Math 112

Chapter 5.2: The Definite Integral

For a generic area defined by $a \leq x \leq b$, and $0 \leq y \leq f(x)$ where f is a positive function, we can make an estimate using a **Riemann Sum**.

The limit of the approximation as $N \to \infty$ is what we call the **definite integral**.

NOTES:

We adopt some *conventions* so that the integral definition makes sense in other situations.

EXAMPLES:

$$\int_0^3 x - 1 \, dx$$

$$\int_1^0 \sqrt{1 - x^2} \, dx$$

$$\int_{-1}^{3} |x| \, dx$$

$$\int_{0}^{2\pi} \cos x \, dx$$

PROPERTIES:

EXAMPLES:

$$\int_{0}^{1} 2x + 3(1 - x^2) \, dx$$

If $\int_2^6 f(x)\,dx = 7$ and $\int_2^3 f(x)\,dx = 5$, find $\int_3^6 f(x)\,dx$

COMPARISON PRINCIPLES:

EXAMPLE:

Give upper and lower bounds on the value of $\int_0^2 e^{-x}\,dx$.

We can always use Riemann sums to *approximate* the value of an integral. In addition to R_N and L_N, we could use other sums such as the **Midpoint Rule**.

EXAMPLE:

Find the M_4 approximation of $\displaystyle\int_0^2 e^{-x}\,dx$.

Approximations do better with larger N.