Math 112
Chapter 11.6: Root and Ratio Tests

Ratio Test:

If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then $\displaystyle\sum_{1}^{\infty} a_n$ converges absolutely.

If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$, then $\displaystyle\sum_{1}^{\infty} a_n$ diverges.

If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

EXAMPLES:

$$\sum_{n=1}^{\infty} \frac{n^2}{5^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^n}{n^n}$$

$$\sum_{n=1}^{\infty} \frac{n^{10}8^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{\sqrt{n^3+2}}$$

$$\frac{1}{4} + \frac{1}{4} \cdot \frac{5}{7} + \frac{1}{4} \cdot \frac{5}{7} \cdot \frac{9}{10} +$$

$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n \ln n}$$

Root Test:

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum_{1}^{\infty} a_n$ converges absolutely.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then $\sum_{1}^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the Root Test is inconclusive.

EXAMPLES:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^n}{n^n}$$

$$\sum_{n=1}^{\infty} \left(\frac{3n}{2n+1}\right)^{4n}$$

$$\sum_{n=1}^{\infty} \left(\frac{n}{\ln n}\right)^{2n}$$

$$\sum_{n=1}^{\infty} \frac{(3n+1)^n}{n^{2n}}$$

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$$