Math 221-AB1: Quiz 3

The transformations $T : \mathbb{R}^2 \to \mathbb{R}^4$ and $S : \mathbb{R}^3 \to \mathbb{R}^2$ are defined as follows:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ x+3y \\ 2x-4y \\ y \end{bmatrix} \qquad S\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x \\ y+z \end{bmatrix}$$

1. (*2 pts*) Find a matrix B so that $T(v) = Bv$ for all $v \in \mathbb{R}^2$.
2. (*3 pts*) Is T a **one-to-one** transformation? Explain your answer clearly.
3. (*3 pts*) Show that S a **linear** transformation.
4. (*2 pts*) Give a basis for $\mathrm{Ker}(S)$.