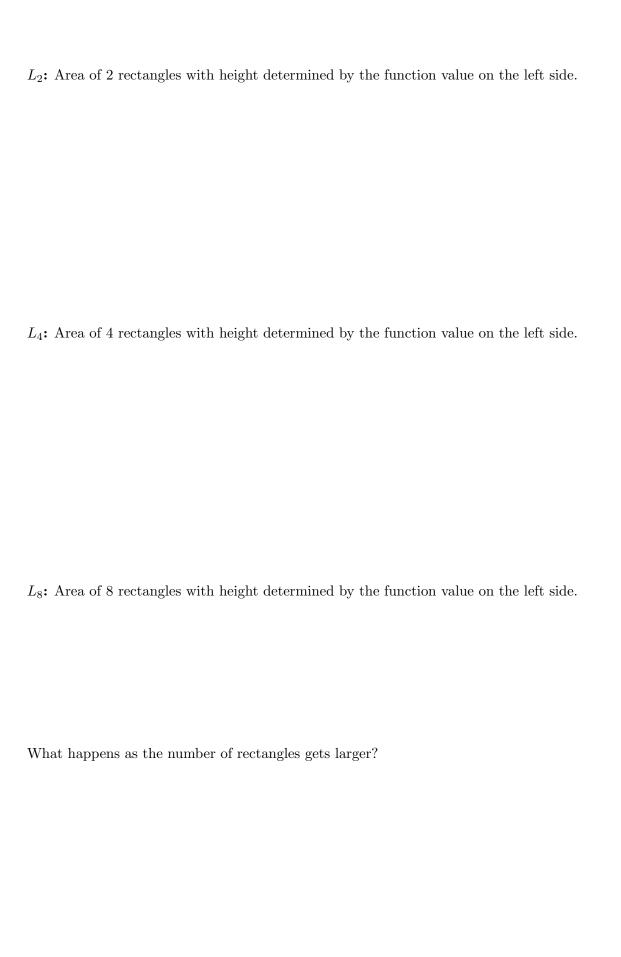
Math 112 Chapter 5.1: The Area Problem

Our goal is to determine a way to calculate areas that are enclosed by curved boundaries. Our starting point is an area defined by $a \le x \le b$, and $0 \le y \le f(x)$ where f is a positive function.
How should we define the area? What do we really $know$ about area?
(EXAMPLE)
Determine the area defined by $0 \le x \le 1$ and $0 \le y \le 1 - x^2$

Idea: Divide the area into pieces and estimate the area of each piece with a simple shape.



Another possibility:
R_2 : Area of 2 rectangles with height determined by the function value on the left side.
R_4 : Area of 4 rectangles with height determined by the function value on the left side.
What happens as the number of rectangles gets larger?
Idea: It may be possible to define the area as a limit on the area as the number of rectangles grows!
Formula for L_N :

Handy formula: $1^2 + 2^2 + 3^2 + ... + M^2 = \frac{M(M+1)(2M+1)}{6}$

$$A = \lim_{N \to \infty} L_N$$

(EXERCISES)

- 1. Find a formula for R_N and calculate $\lim_{N\to\infty} R_N$. You will get the same number
- 2. Repeat the general procedure to find the area bounded defined by $0 \le x \le 2$ and $0 \le y \le x^3$ You will need a handy formula $1^3 + 2^3 + 3^3 + ... + M^3 = \frac{M^2(M+1)^2}{4}$.

Sigma (Summation) Notation

Sigma notation is a compact way to write sums of numbers when there is a pattern in the numbers.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \dots + a_{n-1} + a_n$$

$$(\text{EXAMPLES})$$

$$\sum_{i=1}^{4} i^2$$

$$\sum_{i=3}^{7} i$$

$$\sum_{i=0}^{5} 2^{\frac{1}{2}}$$

$$\sum_{i=2}^{6} 3$$

(NOTES)

Writing L_N and R_N from the previous example.