Math 112
Chapter 5.1: The Area Problem

Our goal is to determine a way to calculate areas that are enclosed by curved boundaries. Our starting point is an area defined by $a \leq x \leq b$, and $0 \leq y \leq f(x)$ where f is a positive function.

How should we define the area? What do we really *know* about area?

(EXAMPLE)

Determine the area defined by $0 \leq x \leq 1$ and $0 \leq y \leq 1 - x^2$

Idea: Divide the area into pieces and estimate the area of each piece with a simple shape.

L_2**:** Area of 2 rectangles with height determined by the function value on the left side.

L_4**:** Area of 4 rectangles with height determined by the function value on the left side.

L_8**:** Area of 8 rectangles with height determined by the function value on the left side.

What happens as the number of rectangles gets larger?

Another possibility:

R_2**:** Area of 2 rectangles with height determined by the function value on the left side.

R_4**:** Area of 4 rectangles with height determined by the function value on the left side.

What happens as the number of rectangles gets larger?

Idea: It may be possible to define the area as a limit on the area as the number of rectangles grows!

Formula for L_N:

Handy formula: $1^2 + 2^2 + 3^2 + ... + M^2 = \dfrac{M(M+1)(2M+1)}{6}$

$A = \lim_{N\to\infty} L_N$

(EXERCISES)

1. Find a formula for R_N and calculate $\lim_{N\to\infty} R_N$. You will get the same number
2. Repeat the general procedure to find the area bounded defined by $0 \leq x \leq 2$ and $0 \leq y \leq x^3$ You will need a handy formula $1^3 + 2^3 + 3^3 + ... + M^3 = \dfrac{M^2(M+1)^2}{4}$.

Sigma (Summation) Notation

Sigma notation is a compact way to write sums of numbers when there is a pattern in the numbers.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + ... + a_{n-1} + a_n$$

(EXAMPLES)

$$\sum_{i=1}^{4} i^2$$

$$\sum_{i=3}^{7} i$$

$$\sum_{i=0}^{5} 2^1$$

$$\sum_{i=2}^{6} 3$$

(NOTES)

Writing L_N and R_N from the previous example.