

5/21/20

Computer Vision.

A1) Linear Shift Invariant

Smoothing filter $= \frac{1}{3} [1 1 1] = w(x)$

$$f(x) = [0, 0, 0, 1, 1, 1, 0, 0, 0]$$

To prove Shift Invariant

$$g(x) = w(x)^* f(x)$$

$$g(x-x_0) = w(x)^* f(x-x_0)$$

Let $x_0 = 1$
shifting $f(x)$ towards left by 1, we get

$$f(x-1) = [0, 0, 1, 1, 1, 0, 0, 0, 0]$$

$$g(x) = \frac{1}{3} [1, 1, 1]^* [0, 0, 0, 1, 1, 1, 0, 0, 0]$$

$$= \underline{\underline{[0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}]}}$$

$$= [0, 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3}, 0, 0]$$

$$(3) g(x-1) = w(x) * f(x-1)$$

To prove the linear property.

$$OP(A+B) = OP(A) + OP(B)$$

$$\text{let } f(A) = [0, 0, 0, 1, 1, 1, 0, 0, 0]$$

$$\text{let } f(B) = [1, 1, 1, 0, 0, 0, 1, 1, 1]$$

$$f(A) + f(B) = [1, 1, 1, 1, 1, 1, 1, 1, 1]$$

$$OP(A+B) \text{ Now } \frac{1}{3}[1, 1, 1] * [1, 1, 1, 1, 1, 1, 1, 1, 1]$$

$$= \frac{1}{3}[2, 3, 3, 3, 3, 3, 3, 3, 2] \quad \text{①}$$

$$\text{Now } OP(A) \\ = \frac{1}{3}[1, 1, 1] * [0, 0, 0, 1, 1, 1, 0, 0, 0] \\ = \frac{1}{3}[0, 0, 0, 1, 2, 3, 2, 1, 0, 0]$$

$$\begin{aligned} OP(B) \\ &= \frac{1}{3}[1, 1, 1] * [1, 1, 1, 0, 0, 0, 1, 1, 1] \\ &= \frac{1}{3}[2, 3, 2, 1, 0, -1, 2, 3, 2] \end{aligned}$$

$$OP(A) + OP(B) = \frac{1}{3}[2, 3, 3, 3, 3, 3, 3, 3, 2] \quad \text{②}$$

$$\text{①} = \text{②}$$

$\frac{1}{3}[1, 1, 1]$ is a smoothing filter which is LSI.

mean filtering with width 3x3 and very smooth

if we search 1 pixel for fifth cell

$$g(x-1) = \begin{bmatrix} 0 & 0 & [0 & 4 & 0 & 2 & 1] & 0 & 0 \end{bmatrix}$$

$\boxed{0 \ 0 \ 1 \ 4}$
 $\boxed{1 \ 0 \ 0 \ 4}$
 $\boxed{0 \ 1 \ 2 \ 4}$
 $\boxed{0 \ 0 \ 1 \ 2}$
 $\boxed{1 \ 1 \ 2 \ 9}$

$$\therefore g(x-1) = [0 \ 0 \ 2 \ 1 \ 2]$$

$$\therefore g(x-1) = w(x) * f(x-1)$$

\therefore Median filter is shift-invariant.

For linearity of Median filter.

$$f_1(x) = [4 \ 0 \ 2 \ 7 \ 9]$$

$$w(x) = \boxed{\text{ }} \quad \text{3 neighborhood median filter}$$

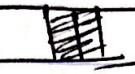
$$w(x) * f_1(x) = [0 \ 0 \ 1 \ 1 \ 2 \ 1 \ 0]$$

$$\text{let } f_2(x) = [3 \ 4 \ 0 \ 2 \ 1 \ 2]$$

$$w(x) * f_2(x) = [3 \ 3 \ 2 \ 0 \ 1 \ 1]$$

$$f_1(x) + f_2(x) = [7 \ 4 \ 2 \ 2 \ 1]$$

~~For median filter.~~

Now taking a 1×3  filter

$$f(x) = [4 \ 0 \ 2 \ 1 \ 9]$$

$$0 \ 0 \ [4 \ 0 \ 2 \ 1 \ 9] \ 0 \ 0$$

$$\boxed{0 \ 4 \ 2}$$

for median filter

$$f(x) = [4 \ 0 \ 2 \ 1 \ 9]$$

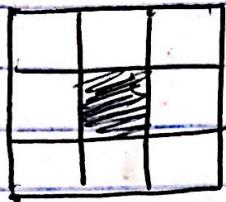
$$\begin{array}{c} 0 \ 0 \ [4 \ 0 \ 2 \ 1 \ 9] \ 0 \ 0 \\ \boxed{0 \ 0 \ 4} \\ \boxed{0 \ 2 \ 4} \\ \boxed{1 \ 1} \\ \boxed{1 \ 2} \\ \boxed{1 \ 1} \end{array}$$

$$[0 \ 2 \ 1 \ 2 \ 1]$$

Checking Shift Invariance

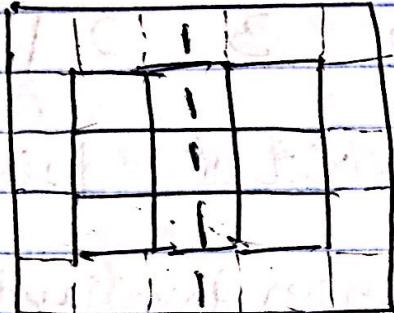
$$f(x-1) = [4 \ 0 \ 2 \ 0 \ 4 \ 0 \ 2 \ 1] \ 9$$

Median filter.

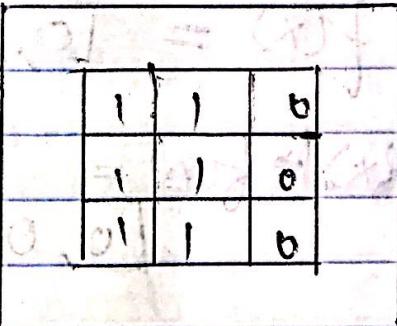
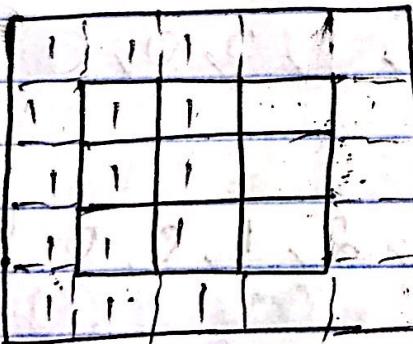
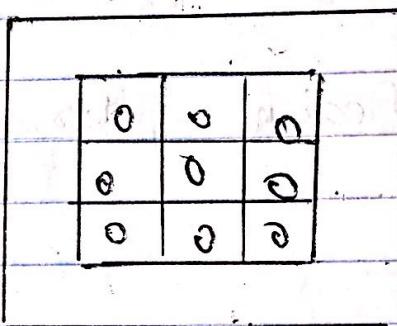
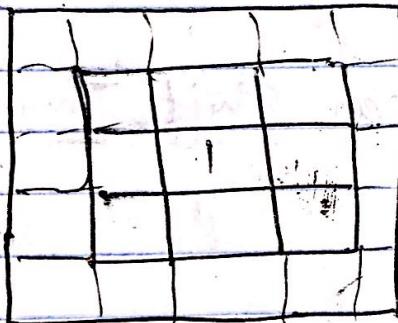
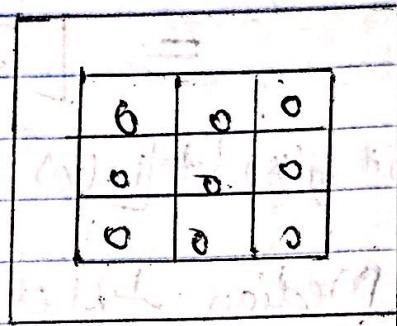


3x3 median filter.

Original Image



Filter Image



for bar, we don't get any shaded region
as it is a median filter.

for a point, median filter does not select it
as pt seems to be an outlier.

$$w(x) * [f_1(x) + f_2(x)]$$

$$= [7 \quad 4 \quad 2 \quad 2 \quad 11]$$

$$= [4 \quad 4 \quad 2 \quad 2 \quad 2]$$

$$(w(x) * f_1(x)) + (w(x) * f_2(x))$$

$$= [3 \quad 5 \quad 2 \quad 3 \quad 2]$$

$$[w(x) * f_1(x)] + [w(x) * f_2(x)] \neq w(x) * [f_1(x) + f_2(x)]$$

\therefore Median filter is not linear invariant

\therefore Median filter is not linear shift invariant.

Q2

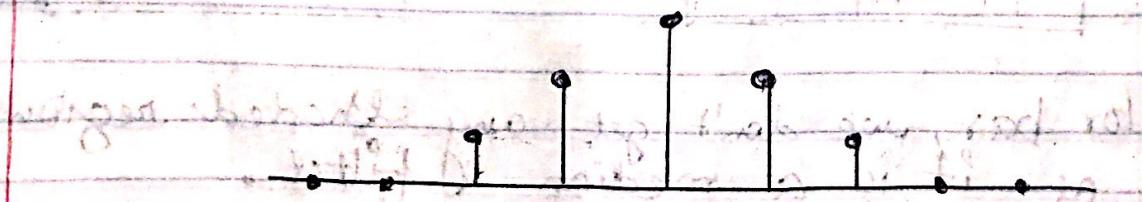
$$w(x) = [1, 1, 1]$$

$$f(x) = [0, 0, 0, 1, 1, 1, 0, 0, 0]$$

$$w(f(x)) * f(x) =$$

$$[0, 0, 1, 2, 3, 2, 1, 0, 0]$$

graph



As we see that for a bar, the intensities are smoothed over the patch, the edges become unsharp.

Note that the intensities are distributed according to what was present in the original signal.

Q4

Separability -

$$2D = \text{Gaussian} \cdot \frac{1}{2\pi\sigma^2} \exp \frac{-x^2+y^2}{2\sigma^2}$$

(a) This 2D gaussian can be written as the product of

$$\left(\frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-x^2}{2\sigma^2} \right) \cdot \left(\frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-y^2}{2\sigma^2} \right)$$

1D gaussian 1D gaussian

(b) If we make an $m \times m$ filter size covering the gaussian into two 1D filters, then the total no. of operations reduce,

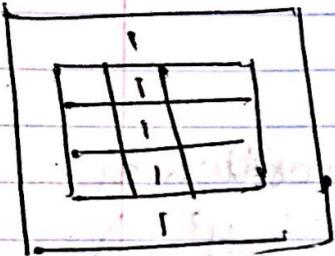
for eg: instead of multiplying with 11×11 matrix which leads to 121 multiplications, if we are able to break it into $2(1 \times 11)$ matrices, we can reduce the no. of multiplications.

For the Intensity bars, we get same values as before. The reason for it is because the intensities are equally distributed.

3×3 linear filter.

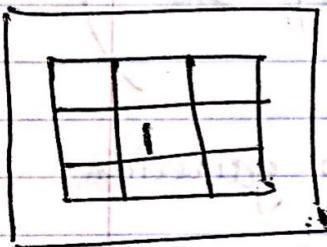
$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Original Image

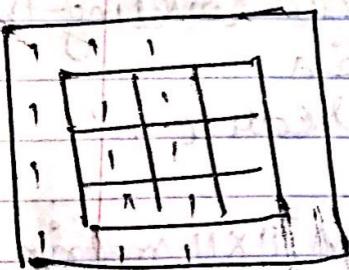


Filter Result.

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$



$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$



$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Q5

$$\frac{\partial}{\partial x} [G(x, y, \sigma)] \text{ of } (x, y) \quad \text{vs} \quad \frac{\partial}{\partial x} [G(x, y, \sigma) \text{ of } (x, y)]$$

Here, we should choose the second option in order to compute the convolution of the image with the gaussian.

Since the Gaussian in 2D is linearly separable, we should separate it into two and convolve it with the image. Then take the derivative.

If we take the derivative first, then the partial derivative of the gaussian cannot be linearly separated into two filters, thus the computations are not reduced, and then we take the derivative.

$$\begin{aligned} &= \frac{\partial}{\partial x} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \frac{\partial}{\partial x} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \quad \left[\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}} \right] \\ &= \frac{\partial}{\partial x} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \\ &= \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \left[-\frac{(2x)}{2\sigma^2} \right] \\ &= \frac{-x}{2\pi\sigma^4} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \end{aligned}$$

This is not linearly separable in x and y .

A6) Canny non-maximum suppression

Non maximum separation is done as the last step in edge detection. After the Canny edge detection is done, there are generally many edges present in the final image. To get the actual edge, we use non maximum separation.

Non Maximum suppression is used to find the local maxima from the these edges and present it as the final edge.

The procedure to do so is the following.

1. Quantize edge directions eight ways according to 8-connectivity (cf. Figures 2.5 and 4.3).
2. For each pixel with non-zero edge magnitude, inspect the two adjacent pixels indicated by the direction of its edge (see Figure 6.10).
3. If the edge magnitude of either of these two exceeds that of the pixel under inspection, mark it for deletion.
4. When all pixels have been inspected, re-scan the image and erase to zero all edge data marked for deletion.