A1) Hough Transform : Parametrization.

The standard ~~paramatrization~~ parametrization of line $y = mx + b$
has issues while going to Hough transform.
In case of vertical lines, m becomes infinity
and cannot be represented in Hough Space.

This is the reason, it did not become a popular
choice.

No, I would not use it because of the above mentioned
problem. While creating discretized Hough Space,
we will not be able to ~~detect~~ divide the
cells on the 'm' (slope) axis because it will be
problem to divide in equal sized intervals.

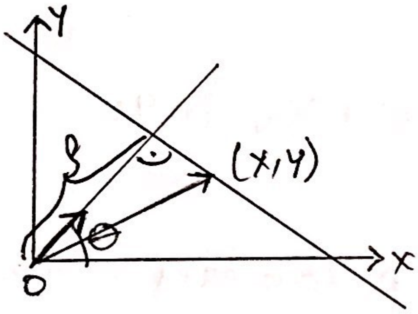
A2) To show that polar representation of a line, $x\cos\theta + y\sin\theta = \rho$ represents a cosine function.

Normal form of a line

$$\rho = x \cdot \cos\theta + y \cdot \sin\theta$$

$(x, y) = \bar{x}$: point coordinate

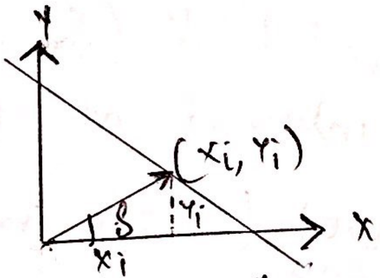
$(\rho, \theta) = \bar{a}$: parameter vector.



$$\rho = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$ | projection of $\bar{x}$ onto direction of normal $\bar{e}$

Here $\cos\rho = \dfrac{x_i}{A_i}$ $\quad A_i = \sqrt{x_i^2 + y_i^2}$

$\sin\rho = \dfrac{y_i}{A_i}$ $\quad \rho = \tan^{-1}\left(\dfrac{y_i}{x_i}\right)$



Now, Given point $(x_i, y_i) \Rightarrow (\rho, \theta)$ are parameters.

$$\rho = x_i\cos\theta + y_i\sin\theta$$

$$\frac{\rho}{A_i} = \frac{x_i}{A_i}\cos\theta + \frac{y_i}{A_i}\sin\theta$$

$$\frac{\rho}{A_i} = \cos\rho_i\cos\theta + \sin\rho_i\sin\theta$$

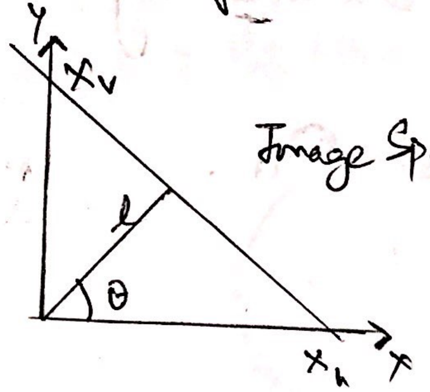
$$\rho = A_i\cos(\theta - \rho_i)$$

$\therefore$ Given $(x_i, y_i) \rightarrow (A_i, \rho_i) \Rightarrow$ cos-function in (ρ, θ) space.

A3) Hough Transform: Polar Representation II.

Polar form of the below line

$\Rightarrow \ell = x\cos\theta + y\cdot\sin\theta$



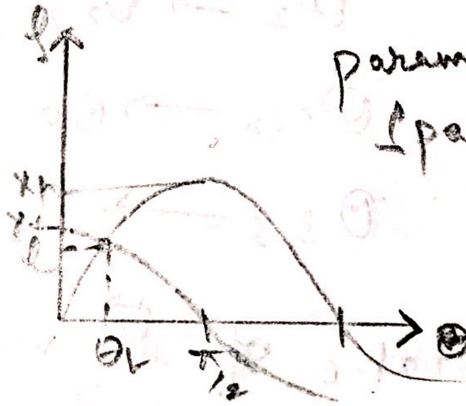
In the parameter space, we get the following curves

① $\vec{X}_1 = \ell_1 = x_v\cos(\theta - \pi/2)$

$\vec{X}_2 = \ell_2 = x_h\cos(\theta - 0)$.

$\vdots$

$\vec{X}_n = \ell_n = \cdots$



There are n points on the line (say, for eg.) and we draw curves for taking two points at a time and finding the high density locations in the parameter space.

We take $\vec{X}_1$ and $\vec{X}_2$ $\rightarrow$

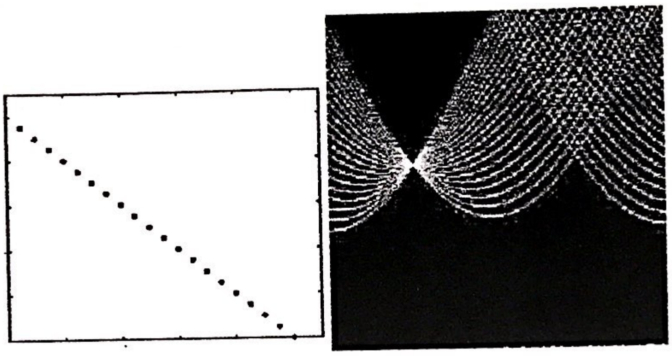
$\vec{X}_1 = \ell_1 = x_v\cos(\theta - \pi/2)$

$\vec{X}_2 = \ell_2 = x_h\cos(\theta - 0)$.

Plotting these curves on the parameter ~~sp~~ space, we will see that these points will intersect at one point between $(0, \pi/2)$. This point will be (θ, ℓ) which is parameters for the polar representation of the line in Image Space.

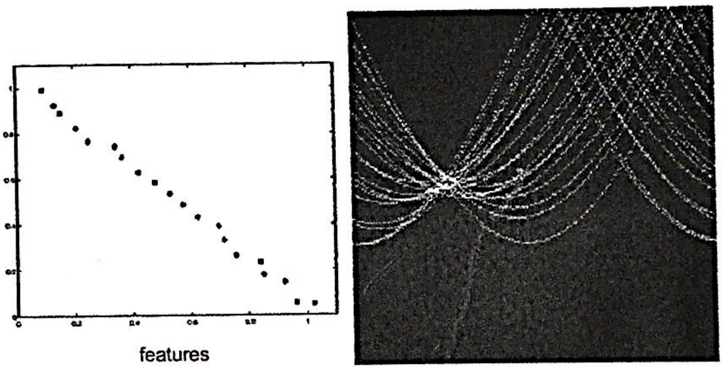
A4)

Case 1: When there is no noise.



If no noise is present in the points, we will get distinct peaks which will be easy to locate with peak detection algorithm..

Case 2: When noise is present



If noise is present in the data points, the peaks are distorted and it becomes difficult to locate for peak finding algorithm.

How to deal with noisy data to find a peak.

1) Choose good grid/discretization.

- Too coarse: large votes obtained when too many different lines correspond to a single bucket
- Too fine: miss lines because some points that are exactly collinear cast votes for different buckets.

2) Increment neighboring bins (Smoothing in accumulator array).

3) Try to get rid of irrelevant features, i.e taking points with significant gradient magnitude.

A5) Hough Transform for Ellipses.

Hough transform can be used ~~to~~ find ellipses in the image space.

The equation for ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Here h & K are the x and y coordinate for the centre of the ellipse and 'a' and 'b' are the horizontal & vertical radii.

For finding general ellipses using Hough transform we would require 4 dimensional space. We plot in 4-dimensions and then use peak finding algorithm in 4-D.

If we know the centre of the ellipse to be fixed ~~origin~~, then equation becomes

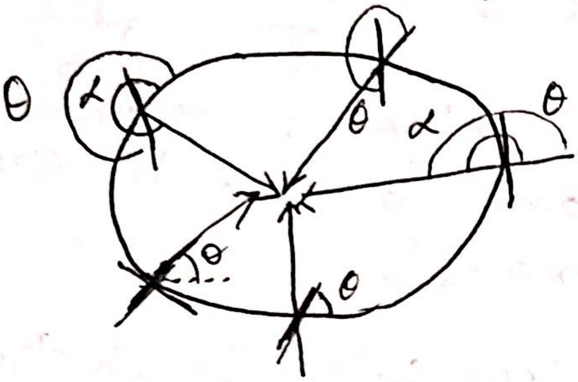
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

For this ellipse in image space, for every point on the edge of this ellipse, we plot in 2 D space, all possible ellipses in Hough Space. We then use, peak finding algorithm to detect the ~~centre of the ellipse~~ centre of the ellipse.

Here, the Hough transform uses 4-D space and it is very sparse & thus peak finding will take a lot of time & computational resources. Hence, we go to Generelized Hough Transform.

Generalized Hough Transform.

GHT offers a solution, for eg:- we have an ellipse



$\theta_i \rightarrow$ angle between the edge normal and positive x-axis at point i.

$\alpha_i \rightarrow$ angle between the vector $\vec{c} - \vec{i} = \vec{r}$ and the positive x-axis.

θ can range from 0 to 359° and there can be more than one $\vec{r}$ with same θ for more than one point on the ellipse. The represented R-table is as:

θ_0	r_{01}, r_{02}, r_{03}
θ_1	r_{11}, r_{12}, r_{13}
θ_2	
$\vdots$	$\ldots$ - -
θ_{359}	- - -

This represents the model for detecting this particular ellipse. To generalize over multiple orientations, we have to rotate the model itself.

A5) - contd.

Say we want to find this model rotated by $10°$ in the positive x direction., then

θ_1 in the model will become θ_{11}.

Rotating by $10° \rightarrow$

$\theta_1 \rightarrow \theta_{11}$

$\theta_{12} \rightarrow \theta_{22}$

$\theta_{350} \rightarrow \theta_0$

$\theta_{355} \rightarrow \theta_5$

The table of $\vec{r}$ vectors will remain unchanged.

Suppose we want to deal with the varying sizes of the ellipse, we scale all the $\vec{r}$ vectors in our R-table, i.e increase or decrease the magnitude of $\vec{r}$ vectors.