Math 111
Chapter 3.3: Derivatives of Trigonometric Functions

Compare graphs of the two basic trigonometric functions $\sin x$ and $\cos x$.

Looks like $\frac{d}{dx}[\qquad\qquad\qquad] =$

Check definition. Need to know that $\sin(x+y) = \sin x \cos y + \sin y \cos x$ and $\lim_{h \to 0} \frac{\sin h}{h} = 1$.

Need also to know that $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$.

Looks like $\frac{d}{dx}[\qquad\qquad\qquad] =$

(OTHER EXAMPLES) For each function, calculate the derivative and then graph the function and the derivative.

1. $f(x) = \tan x$

2. $g(x) = \sec x$

3. $p(x) = \cot x$

(APPLICATION)

Under certain circumstances, the equation that describes the motion of a simple spring-mass system is given by $\frac{d^2s}{dt^2} + s = 0$. Here $s(t)$ is a function that describes the position of the object in units of cm, with time measured in seconds.

1. Show that $s(t) = \sin t$ and $s(t) = \cos t$ are both solutions of the equation.

2. More generally, show that $s(t) = A \sin t + B \cos t$ is a solution.

3. Find values of A and B if the initial postion of the object is 3 cm to the right of equilibrium, and the initial velocity is 0.

4. Graph $s(t)$, $s'(t)$, and $s''(t)$.

5. At what times is the speed of the object greatest? Where is the object at those times?

6. At what times is the acceleration of the object greatest? Where is the object at those times?

(APPLICATION) Quantities that are periodic in nature can often be modeled with a function such as the following.

$$P(t) = A + B\cos\left(\frac{2\pi}{T}(t-\phi)\right)$$

Suppose for example that $P(t)$ represents the hours of daylight in the Fraser Valley as a function of time t in days. We suppose that $t = 0$ corresponds to January 1.

1. Find values for A, B, T, and ϕ if it is assumed that the maximum daylight hours is 16.25, the minimum is 8.25 hours, and the longest day of the year is on June 23 ($t = 174$).

2. What are the number of daylight hours predicted by the model for today ($t = 277$)?

(EXERCISES)

1. Find y' if $y = \dfrac{t \sin t}{1+t}$

2. Find $H''\left(\dfrac{\pi}{3}\right)$ if $H(\theta) = \theta^2 \sin\theta$

3. Find $f'(x)$ if $f(x) = x^4 e^x \tan x$

4. For what values of x does the graph of $g(x) = x + 2\sin x$ have a horizontal tangent line? Sketch a the graph of g.

5. Evaluate $\lim_{\theta \to 0} \dfrac{\sin 8\theta}{3\theta}$

6. Evaluate $\lim_{t \to 0} \dfrac{\tan 6t}{\sin t}$

7. Evaluate $\lim_{x \to 2} \dfrac{\sin(x-2)}{x^2 - 9x + 14}$

8. Suppose again that $s(t) = A \sin t + B \cos t$ describes the position of an oscillating mass.

   (a) Find values of A and B if the initial postion of the object is 2 cm to the right of equilibrium, and the initial velocity is 3 cm/s.

   (b) What is the first time that the object passes through equilibrium?

   (c) What is the farthest distance away from equilibrium that the object reaches?

   (d) At what times is the object moving the fastest?