Math 112
Chapter 11.5: Alternating Series

Alternating Series Test:

If the alternating series

$$\sum_{1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + ... \qquad (b_n \geq 0)$$

satisfies $b_{n+1} \leq b_n$ for large n and $\lim\limits_{n\to\infty} b_n = 0$ then the series converges.

EXAMPLES:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} \quad \text{(Alternating Harmonic Series)}$$

$$\sum_{n=1}^{\infty} \frac{n^2(-1)^{n-1}}{n^3+1}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} n^2 e^{-4/n}$$

$$\sum_{n=1}^{\infty} \frac{5n(-1)^{n-1}}{n^2+9}$$

Remainder Estimate for Alternating Series:

If the alternating series

$$\sum_{1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + ... \qquad (b_n \geq 0)$$

converges to a number s, and $R_n = s - s_n$ is the remainder, then $|R_n| \leq b_{n+1}$.

EXAMPLES:

How large can R_{10} be for the series $\displaystyle\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n!}$?

How large should N be so that $\displaystyle\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n^3} - \sum_{1}^{N}\frac{(-1)^{n+1}}{n^3} < 0.0001$?

How large should N be so that $\displaystyle\sum_{2}^{\infty}\frac{(-\frac{1}{2})^{n+1}}{n} - \sum_{2}^{N}\frac{(-\frac{1}{2})^{n+1}}{n} < 0.005$?

DEFINITIONS:

A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.

A series $\sum a_n$ is called **conditionally convergent** if the series is convergent but not absolutely convergent.

EXAMPLES:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

THEOREM: If a series $\sum a_n$ is absolutely convergent, then it is convergent.

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$$

EXERCISES:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt{n}}{3n+2}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \sin\left(\frac{1}{n}\right)$$

$$\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{4^n}$$

$$\frac{1}{\ln 3} - \frac{1}{\ln 5} + \frac{1}{\ln 7} - \frac{1}{\ln 9} + \ldots.$$