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Special Thanks

Thanks to Michal Kopecky for Oracle database

# Abstract

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# Recommender Systems

Nowadays, Recommender Systems are widely recognized as one of the most beneficial applications of Machine Learning, and there is no doubt that they drive nearly every aspect of our lives. The fundamental objective of these Machine Learning-driven Recommenders is to filter, prioritize and efficiently deliver the necessary information to the consumers in the midst of overwhelmingly numerous choices on the internet. It is also described as:

*“Any system that produces individualized recommendations as output or has the effect of guiding the user in a personalized way to interesting or useful objects in a large space of possible options.”* (Burke, 2002)

Therefore, many companies utilize the recommender systems for the purpose of helping the consumers discover new and relevant items such as movies, musics, jobs, etc. They use the consumer data which may be in explicit or implicit form (e.g., likes, clicks), to comprehensively assess consumers’ preferences and then recommend the relevant items to them. According to various criteria, there are multiple techniques of recommender systems, each of which differs in how a single recommendation is generated. The most common types of recommender systems are described in the following section.

Although there exists several different recommender systems in the literature, we will focus on the three most common ones below:

Diagram

Description automatically generated

Figure 1

## Collaborative Filtering recommender systems

Collaborative Filtering recommender systems are one of the most widely used systems next to the content-based recommender systems. Essentially, these systems create a user profile based on the ratings of various items and then aims to compare these against a wider user group [12]. As the word “collaborative” from the name implies, multiple users come together as group – taste of one user will be similar to the other users of group. Therefore, by utilizing the user’s data which contains their historical preferences on a set of items, system deploys an assumption that the users who have previously agreed are more likely to agree again in the future. So, the system creates the new recommendations by taking the similarities between users based on the ratings into consideration.

Diagram

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Figure : The illustration of Collaborative Filtering recommender systems.

Although, CF recommender systems have been used in the industry for many years [13], they still have a limitation such that they cannot address the cold start problem –they are not able to recommend items which are not rated by any users (e.g., new items). As a result, it is possible that only famous items may get recommended. Furthermore, traditional CF systems are also memory-wise and computationally expensive and suffers from scalability problems.

## Content Based recommender systems

While CF recommender systems, as discussed above, recommend the products or items according to the similarities of user preferences which means that recommendation relies on the user-item interactions, Content Based recommender systems, on the other hand, aims to recommend products or items similar to those a given user has rated positive or liked in the past. So, CB systems generate recommendations based on the comparison between the content of the items and the user profile which was created according to the historical user data. Note that the content of items is described by terms, tags, features, or even plots in case if the items are movies.

An algorithm used to recommend the movies on the Netflix platform is the prominent example which resembles this recommender systems. If a certain user watches and comedy movie and rates it positive via votes or comments, then the new movie recommendations with the same label of that liked movie will be suggested to the user. In other words, based on the content of the consumed item, these recommender system finds other similar items and recommend them. Note that such website platforms often keep the techniques of how the content is labeled and matched against each other as secret [12]. Contrary to the CF systems, CB system doesn’t suffer cold-start problem and they can suggest not only famous or older items, but also the unpopular or new items. In addition to this, they are memory-wise and computationally cheap because there is no need for the data of other users to be able to compute the recommendation for a specific user.

Diagram

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Figure : The illustration of Content Based recommender systems.

## 

## Hybrid recommender systems

Hybrid recommender systems combine two or more types of traditional recommender systems to have better performance by benefiting from complementary advantages of subsystems. Hybrid systems which combine collaborative filtering and content based approaches, achieves state-of-the-art results in many cases and are used in many large scale recommender systems nowadays. Detailed comparison on advantages and disadvantages of Hybrid Recommenders along with Collaborative Filtering and Content Based Recommenders are shown in table below on Table 1**:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Collaborative Filtering** | **Content Based** | **Hybrid** |
| **Number of users** | * Recommendation based on many users having similar interest | * Recommendation based on single user | * Combination of collaborative and content based filtering |
| **Disadvantages** | * Cold start problem * Data sparsity * Scalability * Memory-wise and computationally expensive | * Limited content analysis * Over-specialization | * Increased complexity * Increased expense of implementation |
| **Advantages** | * Serendipitous recommendation * User and item features are not required * Quality may improve over time as more users interact with items * - Minimal domain knowledge required | * User independent * No cold start problem * Interpretable results * Memory-wise and computationally cheap | * Avoids most of the shortcomings of other approaches. |

Table 1

# Probabilistic Models for Recommender Systems

The application of probabilistic modeling to the recommendation problem has a rich history that dates back decades (Breese et al. 1998, Hofmann 2004, Marlin 2004). Many authors incorporated the probabilistic approaches into models which explained the dataset. Initial approaches were probabilistic graphical models such as Bayesian networks and Dependency networks which eventually left their place with subsequent novel topic models such as Latent Dirichlet Allocation (LDA) [3]. The term “latent” is used in their name because LDA is considered a probabilistic topic model, and the topics it aims to find from the corpus are considered latent or hidden variables. A detailed explanation of LDA has been discussed in its own section in later pages as it is an essential part of the model that we will be discussing in this thesis. Also, note that, as LDA can suggest items that have similar content to other items that a user likes, it has been extensively used for Content Based recommender systems.When it comes to the field of Collaborative Filtering recommender systems, the matrix factorization technique had gained decent popularity, especially after being combined with a probabilistic approach [18], [19], [20], [21].

## fLDA

Lately, there has been a lot of interest in combining probabilistic topic modeling with matrix factorization in the field of hybrid recommender systems. One of the major reasons for this is that when the content of an item is represented by topic models, the models benefit from interpretable semantics of the latent space characterized by the topic mixtures, and this leads to more interpretable semantics of the item latent factor[\*\*CTMP\*\*]. Initially, Agarwal and Chen proposed probabilistic topic modeling in matrix factorization with fLDA where the item latent factor took the role of topic proportion in the LDA representation [23]. Despite being an accurate and interpretable model which handles both cold-start and warm-start scenarios, fLDA still had a limitation in dealing with distinguished items where there is an identical topic mixture, but content details that topic mixture cannot cover are of concern to different groups of people. To elaborate on this limitation, consider that we have two articles; A and B. Both articles are about the application of machine learning to social networks. Because both articles are identical in terms of their contents, they will also possess the same topic proportions. Now let’s consider that these two articles are of interest to different kinds of users: Article A provides a prominent machine learning algorithm that is applied to social network applications, whereas article B implements a standard machine learning algorithm, but provides crucial data analysis on social network data. As a result, users who work in machine learning will prefer article A and will hardly be interested in article B, whereas users who work in social networks will be more interested in article B instead of A. However, as the topic proportions of both articles are the same, both will be recommended to both groups of users [24].

## CTR

To tackle the limitation mentioned above, a novel approach called Collaborative Topic Regression (CTR) [24] has been proposed by David M. Blei and Chong Wang. The way CTR addresses that limitation is by allowing the item latent factor to be an offset from topic proportion. So, using this way, an offset may help explain, for instance, an article A is more important to researchers interested in machine learning than it is to those interested in social network analysis. Therefore, CTR allows the item latent factor to also account for user ratings.

Fundamentally, CTR incorporates techniques of both collaborative filtering based on latent factor models and content analysis based on probabilistic topic modeling. According to the CTR model, items are generated by a topic model while users are represented with topic interests [24]. Therefore, CTR is considered one of the excellent hybrid models which shows that the combination of the content modeling with the matrix factorization methods produces more promising results compared to traditional recommender systems. The graphical model of CTR along with its algorithm is shown below. Note that, in machine learning, graphical models are used to represent a repetitive process of the probabilistic model. Essentially, they represent a factorization of the joint distribution of hidden and observed random variables. Nodes are random variables, plate boxes denote the “loop” with a variable shown in the bottom right corner of the plate representing its number of iterations, and edges mean that there is dependence between random variables in the generative process. Note that grey nodes represent observed variables while blank nodes are hidden variables. Figure 4 shows the graphical model of CTR. We assume *U* users and *J* items for the recommender system. The rating variable denotes whether the user *u* likesitem *j* or not. Also, note that can be interpreted in two ways: either user *u* is not interested in item *j* or user *u* does not know about article *j*. For each user, we try to recommend potentially interesting items that are rated yet by this user. Assuming that there is *K* topics in the whole corpus of items, the graphical model and the generative process of the CTR model are shown below:

Diagram

Description automatically generated

Figure : The graphical model for CTR.

Text, letter

Description automatically generated

Algorithm : The generative process for CTR.

Despite its many advantages, the CTR model has significant computational limitations as well. The reason is that the model considers user ratings to have a Gaussian distribution which leads to iterating over all the entries in the rating matrix during training. Because of this, CTR is highly inefficient considering that real-world datasets are very big and sparse. Additionally, CTR is a non-conjugate model [39] which makes it difficult to fit, challenging to work with on sparse data, and challenging to scale without stochastic optimization.

## CTPF

To address CTR’s inefficiency mentioned above, a newer hybrid model called Collaborative Topic Poisson Factorization (CTPF) has been proposed by Prem Gopalan, Laurent Charlin and David M. Blei [25]. Fundamentally, CTPF incorporates concepts from two existing models: Poisson factorization [50] and collaborative topic regression [24].

Poisson factorization substitutes a Poisson likelihood and non-negative representations for the conventional Gaussian likelihood and real-valued representations. In comparison with Gaussian factorization, Poisson factorization possesses more efficient inference, better handling of sparse data, and better predictive performance. So, CTPF model assumes both reader behavior and item texts with Poisson distributions. As a result, CTPF is only concerned with non-zero ratings during training, and therefore it is much more efficient and scalable.

Compared to CTR model, which is a non-conjugate model, CTPF is a conditionally conjugate model which allows us to use standard variational inference with closed-form updates. Moreover, CTPF, because it is based on Poisson and gamma variables, it has a more efficient and simpler-to-implement inference algorithm, and a much better fit to sparse real-world data. It is more scalable and provides significantly better recommendations than collaborative topic regression [25].

We assume we have data containing *U* users and *J* items for the recommender system. CTPF assumes a collection of *K* unnormalized topics . Each topic is a collection of word intensities on a vocabulary of size *V*. Each unnormalized topic component is drawn from a Gamma distribution. CTPF assumes that, given the topics, a document *j* is generated with a vector of *K* latent *topic intensities* and that users are represented by a vector of *K* latent *topic preferences* . In addition, the model assigns each document K latent topic offsets d that represent its deviation from the topic intensities. These deviations happen when a document's content does not sufficiently describe its ratings. Finally, CTPF claims that the conditional probability that a user *u* rated document *j* with rating is derived from a Poisson distribution with rate parameter where is the document topic offset. The graphical model of CTPF along with its algorithm is demonstrated below.

Diagram

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Figure : The graphical model for CTPF.

Text

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Algorithm 2: The generative process for CTPF.

CTPF has two main advantages over previous work; having a conditionally conjugate model which helps to employ standard variational inference with closed-form updates and having built on Poisson factorization which makes the most use of sparsity of user consumption of items, therefore can analyze massive real-world data [25].

Despite the fact that all hybrid models mentioned so far (i.e. fLDA, CTR and CTPF) benefit from the interpretable semantics of the item latent factor, they still have some limits in terms of computational cost or predictive performance. Therefore, in this thesis, we will explore and implement a recent hybrid model called ***Collaborative Topic Model for Poisson distributed ratings (CTMP)*** model which covers the limitation of CTR by considering ratings in Poisson distribution as CTPF does, while modeling contents with LDA [52]. Thus, CTMP outperforms all previous hybrid models in terms of performance. Details of CTMP formalization, graphical model and algorithm is shown in its own section on later pages.

# LDA

## Formalization

In machine learning, topic modeling is a statistical model for discovering a set of topics that occur in a collection of documents [1]. It is also considered as a probabilistic model which offers an interpretable low-dimensional representation of the documents. For many years, implementation of topic models for the purpose of document classification, corpus exploration and information retrieval has been of interest.

There are many topic modeling algorithms, among which Latent Dirichlet allocation (LDA) is the most popular one. LDA is a three-level hierarchical Bayesian model, and its basic idea is that documents are represented as random mixtures over underlying set of topics, where each topic is characterized by a distribution over words which are biased around those associated under a single theme [26]. Therefore, topic probabilities express an explicit representation of each document. This can also be explained as below:

* *Each document is a mixture of topics:*

Each document contains terms/words from some topics in specific proportions. For instance, if we consider that there are 2 topics in the whole corpus, then we might state that some document could be 75% topic A, and 25% topic B, while another document might be consisted of 30% topic A, and 70% topic B.

* *Each topic is a mixture of words:*

Each topic is expressed by the words that explain it most. For example, if we consider that there are 2 topics, namely, “sports” and “education”, in the whole corpus, then the most used words for the sports topic could be “teammate”, “win”, and “play”, while the education topic could contain the words such as “lecture“, “book” and “class”. It is necessary to note that the same words can appear on the multiple topics. For example, the word “time” could participate in both sports and education topics.

By this way, documents can overlap with each other in relationship to their contents, rather than being separated into different individual groups. Generative process and graphical model of LDA for each document in the whole corpus are described below.

## Learning

**Terminology**

* *Word* is a term of the vocabulary, and it is indexed by .
* *Document* is a series of words given by where is the *n*th word inside the document.
* *Corpus* is a collection of a total *J* documents, and it is given by
* *K* is the number of topics to be extracted from the corpus.
* is Dirichlet prior parameter on per-document topic proportions.
* is Dirichlet prior parameter on per-topic word proportion.
* is topic proportions for document *j.*
* is word distribution for topic *k.*
* is topic for *n*-th word in document *j.*
* is specific word.

Diagram

Description automatically generated

Figure : The graphical model for LDA.

Contrary to the original paper of LDA [26], a sparse Dirichlet prior can be employed in order to model the topic-word distribution, based on the idea that the probability distribution over words in a topic is skewed, with only a small subset of words having high probability. This slightly updated model is the most extensively employed variation of LDA today. A graphical model of this slightly modified LDA is shown in Figure 6.

It is also important to emphasize that the overall LDA process is a hidden generative process and according to this process, the model is assumed to generate the observed data (e.g., documents). Obviously, this was just a generative assumption to facilitate the algorithm and it does not illustrate the true process of the real data [17]. The following is how we view the generative process. Documents are represented as random mixtures over latent topics, where each topic is characterized by a distribution over all the words. LDA assumes the following generative process for a corpus *D* consisting of *J* documents:

Text, letter

Description automatically generated

Algorithm : The generative process for LDA.

As seen above, the topics that LDA algorithm tries to find from the whole corpus are treated as hidden variables. Each document of the corpus is represented in terms of topic proportions. Topic proportion is a *K*-dimensional Dirichlet random variable, and its domain is in the (*K* − 1)-simplex. In other words, *K*-vector is in the (*K* − 1)-simplex, therefore, . Also, is *V*-dimensional Dirichlet random variable, and its domain is in (*V*-1)-simplex. Therefore, . Note that the Dirichlet is an exponential family distribution and one of its important properties is that it is conjugate prior to the multinomial distribution [4]. This conjugacy between these two distributions is important, as it facilitates a development of the inference and parameter estimation for LDA which will later be discussed in Variational Inference section.

Diagram

Description automatically generated

Figure 7: The topic simplex for three topics encapsulated in the word simplex for three words.

An example geometry of the latent space of LDA is illustrated in Figure 7 above.

## Inference and Parameter Estimation

Computing the posterior distribution over the latent variables given some documents is the the main inferential problem here, because the posterior inference is intractable to compute:

[*https://courses.engr.illinois.edu/cs598jhm/sp2010/Slides/Lecture07HO.pdf*](https://courses.engr.illinois.edu/cs598jhm/sp2010/Slides/Lecture07HO.pdf)

*START from above2*

Do you learn B? Change WHOLE LDA

As normalization constant which depends on model parameters i.e., marginal probabiliy contains intractable integrals above, the resulting posterior inference also becomes intractable to compute. Therefore, as an exact posterior distribution is not possible, several approximate inference algorithms can be used for LDA. For example, Variational Inference and relevant variational EM algorithm can be used in order to learn the topics and decompose each document of the corpus according to these learnt topics [26]. Details of Variational Inference are discussed in the following section.

# Variational Inference

Variational Bayesian Methods (i.e. Variational Inference) are a group of widely used techniques in the field of statistical Machine Learning. Suppose the following probabilistic model with the joint distribution of the observed variables ***X*** and the hidden variables ***Z***:

Following the *Bayes’ Theorem*, to infer the hidden variables *Z*, the posterior inference is used as follows:

* Prior **–** is the probability of hidden variables before having seen any data. In other words, prior is the probability distribution which expresses one's beliefs about an event before some data is considered.
* Likelihood **–** is the probability of observed variables given hidden variables.
* Posterior – is the probability of hidden variables given the observed variables.
* Normalization constant  **–** is a marginal probability of observed variables or evidence, which does not depend on the hidden variables since it contains integral over all possible sets of hidden variables. It is called the normalization constant because it assures that posterior density integrates into one.

For many interesting models, the denominator is computationally intractable, mainly because of integrals. This means that the exact inference of posterior is not possible. One possible approach is to perform an approximate posterior inference, which is what Variational Inference (VI) offers. The reason behind the popularity of VI methods is that they let us solve statistical inference problem as an optimization problem and solve it for hidden parameters by maximizing their objective function. The most often used VI method is the *Mean-field Variational Inference* which will be discussed on later pages later below. But before this, let’s explore the main idea behind Variational Inference and the forms of statistical models it is applied to.

Diagram

Description automatically generated

Figure 8 The figure illustrates how Variational Inference provides an approximate solution to the inference proble .

Figure 8 above simply illustrates a technique of Variational Inference [52]. Let’s remember that the aim of VI is to approximate the true posterior distribution . To start with, one needs to posit a variational family of distribution over the hidden variables. This variational family is represented as an ellipse area in Figure 8. As seen, it is also parametrized by variational parameters Next, the goal is to find within this family of distributions, such that the corresponding approximate posterior distribution is closest to the true posterior distribution . Note that this closeness is measured by *Kullback-Leibler divergence (KL-divergence)*. Idea is to start at some initial set of variational parameters and then optimize them - i.e., minimize KL divergence [15] to find the point where is closest to .

## KL–divergence derivation

where ***L***above is called **the variational lower bound or evidence lower bound (ELBO)**. We reformulate the equation above as follow:

Because the *KL* divergence is always non-negative (i.e., we get . This proves that *L* is the lower bound on the log marginal probability of the observations. So, the final goal is to, using a coordinate ascent optimization algorithm (e.g., variational EM [43]), *maximize this lower bound L i.e., minimize KL divergence with respect to variational parameters* Note that in the formula above is fixed against all variational parameters .

## Jensen’s inequality derivation

Apart from the derivation mentioned above, there is also an alternative way to arrive at the similar conclusions using the Jensen’s inequality. This is the most widely known ELBO derivation, which clearly demonstrates why the ELBO is a lower bound on the evidence. It states for the *concave log function* as follow:

As shown above, the last term in the equation is the variational lower bound or ELBO. Essentially, it is again shown that *L* is the lower bound of the log marginal probability of the observations and our goal is to maximize the lower bound *L*:

## Mean-field VI

Furthermore, if we consider Mean Field Variational Inference, then the variational distribution over the hidden variables factorizes as follows:

Basically, the mean-field approximation makes a simplifying assumption by partitioning the hidden parameters into independent parts [42]. In other words, this assumption enforces a full independence among all hidden parameters. The reason why this independence is very useful is that, when we use a coordinate ascent optimization algorithm such as variational EM*,* this assumption enables us to compute the update rules for each unknown parameter in isolation by keeping all others fixed [43].

## Mean-field VI in conjugate models

Most importantly, it must be emphasized that there is actually specific form for statistical models in which the coordinate ascent in mean field variational inference yields *closed-form* updates. It is called exponential family conditionals or conditionally conjugate models. *Fundamentally,* *for a model to be conditionally conjugate, a complete conditional of each parameter must be in the exponential family and be in the same family as its prior [25].* A complete conditional is the conditional probability of the hidden variable given all the observed variables and other hidden variables. A generic example for the purpose of understanding conditionally conjugate models is defined in Figure 9 below [46]:

Diagram

Description automatically generated

Figure 9

where are observed variables, are local hidden variables and are global hidden variables. Note that, the main difference between local and global hidden variables is that the *i-*thdata only depends on global on and . In other words, it is not dependent on any other *j*-th local data point. Now, the factorized joint distribution of the model is as follow:

and as usual, our goal is to compute a posterior .

Firstly, for the selected generic model above, following complete conditionals must be in the exponential family:

.

In mathematical terms, an exponential family is expressed as follow:

where is natural parameter, is the sufficient statistics, is the log normalizer and is the base density. In short, if some parameter is in exponential family, then it can be written in the form above.

Secondly, complete conditional must be in the same family as its conjugate prior. This is why the exponential family was a crucial requirement in the first place. The reason is that for the exponential family of distributions, the likelihood distribution is a standardized function of the parameter, and therefore we can make conjugate priors by simulating the likelihood's form. Afterwards, when a likelihood and a prior with same exponential form are multiplied, the posterior maintains the same form, which was required as a second condition above. Fundamentally, an exponential family of distributions provides a beautiful theory around conjugate priors and corresponding posteriors and connects closely to variational inference [45]. Note that, examples for conjugate priors and their corresponding posteriors are shown in the next section below.

## Conjugate Priors and Corresponding Posteriors

The main idea is that given a likelihood distribution, one needs to select a family of prior distributions such that computed posterior distribution is also included in this family. *By this way, chosen conjugate prior enables us to estimate the posterior distribution just by updating the parameters of the prior distribution.*

Exponential family of distributions are the best example for this. The Gaussian, beta, binomial, Dirichlet, multinomial, gamma, Poisson, exponential, geometric, categorical, chi-sequared, log-normal are all in exponential family. Some pairs of conjugate distributions from exponential family are shown below in detail.

### Multinomial distribution and Dirichlet priors

Remember that the multinomial distribution is the probability distribution where outcomes from experiments are discrete and they involve two or more variables. It is also considered as a multivariate generalization of binomial distribution which involves only two outcomes. Mathematically, it is defined as follow:

where, indicates the number of times outcome *i* occurs out of *n* trials, while signifies the probability that outcome *i* occurs.

Now, let’s remember the Dirichlet distribution which is a continuous multivariate probability distribution. It is also considered as a multivariate generalization of beta distribution and defined as follow:

where is a *k*-vector with components is a *k*-dimensional random variable which is in -simplex, therefore Additionally, denotes the gamma function [37], where

According to conjugate Bayesian analysis, the Dirichlet distribution is considered as a conjugate prior to the multinomial distribution. Therefore, when we multiply the likelihood expressed in multinomial form with the prior expressed in Dirichlet form, we get the posterior distribution as follows:

which we can confirm that it has the form of Dirichlet distribution. As shown above, we can estimate the posterior distribution just by updating the parameters of the prior distribution. *Because Dirichlet is a conjugate prior for its multinomial distributed likelihood, it leads to LDA model being conditionally conjugate model, and therefore, having coordinate updates of mean-field variational inference in closed-form [26].*

### Poisson distribution and gamma priors

Let’s now take into account the Poisson distribution from discrete exponential family distributions:

where conjugate prior to this Poisson likelihood must also have the form of Poisson distribution:

This conjugate prior can be easily expressed as *gamma distribution*:

where

denotes the gamma function above. Now, prior-to-posterior update is as follow:

where we can confirm that it has the form of *gamma distribution* which was our the intension from the beginning. Essentially, as seen above, choosing Gamma conjugate prior and multiplying it to Poisson likelihood yielded an the posterior inference which also has Gamma distribution, and therefore we can estimate the posterior distribution just by updating the parameters of the prior distribution, while successfully ignoring the intractable marginal probability in denominator. In other words, if are all identically independently distributed, then conjugate prior for is Gamma(,) and the respective posterior, which is proportional to likelihood multiplied by prior becomes Gamma(+, ). *Because gamma is a conjugate prior for its Poisson distributed likelihood, it leads to CTPF model being conditionally conjugate model, and therefore, having coordinate updates of mean-field variational inference in closed-form [25].*

## Mean-field VI in non-conjugate models

So far, from the previous sections, we have seen that if the model is conditionally conjugate, we can easily use mean-field variational inference to have a closed-form solution. Nevertheless, not all models are conditionally conjugate models; some are non-conjugate. In these models, the mean-field VI approach cannot be applied directly, and practitioners must create their own case-specific variational algorithms. In later sections, we will see that the CTMP model is among these non-conjugate models where its authors developed a co-ordinate ascent algorithm to fit it [53].

# OPE and BOPE

Maximum a posteriori probability (MAP) estimation has a significant impact on doing posterior inference (i.e., estimating hidden parameters) in many probabilistic models. Especially, many interesting MAP problems are continuous, non-convex, and intractable. In the field of non-convex optimization, there have been a variety of different techniques such as Frank–Wolfe [29], Natasha2 [30], Stochastic Majorization-Minimization [31], Concave-Convex procedure [32] which aim to solve the MAP problem [27]. However, non-convex optimization is NP-hard, and the techniques mentioned above may not provide a viable solution for the MAP problem, because they disregard its special underlying structure. Therefore, for solving non-convex MAP problems with a state-of-the-art convergence rate, we will explore two efficient algorithms *Online Maximum a Posteriori Estimation* ***(OPE)*** [27] and its regularized, general, and more flexible version *Bernoulli randomness in Online maximum a Posteriori Estimation* ***(BOPE)*** [28]. First, we introduce MAP estimation as the following task:

(1)

where we denote as the hidden variable, D as the observed data, and denotes domain. Note that there also have been proposed many algorithms which directly try to estimate a full posterior distribution mentioned above, i.e., Variational Bayesian Methods (VBM) [33], Collapsed Gibbs Sampling (CGS) [34], Hessian Approximated Markov Chain Monte Carlo (HAMCMC) [35]. However, these methods provided suboptimal solutions along with a slow convergence rate. We continue by using *Bayes’ Theorem*:

(2)

where we denote as the likelihood of *D,* as x’s prior, and as s marginal probability. Using (2), we rewrite (1) as follows:

(3)

We will focus on the conditions where MAP problem is continuous and non-convex, hence intractable, i.e., is non-convex over the continuous compact domain [28]. As previously mentioned, MAP problem (3) will be treated as an optimization problem. Therefore, objective function defines the complexity of this optimization problem where and . So, our problem (3) becomes as a non-convex constrained optimization problem as follow:

(4)

So, in the following sections we will discuss OPE and BOPE algorithms for solving the optimization problem shown above.

## OPE for solving MAP problem

Online Maximum a Posteriori Estimation (OPE) is considered a type of iterative optimization algorithm, which is the stochastic version of Frank–Wolfe algorithm. The biggest advantage of OPE is that it has a faster convergence rate of to local maximal point compared to the existing stochastic algorithms for nonconvex problems, where signifies the number of iterations during training of its following algorithm [27]:

Text

Description automatically generated

Algorithm : Online Maximum a Posteriori Estimation (OPE) algorithm.

As illustrated above, the OPE algorithm solves a linear program at each iteration, i.e., directing the optimization solution to the good vertex in the convex hull of the compact input domain. In more detail, what OPE does is to develop a sequence of stochastic functions that approximates to by alternatively selecting an from uniformly randomly at each iteration *t*. As proved in its original paper [27], converges to as. .

Despite of fast convergence rate, OPE still has a limitation. As stated in the algorithm, either likelihood or prior is being used during the construction of the approximation function However, when dealing with new samples, we can rely on likelihood if we have seen enough data or rely on prior if there is a lack of data.

## BOPE for solving MAP problem

To overcome the OPE’s limitation mentioned above, a new approximation technique to OPE has been proposed as BOPE which retains all theoretical guarantees of OPE’s convergence while being more general and flexible by using Bernoulli distribution and two stochastic bounds [28]. BOPE solves the equation (4) above by employing Bernoulli distribution with parameter which is supposed to replace the uniform distribution of OPE on likelihood and prior. Furthermore, as seen in Algorithm 6 below, during the procedure, two stochastic sequences are constructed and they converge to objective function : the lower sequence , and the upper sequence . It is worth noting that the Bernoulli parameter determines an impact of likelihood and prior on and . So, during each iteration, using both and stochastic sequences provide further information about , therefore increasing the chances of converging to more quickly [28]. Both lower and upper sequences are guaranteed to converge to as

Text

Description automatically generated

Algorithm : Bernoulli randomness in Online maximum a Posteriori Estimation (BOPE) algorithm.

It’s important to note that one of the reasons why BOPE outperforms OPE is that we can create variants of BOPE by altering the Bernoulli parameter . In addition to this, another property of BOPE is that to prevent overfitting of the learning process which is a widespread issue that affects all machine learning techniques, BOPE employs implicit regularization. Specifically, according to the original paper [28], Bernoulli randomness operates as a regularizer and BOPE uses an implicit prior that is stochastically vanishing with respect to iterations *T*. Note that this implicit prior is not the same as the prior used in MAP estimation. This implicit regularization is very critical, especially in recommender systems where most of the datasets are *sparse* which makes the models prone to overfitting. Therefore, using BOPE instead of OPE in Collaborative Topic Model for Poisson distributed ratings (CTMP) will facilitate the learning procedure and prevent the overfitting.

# Collaborative Topic Model for Poisson distributed ratings

It is worthwhile mentioning that CMTP has been tested in variety of fields where real-world recommendation is the most challenging one, and it has been seen that it outperforms the previous existing models significantly with its main competency being in recommending scientific articles and commercial product recommendations. Recommending movies is also among these, and indeed, this is what we will implement and test in this thesis.

In this section, we describe learning, prediction phases and key properties of CTMP – hybrid, scalable and interpretable probabilistic content-based collaborative filtering model.

## Formalization

Before diving into technical parts, let’s provide some notations:

* *U:* represents the number of users inside the dataset
* *J:* representsthe number of items inside the dasaset
* describes the bag-of-word representation for each item *j* where expresses the frequency of term/word in item *j.*
* represents the vocabulary size of the corpus.
* describes the dataset where is a rating provided by user *u* to item *j,* while is the bag-of-word representation of item *j* as already explained above. represents the ratings given to movies by users. Every rating is expressed as binary 0 or 1. If user *u* liked an item *j,* then . On the contrary, if the user *u* do not know about the item *j* or do not like it, then .
* *K:* represents the number of topics inside corpus.
* describes the topic representation. More precisely, every topic *k* is a distribution over the vocabulary. It is described as and . Note that, lies in the (*k* – 1)-simplex.
* describes the topic proportion of the items. is the vector of the distribution on topics for item *j,* and . Note that, lies in the (*k* – 1)-simplex.

To learn the topics , we use the Latent Dirichlet allocation (LDA) and its Expectation-Maximization (EM) approach which was described in the respective section of LDA. Furthermore, by learning the topic proportion of each item we later describe each item and user in the *K*-dimensional space. Note that these learning procedures will be explained in the further sections below.

Now, we present latent factors for each user and item in terms of *K*-dimensional vectors and , respectively. As discussed in [[[Related Work Section]]], the reason why we consider rather than as the latent factor for item is that in order to have better recommendation system, we allowed an offset between and which accounts for the user-specific preference on the item content that alone can not capture. Therefore we denote that where is an offset term which has Gaussian distribution. Note that in the formula above represents an *K*-dimensional identity matrix , and is a regularization parameter. So, we have .

Furthermore, as shown below, the ratings and users’ latent factors are modeled by Poisson and Gamma distributions, respectively. To put everything together, the generative process and graphical model of CTMP is as follow:

Text, letter

Description automatically generated

Algorithm : The generative process for CTMP.

Note that steps 2(a-b) correspond to LDA.

**CTMP Graphical Model**

Diagram

Description automatically generated

Figure 10: The graphical model for CTMP.

## Learning

Full posterior of latent variables is given as follow:

( )

The problem with this posterior is that it is intractable, and therefore exact inference is impossible. To tackle this problem, we have two methods:

1. Maximum A Posteriori (MAP) for point estimation
2. Bayesian Learning such as MCMC Sampling or Variational Methods for approximate inference

As the prior and posterior distributions of hidden variables and are not conjugate in CTMP model, using Variational Inference Methods to infer these hidden variables does not get us closed-form solution. Therefore, we will carry out the point estimates of and using MAP – coordinate ascent algorithm developed by authors of original paper of CTMP [53].

Furthermore, in order to facilitate the learning, authors added a new auxiliary variable *y,* where Poisson( and . Note that we approximate the posterior of and via mean-field variational inference [\*\*5\*\*]. The mean-field variational inference is a type of Variational Bayesian Method which allows to re-write a statistical inference problem as an optimization problem [42]. Therefore, we can convert the inference problem of CTMP into a full optimization problem where the single objective function which needs to be maximized is as follow:

( )

As shown in *Equation 2*, the term of integration and summation over the whole space causes optimization to be intractable. However, Variational method [\*\*5\*\*] also tackles this problem which will be discussed in detail below.

Note that has Poission distribution, and the *K*-dimensional vector follows multinomial distribution: [6]. So, we get the variational distribution as follow:

( )

where such that is variational parameter of , and are variational parameters of . Note that . Now we get the *evidence lower bound (l)* by applying Jensen’s inequality:

( )

Note that before learning the hidden parameters, are considered fixed parameters in the model.

Next, the lower bound *l*( shp, rte, ) is maximized with respect to shp, rte, . According to Appendix A of the original CTMP paper [53], the terms are expressed in detail as follow:

( )

## Learning Parameters

*Equation 5* is the optimization problem and as mentioned before we solve it by coordinate ascent algorithm. CTMP algorithm for learning and is demonstrated below in Algorithm 7:

Text, letter

Description automatically generated

Algorithm : The algorithm for the CTMP model.

**Learning** . In order to find the point estimate of local topic proportion where

( )

we use Bernoulli randomness for Online Maximum a Posteriori Estimation (BOPE) algorithm [\*\*7\*\*]. Note that in original paper of CTMP [53], authors used a simple Online Maximum a Posteriori Estimation (OPE) algorithm and this difference is the most important one between this and original CTMP paper. By using Bernoulli randomness, BOPE has a faster convergence rate, is more general and flexible compared to OPE. Furthermore, “BOPE implicitely employs a prior which plays a regularization role”[\*\*7\*\*]. Comparison of BOPE with respect to OPE has been carried out in BOPE section in detail. Include this in comparison::: such properties are not found in the common approximate posterior inference methods for topic models, such as Gibbs sampling and variational Bayes. [[[or discuss here???]]] Note that both algorithms tries to lead the solution of the optimization to the closed neighbours of the vertices in the convex hull of compact input domain and they have a fast convergence rate of along with proven quality bound [\*\*8\*\*]. Furthermore, as mentioned earlier too, every topic proportion holds and lies in the (*k* – 1)-simplex. BOPE algorithm for learning is described in Algorithm 2.

**Text

Description automatically generated**

Algorithm : Learning using BOPE.

**Learning** If we know the estimates of other hidden variables, then solving analytically is possible because the objective function regarding the is a *concave* function.

( )

The partial derivative of function with respect to , i.e. for all *k,* is the estimate of . This is also so called the stationary point of . Because is the quadratic function in terms of we can Vieta’s formula for the analytical derivation of the function’s root as follow:

( )

Where,

( )

**Learning** . We use the mean-field variational inference for approximating the conditional posterior of and as in [\*\*9\*\*]. So, in order to solve for the variational parameters of and which are , we solve for the stationary point of with respect to each variational parameter, while holding the others same. The update expression of variational parameters is given in Table/Figure?? [\*\*x\*\*] below. The detailed derivation of these expressions is described in Appedix B and Appendix C. One of the biggest advantages of CTMP algorithm is that whenever , we get and and therefore, we only have to update over non-zero ratings (. This property of our model diminishes the training time significantly, so the total training time is much lower than of other models such as CTR, especially whenever the rating dataset is highly sparse. Because, during each epoch of training, we only consider the positive ratings for updating the expression of and skip all *zero* ratings.

Variational parameter updates

Text

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Figure : The updates of variational parameters.

Note that the function in the update expression of denotes the digamma function [\*\*10\*\*]:

( )

where denotes the gamma function.

**Learning** . So far, we have provided the update expression of the variables regarding both documents and users such as . Now, we have to do the remaining task which is to solve for First, we express the log likelihood of the whole items corpus *C* as in [\*\*10\*\*]:

( )

By using Jensen’s inequality, the last term is derived, due to the fact that Next, the lower bound of is maximized with respect to as in [\*\*10\*\*]:

( )

where Note that the each is separable from each other inside the objective function of . So, we can solve solve each individually. This is carried out by considering the Lagrange function and setting its derivatives to 0 which results in formula of as follow:

( )

## Prediction

We rank the items in order to generate recommendations for each user *u* based on their predictive score after we have learned all the parameters. Because the ratings in the dataset are discrete Poisson variables, can be the expectation of the rate parameter given the observed data i.e. as in CTPF[\*\*CTPF\*\*]. However, the derivation in CTMP is a bit different because CTMP neither aims to approximate solely by point estimate nor require **conjugacy** between the complete **conditional distributions** for the inference as CTPF does [53] :

( )

Note that only is the MAP estimation of complete conditional distribution. Furthermore, is nearly the expectation over the respective variational distributions of ’s:

( )

Note that both and are the estimation of variational parameters that we learned in above section.

## Key properties

1. CTMP implements a fast and scalable coordinate ascent algorithm because it is non-conjugate model.
2. According to the experimental studies we conducted on different real-world datasets in this work, we observe the sparse estimates of topic mixtures can be achieved via learning in spite of the fact that the model specification does not encourage so. Note that the sparsity is very critical property as it leads to an efficient storage of a data by offering compact content representation. (Note that we mostly get sparse estimates rather than dense compared to original paper)

1) by modelling ratings as discrete Poisson variables, all zero entries can be discarded in the estimation of user latent factor η (see Algorithm 1). Therefore, the learning of CTMP is particularly efficient with highly sparse datasets.

2) Furthermore, as we will show by the empirical results in Section 4.2, OPE updates also recover sparse solutions for the estimates . Sparsity is an important property of document-specific topic mixture. On one hand, it reflects closely the number of topics in a real-life document, which would express only a few topics in details, rather than small bits of 50 or 100 topics. On another hand, it largely reduces memory for storage of the content representation, allowing efficient computation of other tasks in industrial settings (e.g. near real-time recommendation for products of the same topics/categories in which the

users are interested).

3) Add 3rd contribution of CTMP in Related work

**.**

**.**

## Interpretable user profiles

**.**

**.**

**.**

## Evaluation

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**.**

# Empirical Studies

We have carried out empirical studies on performance of CTMP algorithm using tech stack of Python, SQL, Git. The repository which contains all the source code is at: <https://github.com/buzzer4mornin/CTMP-ThesisProject>

In order to carry out an empirical study of CTMP algorithm under real-world recommendation context, we use two different datasets, namely **MovieLens 20M** and **NETFLIX**. They are well-known datasets which have been used a lot and considered as stable benchmark datasets for the research purposes. Note that, for our empirical study, we use slightly modified versions of these datasets compared to original versions. Their modified versions are arranged and put on Oracle Database by Michal Kopecky.

MovieLens 20M

**[take citation from https://files.grouplens.org/datasets/movielens/ml-20m-README.html]**

Original dataset describes 5-star rating activity from https://www.movielens.org which is a movie recommendation service. It was created on October 17, 2016 and its raw data consists of movies, users, ratings, and tag applications [47]. Each user is only represented by an id, therefore no other information (e.g. demographic) is taken into account. This dataset’s modified version is arranged by Michal Kopecky and it is also equipped with additional “TT” identifiers which help us to fetch the plots movies from IMDB table. These plots are essential part of our modified dataset , because we will use the plots instead of the tag applications for the document representation. This is the important difference in empirical studies between our work and original CTMP paper, and because of this, we will see slight differences in results too.

NETFLIX

Original dataset was made available by Netflix company for the competition that was held on September 21, 2009, for the best collaborative filtering algorithm to predict user ratings for movies, based on previous ratings without any other information about the users [49]. In other words, the users were not identified, they were only represented by id numbers for the contest. After the competition the dataset became open-sourced, and has since been used by many researchers as a stable benchmark dataset. Along with MovieLens 20M dataset, this NETFLIX dataset’s modified version is also arranged by Michal Kopecky and it is also equipped with addition ”TT” identifiers which help us to fetch the plots movies from IMDB table. These plots are essential part of our modified dataset , because we will use the plots instead of the tag applications for the document representation. This is the important difference in empirical studies between our work and original CTMP paper, and because of this, we will see slight differences in results too.

## Data pre-processing

After fetching both datasets from Oracle database, we continue with data pre-processing steps for transforming the raw data into a useful and efficient format. By utilizing the data wrangling techniques, we make the individual dataframes of user, movie and ratings as below:

|  |  |  |
| --- | --- | --- |
| MovieLens 20M | | |
| df\_user | df\_movie | df\_rating |
|  |  | Text  Description automatically generated with low confidence |

Table 2

|  |  |  |
| --- | --- | --- |
| NETFLIX | | |
| df\_user | df\_movie | df\_rating |
|  |  |  |

Table 3

Note that constructed dataframes are still raw, meaning that ids in df\_user dataframes are not sequentially consistent, some movies in df\_movie dataframes do not have plots (N/A) and ratings in df\_rating dataframes are in 0.5-5.0 scale. So, next section uses these raw datasets for turning them into useful formats.

First, we start cleaning process by representing ratings in binary form in ratings table. This is done by converting ratings bigger or equal to 4 into 1 and remanings into 0. This way, it is presumed that users like or dislike the given movie. As previously stated, we do this because the CTMP model implies that ratings are Poisson distributed. CHECK WHERE IS THIS SENTENCE 🡪 “0/1 helped us to skip 0 rating rows during traninig”

Next, we drop movies that has N/A plot. We also drop duplicates and inconsistent movies. Note that dropping movies results in removal of some rows related to these movies in ratings dataframe. Size of each dataframe before and after preprocessing is shown below:

|  |  |  |  |
| --- | --- | --- | --- |
| **Dataframe** | **Dataset** | **Raw rows size** | **Final rows size** |
| df\_user | MovieLens 20M | 138,493 | 138,493 |
| NETFLIX | 479,870 | 479,870 |
| df\_movie | MovieLens 20M | 27,278 | 25,900 |
| NETFLIX | 9,324 | 7,882 |
| df\_rating | MovieLens 20M | 20,000,263 | 19,994,181 |
| NETFLIX | 90,217,939 | 82,725,788 |

Table 4

### Vocabulary Extraction

We now aim to extract a vocabulary separately for MovieLens 20M and NETFLIX using movie plots in respective df\_movie dataframe. The reason why we want to have vocabulary is that each movie should be numerically represented for our Python model, therefore the vocabulary will be used for building this representation. Before we begin, we merge each movie plot sequentially to get a single long text. Then we extract the vocabulary from this single text. Note that, extracting the vocabulary requires careful investigation, because, for each distinct word, we should decide whether this word should be included in the vocabulary or not. Below are the steps that we follow for vocabulary extraction:

* Removing stop words – they are commonly used words (such as “a”, “an”, “are”, “the”, “about”) of a language which do not add much meaning to a sentence. They can be safely ignored by keeping them out of vocabulary. There are many Python libraries that provide a list of stopwords for many languages. We used NLTK Library’s stop words on English language.
* Removing words of length less than 3 – these words can also be ignored. In English language, there rarely is a word of length less than 3 which adds a meaning to a sentence.
* Removing words with numbers or underscores – we will not include these words into the vocabulary.

It is also worth mentioning that in the domain of Natural Language Processing, well-known techniques such as Stemming and Lemmatization are utilized for a purpose of text normalization. These techniques help a lot when it comes to filtering unwanted words during vocabulary construction. Stemming technique removes the suffix from a word and therefore reduces it to its root/stem. For example, stem of the word “count” is just “count”. Stemming the words such as “counts”, “counting” and “counted” will result in “count”. In contrast to stemming, lemmatization goes beyond word reduction by evaluating a language’s whole lexicon for applying a morphological analysis to words. In other words, lemmatization technique does not simply chop off inflections as stemming does, but instead it relies on a lexical knowledge base for obtaining the correct base forms of words. For example, given the word “mice”, “ran” lemmatization convert them into “mouse” and “run”, respectively. Essentially, by using stemming and lemmatization techniques, one can construct a more concise and accurate vocabulary. However, we don’t use these techniques during our vocabulary construction. The reason behind this is that, according to the recent works, stemming is claimed to reduce the model fit and negligibly affect topic coherence [48]. Utility of lemmatization on topic models is also vague and rather needs further investigation. Therefore, we avoid stemming and lemmatization of the corpus as a data pre-processing step.

**Snippet**

A picture containing table

Description automatically generated A picture containing graphical user interface

Description automatically generated

Figure 12 Figure 13

The figure above is a snippet from the resulting vocabulary of MovieLens 20M dataset. We observe that the word “accident” has many forms with different suffixes. This is because we did not use stemming and lemmatization. We believe that letting vocabulary be rich like this will later facilitate model fit into the corpus. Overall, after extraction, MovieLens 20M vocabulary contains 59,110 words, while the NETFLIX dataset vocabulary has 34,177 words.

### Movie Representation

After extracting vocabulary from corpus, we use it for numerical representation of each movie/plot. Reason for this is that movies should be in a such form that models (e.g., LDA, CTMP) can work with. Therefore, one way is to succinctly represent each movie as a sparse vector of word counts as below:

[M] [word\_1]:[count] [word\_2]:[count] ... [word\_N]:[count]

( 16 )

where [M] denotes the number of unique words in the movie plot. Furthermore, [word\_i] is an integer and it denotes an index of the word inside vocabulary, and [count] associated with this word denotes how many times the word appears in the movie plot. Note that if any word of the plot does not appear on corpus vocabulary, then we disregard it in movie representation.

**Snippet**

Text

Description automatically generated

Figure 14

It is critical to remember that, the authors of both the original CTMP paper [53] and BOPE paper [28] implemented their models using tags for movie representation rather than plots. This is the most significant distinctions between our work and theirs. [[[Precision unmatchiness between our work and original CTMP paper is probably because of this]]]. During the evaluation of results, we will compare our model’s output against theirs and determine whether using the movie plots resulted in different predictive performances.

### Memory usage reduction

This technique helps us to reduce the memory usage of dataframes. It iterates through all integer or float columns of given dataframe, and modifies the data type to reduce the memory usage.

For instance, let’s assume an integer column given as int64 datatype. The way function decided whether to reduce the memory or not is by first getting minimum and maximum values this column. Next, it checks whether these minimum and maximum values are in the range of int32, int16 or int8 datatypes, and if they are, then it will convert the column’s int64 datatype into the respective lower datatype. The same procedure applies to float columns as well. As proof, when we used this technique to reduce the memory storage of df\_rating dataframe of MovieLens 20M dataset, it achieved 58.3% reduction in memory.

## Model Fit

After preparing the corpus vocabulary, converting ratings into binary form, numerically representing movies/plots and using memory reduction technique on all dataframes, we now continue with running and fitting the CTMP model using these dataframes. Note that we will have separate models for MovieLens 20M and NETFLIX. For the model fitting phase, we will use vocabulary, numerical movie representations and settings file.

We use **stratified 5-fold cross-validation** of ratings for training and testing the model. Generally, k-fold cross-validation is widely used technique that assesses the efficacy of machine learning models since it produces a less biased estimate of their effectiveness. The parameter *k*indicates the number of folds that a given dataset is to split into. Pseudo-algorithm for model fitting with k-fold cross-validation is as below:

For each unique fold:

1. Take this fold as test data
2. Take the remaining folds as training data
3. Fit a model on the training data and evaluate its performance on the test data
4. Keep the evaluation score and start from beginning using another unique fold

At the end of entire train-test circle, we average the evaluation scores kept for each fold to obtain a final evaluation score. For our case, we utilize stratified k-fold cross-validation, which has the same purpose as standard k-fold cross-validation but produces stratified folds which are made by preserving the percentage of samples for each class. This way, both train and test folds will contain information about each user, ensuring that the model will be trained and tested with all users. An example of stratified 5-fold cross validation with small dataset is shown below:

Table

Description automatically generated

Figure 15

### Setting Parameters

Along with vocabulary, and numerical movie representations files, we also have a settings file containing the model parameters. It helps us to set the model’s parameters on a per-run basis. Example snippet of settings files and their content are shown below;

Text

Description automatically generated Text

Description automatically generated CHANGE SCREENSHOTS

|  |  |
| --- | --- |
| **Parameter** | **Stands for** |
| *num\_movies* | Number of documents in the corpus |
| *num\_words* | Number of words of vocabulary |
| *num\_topics* | Number of topics we want to discover from corpus |
| *user\_size* | Number of users in dataset |
| *tops* | Number of top words to extact from each topic for eyeballing/analyzing the quality of topics learnt by the model |
| *lamb* ( | Offset precision for documents |
| *e, f* | Gamma priors |
| *alpha* ( | Dirichlet prior |
| *iter\_infer* | Number of inference steps during an estimation of document proportions within BOPE |
| *iter\_train* | Number ofepochs to train the model |
| *bernoulli\_p* | Bernoulli parameter for BOPE |

Table 5

Note that the only parameters which are entirely dependent on the dataset are *num\_movies, num\_words*, and *user\_size*. Thus, they reflect according to the size of MovieLens 20M and NETFLIX datasets.

Other parameters are modifiable, and for our case, their domain is as follow:

|  |  |
| --- | --- |
| **Parameter** | **Domain** |
| num\_topics | {50, 100} |
| tops | {10} |
| lamb |  |
| e, f | {0.3} |
| alpha | {1, 0.1, 0.01} |
| iter\_infer | {50, 100} |
| iter\_train | {25, 50, 100} |
| p | {0.7, 0.9} |

Table 6

We picked Bernoulli parameter and did not include 0.5 since this parameter replaces the uniform distribution of OPE on likelihood and prior, and adjusting it to 0.5 would result in a BOPE algorithm approximating OPE which is not our goal.

### Running the model on Google Cloud

After we have prepared all necessary input data and set input model parameters, we will run the CTMP models on Google Cloud’s Compute Engine which is compute service that lets us create and run virtual machines on Google’s infrastructure. Google Cloud offers a wide range of compute engine variants with a variety of configuration options. For our model, we require a large amount of RAM memory because the dataset are large and we perform 5-fold cross-validation in parallel. Therefore, the most relevant machine type for our work is as follow:

|  |  |
| --- | --- |
| **Google Cloud Virtual Machine** | |
| Machine family | GENERAL-PURPOSE |
| Series | N1 |
| Machine type | n1-highmem-16 |
| CPU | 16x Intel® optimized with Intel® MKL and CUDA 11.0 |
| RAM memory | 104 GB |
| Boot Disk | Standard Persistent Disk 100GB |

Table 8

Generally, time per iteration of the model depends on which dataset and parameters have been selected and the effectiveness of the code written in Python. We have optimized our code by utilizing many techniques to reduce the time complexity and increase the execution speed of the CTMP model. Finally, we arrived at the model that performed as follows:

|  |  |  |
| --- | --- | --- |
| **Dataset** | **Average time per iteration** | **Total average time** |
| MovieLens 20M | ~ 200 seconds | ~ xx hours |
| NETFLIX | ~ (300 max) seconds | ~ xx hours |

netflix

25/175 -- movielens

Reason why NETFLIX takes more time is its # of users.

## Model Evaluation

After running and fitting the model on both datasets with various parameters on each time, we now continue with evaluating the performance of these CTMP models. Performance evaluation will be divided into the following parts:

1. interpretability of learned topics
2. predictive performance
3. sparsity in topic proportion estimates
4. perplexity???? Add new section?
5. sensitivity to hyperparameters

### Interpretability of topics

MovieLens 20M

Lets eyeball the top10 words of first 10 topics learnt by CTMP on MovieLens 20M

with parameters {alpha=1, lamb=1, K=100)

|  |  |
| --- | --- |
| Topic 1 | man, father, young, wife, mother, daughter, son, brother, new, old |
| Topic 2 | new, life, one, love, finds, girl, school, two, job, friend |
| Topic 3 | get, new, johnny, one, house, wife, town, police, killer, back |
| Topic 4 | one, two, begins, friend, girl, mother, man, young, life, friends |
| Topic 5 | new, life, world, story, young, love, find, two, help, husband |
| Topic 6 | life, one, new, father, get, woman, friend, two, mother, family |
| Topic 7 | film, documentary, world, story, life, one, new, interviews, footage, history |
| Topic 8 | life, family, town, one, love, home, old, new, war, man |
| Topic 9 | young, family, one, husband, police, mother, father, wife, finds, woman |
| Topic 10 | town, love, new, family, young, father, man, old, two, find |
| Change to 1.2.3.4.5 … 95,96,97,98,99,100. |  |

NETFLIX

Lets eyeball the top10 words of first 10 topics learnt by CTMP on NETFLIX

with parameters {alpha=1, lamb=1, K=100)

|  |  |
| --- | --- |
| Topic 1 | wife life one team man goes story esmeralda get old |
| Topic 2 | year town takes life friends family michael one must son |
| Topic 3 | life father mother new young family scott son finds friend |
| Topic 4 | find killer new town victims years become one back help |
| Topic 5 | new one time children island get years girl find long |
| Topic 6 | man new one bill father york wife son two love |
| Topic 7 | life love two time get one new find jonathan george |
| Topic 8 | new agent drug back fbi cia help american two friends |
| Topic 9 | find new back one love also two family life get |
| Topic 10 | life man take wants new young back money help school |

### Predictive performance

Model's predictive performance is measured by its ability to recommend in-items and cold-items. Note that in-items are those containing information from user ratings while cold-items do not possess such information. Thus, recommending items which are all in-items is referred as ***in-matrix prediction,*** whereas recommending both in-items and cold-items is called ***out-of-matrix prediction***. Both prediction types are evaluated by recall and precision for all ….

Talk about formulas of them and how in-matrix prediction and out-of-matrix prediction evaluated for our case.

Chart, line chart

Description automatically generated Chart, line chart

Description automatically generated

### Sparsity

Graphical user interface, chart, application, treemap chart

Description automatically generated

### Sensitivity to hyperparameters

.

.

.

## Transfer Learning between MovieLens 20M and NETFLIX

**Movie**: The Naked City (1948)

**Original Plot**: “Amid a semi-documentary portrait of New York and its people, Jean Dexter, an attractive blonde model, is murdered in her apartment. Homicide detectives Dan Muldoon and Jimmy Halloran investigate. Suspicion falls on various shifty characters who all prove to have some connection with a string of apartment burglaries. Then a burglar is found dead who once had an elusive partner named Willie. The climax is a very rapid manhunt sequence. Filmed entirely on location in New York City.”

Note that, on both MovieLens 20M and NETFLIX datasets, this movie is included. We begin by separately training a CTMP model on both datasets. Modifiable model parameters of settings files are as follow:

|  |  |
| --- | --- |
| CTMP on MovieLens 20M | CTMP on NETFLIX |
| **num\_topics: 100** | **num\_topics: 50** |
| tops: 10 | tops: 10 |
| lamb: 1 | lamb: 1 |
| e: 0.3 | e: 0.3 |
| f: 0.3 | f: 0.3 |
| alpha: 1 | alpha: 1 |
| iter\_infer: 100 | iter\_infer: 100 |
| iter\_train: 50 | iter\_train: 50 |

Top-10 for each topic for both MovieLens and NETFLIX

(i.e. topn\_outpu.txt)

Put screenshots from both top10 words, and discuess that they make sense. Then move on

After 50 iterations of training, both models are fit to the respective dataset. Let’s take a look at top10 words of the topics learnt from MovieLens 20M and NETFLIX datasets.

Following that, we compare the topic proportions estimated by both models for “The Naked City (1948)” movie. Estimated topic proportions are as follow:

Chart, diagram, histogram

Description automatically generated

As illustrated in the figure above, CTMP model trained on MovieLens 20M estimates that “The Naked City (1948)” movie has of 92.9% of content related to topic 77 and 7% of content related to topic 7 (the remaining 0.1% comes from other topics, which we disregard because it is insignificant). Now, let's look at what those significant topics are about by looking at the top-10 words of each topic learnt by the model:

* **Topic 77 (92.9%)** – killer, police, detective, serial, one, murder, murders, case, young, two.
* **Topic 7 (7%)** – film, documentary, world, one, life, new, interviews, story, history, footage.

We can see that model estimates that the movie will be mostly centered on the keywords murder, detective, police, killer, and so on. Moreover, it also provides little topic proportion about documentary, cinema, footage, etc. When these results are compared to the movie's original plot, it is clear that the model accurately estimates the topics proportions of the movie. Topic 7 being assigned a 7% share is also a good sign, because related sentences to this topic were only mentioned in the first and last sentences of the original plot, and they were not particularly relevant to the broader narrative.

On the other hand, a CTMP model trained on the NETFLIX dataset suggests that "The Naked City (1948)” movie contains 96.9% conten associated to topic 11 and 3% content belonging to topic 48. (the remaining 0.1 percent comes from other topics, which we disregard because it is insignificant). Let's take a look at what those important topics are about by checking the top-10 words of each topic learnth by the model:

* **Topic 11 (96.9%)** – police, murder, killer, detective, new, man, life, one, crime, father.
* **Topic 48 (3%)** – jimmy, new, one, film, life, father, alison, time, george, make.

It is apparent that this model, too, accurately approximates the movie's content by giving the most weight to Topic 11, which is concentrated on the phrases police, murder, killer, and so on. 3% being assigned to Topic 48 is again a good sign that this model, too, could associate another negligible topic regarding to the first and last sentences of original plot.

Finally, when we compare the outputs of the two models, we can see that they both learned the topic proportions ***accurately and similarly***. They both gave extremely high proportion to the topic, which covers the majority of the movie’s content, and very little weight to the topic, which is not the primary motivation for the movie's plot.

::::::NEW NOTES:::::

* Add Figure x inside texts when it is referred.

Alggorithmic changes:

* Change circles of user-doc. Why? Ans: because of initializations
* Phi normalization

**put (num\_iter, num\_infer) somewhere**

* + sum(mu) =! 1 but it worked, comment on this.
  + CLOB (Large Character Strings) while fetching plots
* Small type during mu update, why we decided to change it, and the challenges it produced when implemented wrongly

|  |  |
| --- | --- |
| **For both MovieLens 20M and NFLX** | |
| **Parameters fixed** | **Parameters changed** |
| p=0.7; k=50; alpha=1 | lamb (1, 10, 100) |
| p=0.7; k=50; alpha=0.1 | lamb (1, 10, 100) |
| p=0.7; k=50; alpha=0.01 | lamb (1, 10, 100) |
| p=0.7; k=100; alpha=1 | lamb (1, 10, 100) |
| p=0.7; k=100; alpha=0.1 | lamb (1, 10, 100) |
| p=0.7; k=100; alpha=0.01 | lamb (1, 10, 100) |
| p=0.9; k=50; alpha=1 | lamb (1, 10, 100) |
| p=0.9; k=50; alpha=0.1 | lamb (1, 10, 100) |
| p=0.9; k=50; alpha=0.01 | lamb (1, 10, 100) |
| p=0.9; k=100; alpha=1 | lamb (1, 10, 100) |
| p=0.9; k=100; alpha=0.1 | lamb (1, 10, 100) |
| p=0.9; k=100; alpha=0.01 | lamb (1, 10, 100) |

**NFLX**

Parameters fixed Parameters changed

p=0.7, k=50, alpha=1 lamb (1, 10, 100) 96.3344%, 91.3271%, **61.0266**%

p=0.7, k=50, alpha=0.1 lamb (1, 10, 100) 97.9888%, 97.8130%, 91.3925%

p=0.7, k=50, alpha=0.01 lamb (1, 10, 100) 97.9878%, 97.8059%, 91.3563%

p=0.7, k=100, alpha=1 lamb (1, 10, 100) 98.3988%, 95.8885%, **80.2701**%

p=0.7, k=100, alpha=0.1 lamb (1, 10, 100) 98.9981%, 98.9496%, 95.9888%

p=0.7, k=100, alpha=0.01 lamb (1, 10, 100) 98.9975%, 98.9480%, 96.1034%

p=0.9, k=50, alpha=1 lamb (1, 10, 100) 95.8686%, 89.9838%, **56.2083**%

p=0.9, k=50, alpha=0.1 lamb (1, 10, 100) 97.9947%, 97.8990%, 91.8691%,

p=0.9, k=50, alpha=0.01 lamb (1, 10, 100) 97.9952%, 97.9076%, 92.3017%

p=0.9, k=100, alpha=1 lamb (1, 10, 100) 98.2607%, 95.1714%, **77.6233**%

p=0.9, k=100, alpha=0.1 lamb (1, 10, 100) 98.9992%, 98.9796%, 96.4929%

p=0.9, k=100, alpha=0.01 lamb (1, 10, 100) 98.9995%, 98.9805%, 96.5180%

MovieLens (float32 used . Delete it when NFLX is used)

p=0.7, k=50, alpha=1 lamb (1, 10, 100) 93.9395%, 90.2419%, **66.3825%,**

p=0.7, k=50, alpha=0.1 lamb (1, 10, 100) 97.9460%, 97.4862%, 92.3768%

p=0.7, k=50, alpha=0.01 lamb (1, 10, 100) 97.9464%, 97.4624%, 92.3967%

p=0.7, k=100, alpha=1 lamb (1, 10, 100) 97.1312%, 94.9307%, **82.0474%**

p=0.7, k=100, alpha=0.1 lamb (1, 10, 100) 98.9878%, 98.8174%, 96.2750%

p=0.7, k=100, alpha=0.01 lamb (1, 10, 100) 98.9873%, 98.8286%, 96.3212%

p=0.9, k=50, alpha=1 lamb (1, 10, 100) 93.7492%, 89.2140%, **63.4297%**

p=0.9, k=50, alpha=0.1 lamb (1, 10, 100) 97.9834%, 97.6863%, 92.7979%

p=0.9, k=50, alpha=0.01 lamb (1, 10, 100) 97.9792%, 97.6967%, 92.8476%

p=0.9, k=100, alpha=1 lamb (1, 10, 100) 96.9735%, 94.4133%, **80.2497%**

p=0.9, k=100, alpha=0.1 lamb (1, 10, 100) 98.9960%, 98.9113%, 96.6508%

p=0.9, k=100, alpha=0.01 lamb (1, 10, 100) 98.9960%, 98.9095%, 96.6926%

WE RUN ON GOOGLE CLOUD. Things to mention

* + - We run each fold in separate terminal to get maximum cpu usage
    - Which CPU, Memory specs used for to be able to run 5 terminal on total.
    - How much time it took to run
    - ***Which optimizations have been carried out to speed up the results***

**Transfer-learning results: k=100 is bad, try k=50. (NFLX)**

**~~Explain how data was retreived from Oracle/~~**

**~~Mention movie plots used as features. Cleaning of plots + vocab construction out of corpus plots~~**

**~~Add description of datasets.~~**

**NFLX is good with k=50, Movielense with k=100 (when we print top10 from each topic, and check words)**

**~~Mention 5-fold cross validation~~**

**Estimate confidence interval with 10% subsampling.**

**~~Python challenges faced: how phi’s are stored 3D matrix.~~**

**Higher lamb 3-30%, but high lamb with high alpha less%. Test it (sparsity)**

* Two most common tasks in recommender systems are predicting the score the user might give for a product (the rating prediction task), and recommending a ranked list of most relevant items (the top-N recommendation task)
* [[Take definitions of “in-matrix” and “out-of-matrix’ from CTR paper]]
* Mention that we run topic modelling LDA on **plots** not other features.
* Discussion on my experiments while implementation on Python
* Put top 20 words for each topic from pure or CTMPs LDA
* Train Time + Machine Specs of Google Cloud (maybe make table on time spent for each hyperparameters)
* Where used “document”, where used “item”. Decide which one.
* Sparsity plots
* ~~Why we choose p {0.7, 0.9} and not 0.5? because 0.5 indicates uniform distribution, which was intention behind OPE.~~
* Include somewhere change of doc and user cycle compared to original paper [include in theory, or in experimental part]
* Compare p=0.7 with p=0.9, is higher better? [in BOPE its mentioned so]
* Python libraries used mostly numpy, pandas [+ numba jit]

Mention that 90% of movies from NFLX is also contained in MovieLens 20M. So, compare LDA results from both datasets, explore the common topics they learn.

**During Model Evaluation, put “Interpretable user profiles” somewhere.**

Refer Pavel thesis http://cbir.utia.cas.cz/homepage/projects/phd\_thesis/vacha\_phd.pdf

**Difference on recall&precision between our and original work may be because we use plots instead of tags/features.**

**Add More**

Put gamma distribution explanation with formula, somewhere

ADD connectivity between sections [e.g., between Probab. Models for RS and LDA]

### Challenges during Python implementation

1. **Initialization;**

In Python, first challenge during the implementation of CTMP algorithm was how to handle an initialization of parameters before starting algorithm. Initializations for some parameters were mentioned in the original paper[53], but not for others:

|  |  |
| --- | --- |
| **Mentioned** | **Not mentioned** |
| *e, f,* , , *K, ,* | , shp, rte |

Table 7

* Initializations of *e, f,* *(lamb)*, *(alpha)*, *K(num\_topics)* havealready been discussed in previous chapters [\*\*chapter xx\*\*], and they are similar to those mentioned on [\*\*BOPE\*\*], [\*\*CTMP\*\*].
* According to original paper [53], initializations of and had to be made by their respective estimates from LDA. Apparently, **this** **didn’t work** in our case, because when and were initialized as LDA-learnt parameters for CTMP algorithm, It was unable to continue updating and fitting these parameters along with others beyond the initial iterations. Therefore, we decided to initialize both and randomly in (k1)-simplex. This resulted in appropriate update and fit of parameters across the iterations.
* We decided to initialize each equal to , and let CTMP algorithm learn the offset for each document across the iterations.
* It has been decided to initialize the user's variational parameters shp and rte to *e* and *f*, respectively, as follow:
* Finally, we need to initializate rating’s variational parameter which is the most difficult of all because it is a three-dimensional matrix with dimensions equal to user\_size, num\_docs and num\_topics, respectively. Unfortunately, **due to memory limits**, we could not directly generate a 3D matrix with this shape in Python. As a result, we discovered a workaround by dividing the entire 3D matrix into smaller 3D matrices that can be constructed within the memory limits. Then, we put all of these smaller 3D matrices sequentially into a list. For example, if we consider running CTMP on MovieLens 20M dataset with *K*=100 topics, then shape of will be as follow:

where, user\_size = 138493 and num\_docs = 25900 for MovieLens 20M. Thus, in order to be able to store , we do the following steps;

* Create 138 empty 3D matrices with shape = [1000, 25900, 100]
* Create 1 remainder empty 3D matrix with shape = [493, 25900, 100]
* Put all created 139 matrices sequentially into a list.

As seen above, with the help of the steps mentioned above, we now are able to represent the whole initial matrix with small chunks of matrices.

Furthermore, note that, in order to reduce the memory storage, all matrices mentioned above are created with the data type being **float32**. Because we have successfully tested that when two separate models are run using float64 and float32, the difference in results is insignificant.

1. Computational Speed

@njit

mostly used Numpy + its vectorization

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In particular, they make a link between this lower bound and parameter estimation via the EM algorithm. ALSO ADD variation EM explanation:::

Text, letter

Description automatically generated