Math 111
Chapter 2.8: Derivatives as functions

(DEFINITION) Given a function f, we may define the **derivative function** f' by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The definition is the same as $f'(a)$ but we now take the point of view that a is an input and $f'(a)$ is an output.

(NOTATIONS) If we use $y = f(x)$ to represent a function, we may use y', $\frac{dy}{dx}$, $\frac{df}{dx}$, $\frac{d}{dx}[f(x)]$ all to mean $f'(x)$.

(EXAMPLE) Find $f'(x)$ if $f(x) = 2x^2 + 5x - 3$. Sketch the graph of f and f'.

(DEFINITION) We say that a function f is **differentiable at** a if $f'(a)$ exists.

(EXAMPLE) $f(x) = 2x^2 + 5x - 3$ is differentiable at all numbers since $f'(a)$ exists for any a.

(EXAMPLE) Find $g'(x)$ if $g(x) = \dfrac{1}{\sqrt{x}}$. Sketch the graph of g and g'. Where is g differentiable?

(EXAMPLE) Find $p'(x)$ if $p(x) = \dfrac{3-x}{x+1}$. Sketch the graph of p and p'. Where is g differentiable?

(EXAMPLE) Find $f'(x)$ if $f(x) = |x|$. Sketch the graph of f and f'. Where is f differentiable?

A function is **not differentiable** at a if any of the following are true:

1.

2.

3.

(THEOREM) If f is differentiable at a, then it is continuous at a.