Computer Vision.

A1) Linear Shift Invariant:

Smoothing filter $= \frac{1}{3}[1\,1\,1] = w(x)$

$f(x) = [0, 0, 0, 1, 1, 1, 0, 0, 0]$

To prove Shift Invariant

$g(x) = w(x) * f(x)$

$g(x-x_0) = w(x) * f(x-x_0)$

let $x_0 = 1$

shifting $f(x)$ towards left by 1, we get

$f(x-1) = [0, 0, 1, 1, 1, 0, 0, 0, 0]$

$g(x) = \frac{1}{3}[1, 1, 1] * [0, 0, 0, 1, 1, 1, 0, 0, 0]$

~~$= [0, 0, \frac{1}{3}, 1 ...]$~~

$= [0, 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3}, 0, 0]$

$g(x-1) = w(x) * f(x-1)$

To prove the linear property.

$Op(A+B) = op(A) + op(B)$

let $f(A) = [0,0,0,1,1,1,0,0,0]$
let $f(B) = [1,1,1,0,0,0,1,1,1]$

$f(A) + f(B) = [1,1,1,1,1,1,1,1,1]$

$op(A+B)$ Now, $\frac{1}{3}[1,1,1] * [1,1,1,1,1,1,1,1,1]$

$= \frac{1}{3}[2,3,3,3,3,3,3,3,2,] \longrightarrow (1)$

Now $op(A)$

$= \frac{1}{3}[1,1,1] * [0,0,0,1,1,1,0,0,0]$

$= \frac{1}{3}[0,0,1,2,3,2,1,0,0]$

$op(B)$

$= \frac{1}{3}[1,1,1] * [1,1,1,0,0,0,1,1,1]$

$= \frac{1}{3}[2,3,2,1,0,1,2,3,2]$

$op(A) + op(B) = \frac{1}{3}[2,3,3,3,3,3,3,3,2] \longrightarrow (2)$

$(1) = (2)$

$\therefore$ $[1,1,1]$ is a smoothing filter which is LSI.

$g(x-1) = 0 \quad 0 \quad [0 \quad 4 \quad 0 \quad 2 \quad 1] \quad 9 \quad 0 \quad 0$

0	0	4

0	0	4

0	2	4

0	1	2

1	2	9

$\therefore g(x-1) = [0 \quad 0 \quad 2 \quad 1 \quad 2]$

$\therefore g(x-1) = w(x) * f(x-1)$

$\therefore$ <u>Median filter is Shift-Invariant.</u>

For Linearity of Median filter:

$f_1(x) = [4 \quad 0 \quad 2 \quad 1 \quad 9]$

$w(x) = [\;\;|\;\;|\;\;]$ - 3 neighborhood median filter

$w(x) * f_1(x) = [0 \quad 2 \quad 1 \quad 2 \quad 1]$

let $f_2(x) = [3 \quad 4 \quad 0 \quad 1 \quad 2]$

$w(x) * f_2(x) = [3 \quad 3 \quad 1 \quad 1 \quad 1]$

$f_1(x) + f_2(x) = [7 \quad 4 \quad 2 \quad 2 \quad 11]$

~~For median filter.~~

~~Now taking a 1×3 filter~~

~~$f(x) = [4\ 0\ 2\ 1\ 9]$~~

~~$0\ 0\ [4\ 0\ 2\ 1\ 9]\ 0\ 0$~~

~~$[0\ 0\ 4]$~~

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for median filter

$f(x) = [4\ 0\ 2\ 1\ 9]$

$0\ 0\ [4\ 0\ 2\ 1\ 9]\ 0\ 0$

$[0\ 0\ 4]$

$[0\ 2\ 4]$

$[\ \ 1\ \ ]$

$[\ \ 2\ \ ]$

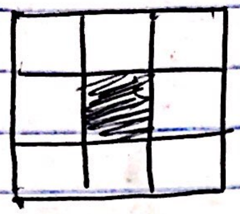
$[\ \ 1\ \ ]$

$[0\ 2\ 1\ 2\ 1]$

Checking Shift Invariance

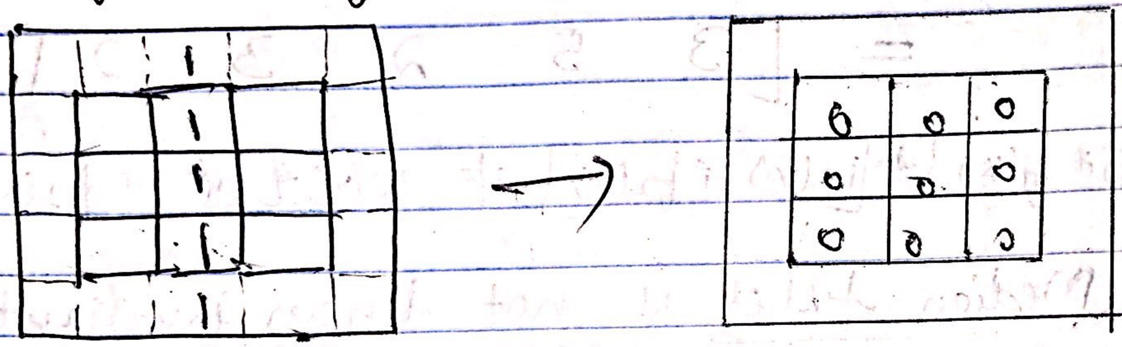
$f(x-1) = \cancel{[4\ 0\ 2}\ [0\ 4\ 0\ 2\ 1]\ 9$

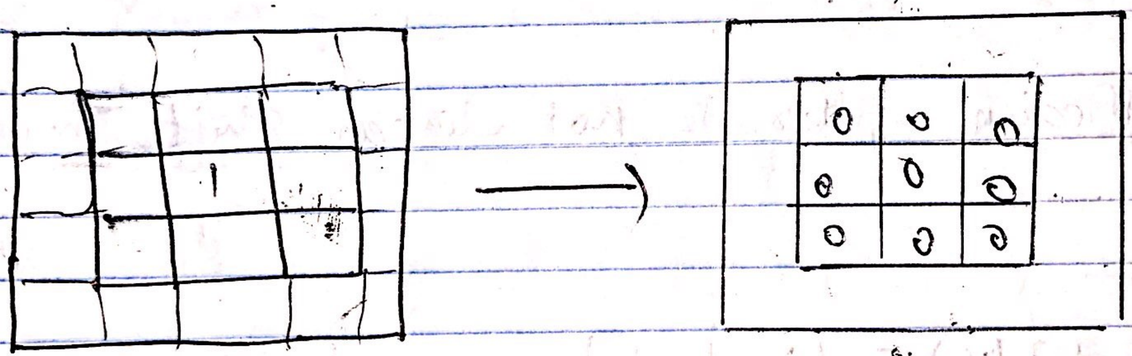
Median filter.

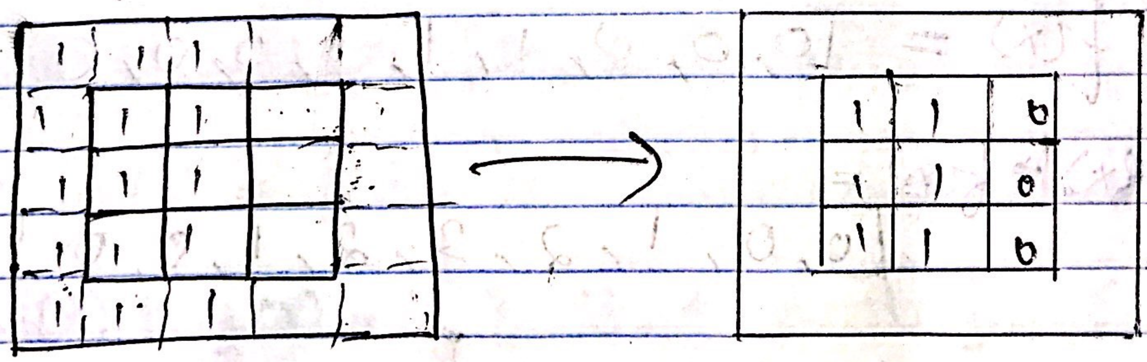


3x3 median filter.

Original Image   Filter Image







For bar, we don't get any shaded region as it is a median filter.
For a point, median filter does not select it as it seems to be an outlier.

$w(x) * [f_1(x) + f_2(x)]$

$= [7\ \ 4\ \ 2\ \ 2\ \ 1\ \ 1]$

$= [4\ \ 4\ \ 2\ \ 2\ \ 2]$

$(w(x) * f_1(x)) + (w(x) + f_2(x))$

$= [3\ \ 5\ \ 2\ \ 3\ \ 2]$

$[w(x) * f_1(x)] + [w(x) * f_2(x)] \neq w(x) * [f_1(x) + f_2(x)]$

∴ <u>Median filter is not linear invariant</u>

∴ Median filter is not linear Shift Invariant.

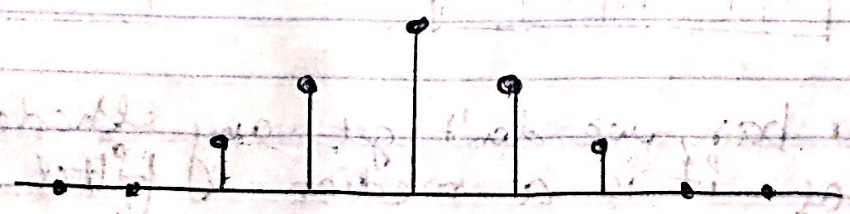
Q2

$w(x) = [1, 1, 1]$

$f(x) = [0, 0, 0, 1, 1, 1, 0, 0, 0]$

$w(x) * f(x) =$
$[0, 0, 1, 2, 3, 2, 1, 0, 0]$

graph



As we see that for a bar, the intensities are smoothed over the patch, the edges become unsharp.
Note that the intensities are distributed according to what was present in the original signal.

Q4 Separability.

2D Gaussian : $\frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$

(a) This 2D gaussian can be written as the product of

$$\underbrace{\left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right)}_{\text{1D gaussian}} \cdot \underbrace{\left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)}_{\text{1D gaussian}}$$

(b) If we make an $m \times m$ filter size covering the gaussian into two 1D filters, then the total no. of operations reduce.

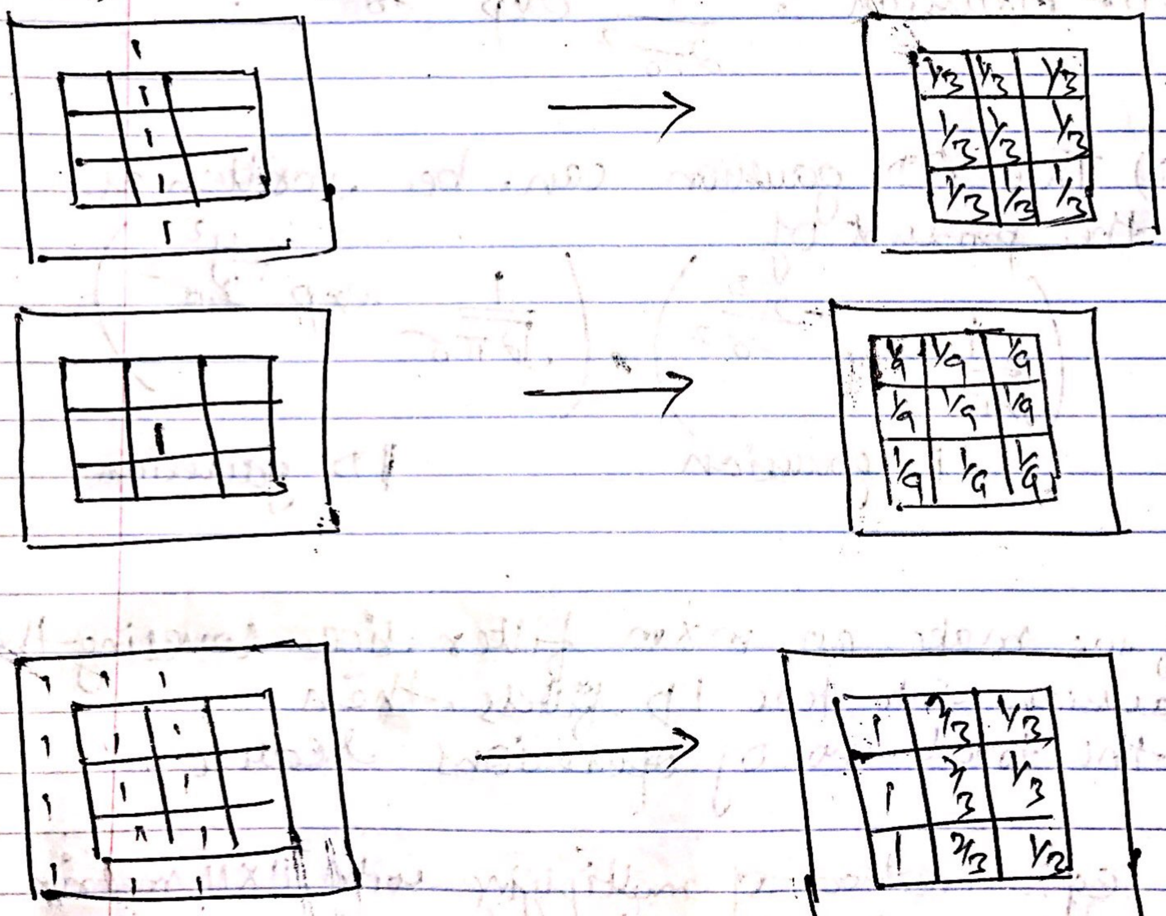
for eg: instead of multiplying with 11×11 matrix which leads to 121 multiplications, if we are able to break it into 2 (1×11) matrices, we can reduce the no. of multiplications.

For the Intensity bar, we get same values as before. The reason for it is because the intensities are equally distributed.

3x3 linear filter.

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Original Image   Filter Result.



Q5

$$\frac{\partial}{\partial x}[G(x,y,\sigma)] \circ f(x,y) \quad \text{vs} \quad \frac{\partial}{\partial x}[G(x,y,\sigma) \circ f(x,y)]$$

Here, we should choose the second option in order to compute the convolution of the image with the gaussian.

Since the Gaussian in 2D is linearly separable, we should separate it into two and convolve it with the image. Then take the derivative.

If we take the derivative first, then the partial derivate of the gaussian cannot be linearly separated into two filters, thus the computations are not reduced, and then we take the derivative.

$$\cancel{= \frac{\partial}{\partial x} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)}$$

$$\cancel{= \frac{1}{\sqrt{2\pi}\sigma} \frac{\partial}{\partial x} e^{-(x^2+y^2)/2\sigma^2}} \qquad \cancel{\left[\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2+y^2}\right.}$$

$$= \frac{\partial}{\partial x} \frac{1}{2\pi\sigma^2} \exp^{-(x^2+y^2)/2\sigma^2}$$

$$= \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \left[-\frac{(\cancel{2}x)}{\cancel{2}\sigma^2}\right]$$

$$= \frac{-x}{2\pi\sigma^4} \exp^{-(x^2+y^2)/2\sigma^2}$$

This is not linearly separable in x and y.