Math 112
Chapter 7.1: Integration by Parts

Integration by parts is the name of the formula we get if we undo the Product Rule.

The formula is used to exchange one integral for another. Ideally, we would like to get a more simple integral in the exchange.

EXAMPLES:

$\int x \cos x \, dx$

$\int x e^{3x} \, dx$

$$\int x \ln x \, dx$$

$$\int e^{2t} \cos t \, dt$$

REDUCTION FORMULAS:

$$\int t^n e^t \, dt = x^n e^x - n \int x^{n-1} e^x \, dx$$

$$\int \cos^n \theta \, d\theta = \frac{1}{n} \sin\theta \cos^{n-1} \theta + \frac{n-1}{n} \int \cos^{n-2} \theta \, d\theta$$

DEFINITE INTEGRALS:

$$\int_1^3 t^3 \ln t \, dt$$

$$\int_0^\pi x^2 \sin x \, dx$$

INTEGRALS OF BASIC FUNCTIONS:

$$\int \ln x \, dx$$

$$\int \arctan x \, dx$$

INTEGRATION BY PARTS + SUBSTITUTION RULE:

$$\int \sin(\ln x) \, dx$$

$$\int \sqrt{x} e^{\sqrt{x}} \, dx$$

SUMMARY:

Integration by parts is *typically* useful if the integrals involve any of the following:

- Products of x^n with trigonometric functions, logarithmic functions, or exponential functions
- Products of exponential and trigonometric functions
- Logarithmic functions
- Inverse trigonometric functions

EXERCISES:

$$\int t \sin 4t \, dt$$

$$\int x^2 \ln 2x \, dx$$

$$\int x \ln x^3 \, dx$$

$$\int (\ln x)^2 \, dx$$

$$\int e^x \sin 5x \, dx$$