Math 111
Chapter 2.1: Rates of Change

If a population grows by 4,360 persons over the course of 6 years, how is it changing on average?

If a hot coffee cools by $8°C$ in a period of 12 minutes, how is it changing on average?

(DEFINITION) The **average rate of change** of a function f over an interval $[x_1, x_2]$ is:

(EXAMPLES)

1. What is the average rate of change of $g(x) = \sqrt{x-2}$ on the interval $[4, 6]$?

   What does this number represent graphically?

2. A falling object has position given by $f(t) = 4.9t^2$. What is the objects **average velocity** from time $t = 0$ to time $t = 3$?

Instantaneous velocity

(BIG QUESTION) How can we determine the velocity of the falling object at one particular point in time? For example, what is the velocity at $t = 3$? (*This is sometimes called instantaneous velocity.*)

We might try computing average velocities for small intervals around $t = 3$.

If the invervals are small, we see that the numbers *approach* a single value.

To understand why we might look at an interval $[3, 3 + \Delta t]$ and see what happens if Δt is small.

(NOTATION)

Tangent problem

(BIG QUESTION) How can we find the equation for a line that is tangent to curve? *Tangent means that the line touches the curve and has the same slope.*
(EXAMPLE) What is the equation for the line that is tangent to the parabola $y = x^2$ at the point $(2, 4)$?

In order to answer, we need the slope of the curve at $(2, 4)$. Let's call $(2, 4)$ P, and lets choose another point Q on the curve and find the slope of the line that joins P and Q.

We find that for Q close to P, the values of the slope are close to:

To understand why, we can look at an arbitrary point Q.

The equation for the tangent line must then be:

Derivative

(BIG IDEA) For a function f and a number a, the rate of change of the function of f at a is the same as the slope of the graph at a.

(BIG DEFINITION) The **derivative of a function f at a number a** is:

(EXAMPLE) If $f(x) = 2^x$, estimate the value of $f'(2)$.