Math 111
Chapter 2.7: Derivatives & Rates of Change

(DEFINITION) The **derivative of a function f at a number** a is:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The meaning of $f'(a)$ is

1.

2.

(EXAMPLE) Find $f'(9)$ if $f(x) = 4 - \sqrt{x}$. Sketch the graph of f and the tangent line at $x = 9$.

(EXAMPLE) The height of a falling object in meters is given by $y(t) = 150 - 4.9t^2$, where time t is measured in seconds. Find the velocity of the object at the time $t = 5$. What are the units of $y'(5)$?

(EXAMPLE) Find the derivative $p'(2)$ if $p(x) = 3/x$. Sketch the graph of p, the line tangent to the graph at $(2, 1.5)$, and give the equation of the line.

(INTERPRETATIONS)

In certain circumstances, the volume of a gas is inversely proportional to the pressure. We could write volume as a function of pressure $V(P) = k/P$ where k is a constant, V is measured in cm^3 and pressure measured in kPa.

1. What does $V'(100)$ measure?
2. What is the sign of $V'(100)$?
3. What are the units $V'(100)$?
4. Which do you think is larger, $V'(100)$ or $V'(500)$?

The garbage G produced by a city is a function of its population P. Suppose P is measured in thousands and G is measured in tons.

1. What does $G'(250)$ measure?
2. What is the sign of $G'(250)$?
3. What are the units $G'(250)$?

Let $A(t)$ represent the total mass of apples (in kg) you gather as a function of time you spend in an orchard (in hours).

1. What is the sign of $A'(6)$?
2. What are the units $A'(6)$?
3. Which do you think is larger $A'(6)$ or $A'(200)$? Why?

Suppose we know that g is a continuous function with $g(1) = 2$, $g'(1) = 0$, $g(4) = 0$, and $g'(4) = 1$. Sketch a possible graph of g.