Math 112
Chapter 11.1: Sequences

A **sequence** as a list of numbers in a definite order. Our goal is to understand infinite sequences and their limits.

NOTATION:

EXAMPLES:

$$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\right\}$$

$$\left\{\frac{(-1)^{n+1}}{n}\right\}_{n=1}^{\infty}$$

$$\left\{\cos\left(\frac{n\pi}{4}\right)\right\}_{n=0}^{\infty}$$

EXERCISES:
Find a formula for a_n in terms of n

$$\left\{3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \dots\right\}$$

$$\left\{\frac{-1}{6}, \frac{3}{10}, \frac{-5}{14}, \frac{7}{18}, \dots\right\}$$

If we can write a formula for a_n in terms of n, we can think of the sequence as a function with domain $\{1, 2, 3, 4, ...\}$.

Some sequences may be easier to describe with recurrence relations (later terms are related to earlier terms)

EXAMPLES:

$a_1 = 0, \quad a_2 = 1, \quad a_n = \dfrac{-a_{n-2}}{n^2}$

$a_1 = 1, \quad a_2 = 1, \quad a_n = a_{n-1} + a_{n-2}$ (Fibonacci sequence)

Let C_n be the amount of caffeine in a person's blood stream in units of bce (Ben's coffee equivalent). It is known that after one hour the body will have absorbed 13% of the caffiene. Suppose that Ben starts his morning with one coffee ($C_0 = 1$) and at the end of each hour quickly drinks another coffee.

We define the **limit** of a sequence similar to the way we define the limit of a function at infinity.

$$\lim_{n\to\infty} a_n = L$$

means that all a_n for which $n > N$ are arbitrarily close to L when N is sufficiently large. We say that L is the **limit** of the sequence.

If the limit exists we say that the sequence **converges**.
If the limit does not exist we say that the sequence **diverges**.

(EXAMPLES)

$$a_n = \frac{3}{2^{n-1}}$$

$$b_n = \frac{2n}{3n^2+1}$$

$$c_n = \cos\left(\frac{n\pi}{4}\right)$$

$$d_n = n^2$$

$$a_n = \frac{\ln n}{n}$$

THEOREM: If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$

EXAMPLE:

$$a_n = \frac{(-1)^{n+1}n^2}{2n^4 + 5}$$

$$b_n = \frac{(-1)^{n+1}n^2}{2n^2 + 5}$$

For what values of r does r^n converge?

OTHER SEQUENCES:

$$a_n = \frac{n!}{2^n}$$

$$\{1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, ...\}$$

But what happens to Ben!? $C_n = 0.87C_{n-1} + 1$

MONOTONE CONVERGENCE THEOREM: Every bounded monotone sequence is convergent.

DYNAMICAL SYSTEMS:

Let P_t represent the population of species in a given habitat in year t. One simple way to model the population is to suppose that the population in the future year, P_{t+1} is a function of P_t, the population during the current year.

$$P_{t+1} = 1.07P_t$$

$$P_{t+1} = \frac{1000}{200 + P_t} P_t$$