Chapter 4.7: Optimization.

(EXAMPLES)

1. A model used for the yield Y of an agricultural crop as a function of the nitrogen level N in the soil is

$$Y = \frac{kN}{1 + N^2}$$

where k is a positive constant. What nitrogen level gives the best yield?

2. The **Shannon index** can be used to measure the diversity of species living in an ecosystem. In the case of two species, it is defined by the formula $H = -a \ln a - b \ln b$, where a is the percentage of species A in the ecosystem and b is the percentage of species B in the ecosystem. What is the maximum value of H and when does it occur?

3. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

4. A rectangular storage container is to have a volume of 10 m^3. The length of its base must be twice the width. Material for the base costs \$10 per m^2. Material for the sides and top costs \$6 per m^2. Find the cost of materials for the cheapest such container.

5. A species of coastal bird nests on the mainland and makes trips to a nearby island to visit a food source. The island is located 5 km offshore a distance of 13 km down the coast. The birds fly down the coast a certain distance before turning and heading straight for the island. (*See diagram.*) It is proposed that the reason for this flight path is that the birds spend more energy when flying over water versus flying over land. If flying over water takes 1.4 times as much energy as flying over land, what is the optimal path for the birds?

(EXERCISES)

1. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$. (*Ans:* $(2.5, \sqrt{2.5})$)

2. A rectangular poster is to have area 180 in^2 with 1 inch margins on the sides and bottom and a 2 inch margin on the top. What dimensions of the poster will give the largest *printed* area? (*Ans:* $2\sqrt{30} \times 3\sqrt{30}$.)

3. A baseball team plays in a stadium that holds 55,000 people. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000. Assume that the relation between price and attendance is linear and determine the ticket price that will maximize the revenue. (*Ans:* \$9.50.)

4. Find the equation of the line that passes through the point $(3, 5)$ that cuts off the least area from the first quadrant. (*Ans:* $5x + 3y - 30 = 0$.)