# Math 111 Chapter 2.5: Continuity

(DEFINITION) A function $f$ is <b>continuous</b> at $x = a$ if $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$
(EXAMPLES)
(DEFINITION) A function $f$ is <b>continuous</b> on an interval $[a,b]$ it is continuous for all numbers in the interval.
(EXAMPLES)
A function $f$ is <b>not continuous</b> at $x = a$ if:
1.
2.
3.

# (EXAMPLES)

## 1. Jump discontinuity

$$g(t) = \begin{cases} 2 - t^2 & t \le -1 \\ e^{-t} & t > -1 \end{cases}$$

# 2. Removable discontinuity

$$p(x) = \frac{x - 4}{x^2 - x - 12}$$

## 3. Infinite discontinuity

$$q(x) = \frac{4}{(x-3)^2}$$

$$r(x) = \frac{x - 4}{x^2 - x - 12}$$

#### (EXAMPLES)

1. What value of b makes h continuous at  $\pi/2$ ?

$$h(t) = \begin{cases} \sin t & t > \pi/2 \\ b - t & t \le \pi/2 \end{cases}$$

2. Is p continuous at x = 0? Why, or why not?

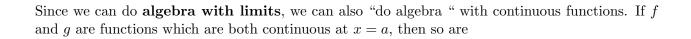
$$p(x) = \begin{cases} |x| & x \neq 0 \\ 4 & x = 0 \end{cases}$$

3. Is it possible to define w(x) so that w is continuous at x = 1?

$$w(x) = \frac{x^3 - 1}{x - 1}$$

# (COMMON CONTINUOUS FUNCTIONS)

- 1. Power functions,  $x^r$ ,
- 2. Exponential functions,  $a^x$
- 3. Logarithmic functions,  $\log_a x$
- 4. Trigonometric functions  $\sin x$ ,  $\cos x$
- 5. Absolute value function |x|



(EXAMPLES)

(THEOREM) If 
$$\lim_{x\to a}g(x)=b$$
 and  $f$  is continuous at  $b$ , then  $\lim_{x\to a}f(g(x))=f(b)$ .

(EXAMPLES)

$$\lim_{x \to 0} \cos(x^2 + \pi)$$