Math 111
Chapter 2.6: Limits at Infinity

(EXAMPLES) Suppose the population of turtles on a remote island is modeled by the following function, where t is measured in years.

$$P(t) = \frac{15000}{50 + 250e^{-0.05t}}$$

What do we expect will happen to the size of the population after many years?

(DEFINITION) Suppose f is a function defined for all $x > a$ for some constant a. Then

$$\lim_{x \to \infty} f(x) = L$$

means $f(x)$ can be made arbitrarily close to L by taking x sufficiently large. We say that L is the **limit** of f as x goes to ∞.

(EXAMPLES)

1. $\lim_{x \to \infty} \frac{x}{x-2} =$

2. $\lim_{x \to -\infty} \frac{x}{x-2} =$

Horizontal Asymptotes

(DEFINITION) The graph of a function f has a **horizontal asymptote** at $y = L$ if either:

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \lim_{x\to-\infty} f(x) = L$$

(EXAMPLES)
Find equations for all vertical and horizontal asymptotes of the graphs of the following functions.

$$g(x) = \frac{x-3}{9x^2-2}$$

$$h(x) = \frac{\sqrt{3x^2+10}}{5x-1}$$

$$f(x) = \frac{5e^x}{e^x-3}$$

(MORE EXAMPLES)

1. $\lim_{x\to\infty} \dfrac{8}{e^x - 2} =$

2. $\lim_{x\to\infty} \dfrac{1 + x^2}{8x^2 + x + 1} =$

3. $\lim_{x\to\infty} \ln\left(x^3 + 1\right)$

4. $\lim_{x\to\infty} \cos x$

5. $\lim_{x\to\infty} \sin\left(\dfrac{\pi}{x}\right)$

If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$ there are 3 possibilities for the limit

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

1. $\lim_{x\to\infty} \frac{f(x)}{g(x)} =$

2. $\lim_{x\to\infty} \frac{f(x)}{g(x)} =$

3. $\lim_{x\to\infty} \frac{f(x)}{g(x)} =$

The outcome is determined by the *rate at which the functions grow* as $x \to \infty$.

(EXAMPLES)