Math 112
Chapter 5: Bonus questions

1. (a) Which is larger, $\left|\int_a^b f(x)\,dx\right|$ or $\int_a^b |f(x)|\,dx$? Why?

   (b) Demonstrate by calculating $\left|\int_0^2 2x-1\,dx\right|$ and $\int_0^2 |2x-1|\,dx$.

   (c) Explain why it must always be true.

   Hint: Use a comparison principle and the fact that $-|f(x)| \le f(x) \le |f(x)|$.

2. Let $g(x) = \int_0^x f(t)\,dt$. Sketch a function f so that g has the following properties:

   (a) g has a local minimum at $x = 2$.
   (b) g is concave up for $1 < x < 4$.
   (c) $g(4) = 0$.

3. Let $A = \int_1^4 \ln x\,dx$. Determine how large N should be so that $R_N - A < 0.0001$.

   Hint: $L_N < A < R_N$ *(why?) so* $R_N - A < R_N - L_N$.