Math 112
Chapter 7.4: Partial Fractions

The goal of this section is to integrate rational functions (ratios of polynomials).

.

EXAMPLE:

$$\frac{1}{x+2} + \frac{3}{x-1}$$

.

Partial fractions is an algebraic method that reverses the addition of rational functions.

EXAMPLE:

$$\int \frac{4x-3}{x^2+3x}\,dx$$

Partial fractions works in general by proposing a possible decomposition with unknown constants, and then applying algebra to solve for the constants. We can break down the study of partial fractions into cases that depend on how the denominator factors.

CASE I: Denominator factors into a number of distinct linear factors.

EXAMPLES:

$$\int \frac{2x+6}{x^2-5x+6}\,dx$$

$$\int \frac{x^2+1}{x^3-2x^2-3x}\,dx$$

CASE II: Denominator factors into a number of linear factors that are not all distinct.

EXAMPLES:

$$\int \frac{x-1}{x^2(x+2)}\,dx$$

$$\int \frac{1}{(x^2-1)^2}\,dx$$

CASE III: Denominator factors into linear factors and distinct irreducible quadratic factors.

EXAMPLES:

$$\int \frac{1}{(x-1)(x^2+5)}\,dx$$

$$\int \frac{x-1}{(x+2)(3x^2+1)}\,dx$$

$$\int \frac{4x^2 - x + 1}{(x^2 + 2x + 2)(x - 3)}\, dx$$

In general the simplest integrals we will get in these three cases have the following forms:

$$\int \frac{A}{x - a}\, dx \qquad \int \frac{A}{(x - a)^n}\, dx \qquad \int \frac{Ax + B}{ax^2 + bx + c}\, dx$$

EXAMPLES: IMPROPER RATIONAL FUNCTIONS

$$\int \frac{2x^3 + x}{x^2 - x}\,dx$$

$$\int \frac{x^3 + 4x^2 + x - 1}{x^3 + x^2}\,dx$$