

Math 112  
Chapter 5.5: Substitution Rule

MORE EXAMPLES:

$$\int \frac{x}{\sqrt{x+3}} dx$$

$$\int \frac{x}{1+x^4} dx$$

EXERCISES:

$$\int \tan^2 \theta \sec^2 \theta d\theta$$

$$\int_1^2 \frac{1}{x^3} \sin\left(\frac{\pi}{x^2}\right) dx$$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$$

Math 112  
Chapter 6.1: Areas

In this section, we extend the calculation of areas to include other possibilities.

EXAMPLES:

1. Find the area bounded by  $x = 0$ ,  $x = 1$ ,  $y = x$ , and  $y = e^x$ .
2. Find the area bounded by  $y = 1$ , and  $y = \cos \pi x$ , between  $x = 0$  and  $x = 2$ .

3. Find the area bounded by  $x = -\frac{1}{2}$ ,  $x = \frac{1}{2}$ ,  $y = 1 - x^2$ , and  $y = 2 + \frac{1}{\sqrt{1 - x^2}}$ .

4. Find the area bounded by  $y = x$  and  $y = (x - 2)^2$ .

5. Find the area bounded by  $y = x\sqrt{4 - x^2}$ , and  $y = -x\sqrt{4 - x^2}$

In some cases upper and lower boundaries change as a function of  $x$ .

6. Find the area bounded by  $y = \sin x$ , and  $y = \cos x$ , between  $x = 0$  and  $x = \pi/2$ .

In some cases it may be simpler to divide the area horizontally, and construct an integral in  $y$ .

7. Find the area bounded by  $y = \ln x$ , and  $x = y^2 - 2$ ,  $y = -1$  and  $y = 1$ .

8. Find the area bounded by  $x = 2y^2$  and  $x = 4 + y^2$ .