Math 221
Class Exercises: Feb. 28

Give a formula for a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$T\left(\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

1. Can you show that the transformation defined by your formula is linear?
2. Can you find a matrix A so that $T(x) = Ax$?

Give a geometric description of the transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Give a geometric description of the transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Suppose $T : \mathbb{R}^2 \to \mathbb{R}^4$ and

$$T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix} \qquad \text{and} \qquad T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

1. Calculate

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$$

2. Calculate

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$