Math 111 Chapter 4.1 Maximum and Minimum Values

(DEFINITIONS)

Let c be a number in the domain D of a function f.

- f(c) is an absolute maximum of f on D if $f(c) \ge f(x)$ for all x in D.
- f(c) is an absolute minimum of f on D if $f(c) \le f(x)$ for all x in D.

NOTE: These values are sometimes called *global maximum* or *global minimum* or the *extreme values* of the function.

(DEFINITIONS)

Let c be a number in the domain D of a function f.

- f(c) is an **local maximum** of f if $f(c) \ge f(x)$ for x near c.
- f(c) is an **local minimum** of f if $f(c) \leq f(x)$ for x near c.

(EXAMPLES)

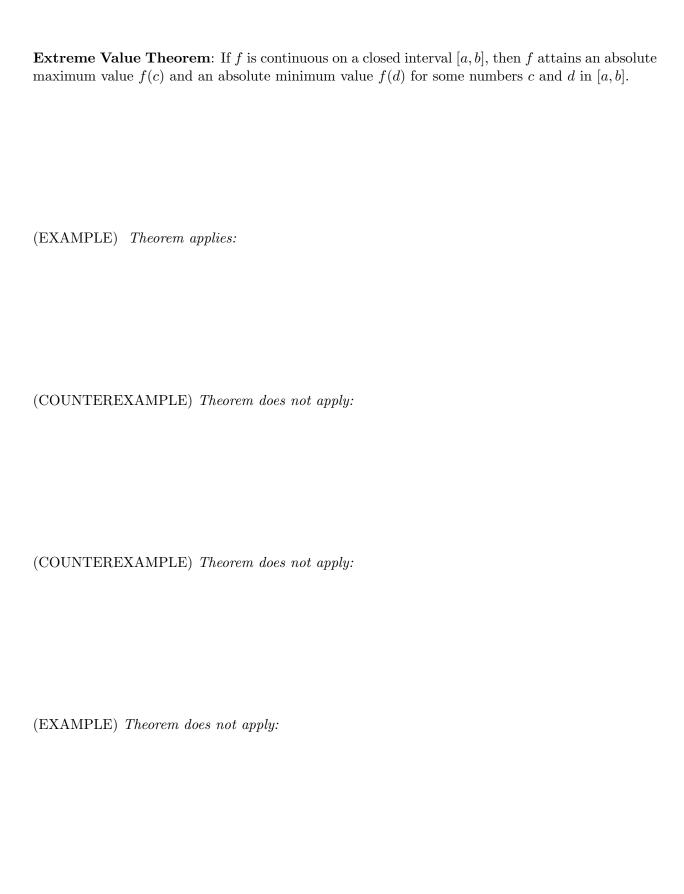
1.
$$f(x) = \sin x$$
 $D = [0, 2\pi]$

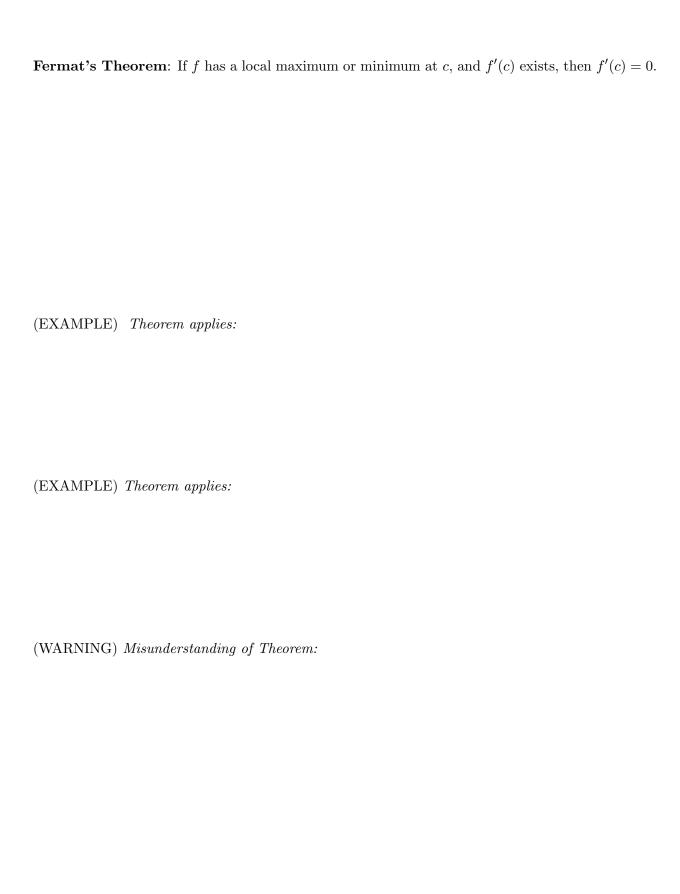
2.
$$g(x) = x^2$$
 $D = (-\infty, \infty)$

3.
$$h(x) = x^3$$
 $D = [-2, 3]$

4.
$$p(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ x - 1 & \frac{1}{2} \le x \le 1 \end{cases}$$
$$D = [0, 1]$$

5.
$$q(x) = x^3 - 2x^2$$
 $D = [-1, 3)$.





(DEFINITION)

A **critical number** of a function f is a number c in the domain of f such that either f'(c) = 0 or f'(c) does not exist.

(EXAMPLES)

Find the critical numbers for each function

1.
$$h(x) = x^4 + 4x^3 + 2$$

2.
$$g(x) = x^{1/3} - x^{-2/3}$$

3.
$$f(x) = |x^2 + 4x + 3|$$

Note: With this new definition, **Fermat's Theorem** says that if a function f has a local maximum or minimum at c, then c is a critical number.

Closed Interval Method for finding the extreme values of a continuous function f on an interval [a, b]:

- 1. Find the values of f at the critical numbers of f in (a,b).
- 2. Find the values of f at the endpoints of the interval.
- 3. The largest value from 1. and 2. is the absolute maximum, and the smallest value is the absolute minimum.

(EXAMPLES) In each case, find the extreme values of the function on the interval.

1.
$$h(x) = x^3 - 3x^2 + 1$$
 on the interval $[-\frac{1}{2}, 4]$

2.
$$f(x) = \ln(x^2 + x + 1)$$
 on the interval [-1, 1]

3.
$$g(x) = xe^{-x^2/8}$$
 on the interval [-1, 4]

4.
$$p(x) = \frac{x}{x^2 - x + 1}$$
 on the interval [-2, 2]

5.
$$q(x) = x - k \arctan x$$
 on the interval $[0, 4]$ $(k > 0 \text{ is a constant})$

6.
$$r(x) = x - 2\cos x$$
 on the interval $[-2, 0]$

7.
$$s(x) = x\sqrt{x - x^2}$$
 on the interval $[0, 1]$