Math 112
Chapter 7.3: Trigonometric Substitutions

Previously, we made substitutions of the form $u = g(x)$. We now consider making substitutions such as $x = h(\theta)$

In the following examples, h will be a trigonometric function, but there is no reason that requires it.

EXAMPLE:

Calculate the area of a circle using an integral.

We will make a choice of substitution based on comparing the integrand to this table.

Form of integrand	Substitution
$a^2 - x^2$	$x = a\sin\theta$
$x^2 - a^2$	$x = a\sec\theta$
$x^2 + a^2$	$x = a\tan\theta$

EXAMPLE:

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

The steps to follow (which are similar to those used to apply the substitution rule) are:

1. Choose $x =$ ______________ and calculate the corresponding dx.
2. Exchange *all* x for θ.
3. Do the integral with trig functions.
4. Draw a triangle if needed to help undo the substitution.

EXAMPLES:

$$\int \frac{\sqrt{x^2-1}}{x}\,dx$$

$$\int \frac{dx}{[(ax)^2+b^2]^{3/2}}$$

$$\int \frac{x}{\sqrt{x^2-1}}\,dx$$

$$\int \frac{\sqrt{16-x^2}}{x}\,dx$$

$$\int \frac{dt}{\sqrt{t^2-6t+13}}$$

$$\int \frac{1}{3x^2+8}\,dx$$

$$\int_0^2 \frac{e^t}{\sqrt{1+e^{2t}}}\,dt$$