Math 112
Chapter 6.5: Integrals and Averages

If we have a set of data x_1, x_2, x_3, ... x_N, we define the average $\bar{x}$ as:

EXAMPLE:

In order to calculate the average outside temperature over a period of 10 hours, we could collect data at a set of times.

Calcuate $\bar{T}$:

For a more accurate average, we can collect more data.

If we let the number of data points N go to infinity, we can produce an integral

DEFINITION:

The **average value** of a function f on an interval $[a, b]$ is $\bar{f}$

EXAMPLE:

Find the average value of $f(x) = \sqrt[3]{x}$ on the interval $[0, 8]$.

APPLICATION:

A growing fish has length that is modeled by the function $L(t) = 20 - \dfrac{20}{t+1}$.

1. What is the average length of the fish over the first 10 years of its life?
2. What is the average length of the fish over the first T years of its life?
3. What happens to this average as T gets large?

APPLICATION:

The velocity of fluid flowing slowly in a cylindrical tube of radius R is a function of the radial distance from the center.

$$v(r) = \frac{P}{4\eta L}(R^2 - r^2)$$

Find the average velocity from $r = 0$ in the center to $r = R$ on the edge. Make a comparision to the maximum velocity.

Geometric meaning:

EXAMPLE:

Find a value of x^* in $[0,\pi]$ so that $\displaystyle\int_0^\pi \sin x\,dx = \pi \sin x^*$.

Mean Value Theorem for Integrals

If f is a continuous function on $[a,b]$, then there exists a number c in $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

EXAMPLE:

Find the value of c described in the theorem for $f(x)=e^x$ on the interval $[-1,3]$.