Math 111
Chapter 3.5: Implicit Differentiation

(PROBLEM)
How can we find the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$?

1. Idea 1

2. Idea 2

3. Idea 3

(EXAMPLES)

1. Find $\dfrac{dy}{dx}$ if $\sqrt{x} + \sqrt{5y} = 1$

2. Find $\dfrac{dy}{dx}$ at the point $(1, 0)$ if $xe^y = x - y$.

3. Find $\dfrac{dy}{dx}$ if $\cos(xy) = 1 + \sin y$

4. Find the equation of the line tangent to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$. Sketch the curve and the tangent line.

(APPLICATION: CHEMISTRY)

In simple scenarios in thermodynamics, the pressure and volume of a gas might be related with the ideal gas law $PV = nRT$, where T is temperature, n is the quantity of gas in moles, and R is a constant. Find the rate of change of volume with respect to pressure.

In a more complex setting, the pressure and volume of a gas might be related through van der Waal's equation $(P + \frac{n^2 a}{V^2})(V - nb) = nRT$. Here a and b are additional constants. Find the rate of change of volume with respect to pressure.

(APPLICATION: INVERSE FUNCTIONS)

If $y = f^{-1}(x)$, then $x = f(y)$. We can find $\dfrac{dy}{dx}$ with implicit differentiation.

(EXAMPLES)

1. Find $\dfrac{dy}{dx}$ if $y = \arcsin(x)$

2. Find $\dfrac{dy}{dx}$ if $y = \arctan(x)$.

(APPLICATION: ORTHOGONAL CURVES)

Using calculus, we can explain why circles ($x^2 + y^2 = r^2$) are orthogonal to lines through their centers ($ax + by = 0$)

Another example of **orthogonal curves** are $y = cx^2$ and $x^2 + 2y^2 = k$.

(EXERCISES)

1. Find y'' if $x^2 + xy + y^2 = 3$.

2. In each case, find $\dfrac{dy}{dx}$ by implicit differentiation

   (a) $2x^2 + y^2x^2 - y = 9$

   (b) $xy = x^2 - \sec y$

   (c) $x - y = e^{x/y}$

3. At what points does the curve described by $y^2 = x^3 + 3x^2$ have horizontal tangent line?

4. Find the derivatives.

(a) $y = (\arcsin x)^4$

(b) $y = \arctan{(1 + e^{5x})^2}$

(c) $y = \dfrac{\arccos t}{t^2}$