Math 112
Practice Problems

1. Write down the R_N approximation for the area below the graph of $f(x) = x^2 + x$ and above the x-axis for $1 \leq x \leq 4$. Use summation notation to write R_N.

2. Write R_N from the previous problem in terms of N.
(*Hint: You will need the handy formulas from the notes.*)

3. If $\displaystyle\int_2^5 y(x)\,dx = 10$, what is the value of $\displaystyle\int_2^5 3y(x) - 6\,dx$

4. If $g(x) = \displaystyle\int_x^{\sqrt{x}} 2 - e^{-t}\,dt$, find $g'(9)$.

5. Evaluate the integrals:

$$\int \frac{x}{e^x}\,dx$$

$$\int_0^3 |x^2 + 2x - 3|\,dx$$

$$\int_0^{1/2} \theta \tan(\pi\theta^2)\,d\theta$$

$$\int \frac{xe^{2x}}{(1+2x)^2}\,dx$$

$$\int \frac{\sec^2\theta}{\tan^2\theta}\,dx$$

$$\int_{\pi/2}^{\pi} \sin^5(3x)\cos(3x)\,dx$$

6. Find the average value of the function $h(x) = \ln\sqrt{x}$ on the interval $[1, 16]$.

7. Calculate $\displaystyle\lim_{N\to\infty} (1/N)\sum_{i=1}^{N} \sqrt{1-(i/N)^2}$

8. Let $A = \displaystyle\int_1^4 \ln x\,dx$. Determine how large N should be so that $R_N - A < 0.0001$ without using the value of A.

(*Hint:* $L_N < A < R_N$ *(why?) so* $R_N - A < R_N - L_N$.)

9. Determine the area of the region described by the inequalities $x - 2y^2 \geq 0$ and $1 - x - |y| \geq 0$. Determine the volume produced by rotating the area around the line $x = 1$.

10. Find the number b so that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two equal areas.

11. For what values of m do the line $y = mx$ and the curve $y = \dfrac{x}{x^2 + 1}$ enclose a region? Find the area of that region in terms of m.

12. Describe the volume represented by the integral $\displaystyle\int_1^3 2\pi y \ln y \, dy$.

13. Suppose g is a function that is increasing and concave up on $[a, b]$.
Which is greater, $\bar{g}$ or $g\left(\dfrac{a+b}{2}\right)$? Why? *Hint: Draw a picture.*