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Hierarchical Structure in Social Networks

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Abstract

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1. Introduction

2. Measuring Hierarchy

2.1. Measuring Hierarchy in Groups

2.2. Measuring Hierarchy in Networks

3. A Model of Network Hierarchy

3.1. Measures of Hierarchy

1. Landau's $h \in [0, 1]$ and Kendall's $K \in [0, 1]$ are both used to compare a network to a perfect linear hierarchy in [1], where $i = 1...N$ is the number of nodes and S_i is the row sum for each node also referred to as the dominance total:

$$h = \frac{12}{N^3-N} \sum_{i=1}^{N} [S_i - \frac{N-1}{2}],$$

IF we let d be the number of cyclic triads defined as: $d = \frac{N(N-1)(2N-1)}{12} - \frac{1}{2} \sum S_i^2$. Then

$$K = 1 - \frac{d}{d_{max}}, \text{ where}$$

$$d_{max} = \begin{cases} \frac{1}{24}(N^3 - N) & \text{if } N \text{ is odd} \\ \frac{1}{24}(N^3 - 4N) & \text{if } N \text{ is even} \end{cases}$$

2. Triangle transitivity is shown to be higher in dominance relationships in [1], but not used as a measure here.

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3. Treeness $T \in [-1, 1]$ is the average of $f(G)$ over the set in $W(G)$, where $W(G)$ is the subset G_C and all of its subsets obtained through a leaf removal algorithm. This measure is defined in [2].

$$f(G) = \frac{H_f(G_C) - H_b(G_C)}{max\{H_f(G_C), H_b(G_C)\}},$$

where H_f, H_b denote the forward and backward path entropies, respectively, where $h_f(v_i) = -\sum P(\pi_k|v_i)logP(\pi_k|v_i)$.
4. Feedforwardness $F \in [0, 1]$ is the average of path weights $F(\pi_k)$ where cyclic modules that are closer to the top get a higher penalty, and defined by [2]. Here the paths under consideration are all paths starting at from the top of G_C denoted π_k. Let $k = 1, ...M$ be this number of paths then:

$$F(G) = \frac{1}{M} \sum_{k=1}^{M} \frac{|v(\pi_k)|}{\sum a_i},$$

where a_i are the weights of each node along the path under consideration (i.e. the number of collapsed nodes from G in the corresponding node of G_C), and $v(\pi_k)$ is the number of nodes along the path π_k.
5. Orderability $O \in [0, 1]$ is the fraction of nodes that do not belong to any cycle and defined by [2]:

$$O(G) = \frac{|v_i \in V_c \cap V|}{|V|}$$

6. Global Reaching Centrality where the graph is unweighted and directed is defined by [3] as:

$$GRC = \frac{\sum_{i \in V} [C_R^{max} - C_R(i)]}{N - 1},$$

where $C_R(i)$ is the local reaching centrality defined as the proportion of all nodes in G that can be reached along outgoing edges from node i.
7. Global Reaching Centrality where the graph is weighted and directed is defined by [3] using the following version for the reaching centrality:

$$C_R^{'}(i) = \frac{1}{N - 1} \sum_{j:0<d^{out}_{(i,j)}<\infty} \left(\frac{\sum_{k=1}^{d^{out}(i,j)} w_i^{(k)}(j)}{d^{out}(i, j)}\right)$$

8. Global Reaching Centrality where the graph is unweighted and undirected is defined by [3] using the following version for the reaching centrality:

$$C_R^{''}(i) = \frac{1}{N - 1} \sum_{j:0<d(i,j)<\infty} \frac{1}{d(i, j)}$$

9. Control Centrality in a weighted and directed graph, defined by [4], identifies the minimum number of nodes need to drive an entire network to a given final state. Consider a directed, weighted network:

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$$

which is the state of each node at time t, and also denoted as $(\boldsymbol{A}, \boldsymbol{B})$. The components of this controllability matrix are: $\boldsymbol{A} \in \mathbb{R}^{N \times N}$, where each element gives the strength that node j can affect node i; and $\boldsymbol{B} \in \mathbb{R}^{N \times M}$, where each element is the strength between the input signal $u_j(t)$ and node i, and M contains independent signals imposed by an outside controller. Defining $\boldsymbol{C} = (\boldsymbol{A},\boldsymbol{B})$, the control centrality of node i is:

$$C_c(i) \equiv rank_g(\boldsymbol{C}^i)$$

4. Data

5. Analysis

6. Conclusions

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