Math 111
Chapter 2.5: Continuity

(DEFINITION) A function f is **continuous** at $x = a$ if $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

(EXAMPLES)

(DEFINITION) A function f is **continuous** on an interval $[a, b]$ it is continuous for all numbers in the interval.

(EXAMPLES)

A function f is **not continuous** at $x = a$ if:

1.

2.

3.

(EXAMPLES)

1. **Jump discontinuity**

$$g(t) = \begin{cases} 2 - t^2 & t \leq -1 \\ e^{-t} & t > -1 \end{cases}$$

2. **Removable discontinuity**

$$p(x) = \frac{x-4}{x^2 - x - 12}$$

3. **Infinite discontinuity**

$$q(x) = \frac{4}{(x-3)^2}$$

$$r(x) = \frac{x-4}{x^2 - x - 12}$$

(EXAMPLES)

1. What value of b makes h continuous at $\pi/2$?

$$h(t) = \begin{cases} \sin t & t > \pi/2 \\ b - t & t \leq \pi/2 \end{cases}$$

2. Is p continuous at $x = 0$? Why, or why not?

$$p(x) = \begin{cases} |x| & x \neq 0 \\ 4 & x = 0 \end{cases}$$

3. Is it possible to define $w(x)$ so that w is continuous at $x = 1$?

$$w(x) = \frac{x^3 - 1}{x - 1}$$

(COMMON CONTINUOUS FUNCTIONS)

1. Power functions, x^r,
2. Exponential functions, a^x
3. Logarithmic functions, $\log_a x$
4. Trigonometric functions $\sin x$, $\cos x$
5. Absolute value function $|x|$

Since we can do **algebra with limits**, we can also "do algebra " with continuous functions. If f and g are functions which are both continuous at $x = a$, then so are

(EXAMPLES)

(THEOREM) If $\lim_{x \to a} g(x) = b$ and f is continuous at b, then $\lim_{x \to a} f(g(x)) = f(b)$.

(EXAMPLES)

$$\lim_{x \to 0} \cos\left(x^2 + \pi\right)$$