Math 112
Chapter 9.2: Euler's Method

The goal of **Euler's Method** is to approximate the solution of a differential equation at at a discrete set of points.

IDEA: We can follow the direction field for a short time in order to appoximate the solution.

Euler's Method:

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}) \quad \text{where} \quad x_n = x_0 + nh$$

EXAMPLES:

Use Euler's Method with $h = 0.1$ to approximate $y(0.3)$ where $y(x)$ is a solution to the initial value problem

$$\begin{cases} y' = y + x \\ y(0) = 1 \end{cases}$$

Use Euler's Method with $h = 0.02$ to approximate $y(1.1)$ where $y(x)$ is a solution to the initial value problem

$$\begin{cases} y' = y\cos(x) \\ y(1) = 2 \end{cases}$$

Chapter 9.3: Separable Differential Equations

A differential equation is called **separable** if can be written in the form

$$\frac{dy}{dx} = g(x)f(y)$$

Solutions can be found by “separating” the variables and then integrating.

EXAMPLES:

$$\frac{dy}{dx} = ky$$

$$\frac{dy}{dx} = 3x^2y$$

Solve the initial value problems:

$$\begin{cases} \dfrac{dy}{dt} = y^2 \sin t \\ y(0) = 4 \end{cases}$$

$$\begin{cases} \dfrac{dy}{dt} = 3 - 4y \\ y(0) = 10 \end{cases}$$

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{-x}{y} \\ y(1) = -3 \end{cases}$$

NOTE: It may not always be possible to “solve” explicity for $y(x)$.

$$\frac{dy}{dx} = \frac{x^2}{1-y^2}$$

APPLICATIONS:

Solve the initial value problem for the velocity of a falling object that is subject to a drag force

$$\begin{cases} m\dfrac{dv}{dt} = kv - mg \\ v(0) = v_0 \end{cases}$$

A tank contains 200 L of water with 0.8 kg of salt. At $t = 0$, a solution with concentration 3.5 g/L begins flowing into the tank at a rate of 2 L/min. If the tank is kept well-mixed and is drained at a rate of 2 L/min, find the amount of salt in the tank as a function of time.

Find the family of curves that is orthogonal to every member of the family $y = \dfrac{k}{x}$.