Math 112
Chapter 5.3: Fundamental Theorem of Calculus

Area function: Let $g(x) = \int_a^x f(t)\,dt$.

EXAMPLE:

$g(x) = \int_0^x (2-t)\,dt$.

Fundamental Theorem of Calculus (Part I)

If f is continuous on $[a, b]$, then $g(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$, differentiable on (a, b), and $g'(x) = f(x)$.

EXAMPLES:

Differentiate the following functions.

$g(x) = \int_1^x t^3\,dt$

$h(x) = \int_\pi^x \cos\sqrt{u}\,du$

NOTES:

MORE EXAMPLES:

$y(x) = \int_x^0 \frac{\sin\theta}{\theta^2 + 1}\,d\theta$

$$z(x) = \int_3^{\sqrt{x}} 2p^2 - p \, dp$$

Proof of FTC I:

Fundamental Theorem of Calculus (Part II)

If f is continuous on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative of f

EXAMPLES:

$\int_0^1 1 - x^2\,dx$

$\int_0^{\pi/2} \cos\theta\,d\theta$

Proof of FTC II:

EXAMPLES:

$$\int_1^4 \frac{1}{x}\,dx$$

$$\int_0^9 \sqrt{2t}\,dx$$

$$\int_{-2}^2 (3u+1)^2\,du$$

Integrals where the integrand is discontinuous?

Find $\displaystyle\int_0^2 g(x)\,dx$ if

$$g(x) = \begin{cases} 3-x & x < 1 \\ -x^2 & x \geq 1 \end{cases}$$

$$\int_{-2}^{1} \frac{1}{x^2}\, dx$$

EXERCISES:

$$\int_{0}^{4} (4-t)\sqrt{t}\, dt$$

$$\int_{1}^{18} \sqrt{\frac{3}{z}}\, dz$$

$$\int_{1}^{2} \frac{u+5u^5}{u^3}\, du$$

$$\int_{-1}^{4} 3^x\, dx$$

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-w^2}}\, dw$$