

Math 221
Class Exercises: Mar. 9

A transformation $T : \mathbb{M}_{2 \times 2} \rightarrow \mathbb{M}_{2 \times 2}$ is defined by

$$T \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & 0 \\ 3c & d \end{bmatrix}.$$

1. Show that T is linear.
2. Find a basis for $\text{Im}(T)$.
3. Is T an isomorphism?
4. Find $[T]_{\alpha}^{\alpha}$ if

$$\alpha = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

5. Find $[T]_{\beta}^{\alpha}$ if

$$\beta = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}.$$