Hierarchy in networks–Measures

Measure	Undirected	Weighted	Global	Local
Landau's h		X	X	
Kendall's K	X	X		X
Reach Degree	X	X		X
Reach Closeness	X	X		X
GRC	X	X	X	X
Rooted Depth			X	
Degree	X	X	X	X
Closeness	X	X	X	X
Betweenness	X	X	X	X
Eigenvector	X	X	X	X

Hierarchy in Networks–Simulated Datasets

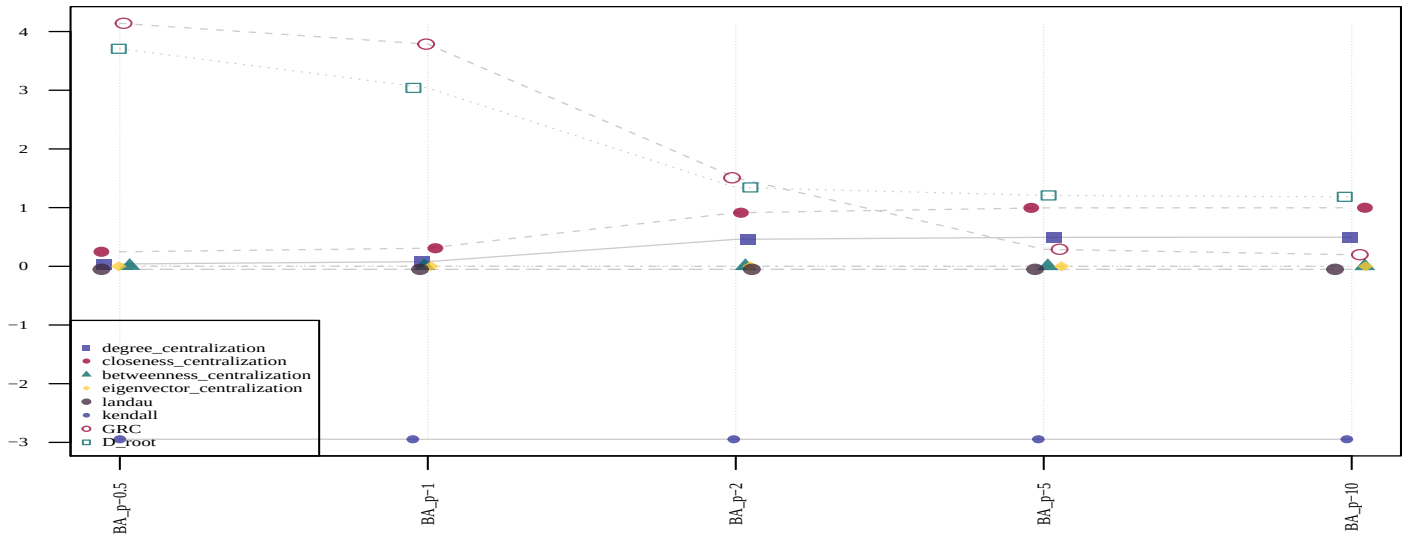
- 7500 Barabasi-Albert (BA) Datasets with node size $50, 200,$ or 500 and preferential attachment parameter $0.5, 1, 2, 5,$ or 10.
- 6000 Tree-Structured (TR) Datasets with node size $50, 200,$ or 500 and children parameter $2, 5, 10,$ or 50.
- 4500 Erdos-Renyi (ER) Datasets with node size $50, 200,$ or 500 and parameter size $0.05, 0.1,$ or 0.2.

BA and TR	Node	Edge	Density
	50	49	0.02
	200	199	0.005
	500	499	0.002

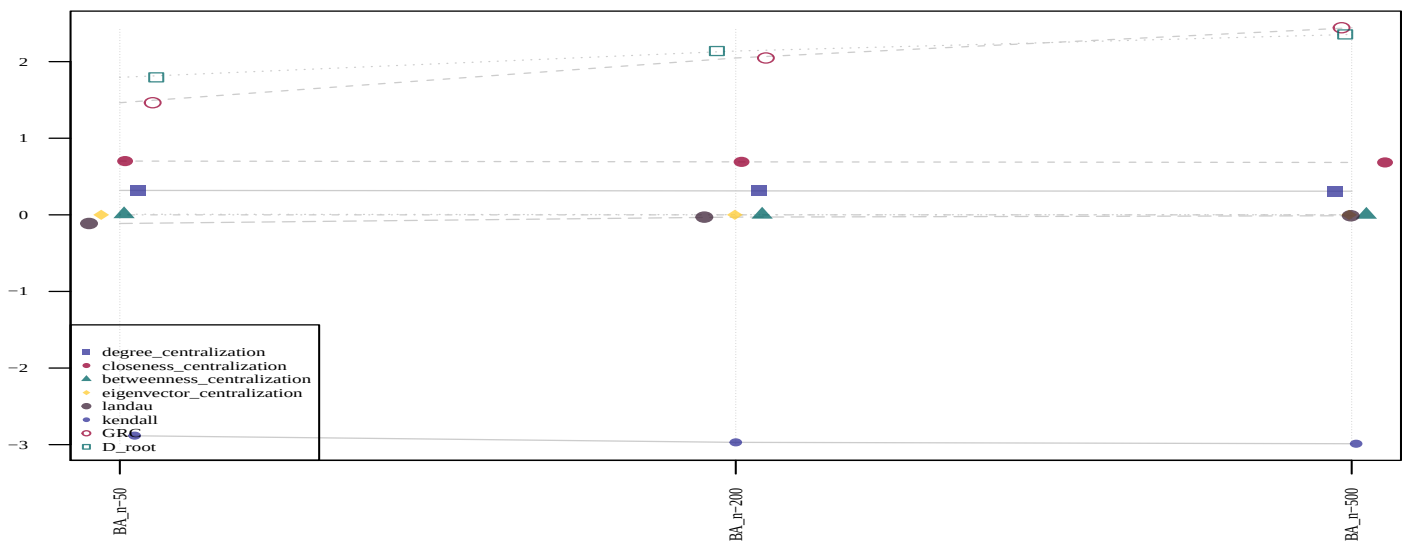
ER	Node	Parameter	Edge	Density	Cluster Coefficient
	50	0.05	122.524	0.050	0.0956
	50	0.1	244.768	0.100	0.188
	50	0.2	490.050	0.200	0.359
	200	0.05	1989.242	0.050	0.098
	200	0.1	3980.776	0.100	0.190
	200	0.2	7952.138	0.120	0.360
	500	0.05	12473.270	0.050	0.097
	500	0.1	24932.824	0.100	0.190
	500	0.2	49892.784	0.200	0.360

Average Global Hierarchy–BA Networks

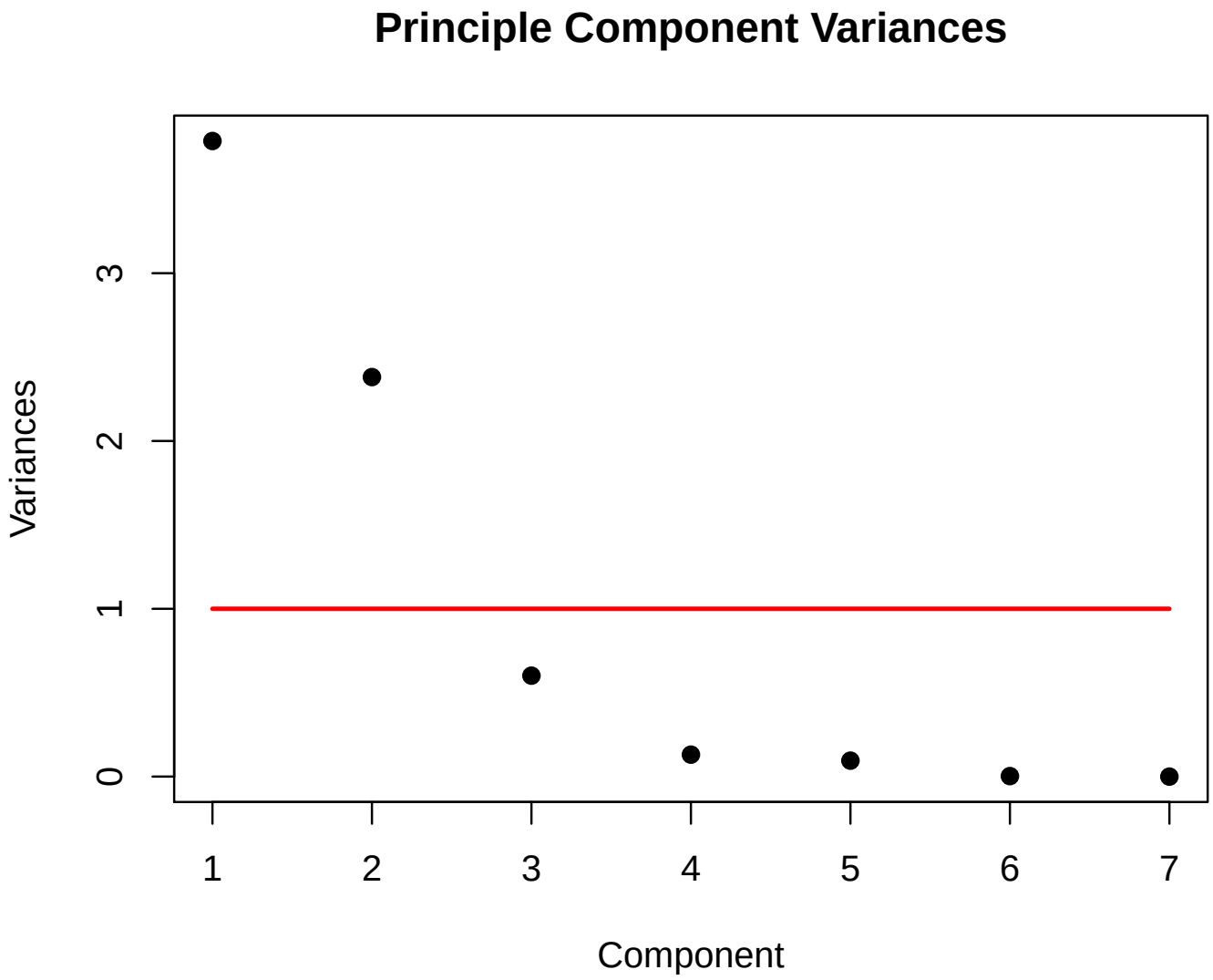
Parameters



Node Size

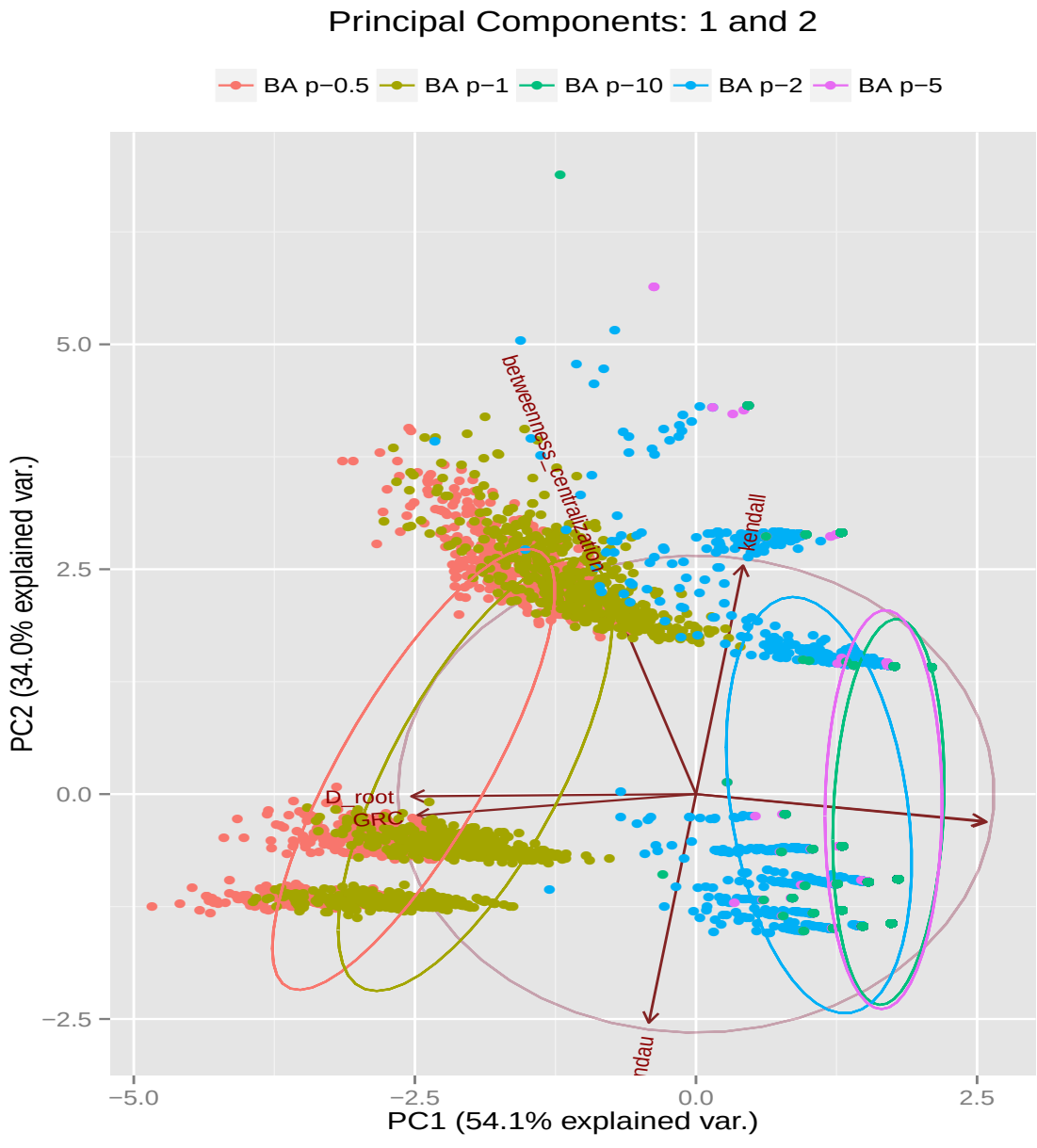


PCA–BA Networks



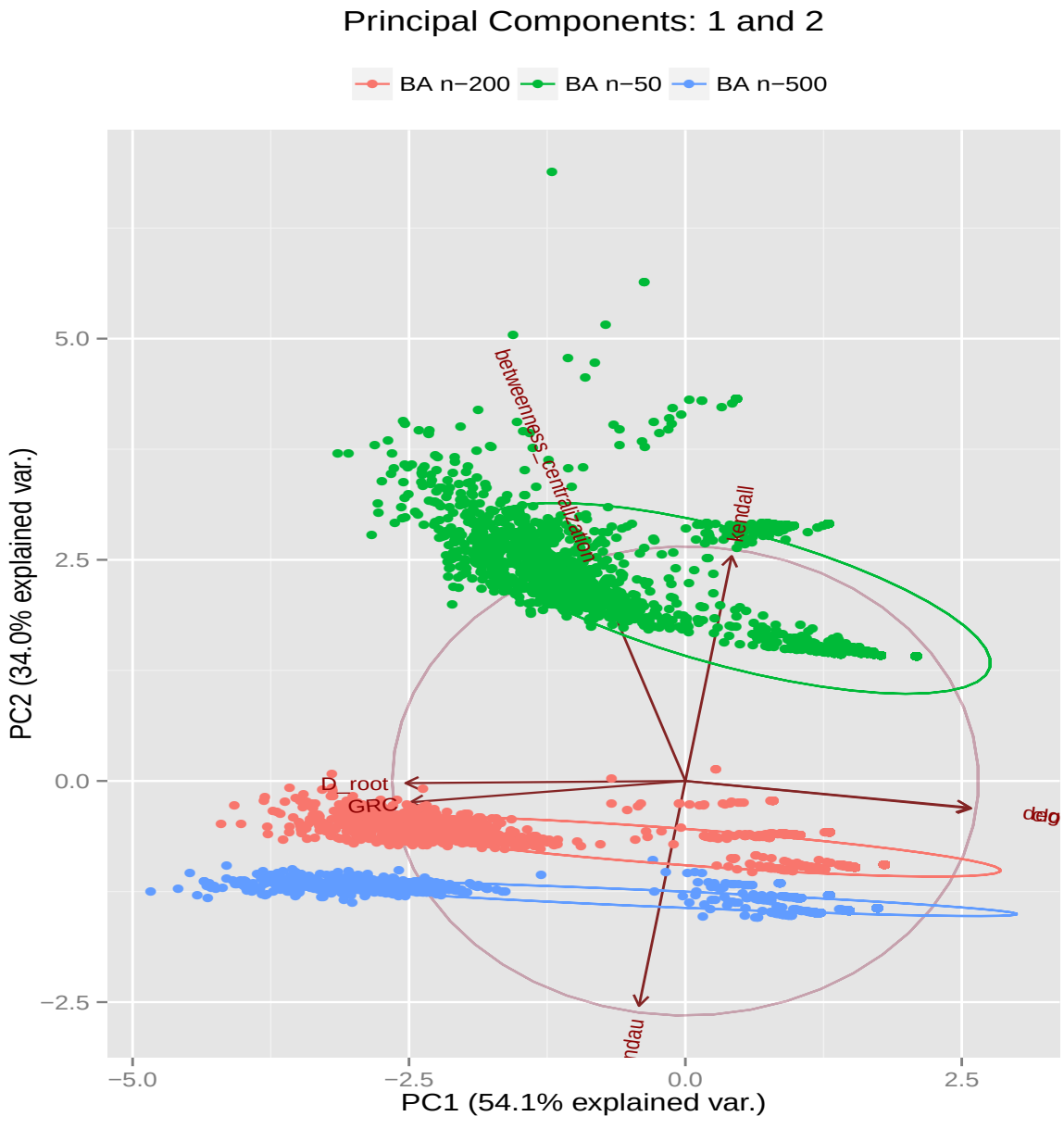
PCA–BA Networks

Parameters



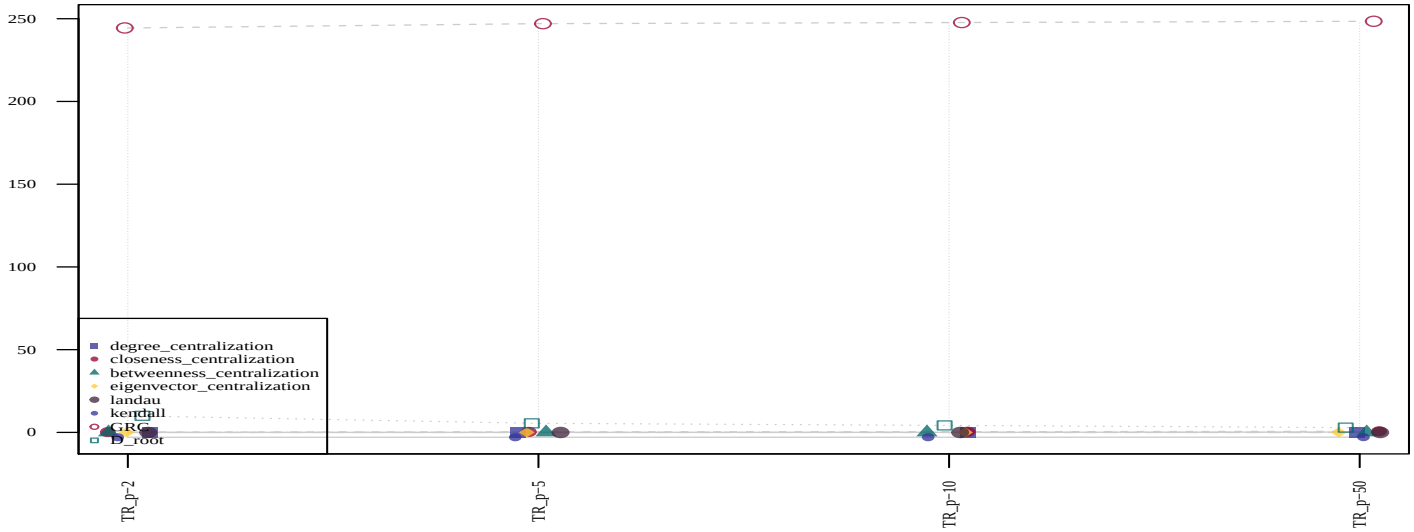
PCA–BA Networks

Node Size

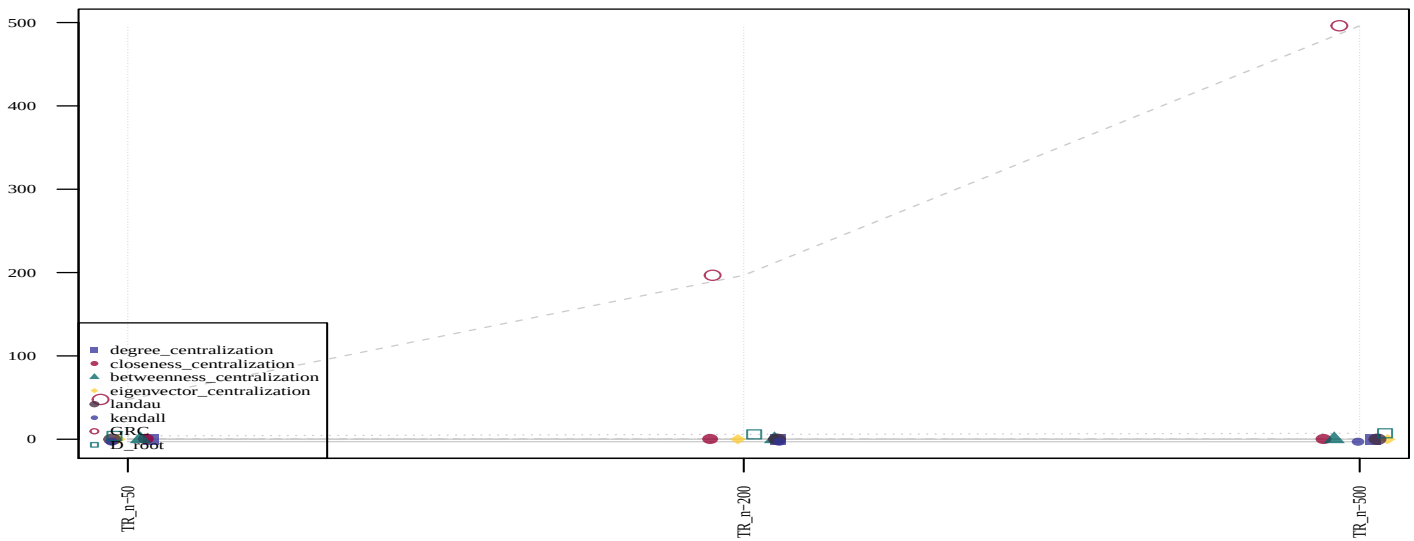


Average Global Hierarchy–TR Networks

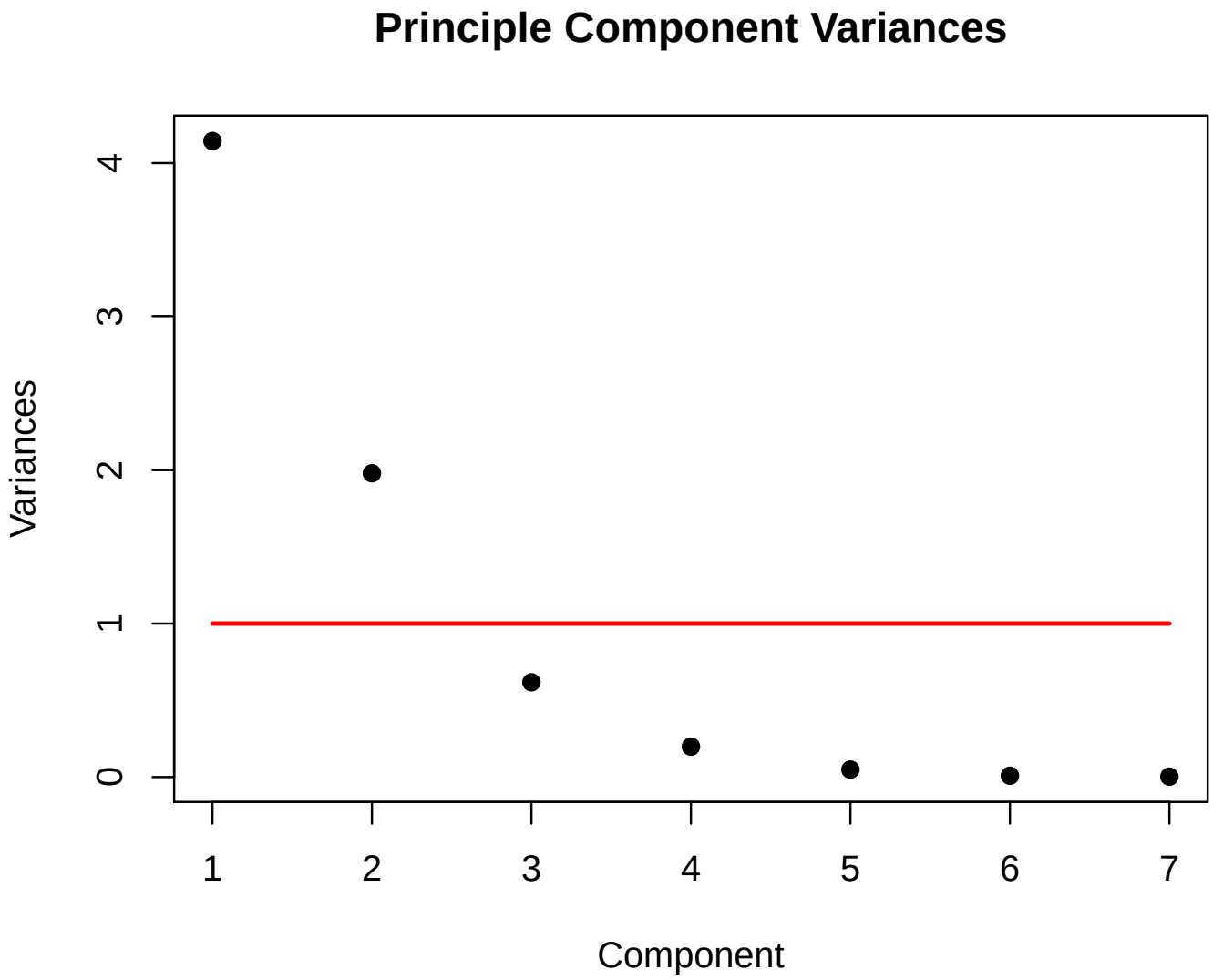
Parameters



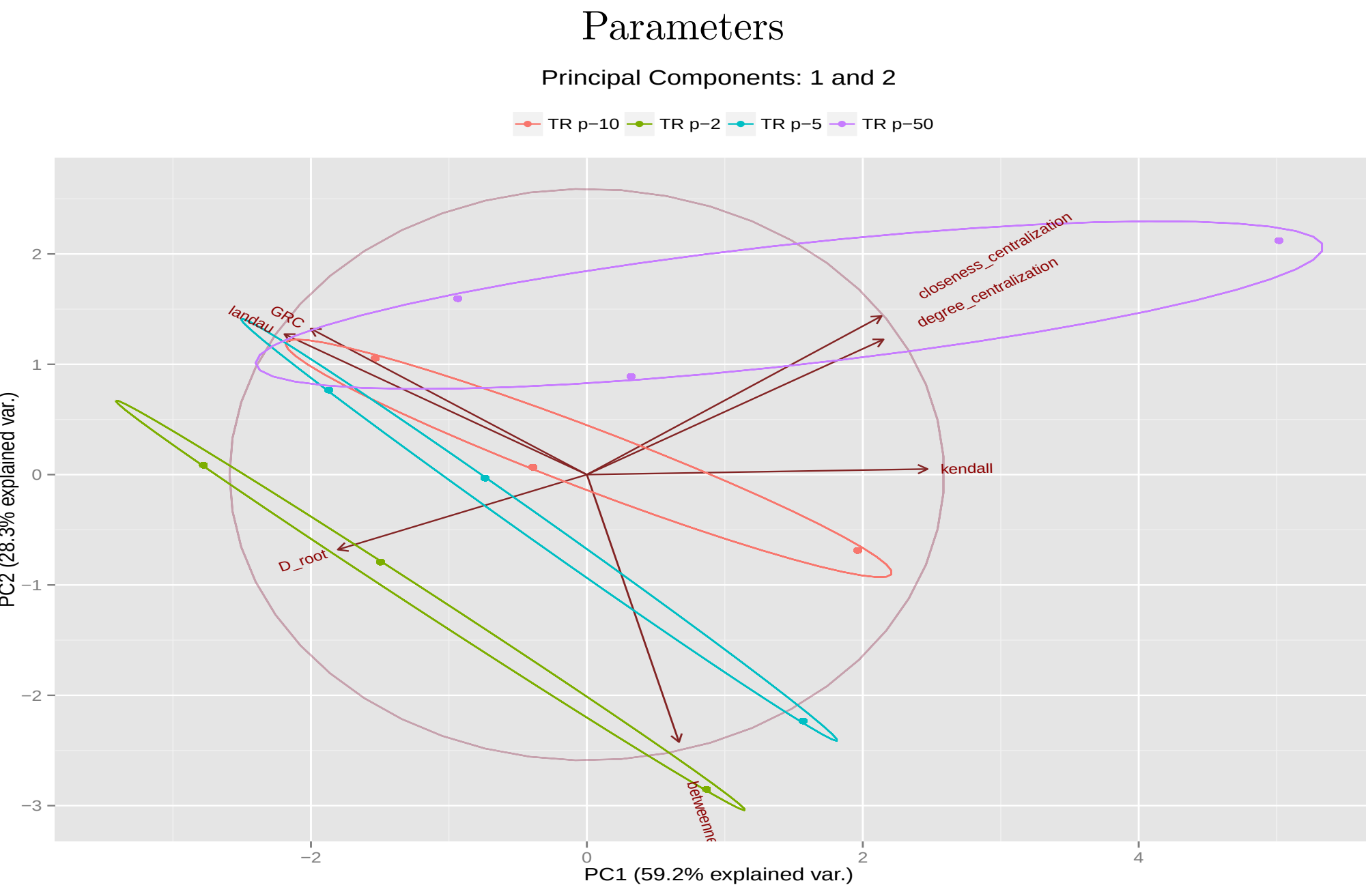
Node Size



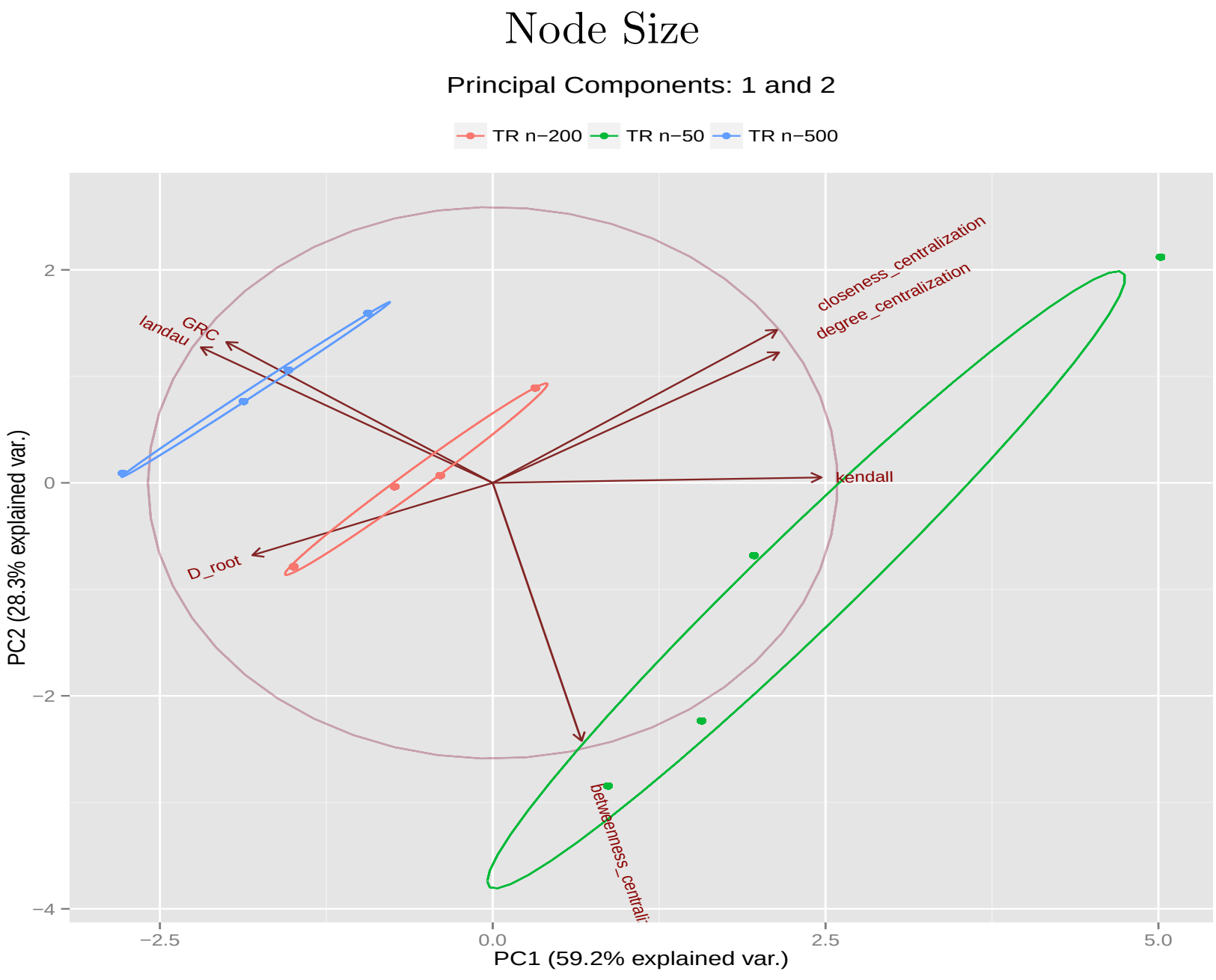
PCA–TR


Principle Component Variances
Variances
0
1
2
3
4
1
2
3
4
5
6
7
Component

PCA–TR

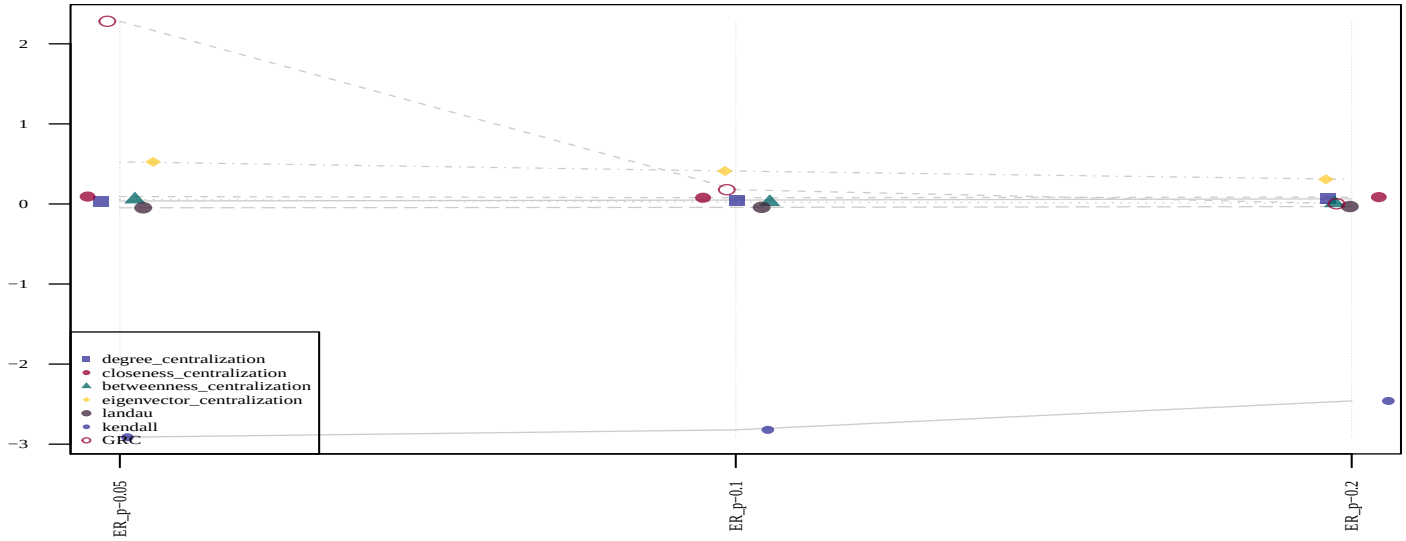

Parameters
Principal Components: 1 and 2
TR p−10
TR p−2
TR p−5
TR p−50
closeness_centralization
degree_centralization
landau
GRC
kendall
D_root
betweenne
PC2 (28.3% explained var.)
PC1 (59.2% explained var.)
2
1
0
−1
−2
−3
−2
0
2
4

PCA–TR

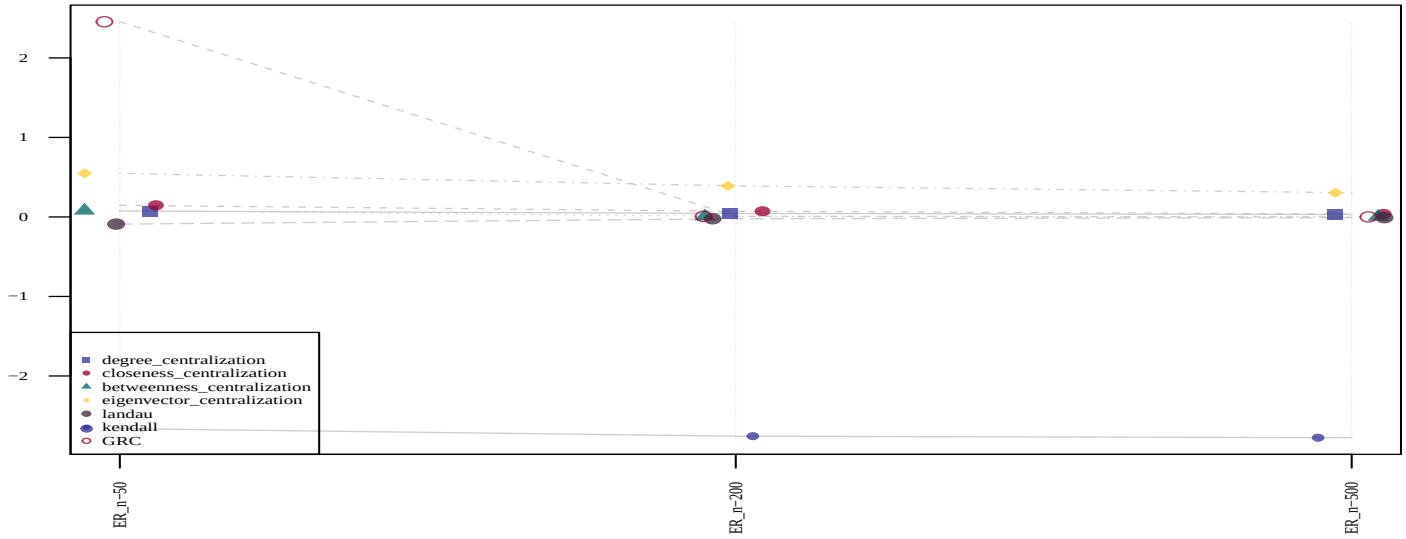

Node Size
Principal Components: 1 and 2
TR n−200
TR n−50
TR n−500
landau
GRC
D_root
kendall
closeness_centralization
degree_centralization
betweenness_centrali
PC2 (28.3% explained var.)
PC1 (59.2% explained var.)
2
0
−2
−4
−2.5
0.0
2.5
5.0

Average Global Hierarchy–ER Networks

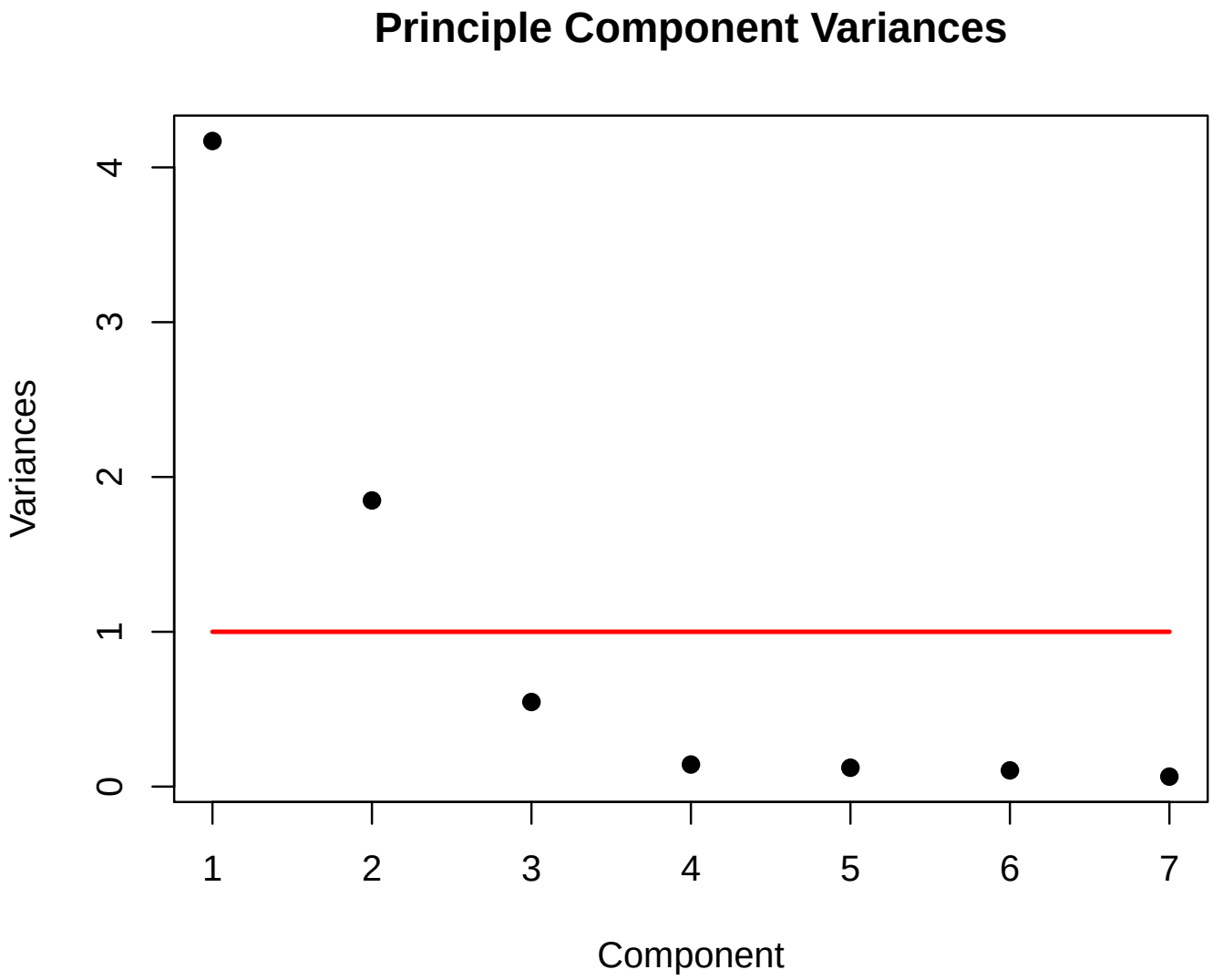
Parameters



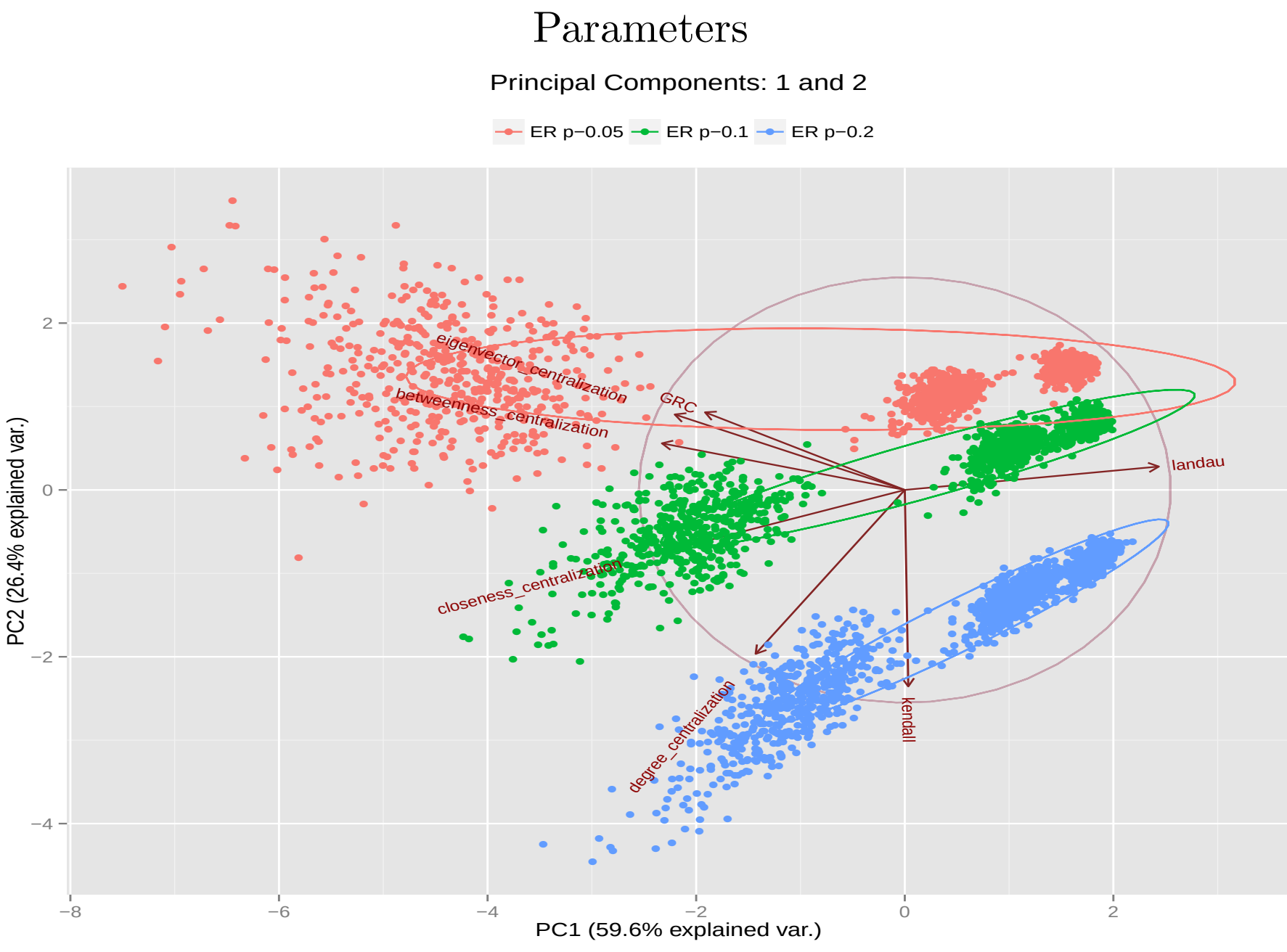
Node Size



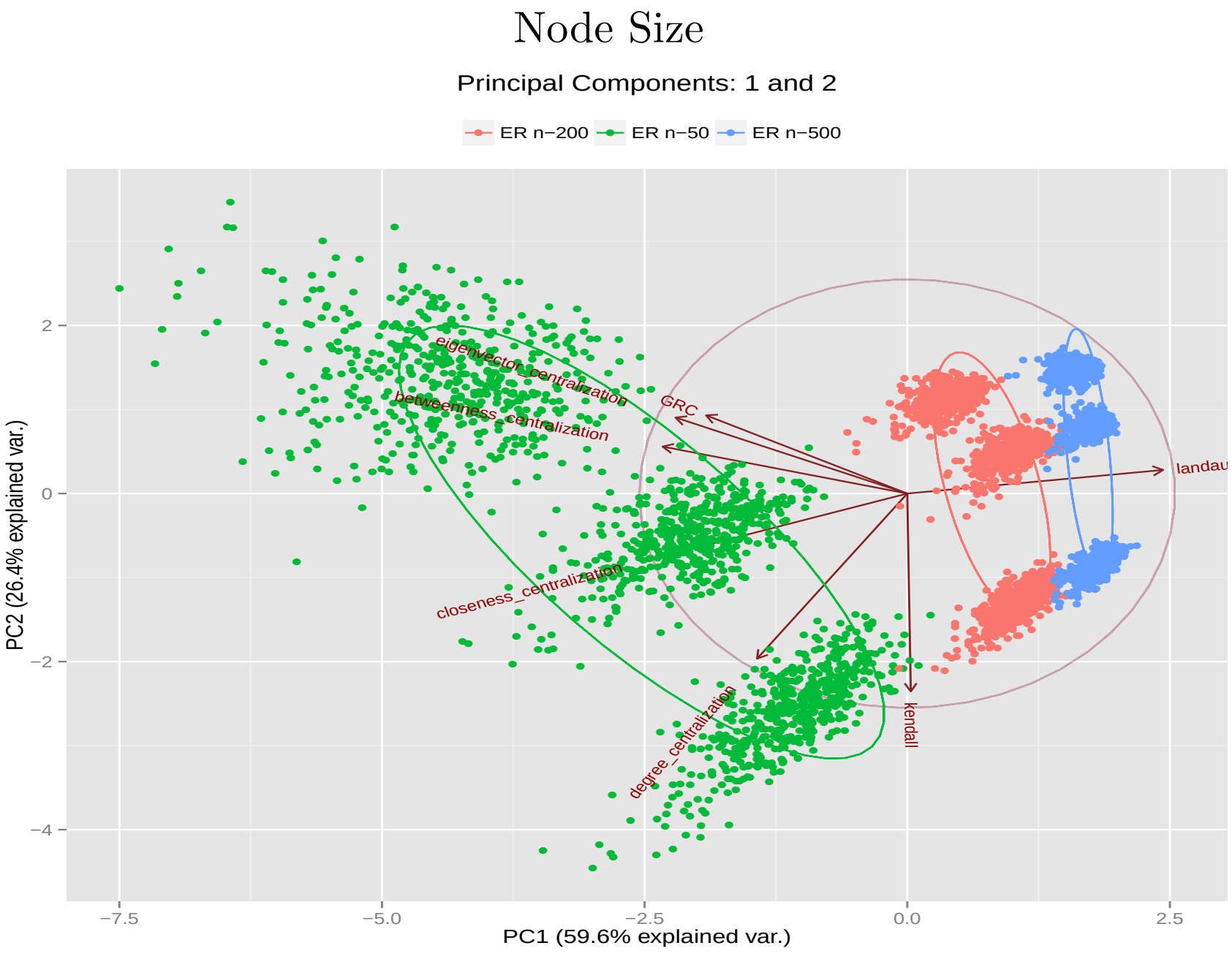
PCA–ER


Principle Component Variances
Variances
0
1
2
3
4
1
2
3
4
5
6
7
Component

PCA–ER


Parameters
Principal Components: 1 and 2
ER p−0.05
ER p−0.1
ER p−0.2
eigenvector_centralization
betweenness_centralization
GRC
landau
closeness_centralization
degree_centralization
kendall
PC2 (26.4% explained var.)
PC1 (59.6% explained var.)
2
0
−2
−4
−8
−6
−4
−2
0
2

PCA–ER


Node Size
Principal Components: 1 and 2
ER n−200
ER n−50
ER n−500
PC2 (26.4% explained var.)
PC1 (59.6% explained var.)
2
0
−2
−4
−7.5
−5.0
−2.5
0.0
2.5
eigenvector_centralization
betweenness_centralization
GRC
landau
closeness_centralization
kendall
degree_centralization

Hierarchy in Networks–Results

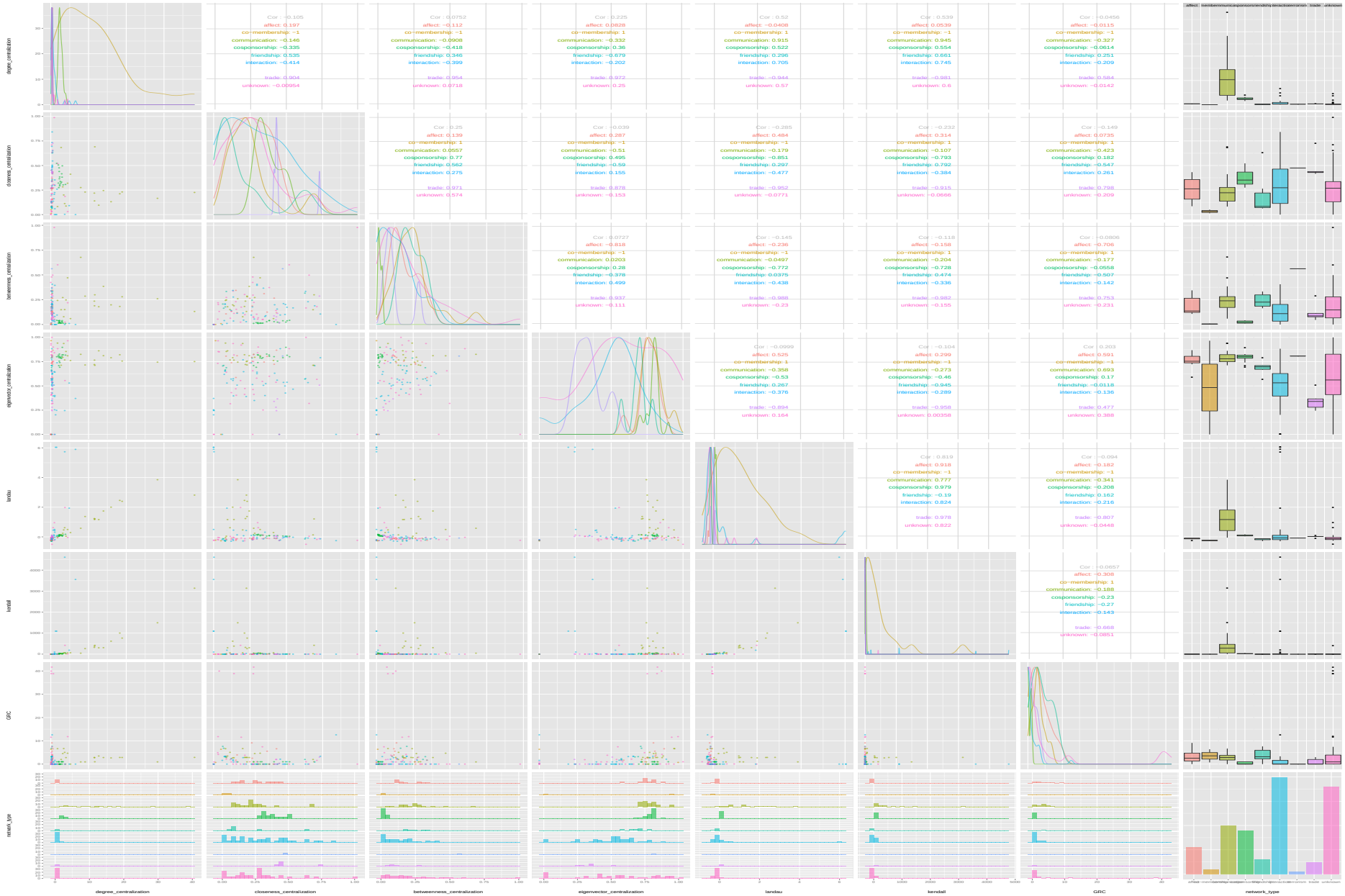
- GRC and D Root are not robust to changes in node size.
  - GRC increases with node size for both BA and TR networks
  - GRC decreases with node size for ER networks.
  - D Root increases with node size for BA networks.
- Only two principle components are needed.
  - For node size, very clear groupings and in all cases, not so for parameters.
  - Closeness and Degree Centrality stay close.
  - Betweenness and Eigenvector Centrality stay close.
  - Landau and Kendall stay opposite.

Hierarchy in Networks–Datasets

- 141 datasets from UCINET, organizational emails, and Congress

Type	Nodes	Edges	Density	Cluster Coeff.
Communication	25.714	2419	3.483	0.558
Cosponsorship	101.222	13358.889	1.317	0.788
Co-membership	22	9267.333	18.318	0.377
Interaction	23.075	1944.95	1.985	0.589
Unknown	76.833	4688.25	0.360	NaN
Friendship	29.833	92	0.122	0.348
Affect	17.636	95.182	0.319	0.329
Terrorism	63	308	0.079	0.361
Trade	24	285.6	0.5170	0.734

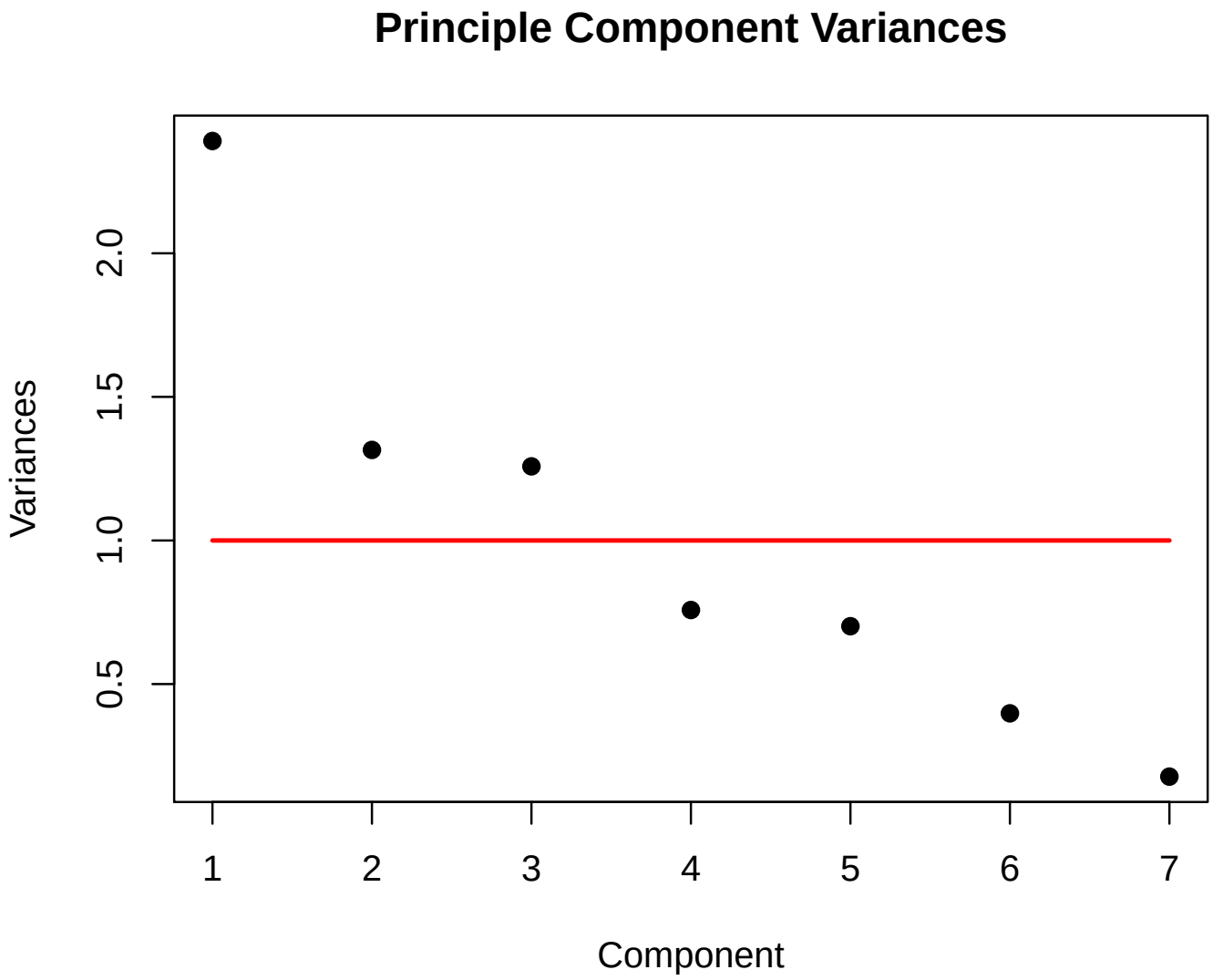
Hierarchy in Networks – Global Measures



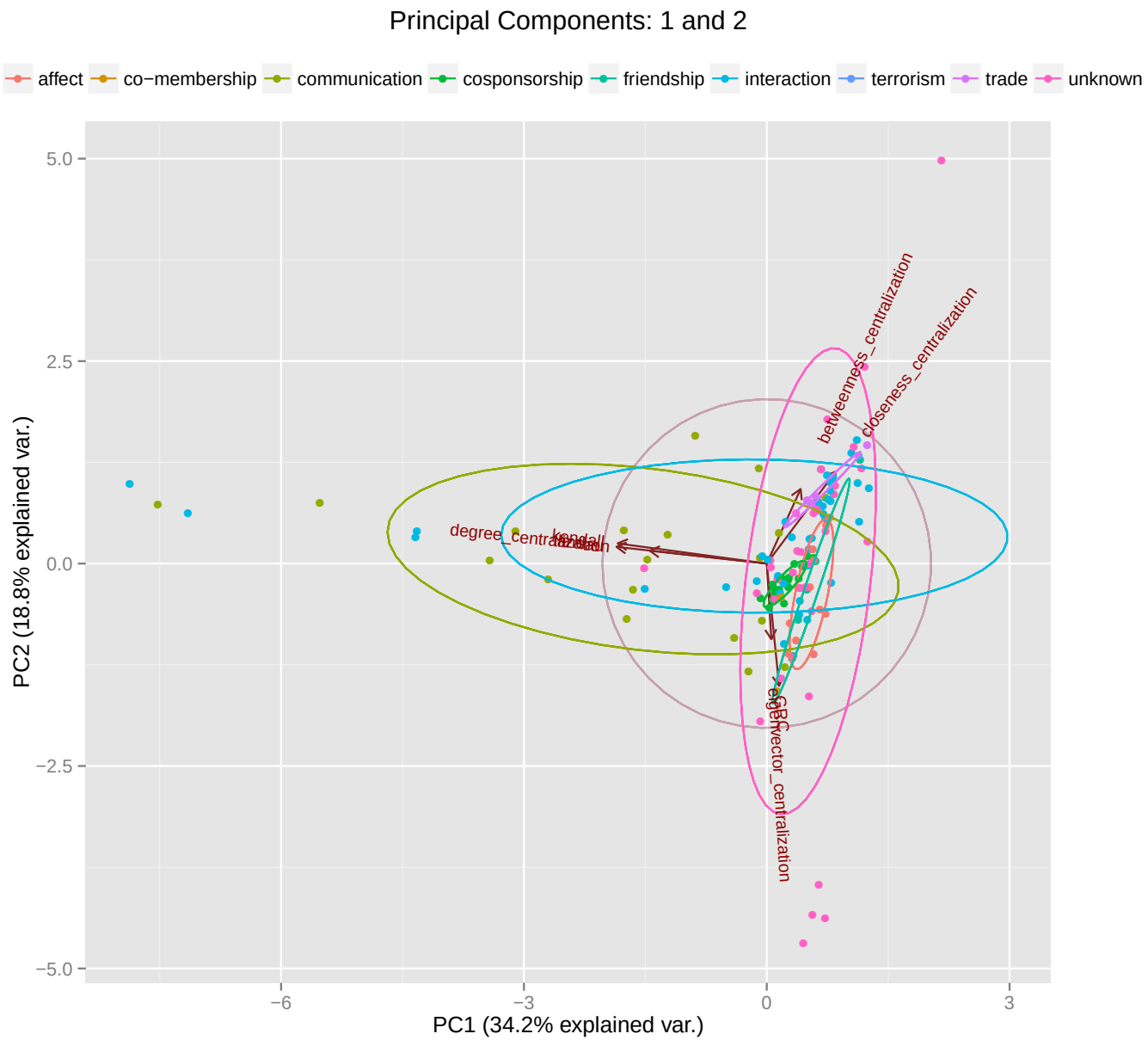
Hierarchy in Networks–Results

- Landau and Kendall's measures have a correlation of 0.819, but all others are below 0.54
- Correlation for co-membership and trade networks are the highest across all measures.

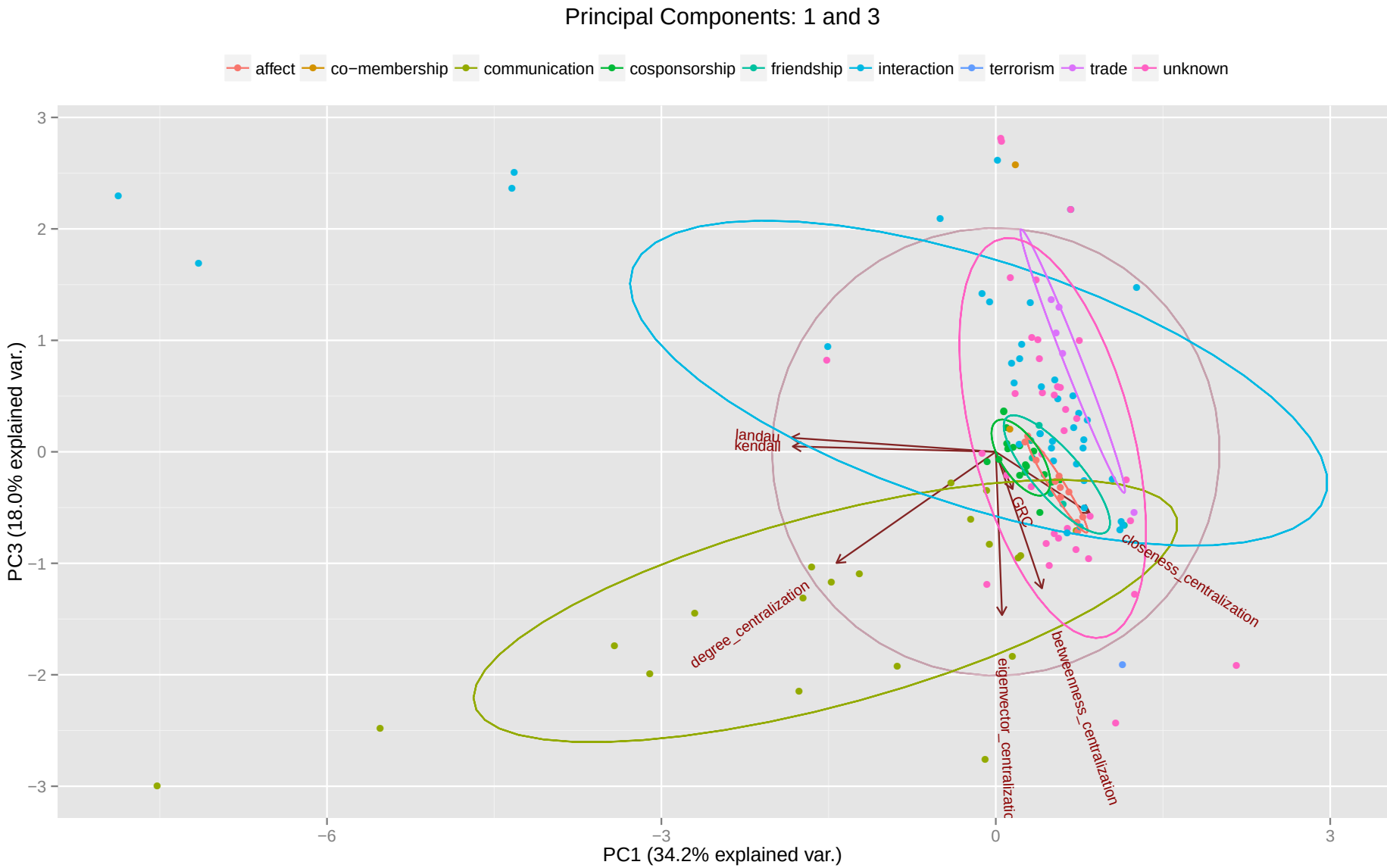
Hiearchy in Networks – PCA



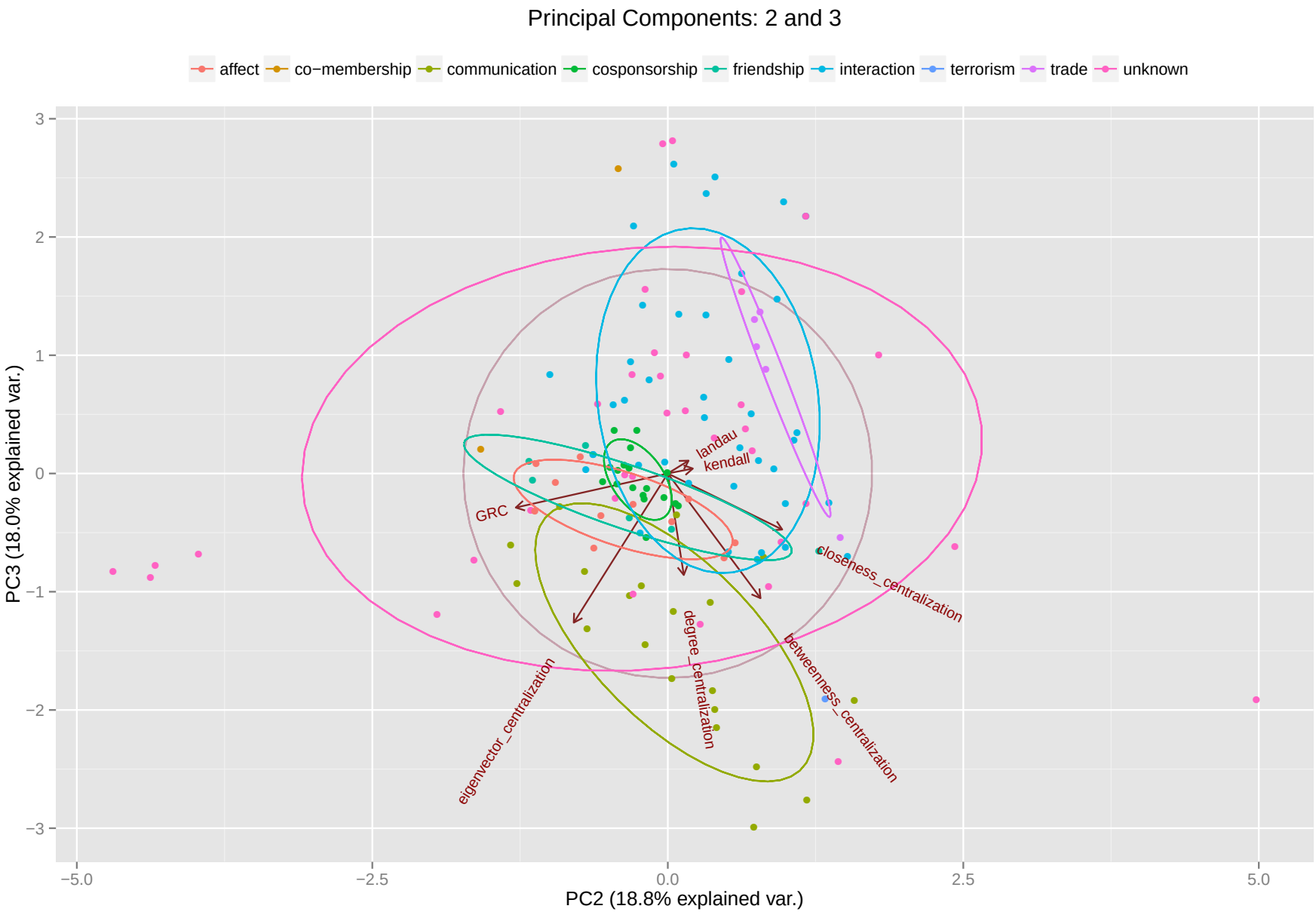
Hierarchy in Networks – PCA



Hierarchy in Networks – PCA



Hierarchy in Networks – PCA



Hierarchy in Networks–Results

- Need first three principle components to explain the variation.
- Both trade and co-membership stay grouped and towards the center.
- Both Landau and Kendall stay in same direction.
- From the first PCA plot (PC1 PC2), we notice that there are three clear groupings of measures.
- PC3 tries to separate the grouping of Betweenness from Landau and Kendall.