Math 111
Chapter 3.9: Related Rates

We wish to consider situations where two or more quantities are related to each and are *also changing with time*. To avoid confusion it is important that we use Leibnitz notation ($\frac{dy}{dt}$) in place of prime notation (y').

EXAMPLE
If $y = a^2 b$, then

$$\frac{dy}{da} = \qquad\qquad\qquad \frac{dy}{db} =$$

If $y = a^2 b$ *and* a and b are both functions of t, then

$$\frac{dy}{dt} =$$

EXAMPLES

1. Suppose a particle is moving around the circle $x^2 + y^2 = 25$, where x and y are in cm. How is the x coordinate of the particle changing at the point $(-3, 4)$ if the y coordinate is increasing at a rate of 2 cm/s?

2. The ideal gas law is $PV = nRT$ where T is temperature in Kelvins, P is pressure in atmospheres, V is the volume of the gas in liters, n is the number of moles of gas, and $R = 0.08121$ is a constant. Suppose that at a certain instant, $P = 8$ atm and is increasing at a rate of 0.1 atm/min, and $V = 10$L and is decreasing at a rate of 0.15 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ mol.

Related Rates

In related rates problems, variables are functions of time. The typical strategy follows four points.

1. Determine those quantities that are changing and label them as variables.
2. Find an equation that relates the variables. (Pictures are helpful if geometry is involved.)
3. Differentiate the equation with respect to time.
4. Solve for the desired quantity.

(EXAMPLES)

1. A spherical shaped cell is growing at a rate of 20 μm^3/hr. How is the radius of the cell increasing when the volume is 500μm^3?

2. A ramp that is 20 feet long rises a total of 5 feet. A person is pushing the box up the ramp at a rate of 3 ft/s. How fast is the box rising?

(EXERCISES)

1. A cylindrical tank of radius 5m is being drained at a rate of $3\text{m}^3/\text{min}$. How fast is the depth of the water in the tank decreasing when the depth is 7m?

2. Police radar is stationed near a road. The officer aims the radar gun at a passing car and the gun is at an angle of 45° to the road If the officer records the distance between the car and the radar is decreasing at a rate of 100 km/hr, how fast is the car travelling?

3. Sand is dumped on a pile in the shape of a cone at a rate of 30 ft^3/min. The diameter and the height of the cone remain equal as the pile grows. How fast is the height of the pile increasing when the height is 10 ft?

4. Two cars start off from the same point. One travels west at 25 mph, the other south at 60mph. At what speed are they moving apart 2 hours later?

5. In a certain species of fish, brain weight as a function of body weight is determied to be $B = 0.007W^{2/3}$. Body weight as a function of body length is determined to be $W = 0.12L^{5/2}$. How fast is the fish brain growing when the fish is 5 cm and growing by 0.1cm/day?

6. If the radius of a cylinder is decreasing by 2cm/min, and the height is increasing at a rate of 10 cm/min, how is the volume of the cylinder changing when the radius is 5 cm and the height is 15 cm?

7. The area between two varying concentric circles is at all times 9π If the area of the larger circle is increasing by 10π in^2/sec, how fast is the circumference of the smaller circle changing?

8. A swimming pool is 12 m long, 7 m wide, 1.2 m deep at the shallow end and 3 m deep at the deep end. Water is being pumped into the pool at a rate of $\frac{1}{3}\text{m}^3/\text{min}$. How fast is the water rising when the water is 1 m deep at the deep end?