Math 221
Class Exercises: Mar. 11

1. A transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ is defined by $T(p(x)) = p(x+1)$.

   (a) Find $[T]_\alpha^\alpha$ if

   $$\alpha = \left\{1, x, x^2\right\}.$$

   (b) Find $[T]_\beta^\beta$ if

   $$\beta = \left\{1 + x, 1 - x, x + x^2\right\}.$$

   (c) Draw a commutative diagram demonstrating your answers with $p(x) = x^2$.

2. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ with

   $$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_3 \\ x_1 \\ 2x_3 \end{bmatrix} \qquad \text{and} \qquad \alpha = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$$

   Find the basis β such that $[T]_\alpha^\beta = I$.