Math 112
Chapter 9.1: Differential Equations

A **differential equation** is a relationship between and unknown function and its derivatives.

EXAMPLES:

$$\frac{dy}{dt} = 4y$$

$$\frac{dy}{dt} = 3t^2 y$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dy}{dt} + y = e^{-t}$$

NOTES:

If we attach an **initial condition** to the differential equation, we get what is called an **initial value problem**.

$$\begin{cases} y' = \cos t \\ y(0) = 6 \end{cases}$$

A differential equation alone has a family of solutions, a well-posed initial value problem has only one solution.

$$\begin{cases} y' + y = e^{-t} \\ y(0) = 2 \end{cases}$$

EXAMPLES OF DIFFERENTIAL EQUATIONS AS MODELS:

1. Let $P(t)$ represent the size of a growing population. Determine an equation for P if it is assumed that rate at which the population grows is proportional to its size.

2. An object falls from a given height with only the force of gravity acting upon it. Find a differential equation for $v(t)$, its velocity as a function of time.

3. An object falls from a given height with only the force of gravity acting upon it. Find a differential equation for $h(t)$, its *height* as a function of time.

4. An object falls from a given height, and in addition to the force of gravity, it experiences a drag force that is proportional to its velocity. Find a differential equation for the velocity $v(t)$.

5. A mass is attached to one end of a spring, while the other end is attached to a wall. The force needed to stretch the spring is proportional to the length it is stretched from equilibrium. Let $x(t)$ be the position of the mass and find a differential equation for $x(t)$.

Direction Fields

If a differential equation can be written in the form

$$\frac{dy}{dx} = f(x, y)$$

we say that f is the rate function. We can use f to draw a **direction field**.

EXAMPLES:

$$\frac{dy}{dx} = y + x$$

$$\frac{dy}{dx} = 4 - 2y$$

$$\frac{dy}{dx} = y \cos x$$