Math 111
Chapter 3.7: Rates of Change

DENSITY

Consider a thin wire and let $m(x)$ represent the mass of wire to the left of position x. Suppose x is in cm and $m(x)$ is in grams.

$$m(x) = \frac{3x}{x+1} \qquad 0 \leq x \leq 2$$

Interpret dm/dx. What are the units?

ELECTRIC CURRENT

Suppose that $Q(t) = t^3 - 2t^2 + 6t + 2$ represents the quantity of charge that has passed a particular position in a wire up to time t. Time is measured in seconds and Q is measured in coulombs. Interpret $Q'(t)$.

FLUID FLOW

Law of laminar flow states:

$$v(r) = \frac{P}{4\eta L}(R^2 - r^2)$$

1. How does v change with respect to radial position r?
2. If a position were fixed, how would v change with respect to pressure? viscosity?

PROJECTILE MOTION

The height of a projectile with initial height h_0 and initial upward velocity v_0 is $h(t) = h_0 + v_0 t - 4.9t^2$. Here h is measured in meters and t in seconds.

1. Show that h satisfies the stated initial conditions.
2. Show that the vertical acceleration of the object is constant.
3. What is the maximum height of the projectile if $h_0 = 2$ and $v_0 = 24.5$?
4. How fast is the object going when it hits the ground if $h_0 = 2$ and $v_0 = 24.5$?

CHEMISTRY

Let $x(t)$ be the concentration of product in a chemical reaction.

$$x(t) = \frac{ak^2t}{1 + akt}$$

Here a and k are positive constants. Suppose x is in units of mol/L and t is in seconds.

1. Find the rate of reaction at time $t = 1$.
2. What happens to x as $t \to \infty$?
3. What happens to the rate of reaction as $t \to \infty$?
4. Show that $dx/dt = k(a - x)^2$?

RUMOR SPREAD

Let $p(t)$ represent the proportion of a population that has heard a rumor. Suppose time t is measured in hours

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

1. Find the values of a and k if p(0)= 0.05 and p(1)=0.12.
2. Determine the rate at which the rumor is spreading at time $t = 3$ hours.
3. When is the rumor spreading the fastest?
4. Show that $dp/dt = kp(1 - p)$.

ECONOMICS

The cost in dollars of producing x units of a certain commodity is given by a cost function.

$$C(x) = 5000 + 10x + 0.08x^2$$

1. Find the average rate of change C with respect to production level x when x increases from 100 to 101.

2. Find the average rate of change C with respect to production level x when x increases from 100 to 105.

3. Find the instantaneous rate of change of C with respect to x when $x = 100$. This is called the *marginal cost*.

4. Use the marginal cost to *estimate* the increase in cost associated with increasing production from $x = 100$ to $x = 105$.