Convergence tests

Test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Integral Test:
Suppose that f is a continuous, positive, decreasing function on $[1,\infty)$ and that $a_n = f(n)$. Then $\int_1^{\infty} f(x)\,dx$ and $\sum_1^{\infty} a_n$ either both converge or both diverge.

Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\sum b_n$ converges and $a_n \leq b_n$ for all n, then $\sum a_n$ also converges. If $\sum b_n$ diverges and $a_n \geq b_n$ for all n, then $\sum a_n$ also diverges.

Limit Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number that is *not zero*, the either both series converge or both diverge.

Alternating Series Test:

If the alternating series $\sum_0^{\infty} (-1)^{n-1} b_n$ with $b_n \geq 0$ satisfies $b_{n+1} \leq b_n$ for large n and $\lim_{n\to\infty} b_n = 0$, then the series converges.

Ratio Test:

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then $\sum_1^{\infty} a_n$ converges absolutely.

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$, then $\sum_1^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

Root Test:

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum_1^{\infty} a_n$ converges absolutely.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then $\sum_1^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the Root Test is inconclusive.