Лабораторные работы №1-6
Вариант №18

№1.1

$$X \xrightarrow{k_1} Y, \quad Y \xrightarrow{k_2} P$$

$$\begin{cases} x_0 = 18 \\ k_1 = \frac{1}{2} \\ k_2 = 13/3 \\ \tau = 6 \end{cases}$$

$$\begin{cases} \frac{dX}{dt} = \frac{1}{\tau}(x_0 - x) - k_1 x \\ \frac{dy}{dt} = -\frac{1}{\tau} y + k_1 x - k_2 y \end{cases}$$

$$\begin{cases} \frac{1}{\tau}(x_0 - x) - k_1 x = 0 \\ -\frac{1}{\tau} y + k_1 x - k_2 y = 0 \end{cases} \qquad \begin{cases} x\left(\frac{1}{\tau} + k_1\right) - \frac{x_0}{\tau} = 0 \\ y\left(\frac{1}{\tau} + k_2\right) - k_1 x = 0 \end{cases}$$

$$\bar{x} = \frac{x_0}{1 + \tau k_1} = 4{,}5$$

$$\bar{y} = \frac{k_1 \bar{x}}{\frac{1}{\tau} + k_2} = 0{,}5$$

$$A = \begin{pmatrix} -\left(\frac{1}{\tau} + k_1\right) & 0 \\ k_1 & -\left(\frac{1}{\tau} + k_2\right) \end{pmatrix}; \quad (A - \lambda E) = \begin{pmatrix} -\left(\frac{1}{\tau} + k_1\right) - \lambda & 0 \\ k_1 & -\left(\frac{1}{\tau} + k_2\right) - \lambda \end{pmatrix}$$

$$\Delta = \left(-\left(\frac{1}{\tau} + k_1\right) - \lambda\right)\left(-\left(\frac{1}{\tau} + k_2\right) - \lambda\right) = 0$$

$$\lambda_1 = -\left(\frac{1}{\tau} + k_1\right) < 0$$

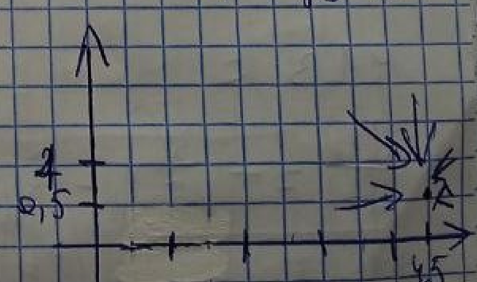
$$\lambda_2 = -\left(\frac{1}{\tau} + k_2\right) < 0$$

$\Rightarrow$ корни действ. и отр $\Rightarrow$

$\Rightarrow$ уст. узел.

$$\frac{x^{n+1} - x^n}{\Delta t} = \frac{1}{\tau}(x_0 - x^n) - k_1 x^n \Rightarrow$$

$$\Rightarrow x^{n+1} = x^n + \Delta t\left(\frac{1}{6}(18 - x^n) - \frac{1}{2} x^n\right)$$



$$\frac{y^{n+1} - y^n}{\Delta t} = -\frac{1}{\tau} y^n + k_1 x^n - k_2 y^n \Rightarrow$$

$$\Rightarrow y^{n+1} = y^n + \Delta t\left(-\frac{y^n}{6} + \frac{1}{2} x^n - 13/3 \cdot y^n\right)$$

$$x^{n+1} = x^n + \Delta t\left(3 - \frac{2}{3} x^n\right) \qquad y^{n+1} = y^n + \Delta t\left(\frac{1}{2} x^n - 4{,}5\, y^n\right)$$

№1.2

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{2}{3}x_1 - 7 \\ \dfrac{dx_2}{dt} = \dfrac{1}{3}x_2 + 4 \end{cases} \qquad \begin{cases} \dfrac{2}{3}x_1 = 7 \\ \dfrac{1}{3}x + 4 = 0 \end{cases} \qquad \begin{cases} \bar{x}_1 = \dfrac{21}{2} \\ x_2 = -12 \end{cases}$$

$$A = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}; \quad (A - \lambda E) = \begin{pmatrix} \frac{2}{3} - \lambda & 0 \\ 0 & \frac{1}{3} - \lambda \end{pmatrix}$$

$$A = \left(\frac{2}{3} - \lambda\right)\left(\frac{1}{3} - \lambda\right) = 0.$$

$$\lambda_1 = \frac{3}{5} > 0$$
$$\lambda_2 = \frac{1}{3} > 0 \Rightarrow \text{не уст. узел.}$$

$$\frac{x_1^{n+1} - x_1^n}{\Delta t} = \frac{2}{3}x_1^{n+1} - 7 \Rightarrow x_1^{n+1} = \frac{2\Delta t}{3}x_1^{n+1} + x_1^n - 7\Delta t \Rightarrow$$

$$\Rightarrow x_1^{n+1} - \frac{2}{3}\Delta t\, x_1^{n+1} = x_1^n - 7\Delta t \Rightarrow \quad x_1^{n+1} = \frac{x_1^n - 7\Delta t}{1 - \frac{2\Delta t}{3}}.$$

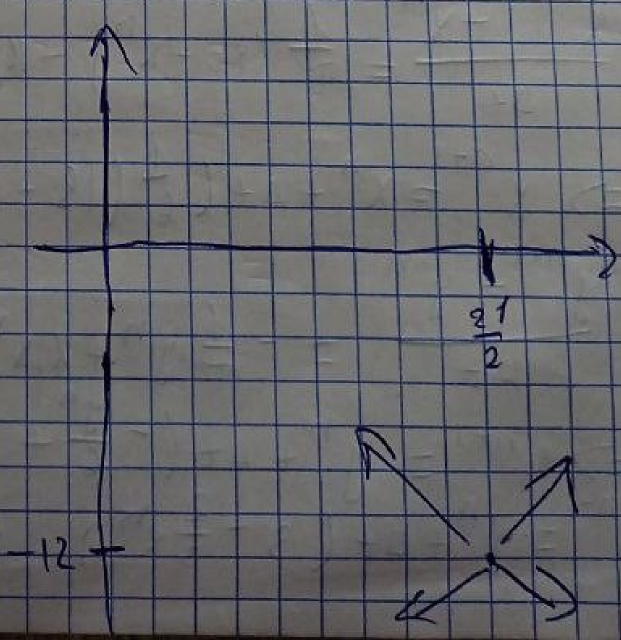
$$\frac{x_2^{n+1} - x_2^n}{\Delta t} = \frac{1}{3}x_2^{n+1} + 4 \Rightarrow x_2^{n+1} - \Delta t\,\frac{1}{3}x_2^{n+1} = x_2^n + 4\Delta t$$

$$\Rightarrow x_2^{n+1} = \frac{x_2^n + 4\Delta t}{1 - \frac{1}{3}\Delta t}.$$

№1.3

$$\frac{dx_1}{dt} = -\frac{3}{5}x_1 - 3$$

$$\frac{dx_2}{dt}$$



№1.3

$$\begin{cases} \dfrac{dx_1}{dt} = 1 - \dfrac{1}{6}x_1 \\ \dfrac{dx_2}{dt} = \dfrac{7}{4}x_2 \end{cases} \qquad \begin{cases} 1 = \dfrac{1}{6}x_1 \\ \dfrac{7}{4}x_2 = 0 \end{cases} \qquad \begin{cases} \bar{x}_1 = 6 \\ \bar{x}_2 = 0 \end{cases}$$

$$A = \begin{pmatrix} -\frac{1}{6} & 0 \\ 0 & \frac{7}{4} \end{pmatrix} \Rightarrow (A - \lambda E) = \begin{pmatrix} -\frac{1}{6} - \lambda & 0 \\ 0 & \frac{7}{4} - \lambda \end{pmatrix} \Rightarrow$$

$$\Rightarrow \Delta = \left(-\frac{1}{6} - \lambda\right)\left(\frac{7}{4} - \lambda\right) = 0$$

$$\lambda_1 = -\frac{1}{6} \quad \lambda_2 = \frac{7}{4} \Rightarrow \text{разн зн} \Rightarrow \text{седло, неуст}$$

$$\frac{x_1^{n+1} - x_1^n}{\Delta t} = -\frac{1}{6}x_1^{n+1} + 1 \Rightarrow x_1^{n+1} + \frac{1}{6}x_1^{n+1} = x_1^n + \Delta t,$$

$$x_1^{n+1} = \frac{x_1^n + \Delta t}{1 + \frac{1}{6}\Delta t}$$

$$\frac{x_2^{n+1} - x_2^n}{\Delta t} = \frac{7}{4}x_2^{n} \Rightarrow x_2^{n+1} = x_2^n + \Delta t \cdot \frac{7}{4}x_2^n$$

№1.4

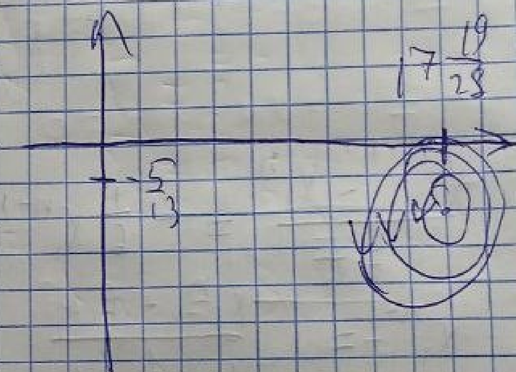
$$\begin{cases} \dfrac{d\mu_0}{dt} = k\mu_1 - b + q \\ \dfrac{d\mu_1}{dt} = \mu_0(n_1 - n_2) + d \end{cases}$$

$$\begin{cases} k = 1/5 = 0,2 \\ b = 17/4 = 4,25 \\ q = 5/7 = 0,71 \\ n_1 = 11/15 = 0,73 \\ n_2 = 1 \\ d = 2/3 = 0,67 \end{cases}$$

$$\begin{cases} k\mu_1 - b + q = 0 \\ \mu_0(n_1 - n_2) + d = 0 \end{cases}$$

$$\mu_1 = \frac{b-q}{k} = 17\frac{19}{28}$$

$$\mu_0 = \frac{-d}{n_1 + n_2} = -\frac{5}{13}$$



$$A = \begin{pmatrix} 0 & k \\ n_1 - n_2 & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow A - \lambda E = \begin{pmatrix} -\lambda & k \\ n_1 - n_2 & -\lambda \end{pmatrix} \Rightarrow$$

$$\Rightarrow \lambda^2 - k(n_1 - n_2) = 0$$

$$\lambda_{1,2} = \pm \frac{2\sqrt{3}}{15} i \quad \text{— центр.}$$

$$\frac{\mu_0^{n+1} - \mu_0^n}{\Delta t} = k\mu_1^n - b + q \Rightarrow \mu_0^{n+1} = \mu_0^n + \Delta t(k\mu_1^n - b + q)$$

$$\mu_0^{n+1} = \mu_0^n + \Delta t(0,2\,\mu_1^n - 4,25 + 0,71)$$

$$\frac{\mu_1^{n+1} - \mu_1^n}{\Delta t} = \mu_0^n(n_1 - n_2) + d \Rightarrow \mu_1^{n+1} = \mu_1^n + \Delta t(\mu_0^n(n_1 - n_2) + d)$$

$$= \mu_1^n + \Delta t\left(-\frac{4}{15}\mu_0^n + 2/3\right)$$

N1.5

$$\begin{cases} \dfrac{dx_1}{dt} = -2x_1 - \dfrac{3}{7}x_2 + 11 \\ \dfrac{dx_2}{dt} = \dfrac{7}{4}x_1 - 2x_2 + 7 \end{cases} \qquad \begin{cases} -2x_1 - \dfrac{3}{7}x_2 = -11 & \Big| \cdot 7 : (-3) \\ \dfrac{7}{4}x_1 - 2x_2 = -7 & \Big| \cancel{4} : 2 \end{cases}$$

$$+\begin{cases} \cancel{14x_1 - 3x_2 = -77} \\ \cancel{7x_1 - 8x_2 = -28} \end{cases}$$
$$\cancel{-7x_1 - 11x_2 = -105}$$

$$+\begin{cases} \dfrac{14}{3}x_1 + x_2 = \dfrac{77}{3} \\ \dfrac{7}{8}x_1 - x_2 = -\dfrac{7}{2} \end{cases}$$

$$\left(\frac{14}{3} + \frac{7}{8}\right)x_1 = \frac{77}{3} - \frac{7}{2}$$

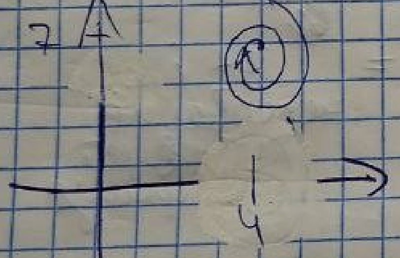
$$x_1 = 4$$
$$x_2 = 7$$

$$A = \begin{pmatrix} -2 & -\frac{3}{7} \\ \frac{7}{4} & -2 \end{pmatrix} \Rightarrow$$

$$\Rightarrow A - \lambda E = \begin{pmatrix} -2-\lambda & -\frac{3}{7} \\ \frac{7}{4} & -2-\lambda \end{pmatrix}$$

$$\cancel{\Delta = (-2-\lambda)^2} \quad \Delta = \frac{19}{4} + 4\lambda + \lambda^2$$

$$\lambda_{1,2} = -2 \pm \frac{\sqrt{3}}{2} i \Rightarrow$$ неподв. т. уст. фокус



$$\frac{x_1^{n+1} - x_1^n}{\Delta t} = -2x_1^{n+1} - \frac{3}{7}x_2^n + 11 \Rightarrow$$

$$\Rightarrow x_1^{n+1} + 2\Delta t\, x_1^{n+1} = x_1^n - \Delta t\left(\frac{3}{7}x_2^n - 11\right)$$

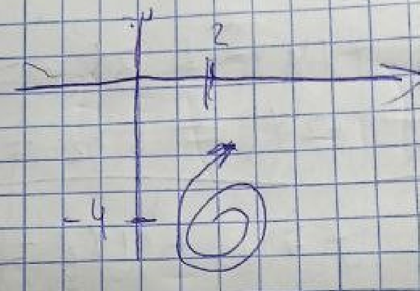
$$x_1^{n+1} = \frac{x_1^n - \Delta t\left(\frac{3}{7}x_2^n - 11\right)}{1 + 2\Delta t}$$

$$\frac{x_2^{n+1} - x_2^n}{\Delta t} = \frac{7}{4}x_1^{n} - 2x_2^{n+1} + 7 \Rightarrow x_2^{n+1} + 2x_2^{n+1}\Delta t = x_2^n + \frac{7}{4}x_1^n \Delta t + 7\Delta t$$

$$x_2^{n+1} = \frac{x_2^n + \Delta t\left(\frac{7}{4}x_1^n + 7\right)}{1 + 2\Delta t}$$

№6.

$$\begin{cases} \dfrac{dx_1}{dt} = 3x_1 - 2x_2 - 14 \\ \dfrac{dx_2}{dt} = \dfrac{1}{2}x_1 + 3x_2 + 11 \end{cases} \qquad \begin{cases} 3x_1 - 2x_2 - 14 = 0 & \mid :2 \\ \dfrac{1}{2}x_1 + 3x_2 + 11 = 0 & \mid :3 \end{cases}$$



$$+\begin{cases} \dfrac{3}{2}x_1 - x_2 - 7 = 0 \\ \dfrac{1}{6}x_1 + x_2 + \dfrac{11}{3} = 0 \end{cases}$$

$$\frac{3}{2}x_1 + \frac{1}{6}x_1 + \frac{11}{3} - 7 = 0 \Rightarrow \bar{x}_1 = 2$$

$$\bar{x}_2 = -4$$

$$A = \begin{pmatrix} 3 & -2 \\ \frac{1}{2} & 3 \end{pmatrix} \Rightarrow (A - \lambda E) = \begin{pmatrix} 3-\lambda & -2 \\ \frac{1}{2} & 3-\lambda \end{pmatrix} \Rightarrow$$

$$\Rightarrow \Delta = (3-\lambda)^2 + 1 = 0.$$

$$\lambda^2 - 6\lambda + 10 = 0 \qquad \lambda_{1,2} = 3 \pm i$$

неподвижные т.; неуст фокус

$$\frac{x_1^{n+1} - x_1^n}{\Delta t} = 3x_1^n - 2x_2^{n+1} - 14 \Rightarrow$$

$$\Rightarrow x_1^{n+1} = x_1^n + \Delta t\left(3x_1^n - 2x_2^{n+1} - 14\right)$$

$$\frac{x_2^{n+1} - x_2^n}{\Delta t} = \frac{1}{2}x_1^n + 3x_2^{n+1} + 11 \Rightarrow$$

$$\Rightarrow x_2^{n+1} = x_2^n + \Delta t\left(\frac{1}{2}x_1^n + 3x_2^{n+1} + 11\right) \Rightarrow$$

$$\Rightarrow x_2^{n+1} - 3x_2^{n+1}\Delta t = x_2^n + \Delta t\left(\frac{1}{2}x_1^n + 11\right)$$

$$x_2^{n+1} = \frac{x_2^n + \Delta t\left(\frac{1}{2}x_1^n + 11\right)}{1 - 3\Delta t}$$