Math 111
Chapter 2.3: Limit Laws

If limits exist we can do **algebra with limits**. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then

$$\lim_{x\to a} f(x) + \lim_{x\to a} g(x) = \lim_{x\to a}(f(x) + g(x))$$

Other simple rules:

$$\lim_{x\to a} x = a \qquad \lim_{x\to a} c = c$$

where c is a constant

(EXAMPLES)

1. $$\lim_{x\to 0} x^2 - 3 =$$

2. $$\lim_{x\to 2} \frac{4x^2(x-2)}{x^2+x-6} =$$

(THEOREM) If $f(x) = g(x)$ when $x \neq a$ then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$ if the limits exist.

(EXAMPLES)

1. $\lim_{h \to 0} \dfrac{(-6+h)^2 - 36}{h}$

2. $\lim_{t \to 0} \dfrac{\sqrt{9+t} - 3}{t}$

3. $\lim_{x \to 1} \dfrac{x^4 - 1}{x^3 - 1}$

(SQUEEZE THEOREM) If $f(x) \leq g(x) \leq h(x)$ when x is near a and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

(EXAMPLE)

$$\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right)$$

(THEOREM) $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^-} f(x) = L$.

(EXAMPLE)

$$\lim_{x\to 5} \frac{|x-5|}{3x-15}$$