Math 111

Chapter 3.3: Derivatives of Trigonometric Functions

Compare graphs of the two basic trigonometric functions $\sin x$ and $\cos x$.

Looks like
$$\frac{d}{dx}$$
[] =

Check definition. Need to know that $\sin(x+y) = \sin x \cos y + \sin y \cos x$ and $\lim_{h\to 0} \frac{\sin h}{h} = 1$.

Need also to know that $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$.

Looks like
$$\frac{d}{dx}$$
[] =

(OTHER EXAMPLES) For each function, calculate the derivative and then graph the function and the derivative.

$$1. \ f(x) = \tan x$$

$$2. \ g(x) = \sec x$$

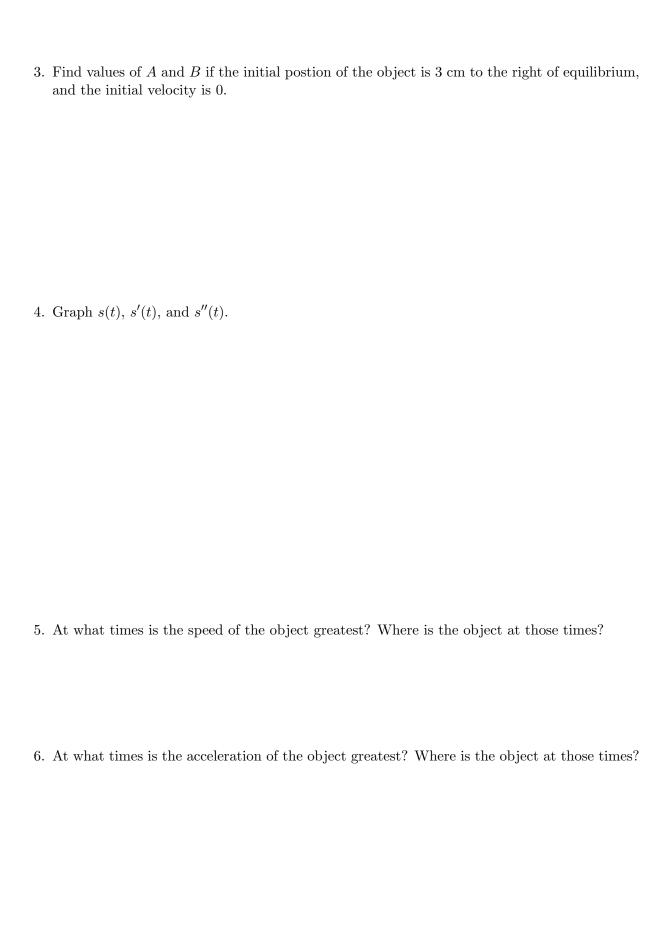
$$3. \ p(x) = \cot x$$

(APPLICATION)

Under certain circumstances, the equation that describes the motion of a simple spring-mass system is given by $\frac{d^2s}{dt^2} + s = 0$. Here s(t) is a function that describes the position of the object in units of cm, with time measured in seconds.

1. Show that $s(t) = \sin t$ and $s(t) = \cos t$ are both solutions of the equation.

2. More generally, show that $s(t) = A \sin t + B \cos t$ is a solution.



(APPLICATION) Quantities that are periodic in nature can often be modeled with a function such as the following.

$$P(t) = A + B\cos\left(\frac{2\pi}{T}(t - \phi)\right)$$

Suppose for example that P(t) represents the hours of daylight in the Fraser Valley as a function of time t in days. We suppose that t = 0 corresponds to January 1.

- 1. Find values for A, B, T, and ϕ if it is assumed that the maximum daylight hours is 16.25, the minimum is 8.25 hours, and the longest day of the year is on June 23 (t = 174).
- 2. What are the number of daylight hours predicted by the model for today (t = 277)?

(EXERCISES)

1. Find
$$y'$$
 if $y = \frac{t \sin t}{1+t}$

2. Find
$$H''\left(\frac{\pi}{3}\right)$$
 if $H(\theta) = \theta^2 \sin \theta$

3. Find
$$f'(x)$$
 if $f(x) = x^4 e^x \tan x$

4. For what values of x does the graph of $g(x) = x + 2\sin x$ have a horizontal tangent line? Sketch a the graph of g.

5. Evaluate
$$\lim_{\theta \to 0} \frac{\sin 8\theta}{3\theta}$$

6. Evaluate
$$\lim_{t\to 0} \frac{\tan 6t}{\sin t}$$

7. Evaluate
$$\lim_{x\to 2} \frac{\sin(x-2)}{x^2 - 9x + 14}$$

- 8. Suppose again that $s(t) = A \sin t + B \cos t$ describes the position of an oscillating mass.
 - (a) Find values of A and B if the initial postion of the object is 2 cm to the right of equilibrium, and the initial velocity is 3 cm/s.
 - (b) What is the first time that the object passes through equilibrium?
 - (c) What is the farthest distance away from equilibrium that the object reaches?
 - (d) At what times is the object moving the fastest?