Math 111
Chapter 3.10: Linear Approximation

Idea: The graph of a function f is very close to the line tangent at a $x = a$, so long as we look *near* $x = a$.

Idea: The value of the function *near* $x = a$ should be *near* the value of the tangent line. In this context, we refer to the tangent line as the **linearization** of f at a.

(EXAMPLE)

Estimate $\sqrt{4.1}$ using the linearization of the square root function.

Note this approach does not do very well to estimate $\sqrt{5}$, or $\sqrt{6}$.

Linearization is also called *linear approximation* or *tangent line approximation*.

(EXAMPLES)

1. Estimate $e^{0.1}$ using a linear approximation.

2. Estimate $\ln 1.2$ using a linearization.

3. Suppose $P(t)$ represents a population, and that the function satisfies the initial value problem.

$$\begin{cases} \dfrac{dP}{dt} = 2.2\sqrt{P} \\ P(0) = 900 \end{cases}$$

Use linearization to estimate $P(1)$.

Another way to compute this approximations is in terms of **differentials**. The idea is that we replace f with the linearization L and let dx be a small change in x. Then dy is the resulting change in y.

$$dy = f'(x)dx$$

(EXAMPLES)

1. Compute dy and Δy if $y = f(x) = x^3 + x^2 - 2x + 1$ and x changes from 2 to 2.05.

2. A sphere is measured and found to be 21 cm with a possible error in the measurement of at most 0.05 cm.

   (a) Estimate the maximum error made in using this measurement to compute the volume of the sphere.

   (b) Estimate the maximum *relative* error made in using this measurement to compute the volume of the sphere.

3. Estimate the volume of a thin cylindrical shell with radius r, height h, and thickness dr.

A BETTER APPROXIMATION

Suppose we are not satisfied with $e^{0.1} \approx 1.1$, or $\sqrt{4.1} \approx 2.025$, can we do better?