Math 111

Chapter 3.2: Derivatives of Products and Quotients

The **Product Rule** for derivatives:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

if f and g are differentiable functions.

(EXAMPLES)

1. $f(x) = x^3 e^x$

2. $w(y) = y^3\left(10 - \dfrac{15}{y^4}\right)$

3. $g(t) = t^5 h(t)$. Find $g'(2)$ if $h(2) = -1$ and $h'(2) = 3$

4. $h(x) = 7\sqrt[3]{x}$

Why does it work?

(APPLICATION)
A pond has a population of tadpoles. It is estimated that the current population of tadpoles is 3000, but that this number is falling by 150 per week. The current average mass of a tadpole is estimated to be 45 grams and is rising by 3.6 grams per week.

1. Estimate the total biomass of tadpoles.

2. Estimate the rate at which the biomass is changing.

(EXERCISE)
Let $f(x) = xe^x$. Find $f'(x)$, $f''(x)$, and $f'''(x)$. Guess a formula for $f^{(n)}(x)$.

The **Quotient Rule** for derivatives:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

if f and g are differentiable functions.

(EXAMPLES)

1. $f(x) = \dfrac{x^2}{e^x}$

2. $u(t) = \dfrac{t^2 - 2t + 5}{t^4 + 1}$

3. $g(x) = \dfrac{10}{x^8}$

4. $h(w) = \dfrac{e^w}{1 - 12e^w}$

5. $f(w) = \dfrac{1 - 12e^w}{e^w}$

6. $y(x) = e^{-x}$

Why does it work?

(APPLICATION) A pond has a population of tadpoles and trout. It is estimated that the current population of tadpoles is 3000, but that this number is falling by 150 per week. The trout population is estimated to be 220 and rising by about 3 trout per week. What is the current estimate of "tadpoles per trout" and how is it changing?

(EXERCISES)

1. Find $f''(x)$ if $f(x) = (x^3 + 1)e^x$.

2. Find an equation of the line tangent to $y = \dfrac{1+x}{1+e^x}$ at the point $\left(0, \dfrac{1}{2}\right)$.

3. Find $p'(x)$ if $p(x) = \dfrac{2 + xe^x}{1 + x^2}$.

4. Determine the **Reciprocal Rule**

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{[g(x)]^2}$$