Math 112
Chapter 5.4: Indefinite Integrals and Net Change

Indefinite integrals are a notation for antiderivatives.

NOTES:

1. Definte integrals are numbers, indefinite integrals are functions.

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2. Fundamental Theorem connects definite and indefinite integrals.

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3. Indefinite integrals represent a family of functions.

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EXAMPLES:

$$\int \sec^2 \theta \, d\theta$$

$$\int \frac{2}{x} + \frac{x}{8} \, dx$$

$$\int \frac{\sin t}{\cos^2 t} \, dt$$

Net Change Theorem (FTC Part II)

The integral of a rate of change equals the net change.

$$\int_a^b F'(x) = F(b) - F(a)$$

EXAMPLES:

1. Let $V(t)$ be the volume of water in a tank as a function of time. Then $\displaystyle\int_a^b \frac{dV}{dt} \, dt = \Delta V$.

2. A colony of bactera starts at population 100 and grows at rate of $200e^t$ bactera/hr. What is the population after 4 hours?

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3. If $x(t)$ represents the position of an object, and $v(t) = x'(t)$ its velocity, then $\int_a^b v(t)\,dt$ calculates the displacement from time a to time b. Suppose $v(t) = 3t^2 - 24t + 36$ m/s.

   (a) What is the displacement during the interval $[0, 2]$?

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   (b) What is the displacement during the interval $[0, 6]$?

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   (c) What is the distance traveled during the interval $[0, 6]$?

Chapter 5.5: The Substitution Rule

In order to compute antiderivatives, we will need to use differentiation rules in reverse.

Chain Rule review: Differentiate the following functions

$h(x) = (x^2 + 4x^4)^{10}$ $\qquad$ $p(t) = \ln(\tan t)$ $\qquad$ $y(u) = \arctan e^u$

Evaluate the integral $\int 2x\sqrt{1+x^2}\,dx$.

Substitution Rule

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

MANY EXAMPLES:

$$\int x^2 e^{x^3}\,dx$$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$$\int e^{2x}\cos e^{2x}\,dx$$

$$\int (3w+1)^3\,dw$$

$$\int \tan\theta\,d\theta$$

$$\int_0^4 \sqrt{4+3x}\,dx$$

$$\int_0^{\sqrt{\pi}} t\sin t^2\,dt$$

$$\int_0^{\ln 3} \frac{e^x}{1+e^x}\,dx$$

$$\int_1^{e^2} \frac{\ln x}{x}\,dx$$

$$\int_3^4 \frac{dx}{(x-2)^2}$$