Math 112
Practice Problems

1. Evaluate the integrals:

$$\int \frac{x+2}{x^2+3x-4}\,dx$$

$$\int \ln\left(1+x^2\right) dx$$

$$\int_0^{\pi} t\cos^2 t\,dt$$

$$\int_0^4 \frac{z}{z-3}\,dz$$ Be careful.

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$\int_0^2 x^3\sqrt{4x^2-x^4}\,dx$$ Use the formula $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\frac{u}{a}+C$

2. The table shows values of a force function $f(x)$. Use Trapezoid, Midpoint, and Simpson's Rules to estimate the work done in moving an object from $x=0$ to $x=18$.

t	0	3	6	9	12	15	18
$r(t)$	9.2	8.9	8.6	8.0	7.7	7.6	7.0

3. Consider the following definite integral.

$$\int_1^4 \frac{1}{\sqrt{x}}\,dx$$

(a) Calculate the T_6, M_6, and S_6 approximations using Trapezoid, Midpoint, and Simpsons Rule.

(b) How large should N be so that the error in the Midpoint Rule approximation is less than 0.0005?

4. If $f(t)$ is continuous for $t \geq 0$, the *Laplace transform* of f is defined as

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

   (a) Find a formula and domain for $F(s)$ if $f(t) = t$.
   (b) Find a formula and domain for $F(s)$ if $f(t) = e^{2t}$.

5. Calculate the second Taylor polynomial, $T_2(x)$, for the function $f(x) = \sec x$ at $a = 0$.

6. Find the sum of each series, or explain why the series diverges.

$$1 - \frac{2}{7} + \frac{4}{49} - \frac{8}{243} + \ldots$$

$$\frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \frac{1}{17} + \ldots$$

$$1 + 2 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!}\ldots$$

$$3 + \frac{5}{3} + \frac{7}{5} + \frac{9}{7}\ldots$$

$$\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right)$$

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

7. Explain why the radius of convergence for $\sum c_n(x-a)^n$ is $\lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$, if this limit exists.

8. For what values of x do the following series converge

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(x-1)^n$$

$$\sum_{n=1}^{\infty} \frac{3^n}{x^n}$$

9. Suppose a and b are real numbers with $a < b$. Find a power series that converges on $(a, b]$.

10. Find all positive values of b for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges.