_\pi^2\tilde m_\rho^4m_\phi^2-2\tilde m_\rho^6m_\phi^2+m_\pi^4m_\phi^4+\tilde m_\rho^4m_\phi^4)\\
&\quad\times\left(\frac{1}{|D_\rho(m_\rho)|^2}+\frac{1}{|D_\rho(\tilde m_\rho^*)|^2}\right)+(m_\phi^2-m^2)(m^2-2m_\pi^2+2\tilde m_\rho^2-m_\phi^2)(2m^2m_\pi^2+2m_\pi^2m_\phi^2-m^4)\frac{1}{|D_\rho(\tilde m_\rho^*)|^2}\\
&\quad+2{\rm Re}\left(\frac{1}{D_\rho(m_\rho)D_\rho^*(\tilde m_\rho^*)}\right)(m_\pi^8-m^6\tilde m_\rho^2+2m^4m_\pi^2\tilde m_\rho^2+2m^2m_\pi^4\tilde m_\rho^2-4m_\pi^6\tilde m_\rho^2-4m^2m_\pi^2\tilde m_\rho^4+6m_\pi^4\tilde m_\rho^4+2m^2\tilde m_\rho^6\\
&\quad-4m_\pi^2\tilde m_\rho^6+\tilde m_\rho^8+m^2m_\pi^4m_\phi^2-2m_\pi^6m_\phi^2+2m^4\tilde m_\rho^2m_\phi^2-4m^2m_\pi^2\tilde m_\rho^2m_\phi^2+2m_\pi^4\tilde m_\rho^2m_\phi^2-m^2\tilde m_\rho^4m_\phi^2+2m_\pi^2\tilde m_\rho^4m_\phi^2\\
&\quad-2\tilde m_\rho^6m_\phi^2-m_\pi^4m_\phi^4-m^2\tilde m_\rho^2m_\phi^4+2m_\pi^2\tilde m_\rho^2m_\phi^4+\tilde m_\rho^4m_\phi^4)\Big\}\end{aligned}\tag{18}$$

and

$$\tilde m_\rho^2=m_\pi^2+\frac{(m_\phi^2-m^2)}{2}\left(1-x\sqrt{1-\frac{4m_\pi^2}{m^2}}\right),$$

$$\tilde m_\rho^{*2}=m_\phi^2+2m_\pi^2-m^2-\tilde m_\rho{}^2.\tag{19}$$

The interference between the amplitudes of the background process and the signal has the form

$$\frac{d\Gamma_{\rm int}(m)}{dm}=\frac{1}{\sqrt2}\,\frac{\sqrt{m^2-4m_\pi^2}}{256\pi^3m_\phi^3}\int_{-1}^{1}dxA_{\rm int}(m,x),\tag{20}$$

where

$$\begin{aligned}A_{\rm int}(m,x)&=\frac{2}{3}{\rm Re}\sum M_fM^*_{\rm back}\\
&=\frac{1}{3}{\rm Re}\Big\{g(m)e^{\delta_b}g_{\phi\rho\pi}g_{\rho\pi^0\gamma}\\
&\quad\times\left(\sum_{R,R'}g_{RK^+K^-}G^{-1}_{RR'}g_{R'\pi^0\pi^0}\right)\\
&\quad\times\left(\frac{[(\tilde m_\rho^2-m_\pi^2)^2m_\phi^2-(m_\phi^2-m^2)^2\tilde m_\rho^2]}{D_\rho^*(\tilde m_\rho)}\right.\\
&\quad\left.+\frac{[(\tilde m_\rho^{*2}-m_\pi^2)^2m_\phi^2-(m_\phi^2-m^2)^2\tilde m_\rho^{*2}]}{D_\rho^*(\tilde m_\rho^*)}\right)\Big\}.\end{aligned}\tag{21}$$

The factor 1/2 in Eq. (17) and the factor $1/\sqrt2$ in Eq. (20) take into account the identity of pions. In Eq. (16), the identity of pions is taken into account by the definition of the coupling constant $g_{R\pi^0\pi^0}=g_{R\pi^+\pi^-}/\sqrt2$. For the fitting of the experimental data we use the model of $\pi\pi$ scattering considered in Ref. [3]. The phase of the elastic background of

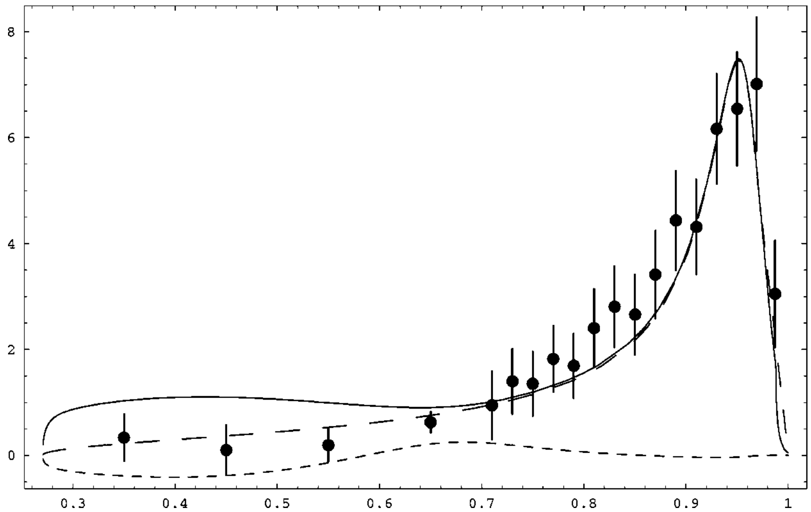

FIG. 2. Fitting of $d\mathrm{BR}(\phi\to\gamma\pi^0\pi^0)/dm$ with the background is shown with the solid line, the signal contribution is shown with the dashed line. The dotted line is the interference term. The data are from the SND detector.

$\pi\pi$ scattering is taken in the form $\delta_B=b\sqrt{m^2-4m_\pi^2}$. We fit simultaneously the phase of $\pi\pi$ scattering and the experimental data on the decay $\phi\to\gamma\pi^0\pi^0$.

The fit of the experimental data [10], obtained using the total statistics of SND detector, and the data on the $\pi\pi$ scattering phase [21–25], taking the value $g_{\rho\pi^0\gamma}=0.295\pm0.037\ \mathrm{GeV}^{-1}$ obtained from the data on the $\rho\to\pi^0\gamma$ decay [19] with the help of the following expression:

$$\Gamma(\rho\to\pi^0\gamma)=\frac{g^2_{\rho\pi^0\gamma}}{96\pi m_\rho^3}(m_\rho^2-m_\pi^2)^3, \tag{22}$$

gives the constructive interference and the following parameters:

$$g_{f_0K^+K^-}=4.021\pm0.011\ \ \mathrm{GeV}, \tag{23}$$

$$g_{f_0\pi^0\pi^0}=1.494\pm0.021\ \ \mathrm{GeV},\quad m_{f_0}=0.996\pm0.0013\ \ \mathrm{GeV},$$

$$g_{\sigma K^+K^-}=0,\quad g_{\sigma\pi^0\pi^0}=2.58\pm0.02\ \ \mathrm{GeV},$$

$$m_\sigma=1.505\pm0.012\ \ \mathrm{GeV},$$

$$b=75\pm2.1\ \ (1°/\mathrm{GeV}),\quad C=0.622\pm0.04\ \ \mathrm{GeV}^2,$$

$$g^2_{f_0K^+K^-}/4\pi=1.29\pm0.017\ \ \mathrm{GeV}^2.$$

The constant C takes into account effectively the contribution of multi particle intermediate states in the $f_0\leftrightarrow\sigma$ transition in $G_{RR'}$ matrix, see Ref. [3], and incorporates the subtraction constant for the $R\to(0^-0^-)\to R'$ transition. We treat this constant as a free parameter.

The total branching ratio, with interference being taken into account, is $\mathrm{BR}[\phi\to(\gamma f_0+\pi^0\rho)\to\gamma\pi^0\pi^0]=(1.26\pm0.29)\times10^{-4}$, the branching ratio of the signal is $\mathrm{BR}(\phi\to\gamma f_0\to\gamma\pi^0\pi^0)=(1.01\pm0.23)\times10^{-4}$, the branching ratio of the background is $\mathrm{BR}(\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0)=0.18\times10^{-4}$. The results of fitting are shown in Figs. 2 and 3. Note, that for our aim, the phase in the region $m_{\pi\pi}<1.1$ GeV is important.

The authors of Ref. [10] fit the data taking into account the background reaction $\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0$. The parameters found [10] are $m_{f_0}=0.9698\pm0.0045$, $g^2_{f_0K^+K^-}/4\pi=2.47\pm^{0.73}_{0.51}\ \mathrm{GeV}^2$ $g^2_{f_0\pi^+\pi^-}/4\pi=0.54\pm^{0.09}_{0.08}\ \mathrm{GeV}^2$. They are different from the parameters found in our fitting. The difference

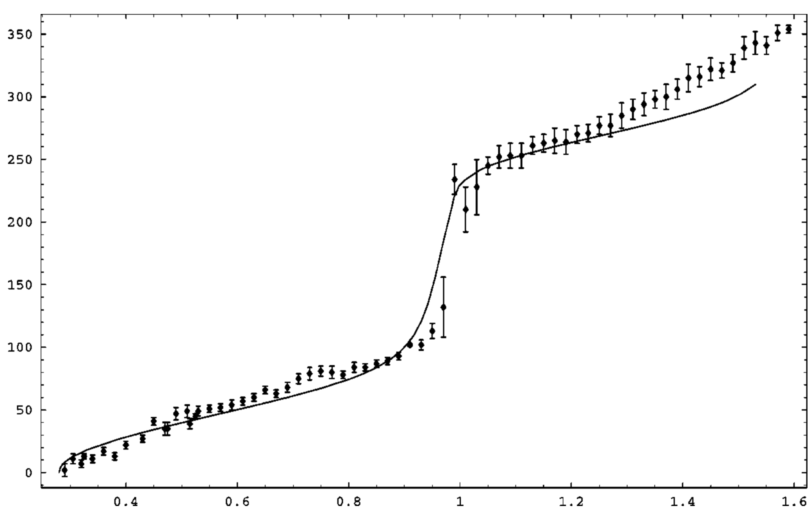

FIG. 3. Fitting of the phase δ_0^0 of $\pi\pi$ scattering.

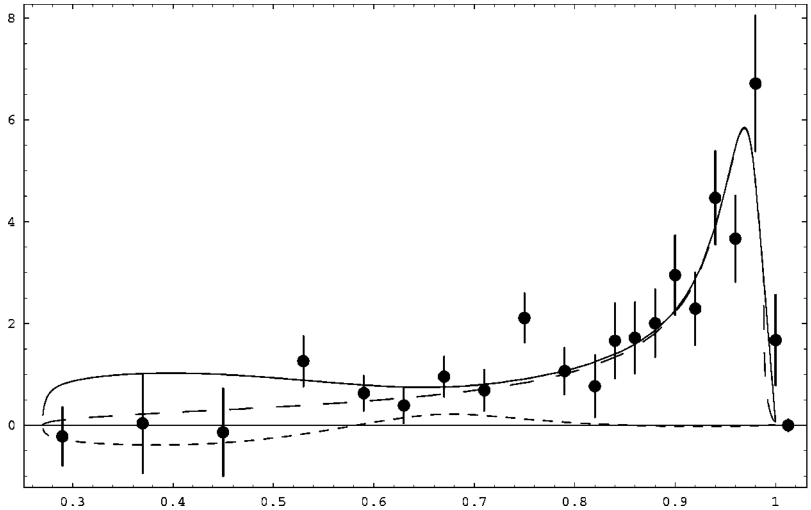

FIG. 4. Fitting of $d\mathrm{BR}(\phi\to\gamma\pi^0\pi^0)/dm$ with the background is shown with the solid line, the signal contribution is shown with the dashed line. The dotted line is the interference term. The data are from the CMD-2 detector.

is due to the fact that we perform the simultaneous fitting of the data on the decay $\phi\to\gamma\pi^0\pi^0$ and the data on the S-wave phase of $\pi\pi$ scattering, taking into account the mixing of f_0 and σ mesons.

In addition, in Ref. [10], the interference between the background and signal is found from the fitting; meanwhile in our paper the interference is calculated. The branching ratio of the background $\mathrm{BR}(\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0)=0.12\times10^{-4}$ used in Ref. [10] is taken from Ref. [17] in which the coupling constant $g_{\rho^0\pi^0\gamma}$ is less by 25% than resulting from the experiment. In our paper, the background is calculated on the basis of experiment and is accordingly larger, $\mathrm{BR}(\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0)=0.18\times10^{-4}$. Note that in Ref. [10], in contrast to us, the fitting is performed taking into account the event distribution inside each bin.

The fitting of the experimental data of the CMD-2 detector [11] and the data on the $\pi\pi$ scattering phase [21–25] gives the constructive interference and the following parameters:

$$g_{f_0K^+K^-}=3.874\pm0.17\ \ \mathrm{GeV}, \tag{24}$$

$$g_{f_0\pi^0\pi^0}=0.536\pm0.03\ \ \mathrm{GeV},\quad m_{f_0}=1.0019\pm0.002\ \ \mathrm{GeV},$$

$$g_{\sigma K^+K^-}=0,\quad g_{\sigma\pi^0\pi^0}=2.61\pm0.1\ \ \mathrm{GeV},$$

$$m_\sigma=1.585\pm0.015\ \ \mathrm{GeV},$$

$$b=70.7\pm2.0\ \ (1°/\mathrm{GeV}),$$

$$C=-0.593\pm0.06\ \ \mathrm{GeV}^2,$$

$$g^2_{f_0K^+K^-}/4\pi=1.19\pm0.03\ \ \mathrm{GeV}^2.$$

The total branching ratio, taking into account the interference, is $\mathrm{BR}[\phi\to(\gamma f_0+\pi^0\rho)\to\gamma\pi^0\pi^0]=(0.98\pm0.21)\times10^{-4}$, the branching ratio of the signal is $\mathrm{BR}(\phi\to\gamma f_0\to\gamma\pi^0\pi^0)=(0.74\pm0.2)\times10^{-4}$, the branching ratio of the background is $\mathrm{BR}(\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0)=0.18\times10^{-4}$. The results of fitting are shown in Figs. 4 and 5.

The parameters found in Ref. [11], which are $m_{f_0}=0.969\pm0.005$, $g^2_{f_0K^+K^-}/4\pi=1.49\pm0.36$ GeV^2 $g^2_{f_0\pi^+\pi^-}/4\pi=0.4\pm0.06$ GeV^2, are different from the parameters found in our fitting. The difference is due to the fact that we perform the simultaneous fitting of the data on the decay $\phi\to\gamma\pi^0\pi^0$ and the data on the S-wave phase of the

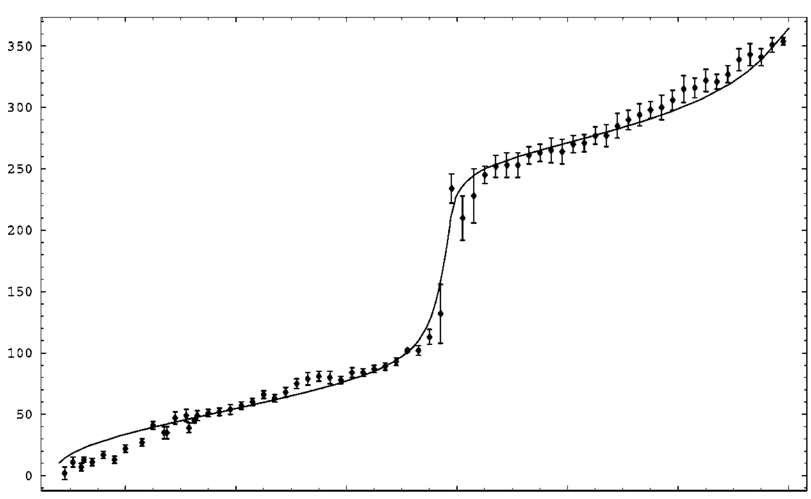

FIG. 5. Fitting of the phase δ_0^0 of $\pi\pi$ scattering.

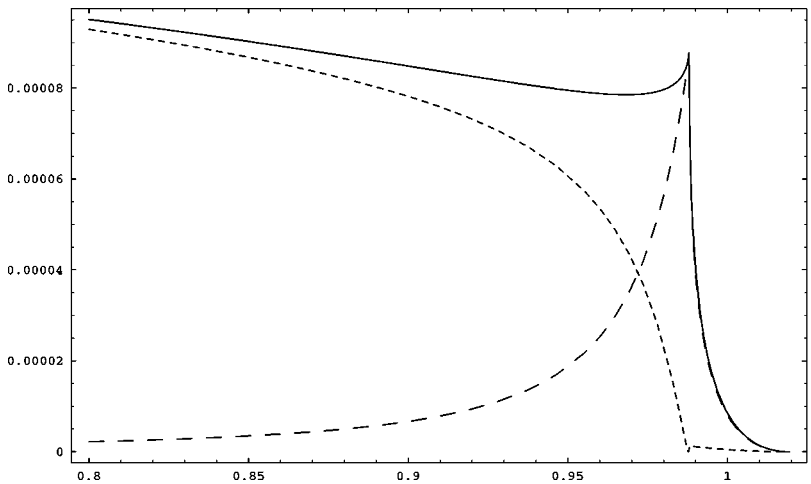

FIG. 6. The function $|g(m)|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

$\pi\pi$ scattering, taking into account the mixing of f_0 and σ mesons and taking into account the background reaction $\phi\to\rho^0\pi^0\to\gamma\pi^0\pi^0$.

One can see from Figs. 2 and 4 that the influence of the background process on the spectrum of the $\phi\to\gamma\pi^0\pi^0$ decay is negligible in the wide region of the $\pi^0\pi^0$ invariant mass, $m_{\pi\pi}>670$ MeV, or when photon energy less than 300 MeV.

In the meantime, the difference from the experimental data is observed in the region $m_{\pi\pi}<670$ MeV. We suppose this difference is due to the fact that in the experimental processing of the $e^+e^-\to\gamma\pi^0\pi^0$ events the background events $e^+e^-\to\omega\pi^0\to\gamma\pi^0\pi^0$ are excluded with the help of the invariant mass cutting and simulation, in so doing the part of the $e^+e^-\to\phi\to\rho\pi^0\to\gamma\pi^0\pi^0$ events is excluded as well.

It should be noted that the SND and CMD-2 data on the branching ratios of the $\phi\to\gamma\pi^0\pi^0$ decay are quite consistent, in the meantime, the SND and CMD-2 data on the shapes of the spectra of the $\pi^0\pi^0$ invariant mass are rather different. The CMD-2 shape is noticeably more narrow, compare Figs. 2 and 4. This difference reflects on the coupling constant $g_{f_0\pi^0\pi^0}$ and the constant C, which are quite different, see Eqs. (23) and (24). In all probability, this difference will disappear when the CMD-2 group processes the total statistics.

IV. CONCLUSION

The experimental data give evidence not only in favor of the four-quark model but in favor of the dynamical model suggested in Ref. [1], in which the discussed decays proceed through the kaon loop, $\phi\to K^+K^-\to\gamma f_0(a_0)$. Indeed, according to the gauge invariance condition, the transition amplitude $\phi\to\gamma f_0(a_0)$ is proportional to the electromagnetic tensor $F_{\mu\nu}$ (in our case to the electric field). Since there are no pole terms in our case, the function $g(m)$ in Eqs. (2) and (14) is proportional to the energy of photon $\omega=(m_\phi^2-m^2)/2m_\phi$ in the soft photon region. To describe the experimental spectra, the function $|g(m)|^2$ should be smooth (almost constant) in the range $m\leqslant 0.99$ GeV, see Eqs. (4) and (16). Stopping the function ω^2 at $\omega_0=30$ MeV, using the form factor of the form $1/(1+R^2\omega^2)$, requires $R\approx 100$ GeV^{-1}. It seems to be incredible to explain the formation of such a huge radius in hadron physics. Based on the large, by hadron physics standard, $R\approx 10$ GeV^{-1}, one can obtain an effective maximum of the mass spectra under discussion only near 900 MeV. In the meantime, the K^+K^- loop gives the natural description to this threshold effect, see Fig. 6.

To demonstrate the threshold character of this effect we present Fig. 7 in which the function $|g(m)|^2$ is shown in the case of K^+ meson mass is 25 MeV less than in reality. One can see that in the region 950–1020 MeV the function

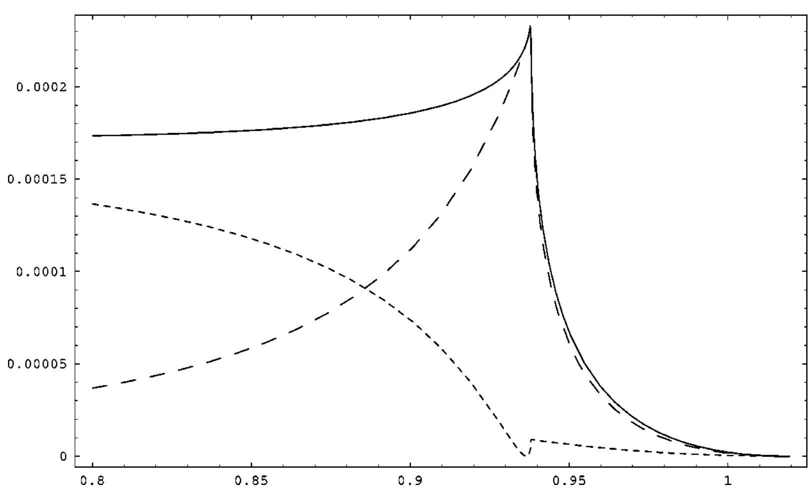

FIG. 7. The function $|g(m)|^2$ for $m_K=469$ MeV is drawn with the solid line. The contribution of the imaginary is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

$|g(m)|^2$ is suppressed by the ω^2 low. In the mass spectrum this suppression is increased by one more power of ω, see Eqs. (4) and (16), so that we cannot see the resonance in the region 980–995 MeV. The maximum in the spectrum is effectively shifted to the region 935–950 MeV. In truth this means that $a_0(980)$ and $f_0(980)$ resonances are seen in the radiative decays of ϕ meson owing to the K^+K^- intermediate state, otherwise the maxima in the spectra would be shifted to 900 MeV.

It is worth noting that the K^+K^- loop model is practically accepted by theorists, compare, for example, Eq. [26] with Eq. [27]; true there is exception [28].

It was noted already in paper [1] that the imaginary part of the K^+K^- loop is calculated practically in a model independent way making use of the coupling constants $g_{\phi K^+K^-}$ and $g_{a_0(f_0)K^+K^-}$ due to the Low's theorem [29] for the photons with energy $\omega < 100$ MeV which is soft by the standard of strong interaction. In the same paper it was noted that the real part of the loop [with accuracy up to 20% in the width of the $\phi\to\gamma f_0(a_0)$ decay] is practically not different for the pointlike particle and the compact hadron with form factor which has the cutting radius in the momentum space about the mass of ρ meson ($m_\rho = 0.77$ GeV). In contrast to the four-quark state which is the compact hadron [12], the bound $K\bar{K}$ state is the extended state with the spatial radius $R \sim 1/\sqrt{m_K\epsilon}$, where ϵ is the binding energy. Corresponding form factor in the momentum space has the radius of the order of $\sqrt{m_K\epsilon}\approx 100$ MeV for $\epsilon = 20$ MeV [30]. The more detailed calculation [2] gives for the radius in the momentum space the value $p_0 = 140$ MeV. As a result, the real part of the loop of the bound state is negligible [4]. It leads to the branching ratio much less than the experimental one, as it was noted above. In addition, the spectrum is much narrower in the $K\bar{K}$ molecule case that contradicts to the experiment, see the behavior of the imaginary part contribution in Fig. 6 and in corresponding figures in Ref. [4].

Of course, the two-quark state is as compact as four-quark one. The question arises, why is the branching ratio in the two-quark model suppressed in comparison with the branching ratio in the four-quark model? There are two reasons. First, the coupling constant of two-quark states with the $K\bar{K}$ channel is noticeably less [3,13] and, second, there is the Okubo-Zweig-Iizuka (OZI) rule that is more important really.

If the isovector $a_0(980)$ meson is the two-quark state, it has no strange quarks. Hence [1,3,15], the decay $\phi\to\gamma a_0$ should be suppressed according to the OZI rule. On the intermediate state level, the OZI rule is formulated as compensation of the different intermediate states [31–33]. In our case these states are $K\bar{K}$, $K\bar{K}^*+\bar{K}K^*$, $K^*\bar{K}^*$ and so on. Since, due to the kinematical reason, the real intermediate state is the only K^+K^- state, the compensation in the imaginary part is impossible and it destroys the OZI rule. The compensation should be in the real part of the amplitude only. As a result, the $\phi\to\gamma a_0$ decay in the two-quark model is mainly due to the imaginary part of the amplitude and is much less intensive than in the four-quark model [1,3]. In addition, in the two-quark model, $a_0(980)$ meson should appear in the $\phi\to\gamma a_0$ decay as a noticeably more narrow resonance than in other processes, see the behavior of the imaginary part contribution in Fig. 6.

As regards to the isoscalar $f_0(980)$ state, there are two possibilities in the two-quark model. If $f_0(980)$ meson does not contain the strange quarks the all abovementioned arguments about suppression of the $\phi\to\gamma a_0$ decay and the spectrum shape are also valid for the $\phi\to\gamma f_0$ decay. Generally speaking, there could be the strong OZI violation for the isoscalar $q\bar{q}$ states (mixing of the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states) with regard to the strong mixing of the quark and gluon degree of freedom which is due to the nonperturbative effects of QCD [34]. But, an almost exact degeneration of the masses of the isoscalar $f_0(980)$ and isovector $a_0(980)$ mesons excludes such a possibility. Note also, the experiment points directly to the weak coupling of $f_0(980)$ meson with gluons, $B(J/\psi\to\gamma f_0\to\gamma\pi\pi) < 1.4\times 10^{-5}$ [35].

If $f_0(980)$ meson is close to the $s\bar{s}$ state [15,36] there is no suppression due to the the OZI rule. Nevertheless, if $a_0(980)$ and $f_0(980)$ mesons are the members of the same multiplet, the $\phi\to\gamma f_0$ branching ratio $\mathrm{BR}(\phi\to\gamma\pi^0\pi^0) = (1/3)\mathrm{BR}(\phi\to\gamma\pi\pi)\approx 1.8\times 10^{-5}$, is significantly less than that in the four-quark model, due to the relation between the coupling constants with the $K\bar{K}$, $\pi\eta$ and $K\bar{K}$, $\pi\pi$ channels inherited in the two-quark model, see Refs. [1,3].

Only in the case when the nature of $f_0(980)$ meson in no way related to the nature of $a_0(980)$ meson (which, for example, is the four-quark state) the branching ratio experimentally observed $\phi\to\gamma f_0$ could be explained by $s\bar{s}$ nature of $f_0(980)$ meson. But, from the theoretical point of view, such a possibility seems awful [15].

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