Math 111
Chapter 3.6: Logarithmic Differentiation

The derivative of the **natural logarithm function**:

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

The derivative of logarithm functions of other bases:

$$\frac{d}{dx}\left[\log_b x\right] = \frac{1}{x \ln b}$$

(EXAMPLES)

1. Find $h'(x)$ if $h(x) = \ln(\tan x)$

2. Find $p'(v)$ if $p(v) = v^3 \ln v$

3. Find $y'(t)$ if $y(t) = \dfrac{1}{\log_3 t}$

4. Find $f'(x)$ if $f(x) = \ln(e^x + \sin x)$

5. Find $g'(t)$ if $g(t) = \ln(\ln 10t)$

6. Find $f'(x)$ if $f(x) = \ln|x|$

7. Find $\dfrac{dy}{dx}$ if $y = \ln(3x + y^2)$

8. Find $f'(x)$ if $f(x) = \ln\left(\dfrac{\sqrt{x}\cos x}{2 - x^3}\right)$

   Hint: Algebra of logaritms:

   - $\ln(ab) = \ln a + \ln b$
   - $\ln(a/b) = \ln(a) - \ln(b)$
   - $\ln(a^r) = r\ln(a)$

Logarithmic Differentiation

In the following examples, find $\dfrac{dy}{dx}$ by differentiating $\ln y$ using implicit differentiation. The final answer can be written in terms of x.

1. $y = \dfrac{(2x+1)^8 \tan^3 x}{e^x - x}$

2. $y = \dfrac{x^{2/5}\sqrt{x^2+5x}}{(x-1)^4}$

3. $y = e^{\sin x}(x^2-1)^8(\ln x)^3$

4. $y = x^{\tan x}$

5. $y = (\ln x)^x$

In summary, there are 4 different cases of functions involving exponents. Suppose b, and n are constants, while f and g are functions. Consider the derivatives of the following functions:

- b^n
- $[f(x)]^n$
- $b^{g(x)}$
- $[f(x)]^{g(x)}$

A limit definition of e:

$$\lim_{x \to 0} (1 + x)^{1/x}$$

(*Note that our previous definition of e did not give us a way to approximate it's value.*)

OR

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$