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Tomography problem in polychromatic conditions

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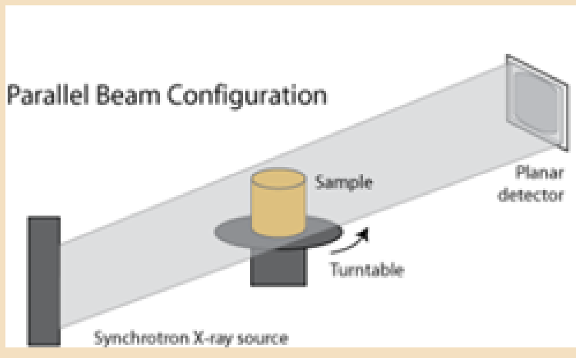
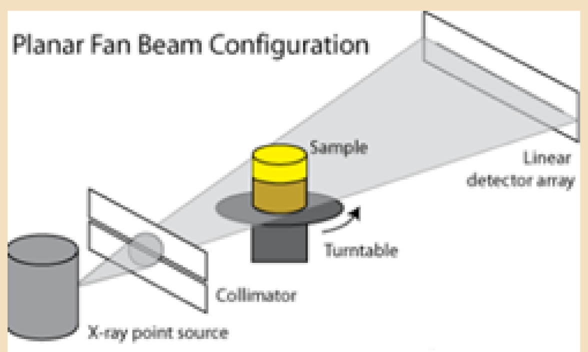
Statement of the problem

X-ray polychromatic tomography methods can be based on the use of polychromatic radiation to probe the sample or to register the polychromatic radiation with monochromatic beam scanning. From mathematical point of view both reconstruction problems are generally very similar to each other. For co-planar case (parallel and fan beams) of X-ray polychromatic tomography the reconstruction problem is expressed in terms of algebraic expressions. We present the solution of reconstruction obtained with an algebraic algorithm based on this philosophy.

How to register

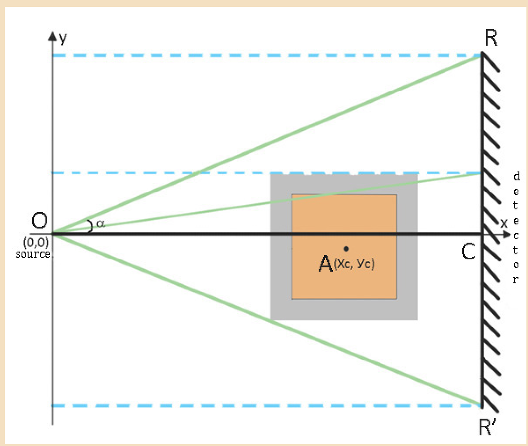
In 2D case CT tomograph may have fan or parallel scheme of the experiment. Only scheme with fan beam may be used in polychromatic case. Because of this an approach of translation fan from measuring scheme to a parallel one was proposed since reconstruction algorithms for the parallel scheme are studied more detailed.





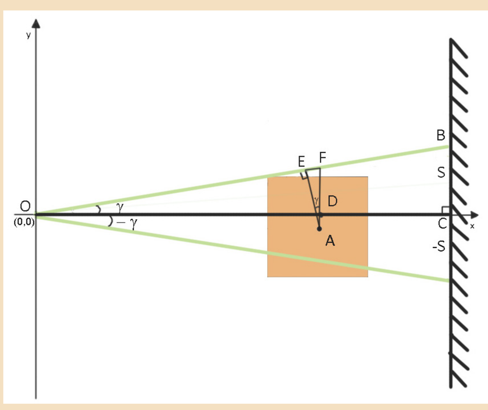
Scheme of the experiment

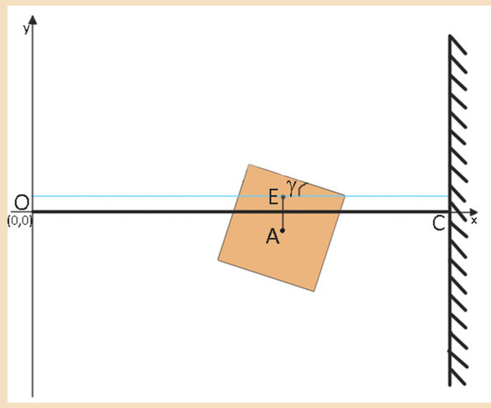
X-ray source is located at the point $O(0,0)$ in fan scheme. In the parallel case x-ray radiation extends in the direction of the axis OX. Detector is located at the OC distance from the center of coordinate system. The point $A(x_c, y_c)$ - rotation centre of the object. Path of the rays is shown with solid line for fan scheme and with dotted line for parallel scheme. Aperture of the fan beam is $\alpha = 15.089$, detector size equals $RR' = 1024$ pixels.



Analytical solution

The right figure shows the beam forming scheme in the fan scheme, and The left figure shows beam in a parallel scheme for the same rotation of the sample. For translation one sinogram to another we need to know only the angle of incidence beam.





If (α_f, S_f) - point in fan scheme; α_f - angle of rotation sample, S_f - detector cells. (α_p, S_p) - point in parallel scheme. Formula of the coordinates translation:

$$\alpha_p = \alpha_f + \gamma;$$
$$S_p = (y_c S_f/OC - x_c)\cos(S_f/OC).$$

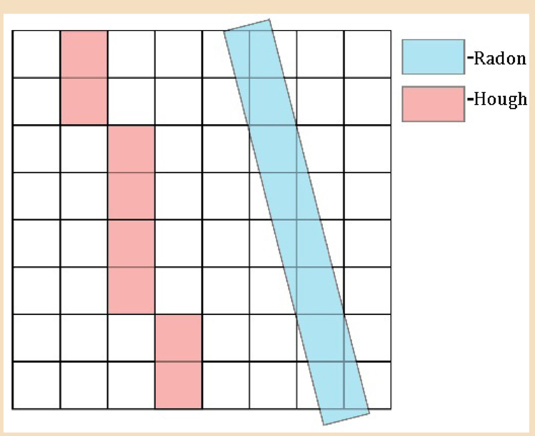
To inverse problem

$$\mu(x,y,k) = \sum_{l=1}^{N} c(x,y,l)\mu_{tabel}(l,k);$$
$$\bar{\mu}(x,y) = \hat{\mu}_{tabel}\vec{c}(x,y);$$
$$I_0(k) \xrightarrow[\mu(k)]{} I(k);$$
$$s \to shift,$$
$$t \to tanget,$$
$$\hat{H} \to Normalized\ Hough\ Transform;$$

$$\mu_{integral}(s,t,k) = \hat{H}\mu(x,y,k)$$
$$\log I_{teor}(s,t,k) = \mu_{integral}(s,t,k) + \log I_0(k)$$
$$\log \vec{I}_{teor}(s,t) = \vec{\mu}_{integral} + \log I_0$$
$$P_{teor} \to registersignal$$
$$P_{teor}(s,t) = \sum_k exp(\log I_{teor}(s,t,k))$$
$$\|p_{teor} - P_{expect}\| \xrightarrow[\vec{c}(x,y)]{} min$$

Fast Hough transform

The figure shows the differences in the representation of directions in the coordinates of the Radon and Hough systems. We write algebraic method (ART) for reconstruct the linear absorption coefficient for the parallel case using the Hough transform. Thus, thanks to usage of the fast Hough transform, we've managed to reduce complexity of one iteration from $O(n^3)$ to $O(n^2 \log n)$ and also it is possible to approximate the ART reconstruction on speed to FBP.



Mixing chemical compounds

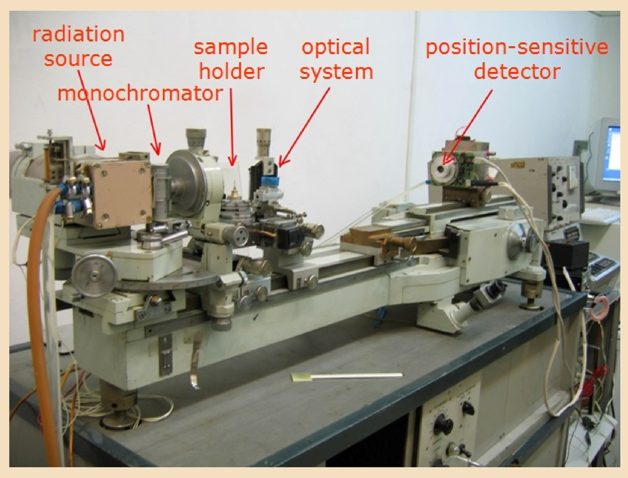
The linear attenuation coefficient of the mixture at energy E is given by:

$$\mu_L(E) = \sum_c \sum_Z \frac{m(c,Z)\mu(Z,E)}{V}$$

where $m(c,Z)$ is molal mass of element Z in compound and $\mu(Z,E)$ is linear attenuation coefficient of element Z for energy E. The sample contains a sequence of patches with a particular order. Pixels with $\mu_L(E) = 0$ are considered to be atmosphere. We compute the linear attenuation coefficient for compounds by using the xraylib library (`https://github.com/tschoonj/xraylib/wiki30/04/2014`).

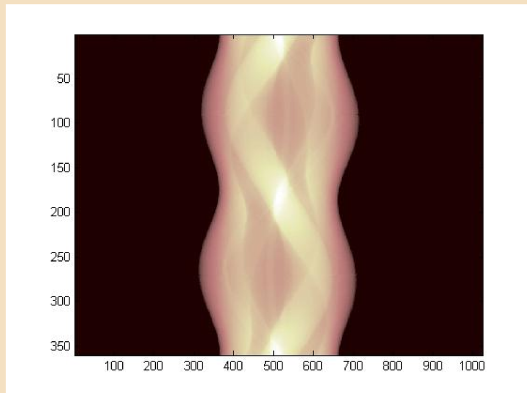
Prototype CT scanner

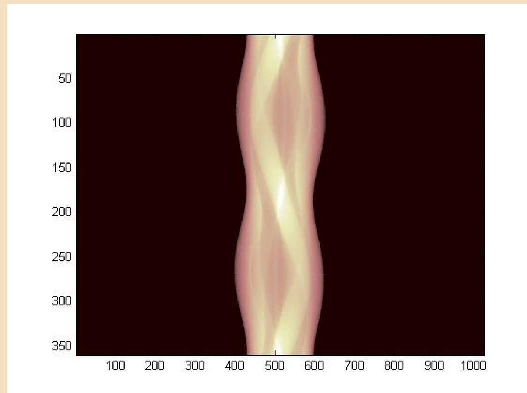
The prototype CT scanner that is running all necessary experiments is located at Shubnikov Institute of Crystallography RAS. This scanner has been made by our group itself.



Result of the translation

The left figure shows sinogram for Shepp-Logan phantom with resolution of 2 degree, so sinogram includes 180 angles of the sample orientation. The right figure shows translation sinogram for the parallel scheme.

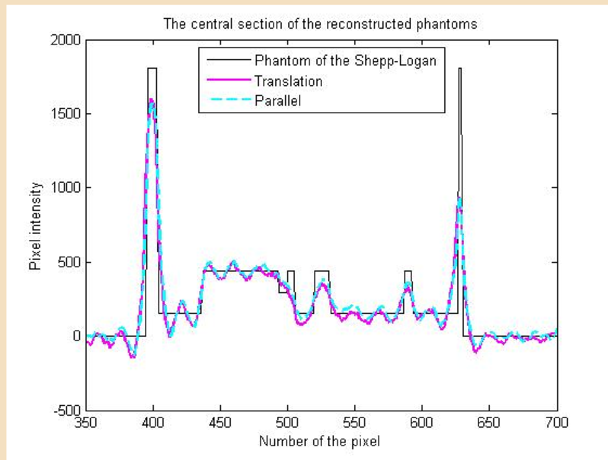




Since the object is the same for parallel and fan schemes of the experiment, it is expected, that the sinogram for parallel scheme gets even thinner.

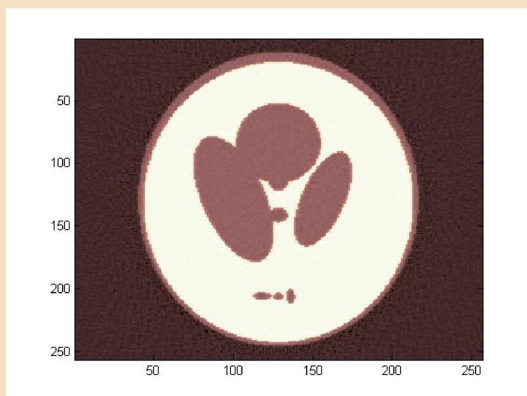
Result of the reconstruction

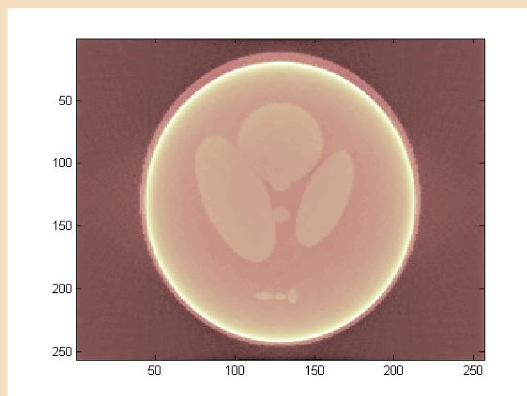
As can be seen from the cross sections, the results of reconstruction are almost identical, that indicates correct implementation of the sinogram translation from one to another.



What is a problem?

Figures are presented results of the experiment for 2 elements: Germanium (Z=32) and Chromium (Z=24); anode Cu. Using our algorithm for monochromatic case we reconstructed the sample for monochromatic source energy 8 kEv (in the right figure) and the sample with the fluorescence spectrum.





Statement of the polychromatic problem

Equation of Bouguer's law for the polychromatic case can be written as:

$$I(\theta, s) = \int_S I_0(E)\exp\left(-\int_L f(x,y,E)\delta(x\cos(\theta) + y\sin\theta - s)dxdy\right)dE$$

where S is energy of fluorescent spectrum $I_0(E)$. We'we added integral over energy in the previous equation, L is area of sample. In discrete case the integrals are to be replaced with the sums.

Our solution

Consider one pixel of the image j. N is the number of elements taken from the fluorescence spectrum recorded for whole sample before the start scanning. In each pixel absorption coefficients of the N elements compound are written as vector $(c_1^j, c_2^j, ..., c_n^j)$. So each cell in sinogram is p_i^h, where h is current rotate angle and i is the detector cell.

$$p_i^h = \sum_{k=1}^{R} I_0(k) e^{-\sum_{j=1}^{M}\sum_{j=1}^{M} c_i^j w^{jh} \mu^{jk}}$$

for $i = 1...V$, $h = 1...Q$. Where k is the index of current energy level. So we've obtained the linear system with $Q \times V$ equation and $M \times M \times N$ unknowns. If $Q \times V > M \times M \times N$ we can solved this system.

Conclusions and plans

- The first step is to translate the sinogram for fan scheme to sinogram for parallel scheme. The results show that the translated is made accurately.
- Inconsistencies in amplitude are less than 0.5% due to the different passing beam through the pixels in fan and parallel schemes of the experiment.
- We propose reconstruction obtained with an algebraic algorithm based on Hough transform and our next step is to implement algorithm using C++ language.