Math 111
Chapter 3.1: Derivatives of Polynomials and Exponential Functions

The **Power Rule** for derivatives:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

for any constant n

(EXAMPLES)

1. $f(x) = x^8$

2. $p(x) = x^{100}$

3. $g(t) = t^{22}$

Why does it work?

The **derivative of a constant function**:

$$\frac{d}{dx}[c] = 0$$

for any constant c

(EXAMPLE)

(EXAMPLES) Power Rule applies to *any* power!

1. $f(x) = \dfrac{1}{x^2}$

2. $g(x) = \dfrac{1}{\sqrt{x}}$

3. $p(s) = s^{4.6}$

The **Constant Multiple Rule** for derivatives:

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x)$$

if f and is a differentiable functions and c is a constant

(EXAMPLES)

1. $f(x) = 1.5x^{12}$

2. $g(x) = \dfrac{7}{x^4}$

3. $p(s) = \pi\sqrt{s}$

The **Sum and Difference Rules** for derivatives:

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}\left[f(x) - g(x)\right] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

if f and g are differentiable functions.

Why do these rules work?

(EXAMPLES)

1. $f(x) = x^9 + 5x^7 - 3x^2 + 13$

2. $h(x) = \dfrac{x^2 - 3}{\sqrt{x}}$

3. $p(s) = A\sqrt{s} + \dfrac{B}{\sqrt[4]{s}}$

4. $w(y) = y^3 \left(10 - \dfrac{15}{y^4}\right)$

(APPLICATIONS)

1. Find the equation for the line tangent to $y = x^2 - x^4$ at the point $(1, 0)$. Find the equation for the line normal to $y = x^2 - x^4$ at the point $(1, 0)$.

2. Suppose $s(t) = t^3 - 12t$ represents the position of a moving object in cm, with time measured in seconds.

   (a) Find the velocity of the object at $t = 1$.

   (b) Find the acceleration of the object at $t = 1$.

   (c) Find the distance traveled from $t = 0$ to $t = \sqrt{12}$.

   (d) At what time does the object have greater speed, $t = 0$ or $t = \sqrt{6}$.

The **derivative of the natural exponential function**:

$$\frac{d}{dx}\left[e^x\right] = e^x$$

Why? Because e is special!

(EXERCISES)

1. Find the equation of the line tangent to the curve $y = 3x\sqrt{x}$ when $x = 1$.

2. Find the line that is tangent to the curve $y = e^x$ and parallel to the line $y = 5x$.

3. If $y = Ax^2 + Bx + C$, find numbers A, B, and C so that $y'' + 2y = x^2 - 4$.

4. If $f(x) = |x^2 - 9|$, find a formula for $f'(x)$ all points where f is differentiable.

5. Find a number c so that $y = c\sqrt{x}$ is tangent to $y = 1.5x + 6$