

Math 112  
Chapter 6.2: Volumes

The goal of this section is to use integrals to compute volumes. We start by looking at **volumes of revolution**.

$V = \int_a^b A(x) dx$  where  $A(x)$  is the area of the cross-section at  $x$ .

EXAMPLES:

1. An area is bounded by  $y = 0$ ,  $y = \sqrt{x}$ , and  $x = 1$ . Find the volume generated by rotating the area around the  $x$ -axis.

2. An area is bounded by  $y = 0$ ,  $y = \sec x$ ,  $x = -\pi/6$  and  $x = \pi/6$ . Find the volume generated by rotating the area around the  $x$ -axis.

3. An area is bounded by  $y = x^3$ ,  $y = 8$ , and  $x = 0$ . Find the volume generated by rotating the area around the  $y$ -axis.

4. Find the volume of solid obtained by rotating the the area bounded by  $y = x^2 + 1$  and  $y = 9 - x^2$  around the  $x$ -axis.

5. Find the volume of solid obtained by rotating the the area bounded by  $x = y^2 + 1$  and  $y = x - 3$  around the  $y$ -axis.

6. The volume of a sphere of radius  $r$  is  $V = \frac{4}{3}\pi r^3$ . Derive this formula with an integral.

7. The volume of a cone with a height  $h$  and base radius  $r$  is  $V = \frac{1}{3}\pi r^2 h$ . Derive this formula with an integral.