

# Hierarchy in networks–Measures

Measure	Undirected	Weighted	Global	Local
Landau's h		X	X	
Kendall's K	X	X		X
Reach Degree	X	X		X
Reach Closeness	X	X		X
GRC	X	X	X	X
Rooted Depth			X	
Degree	X	X	X	X
Closeness	X	X	X	X
Betweenness	X	X	X	X
Eigenvector	X	X	X	X

# Hierarchy in Networks–Simulated Datasets

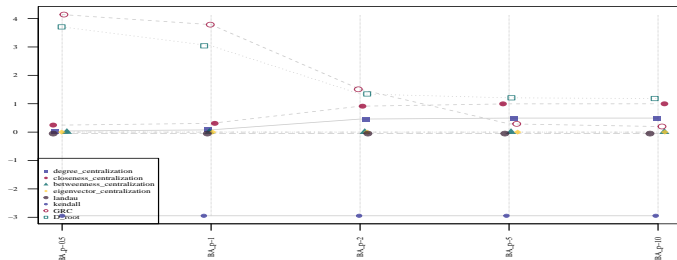
- ▶ 7500 Barabasi-Albert (BA) Datasets with node size 50, 200, or 500 and preferential attachment parameter 0.5, 1, 2, 5, or 10.
- ▶ 6000 Tree-Structured (TR) Datasets with node size 50, 200, or 500 and children parameter 2, 5, 10, or 50.
- ▶ 4500 Erdos-Renyi (ER) Datasets with node size 50, 200, or 500 and parameter size 0.05, 0.1, or 0.2.

BA and TR	Node	Edge	Density
	50	49	0.02
	200	199	0.005
	500	499	0.002

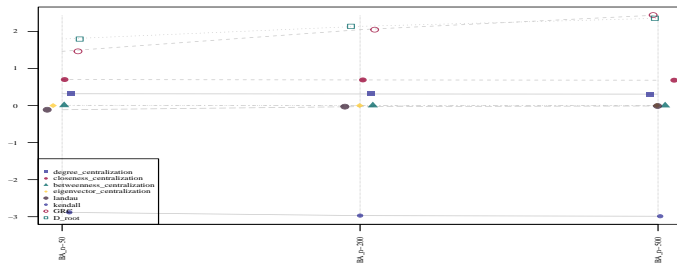
ER	Node	Parameter	Edge	Density	Cluster Coefficient
	50	0.05	122.524	0.050	0.0956
	50	0.1	244.768	0.100	0.188
	50	0.2	490.050	0.200	0.359
	200	0.05	1989.242	0.050	0.098
	200	0.1	3980.776	0.100	0.190
	200	0.2	7952.138	0.120	0.360
	500	0.05	12473.270	0.050	0.097
	500	0.1	24932.824	0.100	0.190
	500	0.2	49892.784	0.200	0.360

# Average Global Hierarchy-BA Networks

## Parameters

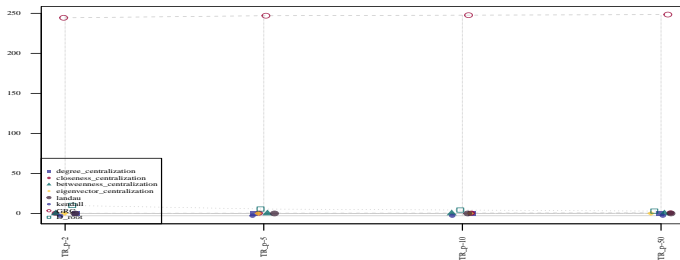


## Node Size

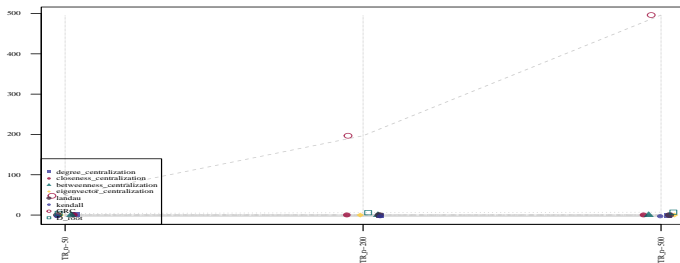


# Average Global Hierarchy–TR Networks

## Parameters

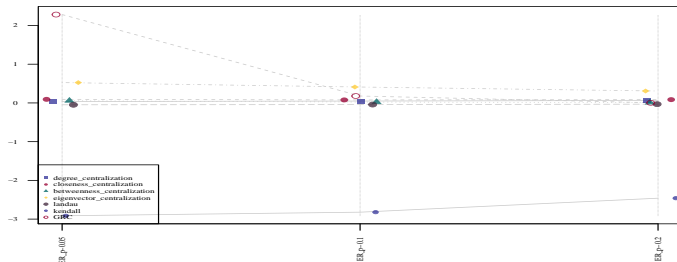


## Node Size

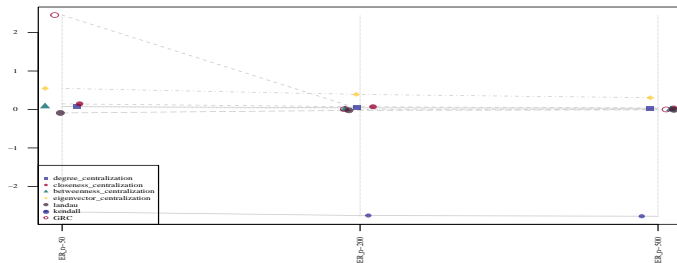


# Average Global Hierarchy–ER Networks

## Parameters



## Node Size

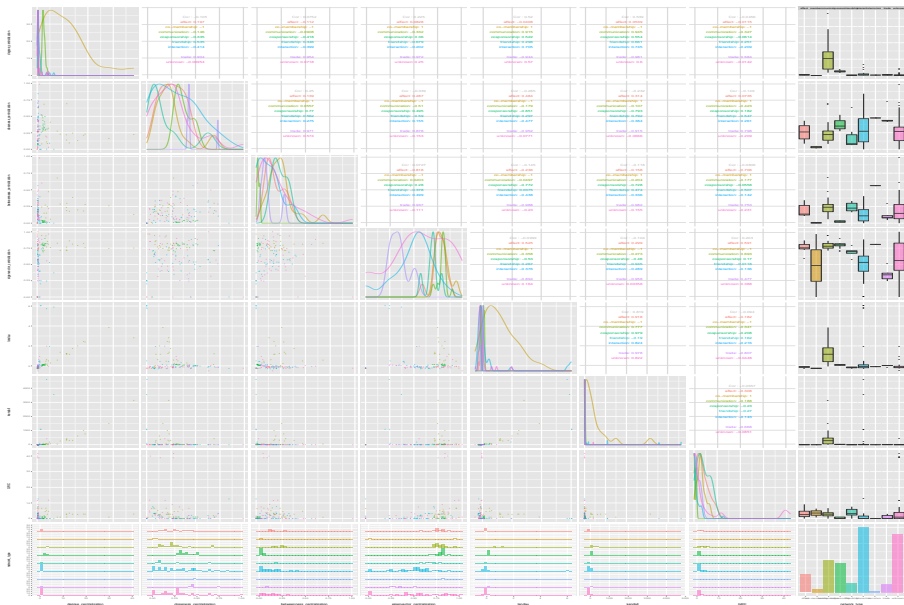


# Hierarchy in Networks–Datasets

- 141 datasets from UCINET, organizational emails, and Congress

Type	Nodes	Edges	Density	Cluster Coeff.
Communication	25.714	2419	3.483	0.558
Cosponsorship	101.222	13358.889	1.317	0.788
Co-membership	22	9267.333	18.318	0.377
Interaction	23.075	1944.95	1.985	0.589
Unknown	76.833	4688.25	0.360	NaN
Friendship	29.833	92	0.122	0.348
Affect	17.636	95.182	0.319	0.329
Terrorism	63	308	0.079	0.361
Trade	24	285.6	0.5170	0.734

# Hierarchy in Networks – Global Measures

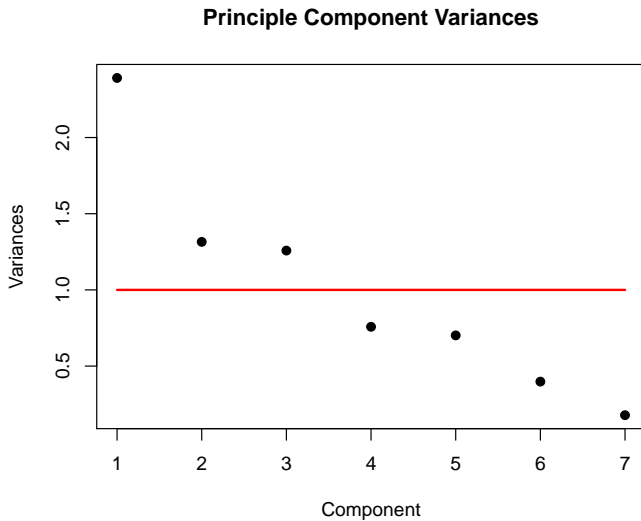


# Hierarchy in Networks–Results

- ▶ Landau and Kendall's measures have a correlation of 0.819, but all others are below 0.54
- ▶ Correlation for co-membership and trade networks are the highest across all measures.

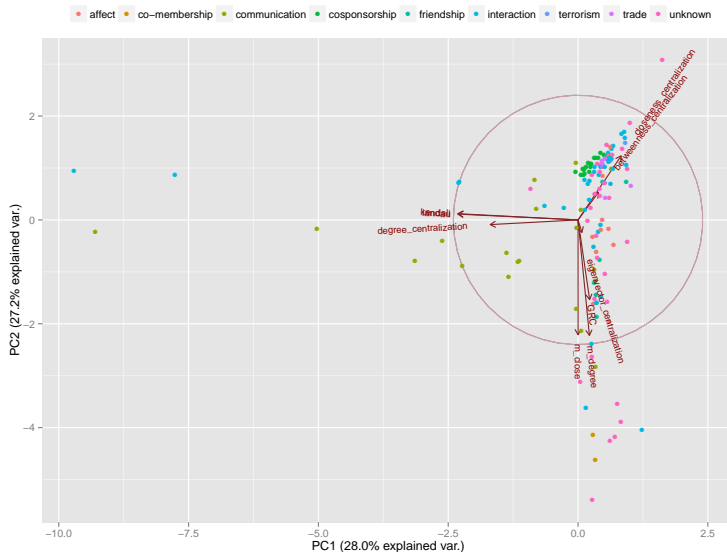


# Hierarchy in Networks – PCA



# Hierarchy in Networks – PCA

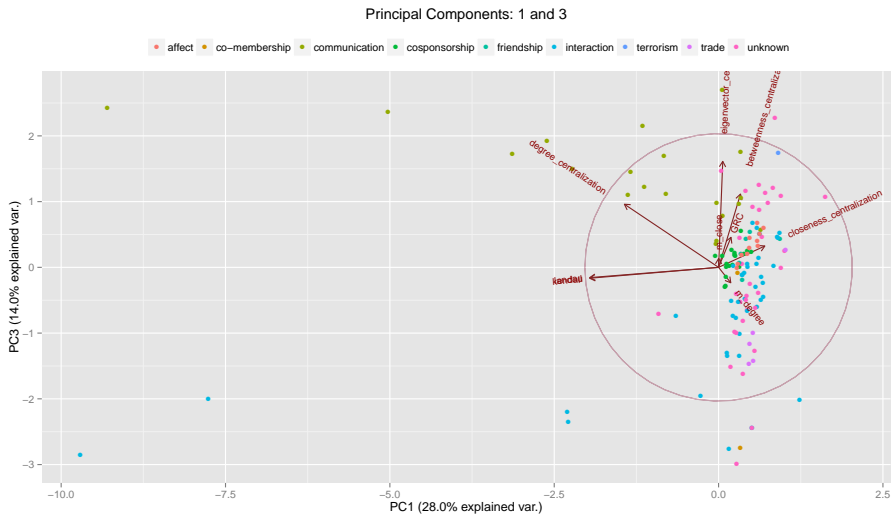
Principal Components: 1 and 2



# Interpretation for Components 1 and 2

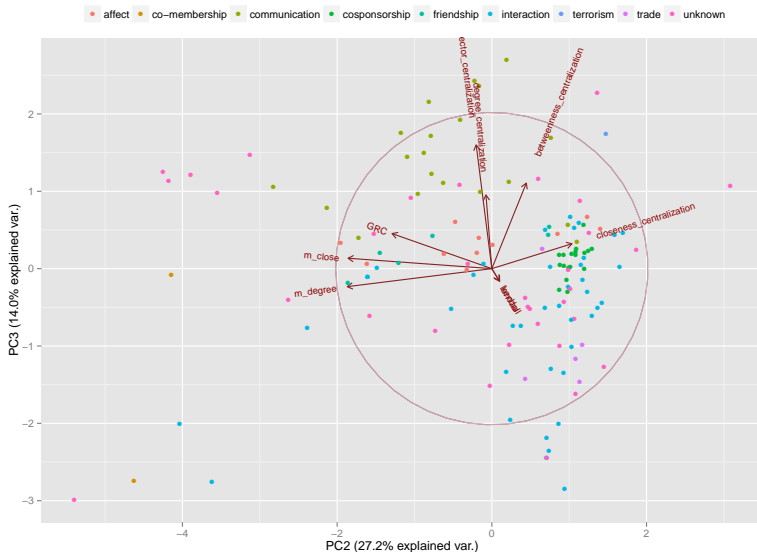
- ▶ These two components explain over half of the variation
- ▶ There is a significant dropoff in the variation accounted for after PC2
- ▶ three distinct groupings:
  - Control** Kendall's K, Landau's H, Degree Centralization
  - Reach** M Degree, M Close, GRC
  - Centrality** ...we'll come back to this

# Hierarchy in Networks – PCA



# Hierarchy in Networks – PCA

Principal Components: 2 and 3



## Interpretation for Components 2 and 3

- ▶ Component 3 is primarily Eigenvector centralization
- ▶ the rest of component 3 is split between the various centralization measures
- ▶ back to centrality...
- ▶ the centrality measures are split evenly between the components
- ▶ we believe this is because centralization is a *related* but *distinct* concept from hierarchy

# Hierarchy in Networks–Results

- ▶ Need first three principle components to explain the variation.
- ▶ Both trade and co-membership stay grouped and towards the center.
- ▶ Both Landau and Kendall stay in same direction.
- ▶ From the first PCA plot (PC1 PC2), we notice that there are three clear groupings of measures.
- ▶ we believe these correspond to two distinct models of hierarchy: dictatorial control, and decentralized reach