Math 111
Chapter 1: Precalculus Review

(DEFINITION)

A **function** is:

(EXAMPLES)

Relationships that we might think of as a function:

Ways that we might describe a function:

Relationships that are *not* functions

(NOTATION)

If $f(x) = \dfrac{x}{2} - 1$ and $g(p) = p^2 + 2$

Then $f(4) =$ $\qquad\qquad$ $g(3) =$

$f(a) =$ $\qquad\qquad$ $g(p+h) =$

(DEFINITION)

The **domain** of a function is:

(DEFINITION)

The **range** of a function is:

(DEFINITION)

The **graph** of a function is:

(EXAMPLES)

What are the domain and range of the following functions? Sketch the graph and label two points on the graph.

$y(x) = \sqrt{x-5}$

$$f(x) = \frac{1}{x^2 - 9}$$

$p(x) = \sqrt{4 - x^2}$

Linear functions

(DEFINITIONS)

A function f is **linear** if:

Slope

(EXAMPLES)

Let C be a temperature measured in degrees Celsius and F be the same temperature measured in degrees Fahrenheit. We know F is a linear function of C and that $F(0) = 32$ and $F(100) = 212$. Find a formula for $F(C)$.

1. Find a formula for $F(C)$.
2. Give an interpretation of the slope. What are the units?

Let $T(t)$ represent the temperature of a lake as a function of time. T is measured in degrees Celsius and t is measured in hours. Suppose that we know that T is a linear function and that $T(4) = 6$ and $T(8) = 7$.

1. Find $T(t)$ and give an interpretation of the slope. What are the units?
2. How much does T change in 10 hours?

For small mammals it has been determined that body mass is proportional to heart mass. If a 4.7 kg dog has a heart mass of 33 g, find the mass of a 1.8 kg cat. Present your solution in terms of a linear function.

Piecewise defined functions

In some cases it is useful to describe functions by giving different output rules depending on (EXAMPLES)

$$g(x) = \begin{cases} x^2 & x \geq 1 \\ 2 - x & x < 1 \end{cases}$$

$$f(x) = \begin{cases} x - 1 & x > 0 \\ 3 & -2 \leq x \leq 0 \\ 2 - x & x < -2 \end{cases}$$

$$h(x) = |x|$$

(EXAMPLES) Some situations where piecewise defined functions might make sense:

Algebra of functions

If f and g are functions, then so are $f+g$, $f-g$, fg, and $\frac{f}{g}$ for the appropriate domain

(EXAMPLE)

Suppose $a(t) = t^2 + 1$ represents the population of fish species A, $b(t) = \frac{500}{1+2t}$ represents the population of fish species B, and $p(t)$ represents the average individual mass of a fish of species A, then:

1. $(a+b)(t) =$ and represents
2. $(a-b)(t) =$ and represents
3. $(ap)(t) =$ and represents

If f and g are functions, then so does $f \circ g$ on the appropriate domain

(EXAMPLE)

1. Suppose $r(x) = 100(1 - 1/x)$ represents a rabbit population as a function of x, the amount of edible vegetation in a habitat and $f(r) = \sqrt{r}$ represents the population of foxes as a function of the rabbit population, then:

   $f \circ r =$ and represents

2. If $f(x) = \frac{x}{x+3}$ and $g(x) = \frac{1}{x}$, then
   $f(g(x)) =$ with domain

   $g(f(x)) =$ with domain

3. If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 3x$, then
   $f(g(x)) =$ with domain

   $g(f(x)) =$ with domain

Inverse functions

(DEFINITION) If f is a one-to-one function, then f^{-1} is the function that 'undoes' f

(EXAMPLES)

1. Let C be a temperature measured in degrees Celsius and F be the same temperature measured in degrees Fahrenheit. Let f be the function that associates C to F. We found earlier that $f(C) = \frac{9}{5}C + 32$.
   Suppose now we know the temperature is 45 degrees Fahrenheit and want to compute the temperature in degrees Celsius. We can make use of f^{-1}.

2. If $g(x) = x^3$ then $g^{-1}(x) =$

   If $h(w) = 3w$ then $g^{-1}(w) =$

   If $f(y) = \frac{1}{y}$ then $f^{-1}(y) =$

3. If $g(x) = x^3 + 4$ then $h^{-1}(x) =$

4. If $h(x) = \dfrac{4x - 1}{2x + 3}$ then $h^{-1}(x) =$

5. If $f(x) = x^2 - x$ for $x \geq \dfrac{1}{2}$, then $f^{-1}(x) =$