Math 111
Chapter 3.4: Chain Rule

The **Chain Rule** for derivatives:

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\frac{dg}{dx}$$

$$[f(g(x))]' = f'(g(x))g'(x)$$

if f and g are differentiable functions.

(EXAMPLES)

1. Find $h'(x)$ if $h(x) = \sqrt{1+x^3}$

2. Find $p'(x)$ if $p(x) = e^{10x-x^2}$

3. Find $y'(x)$ if $y(x) = (2x+e^x)^3$

4. Suppose $r(x) = 100(1-1/x)$ represents a rabbit population as a function of x, the amount of edible vegetation in a habitat and $f(r) = \sqrt{r}$ represents the population of foxes as a function of the rabbit population.

5. Previously we had written a function to model the hours of daylight in the Fraser Valley as a function of time t in days.

$$P(t) = 12.25 + 4\cos\left(\frac{2\pi}{365}(t - 174)\right)$$

Calculate the rate at which the days are getting shorter as of today ($t = 282$). What are the units?

6. Find $g''(t)$ if $g(t) = \sec^6(t)$

7. Find $f'(x)$ if $f(x) = e^{5x}(x^2 - 3x - 4)^{12}$

8. Find $h'(t)$ if $h(t) = \sin(\tan(2t - 4))$

9. Find $P'(t)$ if $P(t) = \dfrac{1}{1 + 4e^{-kt}}$

The derivative of a general exponential function:

$$\frac{d}{dx}\left[a^x\right] = a^x \ln a$$

where $a > 0$.

(EXAMPLES)

1. Find $f'(x)$ if $f(x) = 7^x$

2. Find $w'(\theta)$ if $w(\theta) = 10^{\cos \pi\theta}$.

(EXERCISES)

1. Find an equation of the line tangent to $y = \sin(\sin x)$ at the point $(\pi, 0)$.

2. Find $u'''(x)$ if $u(x) = 2\cos(3x)$. (BONUS: Find $u^{(15)}(x)$)

3. Find $h'(4)$ if $h(y) = \dfrac{1}{(\sqrt{y}+8)^{10}}$.

4. Find $f'(x)$ if $f(x) = 3^x \tan x \sin x$.

5. Determine where on the curve $y = e^{-x^2}$ the tangent line has greatest slope. Sketch the curve and the tangent line with greatest slope.

6. Suppose f is a differentiable function with $f(0) = 3$, $f'(0) = 5$, and $f''(0) = -2$, and suppose that $g(x) = e^{-x} f(x)$. Calculate $g'(0)$, and $g''(0)$.