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The HBT-interferometry of expanding sources

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Abstract

The structure of the bosonic correlation function for expanding thermalized systems is obtained using the conception of the system's lengths of homogeneity. The analysis of the p_T-behavior of the *long-*, *out-* and *side-*interferometry radii is performed for radiating sources with relativistic transversal and longitudinal flows. Simple analytical formulas for all interferometry radii are obtained for typical classes of transversal flows.

1. Introduction

The spectra and correlations in small thermal relativistic quantum-field systems are not trivial and can be understood on the base of space-time scales [1,2]. This includes the total geometrical length which the thermalized system occupies, $\bar{R}_i$, the hydrodynamic length, $\bar{\lambda}_{\mathrm{hydr},i}$, the local length of homogeneity $\bar{\lambda}_i$, and the wavelength of the quanta, $\lambda_p \propto 1/p^0$. First it has been shown in Ref. [3] that in contrast to small homogeneous systems, $\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{R}}$, when the interferometry radii R coincide with the geometrical radii of the system, $\bar{R} = R$, for longitudinally expanding system where $\bar{R}_{\mathrm{L}} \gg \bar{\lambda}_{\mathrm{L}}$, the longitudinal interferometry radius is defined by the hydrodynamic length, $R_{\mathrm{L}} \propto \bar{\lambda}_{\mathrm{L}} = \bar{\lambda}_{\mathrm{hydr}}$. In Ref. [2] the *out-*, *side- and long-*interferometry radii have been expressed through the corresponding lengths of homogeneity for 3-dimensionally expanding systems.

The main aim of this paper is to consider the general case of relativistic transversal expansion and to find typical analytical approximations of the interferometry radii for the different classes of transversal flows.

2. The physical assumptions and formalism

The theory of bosonic spectra and correlation functions for inhomogeneous thermalized systems has been proposed in Ref. [1]. It has been shown that if the wavelength of the quanta, $\lambda_p \propto 1/p^0$, in a weakly interacting bosonic gas, is much smaller than the system's length of homogeneity, $\bar{\lambda}$, the Wigner function coincides approximately with the locally equilibrium Bose–Einstein distribution function

$$f(x, p) = (2\pi)^{-3} \times \{\exp[\beta p \cdot u(x) - \beta\mu] - 1\}^{-1}, \quad (1)$$

where $u^{\mu}(x)$ is the 4-velocity of the hadron gas, $\beta(x)$ is the inverse of the temperature, $\mu(x)$ is the chemical potential. Then the single- and double-particle inclusive spectra are expressed through the

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products of the thermal averages of the creation and annihilation operators. They can be expressed through the Wigner function (with the momentum argument $p=(p_1+p_2)/2$) by the integration of it over the freeze-out hypersurface σ:

$$\langle a^+(p_1)a(p_2)\rangle_\sigma = \int \mathrm{d}\sigma_\mu p^\mu \exp(\mathrm{i}\Delta p\cdot x) f(x,p). \tag{2}$$

We suppose that hadron emission occurs mostly near the surface of the tube, at least at the initial stage when the system is very dense inside. So, we can describe the freeze-out hypersurface σ using the proper time τ as $\tau(r)$. Our approach corresponds to the main approximation based on the saddle point method.

In a cylindrically symmetric model with longitudinal boost-invariance we use the following notations: $\tau=\sqrt{t^2-z^2}$ is the freeze-out proper time, y_{L} is the longitudinal rapidity; y_{T} is the transversal rapidity; r is the two-dimensional transversal radius-vector. The particle 4-momenta are expressed through the longitudinal rapidity θ: $p^\mu=(m_{\mathrm{T}}\cosh\theta, p_{\mathrm{T}}, m_{\mathrm{T}}\sinh\theta)$.

The function $f(x,p)$ in Eq. (2) taking into account the finite average transversal radius of the system, has the form $f(x,p)=f_{\mathrm{W}}(x,p)\rho(r)$. The distribution $\rho(r)$ is chosen usually as a Gaussian [4,5]. To simplify the analytical analysis we introduce a weakly modified argument in Gaussian form. In the relativistic covariant form it looks like

$$\begin{aligned}\rho(x) &\propto \exp\left[\tfrac{1}{2}\alpha\left(u(r,y_{\mathrm{L}})-u(0,y_{\mathrm{L}})\right)^2\right]\\ &=\exp\left[-\alpha\left(\cosh y_{\mathrm{T}}(r)-1\right)\right]\\ &\approx \exp\left(-r^2/2\bar{R}_{\mathrm{T}}^2\right).\end{aligned} \tag{3}$$

Indeed, $v_{\mathrm{T}}(0)=0$ because of symmetry and supposing that $v_{\mathrm{T}}'(0)\equiv R_v^{-1}\neq 0$ we have at small r the following:

$$y_{\mathrm{T}}(r)\approx v_{\mathrm{T}}\approx v_{\mathrm{T}}'(0)r=\frac{r}{R_v}$$

$$\cosh y_{\mathrm{T}}(r)\approx 1+\tfrac{1}{2}y_{\mathrm{T}}^2(r)\approx 1+\frac{1}{2}\frac{r^2}{R_v^2}.$$

So, the last equality in (3) occurs for flows that are approximately linear at $r\ll \bar{R}_v$. The physical meaning of the parameter α is

$$\alpha=R_v^2/\bar{R}_{\mathrm{T}}^2. \tag{4}$$

The numerical analysis for different types of transversal flows with Gaussian transversal distribution and with the modified one (3) demonstrates the closeness of the transversal radii in a typical momentum region for the current interferometry analysis, even for essentially relativistic flows. The main advantage of the distribution (3) linking dynamic and geometrical properties of a system at large $r>R_v$ is that it allows to obtain analytical solutions for relativistic transversal flows. In the non-relativistic limit they coincide with the analogous results obtained in Refs. [4,5].

3. The structure of the correlation functions

The form of the correlation function is

$$C(p_1,p_2)=1+\langle a_{p_1}^+a_{p_2}\rangle\langle a_{p_2}^+a_{p_1}\rangle / \langle a_{p_1}^+a_{p_1}\rangle\langle a_{p_2}^+a_{p_2}\rangle. \tag{5}$$

The average of the operators is defined by Eqs. (2). To calculate Eq. (5), we use the saddle-point method at the large parameter $\beta p_0\gg 1$ in the Wigner function (1). This corresponds to the Boltzmann approximation. Then (see Refs. [2,6]) the correlation function at $\Delta p_i^2\le 1/R_i^2$ can be expressed through the lengths of the homogeneity in a radiating system:

$$\left(\bar{\lambda}_i(x_0)\right)^{-2}=\left|\frac{\partial^2 f(p,x)}{\partial^2 x_i}\right| \Big/ 2f(p,x)\Bigg|_{\bar{x}(p)},$$

$$\left.\frac{\partial f(x,p)}{\partial x_i}\right|_{\bar{x}}=0, \tag{6}$$

where $i=long, side, out$ are the standard directions for interferometry analysis [7]. Then we have for the correlation function (see Refs. [2,6] for details) the following:

$$C(p,q)=1+\exp\left(-\tfrac{1}{2}R_{out}^2q_{out}^2-\tfrac{1}{2}R_{side}^2q_{side}^2-\tfrac{1}{2}R_{long}^2q_{long}^2+R_{out,long}^2q_{out}q_{long}\right), \tag{7}$$

where the corresponding interferometry radii are

$$R_{out}^2 = \bar{\lambda}_{out}^2 \left(1 + \frac{p_T}{m_T} \left| \frac{d\tau}{dr}(\bar{r}) \right| \right)^2 + \bar{\lambda}_{long}^2 \left(\frac{p_T}{m_T} \frac{\tanh\theta}{\cosh\theta} \right)^2,$$

$$R_{side}^2 = \bar{\lambda}_{side}^2,$$

$$R_{long}^2 = \bar{\lambda}_{long}^2 \cosh^{-4}\theta,$$

$$\bar{R}_{out,long}^2 = \bar{\lambda}_{long}^2 \left(\frac{p_T}{m_T} \right) \sinh\theta \cosh^{-4}\theta. \tag{8}$$

The existence of the crossing *out-long* term in the correlation function (7) was recently discussed in Ref. [5]. The form of the crossing *out-long* term is obvious from Eq. (8):

$$R_{out,long}^2 = R_{long}^2 \frac{p_T}{m_T} \sinh\theta. \tag{9}$$

In the boost-invariant approach it is equal to zero if $p_L = 0$ or $p_T = 0$. In other situations it is present in the correlation function. If one is performing the interferometry analysis in the LCMS ($p_L = 0$) for each i-bin in the Lab system: $(\theta_i - \Delta\theta, \theta_i + \Delta\theta)$, $(p_{T,i} - \Delta p_T, p_{T,i} + \Delta p_T)$, it is possible to neglect of this crossing term if $(p_{T,i}/m_{T,i}) \sinh\Delta\theta \ll 1$.

4. The lengths of the homogeneity for typical physical scenarios

The model-independent expressions (7), (8) define the correlation functions by means of the lengths of homogeneity in the decaying system.

The most realistic one is the scenario when the freeze-out hypersurface is characterized by an almost constant temperature. If such a system possesses essential transversal flows, it is easy to obtain the lengths of homogeneity using Eq. (6) and the supposition that $y'_T(r) \neq 0$:

$$\bar{\lambda}_{out}^2 = \frac{2\bar{v}_T}{\beta p_T \bar{y}_T'^2} \left(1 - \bar{v}_T^2\right)^{-1/2},$$

$$\bar{\lambda}_{side}^2 = \frac{2\bar{r}^2}{\beta p_T \bar{v}_T} \left(1 - \bar{v}_T^2\right)^{1/2},$$

$$\bar{\lambda}_{long}^2 = \frac{2\bar{\tau}^2 \cosh^2\theta}{\beta m_T} \left(1 - \bar{v}_T^2\right)^{1/2}, \tag{10}$$

and the spectra

$$p_0 \frac{d^3N}{d^3p} \propto \exp\left[-(\beta m_T + \alpha)\left(1 - \bar{v}_T^2\right)^{1/2} \right], \tag{11}$$

where α is defined by Eq. (4), $\bar{y}_T \equiv y_T(\bar{r})$. At $\alpha \ll m/m_T$ the values (10) are hydrodynamic lengths and coincide with the results obtained in Ref. [2] in the analogous limit for models with a pure Gaussian distribution $\rho(r)$. It is important to note that the transversal velocity at the saddle point as well as the longitudinal length of homogeneity, $\bar{\lambda}_{long}(\bar{x})$, and the exponential factor in the single-particle spectra do not depend on the specific model of transversal flows (i.e., on the form of the function $v_T(r)$ on a freeze-out hypersurface):

$$\bar{v}_T \equiv \tanh\bar{y}_T = \frac{\beta p_T}{\beta m_T + \alpha}. \tag{12}$$

The *out*- and *side*-lengths of homogeneity depend on the specific model of transversal expansion. But in two limited situations we can conclude about its behavior.

– *Nonrelativistic transversal flows.* The corresponding condition is

$$\bar{v}_T = \frac{\beta p'_T}{\beta m_T + \alpha} \ll 1 \Rightarrow \bar{v}_T \approx y_T(\bar{r}) \approx \frac{\bar{r}}{R_v} \ll 1. \tag{13}$$

The parameter $\alpha = R_v^2/\bar{R}_T^2$ according to (4). Using this conditions we obtain from Eqs. (10), (11) the following:

$$\bar{\lambda}_{out}^2 = \bar{\lambda}_{side}^2 \cong 2\left(\frac{\beta m_T}{R_v^2} + \frac{1}{R_T^2} \right)^{-1},$$

$$\bar{\lambda}_{long}^2 \cong \frac{2\bar{\tau}^2 \cosh^2\theta}{\beta m_T},$$

$$p_0 \frac{d^3N}{d^3p} \propto \exp(-\beta m_T). \tag{14}$$

It is important to emphasize that according to Eq. (13) the applicability of expressions (14) in the relativistic momentum region, $P_T \approx m_T$, is limited by the conditions $\beta m_T/R_v^2 \ll 1/\bar{R}_T^2$. If this conditions is violated, the behavior of the lengths of homogeneity, as we will show, changes dramatically as compared with the results (14) or the analogous ones from Refs. [4,5]. We note also that in the non-relativistic limit $\bar{\lambda}_{out}$ and $\bar{\lambda}_{side}$ are expressed through geometrical and hydrodynamic lengths. This is not true for the general relativistic case (see Eqs. (10)).

– *Ultrarelativistic transversal flows.* In the framework of the ultrarelativistic approach we suppose $\bar{v}_T \approx 1$, that means according to Eqs. (12), (4) that $\beta m_T/R_v^2 \gg 1/\bar{R}_T^2$ and $\bar{v}_T \approx p_T/m_T = \tanh \eta_T \approx 1$. It is easy to show that for monotonous functions $y_T(r)$ is growing faster than $\ln \ln(r/R_v)$, the ratio *out*- to *side*-lengths of homogeneity in the ultrarelativistic limit has the form (with accuracy to the powers of $\ln(p_T/m)$)

$$\frac{\bar{\lambda}_{out}(\bar{x})}{\bar{\lambda}_{side}(\bar{x})} \propto \left(1 - \bar{v}_T^2\right)^{-1/2}. \tag{15}$$

The detailed behavior of the transversal lengths of homogeneity and the radii depends on the specific model of the transversal expansion. We will demonstrate this in two typical scenarios.

5. The momentum dependence of the interferometry radii for a three-dimensional expansion

(i) First we consider the scenario with "hard" transversal flows:

$$y_T(r) = \frac{r}{R_v}. \tag{16}$$

Then we have from Eqs. (10), (12) the solution for the saddle point:

$$\bar{r} = \frac{R_v}{2} \ln\left(\frac{1 + \bar{v}_T}{1 - \bar{v}_T}\right), \tag{17}$$

and for the lengths of homogeneity:

$$\bar{\lambda}_{out}^2 = \frac{2R_v^2}{\beta m_T + \alpha}\left(1 - \bar{v}_T^2\right)^{-1/2},$$

$$\bar{\lambda}_{side}^2 = \frac{R_v^2}{2\beta p_T \bar{v}_T}\left(1 - \bar{v}_T^2\right)^{1/2} \ln^2\left(\frac{1 + \bar{v}_T}{1 - \bar{v}_T}\right). \tag{18}$$

The transversal velocity is expressed through the momentum variables by Eq. (12).

(ii) Let us consider the model realizing the scenario of "soft" transversal flows:

$$y_T(r) = \ln\frac{\sqrt{R_v^2 + r^2} + r}{R_v} \Rightarrow v_T(r) = \frac{r}{\sqrt{r^2 + R_v^2}}. \tag{19}$$

The gradient of transversal rapidity for the "soft" flow is smaller everywhere (at $r \neq 0$) than for the "hard" flow. Because of the *out*-radius being defined by this gradient according to Eq. (10), it may differ essentially from the *out*-radius in the "hard" flow model, even at $\bar{r} < \bar{R}_T$. A model of such type has been used in Ref. [4] with $R_v \equiv \tau$. We have from Eqs. (10), (12) the solution for the saddle point:

$$\bar{r} = R_v \frac{\bar{v}_T}{\left(1 - \bar{v}_T^2\right)^{1/2}}, \tag{20}$$

and homogeneity lengths

$$\bar{\lambda}_{out}^2 = \frac{2R_v^2}{\beta m_T + \alpha}\left(1 - \bar{v}_T^2\right)^{-3/2},$$

$$\bar{\lambda}_{side}^2 = \frac{2R_v^2}{\beta m_T + \alpha}\left(1 - \bar{v}_T^2\right)^{-1/2}. \tag{21}$$

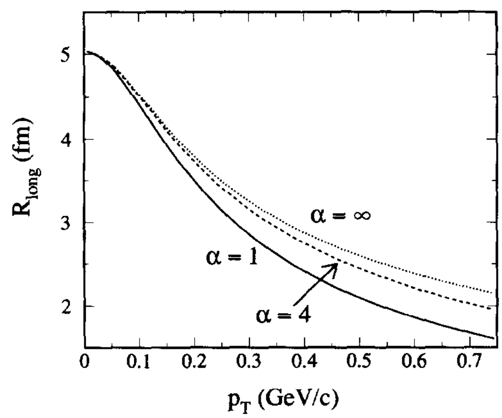

Fig. 1. The p_T-behavior of the longitudinal *long*-radius of the pion that is universal for both classes of transversal flows. The results are presented for the ratio of the transversal hydrodynamic length to the transversal radius of the system: $R/\bar{R}_T = \sqrt{\alpha} = 1$ (solid line) and 2 (dashed line). The *long*-radius without transversal flows ($\alpha = 4$) is presented by the dotted line. The proper time of the longitudinally expanding source when emission is maximal is $\bar{t} = 36$ fm/c. The freeze-out temperature $T = m_0$.

Note that $\overline{\lambda}_{out}(\bar{x})$ for "hard" flows coincides formally with $\overline{\lambda}_{side}(\bar{x})$ for "soft" flows.

The slope k of the transversal spectra $\propto \exp(-km_T)$ in the region $m_T/m \gg 1$ is $k = T^{-1}\sqrt{-\bar{v}_T^2}$ according to Eq. (11). The freeze-out temperature is $T \equiv T_{\text{f.o.}} \approx m_\pi$ [8]. The saddle-point velocity $\bar{v}_T$ is depending on α according to Eq. (12). It gives the possibility to extract the parameter α of the transversal flow intensity from the slope of a single particle spectrum.

In Figs. 1–3 we demonstrate the p_T-behavior of the *long*-, *out*-, and *side*-lengths of homogeneity (in the LCMS) at different α for both types of flows: "hard" and "soft". If $d\tau(\bar{r})/dr \ll 1$, the lengths of homogeneity correspond to the interferometry radii: R_{out}, R_{side} and R_{long}. The value $\alpha = 1$ corresponds to developed transversal flows when the transversal hydrodynamic lengths is compared with the transversal size of the system, $\alpha = 4$ means slow enough transversal flow, $\alpha = \infty$ corresponds to a one-dimensional expansion without transversal flows. We noted that for "soft" developed flows there is big difference in p_T-behavior of the out- and side-correlator: the first one grows, the second one decreases sharply. The current experimental data from the NA-35 and NA-44 SPS CERN display another tendency: the similarity of momentum behavior for both transversal radii. The "hard" flow approach is more suitable for fitting the experimental data.

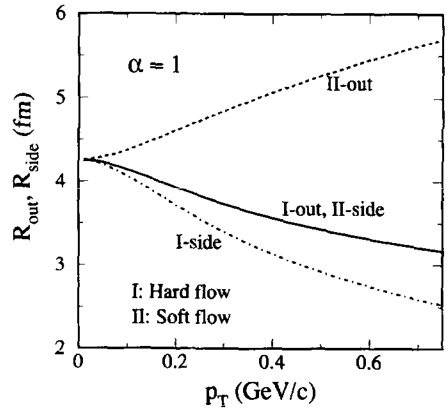

Fig. 2. The p_T-behavior of the transversal *out*- and *side*-interferometry radii of the pion for "hard" (I, dash-dotted line) and "soft" (II, dashed line) transversal expansion. The *out*-radius for the Ist class of the flows coincides with the *side*-radius for the IInd one (solid line). The parameter of the intensity of transversal flows $\alpha = 1$ (strong flow). The freeze-out temperature $T = m_0$, $\bar{R}_T = 4.2$ fm.

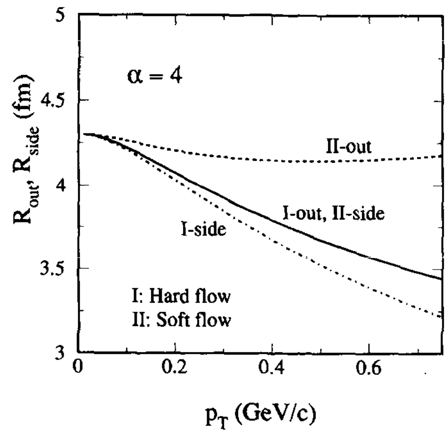

Fig. 3. The p_T-behavior of the transversal *out*- and *side*-interferometry radii of the pion for "hard" (I, dash-dotted line) and "soft" (II, dashed line) transversal expansion. The *out*-radius for the Ist class of the flows coincides with the *side*-radius for the IInd one (solid line). The parameter of the intensity of transversal flows $\alpha = 4$ (slow flow). The freeze-out temperature $T = m_0$, $\bar{R}_T = 3.4$ fm.

The region of applicability for the result presented is limited by the validity of the saddle-point method and is approximately $p_T > m_\pi$. Otherwise the method gives an overestimation of (10–20)% At the same time at small $p_T \leq m_\pi$ the contribution of resonance decays to the observed pion interferometry radius can be significant.

6. Conclusions

We show that the experimentally observed particle spectra and correlation are very sensitive to the space-time structure of the emitting matter. The local lengths of homogeneity in the system define the behavior of the spectra and correlations. This is reflected in the different behavior of the interferometry radii depending on the momentum regions. In particularly, strong transversal expansion leads to a more quick decrease of the longitudinal interferometry radius than the inverse of $\sqrt{m_T}$. At the same time the *side*-radius decreases more quickly than the *out*-radius when the transversal momentum increases.

The simple analytical behavior of the *long-*, *side-* and *out-*interferometry radii, depending on the transversal momenta, has been found for typical classes of transversal flows. The analytical approximations for spectra and correlations allow to clear up experimentally the character of the transversal flows in systems formed in ultra-relativistic nucleus–nucleus collisions and the details of its evolution.

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