Math 112
Chapter 11.2: Series

A **series** is an infinite sum of numbers.

$$\sum_{n=1}^{\infty} a_n$$

EXAMPLES:

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

To understand if the series are finite, we define the sequence of **partial sums**

$$s_n = \sum_{k=0}^{n} a_k$$

If the limit of the s_n exists we say that the series $\sum a_n$ **converges**.
If the limit of the s_n does not exist we say that the series $\sum a_n$ **diverges**.

TELESCOPING SERIES

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

GEOMETRIC SERIES

$$\sum_{n=0}^{\infty} r^n$$

EXAMPLES

$$20 + 5 + \frac{5}{4} + \frac{5}{16} + ...$$

$$6 + 9 + \frac{27}{2} + \frac{81}{4} + ...$$

$$\sum_{n=1}^{\infty} \frac{2^{n-3}}{5^n}$$

$$1 - \frac{1}{7} + \frac{1}{7^2} - \frac{1}{7^3} + ...$$

$$\sum_{n=0}^{\infty} \frac{1}{\pi^n}$$

For what values of x does the series converge?

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{5^n}$$

For what values of x does the series converge?

$$\sum_{n=0}^{\infty} \left(\frac{2}{x}\right)^n$$

REPEATING DECIMAL EXPANSIONS

Use a geometric series to find the fraction represented by 0.72727272....

Use a geometric series to find the fraction represented by 0.135135...

Use a geometric series to find the fraction represented by 0.999...

GENERAL SERIES

For most series, it is difficult or impossible to find a formula for the partial sums. We will instead rely on a series of **convergence tests**. These tests are rules that we can apply to determine if a series converges.

THEOREM: If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

EXAMPLES:

$$\sum_{n=1}^{\infty} \sqrt{n}$$

$$\sum_{n=1}^{\infty} \frac{n^2}{2n^2+5}$$

$$\sum_{n=1}^{\infty} \frac{n}{\ln n}$$

$$\sum_{n=1}^{\infty} e^{1/n}$$

WARNING: If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ may converge *or* diverge.

EXAMPLES:

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$