Math 221
Class Exercises: Mar. 2

A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + 3x_3 \end{bmatrix}.$$

Show that T is **surjective** (onto).

If $S : \mathbb{R}^2 \to \mathbb{R}^4$, explain why S cannot be **onto**.

A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_2 \\ x_2 - x_3 \\ x_1 + 3x_3 \end{bmatrix}.$$

Show that T is **injective** (one-to-one).

Suppose $T : \mathbb{R}^3 \to \mathbb{R}^4$ and $S : \mathbb{R}^4 \to \mathbb{R}^2$ are defined as follows.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 \\ x_2 - x_3 \\ x_1 - x_2 \end{bmatrix} \qquad \text{and} \qquad S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 + x_4 \\ x_2 + 3x_4 \end{bmatrix}$$

1. Find a matrix B so that $S \circ T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $(S \circ T)(x) = Bx$.
2. Is $(S \circ T)$ one-to-one? Explain.