Math 111
Chapter 4.1 Maximum and Minimum Values

(DEFINITIONS)

Let c be a number in the domain D of a function f.

- $f(c)$ is an **absolute maximum** of f on D if $f(c) \geq f(x)$ for all x in D.
- $f(c)$ is an **absolute minimum** of f on D if $f(c) \leq f(x)$ for all x in D.

NOTE: These values are sometimes called *global maximum* or *global minimum* or the *extreme values* of the function.

(DEFINITIONS)

Let c be a number in the domain D of a function f.

- $f(c)$ is an **local maximum** of f if $f(c) \geq f(x)$ for x near c.
- $f(c)$ is an **local minimum** of f if $f(c) \leq f(x)$ for x near c.

(EXAMPLES)

1. $f(x) = \sin x \qquad D = [0, 2\pi]$

2. $g(x) = x^2 \qquad D = (-\infty, \infty)$

3. $h(x) = x^3 \qquad D = [-2, 3]$

4. $$p(x) = \begin{cases} x & 0 \leq x < \frac{1}{2} \\ x - 1 & \frac{1}{2} \leq x \leq 1 \end{cases}$$

   $D = [0, 1]$

5. $q(x) = x^3 - 2x^2 \qquad D = [-1, 3)$.

Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some numbers c and d in $[a, b]$.

(EXAMPLE) *Theorem applies:*

(COUNTEREXAMPLE) *Theorem does not apply:*

(COUNTEREXAMPLE) *Theorem does not apply:*

(EXAMPLE) *Theorem does not apply:*

Fermat's Theorem: If f has a local maximum or minimum at c, and $f'(c)$ exists, then $f'(c) = 0$.

(EXAMPLE) *Theorem applies:*

(EXAMPLE) *Theorem applies:*

(WARNING) *Misunderstanding of Theorem:*

(DEFINITION)
A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

(EXAMPLES)
Find the critical numbers for each function

1. $h(x) = x^4 + 4x^3 + 2$

2. $g(x) = x^{1/3} - x^{-2/3}$

3. $f(x) = |x^2 + 4x + 3|$

Note: With this new definition, **Fermat's Theorem** says that if a function f has a local maximum or minimum at c, then c is a critical number.

Closed Interval Method for finding the extreme values of a continuous function f on an interval $[a,b]$:

1. Find the values of f at the critical numbers of f in (a,b).
2. Find the values of f at the endpoints of the interval.
3. The largest value from 1. and 2. is the absolute maximum, and the smallest value is the absolute minimum.

(EXAMPLES) In each case, find the extreme values of the function on the interval.

1. $h(x) = x^3 - 3x^2 + 1$ on the interval $[-\frac{1}{2}, 4]$

2. $f(x) = \ln(x^2 + x + 1)$ on the interval $[-1, 1]$

3. $g(x) = xe^{-x^2/8}$ on the interval $[-1, 4]$

4. $p(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[-2, 2]$

5. $q(x) = x - k \arctan x$ on the interval $[0, 4]$ ($k > 0$ *is a constant*)

6. $r(x) = x - 2\cos x$ on the interval $[-2, 0]$

7. $s(x) = x\sqrt{x - x^2}$ on the interval $[0, 1]$