# Math 112
## Chapter 9.5: Linear Differential Equations

A differential equation is called **linear** if can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P$ and $Q$ are continuous functions on some interval.

Solution method is based on the Product Rule

$$\frac{d}{dx}[xy] = y + x\frac{dy}{dx}$$

EXAMPLE:

$$x\frac{dy}{dx} + y = x$$

More generally, we will need to multiply by an **integrating factor** $u(x)$ in order to use the Product Rule.

$$y' - 2y = 3e^x$$

Solve the initial value problem:

$$\begin{cases} xy' + 2y = 4x^2 \\ y(1) = 2 \end{cases}$$

We can write down the formula for a solution in terms of $P$ and $Q$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

APPLICATIONS:

A tank contains 1000 L of whiskey 35% alcohol. At $t = 0$, a whiskey with concentration 45% alcohol begins flowing into the tank at a rate of 2 L/min. If the tank is kept well-stirred and is drained at a rate of 5 L/min, find the concentration of the whiskey blend in the tank as a function of time.

The following modification of Newton's Law of Cooling can be used in the case that the ambient temperature varies in time.

$$\begin{cases} \dfrac{dT}{dt} = k(A(t) - T) \\ T(0) = T_0 \end{cases}$$

Suppose for example that $A(t)$ represents the outside temperature and $T(t)$ is the interior temperature of a building with no climate control. If we measure $T$ in hours, it might be reasonable to assume $A$ is a periodic function such as $A(t) = 14 + 5\cos\dfrac{\pi t}{12}$.

Let's take $T_0 = 19$, $k = 0.15/\text{hr}$, and see what the model predicts.