$\begin{array}{c} {\rm Math~112} \\ {\rm Chapter~5.2:~The~Definite~Integral} \end{array}$

For a generic area defined by $a \le x \le b$, and $0 \le y \le f(x)$ where f is a positive function, we can make an estimate using a Riemann Sum .
The limit of the approximation as $N \to \infty$ is what we call the definite integral .
NOTES:

We adopt some *conventions* so that the integral definition makes sense in other situations.

EXAMPLES:

$$\int_0^3 x - 1 \, dx$$

$$\int_{1}^{0} \sqrt{1 - x^2} \, dx$$

$$\int_{-1}^{3} |x| \, dx$$

$$\int_0^{2\pi} \cos x \, dx$$

PROPERTIES:

EXAMPLES:

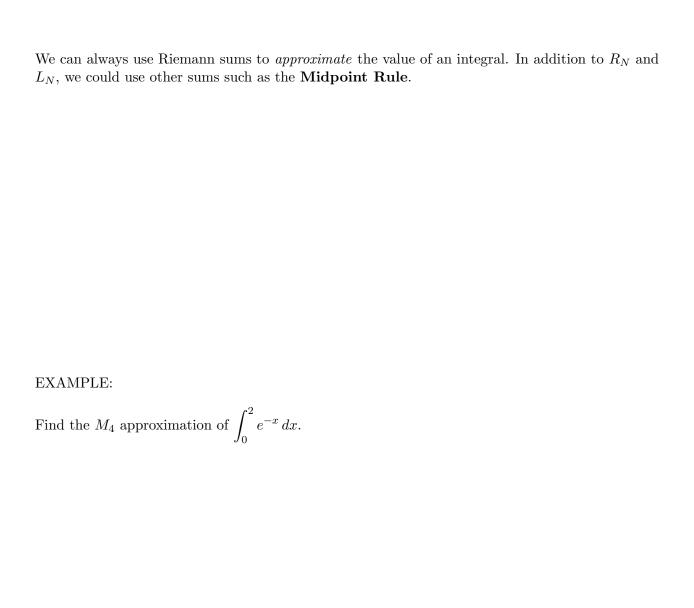
$$\int_0^1 2x + 3(1 - x^2) \, dx$$

If
$$\int_{2}^{6} f(x) dx = 7$$
 and $\int_{2}^{3} f(x) dx = 5$, find $\int_{3}^{6} f(x) dx$

COMPARISON PRINCIPLES:

EXAMPLE:

Give upper and lower bounds on the value of $\int_0^2 e^{-x} dx$.



Approximations do better with larger N.