

Hierarchy in networks–Measures

Measure	Undirected	Weighted	Global	Local
Landau's h		X	X	
Kendall's K	X	X		X
Reach Degree	X	X		X
Reach Closeness	X	X		X
GRC	X	X	X	X
Rooted Depth			X	
Degree	X	X	X	X
Closeness	X	X	X	X
Betweenness	X	X	X	X
Eigenvector	X	X	X	X

Hierarchy in Networks—Simulated Datasets

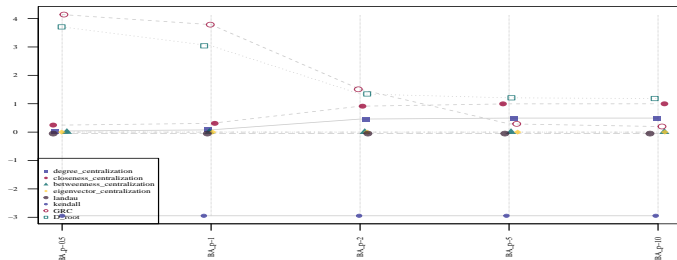
- ▶ 7500 Barabasi-Albert (BA) Datasets with node size 50, 200, or 500 and preferential attachment parameter 0.5, 1, 2, 5, or 10.
- ▶ 6000 Tree-Structured (TR) Datasets with node size 50, 200, or 500 and children parameter 2, 5, 10, or 50.
- ▶ 4500 Erdos-Renyi (ER) Datasets with node size 50, 200, or 500 and parameter size 0.05, 0.1, or 0.2.

BA and TR	Node	Edge	Density
	50	49	0.02
	200	199	0.005
	500	499	0.002

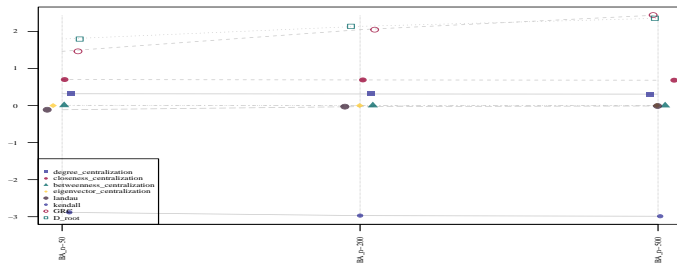
ER	Node	Parameter	Edge	Density	Cluster Coefficient
	50	0.05	122.524	0.050	0.0956
	50	0.1	244.768	0.100	0.188
	50	0.2	490.050	0.200	0.359
	200	0.05	1989.242	0.050	0.098
	200	0.1	3980.776	0.100	0.190
	200	0.2	7952.138	0.120	0.360
	500	0.05	12473.270	0.050	0.097
	500	0.1	24932.824	0.100	0.190
	500	0.2	49892.784	0.200	0.360

Average Global Hierarchy-BA Networks

Parameters

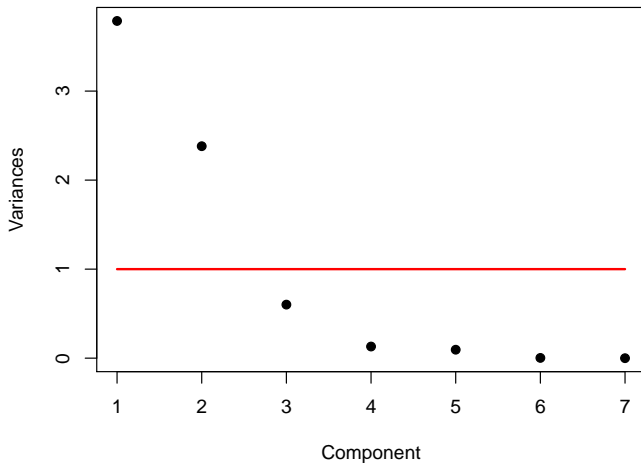


Node Size



PCA-BA Networks

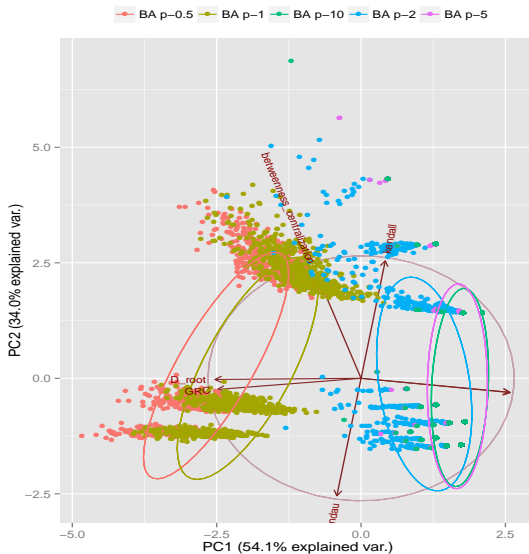
Principle Component Variances



PCA-BA Networks

Parameters

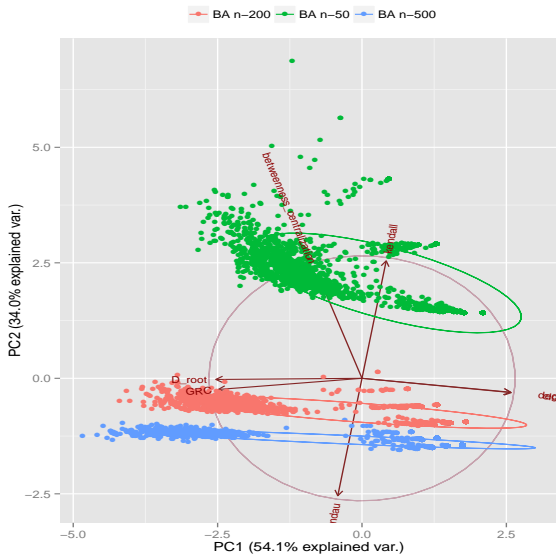
Principal Components: 1 and 2



PCA-BA Networks

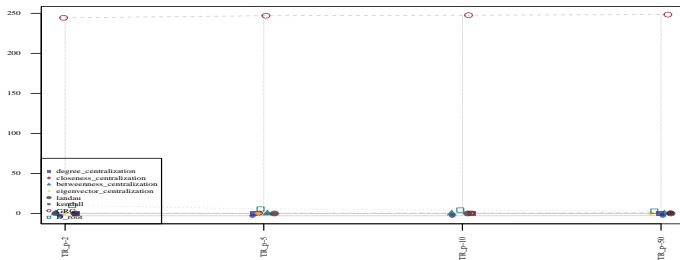
Node Size

Principal Components: 1 and 2

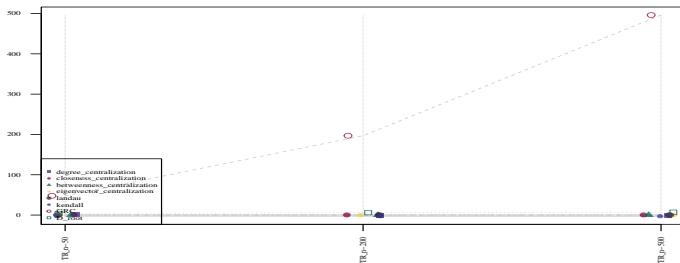


Average Global Hierarchy-TR Networks

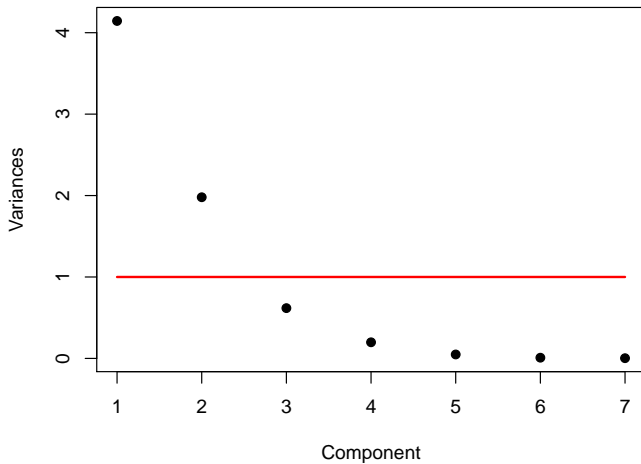
Parameters



Node Size



Principle Component Variances

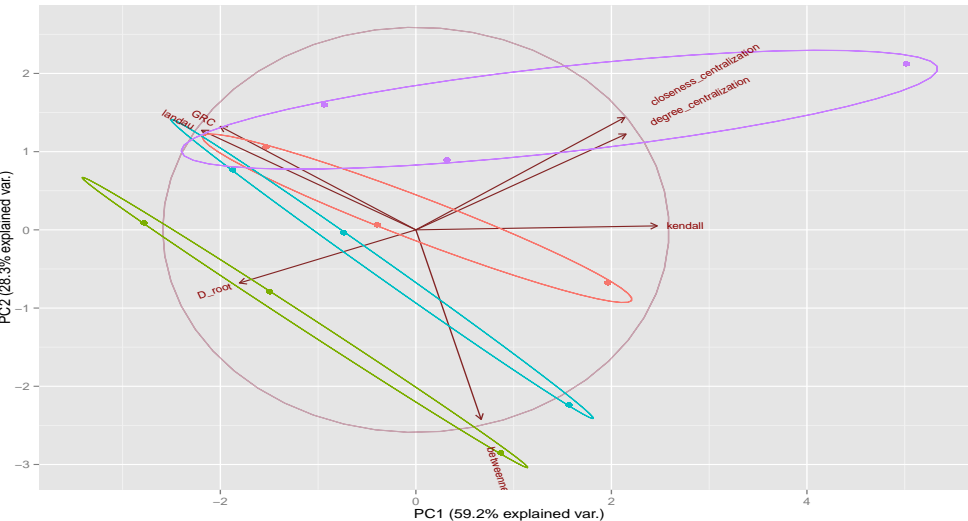


PCA-TR

Parameters

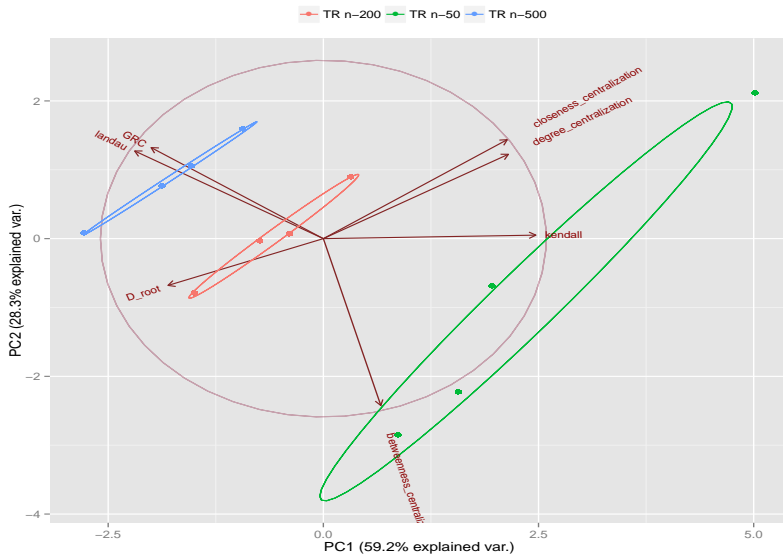
Principal Components: 1 and 2

TR p=10 TR p=2 TR p=5 TR p=50



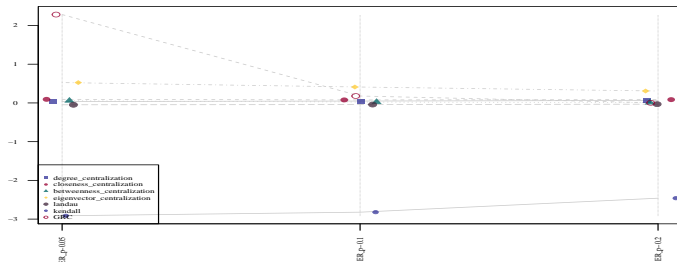
Node Size

Principal Components: 1 and 2

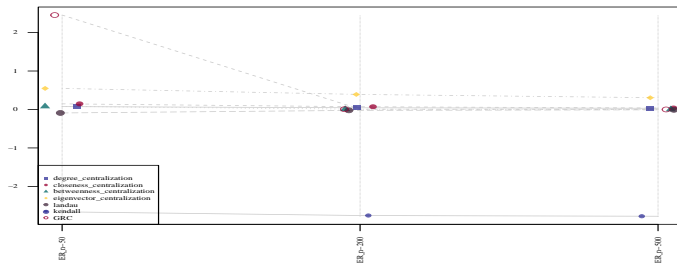


Average Global Hierarchy–ER Networks

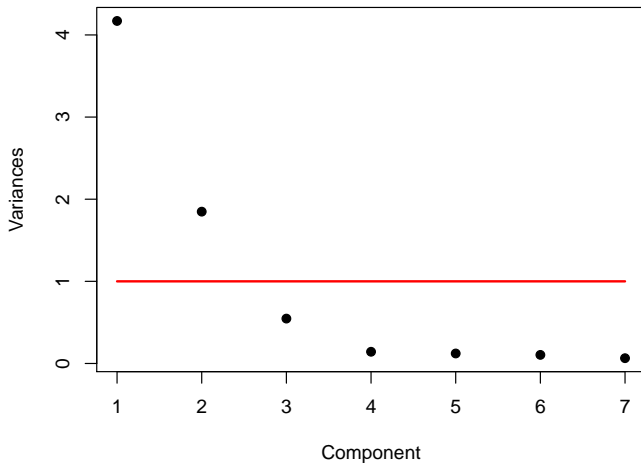
Parameters



Node Size



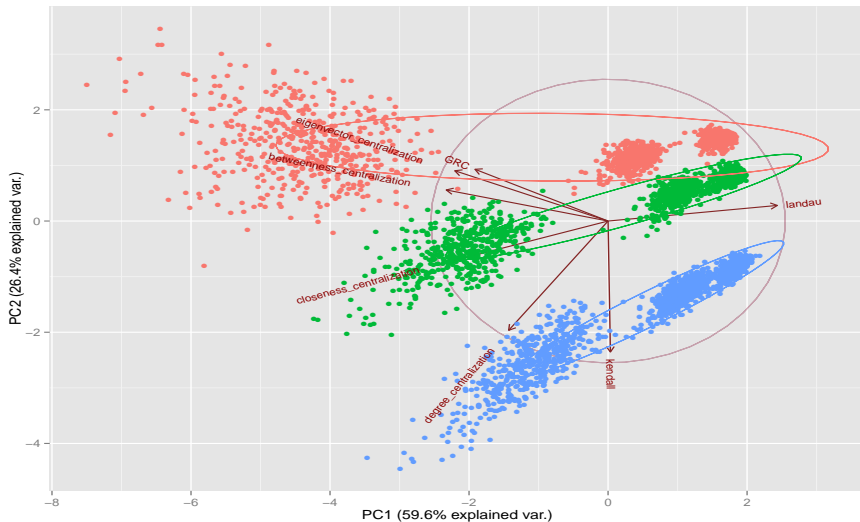
Principle Component Variances



Parameters

Principal Components: 1 and 2

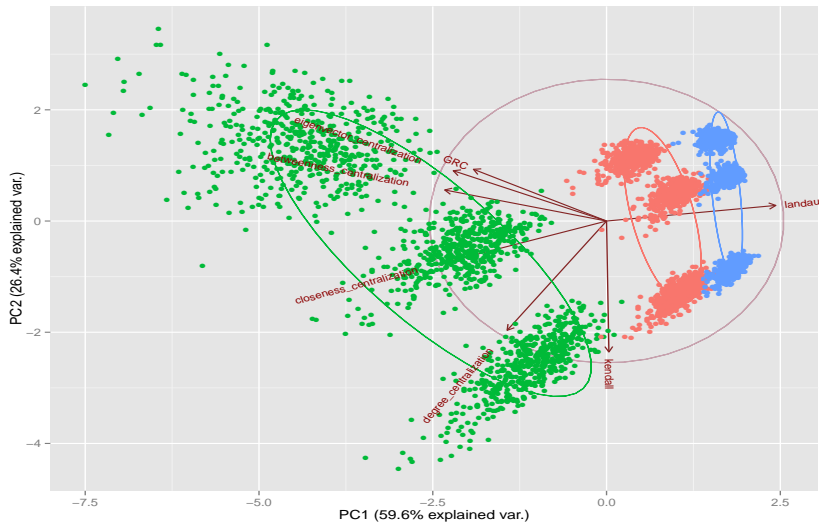
ER p=0.05 ER p=0.1 ER p=0.2



Node Size

Principal Components: 1 and 2

ER n=200 ER n=50 ER n=500



Hierarchy in Networks–Results

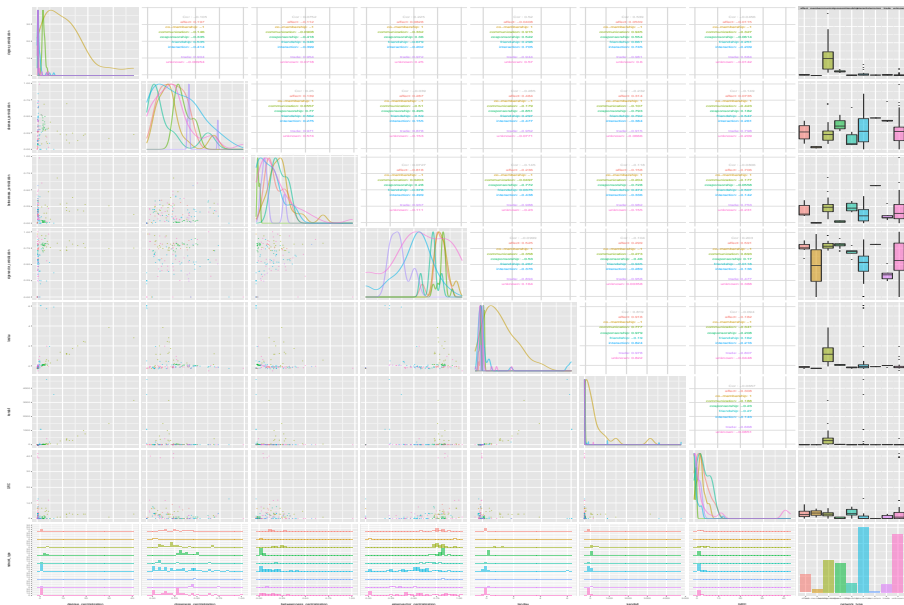
- ▶ GRC and D Root are not robust to changes in node size.
 - ▶ GRC increases with node size for both BA and TR networks
 - ▶ GRC decreases with node size for ER networks.
 - ▶ D Root increases with node size for BA networks.
- ▶ Only two principle components are needed.
 - ▶ BA
 - ▶ TR
 - ▶ ER

Hierarchy in Networks–Datasets

- 141 datasets from UCINET, organizational emails, and Congress

Type	Nodes	Edges	Density	Cluster Coeff.
Communication	25.714	2419	3.483	0.558
Cosponsorship	101.222	13358.889	1.317	0.788
Co-membership	22	9267.333	18.318	0.377
Interaction	23.075	1944.95	1.985	0.589
Unknown	76.833	4688.25	0.360	NaN
Friendship	29.833	92	0.122	0.348
Affect	17.636	95.182	0.319	0.329
Terrorism	63	308	0.079	0.361
Trade	24	285.6	0.5170	0.734

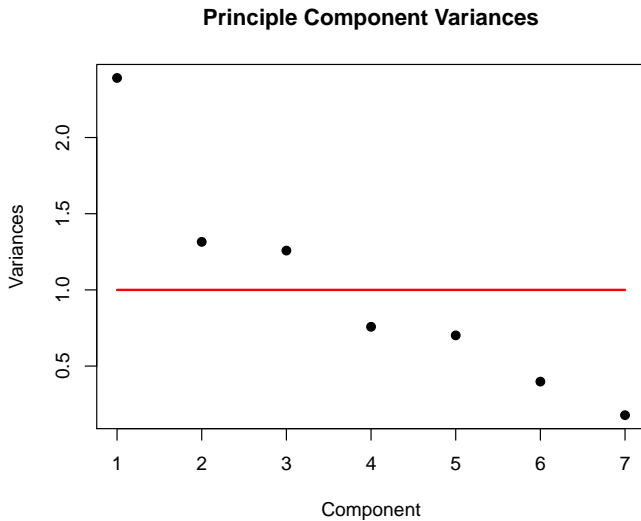
Hierarchy in Networks – Global Measures



Hierarchy in Networks–Results

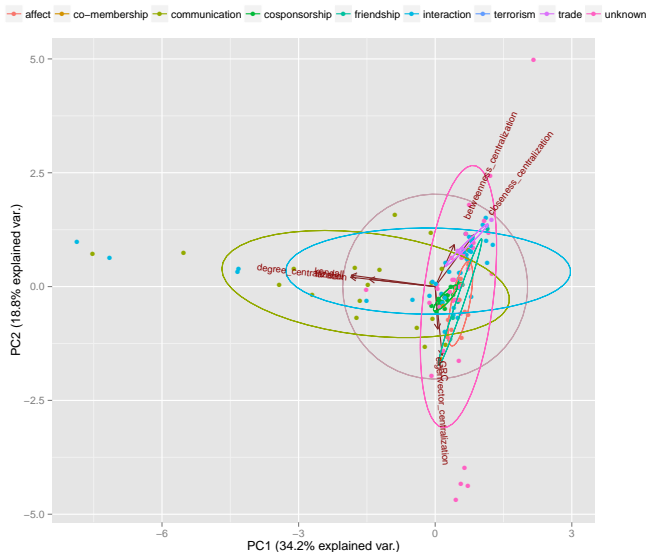
- ▶ Landau and Kendall's measures have a correlation of 0.819, but all others are below 0.54
- ▶ Other from above plot...

Hierarchy in Networks – PCA

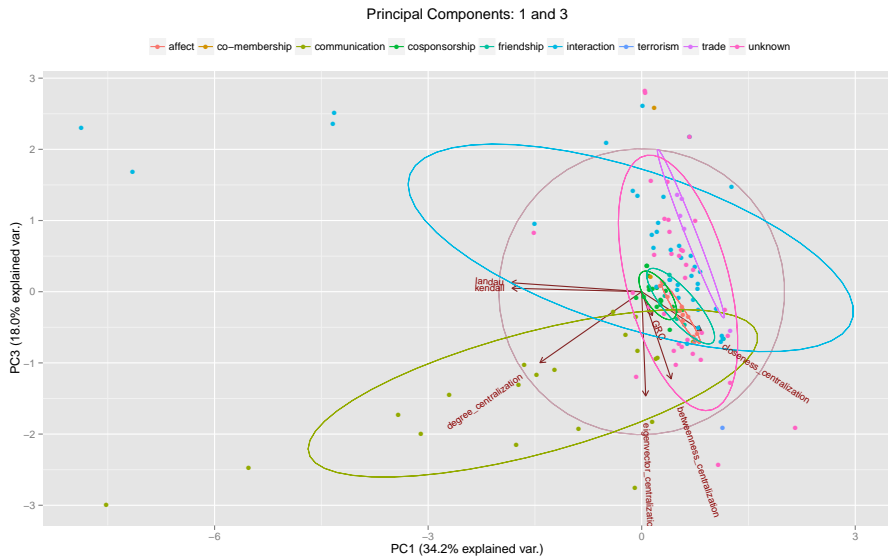


Hierarchy in Networks – PCA

Principal Components: 1 and 2

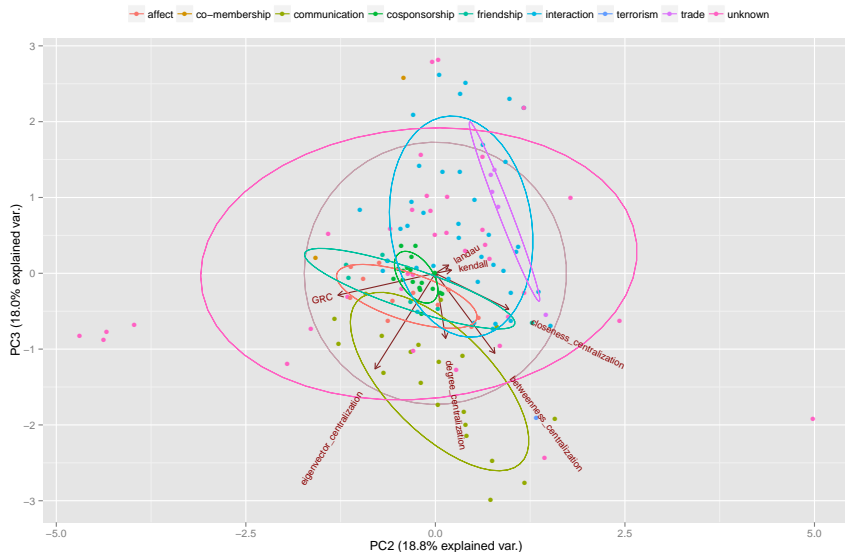


Hierarchy in Networks – PCA



Hierarchy in Networks – PCA

Principal Components: 2 and 3



Hierarchy in Networks–Results

- ▶ Need first three principle components to explain the variation.
- ▶ more...
- ▶ All measures are not created equal.