CONTROLS NOTES

(Modified)

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Control Systems (15M)

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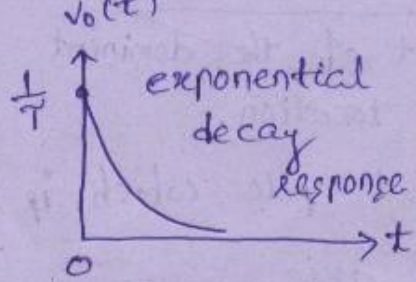
The LTI System is nothing but RLC n/ω. because the RLC components gives the linear transfer char.s and its values are not changes w.r.t time [Time invarient].

* $L[t^n] = \frac{n!}{s^{n+1}}$
* $L[t^n . e^{\pm at}] = \frac{n!}{(s \mp a)^{n+1}}$
* $L[e^{\pm at}] = \frac{1}{s \mp a}$
* $L[\sin bt] = \frac{b}{s^2+b^2}$
* $L[\cos bt] = \frac{s}{s^2+b^2}$
* $L[e^{\pm at} \sin bt] = \frac{b}{(s \mp a)^2 + b^2}$
* $L[f(t-\tau)] = e^{-s\tau} F(s)$

→ pole may effect s.s. stability but not a zero.

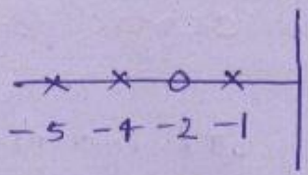
Ex:- $V_0(s) = \frac{1}{s\tau+1} \Rightarrow \frac{1}{\tau(s+\frac{1}{\tau})}$

$\Rightarrow \frac{1}{\tau} e^{-t/\tau} = V_0(t)$.

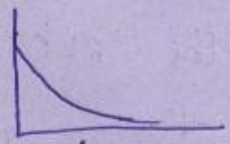


⟹ one pole on the -ve real axis giving an exponential decay response.

$$\frac{(s+2)}{(s+1)(s+4)(s+5)}$$



$$\Rightarrow \frac{k_1}{s+1} + \frac{k_2}{s+4} + \frac{k_3}{s+5}$$



⇒ If many poles are located on the -ve real axis at different locations then the system response exponential decay irrespective of positions of zeros.

⇒ The movement of pole in s-plane is nothing but varying the system component value in RLC.

↳ **Conditional stable System:-**

A System is stable for certain range of system components

Eg:- R value from 10k to 100kΩ.

↳ **Absolute stable System:-**

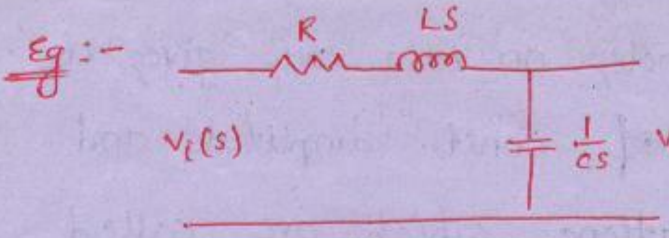
The System is stable for all the values in the system components.

* Time constant $= \dfrac{-1}{\text{Real part of the dominent pole location.}}$

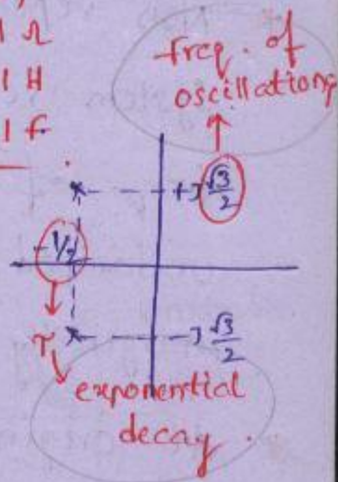
* Dominent pole is nothing but pole which is nearer to the imaginary axis.

* Insignificant pole, the pole which is located in the left most.

Eg:-



Let,
$R = 1\,\Omega$
$L = 1\,H$
$C = 1\,f$



$$\frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R + LS + \frac{1}{cs}} = \frac{1}{s^2 + s + 1}$$

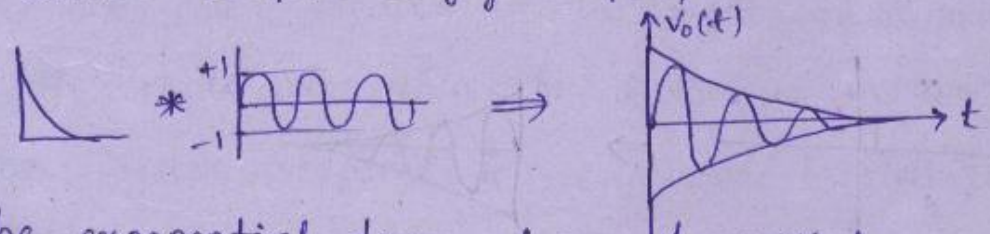
System is stable.

$$\Longrightarrow V_o(t) = k \cdot e^{-\frac{1}{2}t} \cdot \sin\left(\frac{\sqrt{3}}{2}\right)t$$

* In a complex conjugate poles, the real part gives the exponential decay where as ima. part gives the freq. of oscillations. The total system response is called exponential decay freq. of oscillations.

$$\Longrightarrow V_o = k \cdot e^{(\text{real part})t} \cdot \sin/\cos\,(\text{ima. part}) \cdot t$$

whenever complex conjugate poles.

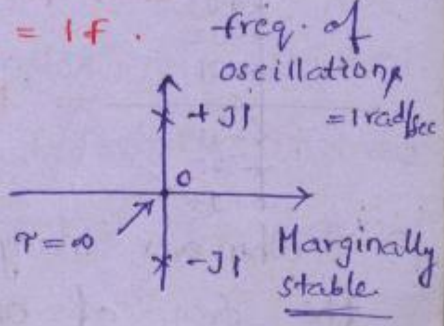


The exponential decay freq. of oscillations are called damped oscillations.

* A System which produces damped oscillations is called under damped system.

Case 2): $R = 0\,\Omega$, $L = 1H$, $C = 1f$.

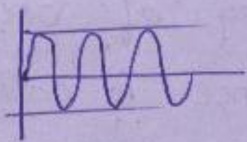
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 LC + sCR + 1} = \frac{1}{s^2 + 1}$$

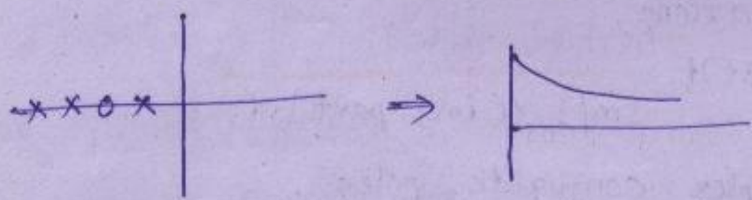


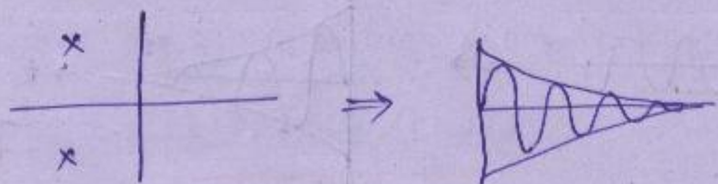
* Non repeated poles on ima. axis gives the system response of const. amplitude and freq. of oscillations, which are called undamped oscillations and the system is marginally stable.

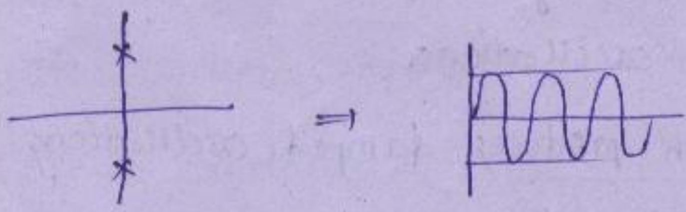
* At origin $\tau = \infty$. , As poles moves towards left hand side the system τ decreases, stability improves and system gives very quick response.

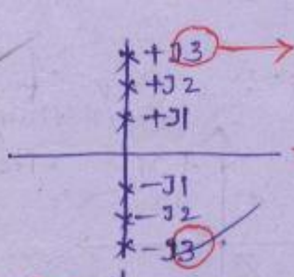
→ $v_o(t) =$ sin t









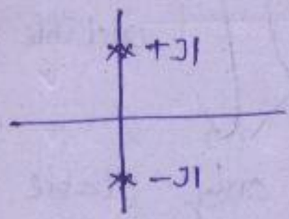


→ freq. of oscillations

⇒ Marginally stable.

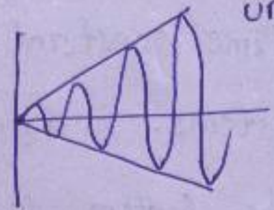
→ when ever many poles lies on ima. axis then the freq. of oscillations are nothing but largest intersection point ~~with~~ on the ima. axis.

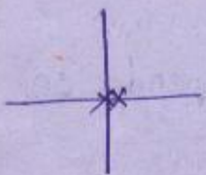
Repeated poles on ima axis



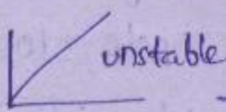
$\Rightarrow \dfrac{1}{(s^2+1)^2} \Rightarrow t \cdot \sin t$

unstable

$\Rightarrow$ 



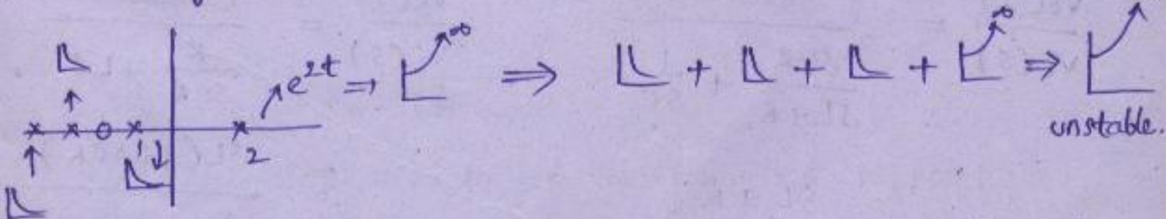
$\Rightarrow \dfrac{1}{s^2} \Rightarrow t$

$\Rightarrow$ 

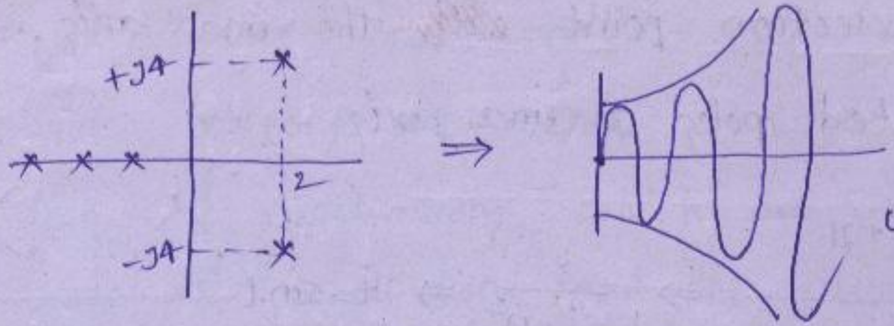
$\rightarrow$ ramp.

* Repeated poles on ima. axis $\Rightarrow$ System is unstable b'coz system response is increasing amplitude oscillations. (If finite ima term).

$\longrightarrow$ Many poles are lies in the left of s-plane but one pole located in the right of s-plane on the real axis then the system is unstable, b'coz system response is exponential rise to infinity.



$\longrightarrow$ Many poles are located in the left of s-plane but one pair of complex poles located in the right of s-plane $\Rightarrow$ System is unstable b'coz system response is exponential rise freq. of oscillations.

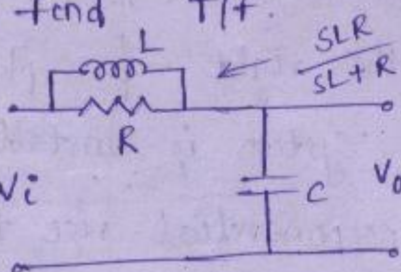


$e^{2t}\sin(4)t$

unstable.

* Time constant is defined for only stable systems.
* The system stability / response is depends on the right most side pole location.
* If right most side pole location in the left hand side ⇒ stable.
* If it is on ima. axis ⇒ Marginally stable.
* If it is on right hand side ⇒ unstable.

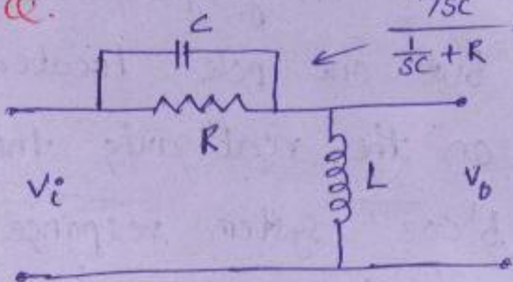
Q. find T/f.



$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{\frac{SLR}{SL+R} + \frac{1}{SC}}$$

$$= \frac{SL+R}{S^2LCR + SL + R}$$

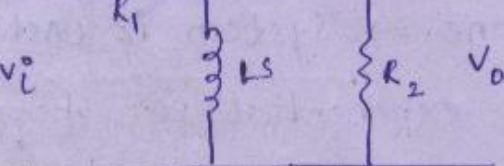
Q.



$$\frac{V_o(s)}{V_i(s)} = \frac{SL}{\frac{R}{SCR} + LS}$$

$$= \frac{SL(1+SCR)}{S^2LCR + SL + R}$$

Q.



$$\frac{V_o(s)}{V_i(s)} =$$

$$\frac{V_o(s)}{V_i(s)} = \frac{SL \cdot R_2}{R_1\left[SL+\frac{1}{SC}+R_2\right]+SL\left[\frac{1}{SC}+R_2\right]}$$

find T/f for, $\frac{d^3y}{dt^3} + 6\cdot\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 10\frac{dx}{dt}$

$\Rightarrow$ x : i/p $\quad s^n = \frac{d^n}{dt^n}$

y : o/p

$$\Rightarrow y(s)\left[s^3+6s^2+3s+2\right] = 10\,x(s)\cdot s$$

$$\Rightarrow \frac{y(s)}{x(s)} = \frac{10\cdot s}{s^3+6s^2+3s+2}$$

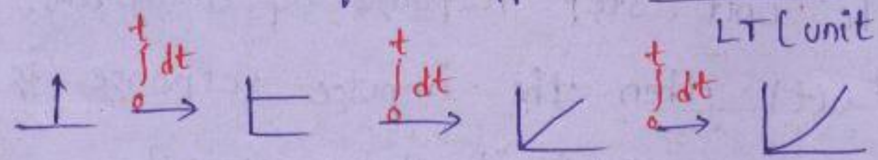
Given $\qquad$ find

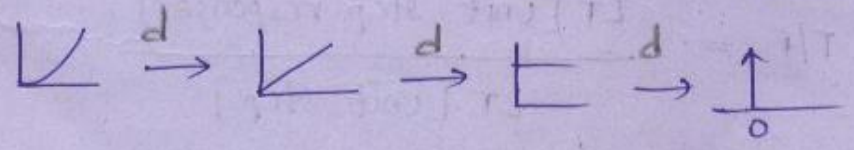
Q1. Signal response $\longrightarrow$ T/f.

$$T/f = \frac{LT[\text{given signal response}]}{LT[\text{given i/p}]}.$$

Eg:- Given: unit ramp response

then to find $T/f = \frac{LT[\text{unit ramp res.}]}{LT[\text{unit ramp}]}$

Q2. 

Q3. 

Step1 : find T/f.

step2 : Sub. U(s) to get the required response.

step3 : find partial fractions and apply ILT.

Q. The unit step response of the system is

$y(t) = 5/2 - 5/2\, e^{-2t} + 5t$, find its T/f.

$$T/f = \frac{LT[\text{unit step response}]}{LT[\text{unit step}]}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{5}{2s} - \frac{5}{2}\cdot\frac{1}{s+2} + \frac{5}{s^2}}{1/s}$$

$$= \frac{\frac{2.5\,s(s+2) - 2.5\,s^2 + 5(s+2)}{s^2(s+2)}}{1/s}$$

$$= \frac{10(s+1)}{s(s+2)}.$$

Q. The unit impulse response of a system is $c(t) = -4e^{-t} + 6e^{-2t}$ $(t \geq 0)$. The step response is — ?

Ans: $\int_0^t (-4e^{-t} + 6e^{-2t})\,dt$

$$= \left[4e^{-t} + 6\cdot\frac{e^{-2t}}{-2}\right]_0^t$$

$$= 4e^{-t} - 3\cdot e^{-2t} - (4-3)$$

Q. The unit step response of a system is $e^{-5t}u(t)$. then the impulse response is — ?

Ans: $T/F = \frac{LT[\text{unit step response}]}{LT[\text{unit step}]}$

$$= \frac{\frac{1}{s+5}}{\frac{1}{s}} = \frac{s}{s+5}.$$

$U(s) = 1$

$$Y(s) = \frac{s+5-5}{s+5}$$

$$= 1 - \frac{5}{s+5}$$

$$\Rightarrow y(t) = \delta(t) - 5e^{-5t}.$$

* SAT. AUG. 16. 2008 *

* For the +ve f/b system, the ph. shift b/w input & f/b signal is 0° or ±360°, where as for -ve f/b, the ph. shift b/w i/p & f/b signal is ±180°.

* G(S). H(S) ⟶ O/L gain. (Actual system gain)
↳ Loop gain (open).

⇒ Comparisions b/w O/L & C/L CS's:-

* The stability of C/L system depends on loop gain. If loop gain (GH) = -1. Then the C/L system stability effected. If loop gain > 0, then the C/L system is more stable than O/L system.

* The C/L system is more accurate than O/L system when the H(S) gives the stable value. ie The accuracy of C/L system depends on the f/b n/w H(S). where as O/L system accuracy depends on i/p & process.

* The O/L system is more sensitive for noise b'coz whatever changes occurs in G(S) the same changes occurs in o/p. where as in C/L system % of change in o/p with disturbance & noise is ≤ 1%. that means even though disturbance & noise occurs in the system, the change in o/p is very less, which is called improving sensitivity with f/b.

* Reliability depends on no. of discrete comp. & the o/L control system has the less no. of components, hence o/L system is more reliable.

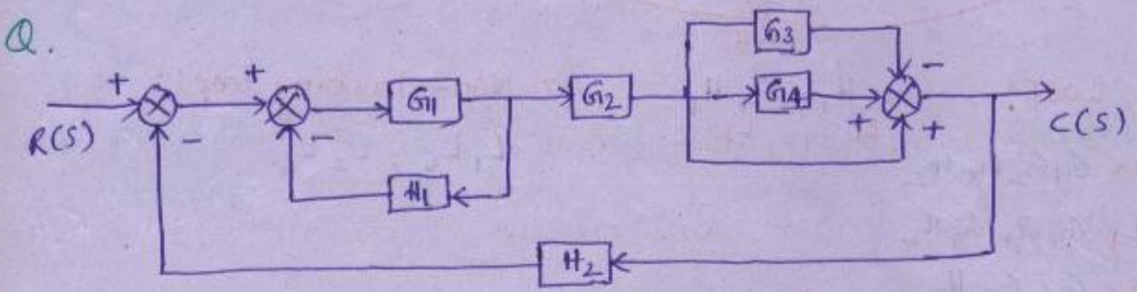
* for any perticular system the gain, bandwidth product is const. With f/b the band width is increased by the factor of $1 + G(s) \cdot H(s)$.

* Band width represents the speed of the response, as BW increases the system gives the quick response.

$\uparrow BW \propto \frac{1}{t_r}\downarrow \propto$ (response speed)

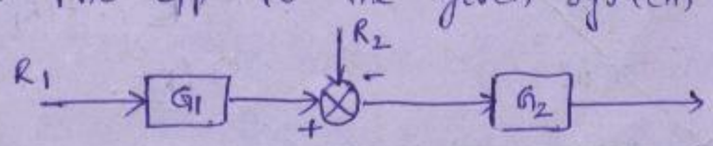
* In o/L System it is not necessary to measure the o/p., where as in c/L cs the o/p must be measured.

Q.



Sol. $$\frac{C(S)}{R(S)} = \frac{G_1 G_2 (1 + G_4 - G_3)}{1 + G_1 H_1 + [G_1 G_2 (1 + G_4 - G_3)] H_2}$$

Q. The o/p to the given System is −?



Sol. $G_1 G_2 R_1 - G_2 R_2$

$\Rightarrow$ To get the o/L T/f from c/L T/f, Subtract the numerator in the denominator, when $H(s) = 1$.

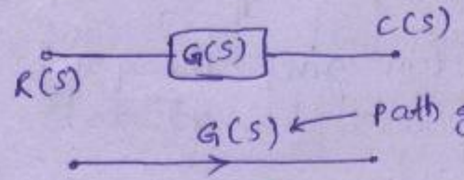
$= \frac{G}{1 + G - G}$

⟹ To get the C/L T/f from O/L, add the numerator term in the denominator when $H(s) = 1$.

$$= \frac{G}{1+G}.$$

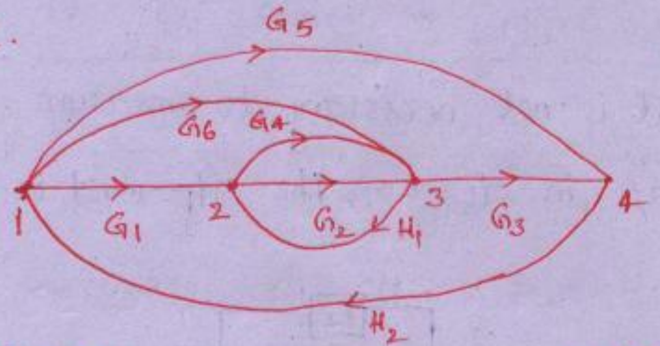
⟹ **Signal flow Graphs:**

⟹ Set of Linear Algebraic eq.s represents the system.



$G(s) \leftarrow$ path gain (or) Transmittance

Q.



forward paths:

$G_1 G_2 G_3$

$G_6 G_3$

$G_1 G_4 G_3$

G_5

Loops: $\underbrace{G_2 H_1}_{L_1}$, $\underbrace{G_4 H_1}_{L_2}$

L_3 $G_1 G_2 G_3 H_2$

L_4 $G_1 G_4 G_3 H_2$

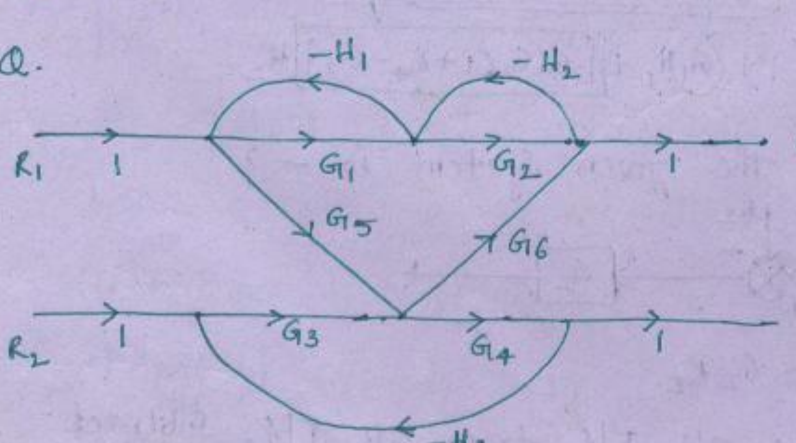
L_5 $G_6 G_3 H_2$

L_6 $G_5 H_2$

2 Non-touching loops:

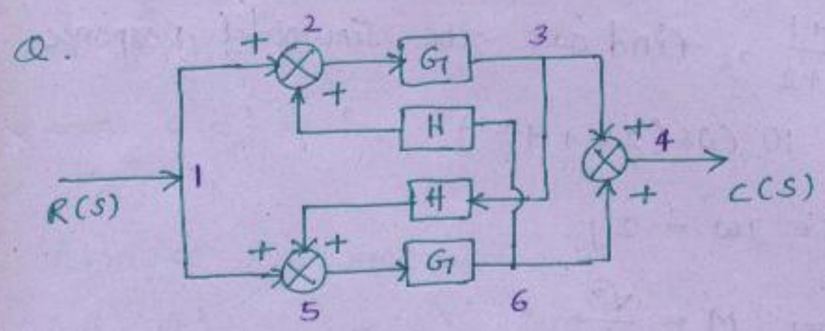
$L_1 L_6$, $L_2 L_6$

Q.



⟹ find C_1/R_1, C_1/R_2, C_2/R_1 and C_2/R_2.

Q.



forward paths:

$P_1 = 1234 \rightarrow G$

$P_2 = 123564 \rightarrow G^2H$

$P_3 = 1564 \rightarrow G$

$P_4 = 156234 \rightarrow G^2H$

Loops:

$L_1 = 23562$

$$T/f = \frac{G + G^2H + G + G^2H}{1 - G^2H^2}$$

$$= \frac{2G}{1 - GH}.$$

→ Procedure to draw signal-flow graph for Electrical Network :-

(1). Select the nodes as a series branch var-s in the same order.

(2). Each component in an electrical n/w gives one f. path & one -ve f/b path except the last element where we takes o/p.
Last element gives only f. path.

(3). Take the ratio of impedance for series branch elements as a path gain and take the same impedance for shunt branch elements.

Time Domain Analysis ⇒

Q. Identify $c_{ss}(t)$ & $c_{tr}(t)$ in the following response.

$$c(t) = \underbrace{5 + 2\sin 3t}_{c_{ss}(t)} + \underbrace{e^{-10t} + e^{-10t}\cdot t + e^{-5t}\sin 3t}_{c_{tr}(s)}$$

Q. $\dfrac{C(S)}{R(S)} = \dfrac{S+1}{S+2}$, find out the sinusoidal response for $r(t) = 10\cos(2t+45^\circ)$.

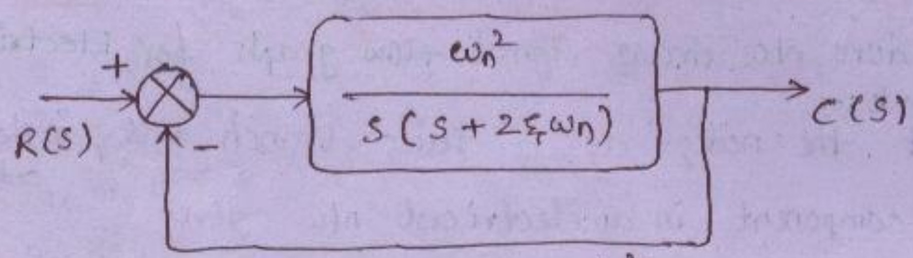
$\omega = 2, \quad S = j\omega = 2j.$

$$\Rightarrow \frac{2j+1}{2j+2} \Rightarrow M = \frac{\sqrt{5}}{\sqrt{8}}$$

$$\phi = \frac{\tan^{-1}(2/1)}{\tan^{-1}(2/2)} = 18.43^\circ.$$

$$\therefore\ c(t) = 10\times\sqrt{\frac{5}{8}}\cdot\cos(2t + 45^\circ + 18.43^\circ)$$

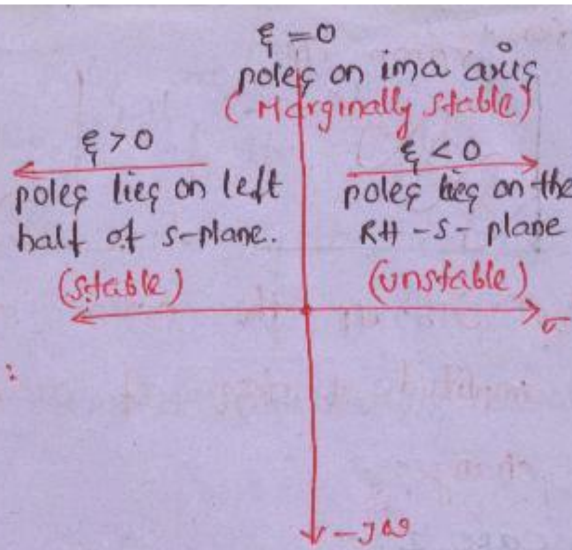
SECOND ORDER SYSTEMS :



$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

* The II order system response completely depends on ξ.
* II order system is stable for all +ve of ξ, $0 < \xi \le \infty$.

B'coz for +ve values of ξ, the poles lies on the left half of s-plane.



IMPULSE RESPONSE:

$r(t) = \delta(t)$

$\Rightarrow R(s) = 1.$

case. 1:

Impulse Response :-

$\xi = 0$:- $r(t) = \delta(t)$

$R(s) = 1.$

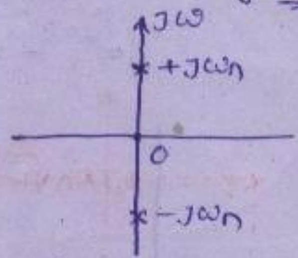
$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$\Rightarrow c(t) = \omega_n \cdot \sin \omega_n t.$

[Non-repeated poles on imaginary axis].

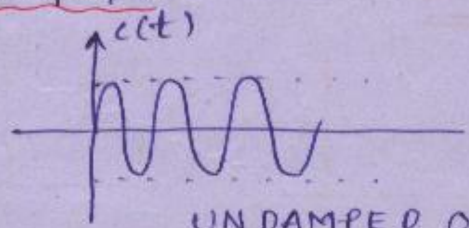
marginally stable



* freq of oscillations $= \omega_n$ rad/sec

* $\tau = -\frac{1}{0} = \infty.$

Response:

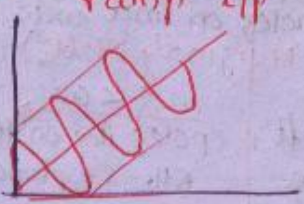


UNDAMPED OSCILLATIONS. $= \omega_n$ rad/sec.

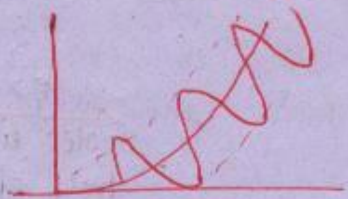
when $\xi = 0$, the poles on the ima. axis which are not repeated, the System is M.stable and the system response is const. amplitude and freq. of oscillations, which are called undamped oscillations. The system which gives undamped oscillations is called undamped system.

when $\xi = 0$, irrespective of all i/p's, Response is undamp.

for ramp i/p



for parabolic i/p



for $\xi = 0$, with i/p the nature of the system (const. amplitude & freq. of oscillations around i/p not changes.

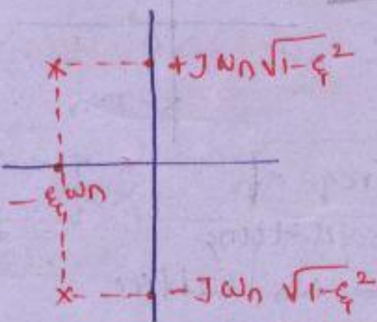
Case 2:-

$0 < \xi < 1$:-

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}, \quad \xi > 1$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}, \quad \xi < 1$$



$\tau = \frac{1}{\xi\omega_n}$

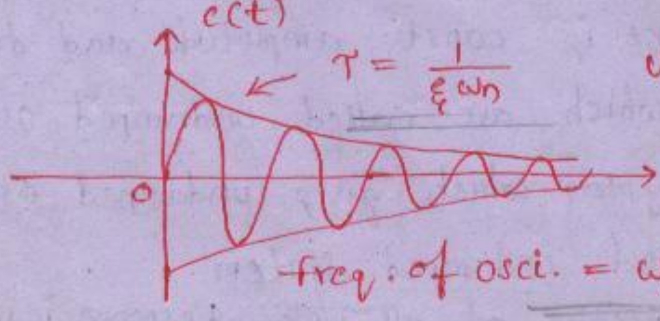
freq. of oscillation $= \omega_n\sqrt{1-\xi^2}$ (ω_d)

Stable.

Response:

$$c(s) = \frac{\omega_n^2}{(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})}$$

$$\Rightarrow c(t) = K.e^{-(\xi\omega_n)t} \cdot \sin(\omega_n\sqrt{1-\xi^2})t$$



under damped system

freq. of osci. $= \omega_d = \omega_n\sqrt{1-\xi^2}$

when $0<\xi<1$, the poles are complex conju. which are at the Left half of the s-plane. The system is stable. The system response is exponential dacay freq of oscillations which are called damped oscillations ie ω_d.

$\omega_d = \omega_n\sqrt{1-\xi^2}$.

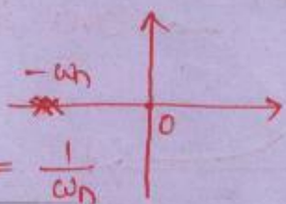
* A system which produce damped oscillations are called under damped system.

case 3:-

$\xi = 1$:-

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{s^2+2\omega_n s+\omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\omega_n)^2}$$



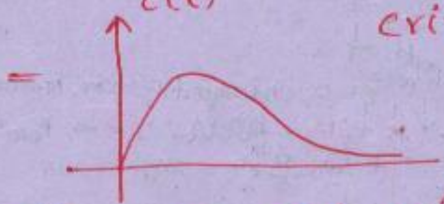
$\Rightarrow c(t) = \omega_n^2 \cdot t \cdot e^{-\omega_n t}$

$\tau = \frac{1}{\omega_n}$

Response:

freq. of osci. = 0

stable



critical damped system

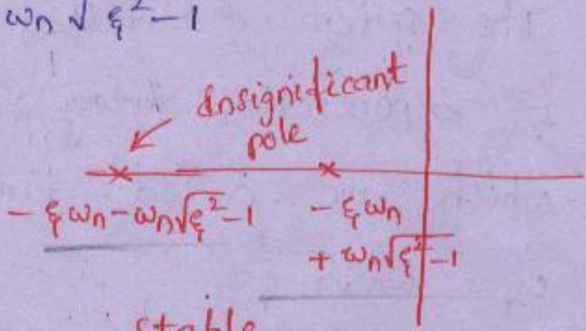
when $\xi = 1$, the pole on the $-ve$ real axis at the same location, the system is stable, the system response is critical damped. b'coz it generates critically or hardly one damped oscillation.

Case 4 :-

$\xi > 1$:-

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$



stable

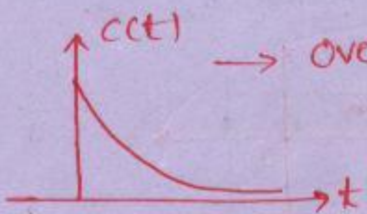
freq. of oscillations = 0. rad./sec.

Response:

$$C(s) = \frac{\omega_n^2}{(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})}$$

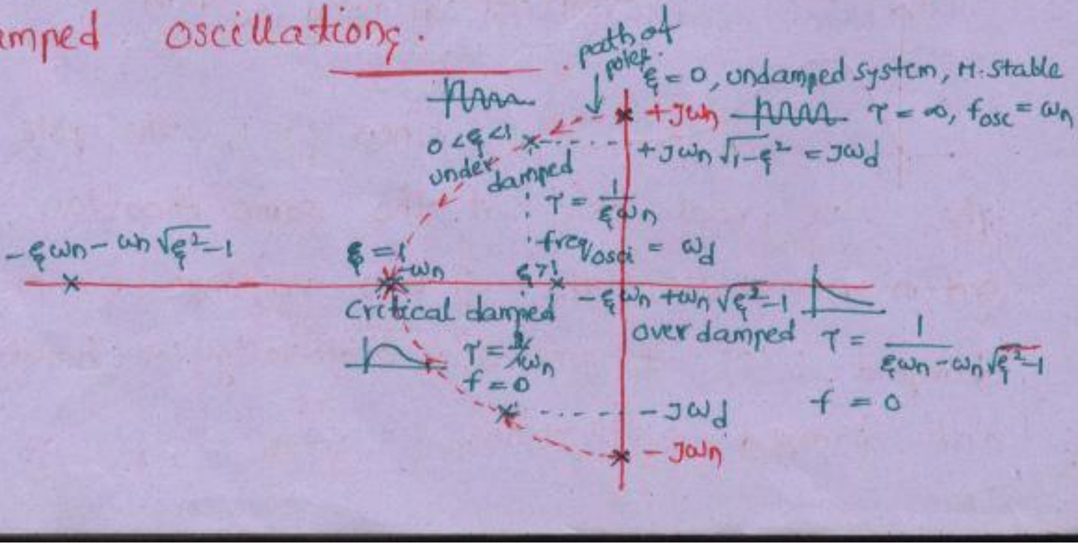
insignificant pole

$$\rightarrow c(t) = k \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}$$

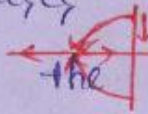


→ over damped.
↳ means over comes

When $\xi > 1$, the poles on the -ve real axis at different locations, the system is stable, the system response is over damped b'coz the system response elemenates or over comes the damped oscillations.



⇒ when ξ increases from 0 to 1, the poles moves towards L.H.S and near to the real axis. Hence the system time constant & freq. of oscillations are decreased. when $\xi \geq 1$ and increases ^one pole moves on –ve real axis towards imaginary axis the freq. of oscillations becomes zero b'coz the poles lies on the real axis only. and hence system τ increases, & FOO are reduced to zero.



when $\xi > 1$ and increases the system τ increases b'coz one poles moves towards the origin on the real axis.

order of Time constants:

$\tau_{undamped} > \tau_{overdamped} > \tau_{underdamped} > \tau_{critical\ damped}$

$\tau_{undamped}$	$\tau_{overdamped}$	$\tau_{underdamped}$	$\tau_{critical\ damped}$
(∞)	$\left(\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right)$	$\left(\frac{1}{\xi\omega_n}\right)$	$\left(\frac{1}{\omega_n}\right)$
(Marginal stable)	(stable)	(stable)	(stable)
	Largest τ	Medium τ	lowest τ

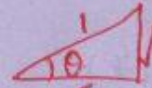
Large Time const. ⟶ slow response (or) sluggish

Unit Step Response :- $r(t) = 1 \cdot u(t)$

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s\cdot(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

when $\xi \geq 0$, & $\xi < 1$ $(0 \leq \xi < 1)$ the unit step response of the system



$\cos\theta = \xi$

$\sin\theta = \cos\omega$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left\{(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}$$

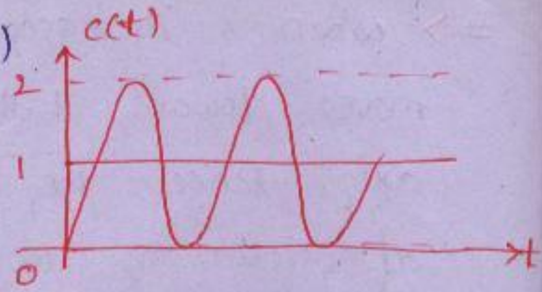
or $\cos^{-1}\xi$

Case. 1 :-

$\xi = 0$: $c(t) = 1 - \sin(\omega_n t + \pi/2)$

$\Rightarrow c(t) = 1 - \cos(\omega_n t)$

Undamped, Marginally stable.



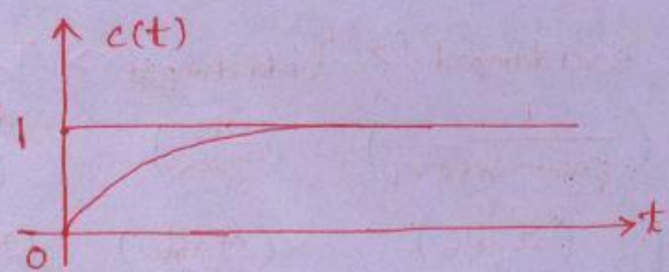
Case. 3

$\xi = 1$:-

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{(s+\omega_n)}$$

$$\Rightarrow c(t) = \left(1 - \omega_n \cdot t \cdot e^{-\omega_n t} - e^{-\omega_n t}\right) \cdot u(t)$$



Case. 4

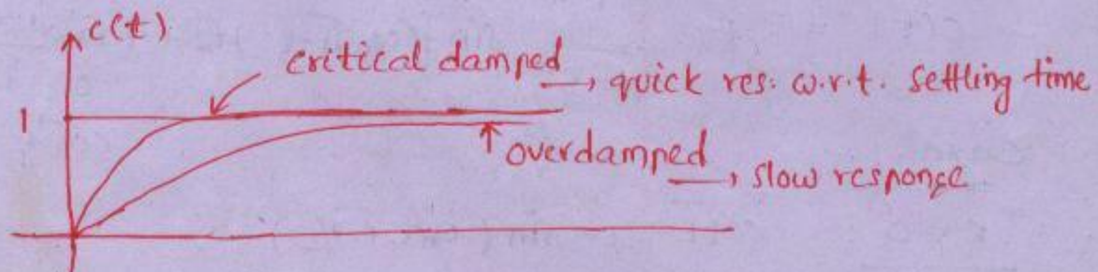
$\xi > 1$:-

$$C(s) = \frac{\omega_n^2}{s(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1})(s+\underline{\xi\omega_n + \omega_n\sqrt{\xi^2-1}})}$$

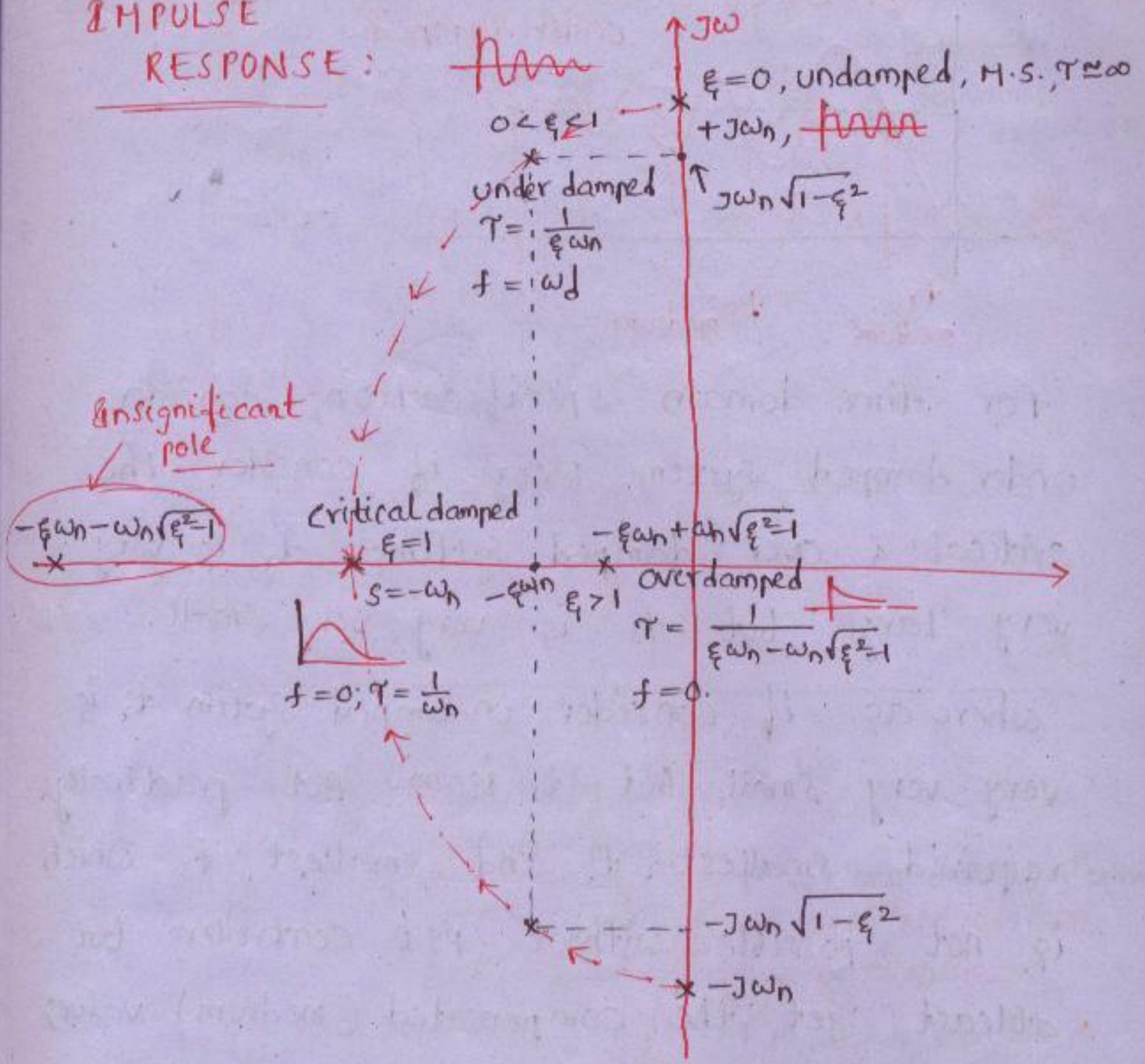
Insignificant pole

$$= \frac{1}{s} - \frac{K}{s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$

$$\Rightarrow c(t) = \left(1 - K \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}\right) u(t)$$

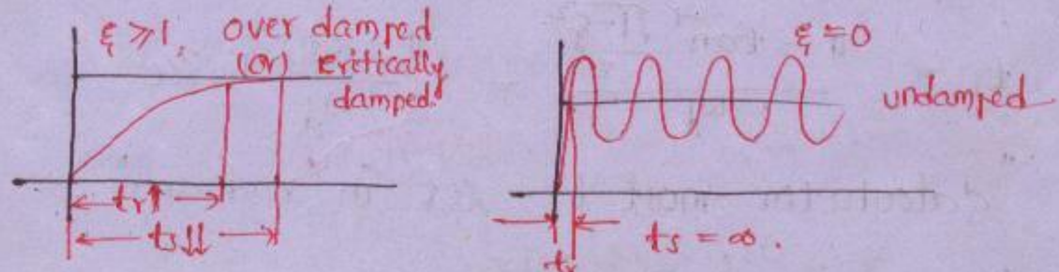


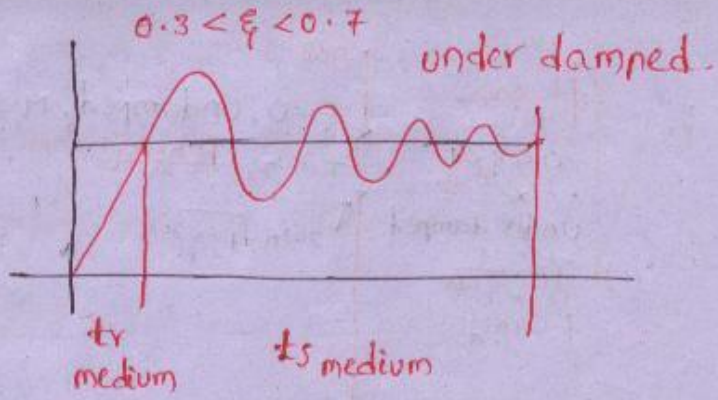
IMPULSE RESPONSE:



TIME DOMAIN SPECIFICATIONS:

Test signals	Tr.res.	SS.res.	Stability	
Impulse	✓	×	✓	→ practically not exist
Step	✓	✓	✓	→ BOUNDED widely used
Ramp	✓	✓	×	UNBOUNDED
Parabolic	✓	✓	×	UNBOUNDED





for time domain specifications consider under damped system. b'coz if consider the critical & over damped systems, t_r is very very large but t_s is very very small.

where as if consider undamped system t_r is very very small but t_s is ∞. But practically required smallest t_r and smallest t_s which is not possible without P&D controllers but atleast get the compensated (medium) values of t_r & t_s are possible when selected ξ 0.3 to 0.7.

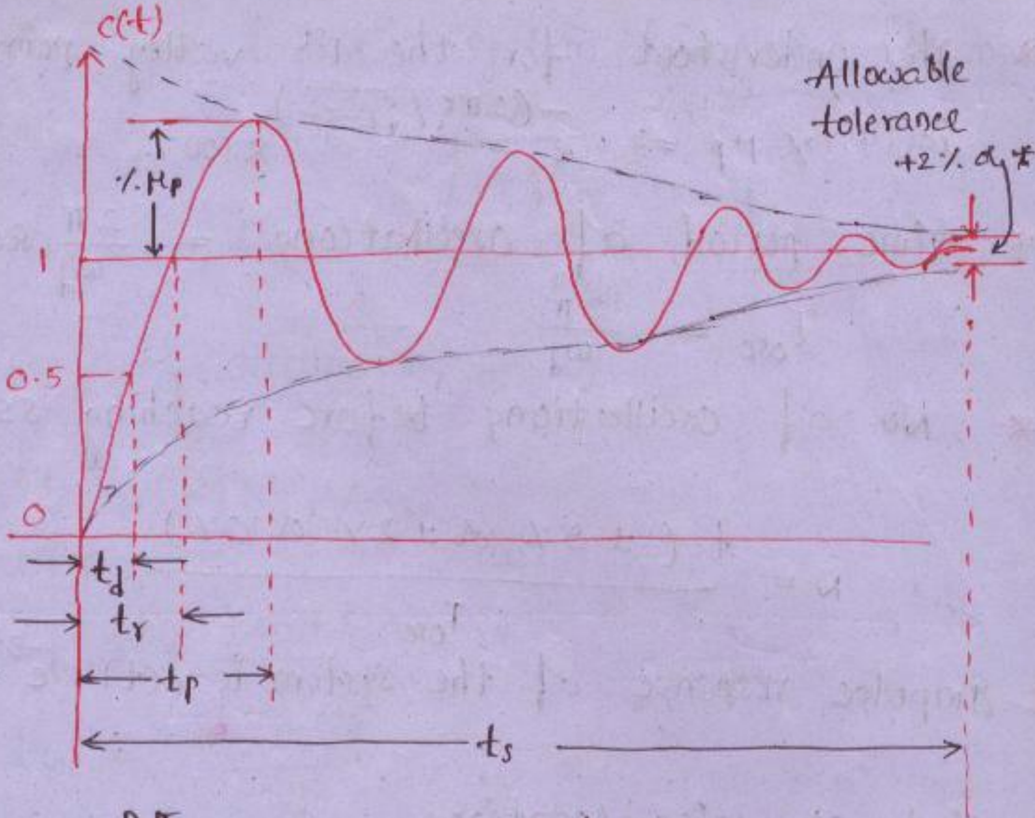
* when $\xi > 0$ & $\xi < 1$ the unit step res. of the system is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

* $t_d = \dfrac{1+0.7\xi}{\omega_n}$ sec

* $t_r = \dfrac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d} = \dfrac{\pi - \cos^{-1}\xi}{\omega_d}$ sec

"calculator must be set in radians".

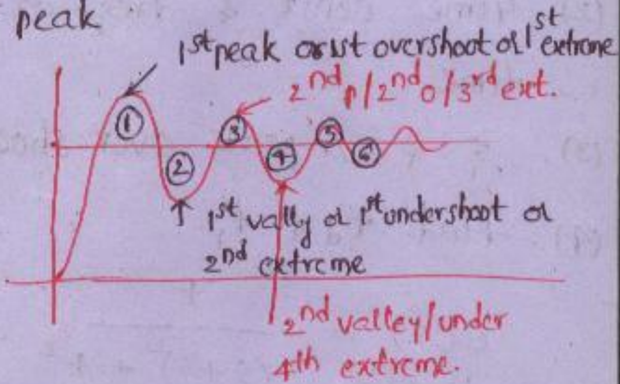


$$* \; t_p = \frac{n\pi}{\omega_d} \; ; \; n = 1 \text{ by default}$$

$$= \frac{\pi}{\omega_d} \; , \; 1^{st} \text{ peak}$$

$$= \frac{3\pi}{\omega_d} \; , \; 2^{nd} \text{ peak}$$

$$= \frac{2\pi}{\omega_d} \; , \; 1^{st} \text{ valley}$$



$$* \; t_s = 3\tau = \frac{3}{\xi\omega_n} \longrightarrow \pm 5\%$$

$$t_s = 4\tau = \frac{4}{\xi\omega_n} \longrightarrow \pm 2\%$$

$$t_s = 5\tau = \frac{5}{\xi\omega_n} \longrightarrow 0\%$$

$$* \; \% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

$$= [c(t_p) - 1] \times 100\%$$

$$= e^{-(n\pi\xi/\sqrt{1-\xi^2})} \times 100\%$$

$$= e^{-(\pi\xi/\sqrt{1-\xi^2})} \times 100\% \; , \; 1^{st} \text{ peak.}$$

* The undershoot for the 1st valley point is $\% M_p = e^{-(2\pi\xi/\sqrt{1-\xi^2})} \times 100\%$.

* Time period of oscillations $= \frac{2\pi}{\omega_d}$ sec.

$$T_{osc} = \frac{2\pi}{\omega_d}$$

* No. of oscillations before reaching SS is

$$N = \frac{t_s(\pm 5\% \text{ or } \pm 2\% \text{ or } 0\%)}{T_{osc}}$$

Q. Impulse response of the system is $c(t) = e^{-3t} \cdot \sin 4t$

(1). Find C/L poles locations

(2). Time const. & freq. of oscillations ie ω_d, peak time

(3). ξ & % peak over shoot.

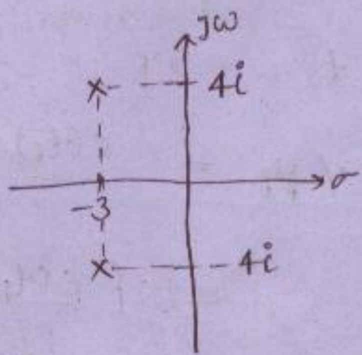
(4). find t_d & t_r.

$$C(s) = \frac{4}{(s+3)^2 + 4^2}$$

$$= \frac{4}{s^2 + 6s + 25}$$

$\Rightarrow \omega_n = 5; \quad 2\xi(5) = 6$

$\rightarrow \xi = 3/5$.

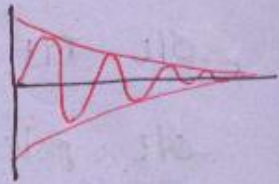


C/L poles:

$s^2 + 6s + 25 = 0$

$\Rightarrow s = -3 \pm 4i$

for underdamped system, impulse response will be $c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot e^{-\xi\omega_n t} \cdot \sin\omega_d t$



Time constant $\tau = \frac{1}{\xi\omega_n}$.

freq. of oscillations $= \omega_d$

$$t_p = \frac{\pi}{\omega_d}$$

$$\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$$

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

ROOT LOCUS:

Relationship b/w o/l T/F poles & zeros with c/l T/F poles:

o/l T/F $= G(s) \cdot H(s) = k \cdot \frac{N(s)}{D(s)}$.

o/l poles $D(s) = 0$ —①

o/l zeros $N(s) = 0$ —②

A c/l system stabilty is given by char. eq.

$1 + G(s) \cdot H(s) = 0.$

$1 + k \cdot \frac{N(s)}{D(s)} = 0$

→ char. eq. ⇒ c/l T/F poles:

$D(s) + k\,N(s) = 0.$

The c/l poles are nothing but sum of o/l poles and o/l zero's with the fun. of system gain k.

Case (1):

$k = 0;$

$k = \left| -\frac{D(s)}{N(s)} \right| = 0.$

c/l poles $D(s) = 0.$

when $k = 0$, o/l poles = c/l poles.

case (2):

$k = \infty;$ c/l pole $= N(s) = 0$

when $k = \infty$, c/l poles = o/l zero's.

The RL diagram start at o/l pole, where $k = 0$ and end at o/l zeros where $k = \infty$.

Q. find where RL diagram starts & ends.

for $G(s).H(s) = \dfrac{K(s+1)}{s(s+3)(s+5)}$

Starts at o/L pole $(k=0) \Rightarrow 0, -3, -5$.

Ends at o/L zeros $(k=\infty) \Rightarrow -1, \underbrace{\infty, \infty}_{\text{Along Asympto.}}$

Q. check whether the following points lies on RL or not ?

$$G(s).H(s) = \frac{k}{s(s+3)(s+5)}$$

(i). $s=-2$ (ii). $s=-4$.

$$\angle GH\Big|_{s=-2} = \frac{\angle k}{\angle s\ \angle(s+3)\ \angle(s+5)}$$

$$= \frac{\angle k}{\angle -2 . \angle +1 . \angle +3}$$

$$= \frac{0^\circ}{\pm 180^\circ . 0^\circ . 0^\circ} = \pm 180^\circ.$$

$\therefore$ Satisfies Angle condition, so $s=-2$ lies on RL.

(ii). $$\angle GH\Big|_{s=-4} = \frac{\angle k}{\angle -4 . \angle -1 . \angle 1}$$

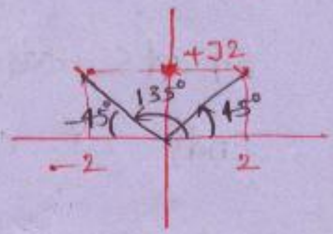
$$= \frac{0^\circ}{\pm 180 . \pm 180^\circ . 0^\circ}$$

$$= \mp 360^\circ \longrightarrow \text{Not Satisfy.}$$

So $s=-4$ is not lie on RL.

Q. $G(s).H(s) = \dfrac{k}{s(s+4)}$.

(i). $s = -2+j2$ find system gain at given point.



$$\angle GH = \frac{\angle k}{\angle(-2+j2).\angle(2+j2)}$$

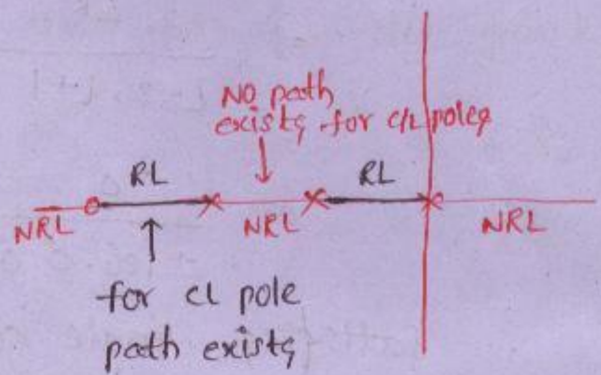
$$= \frac{0}{135^\circ . 45^\circ} = -180^\circ, \text{ satisfy. AC.}$$

$$MC := \left|\frac{k}{s(s+4)}\right| = 1$$

$$\Longrightarrow \left|\frac{k}{(-2+j2)(2+j2)}\right| = 1$$

$$\longrightarrow k = 8 \qquad \frac{k}{\sqrt{8}.\sqrt{8}} = 1.$$

③. Real axis loci :-



Q. Identify which are on RL.

$$GH = \frac{k(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)}$$

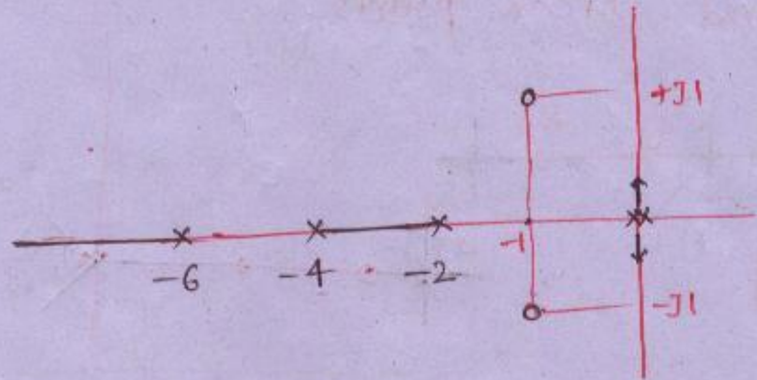
(1). $s = 0$ ✓ (2). $s = -1$ ✗, (3). $s = -2$ ✓

(4). $s = -3$ ✓ (5). $s = -4$ ✓, (6). $s = -6$ ✓

(7). $s = -\infty$ ✓ (8). $s = -1+j1$ ✓ (9). $s = -1-j1$ ✓

(10): $s = -5$ ✗

$$GH = \frac{K(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)} \quad s = -1 \pm j.$$



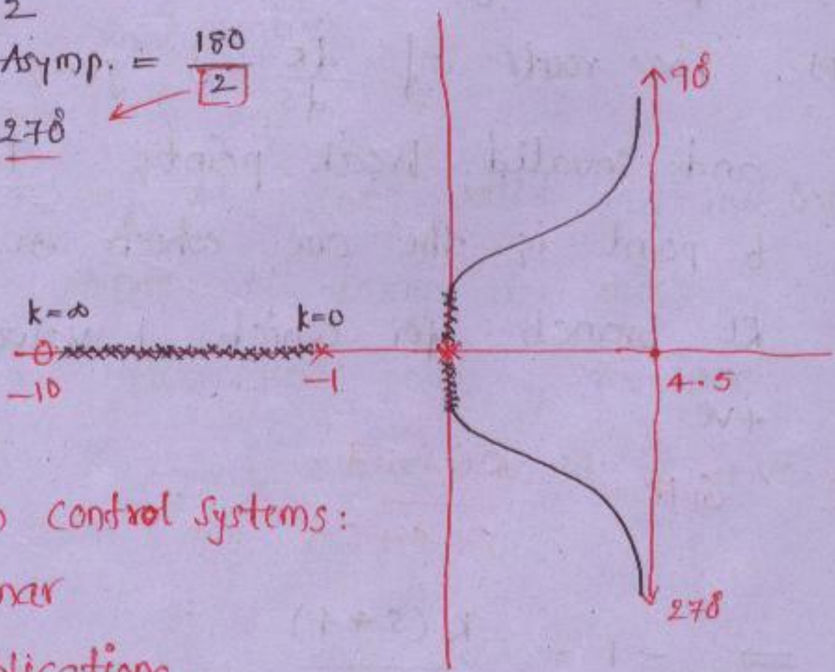
At the position of all poles & zeros, there must be a root locus branch b'coz RL branch start at o/L poles and ends at o/L zeros.

Q. Draw RL $GH = \frac{K(s+10)}{s^2(s+1)}$

$$\sigma = \frac{-1-(-10)}{2} = 4.5.$$

Angle of Asymp. $= \frac{180}{\boxed{2}}$

$= 90^\circ, 270^\circ$



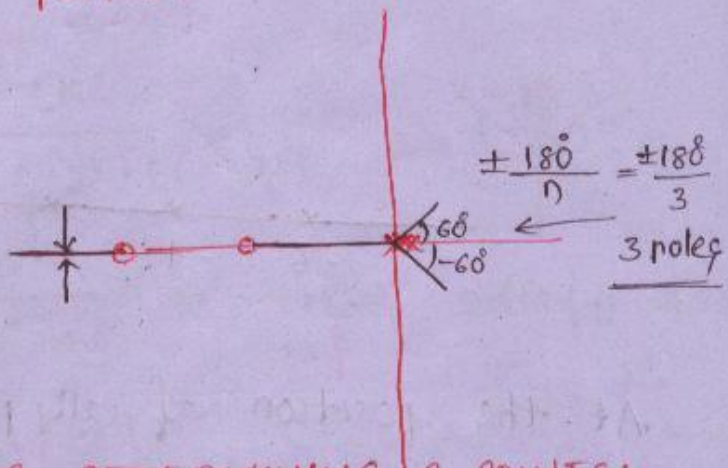
* Problems in Control Systems:

By, Ashok kumar

Sigma publications.

Q. $GH = \frac{K(s+2)(s+4)}{s^3}$.

find Break points.



PROCEDURE FOR DETERMINING B. POINTS:-

(1). $G(s)H(s)$ replace by -1

(2). Re arrange above eq. in the form of $K = f(s)$.

(3). defferentiate k w.r.t s and make it equal to zero.

(4). The roots of $\frac{dk}{ds} = 0$ gives the valid and invalid break points. The valid b. point is the one which must be on RL branch for which k value should be +ve.

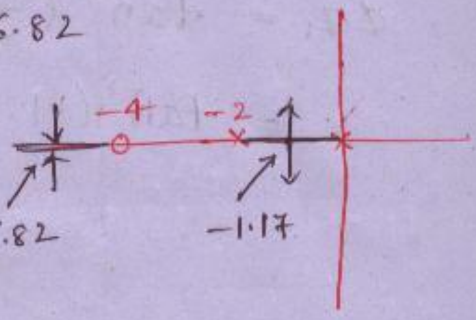
Q $GH = \frac{K(s+4)}{s(s+2)}$

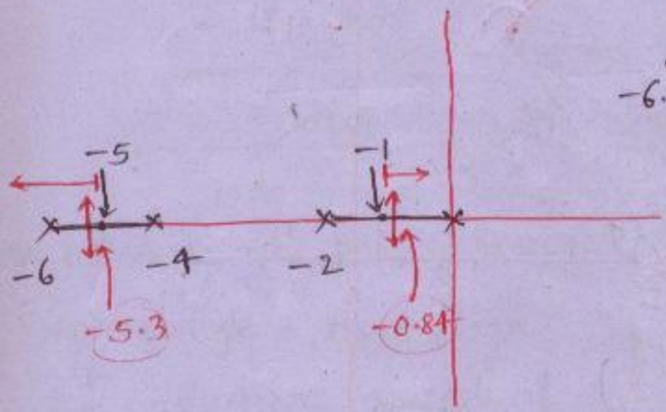
$$\Rightarrow -1 = \frac{K(s+4)}{s(s+2)}$$

$$\Rightarrow K = \frac{-s^2 - 2s}{s+4}$$

$$\Rightarrow \frac{dk}{ds} = \frac{(-2s-2)(s+4)+s^2+2s}{(s+4)^2} = 0$$

$$= -1.17,\ -6.82$$





PROCEDURE TO FINDOUT INTERSECTION POINT ON IMAGINARY AXIS:

(1). form Char. eq.

(2). write Routh tabular form

(3). find $k_{marginal}$ value.

(4). form Auxiliary eq.

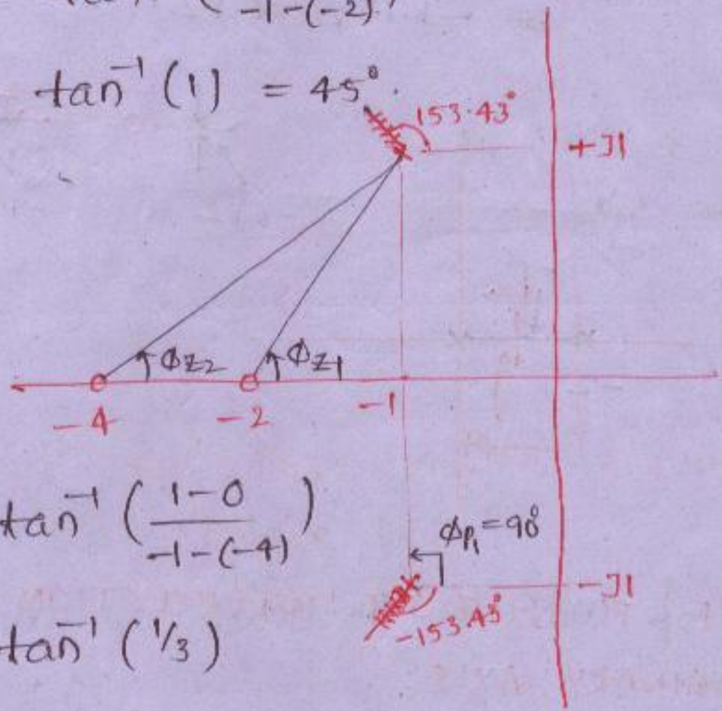
The roots of AE, gives valid and invalid intersection points with imaginary axis.

The valid intersection point is the one for which $k_{marginal}$ value should be +ve.

$$G(s)H(s) = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$$

$$\phi_{z_1} = \tan^{-1}\left(\frac{1-0}{-1-(-2)}\right)$$

$$= \tan^{-1}(1) = 45^\circ.$$



$$\phi_{z_2} = \tan^{-1}\left(\frac{1-0}{-1-(-4)}\right)$$

$$= \tan^{-1}(1/3)$$

$$= 18.43^\circ$$

$$\therefore \phi_{\pm} = \phi_{p_1} - (\phi_{z_1} + \phi_{z_2})$$

$$= 90 - (45 + 18.43)$$

$$= 26.57^\circ.$$

$$\phi_d = 180^\circ - \phi$$

$$= 153.43^\circ.$$

$$G(s).H(s) = \frac{K(s^2+2s+2)}{(s+2)(s+4)}$$

$$\phi_{p_1} = 45^\circ$$

$$\phi_{p_2} = 18.43^\circ.$$

$$\phi_{z_1} = 90^\circ.$$

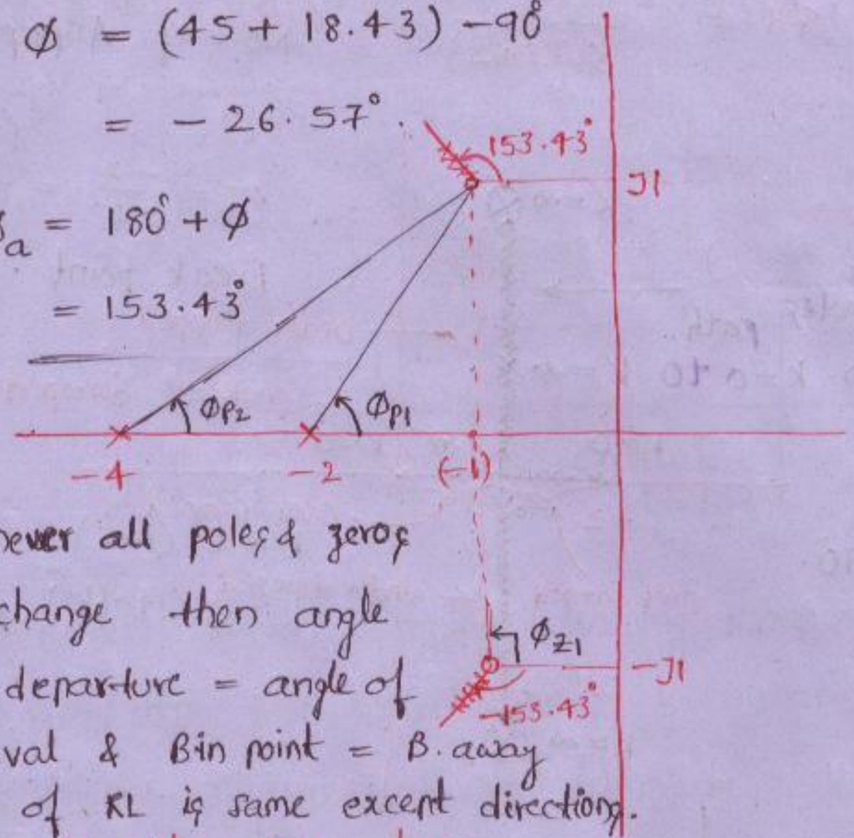
$$\therefore \phi = (\phi_{p_1} + \phi_{p_2}) - \phi_{z_1}$$

$\Rightarrow \phi = (45 + 18.43) - 90°$

$= -26.57°$.

$\therefore \phi_a = 180° + \phi$

$= 153.43°$



* whenever all poles & zeros interchange then angle of departure = angle of arraival & B.in point = B. away shape of RL is same except direction.

Q. Draw the RL for,

① $GH = \dfrac{k}{s(s+2)}$

② $\dfrac{k}{s(s^2+2s+2)}$

③ $\dfrac{ks}{s^2+4}$

④ $\dfrac{k(s+2)(s+4)}{(s^2+2s+2)}$

⑤ $\dfrac{k(s^2+2s+2)}{(s+2)(s+4)}$

⑥ $\dfrac{k}{s}, \dfrac{k}{s^2}, \dfrac{k}{s^3}, \dfrac{k}{s^4}$

⑦ $\dfrac{k}{s(s+1)^2(s+2)}$

⑧ $\dfrac{k(s+1)^2}{s(s+2)}$

⑨ $\dfrac{k}{s(s+k_1)(s^2+2s+2)}$

$k_1 > 2,\ k_1 < 2,\ k_1 = 2$

⑩ $\dfrac{k(s+1)}{s^2(s+k_1)}$

① $GH = \frac{K}{s(s+2)}$

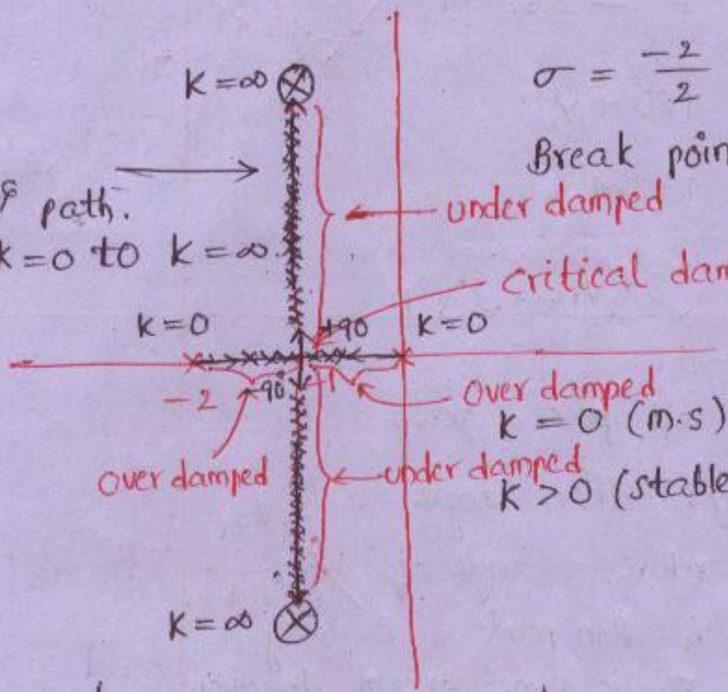
No. of Asymp. $= P - Z$
$= 2$
$(90°, 270°)$

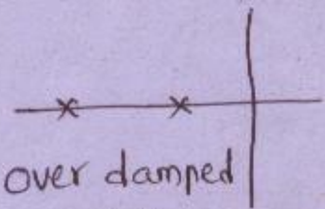
$\sigma = \frac{-2}{2} = -1.$

Break point

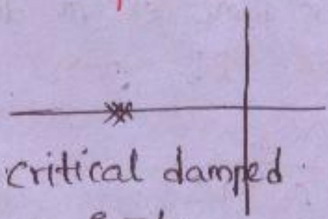
C/L poles path from $k=0$ to $k=\infty$.

$\frac{\pm 180°}{2} = \pm 90°.$

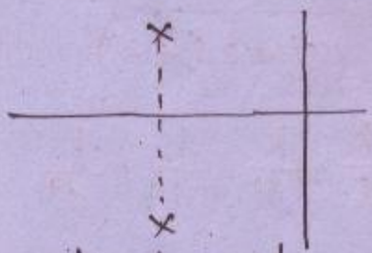




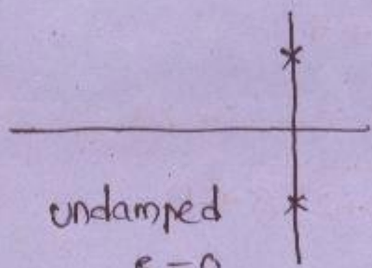
over damped
$\xi > 1$



critical damped
$\xi = 1.$



under damped
$0 < \xi < 1$



undamped
$\xi = 0$

find K value at for Break point? by using magnitude condition.

$$\left| \frac{K}{s(s+2)} \right|_{s=-1} = 1$$

$\Rightarrow K = 1.$

$0 < K < 1$, overdamped
$K = 1$, critical
$K > 1$, under damped

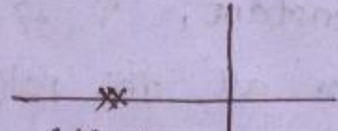
for $k = 1$, CHECKING for $k = 2$,

$$GH = \frac{1}{s(s+2)} \qquad C/L\ T/F = \frac{2}{s^2+2s+2}$$

under damped

$$\rightarrow C/L\ T/F\ \frac{C(s)}{R(s)} = \frac{1}{s^2+2s+1}$$

$$= \frac{1}{(s+1)^2}$$

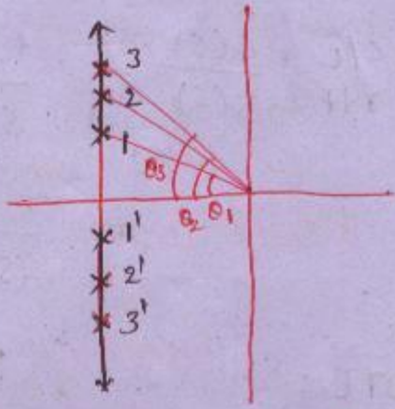


critical damped.

NOTE :

* whenever a RL diagram having more than or equal to two real axis RL branches, then the system is over damped.
* whenever RL diagram having b. points, the should have the critical damped nature.
* whenever RL diagram having break away or break in then the system should have the under damped nature.
* whenever the system having angle of departure or angle of arraival or angle of asymptotes direction towards imaginary axis, the system should have undamped nature.

Q. find the variations in time domain specifications to the given poles path in the s-plane.



As the real part is constant, σ is constant for all the poles hence settling time also same for all poles.

As damped freq. of oscillations increases, time specifications t_r, t_d, t_p must be decreases.

As the inclination of poles increases the damping factor 'ξ' decreases. ($\xi = \cos\theta$)

As damping factor ξ decreases, the % M_p increases ∴ system becomes less stable.

As $\xi \downarrow\downarrow \longrightarrow$ % $M_p \uparrow\uparrow$.

The optimum value of % M_p is 5% to 40% w.r.t t_s

If it is > 40% ⟶ unstable (less stable)

< 5% ⟶ response becomes slow w.r.t. t_r. ~~t_s~~

Q. find % M_p for a given clt T/F.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+25}$$

$\xi = 0 \longrightarrow$ undamped.

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$$

$$= 100\%.$$

Q. for $C(s) = \frac{100}{s^2+20s+100}$

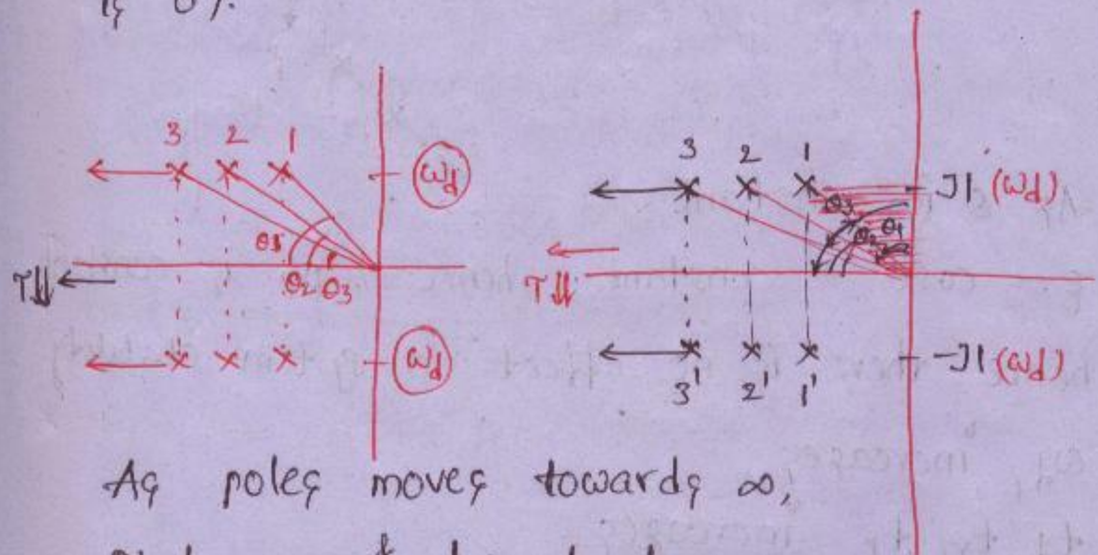
$\omega_n = 10, \quad \xi = 1. \longrightarrow$ critical damped

$\% M_p = e^{-\pi\xi/0} \times 100\,\%$

$= 0\,\%$.

As ξ increases, % of M_p decreases. and system becomes more stable.

when $\xi \geq 1$, and increases the $\% M_p$ is 0%.



As poles moves towards ∞, τ decreases & also t_s decreases.

As ima. part is constant, the peak time is const. but there exists a slight variations in delay & rise time.

ω_d — const.

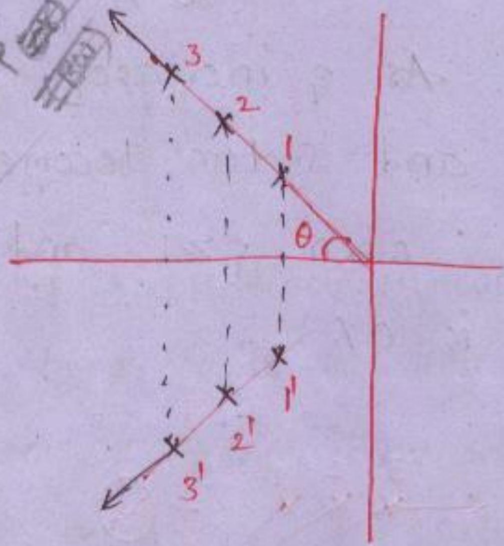
$t_p = \frac{\pi}{\omega_d} \rightarrow$ const.

But $t_d = \frac{1+0.7\xi}{\omega_n}$

As inclination of the poles decreases then ξ increases. $\xi = \cos\theta$

Hence % M_p decreases $\longrightarrow$ system becomes more stable.

$\theta \rightarrow \xi$
$\xi \rightarrow$ % M_p
stability $\rightarrow$ % M_p
$\omega_d \rightarrow t_r, t_p, t_d$
$t_r \rightarrow BW$
$\tau \rightarrow t_s$
speed of res. $\rightarrow \tau$



As θ is constant.

$\xi = \cos\theta$ = constant. hence % M_p is constant.

hence there is no effect on system stability.

ω_d increases,

t_d, t_r, t_p ~~increases~~.

As poles moves to left, $\tau \downarrow$, $t_s \downarrow$.

$\Longrightarrow$ t_s depends only on τ

τ depends on real part of poles.

t_p depends only on ω_d

ω_d depends on ima. part

% M_p depends on ξ

ξ depends inclination of poles. (θ).

Q. find time domain specifications for unity f/b system.

$$G(s) = \frac{25}{s(s+4)}.$$

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25}.$$

$\omega_n = 5.$

$\xi = 0.4$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 4.5 \text{ rad/sec}$

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

$= 0.256$ sec

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$= 0.44$ sec

$t_p =$

$= 0.698$ sec

$\% M_p =$

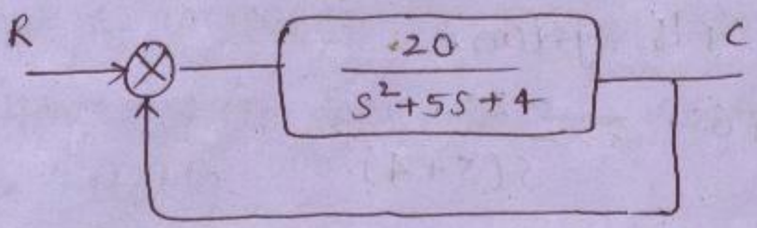
$= 25.8\%$

$t_s =$

$= 2$ secs $(\pm 2\%)$

$$c(t) = 1 - \frac{e^{-4t}}{\sqrt{1-0.16}} \sin(4.5t + \cos^{-1}0.4)$$

Q. find



$$\frac{C}{R} = \frac{20}{s^2+5s+24}$$

$$= \frac{20}{24} \cdot \left[\frac{24}{s^2+5s+24} \right]$$

↑

Affect the steady state value but not any time domain specifications.

$\omega_n = 4.89$ rad/sec

$\xi = 0.511$

$\omega_d = 4.2$ rad/sec

$t_d = 0.277$ sec

$t_r = 0.501$ sec

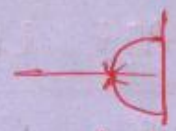
$t_p = 0.745$ sec

$t_s = 1.6$ sec $(\pm 2\%)$

$\% M_p = 15.5\%$

$$c(t) = \frac{20}{24}\left(1 - \frac{e^{-2.5t}}{\sqrt{1-0.511^2}} \sin(4.2t + \cos^{-1}0.511)\right)$$

As $\xi \uparrow\uparrow$



$\rightarrow \tau \downarrow\downarrow \rightarrow t_s \downarrow\downarrow$

$\rightarrow \omega_d \downarrow\downarrow \rightarrow t_d, t_r, t_p \uparrow\uparrow$

$\rightarrow \% M_p \downarrow\downarrow$

As ξ increases poles moves towards left and near to real axis, hence

(1). damped freq. of oscillations ω_d ≢, decreases hence time specifications t_d, t_r, t_p increases.

(2). As poles moves towards left, $\tau \downarrow$ and $t_s \downarrow$.

(3). As $\xi \uparrow$, the $\% M_p$ decreases it shows that the system becomes more stable.

Q. find time domain specifications,

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$$

where $y \rightarrow$ o/p

$x \rightarrow$ i/p.

$$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

STEADY STATE ERROR

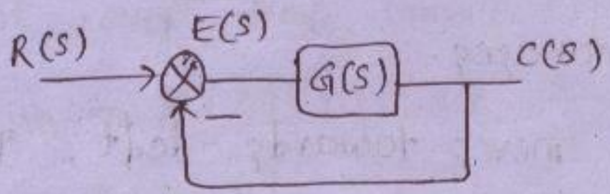
Error is nothing but deviation of the o/p from the reference i/p.

Error at $t \to \infty$ is called ss error.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{s \to 0} s \cdot E(s)$$

$$= \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s)}$$



$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)}.$$

ss errors depends on two factors,

1. type of i/p (R(s))
2. Type of system (G(s))

* ss errors are valid for unity F/B systems only

* ss errors calculated to c/l stable systems only

* ss. errors are calculated to c/l system by using o/l T/F.

we required to calc. ss error to only 3 cases.

(1). Type 0 – step i/p

(2). Type 1 – ramp i/p

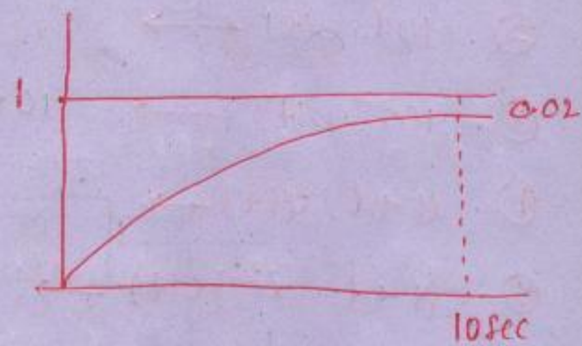
(3). Type 2 – parabolic i/p.

Q. The unit step response of the system is shown in fig.

(1). find τ.

(2). find t_d, t_r.

(3). find t_p, $\% M_p$



Tolerance is $\pm 2\% = 0.02$

$t_s = 4\tau$

$\rightarrow 10 = 4\tau$

$\Rightarrow \tau = 2.5$ sec.

$c(t) = k \cdot (1 - e^{-t/\tau})$

At $t = t_d$, $c(t) = 0.5$.

$\Rightarrow 0.5 = 1 \cdot (1 - e^{-t_d/2.5})$

$\Rightarrow t_d = 1.78$ sec.

$t_r \longrightarrow 10\%$ to 90%

$t_r = 2.2\,\tau$ — (?)

$\rightarrow t_r = 5.5$ sec.

The given response not consists the peak, hence no peak time & peak over shoot.

find steady state error for

$$G(s) = \frac{s+1}{s^2(s+5)(s+10)}$$

① . $10\,u(t) \longrightarrow$

② . $10t \cdot u(t) \longrightarrow$

③ . $10t^2 \cdot u(t) \longrightarrow \quad 10 \times 2 \cdot \frac{t^2}{2} = 20 \cdot \frac{t^2}{2} u(t)$

④ . $(1+t)\,u(t) \longrightarrow$

$$\downarrow \frac{A}{K} = \frac{20}{1/50} = 1000$$

⑤ . $(1+t+t^2)\,u(t) \longrightarrow$

$$\downarrow 2 \cdot \frac{t^2}{2} \qquad e_{ss} = 0 + 0 + \frac{2}{1/50} = 100$$

Q. $G(s) = \dfrac{1}{s^2(s+2)(s+10)}$

char. eq $\Rightarrow s^4 + 15s^3 + 50s^2 + 1 = 0$

In char eq, s term is missing and so the system is unstable.

SS error is valid only for c/L stable system.

The O/L T/F of unity F/B system given by $G(s) = \frac{k}{s(s+1)(s+3)}$.

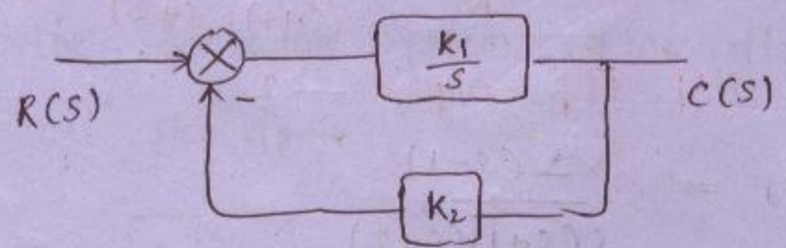
Determine value of k to get ss error $= 0.1 E$.

$$e_{ss} = \frac{A}{K}$$

$$= \frac{1}{K/3} = 0.1$$

$$\Rightarrow k = 30.$$

for the following system ss gain = 2 and system $\tau = 0.4$ sec. The values of k_1 & k_2 are — ?



$$\frac{C}{R} = \frac{k_1}{s + k_1 k_2} = \frac{k_1}{k_1 k_2 \left(\frac{s}{k_1 k_2} + 1\right)}$$

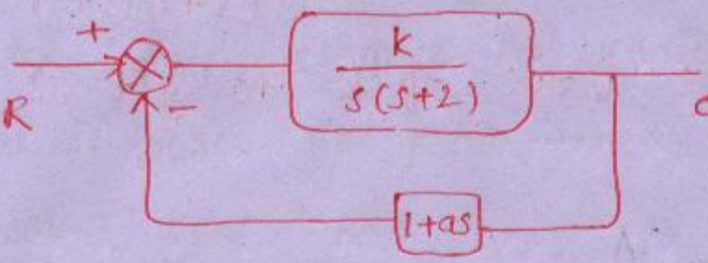
$$= \frac{1/k_2}{(1 + s/k_1 k_2)}$$

Comparing with std form $\frac{k}{1+sT}$ (Of system gain/ ss gain is given otherwise $k = 1$).

$$\Rightarrow 1/k_2 = 2 \Rightarrow k_2 = 0.5 \qquad \frac{2}{1+s(0.4)}$$

$$\frac{1}{k_1 k_2} = 0.4 \Rightarrow k_1 = 5.$$

Q. $\xi = 0.7$, ω_n (undamped natural freq) $= 4$ rad/sec. find k & a for



$$\frac{C}{R} = \frac{k}{(s^2+2s)+k(1+as)}$$

$$= \frac{\cancel{k}^{\to 16}}{s^2+s(2+ka)+\cancel{k}^{\to 16}} = \frac{\cancel{\omega_n^2}^{\to 16}}{s^2+2\xi\omega_n s+\cancel{\omega_n^2}^{\to 16}}$$

Q. The c/c T/F $\frac{C}{R} = \frac{2(s-1)}{(s+1)(s+2)}$ for the unit step i/p, o/p is —?

$$C(s) = \frac{2(s-1)}{s(s+1)(s+2)}$$

$$= -\frac{1}{s} + \frac{4}{s+1} - \frac{3}{s+2}$$

$$= -1 + 4e^{-t} - 3.e^{-2t}$$

The LT of $f(t)$ is $F(s)$. Given $F(s) = \frac{\omega}{s^2+\omega^2}$

The final value of $f(t)$ is —?

(a). 0 (b). ∞ (c). 1. ~~(d)~~. None

Never apply final value theorem for sin & cos term.

Q. The control described by,

$$\frac{d^2y}{dt^2} + 10 \cdot \frac{dy}{dt} + 5y = 12(1 - e^{-2t}).$$ The response of the system at $t \to \infty$ is ?

(a). $y = 2$ (b). $y = 6$ (c). $y = 2.5$

(d). $y = -2$

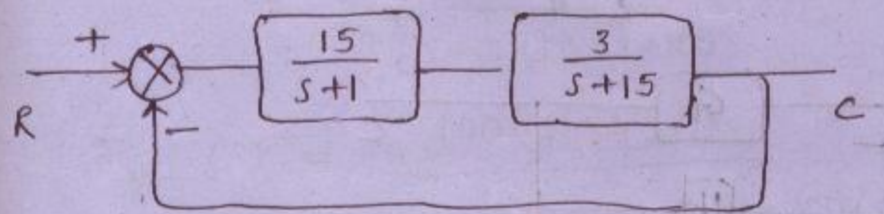
$$\underset{t \to \infty}{\text{lt}}\ y(t) = \underset{s \to 0}{\text{lt}}\ s \cdot Y(s).$$

$$Y(s)\left[s^2 + 10s + 5\right] = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$\Rightarrow Y(s) = \frac{24}{s(s+2)(s^2+10s+5)}$$

$$\underset{t \to \infty}{\text{lt}}\ y(t) = \underset{s \to 0}{\text{lt}}\ s \cdot Y(s) = 2.4.$$

Q. For the following system, find the e_{ss} for unit step i/p — ?



(a). 6 %. (b). 25 %. (c). 33 %. (d). 75 %.

SS errors are evaluated for c/l system by using o/l T/F.

$$G(s) = \frac{45}{(s+1)(s+15)}$$

$$\% \, e_{ss} = \frac{A}{1+k} \times 100\,\%$$

$$= \frac{1}{1 + 45/15} \times 100\,\% = 25\,\%.$$

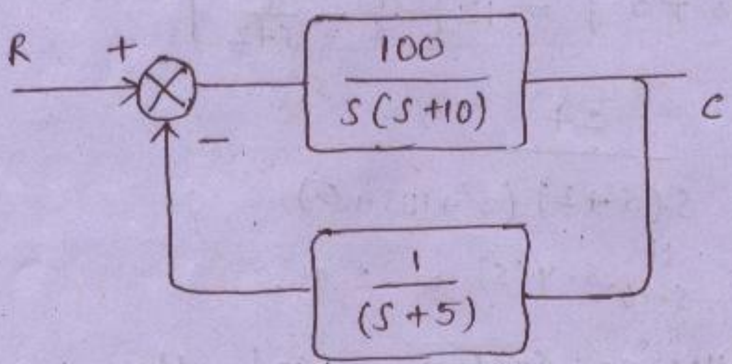
Q. The o/L T/F of unity f/b system is $G(s)$. $e_{ss} = 0$ for

(a). step i/p, Type-0 (b). Ramp i/p - Type-0

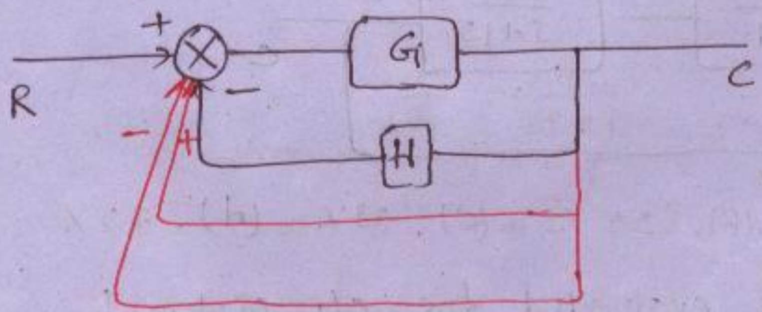
(c). step i/p, Type-1 (d). Ramp i/p - Type-1

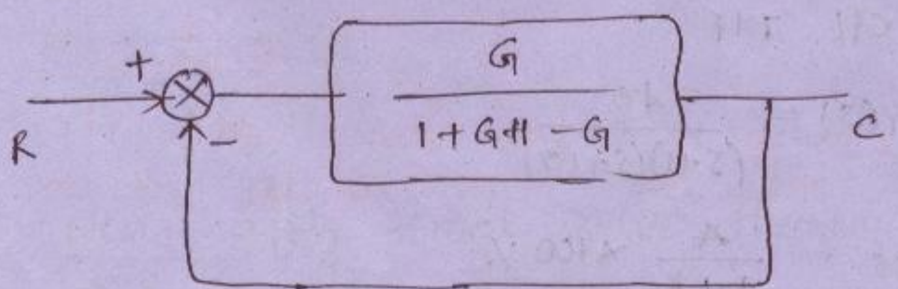
SS. errors to Non-unity f/b System

Q. find ss errors ~~·/·/·/·/~~ to the given Non-unity f/b system for the unit step i/p —?



Given non-unity f/b system should be converted into unity f/b system.





$$G_{NUF}(s) = \frac{G}{1+GH-G}$$

Equilavent o/L T/F for non-unity f/b system

Step 1: find c/L T/F : $\frac{C}{R} = \frac{G}{1+GH}$

Step 2: find $G_{NUF}(s)$ by subtracting numerator in the denominator.

$$G_{NUF}(s) = \frac{G}{1+GH-G}$$

Step 3: Compute e_{ss} for type of i/p.

$$\frac{C(s)}{R(s)} = \frac{100}{s(s+10) + \frac{100}{s+5}}$$

$$= \frac{100(s+5)}{s(s+10)(s+5)+100}$$

$$G_{NUF}(s) = \frac{100(s+5)}{s(s+10)(s+5)+100-100(s+5)}$$

$$= \frac{100(s+5)}{s^3+15s^2-50s-400}$$

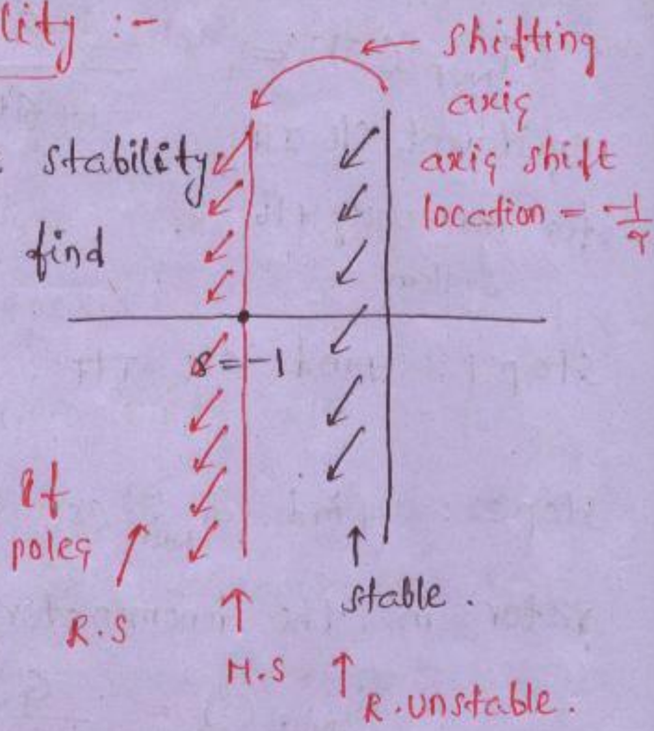
$$e_{ss} = \frac{A}{1+K}$$

$$= \frac{1}{1+\frac{500}{-400}}$$

$$= -4.$$

Relative Stability :-

By using Relative stability concept, we can find system time constant and settling time and time required to reach steady state.



RH CRITERIA:

$as^2+bs+c=0$

If $a,b,c>0 \longrightarrow$ stable

if $a,b,c<0 \longrightarrow$ stable.

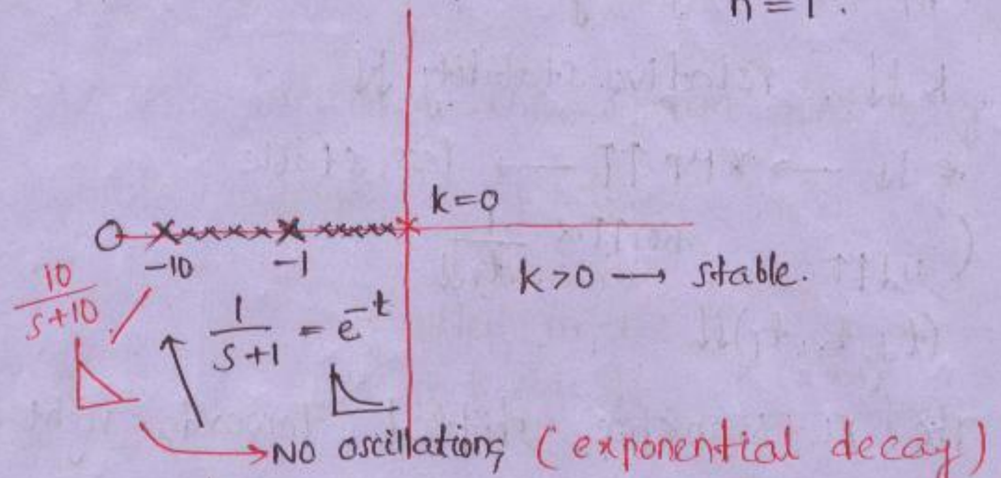
If $b=0,\ a,c>0 \longrightarrow$ M.S.

* Addition of poles & zeros should be in the left of s-plane only.
* If zeros are added in the left most side then steady state performance is improved.

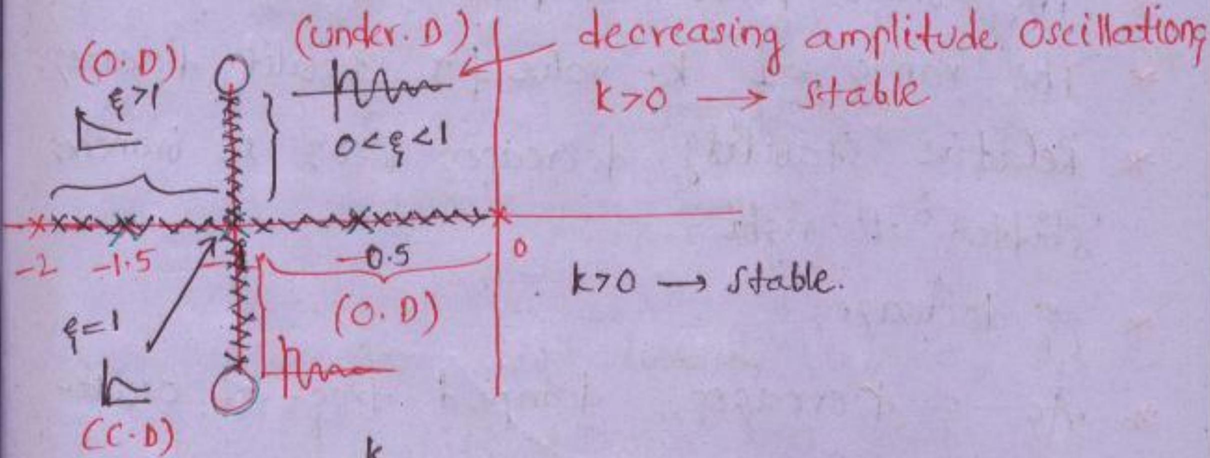
Addition of poles:

$\Rightarrow GH = \frac{k}{s}$

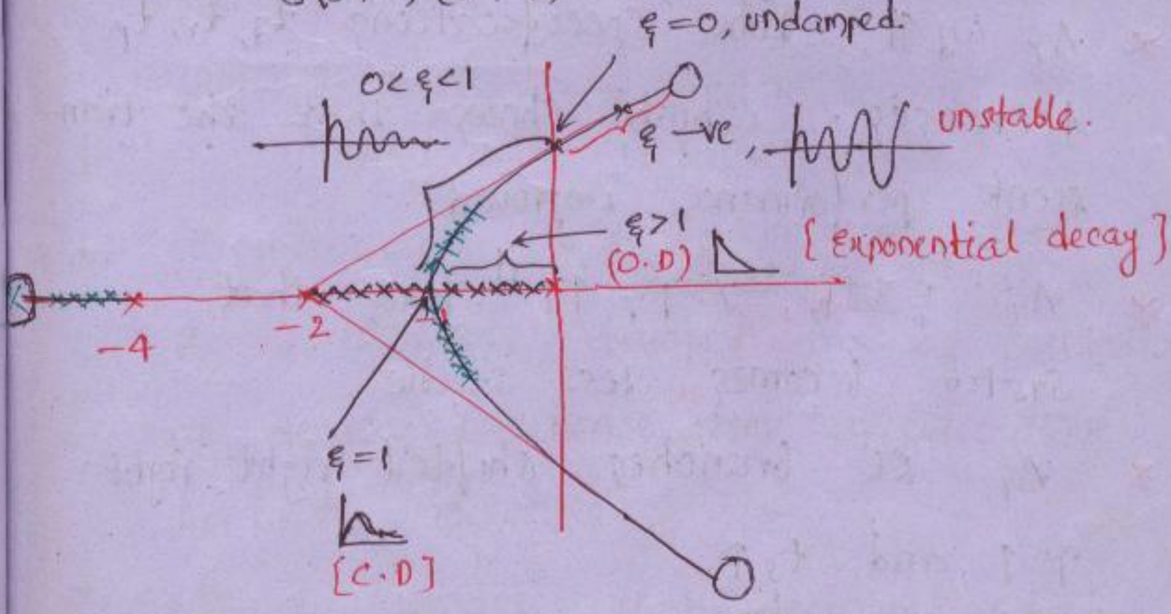
$n = 1.$



$\Rightarrow GH = \frac{k}{s(s+2)}$



$\Rightarrow GH = \frac{k}{s(s+2)(s+4)}$



RL $\rightarrow$ RH - S·plane

BP $\rightarrow$ Imaginary

K $\downarrow\downarrow$, relative stability $\downarrow\downarrow$

$\xi \downarrow\downarrow \rightarrow \% M_p \uparrow\uparrow \rightarrow$ less stable

($\omega_d \uparrow\uparrow$ $\quad BW \uparrow\uparrow \propto \frac{1}{t_r \downarrow\downarrow}$

$(t_d, t_r, t_p) \downarrow\downarrow$

* The RL branches shifted towards right of S-plane.
* The break point shifted towards ima. axis
* The range of k-value for stability decreases.
* Relative stability decreases b'coz RL branches shifted to right.
* ξ decreases
* As ξ decreases, damped freq. of oscillations are increased
* As $\omega_d \uparrow\uparrow$, Time specifications t_d, t_r, t_p decreases. which shows that the transient performance improved.
* As $\xi \downarrow\downarrow$, $\% M_p \uparrow\uparrow$ it shows that System becomes less stable.
* As RL branches shifted right right $\gamma \uparrow$ and $t_s \uparrow$.
* More Oscillatory

* The BW increases b'coz t_r decreases.

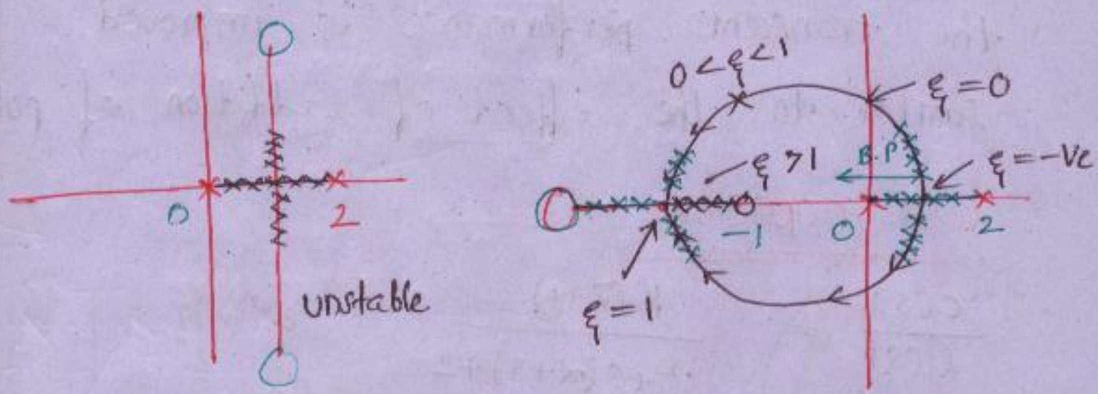
$BW \propto \frac{1}{t_r}$.

* As BW increases the system gives very quick response w.r.t t_r.

Effect of Addition of Zero's:-

~~[zero added in the~~ left most side].

$$\Rightarrow \frac{K}{s(s-2)} \Longrightarrow \frac{K(s+1)}{s(s-2)}$$



1. RL branches shift towards left of s.plane.
2. B. point shift towards ima. axis. zero.
3. The range of k value for system stability increases.
4. RS increases.

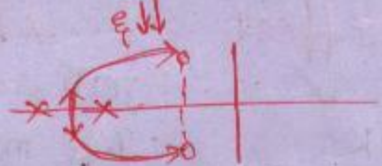
* System becomes more relatively stable.

5. ξ increases

* System becomes less oscillatory.

6. As ξ increases, damped freq. of oscillations (ω_d) are decreased hence as ω_d ↓, time specifications t_r, t_p, t_d increases.
7. As ξ increases, % M_p decreases which shows that system becomes more stable.

* As RL branches turn towards left time const. decreases and also $t_s \downarrow$.
* The BW is decreases b'coz $t_r \uparrow$.
* As BW decreases, system response becomes slow w.r.t t_r.



NOTE :

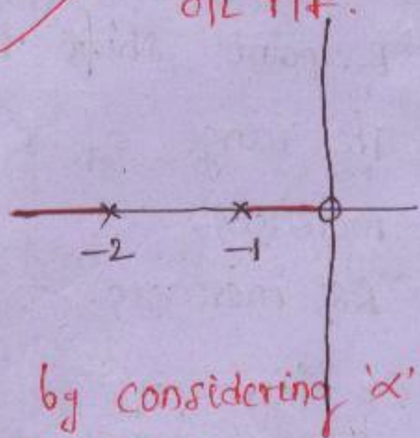
If zero added near to imaginary axis the transient performance is improved similar to the effect of addition of poles.

ROOT CONTOUR :-

$$\frac{C(s)}{R(s)} = \frac{K(s+1)}{s^2+s(\alpha+3)+2} \qquad "\alpha" = ?$$

CE : $s^2+s(\alpha+3)+2 = 0$ ← converting into O/L T/F.

$$GH = \frac{\alpha s}{s^2+3s+2}$$



At pole $\alpha = 0$

At zero $\alpha = \infty$

↑ system gain.

Draw the RC diagram by considering 'α' as system Gain for the given C/L T/F.

ROOT CONTOUR :-

If T/F of char. eq consists more than one unknown parameter, by varying all parameters from 0 to ∞ drawing a RL diagram is nothing but RC.

Draw RC for the following char. eq.

$s^2+as+k=0$.

case 1

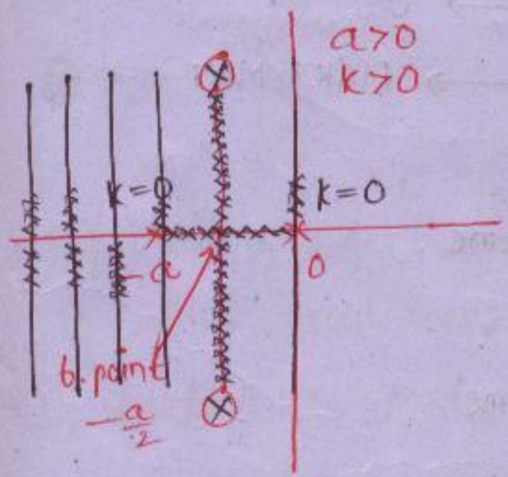
consider,

k as system gain

& a – const.

$$GH(s) = \frac{k}{s^2+as}$$

$$= \frac{k}{s(s+a)}$$



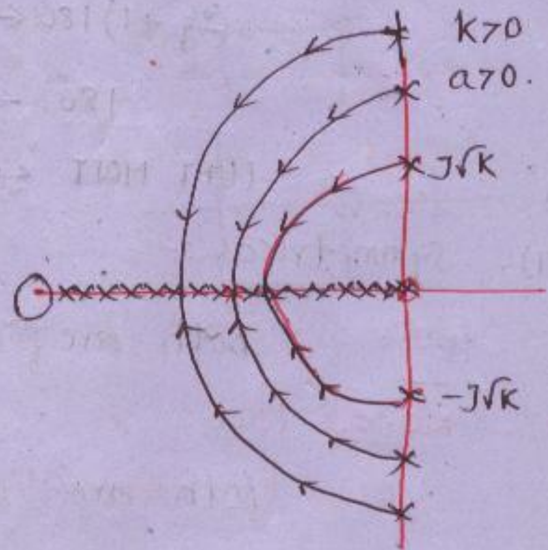
case 2

consider,

'a' as system gain

& k – const.

$$GH(s) = \frac{as}{s^2+k}$$

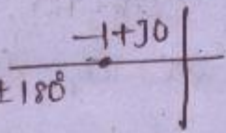


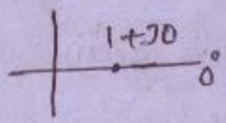
180° RULES DIRECT ROOT LOCUS — 0° RULES INVERSE ROOT LOCUS

(1). $K \longrightarrow 0$ to ∞ $\qquad$ $K \longrightarrow 0$ to $-\infty$

$-ve$ f/b, $\xrightarrow{CE} 1 + GH = 0$ $\qquad$ $+ve$ f/b $\xrightarrow{CE} 1 - GH = 0$

$\angle GH = \angle -1 + j0$





$\angle GH = \angle 1 + j0$

$= \pm(2q+1)180 \pm 180°$ $\qquad$ $= \pm 2q(180°)$

= odd multiples of ± 180 $\qquad$ = Even multiples of ± 180.

DRL $\xleftrightarrow{\text{Replaced}}$ IRL

ODD $\longleftrightarrow$ Even

$(2q+1)180° \longleftrightarrow 2q(180°)$

$180° \longleftrightarrow 0°$.

LEFT MOST $\longleftrightarrow$ RIGHT MOST

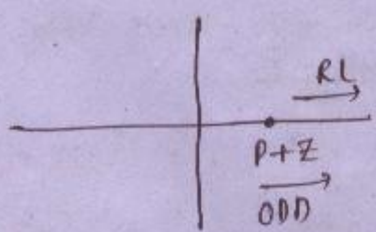
(1). Symmetrical :-

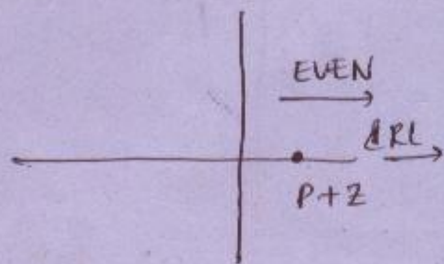
Both are same.

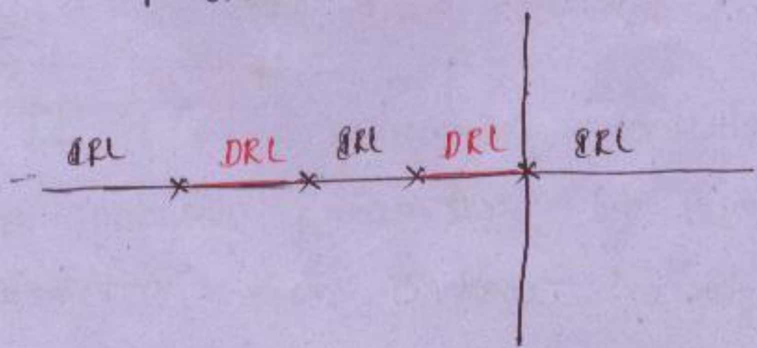
(2). No. of loci:

Both are same.

(3). Real axis loci:







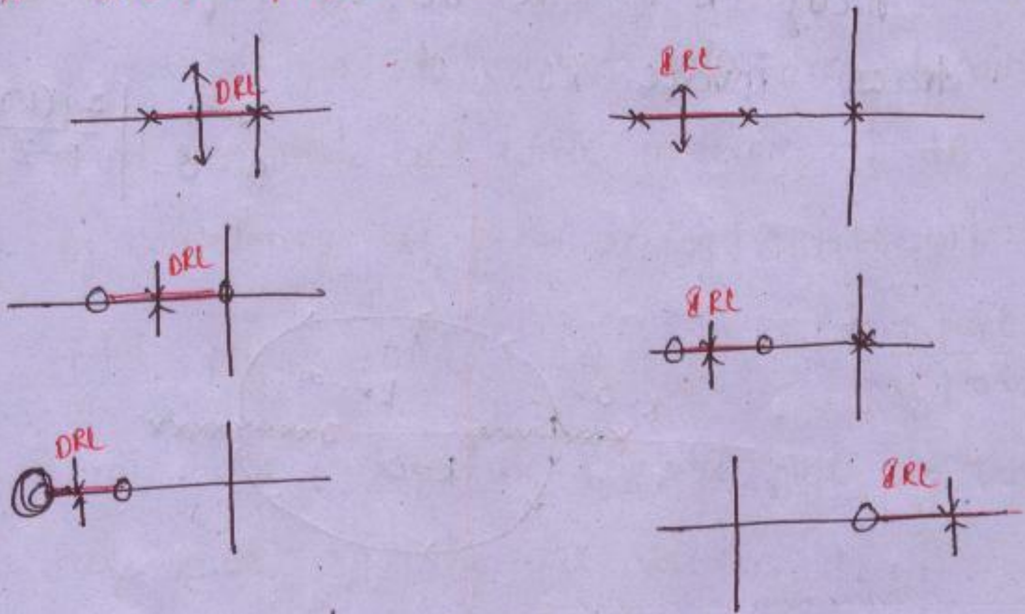
NOTE :

wherever direct RL not exists there must be a IRL.

(4). Asymptotes & centroid :

for both same.

(5). Break point :



procedure for finding B. points is same in both root locus.

(6). Intersection point with imaginary axis :

procedure is same.

for valid intersection point with ima. axis

$K_{marginal}$ is +ve $\qquad\qquad$ $K_{marginal}$ is -ve.

(7). Angle of departure :

$$\phi_d = 180^\circ - \phi \qquad\qquad \phi_d = 0^\circ - \phi$$

Angle of arraival :

$$\phi_a = 180 + \phi \qquad\qquad \phi_a = 0^\circ + \phi$$

Draw the RL for — $G(S) = \frac{k \cdot e^{-s}}{s(s+1)}$

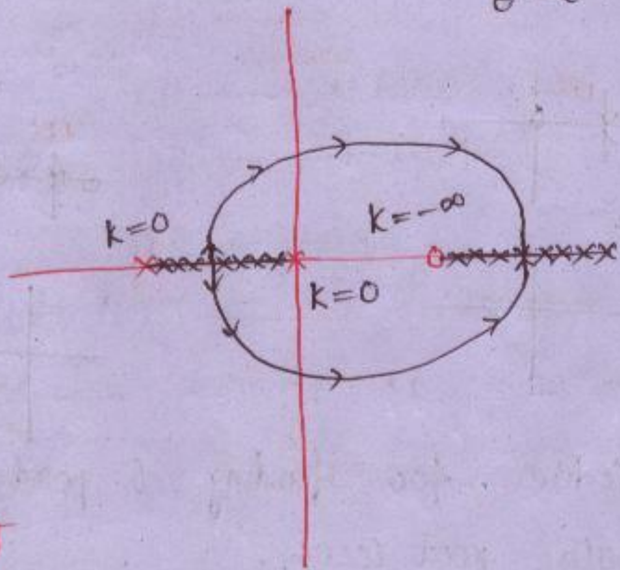
$$G(s) = \frac{k(1-s)}{s(s+1)}$$

$$= \frac{-k(s-1)}{s(s+1)}$$

B'coz k is -ve so we required to draw inverse RL.

N = 1
θ = 0.

$\frac{2q(180)}{P-Z}$



BODE PLOT

To draw freq. response of O/L T/F.

To find c/l system stability by using ph. margin, k_{margin}, $k_{cross\,over\,freq}$ & ph. COF.

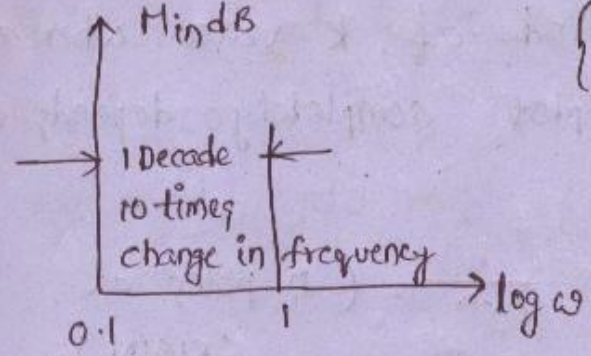
To find relative stability.

The largest gain margin & ph. margin gives more relative stability.

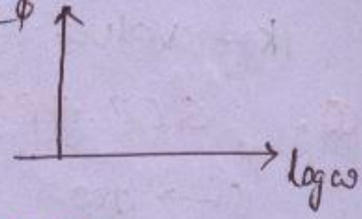
Bode plot consists two plots

(1). Magnitude vs phase ⟶ Magnitude plot

(2). phase plot.



{ Octave → 2 times change in freq. }

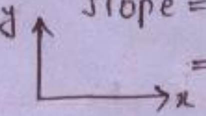


PROCEDURE TO DRAW BODE PLOT:

1. s replaced $j\omega$ to convert into freq domain.
2. find magnitude and write in terms of dB.
   by considering $M_{in\,dB} = 20 \log |G(j\omega) H(j\omega)|$
3. find phase angle $\angle\phi = \tan^{-1}\left(\frac{\text{ima. part}}{\text{real part}}\right)$
4. Draw the magnitude & phase plot by varying freq from min. to max. value.

Q. Draw bode plot for $GH = k$.

$s \rightarrow j\omega$



$\text{Slope} = \frac{dy}{dx} = \frac{dM}{d\log\omega}$

$G(j\omega) \cdot H(j\omega) = k.$

$M = K$

$M_{in\,dB} = 20 \log k.$

$\rightarrow k = 1, \Rightarrow M = 0$

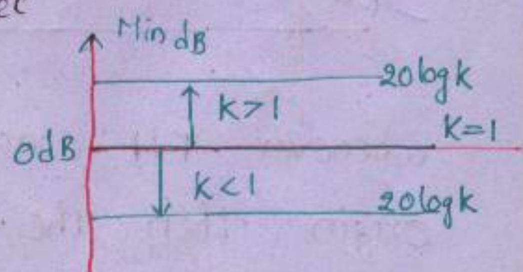
$\rightarrow k > 1 \; (k = 10) \Rightarrow M = +20\,dB$

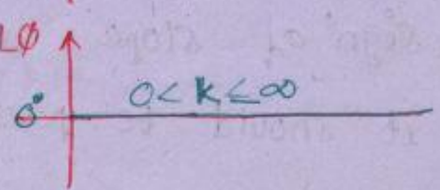
$\rightarrow k < 1 \; (k = 0.1) \Rightarrow M = -20\,dB.$

$S = \frac{dM}{d\log\omega} = 0\,dB/dec$

$\angle\phi = \angle k = 0°.$

shift depends on k value ($20 \log k$).





ph. plot is ind. of k value whereas shift in mag. plot completely depends on k - value.

Q. $G(s).H(s) = \frac{1}{s^n}$ (n poles at ORIGIN).

$s \rightarrow j\omega$

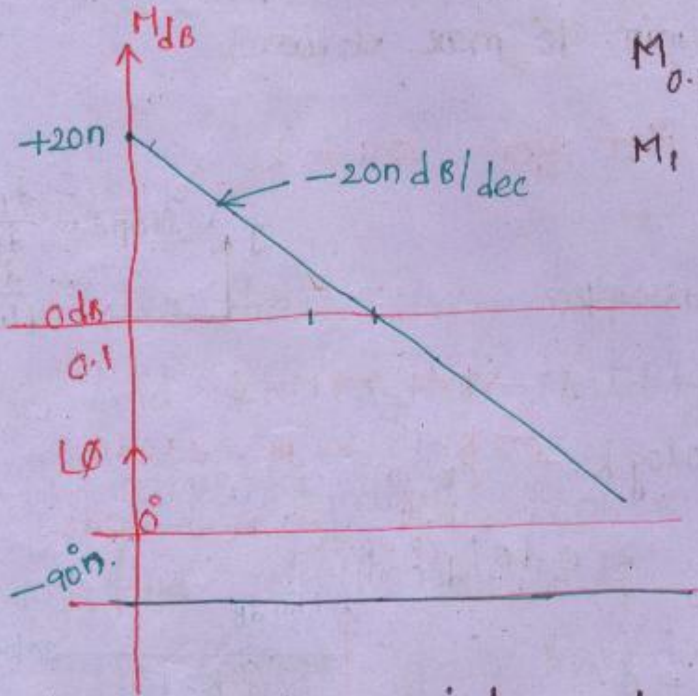
$G(j\omega).H(j\omega) = \frac{1}{(j\omega)^n}$

$M = \frac{1}{\omega^n}$

$M_{dB} = -20n \log \omega$

$S = \frac{dM}{d \log\omega} = -20n \text{ dB/dec}$

$\angle\phi = \frac{\angle 1}{\angle j\omega \ldots n \text{ time}} = -90.n$



$M_{0.1} = -\text{slope}$

$M_1 = 0.$

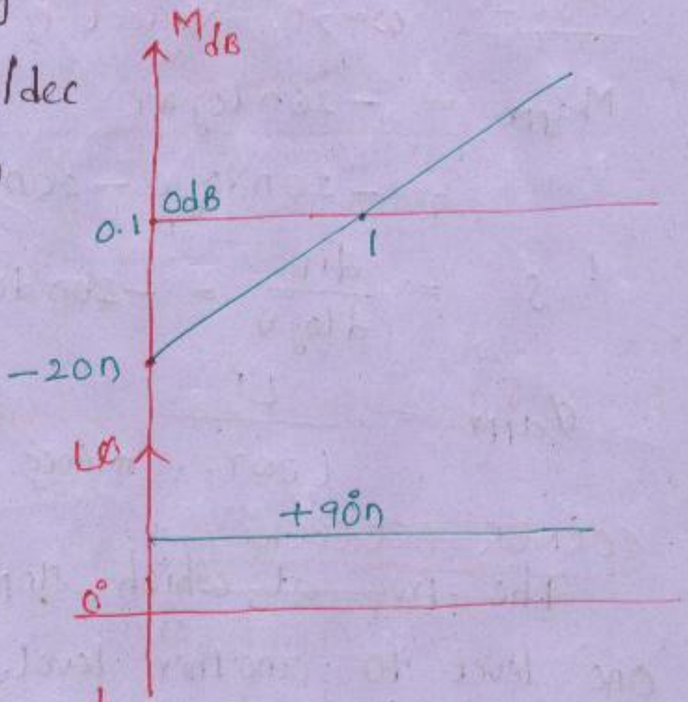
whenever T/F consists poles & zeros at origin then the plot start with opposite sign of slope at a freq. of 0.1 and it should be passes through 0 dB line, intersect at $\omega = 1$ and extendended upto first corner freq if existed, otherwise extended upto ∞. (when $k=1$ only).

Q. $G(s).H(s) = s^n$. (n ~~poles~~ ZEROS at ORIGIN).

$M_{dB} = 20n \log\omega$

$S = 20n$ dB/dec

$\angle\phi = +90^\circ . n$



Q. $G(s).H(s) = \dfrac{1}{(1+sT)^n}$

$s \rightarrow j\omega$

$G(j\omega).H(j\omega) = \dfrac{1}{(1+j\omega T)^n}$.

$M = \left[\dfrac{1}{\sqrt{1+(\omega T)^2}}\right]^n$.

M_{dB} Actual $= -20 \cdot n \log\sqrt{1+(\omega T)^2}$

$\phi_{actual} = \dfrac{\angle 1}{\angle((1+j\omega T)\ldots n\text{ times}}$

$= -n . \tan^{-1}(\omega T)$.

Asymptotic or Appr. analysis :-

Case 1 : $\omega\tau < 1$, Neglect $(\omega\tau)$. $\omega < 1/\tau$.

$M_{Appr.} = 0\ dB/dec$, $S = 0\ dB/dec$.

$\phi_{appr.} = 0^{\circ}$.

Case 2 : $\omega\tau > 1$. Neglect 1, $\omega > \frac{1}{\tau}$

$M_{appr.} = -20n \log \omega\tau$

$= -20n \log \omega - \cancel{20n \log \tau} \rightarrow 0$

$S = \frac{dM}{d\log\omega} = -20n\ dB/dec.$

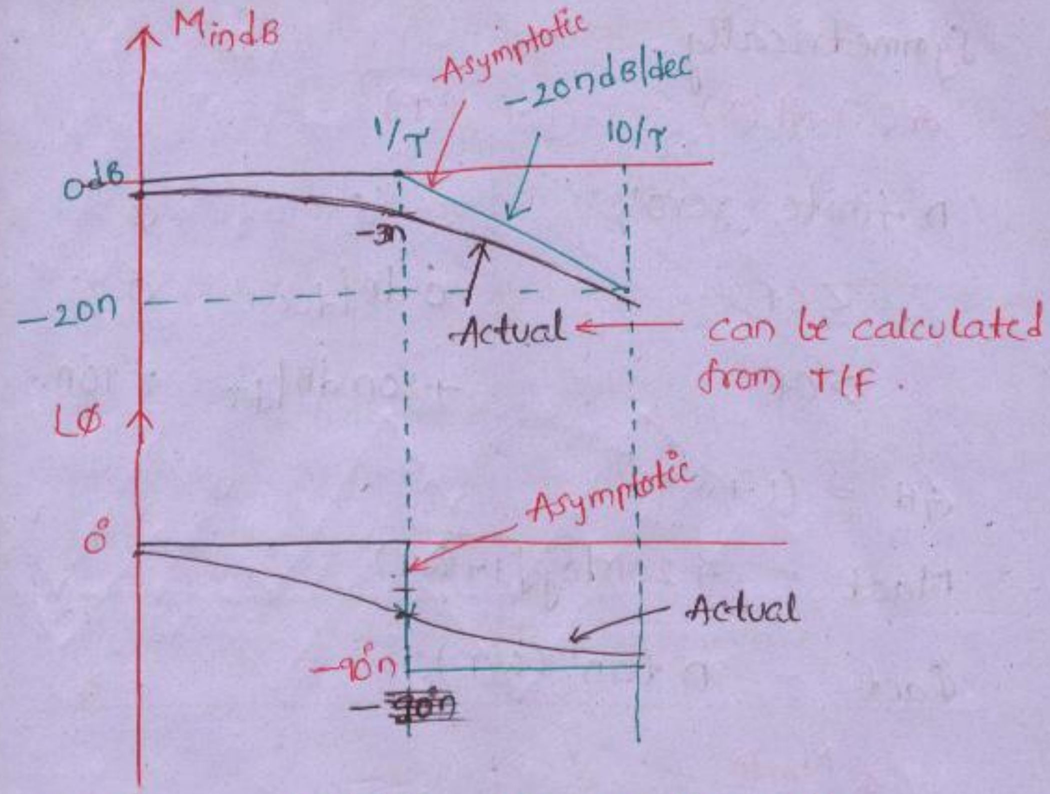
$\phi_{appr.} = \frac{\angle 1}{(j\omega\tau \ldots n\text{ times}} = -90^{\circ}n.$

CORNER FREQUENCY :-

The freq. at which slopes changes from one level to another level is called corner freq.

The corner freq.s are nothing but finite poles & finite zeros location in the form of magnitude.

n finite poles	S	ϕ
< CF	0 dB/dec	0°
> CF	$-20n$ dB/dec	$-90^{\circ}n$.



Error = Actual − Asymptotic value

$E_{at\ \omega=\frac{1}{\tau}}$:

$$M_{act\ \omega=1/\tau} = -20n \log\sqrt{1+(\omega\tau)^2}$$

$$= -20n \log\sqrt{2}$$

$$= -3n\,dB$$

$$M_{asy} = 0\,dB.$$

$$E = -3n\,dB.$$

Error in phase plot:

$$\phi_{\omega=1/\tau} = -n\tan^{-1}(\omega\tau)$$

$$= -45^\circ n.$$

NOTE:

Error is max. at corner freq. either above or below corner freq.s the error is decreases

Symmetrically.

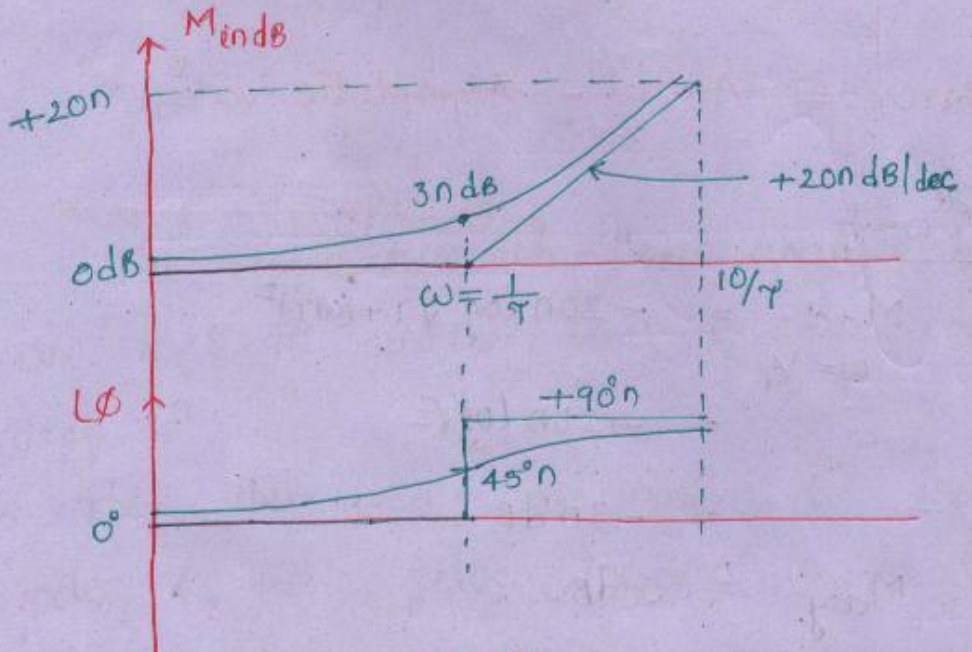
Q. $G(s).H(s) = (1+s\tau)^n$.

n finite zero's	S	ϕ
$\leq$ cf	0 dB/dec	0
> cf	+20n dB/dec	$+90^\circ n$.

$GH = (1+s\tau)^n$.

$M_{act} = +20n\log\sqrt{1+(\omega\tau)^2}$

$\phi_{act} = n\tan^{-1}(\omega\tau)$.

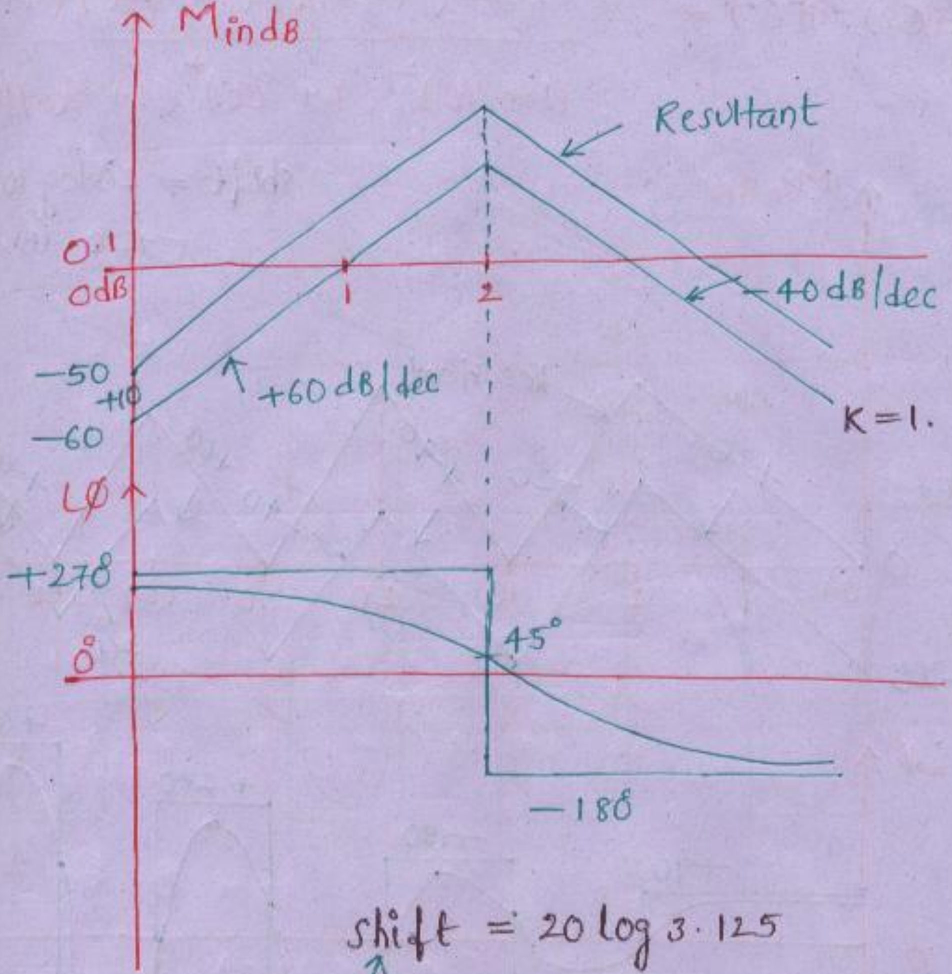


Q. $G(s).H(s) = \dfrac{100.s^3}{(s+2)^5}$

$$\xrightarrow[\text{form}]{\text{Time const.}} \frac{100.s^3}{2^5(1+s/2)^5}$$

$$= \frac{3.125\,s^3}{(1+s/2)^5}$$

cf.



$$\text{shift} = 20 \log 3.125$$
$$\underset{(20 \log k)}{\uparrow} \qquad = +10\, dB.$$

$$\angle\phi\big|_{\omega=2} = 270^\circ - 5\tan^{-1}\left(\frac{\omega}{2}\right)$$
$$= 270^\circ - 5 \times 45^\circ$$
$$= 45^\circ.$$

$+20 \longrightarrow +90^\circ$

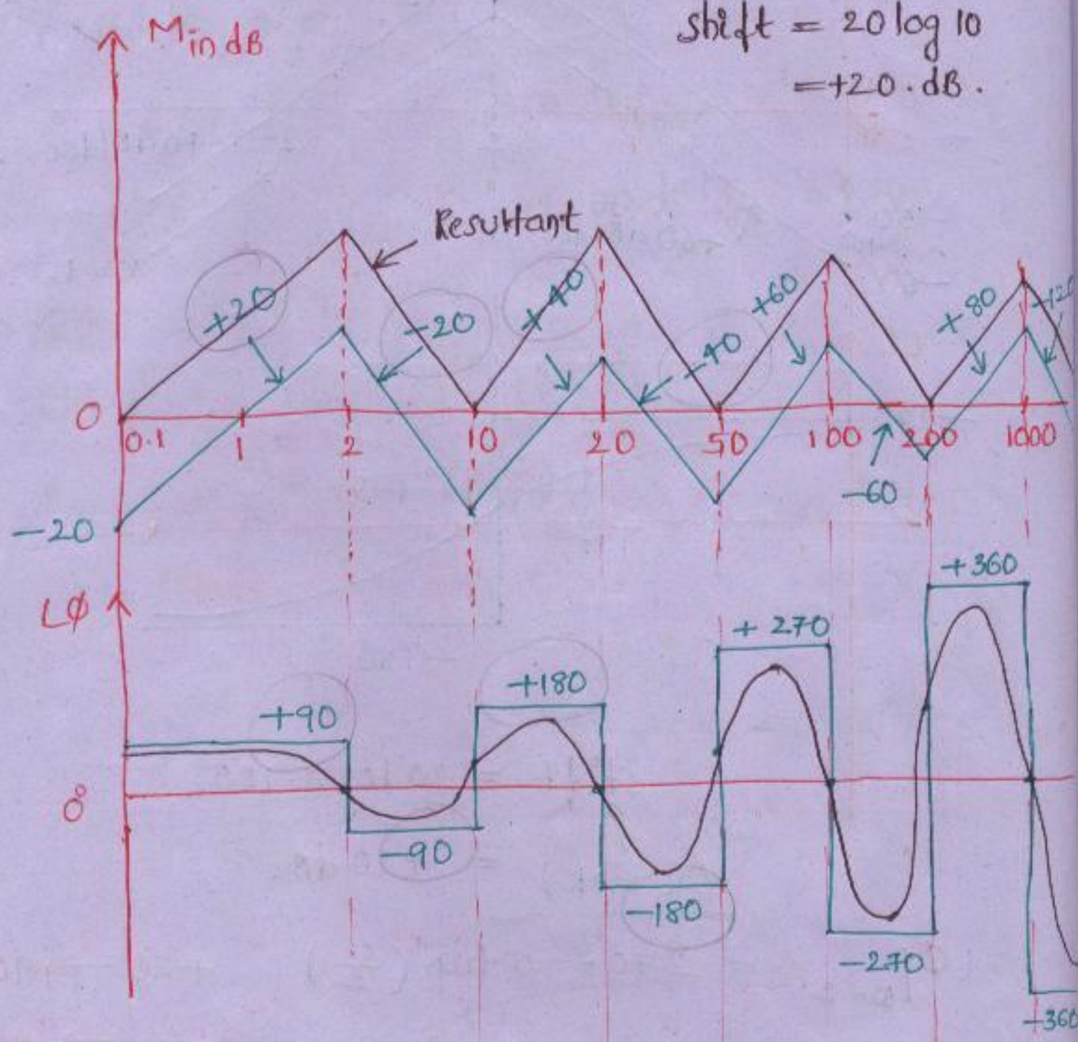
$+60 \longrightarrow +90^\circ \times 3$
(3×20)

$-80 \longrightarrow -360^\circ$

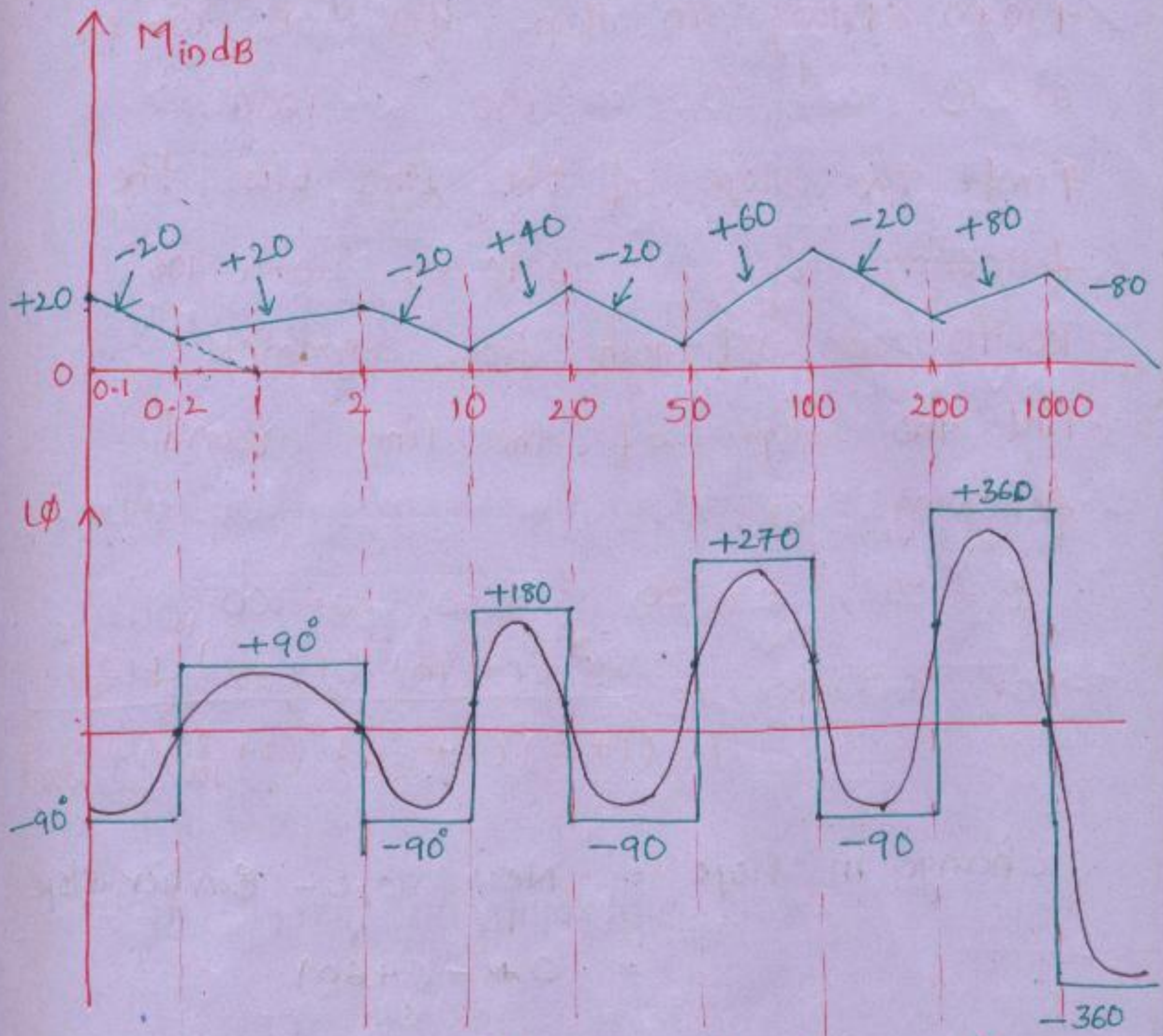
* Initial slope is given by poles & zeros located at origin.

$$G(s).H(s) = \frac{10.s\,(1+s/10)^3(1+s/50)^5(1+s/200)^7}{(1+s/2)^2(1+s/20)^4(1+s/100)^6(1+\frac{s}{1000})^{10}}$$

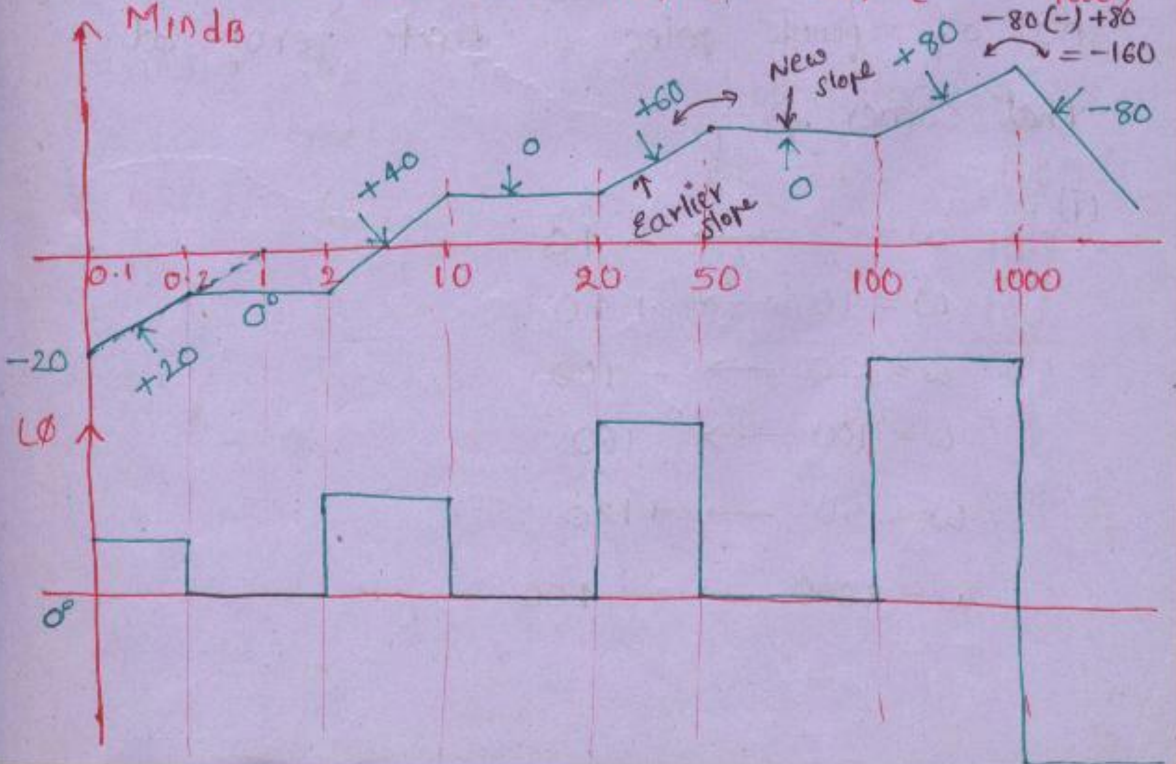
shift = 20 log 10
= +20 dB.



$$G(s)H(s) = \frac{(1+s/0.2)^2(1+s/10)^3(1+s/50)^4(1+s/200)^5}{s.\,(1+s/2)^2(1+s/20)^3(1+s/100)^4(1+s/1000)^8}$$



$$G(s)H(s) = \frac{s\left(1+s/2\right)^2\left(1+s/20\right)^3\left(1+s/100\right)^4}{\left(s/0.2+1\right)\left(1+s/10\right)^2\left(1+s/50\right)^3\left(1+s/1000\right)^8}$$



find change in slope for cf, $\omega = 2$, $\omega = 10$, $\omega = 20$, $\omega = 100$, $\omega = 1000$.

Find the slope of the line b/w the following cf's. 20 to 50, 50 to 100, 100 to 200 and high freq. asymptote.

Find the slopes of the lines around following cf's.

$\omega = 2$, $\omega = 20$, $\omega = 50$, $\omega = 100$.

for $$G(S)H(S) = \frac{S^3 (1+S/10)^2 (1+\frac{S}{200})^{10} (1+\frac{S}{50})^6}{(1+\frac{S}{2})^2 (1+\frac{S}{20})^5 (1+\frac{S}{100})^8 (1+\frac{S}{1000})^{20}}$$

change in slope = New slope − Earlier slope

$= 0 - (+60)$

$= -60$

change in slope is nothing but slopes of no. of finite poles & finite zeros at that corner freq.

(i).

$\omega = 2 \longrightarrow -40$

$\omega = 10 \longrightarrow +40$

$\omega = 20 \longrightarrow -100$

$\omega = 100 \longrightarrow 160$

$\omega = 50 \longrightarrow +120$

$\omega = 1000 \longrightarrow -400$

(include) (exclude)
≥ 20 to < 50

3P
6Z

3Z ⟹ +60

≥ 100 to < 1000

6P
10Z

4Z ⟹ +80

> 2 to < 10

2P
3Z

1Z ⟶ +20

(include) (exclude)
≥ 50 to < 100

7P
11Z

4Z ⟹ +80

100 to 200

15P
11Z

4P ⟹ −80

for All this type of questions consider only below slope of given P & Z's in that include or exclude

High freq. Asymptote:

> 1000

35P
21Z

14P ⟹ −280

Around 10

≤ 10 (exclude) ≥ 10 (include)

≤ 10 (exclude)	≥ 10 (include)
1P	3P
3Z	3Z
2Z ⇓ +40	0

Around 1000

≤ 1000 (E)	≥ 1000 (I)
10Z	10Z
6P	14P
4Z ⇓ +80	4P ⇓ −80

(E) Around 2 (I)

$<2 \quad >2$

$3Z \rightarrow +60$

$3Z$
$2P$
$1Z \rightarrow +20$

(E) Around 20 (I)

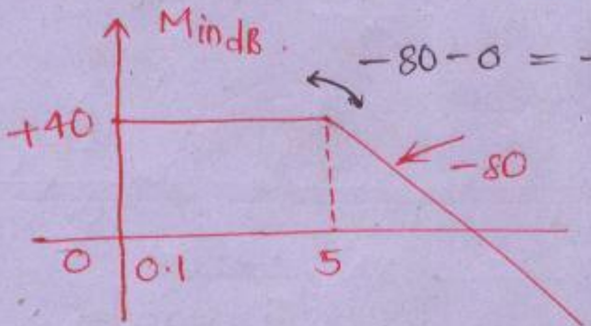
$<20 \quad >20$

$5Z$
$2P$
$3Z \rightarrow 60$

$5Z$
$7P$
$2P \rightarrow -40$

INVERSE PLOTS:

Q. find T/f for given magnitude plot.



$-80-0 = -80$

↑ 4 poles

Step1: Observe the initial slope which gives no. of p & z's at origin.

S2: find change in slope at each & every CF and write the no. of finite poles or finite zeros at that CF.

S3: find the k value.

Initial slope is '0'. So no poles & zeros at origin.

change in slope

$$\frac{k}{(1+s/5)^4}$$

$$40\Big|_{\omega=0.1} = \frac{k}{(1+s/5)^4}$$

$$40\Big|_{\omega=0.1} = 20\log k - 80\log\sqrt{1+\cancel{\left(\frac{\omega}{5}\right)^2}}$$

Neglect

$$= 20\log k - 80\log 1 \to 0$$

$$\Rightarrow k = 10^2 = 100$$

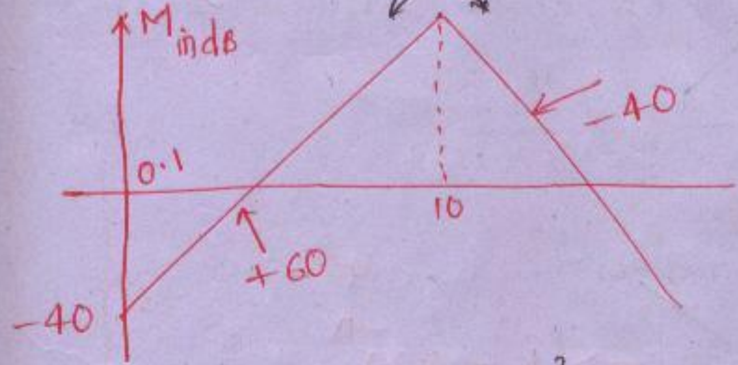
Don't write terms in dB if CF ≥ 0.1

$$TF = \frac{100}{(1+s/5)^4}.$$

Q.-find T/F —

$-40-60 = -100$

5 poles

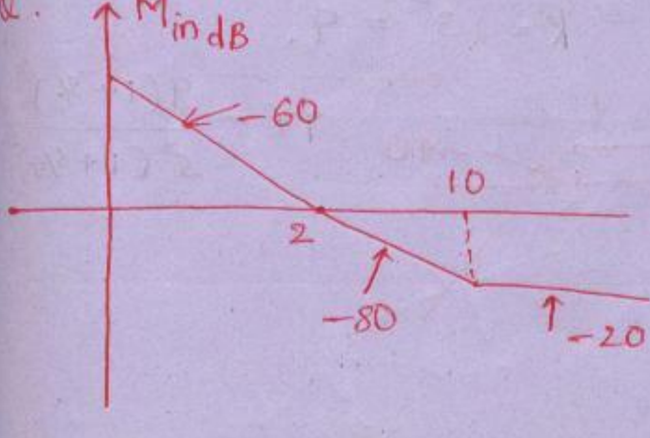


$$-40\Big|_{\omega=0.1} = \frac{k.s^3}{(1+s/10)^5}$$

$$\rightarrow -40 = 20\log k + 60\log \omega^{\to 0.1}$$

$$\Rightarrow 20 = 20\log k \Rightarrow k = 10$$

Q.

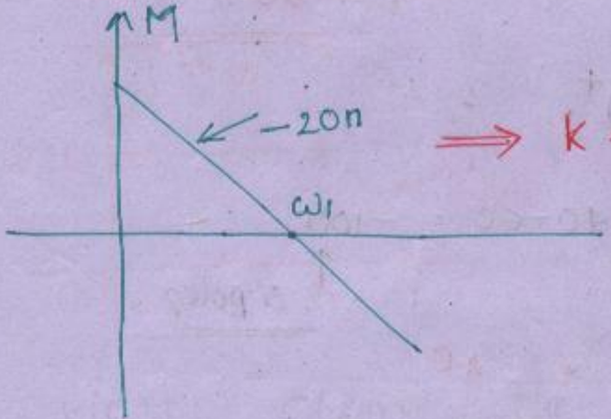


$$0\Big|_{\omega=2} = \frac{k\left(1+s/10\right)^{3}}{s^{3}\left(1+s/2\right)}$$

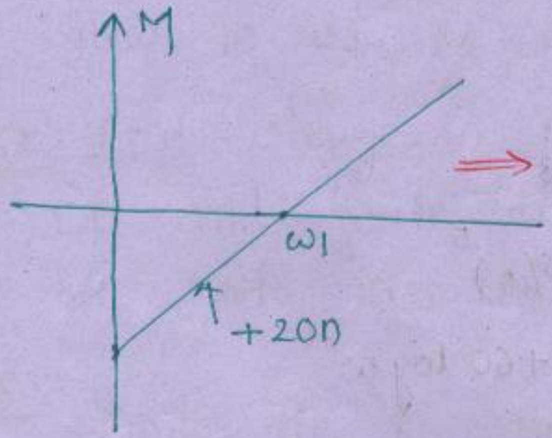
$$\Rightarrow 0 = 20\log k - 60\log 2$$

$$\Rightarrow 20\log k = 20\log 2^{3}$$

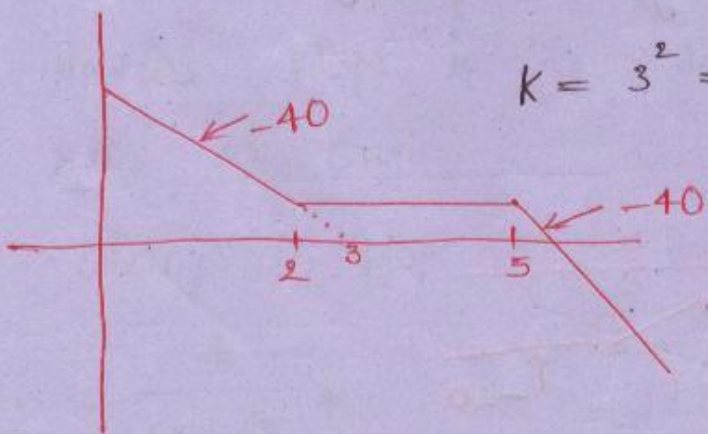
$$\Rightarrow \underline{k = 8}$$



$$\Rightarrow k = (\omega_1)^{n}$$



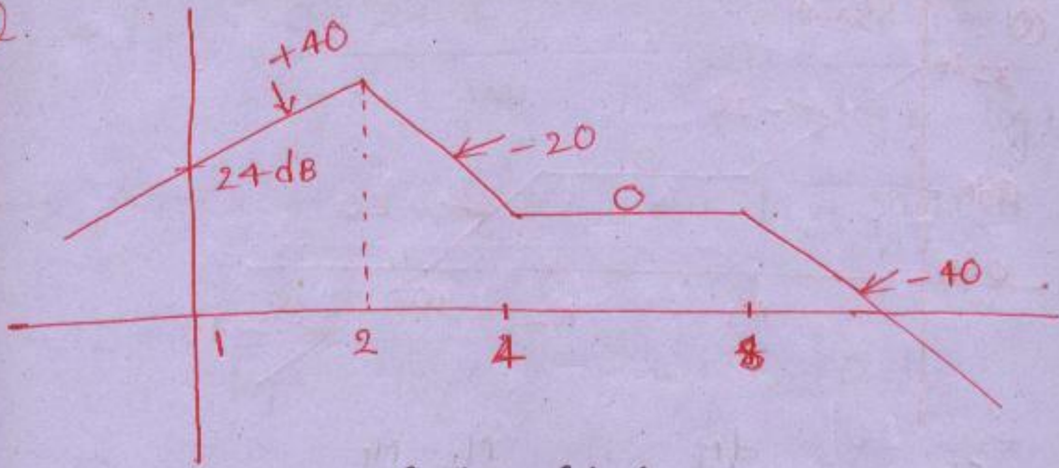
$$\Rightarrow k = (\omega_1)^{-n}$$



$$k = 3^{2} = 9.$$

$$Tf = \frac{9\left(1+s/2\right)^{2}}{s^{2}\left(1+s/5\right)^{2}}$$

Q.



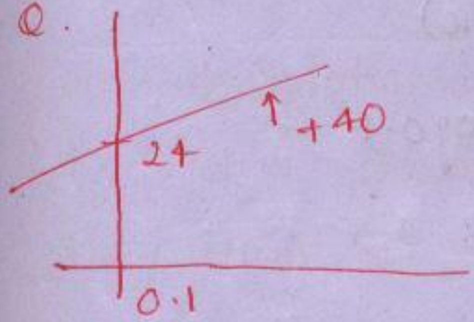
$$24\Big|_{\omega=1} = \frac{K \cdot s^2 (1+s/4)}{(1+s/2)^3 (1+s/8)^2}$$

$$\Rightarrow 24 = 20\log K + 40\cancel{\log 1}^{\to 0}$$

$$\Rightarrow K = 10^{1.2} = 15.24$$

$$\simeq 16$$

Q.

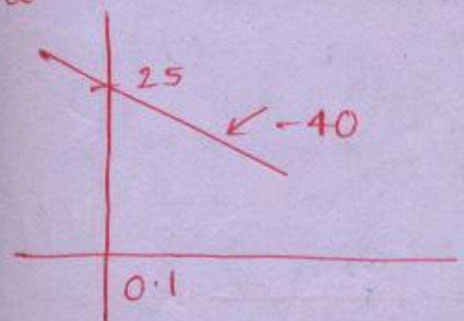


$$24\Big|_{\omega=0.1} = K \cdot s^2$$

$$24 = 20\log K + 40\log 0.1$$

$$\Rightarrow K = 10^{3.2}$$

Q.

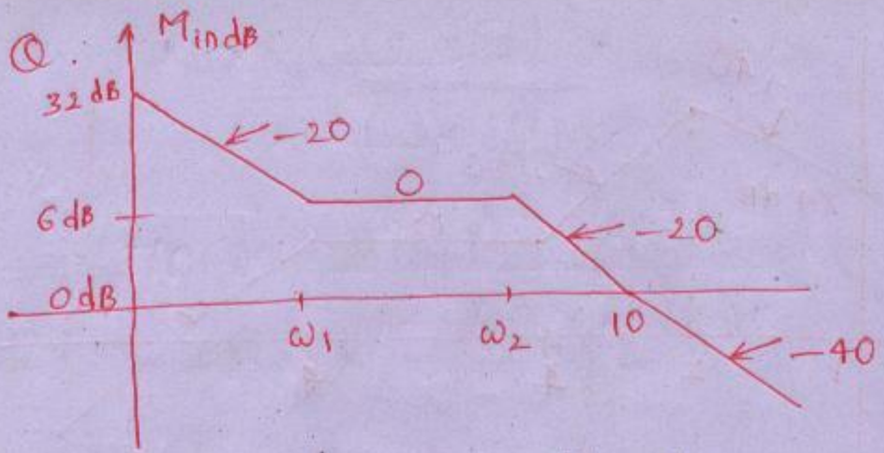


$$25\Big|_{\omega=0.1} = \frac{K}{s^2}$$

$$25 = 20\log K - 40\log 0.1$$

$$\Rightarrow K =$$

Q.



$$\text{Slope} = \frac{dM}{d\log\omega} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$-20 = \frac{6-32}{\log\omega_1 - \log 0.1} \quad ; \quad -20 = \frac{0-6}{\log 10 - \log\omega_2}$$

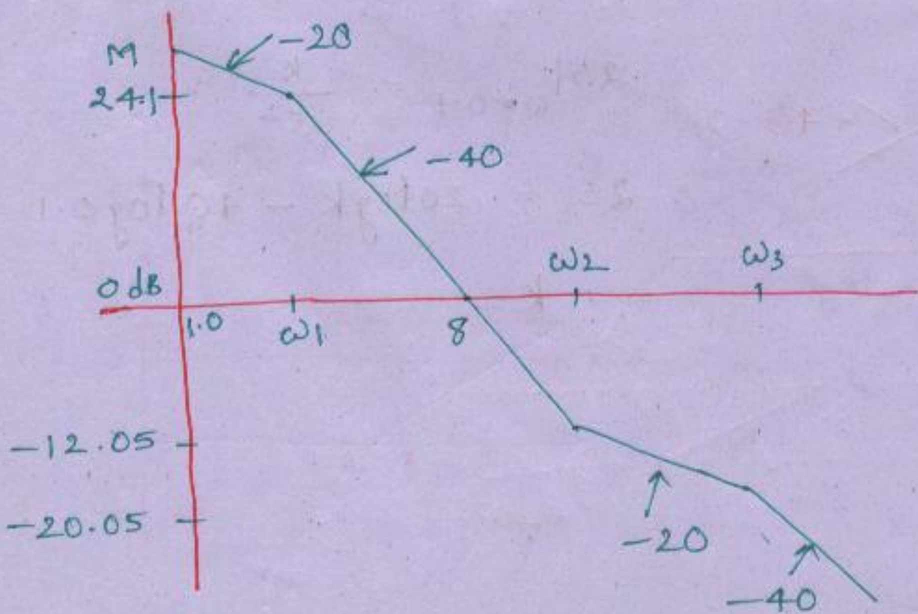
$$\Rightarrow \quad \omega_1 = 2 \text{ rad/sec} \quad \rightarrow \quad \omega_2 = 5 \text{ rad/sec}$$

$$TF = \frac{K(1+S/2)}{S(1+S/5)(1+S/10)}$$

$$32|_{0.1} = 20\log k - 20\log 0.1$$

$$\Rightarrow \quad k = 10^{0.6} \simeq 4$$

Find magnitude M, $\omega_1, \omega_2, \omega_3$ & TF



$$-40 = \frac{0-24.1}{\log 8 - \log\omega_1} \quad ; \quad -20 = \frac{24.1 - M}{\log 2 - \log 1}$$

$\Rightarrow \omega_1 = 2$ $\qquad \Rightarrow M = 30.12\, dB.$

$$-40 = \frac{-12.05 - 0}{\log\omega_2 - \log 8} \quad ; \quad \omega_3 = 40.$$

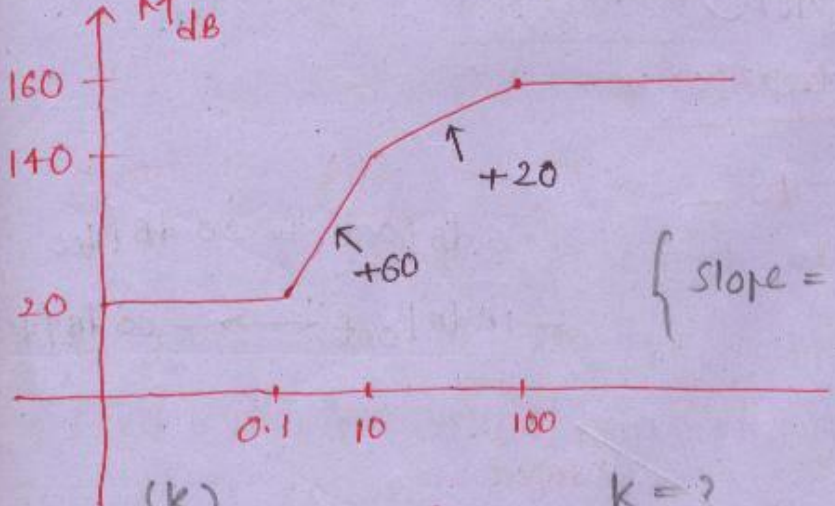
$\Rightarrow \omega_2 = 16$

$$T/f = \frac{k(1+s/16)}{s(1+s/2)(1+s/40)}$$

$$30.12\Big|_{\omega=1} = 20\log k - 20\log 1$$

$\Rightarrow k = 32.06$

Q. The Asymptotic appr. bode mag plot of a minimum ph. system shown in fig. The T/F of the system is —?



$$\left\{ \text{slope} = \frac{dM}{d\log\omega} \right\}$$

$$= \frac{140-20}{\log 10 - \log 0.1}$$

$k = ?$

(a). $\frac{(k)\ \cancel{10^6}\ (s+0.1)^3}{(s+10)^2 (s+100)}$ $\quad$ (b) 10^8 " $\quad$ (c). 10^7 "

(d). 10^9 "

2dec $\longrightarrow$ 120

1dec $\longrightarrow$? 60

$$T/f = \frac{k(1+s/0.1)^3}{(1+s/10)^2(1+s/100)}$$

$$\Rightarrow 20\Big|_{0.1} = 20\log k$$

$$\Rightarrow k = 10$$

$$160\Big|_{\omega=100} = 20\log k + 60\log\sqrt{1+\left(\frac{\omega}{0.1}\right)^2} - 40\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

1000, 100, 1000, Neg.

10, 100, Neg., 10

$$\Rightarrow 160 = 20\log k + 180 - 40$$

$$\Rightarrow k = 10.$$

$$\frac{10\times10^2\times100\times10^3}{(0.1)^3} \Rightarrow k = 10^8.$$

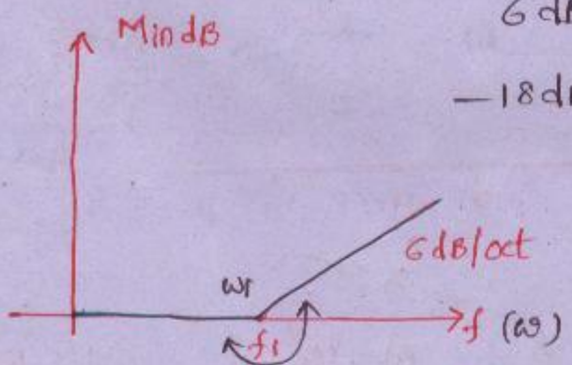
$(1/10)^3$

* MON. 22/12/08 *

find The T/f to –

$6\,dB/oct = 20\,dB/dec$

$-18\,dB/oct \longleftrightarrow -60\,dB/dec$



$$\cancel{k}(1+s/\omega_1) = 1+\frac{j\omega}{\omega_1}$$

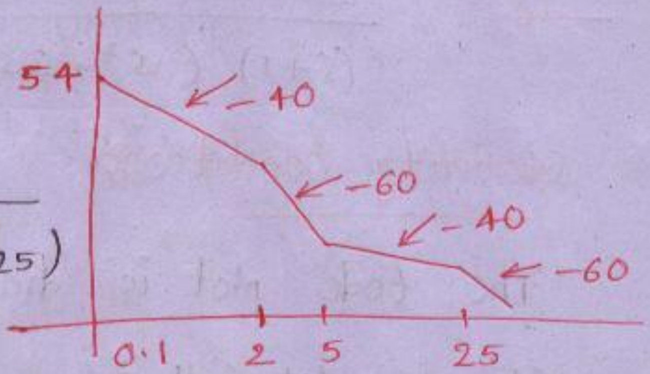
1

$$= \left(1 + \frac{j\omega}{\omega_1}\right)$$

$$= \left(1 + \frac{j2\pi f}{2\pi f_1}\right) = \left(1 + \frac{jf}{f_1}\right)$$

Q. The asymptote appr. of log-mag vs freq of a min. ph. system shown in fig. Its T/F — ?

$$\frac{K(1+S/5)}{S^2(1+S/2)(1+S/25)}$$



$$54\Big|_{\omega=0.1} = 20\log k - 40\log 0.1$$

$$\Rightarrow k = 10^{0.7} = \frac{5\times 2\times 25}{5}$$

$$= 50.$$

$$\therefore \quad T/F = \frac{50(s+5)}{s^2(s+2)(s+25)}$$

Minimum ph. System:

A system in which all the finite poles & zeros lies in the left half of s-plane then it is called min. ph. system.

eg: $\frac{(s+1)}{(s+2)(s+3)}$

ALPS:

A system in which zeros lies in right of s-plane & poles lies in left half s-plane which are symmetrical about ima. axis then it is called All pass system.

Eg: $$\frac{(s-1)(s^2-2s+2)}{(s+1)(s^2+2s+2)}$$

Stability conditions:

The bode plot is drawn for o/L T/F.

$CE = 1 + GH = 0$

$\Rightarrow GH = -1 + j0$

The above eq. gives two conditions:

Angle condi: $\angle GH = \angle -1+j0$

$= -180^\circ \rightarrow \omega_{pc}$

Mag. condi:

$|G(j\omega) \cdot H(j\omega)| = 1$

$M_{in\,dB} = 0\,dB \longrightarrow \omega_{gc}$

$$GM = \frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}} = -20\log|G(j\omega)H(j\omega)|\bigg|_{\omega=\omega_{pc}}$$

$$PM = 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}$$

$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$\Rightarrow$ GM > 1 (L) / +ve (in dB) } & PM $\rightarrow$ +ve

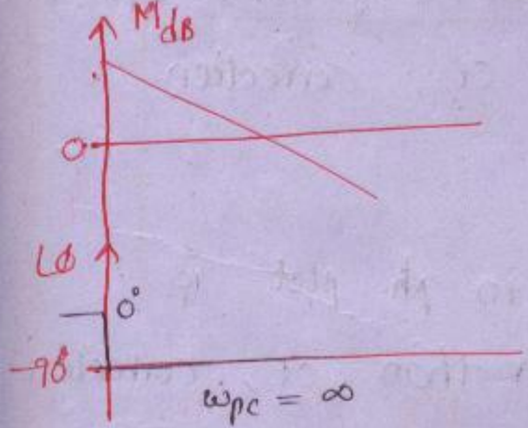
$\omega_{pc} = \omega_{gc} \longrightarrow$ M.S.

$\rightarrow$ GM $= 1$ (L) / $= 0$ (dB) } & PM $= 0°$

$\omega_{pc} < \omega_{gc} \longrightarrow$ unstable.

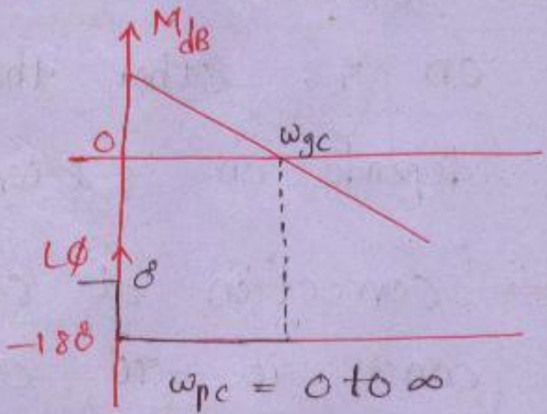
$\Rightarrow$ GM < 1 (L) / $-$ve (dB) } & PM $\rightarrow$ $-$ve.

Q. find system stability ?



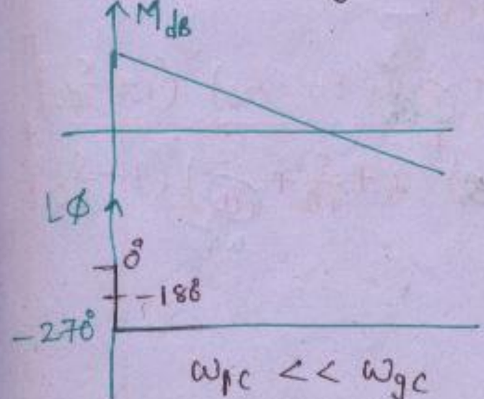
stable

$\omega_{pc} >> \omega_{gc}$



$\omega_{pc} = \omega_{gc}$

M.S.



unstable

* whenever plot maintains less -ve angle than $-180°$ at all the freq's then the $\omega_{pc} = \infty$.

* whenever system gives $-180°$ at all the freq's then the value of ω_{pc} may be any value b/w 0 to ∞. In this case the value of ω_{pc} decided by ω_{gc}.

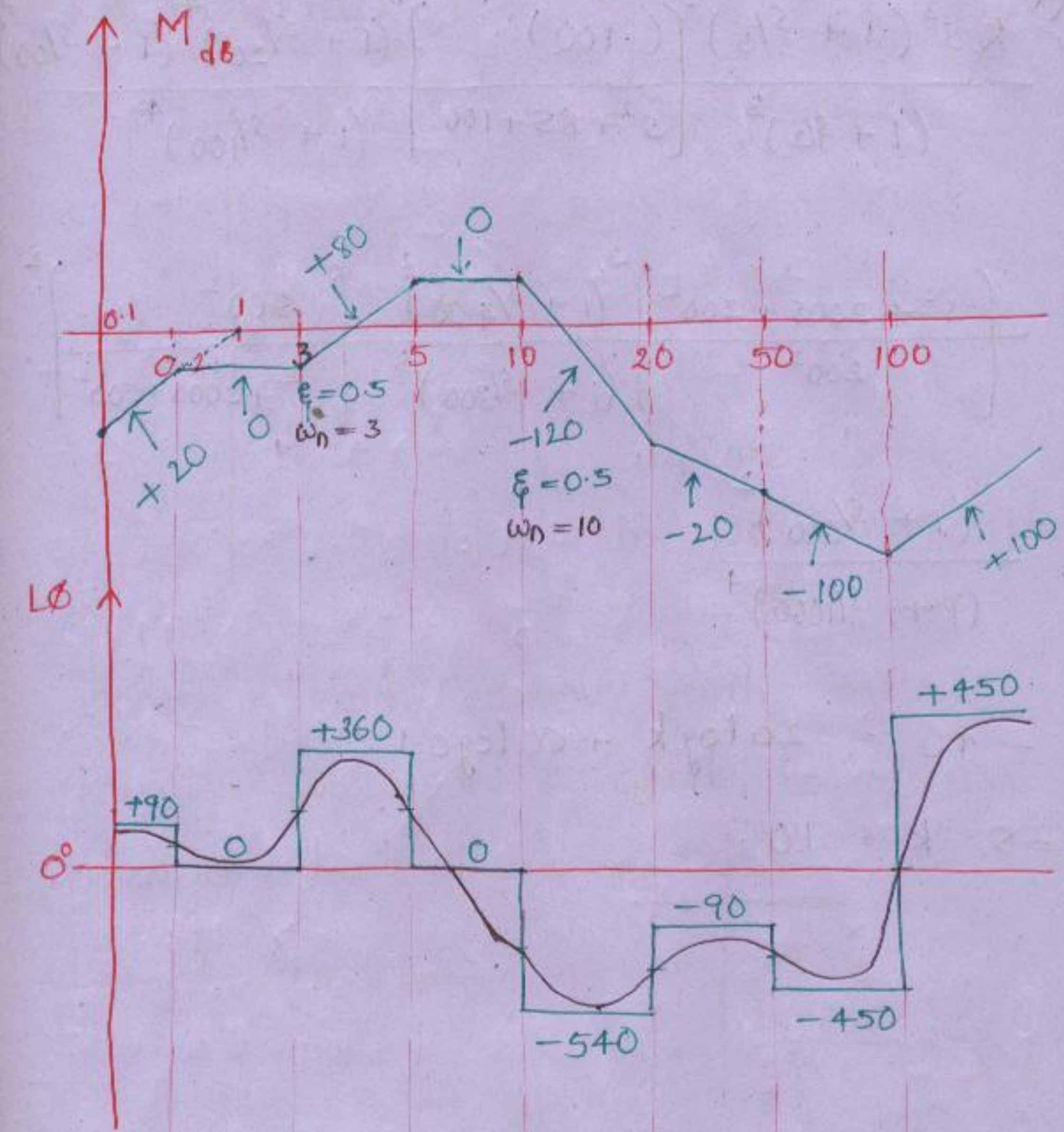
* whenever plot maintains the more -ve than $-180°$ then $\omega_{pc} = 0$

→ Correction at CF in mag. plot depends on ξ, other than CF, correction depends on ξ & ω_n.

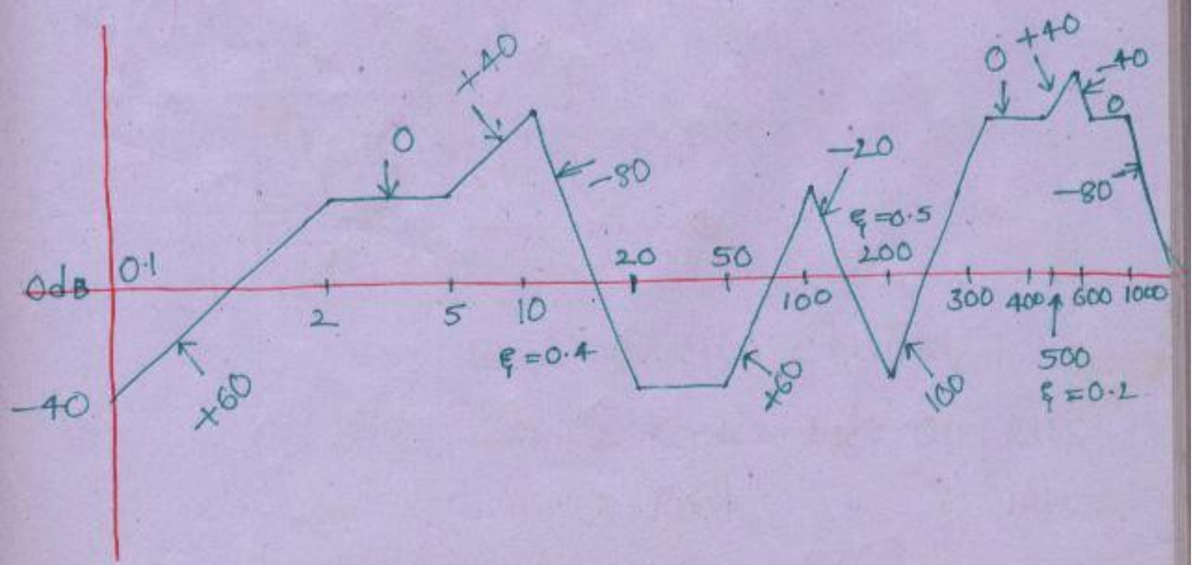
→ Correction at CF in ph. plot is const. ie $-90°$. other than CF, correction depends on ξ & ω_n.

Q. Draw BODE PLOT for –

$$G(s)H(s) = \frac{s\left(1 + s/3 + \frac{s^2}{\underbrace{9}_{\omega_n^2}}\right)^2 (1 + s/20)^5 \left(1 + \frac{s}{100}\right)^{10}}{1 + \left(\frac{s}{0.2}\right)\left(1 + s/5\right)^4 \left(1 + \frac{s}{10} + \frac{s^2}{\underbrace{100}_{\omega_n^2}}\right)^3 \left(1 + \frac{s}{50}\right)^4}$$



Q. find the T/f to the given mag. plot.



TF:

$$\frac{K s^3 (1 + s/5)^2}{(1 + s/2)^3} \left[\frac{(100)}{s^2 + 8s + 100}\right]^3 \frac{(1 + s/20)^4 (1 + s/50)^3}{(1 + s/100)^4}$$

$$\left[\frac{s^2 + 200s + 200^2}{200^2}\right]^3 \frac{(1 + s/400)^2}{(1 + s/300)^5} \left[\frac{500^2}{s^2 + 200s + 500^2}\right]^2$$

$$\frac{(1 + s/600)^2}{(1 + s/1000)^4}$$

$$-40 = 20 \log k + 60 \log 0.1$$

$$\Rightarrow \quad k = 10.$$

R.H. CRITERIA:

Q. $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$

s^4	1	2	8
s^3	$\cancel{2}$ 1	$\cancel{4}$ 2	
s^2	$\cancel{0}$ ϵ	8	
s^1	$\frac{2\epsilon - 8}{\epsilon}$		
s^0	8		

If any one coe. is zero in 1st column, replace zero by smallest +ve const. ϵ and continue routh table. finally sub. $\epsilon = 0$ & check no. of sign changes.

s^4	1
s^3	1
s^2	$\epsilon \to 0$
s^1	$\frac{2\epsilon - 8}{\epsilon} \to 2 - \frac{8}{\epsilon \to 0} = -\infty$
s^0	8

$\Longrightarrow$ 2 sign changes.

Q. $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$.

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}$ ϵ	-12	
s^2	$\boxed{\frac{2\epsilon + 12}{\epsilon}} \to K$	15	
s^1	$\frac{-12K - 15\epsilon}{K} \to -ve$		
s^0	15		

$\Longrightarrow$ 2 sign changes.

$\longrightarrow$ 2 Roots Lies on RHS

$\longrightarrow$ 3 Roots " " LHS.

Q $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 2 \\
s^4 & 1s^4 & 3s^2 & 2s^0 \\
s^3 & \cancel{0}\ 4 & \cancel{0}\ 6 & \cancel{0}\ 0 \\
s^2 & 3/2 & 2 & \\
s^1 & 2/3 & & \\
s^0 & 2 & &
\end{array}$$

AE

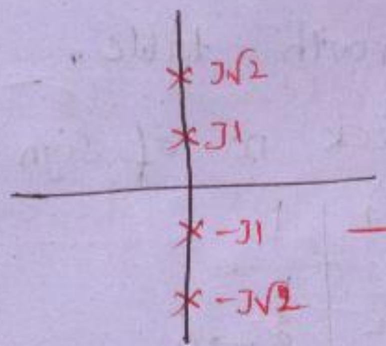
$s^4 + 3s^2 + 2 = 0$

$\xrightarrow{d} 4s^3 + 6s = 0.$

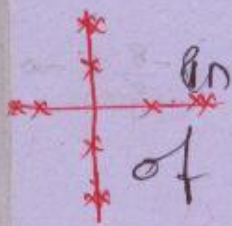
AE: $s^4 + 3s^2 + 2 = 0$

$\Rightarrow (s^2+2)(s^2+1) = 0$

$\rightarrow s = \pm j1, \pm j\sqrt{2}.$



$\longrightarrow$ Non-repeated roots on Ima. axis $\Rightarrow$ M.S.



In Routh tabular form, the row of zeros occurs only when poles are located symmetrical about the origin.

In Routh, the AE consists only even powers b'coz roots of AE must be symmetrical about origin.

$$\begin{array}{|ccc}
+ & & \\
+ & & \\
+ & & \\
0 & 0 & 0 \\
+ & & \\
+ & & \\
+ & &
\end{array} \Rightarrow \text{Marginally stable.}$$

* whenever only once one row of zeros occurs and all coe.s in 1st column are +ve then the system must be marginal stable. b'coz the poles must be on ima axis which are non repeated.

Q. $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$

s^6	1	4	5	2
s^5	$\cancel{3}\ 1$	$\cancel{6}\ 2$	$\cancel{3}\ 1$	
s^4	$\cancel{\tfrac{2}{3}}\ 2s^4$	$4s^2$	$2s^0$	
s^3	$\cancel{0}\ 8$	$\cancel{0}\ 8$	$\cancel{0}$	
s^2	$2s^2$	$2s^0$		
s^1	$\cancel{0}\ 4$	$\cancel{0}$		
s^0	2			

AE_1 : $2s^4 + 4s^2 + 2 = 0$

$\Rightarrow (s^2+1)^2 = 0$

$\rightarrow s =$

AE_2 : $2s^2 + 2 = 0$

$s = \pm j1.$

* whenever many times rows of zeros occurs & all coe.s are +ve, the system must be unstable. b'coz the poles on ima. axis which are repeated.

Q. Identify the location of poles for

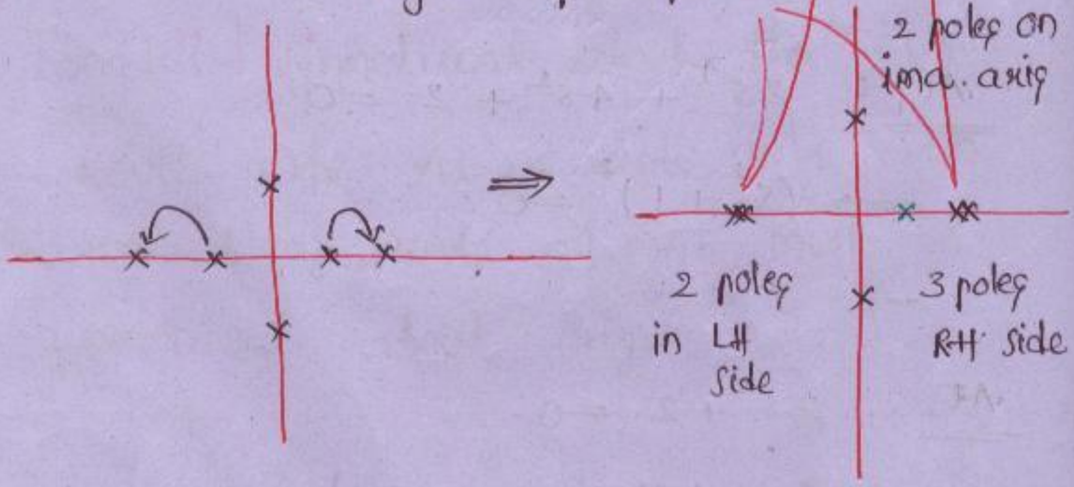
(1).

s^7	−
s^6	+
s^5	~~0 0 0~~
s^4	+
s^3	~~0 0 0~~
s^2	+
s^1	−
s^0	+

$\longrightarrow AE_1 \longrightarrow$ 6th order ie 6 poles are symmetrical about origin.

2 times row of zeros $\longrightarrow$ 2 poles repeated

2 sign changes
↓
2 roots in Right of s-plane.



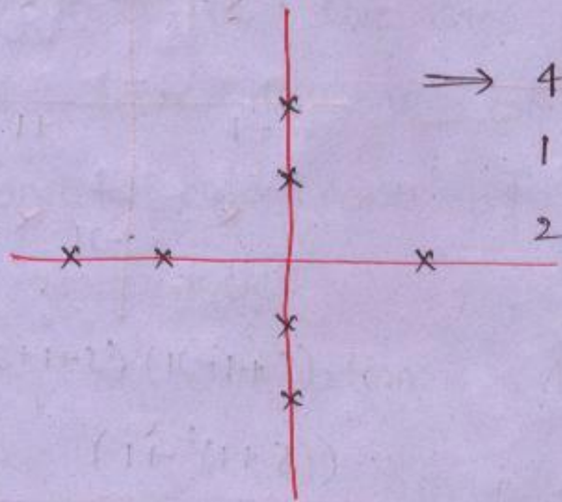
(2).

s^7	+
s^6	+
s^5	~~0 0 0~~
s^4	+
s^3	+
s^2	+
s^1	+
s^0	−

s^6 row → AE : → 6 th order.
6 poles symmetrical about origin.

One time row of zeros
→ so no repeated poles.

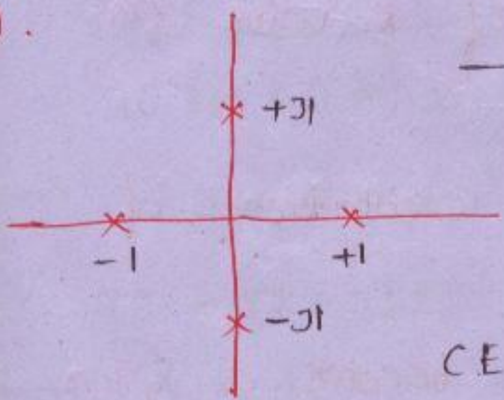
s^1, s^0) 1 sign changes.
→ one pole on right side



⟹ 4 pole → on ima. axis
1 pole → RH
2 poles → LH.

Q. Identify the routh tabular form for–

(1).



→ 1 time row of zeros.

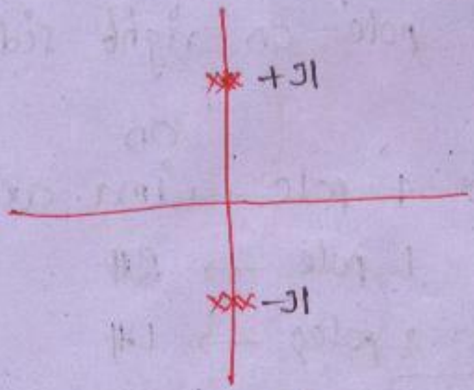
→ 1 sign changes.

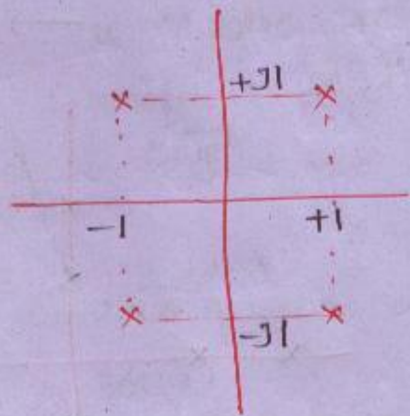
CE: $(s^2-1)(s^2+1) = 0$

$\rightarrow s^4 - 1 = 0.$

$s^4 - 1 = 0$

s^4	$1s^4 \quad 0s^2 \quad -1s^0$
s^3	~~$0^4 \quad 0^0 \quad 0$~~
s^2	$\cancel{0} \rightarrow \epsilon \quad -1$
s^1	$4/\epsilon$
s^0	-1

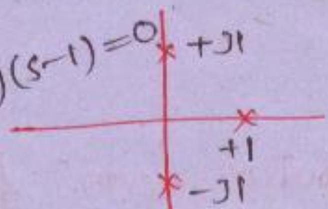




CE: $(s^2+1)^3 = 0$

$(s+1-j1)(s+1+j1)$

CE:
$(s^2+1)(s-1)=0$



$((s+1)^2+1)$

$(s^2+2s+2)(s^2-2s+2)=0$

Find the range of k value for system stability.

find the freq. of oscillations, if system is m.s.

find k value to become a system m.s.

(1). $s^3 + 5s^2 + 8s + k = 0$.

$(0 < k < 40)$

$8 \times 5 > k \times 1 \Rightarrow k < 40 \longrightarrow$ Stable.

$8 \times 5 = k \times 1 \Rightarrow k_{mar} = 40$.

for m.s, not consider s^0 coe. b'coz if s^0 coe. $= 0$ then row of zeros occurs. for this case if form the AE, which consists odd power of s terms, which is undesirable.

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 5 & k \\ s^1 & \frac{40-k}{5} & \\ s^0 & k & \end{array}$$

$\frac{40-k}{5} = 0 \rightarrow$ M.S.

$\frac{40-k}{5} > 0$, $k > 0$ $\rightarrow$ (S)

freq. of oscillations : AE : $5s^2 + 40 = 0$

$\Rightarrow s = \pm j\sqrt{8}$.

Q. $2s^3 + 9s^2 + 10s + (k+5) = 0$.

$90 > (k+5)2$ & $k+5 > 0$

$\Rightarrow -5 < k < 40 \longrightarrow$ stable.

$90 = (k+5)2$

$\Rightarrow k = 40 \longrightarrow$ M.S.

AE: $9s^2 + (\overset{40}{\cancel{k}}+5) = 0$

$\Rightarrow s = \pm j5 \longrightarrow$ freq of oscillations.

Q $GH = \dfrac{k}{s(s+2)(s+4)(s+6)}$

CE: $s(s+2)(s+4)(s+6) + k = 0$

$\Rightarrow s\left[s^3 + 12s^2 + 44s + \cancel{32}^{\,48}\right] + k = 0 \quad (8+12+24)$

$\Rightarrow s^4 + 12s^3 + 44s^2 + \cancel{32}^{\,48}s + k = 0.$

s^4	1	44	k
s^3	$\cancel{12}$ 1	$\cancel{48}$ 4	
s^2	40	K	
s^1	$\frac{160-k}{40} > 0$		
s^0	K >0		

AE: $40s^2 + 160 = 0$
$\rightarrow s = \pm j2$ (FOO)
$k_{mar} = 160$

$= 0 \rightarrow$ M.S

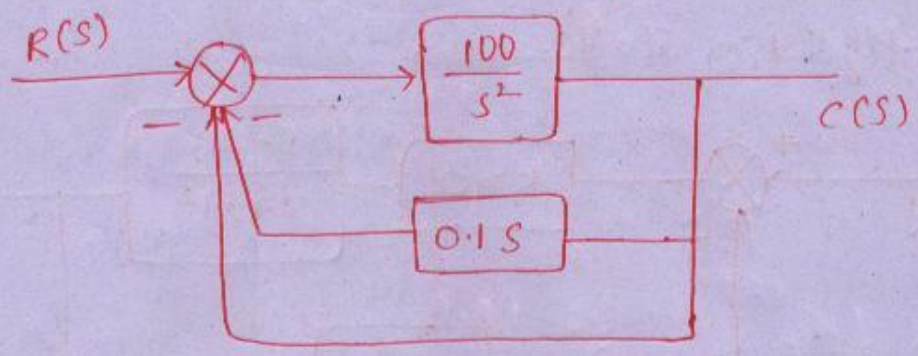
$\rightarrow$ Stable $= 0 < k < 160$

Q. Determine k & b, oscillates at a freq. of 2 rad/sec

$G(s) = \dfrac{k(s+1)}{s^3 + bs^2 + 3s + 1}$ & $H(s) = 1$.

The system oscillates at 2 rad/sec means system is M.S.

Q. find system stability — ?



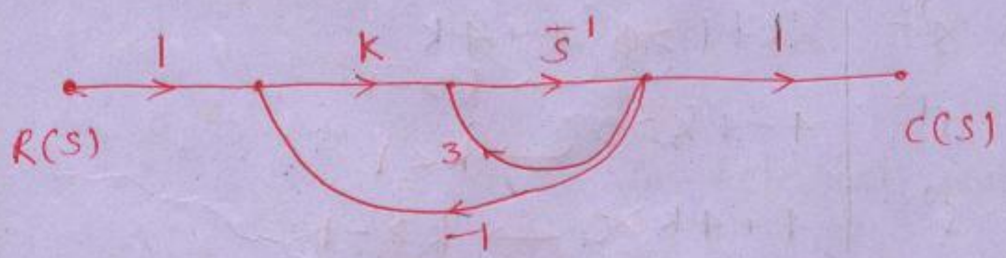
$$T/f = \frac{C(S)}{R(S)} = \frac{100}{s^2+10s+100}$$

$s^2+10s+100=0$.

a, b, c >0 $\Longrightarrow$ stable.

$$\begin{array}{c|c} s^2 & 1 \quad 100 \\ s^1 & 10 \\ s^0 & 100 \end{array} \Bigg\} +ve$$

Q. The system shown in fig. remain stable when (a). $k<-1$ (b). $k<1$ & $k>-1$ (c). $1<k<3$ (d). $k>3$.



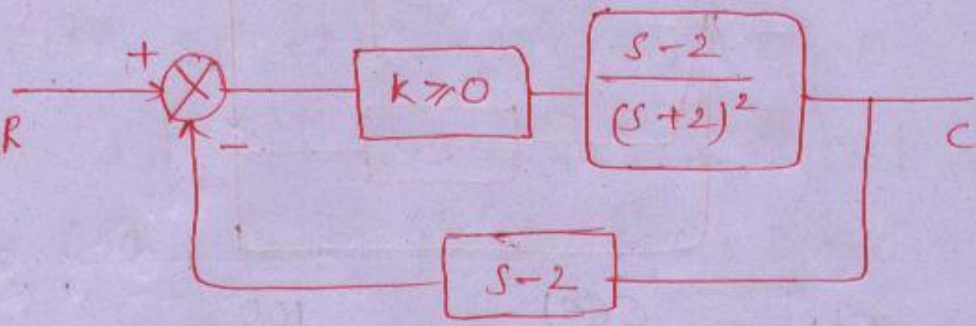
$$\frac{C}{R} = \frac{k/s}{1-3/s+\frac{k}{s}}$$

$$= \frac{k}{s-3+k}$$

$$\begin{array}{c|c} s^1 & 1 \\ s^0 & k-3>0 \end{array}$$

$\Longrightarrow k>3$.

Q. The -ve fb control system shown in fig.. is stable.



(a). ∀ k ≥ 0 (b). only if k ≥ 0

(c). only if 0 ≤ k < 1 (d). only if 0 ≤ k ≤ 1.

$$\frac{C}{R} = \frac{k(s-2)}{(s+2)^2 + k(s-2)^2}$$

CE: $s^2(1+k) + s(4-4k) + 4+4k = 0$

s^2	$k+1>0$ $\quad 4+4k$ $\quad \rightarrow k>-1$
s^1	$4-4k>0 \rightarrow k<1$
s^0	$4+4k>0 \rightarrow k>-1$

Ans: $0 \le k < 1$

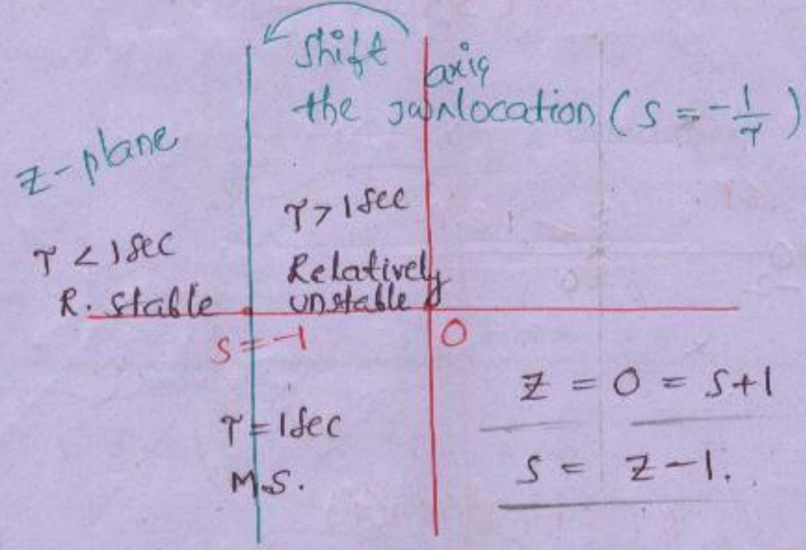
Relative stability is applicable only for c/l system.

Q. A system has $G(s) = \frac{2}{s(s+1)(s+2)}$

$H(s) = 1$. with RH criteria Determine

RS about the point or line $s = -1$.

CE: $s^3 + 3s^2 + 2s + 2 = 0 \rightarrow$ stable.



$Z = 0 = s+1$

$s = Z - 1$.

$\rightarrow (Z-1)^3 + 3(Z-1)^2 + 2(Z-1) + 2 = 0$

$\Rightarrow Z^3 - Z + 2 = 0 \longrightarrow$ Relatively unstable.

Z^3	1	-1
Z^2	$\cancel{0}\ \varepsilon$	2
Z^1	$\frac{-\varepsilon-2}{\varepsilon} \rightarrow -\infty$	
Z^0	2	

2 roots b/w 0 & -1.

1 root at left of $s = -1$

$\therefore$ Relatively unstable.

Q. Check whether the τ is greater or lesser or equal to 1 sec, for $s^3 + 7s^2 + 25s + 39 = 0$.

CE: $s^3 + 7s^2 + 25s + 39 = 0$.

$s = z +$ axis shift location $[s = -\frac{1}{\tau}]$

$s = -1$.

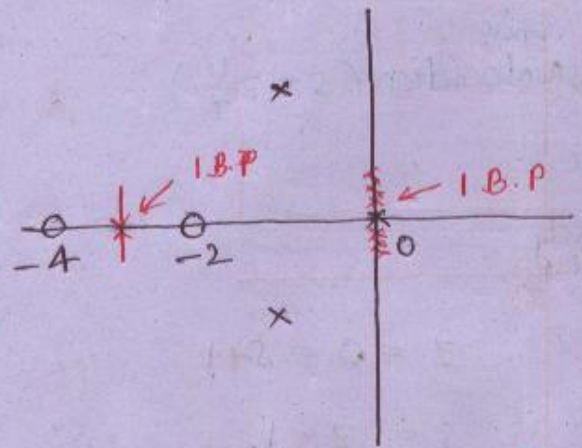
$s = z - 1$

$\Rightarrow z^3 + 4z^2 + 14z + 20 = 0$.

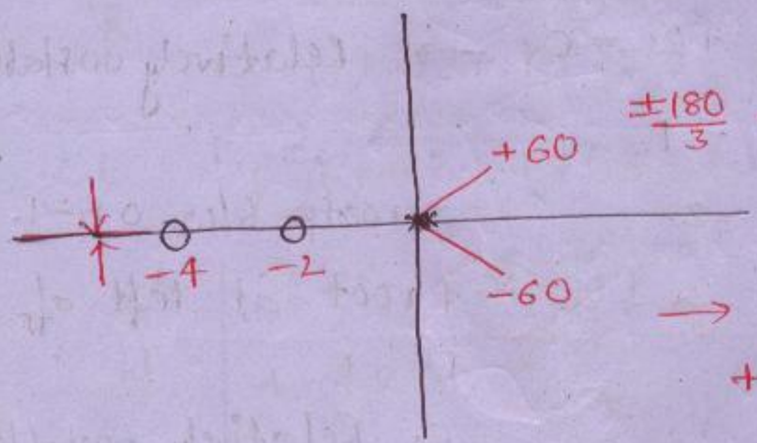
$\longrightarrow$ Relatively stable $\therefore \tau < 1$sec.

Q. Determine no. of break points.

$$GH(s) = \frac{K(s+2)(s+4)}{s^2(s^2+2s+2)}$$



$$GH(s) = \frac{K(s+2)(s+4)}{s^3}$$

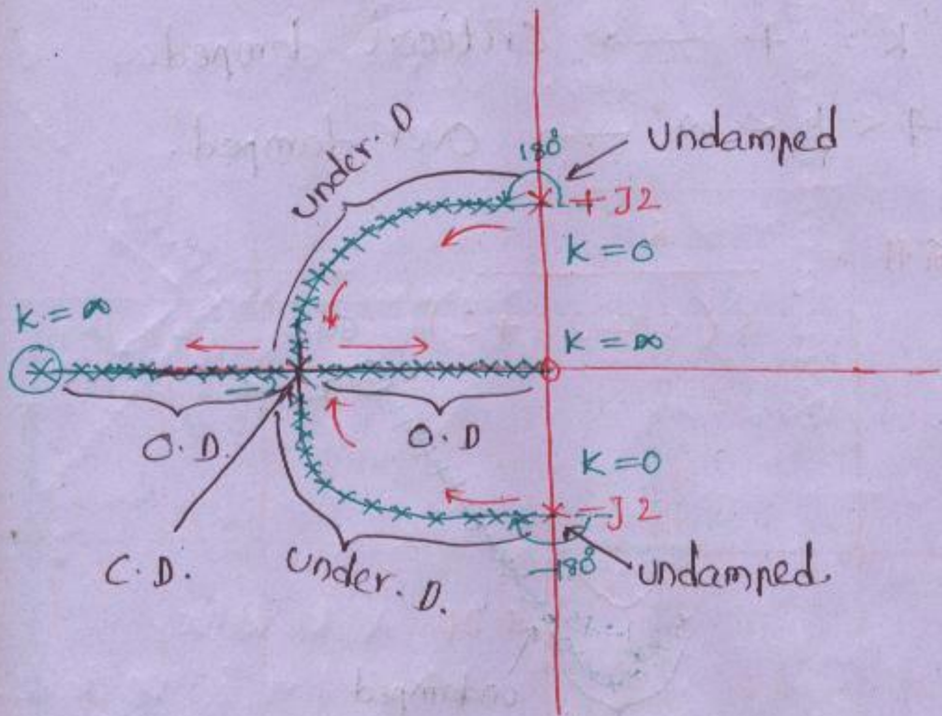


$\frac{\pm 180}{3} = \pm 60$

$\longrightarrow$ 2 B. points
+1 Break in

Q. $\frac{ks}{s^2+4} \longrightarrow$ Break point $= -2$.

$(\cancel{+2}, -2)$



Angle of departure $\phi_d : 180^\circ - \phi$

$$\phi = 90^\circ - 90^\circ = 0.$$

$$\therefore \phi_d = 180^\circ.$$

$k > 0 \longrightarrow$ stable.

$\Rightarrow \quad 0 < k < \infty \longrightarrow$ stable.

$k = 0 \longrightarrow$ M.S.

when $k = 0 \longrightarrow$ UNDAMPED

$$\left| \frac{ks}{s^2+4} \right|_{s=-2} = 1 \longrightarrow$$ {for finding, k value at break point.}

By using magnitude condition.

$\Rightarrow \quad k = 4$

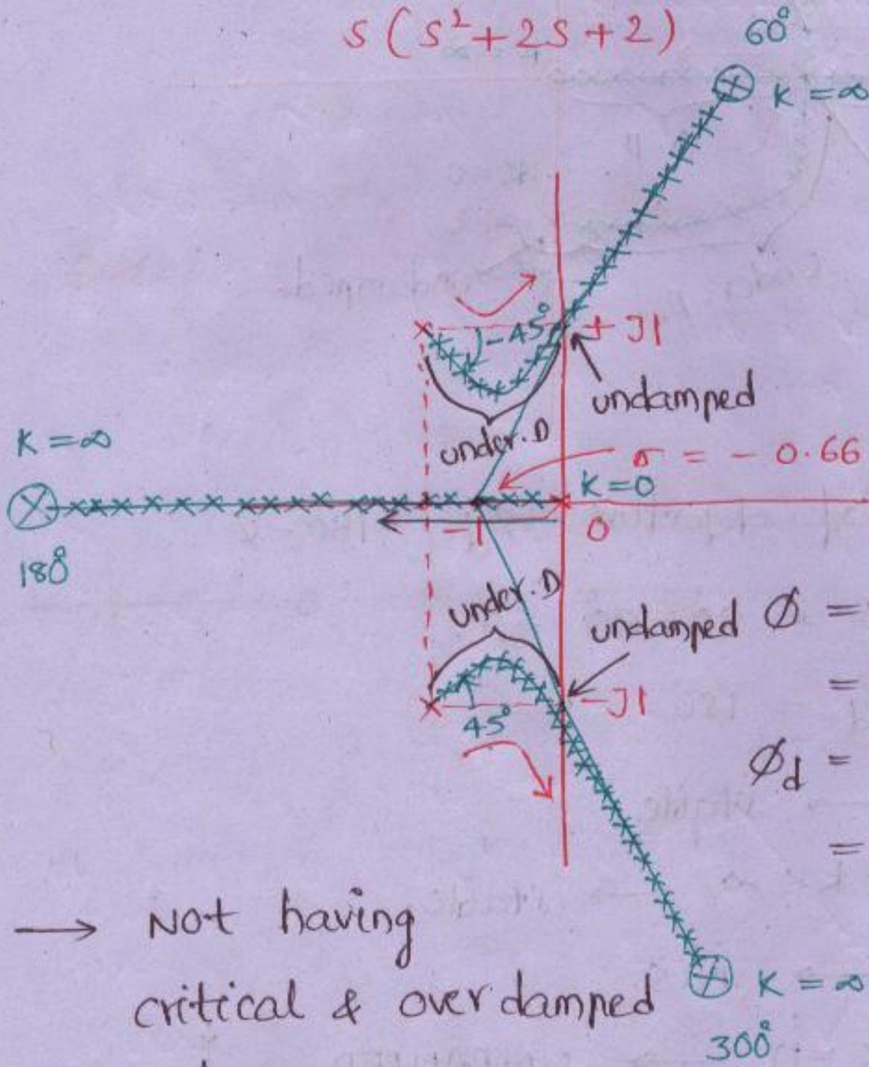
$\therefore$ $k = 0 \longrightarrow$ undamped.

$0 < k < 4 \longrightarrow$ under damped.

$k = 4 \longrightarrow$ critical damped

$4 < k < \infty \longrightarrow$ over damped

Q. $GH = \dfrac{k}{s(s^2+2s+2)}$



$\phi = 135 + 90$

$= 225$

$\phi_d = 180 - 225$

$= -45°$.

$\longrightarrow$ Not having critical & over damped natures.

CE: $s^3 + 2s^2 + 2s + k = 0$

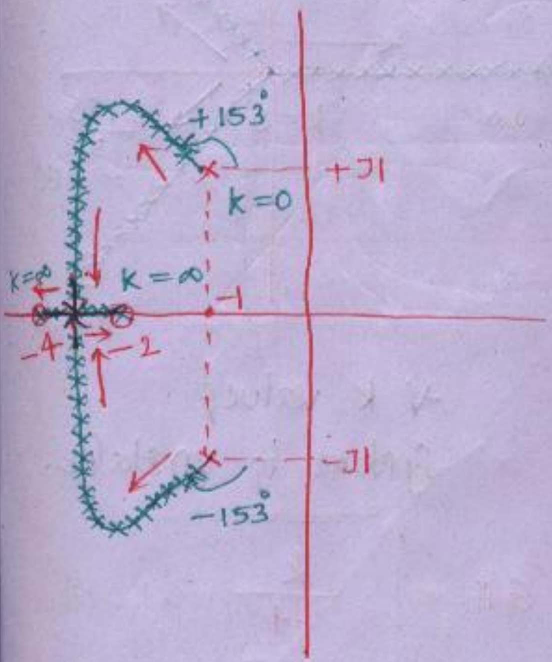
$k_{max} = 4$

$0 < k < 4 \longrightarrow$ under damped (stable)

$k = 4 \longrightarrow$ undamped (M.S)

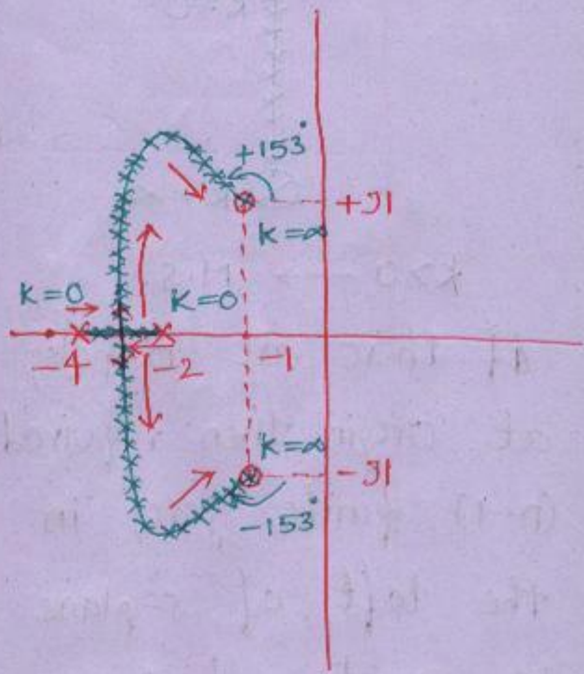
Q.

$$GH = \frac{k(s+2)(s+4)}{(s^2+2s+2)}$$



Q.

$$GH = \frac{k(s^2+2s+2)}{(s+2)(s+4)}$$



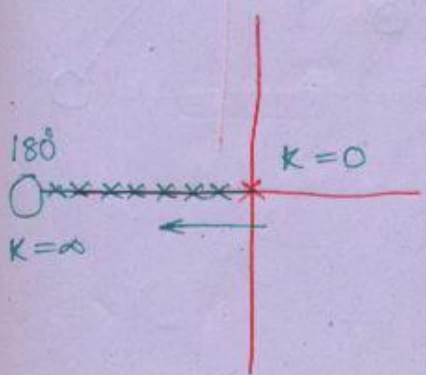
Q. $\frac{k}{s}$, $\frac{k}{s^2}$

NOTE: whenever T/F consists pole at origin then RL diagram is nothing but angle of Asymptote line.

$GH = \frac{k}{s}$

No. of Asymptotes (N) = 1

$(\theta) = 180°$

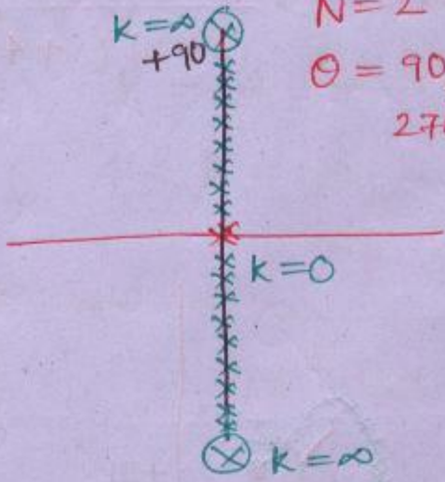


$k > 0 \longrightarrow$ stable.

$$GH = \frac{k}{s^2}$$

$N = 2$

$\theta = 90, 270.$

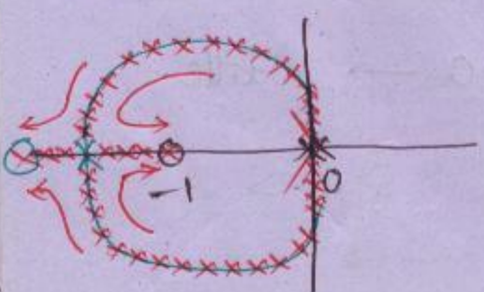


$k > 0 \longrightarrow$ M.S.

If there are n poles at origin then required (n−1) finite zeros in the left of s-plane to avoid affect on stability.

So by adding one finite zero, the above system becomes stable.

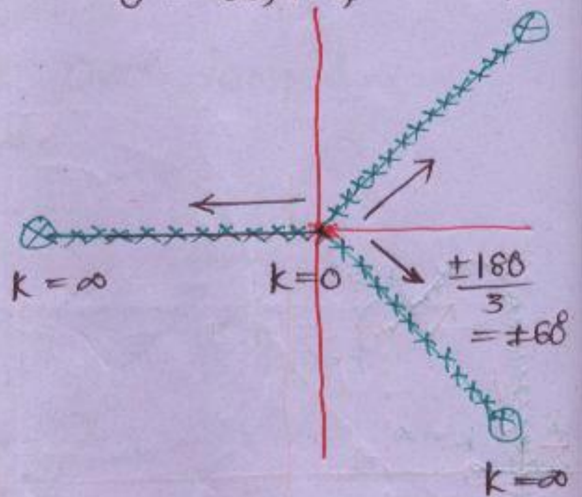
eg: $GH = \dfrac{k(s+1)}{s^2}$



$$GH = \frac{k}{s^3}$$

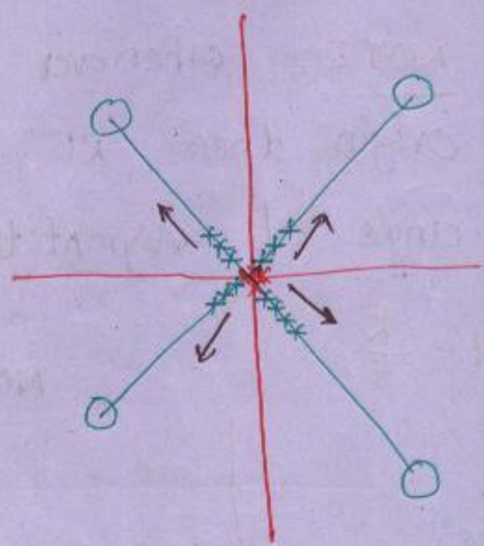
$N = 3.$

$\theta = 60, 180, 300$



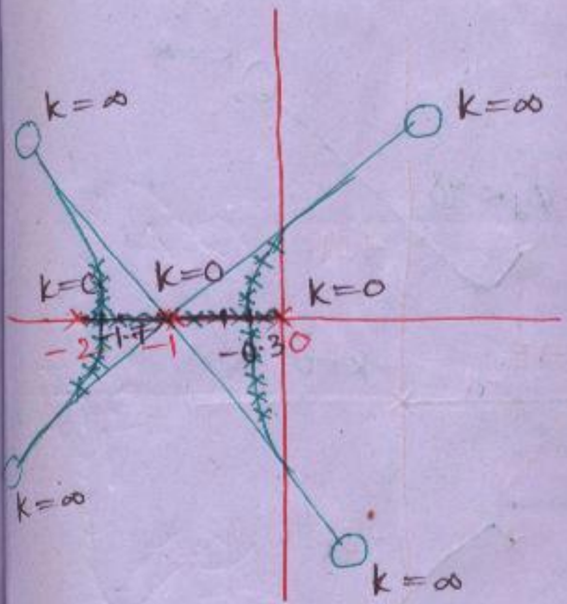
∀ k values system is unstable.

$$GH = \frac{k}{s^4}$$



Q.

$$GH = \frac{k}{s(s+1)^2(s+2)}$$



Break points :

$$s(s+1)^2(s+2) = 0$$

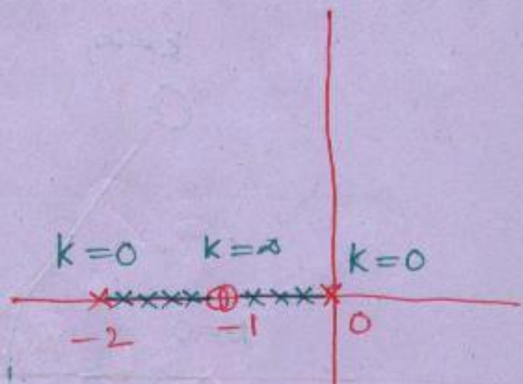
$$\Rightarrow 4s^3 + 12s^2 + 10s + 2 = 0$$

3 B.P's.

$\rightarrow$ $-0.3, -1, -1.7$

Q.

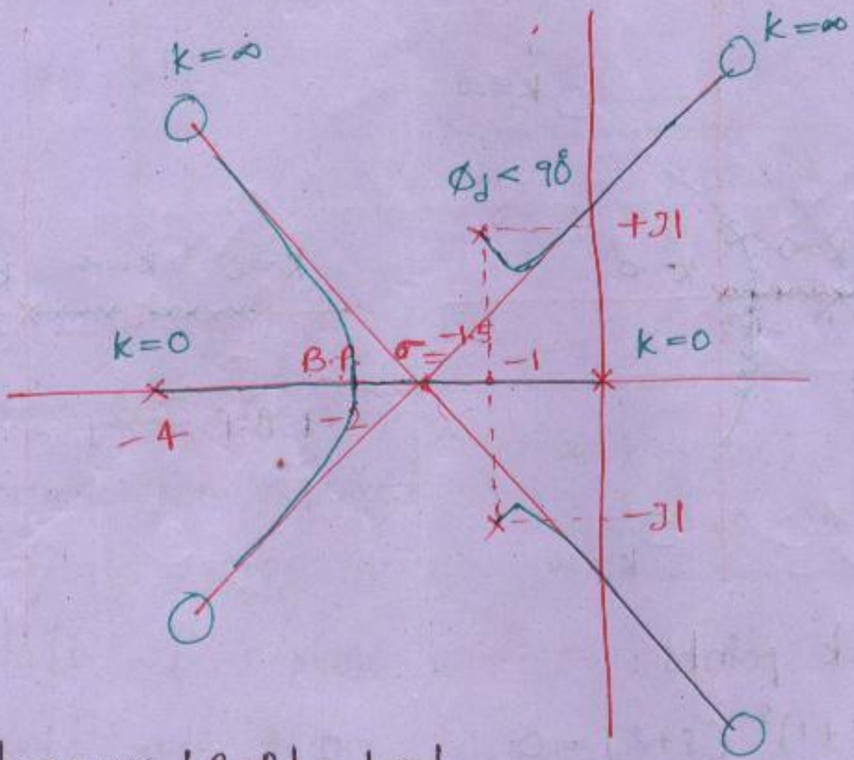
$$GH = \frac{k(s+1)^2}{s(s+2)}$$



1 B.P = -1

Q. $GH = \dfrac{K}{s(s+k_1)(s^2+2s+2)}$

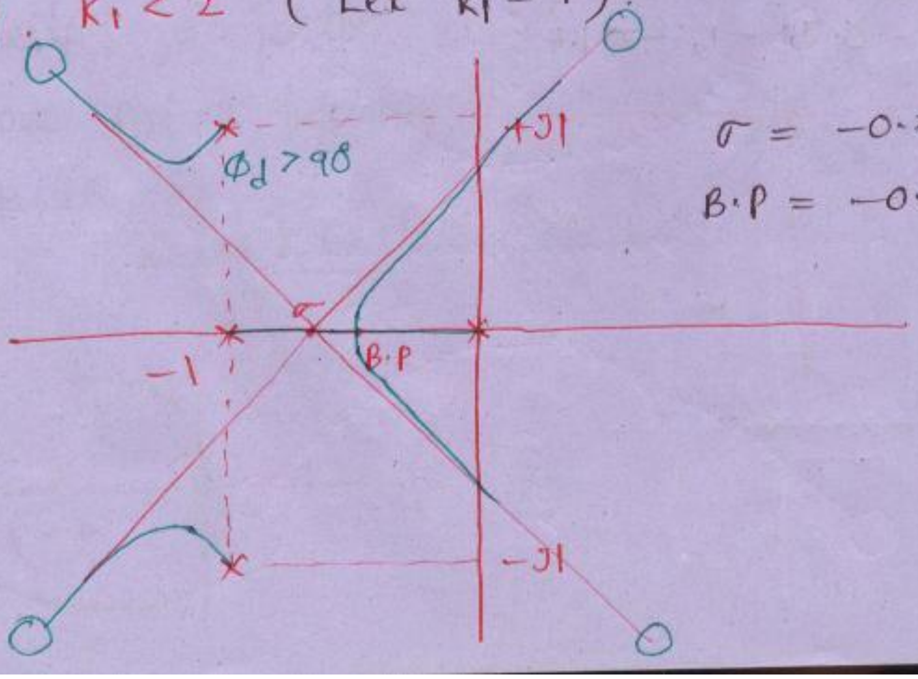
(i). $k_1 > 2$ (Let $k_1 = 4$) ; B.P $= -2$.



whenever $|B.P| > |\sigma|$

then $\phi_d < \mp 90°$.

(ii). $k_1 < 2$ (Let $k_1 = 1$).



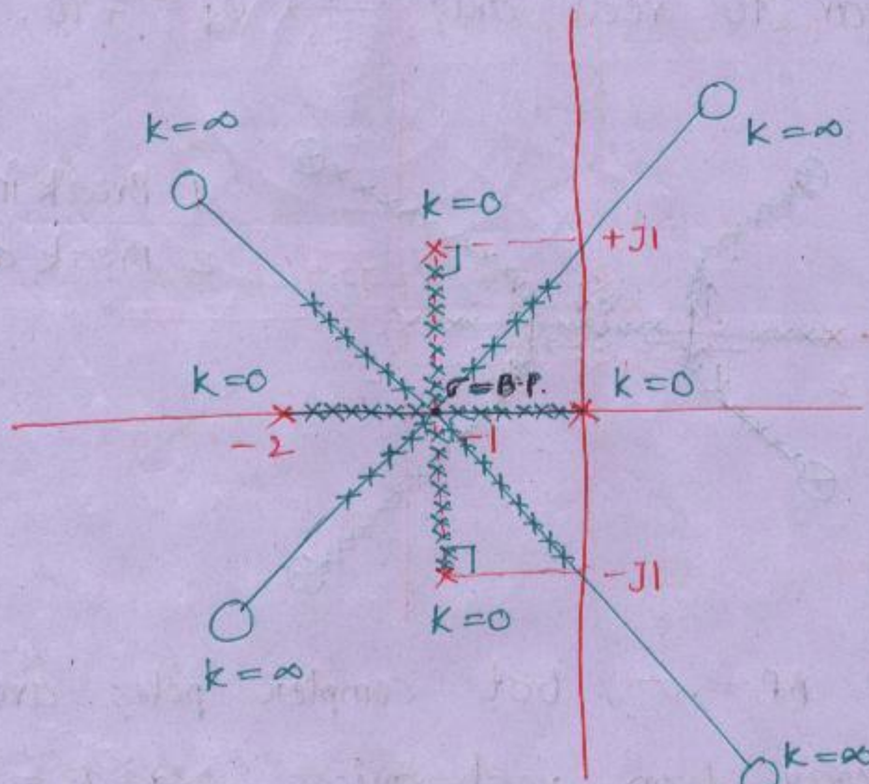
$\sigma = -0.75$

B.P $= -0.5$

whenever $|B.P| < |\sigma|$ then $\phi_d > \mp 90°$.

(iii). $k_1 = 2$.

$$GH = \frac{k}{s(s+2)(s^2+2s+2)}$$



whenever $B.P = \sigma$ then $\phi_d = \mp 90°$.

In above system, the no. of poles meet at $B.P = 4$.

The k value at B.P is 1.

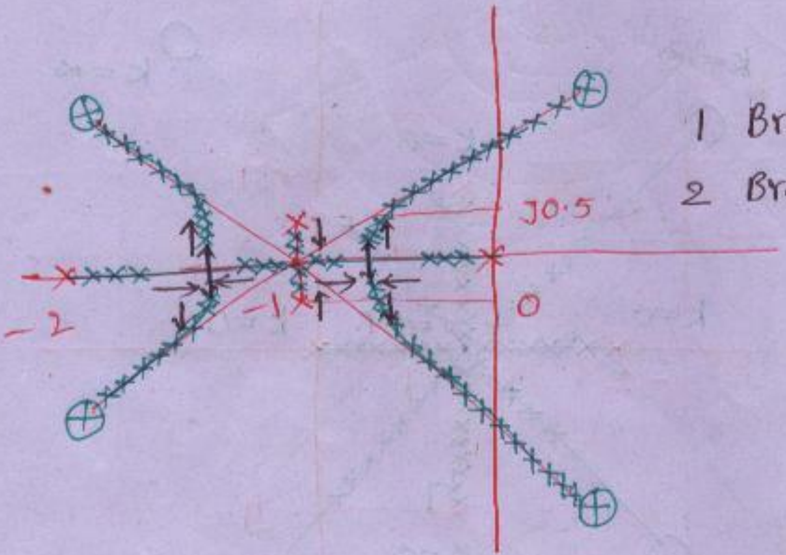
$$\left| \frac{k}{s(s+2)(s^2+2s+2)} \right|_{s=-1} = 1$$

$\rightarrow k = 1$.

The C/L TF at B.P is $\frac{1}{(S+1)^4}$

$$\left\{ \frac{1}{S(S+2)(S^2+2S+2)+1} \right\}$$

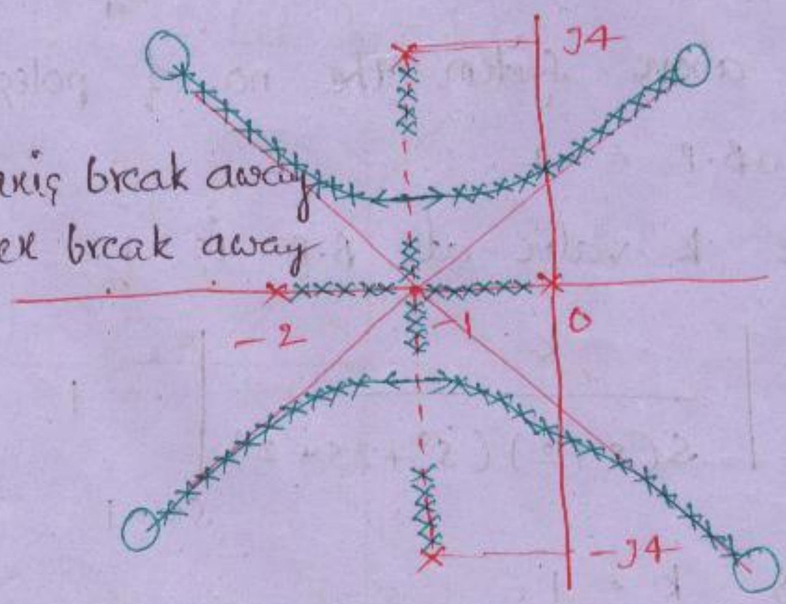
** B.P = σ, but complex poles very near to real axis $\longrightarrow \phi_d = \mp 90$.



1 Break in
2 Break away

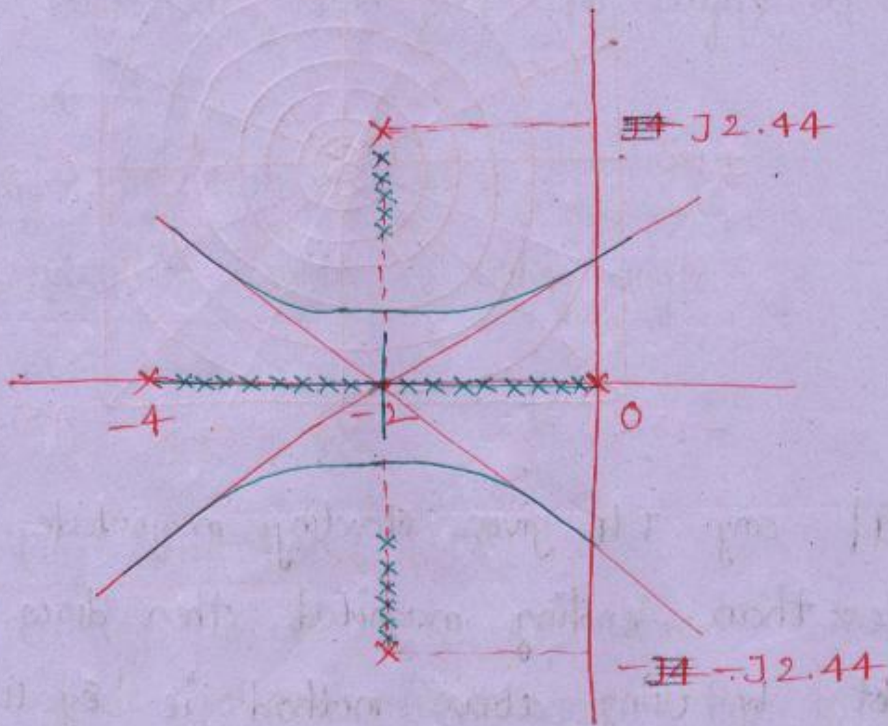
** BP = σ, but complex poles are away from real axis. $\longrightarrow \phi_d = \mp 90$

1 real axis break away
2 complex break away



whenever centroid = real part of a complex pole

Q. Draw RL for $GH = \frac{k}{S(S+4)(S^2+4S+20)}$



$S^4 + 8S^3 + 36S^2 + 80S$

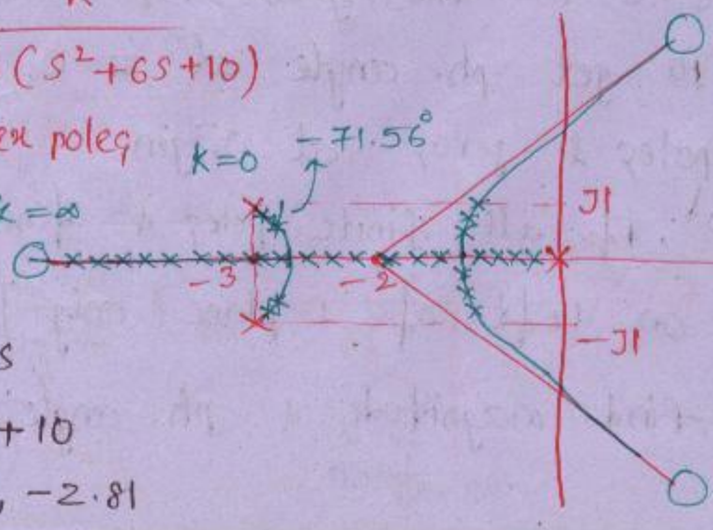
$\xrightarrow{d} 4S^3 + 24S^2 + 80 = 0$

$\longrightarrow S = -2 \pm J2.44, -2$

$\sigma = B.P = -2.$

Q. $GH = \frac{K}{S(S^2+6S+10)}$

BP ≠ σ & complex poles near to real axis.



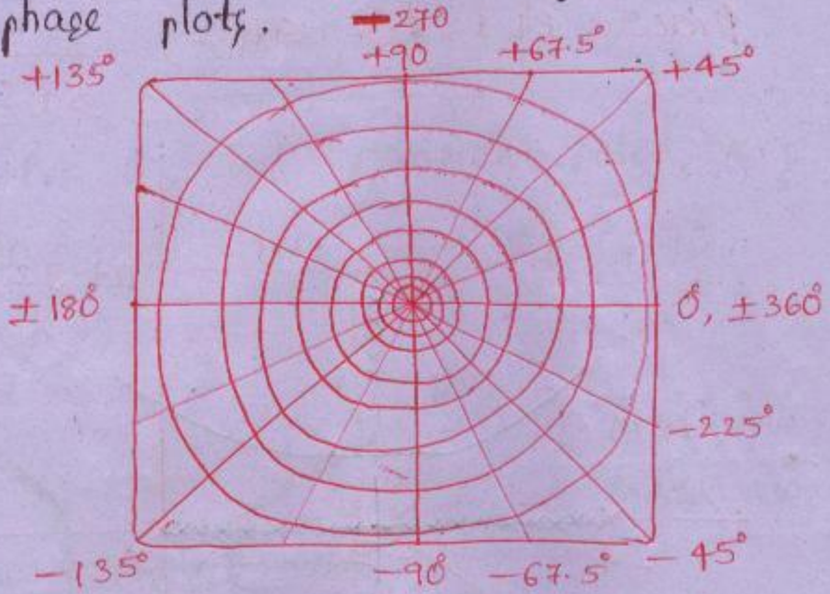
$S^3 + 6S^2 + 10S$

$\xrightarrow{d} 3S^2 + 12S + 10$

$\longrightarrow S = -1.18, -2.81$

POLAR PLOTS:

→ polar plots are nothing but Magnitude Vs phase plots.



If any T/F gives starting magnitude is less than ending magnitude then draw polar plot by using above method ie Eg (i) & (ii) $\left\{GH = \frac{1}{1+ST}\right\}$ in old notes.

PROCEDURE TO DRAW POLAR PLOT :-

* find magnitude & ph. angle at $\omega = 0$.

To get magnitude at $\omega=0$, simply substitute $s=0$ in the given T/F.

To get ph. angle at $\omega=0$, consider no. of poles & zeros at origin.

[valid, If all finite poles & finite zeros lies on left of s-plane only].

* find magnitude & ph. angle at $\omega=\infty$.

To find magnitude at $\omega = \infty$, simply substitute $s = \infty$ in the given T/F.

To get ph. angle at $\omega = \infty$, consider the algebraic sum of ph. angles of all poles & zeros.

* Ending direction : [ED]

= starting angle − ending angle

= +ve $\longrightarrow$ cw

= −ve $\longrightarrow$ ccw

* Starting direction : [SD]

Starting direction is considered when all finite poles & finite zeros lies in the 1st quadrant only.

If finite pole is near to ima. axis, the plot push towards cw dirc.

If finite zero is near to ima. axis then plot push towards ccw dirc.

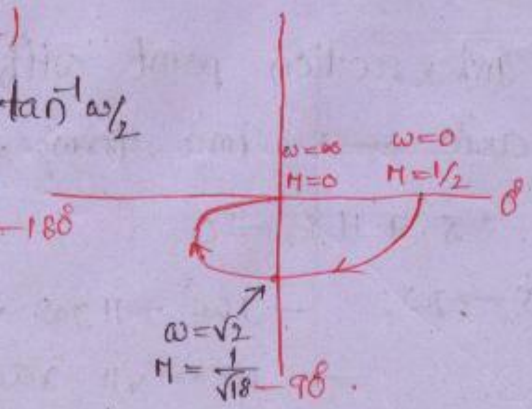
Q. $GH = \dfrac{1}{(s+1)(s+2)}$

$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\omega/2$

$90 = \tan^{-1}\left(\dfrac{\omega + \omega/2}{1 - \omega^2/2}\right)$

$\Rightarrow \infty = \dfrac{\omega + \omega/2}{\left(1 - \frac{\omega^2}{2}\right)} \rightarrow = 0$

$\Longrightarrow \omega = \sqrt{2}$.



$$M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{(1+\omega^2)(4+\omega^2)}}$$

$$= \frac{1}{\sqrt{18}}$$

Intersection point $= \left(0, -\frac{j}{\sqrt{18}}\right)$

$$\frac{K}{(1+sT_1)(1+sT_2)} \; ; \quad M = \frac{K\sqrt{T_1 T_2}}{T_1+T_2}$$

$$= \frac{0.5\sqrt{1\times 1/2}}{1+1/2}$$

$$= \frac{1}{\sqrt{18}}.$$

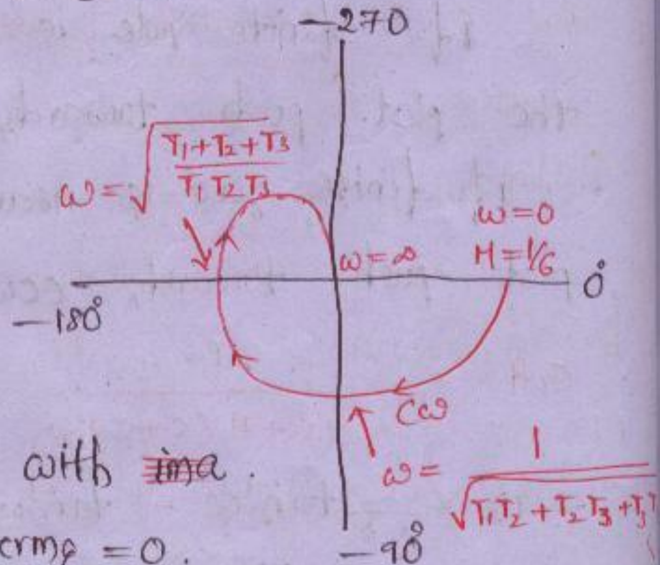
Q. $GH = \dfrac{1}{(s+1)(s+2)(s+3)}$

At $\omega = 0$, $\frac{1}{6}\angle 0^\circ$.

At $\omega = \infty$, $0\angle -270^\circ$

ED: $\longrightarrow$ CW

SD: $\longrightarrow$ CW



Expand terms:

$s^3 + 6s^2 + 11s + 6$

Intersection point with ~~ima~~ real axis $\longrightarrow$ ima. terms $= 0$.

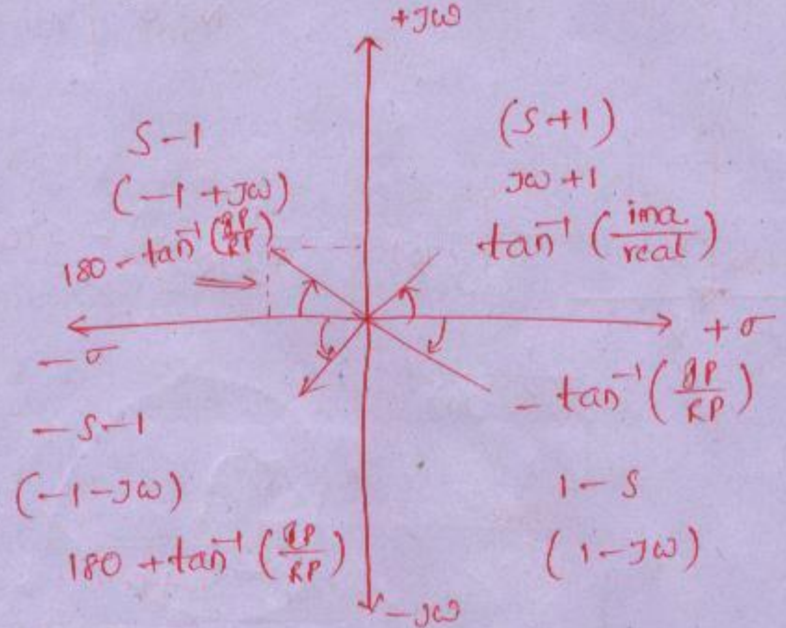
$s^3 + 11s = 0$

$s \to j\omega$; $\quad -j\omega^3 + 11j\omega = 0$

$\Rightarrow \omega = \sqrt{11}$ rad/sec.

$$M = \frac{1}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}}\Bigg|_{\omega=\sqrt{11}}$$

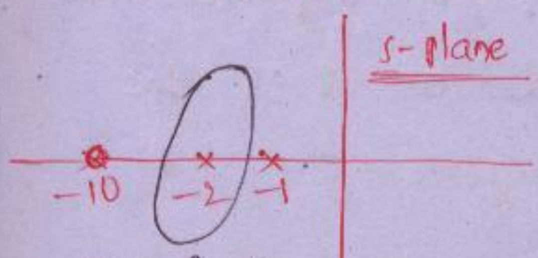
$$= \frac{1}{60}.$$



NYQUIST PLOTS:

* To draw complete freq. response of O/L T.F.
* To findout C/L system stability.
* To findout no. of C/L poles in the right of s-plane.
* To find range of k-value for system stability.
* To find gain margin, ph. margin, ω_{gc}, ω_{pc}.

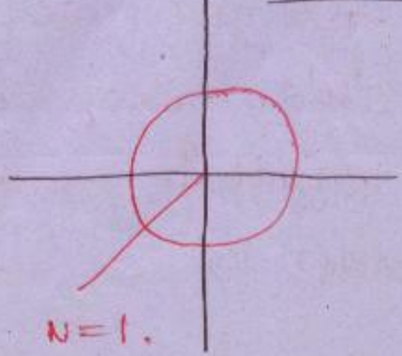
$$GH = \frac{(S+10)}{(S+1)(S+2)}$$

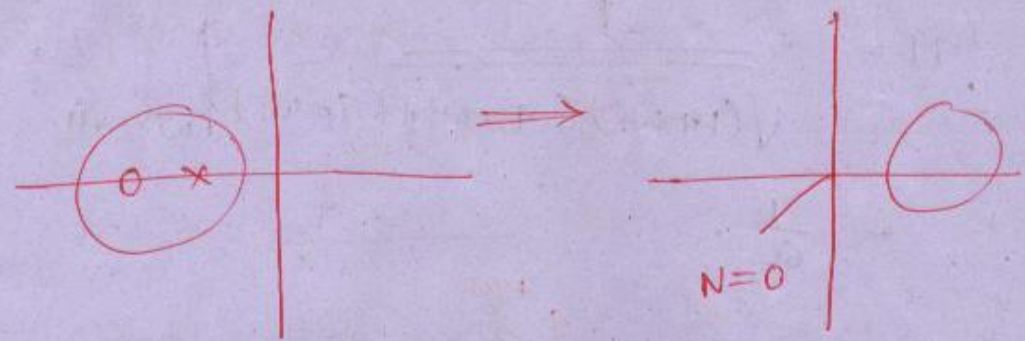


$N = P - Z$

$= 1$

G(S)H(S) plane





$+j0$: $\quad 1/s^3 = \frac{1}{3(\angle +j0)} = \frac{1}{+270} = -270^\circ$.

$-j0$: $\quad 1/s^3 = \frac{1}{3(\angle -j0)} = \frac{1}{-270^\circ} = +270^\circ$

$$540 = (180 \times 3)$$

→ no. of infinite radius half circle's = no. of poles at origin.

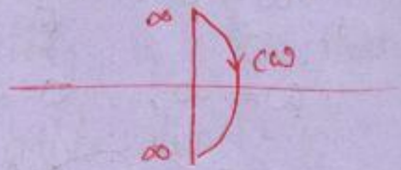
$+j0$: $\quad 1/s = \frac{1}{\angle +j0} = \frac{1}{+90} = -90^\circ$

$-j0$: $\quad 1/s = \frac{1}{\angle -j0} = \frac{1}{-90} = +90$

$$180 = (180 \times 1)$$

→ for zeros we dont get infinite radius circle's. b'coz at $\omega = 0-$ & $0+$ the magnitude becomes zero.

→ The infinite radius half c'le dirc. always in the ccw b'coz the infinite radius half c'le dirc. completely depends on Nyquist contour dirc.



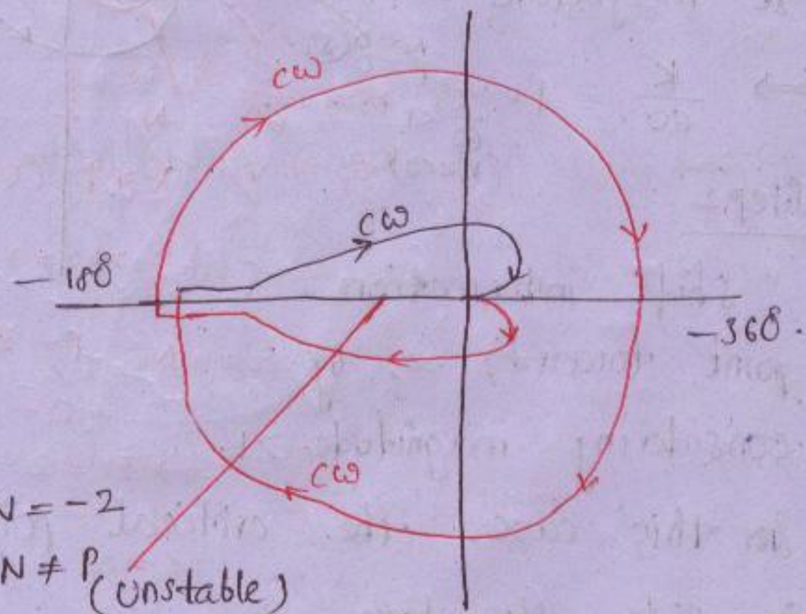
Q. $GH = \frac{10}{s^2(s+1)(s+2)}$ → OL (P=0).

$\omega = 0 \rightarrow \infty \angle -180^\circ$

$\omega = \infty \Rightarrow 0 \angle -360$

ED → cω

SD → cω.



The no. of cl poles given in RH s-plane is given by principle of arguments.

$N = P - Z$

$\Rightarrow -2 = 0 - Z \Rightarrow Z = 2$ [cl poles in RH s plane].

Q. find range of k-value for system stability. $GH = \frac{k}{(s+1)(s+2)(s+3)}$

$\omega = 0, \quad \frac{k}{6} \angle 0$

$\omega = \infty, \quad 0 \angle -270$

ED ⟶ CW

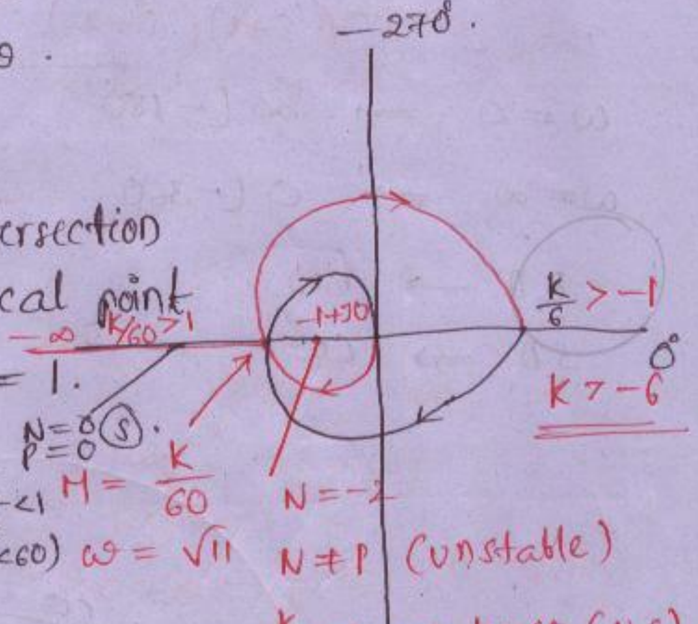
SD ⟶ CW

Step 1 :

Assume intersection point = critical point ie magnitude = 1.

$\Rightarrow \frac{k}{60} = 1$. $\frac{k}{60} < 1$ $(k < 60)$ $M = \frac{k}{60}$ $\omega = \sqrt{11}$

$N = 0$ (S). $P = 0$



$N = -2$

$N \neq P$ (unstable)

$\frac{k}{60} > 1 \Rightarrow k > 60$ (U.S).

$N = P - Z$

$-2 = 0 - Z$

$\Rightarrow Z = 2$ CL poles on RH s-plane.

Step 2 :

shift intersection point towards ∞, by considering magnitude >1. In this case, the critical point lies in side the loop.

for this case find no. of encirclements and get one condi. for stability.

Step 3:

shift intersection point towards origin by considering mag. < 1, in this case critical point is outside the loop, for this case find no. of encirclements and get the condition for stability.

If the condi. for stability is less than certain value then the other limit is given by intersection point with 0° by considering magnitude of intersection point > -1.

$\Longrightarrow \quad -6 < k < 60 \longrightarrow$ stable.

Q. $GH(s) = \dfrac{k(s+2)}{(s+1)(s-1)} \longrightarrow P = 1$

$\phi = -\tan^{-1}(\omega) - (180 - \tan^{-1}\omega) + \tan^{-1}\frac{\omega}{2}$.

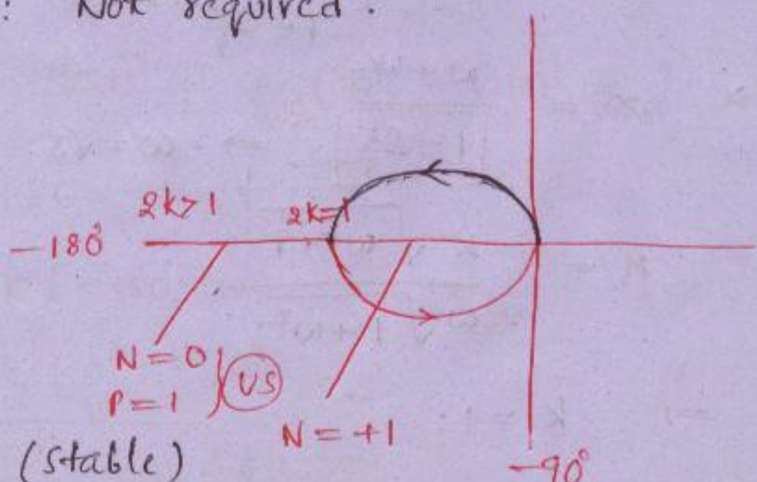
$= -180 + \tan^{-1}\frac{\omega}{2}$.

$\omega = 0$; $\quad 2k \angle -180°$

$\omega = \infty$; $\quad 0 \angle -90°$

ED : ACW

SD : Not required.



$N = P$

$2k > 1$

$\longrightarrow k > 1/2$ (stable)

$N = P - Z$

$0 = 1 - Z$

$\Rightarrow Z = 1$ (CL RH pole).

Q. $GH = \dfrac{K(s+3)}{s(s-1)}$ → $\left.\begin{array}{l} P=1 \\ N=1 \end{array}\right\}$ (S)

(1) half ole. $K > 1 \rightarrow$ (S).

$\phi = -90 - (180 - \tan^{-1}\omega) + \tan^{-1}\left(\frac{\omega}{3}\right)$

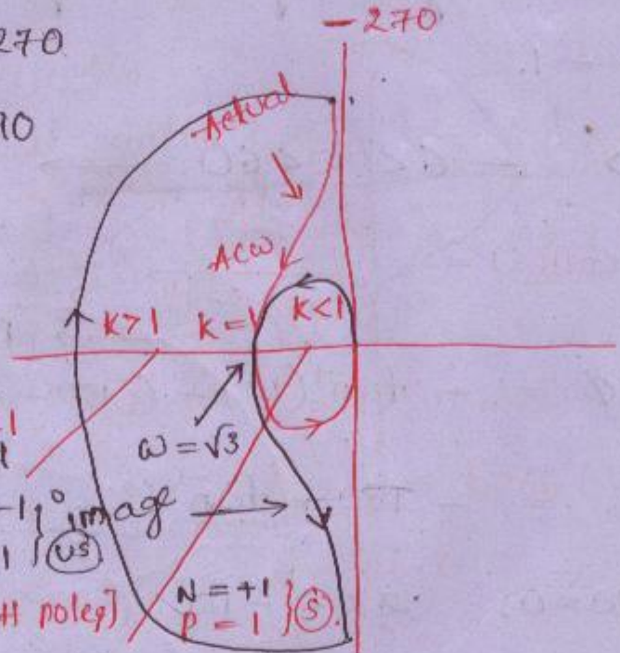
$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/3$.

$\omega = 0;\quad \infty \angle -270$

$\omega = \infty;\quad 0 \angle -90$

ED : ACW

SD : X



K<1

N = -1 } image

P = 1 } (US)

$N = P - Z$

$\Rightarrow -1 = 1 - Z$

$\Rightarrow Z = 2$ [2 CL RH poles]

$-180 = -270 + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}$ -90

$90 = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}$

$\Rightarrow 90 = \tan^{-1}\left[\dfrac{\omega + \omega/3}{1 - \frac{\omega^2}{3}}\right]$

$\Rightarrow \infty = \dfrac{\omega + \omega/3}{\boxed{1 - \frac{\omega^2}{3}}} \rightarrow 0 \Rightarrow \omega = \sqrt{3}$.

$M = \dfrac{K\sqrt{\omega^2 + 9}}{\omega\sqrt{1 + \omega^2}}$

$\Rightarrow K = 1$.

Q. $GH = \frac{K(S+2)}{(S-2)} \longrightarrow \left.\begin{matrix} P=1 \\ N=1 \end{matrix}\right\}$ (S)

K>1 (S).

$\phi = -(180 - \tan^{-1}\frac{\omega}{2}) + \tan^{-1}\frac{\omega}{2}$.

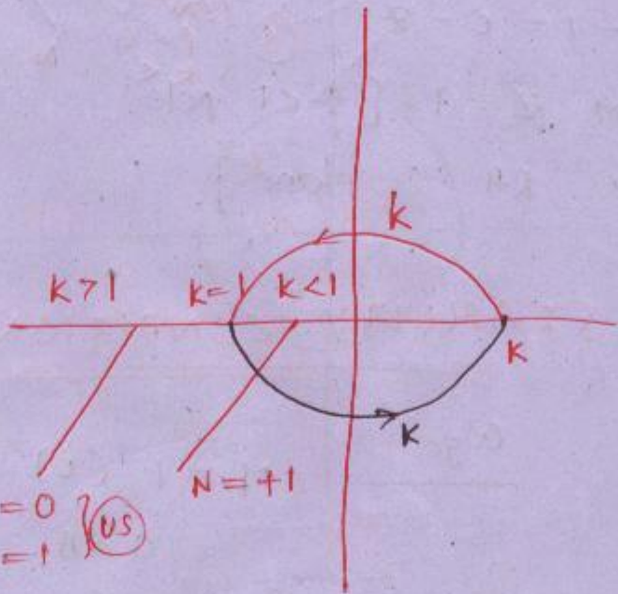
$= -180 + 2.\tan^{-1}\frac{\omega}{2}$.

$\omega = 0; \quad K\angle{-180}$

$\omega = \infty; \quad K\angle{0}$

ED: ACW

SD: ×



$\left.\begin{matrix} N=0 \\ P=1 \end{matrix}\right\}$ (US)

$N = P - Z$

$0 = 1 - Z$

$\Rightarrow Z = 1$ [1 CL pole in RH S-plane]

Q. $GH = \frac{K(S-2)}{(S+2)}$

$\phi = -\tan^{-1}\frac{\omega}{2} + (180 - \tan^{-1}\frac{\omega}{2})$

$= 180 - 2.\tan^{-1}\frac{\omega}{2}$.

$\omega = 0; \quad K\angle{-180^\circ}$

$\omega = \infty; \quad K\angle{0^\circ}$

ED: CW SD: ×.

$$k\,\frac{(s-2)}{(s+2)} \longrightarrow P = 0$$

$N = -1$

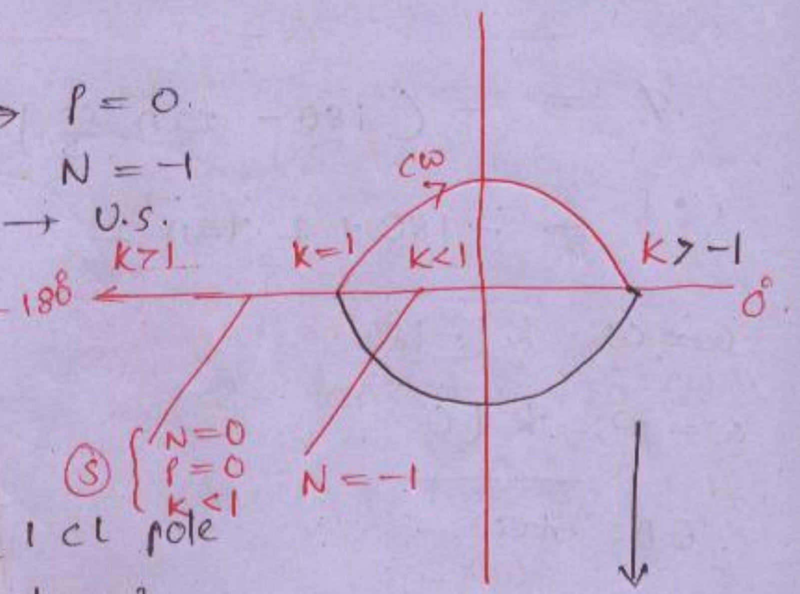
$\Rightarrow N \neq P \rightarrow$ U.S.

$K > 1 \rightarrow$ U.S

$N = P - Z$

$-1 = 0 - Z$

$\Rightarrow Z = 1$ [1 CL pole RH s-plane].



$-1 < K < 1$

* 29/12/08 *

STABILITY CONDITIONS:

$\omega_{gc} \longrightarrow M = 1$ (L)

$= 0$ dB.

$\omega_{pc} \longrightarrow -180^\circ$.

$$GM = \frac{1}{|M|\big|_{\omega=\omega_{pc}}} \quad (L)$$

$$= -20\log |GH(j\omega)|\big|_{\omega=\omega_{pc}}.$$

$$PM = 180 + \angle GH\big|_{\omega=\omega_{gc}}$$

$\omega_{pc} > \omega_{gc} \longrightarrow$ S $\left.\begin{array}{l} GM>1 \\ +\text{ve dB}\end{array}\right\}$ PM +ve

$\omega_{pc} = \omega_{gc} \longrightarrow M.S$ $\quad$ $\left.\begin{matrix} GM = 1 \\ 0\,dB \end{matrix}\right\}$ $PM = 0$.

$\omega_{pc} < \omega_{gc} \longrightarrow U.S$ $\quad$ $\left.\begin{matrix} GM < 1 \ (L) \\ -ve \text{ in } dB \end{matrix}\right\}$ PM -ve

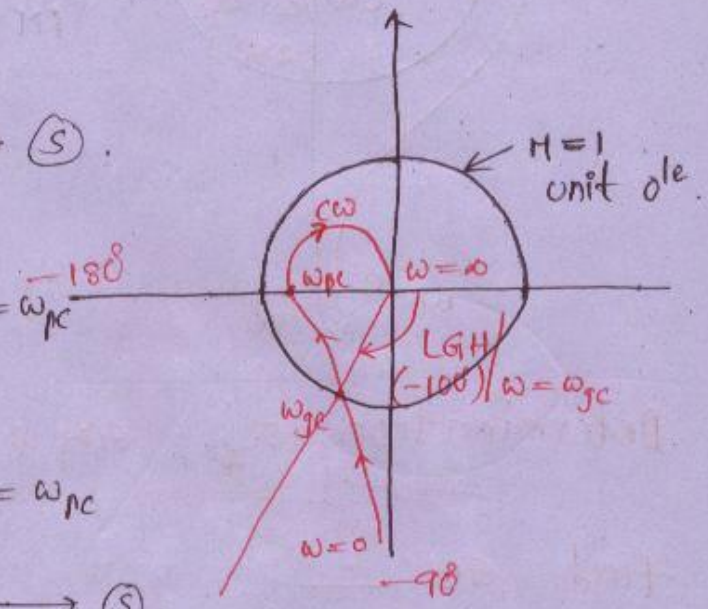
Q. Identify stability.

$\omega_{pc} > \omega_{gc} \longrightarrow$ (S).

$$GM = \frac{1}{|M|/_{\omega=\omega_{pc}}}$$

$$= \frac{1}{<1/_{\omega=\omega_{pc}}}$$

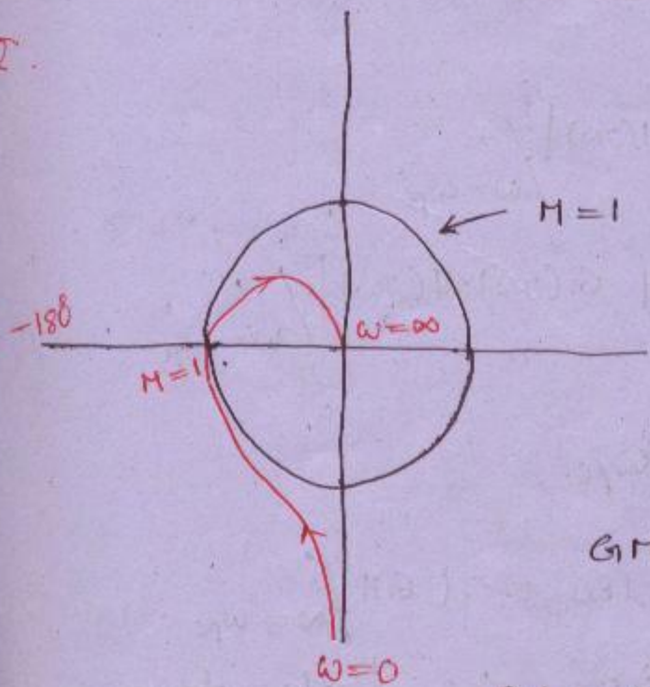
$\Longrightarrow$ $GM > 1 \longrightarrow$ (S).



$PM = 180 - 100$

$= +80$

Q.

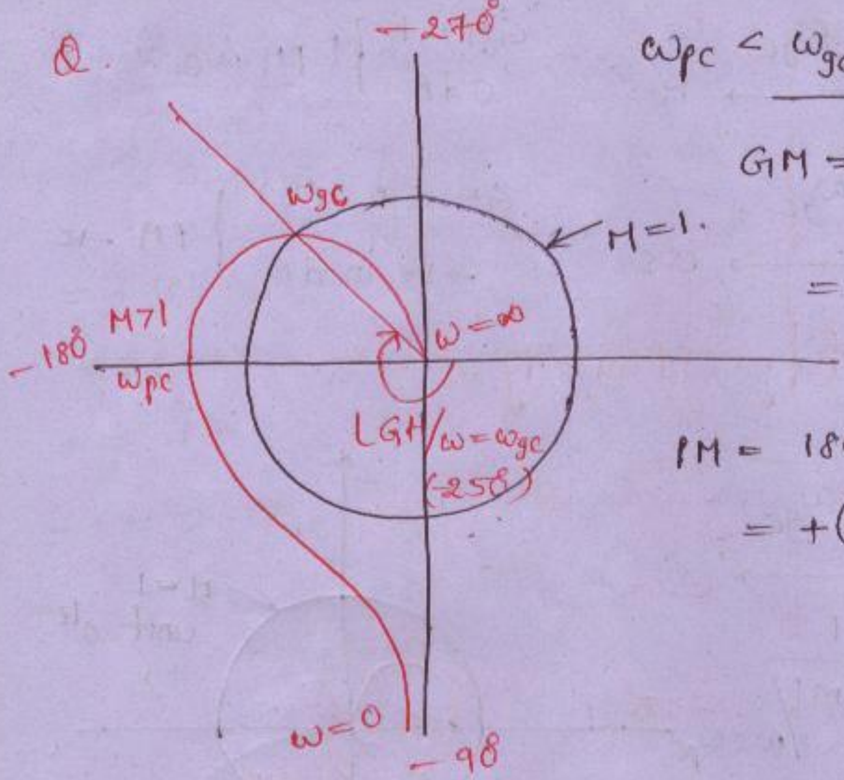


$\omega_{pc} = \omega_{gc} \longrightarrow M.S.$

$$GM = \frac{1}{1/_{\omega=\omega_{pc}}} = 1$$

$PM = 180 - 180 = 0°$.

Q.



$\omega_{pc} < \omega_{gc} \longrightarrow$ U.S.

$$GM = \frac{1}{M/_{\omega=\omega_{pc}}}$$

$$= \frac{1}{>1} = <1 \text{ (U.S)}$$

$$PM = 180 - 250°$$

$$= +(-70) \text{ (U.S)}$$

Determination of GM & PM:

find GM —

$$GH = \frac{1}{s(s+1)(s+2)}$$

$$GM = \frac{1}{|G(j\omega)H(j\omega)|/_{\omega=\omega_{pc}}}$$

$$= -20\log|G(j\omega)H(j\omega)|/_{\omega=\omega_{pc}}$$

Step 1 : find ω_{pc},

Method 1 : $-180 = \angle GH/_{\omega=\omega_{pc}}$

if T/F consists ≤ 2 finite terms.

(or) T/F consists exponential terms.

Method 2 :

Expand the terms and make odd power s - terms = 0. [if T/F consists ≥ 3 finite terms].

$$-180 = \angle GH \Big/_{\omega = \omega_{pc}}$$

$$\Rightarrow -180 = -90 - \tan^{-1}\omega - \tan^{-1}\omega/_2$$

$$\Rightarrow 90 = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2}$$

$$\Rightarrow 90 = \tan^{-1}\left[\frac{\omega + \omega/_2}{1 - \omega^2/_2}\right]$$

∞ $\rightarrow 0$

$$\Rightarrow \omega_{pc} = \sqrt{2}.$$

$$M = \frac{1}{\omega\sqrt{(1+\omega^2)(4+\omega^2)}} \Big/_{\omega_{pc} = \sqrt{2}}$$

$$= \frac{1}{\sqrt{36}} = \frac{1}{6}.$$

$$GM = \frac{1}{M} \Big/_{\omega = \omega_{pc}} = 6.$$

Q. Calc. PM ___

$$GH = \frac{1}{s(s+1)}.$$

$$PM = 180 + \angle GH \Big/_{\omega = \omega_{gc}}$$

Step 1 : find ω_{gc} by using magnitude condi.

$$GH = \frac{1}{s(s+1)}$$

$$PM = 180 + \angle GH / \omega = \omega_{gc}$$

$$\xrightarrow{\omega_{gc}} |M| = 1$$

$$\frac{1}{\omega\sqrt{1+\omega^2}} = 1$$

$$\Rightarrow \omega^4 + \omega^2 = 1$$

$$\Rightarrow \text{let } x = \omega^2$$

$$\Rightarrow \omega_{gc} = 0.786 \text{ rad/sec.}$$

$$PM = 180 - 90 - \tan^{-1}\omega \to 0.786$$

$$= 52^\circ.$$

Q. Calc. k value to get PM = 30

for $GH = \frac{k}{s(s+1)}$ [k - System gain]

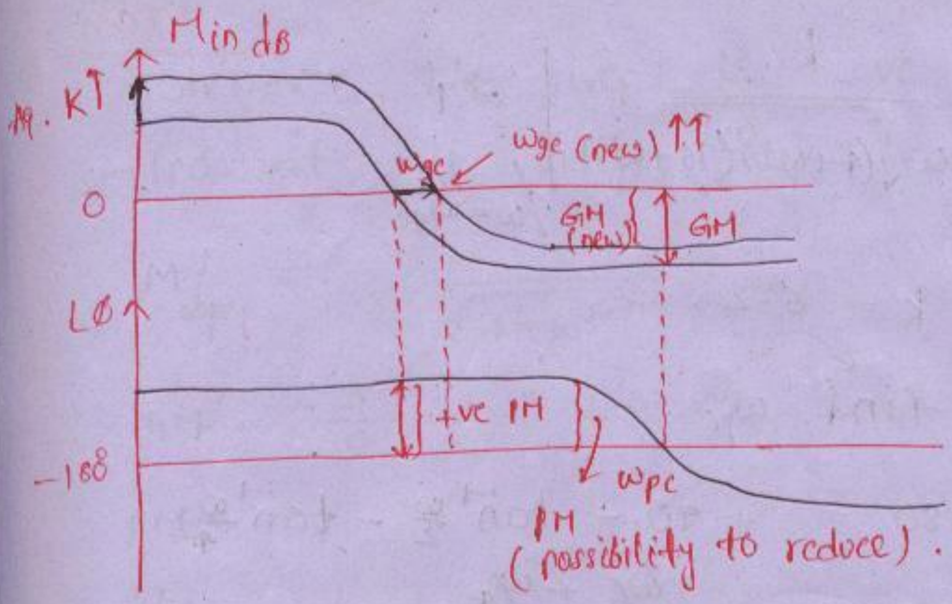
$$PM = 30$$

$$\Rightarrow 30 = 180 - 90 - \tan^{-1}\omega /_{\omega = \omega_{gc}}$$

$$\Rightarrow \omega_{gc} = \sqrt{3} \text{ rad/sec.}$$

$$\left| \frac{k}{\omega\sqrt{1+\omega^2}} \right|_{\omega = \omega_{gc}} = 1$$

$$\Rightarrow k = 2\sqrt{3}.$$



As Gain k increases;

→ ω_{gc} ↑↑

→ No change in ω_{pc}.

→ GM ↓↓

→ PM ↓↓.

Q. find k value to get PM = 60.

find k value to get GM = 20 dB.

for . $GH = \frac{k}{s(s+2)(s+4)}$

PM = 60.

$$\Rightarrow 60 = 180 + \left[-90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}\right]_{\omega=\omega_{gc}}$$

$$\Rightarrow 30 = \tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{4}}{1-\frac{\omega^2}{8}}\right)$$

$\Rightarrow \omega_{gc} = 0.72$ rad/sec.

$$\left| \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega=\omega_{gc}} = 1$$

$$\Rightarrow \quad k = 6.2.$$

(ii). find ω_{pc}.

$$-180 = -90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$$

$$\Rightarrow \quad \tan 90 = \frac{\omega/_2 + \omega/_4}{1 - \omega^2/_8} \rightarrow 0$$

$$\Rightarrow \quad \omega_{pc} = \sqrt{8}.$$

$$GM = -20\log|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}=\sqrt{8}}$$

$$\Rightarrow 20 = -20\log\left| \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega=\sqrt{8}}$$

$$\Rightarrow \quad 0.1 = \frac{k}{\sqrt{8}\sqrt{12\times 24}}$$

$$\Rightarrow \quad k = 4.8.$$

Q. Calc. gain margin & phase margin

$$GH = \frac{1}{s+2}$$

$$\xrightarrow{\omega_{pc}} \quad -180 = \angle GH/_{\omega=\omega_{pc}}.$$

$$-180 = -\tan^{-1}\omega/_2 \Big|_{\omega=\omega_{pc}}$$

$$\Rightarrow \quad \omega_{pc} = \infty.$$

b'coz $\quad \omega = 0 \Rightarrow 0^\circ.$

$\quad\quad\quad \omega = \infty \rightarrow -90^\circ.$

NOTE :

whenever T/F gives less -ve than -180 at all freq-s then ω_{pc} becomes ∞.

$$M\Big|_{\omega_{pc}} = \frac{1}{\sqrt{4+\omega^2}}_{\omega^2 \to \infty} = 0.$$

$$GM = \frac{1}{0} = \infty.$$

PM :

$$\xrightarrow{\omega_{gc}} M = 1$$

$$\frac{1}{\sqrt{4+\omega^2}} \stackrel{?}{=} 1$$

$$\omega = 0 \longrightarrow 0.5$$

$$\omega = \infty \longrightarrow 0$$

$$\Longrightarrow \omega_{gc} = 0.$$

NOTE:

whenever T/F gives less magnitude than 1 at all freq-s then $\omega_{gc} = 0$.

$$PM = 180 - \tan^{-1} \cancel{\omega}/_2{}^{0}$$

$$= 180 \longrightarrow (\infty) \quad \text{(S). } \underline{\text{stable}} \qquad \text{Ans: } (\infty, \infty)$$

Q. $GH = \frac{1}{s}$.

GM :

$$\xrightarrow{\omega_{pc}} -180 = \angle GH$$

$$-180 = -90$$

$$\Longrightarrow \omega_{pc} = \infty.$$

$M|_{\omega_{pc}} = \frac{1}{\omega} = 0$

$GM = \frac{1}{0} = \infty$

PM:

$\xrightarrow{\omega_{gc}} M = 1.$

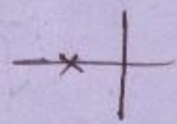
$\frac{1}{\omega} = 1$

$\longrightarrow \omega_{gc} = 1.$

OL T/f $\frac{1}{s}$

$PM = 180 - 90$

$= 90^\circ \longrightarrow$ (S). $\left(\text{CL T/F } \frac{1}{s+1}\right)$



Q. $GH = \frac{1}{s^2}$

GM:

$\xrightarrow{\omega_{pc}} -180 = -180$

$\Rightarrow \omega_{pc} = 0 \text{ to } \infty.$

In this case ω_{pc} is decided by ω_{gc}.

$\omega_{gc} = \frac{1}{|\omega^2|} = 1$

$\Rightarrow \omega_{gc} = 1 = \omega_{pc}.$

CL T/f.

$\frac{1}{s^2+1}$

$GM = \frac{1}{1} = 1$ (L)

$PM = 180 - 180$

$= 0$ (M.S).

Q. $GH = \frac{1}{s^3}$.

GM:

$\xrightarrow{\omega_{pc}}$ $-180^\circ = -270^\circ$

$\Rightarrow \omega_{pc} = 0$

whenever system gives more −ve than −180 at all freq's then $\omega_{pc} = 0$.

$M/_{\omega_{pc}} = \frac{1}{\omega^3} = \infty$

$\Rightarrow GM = \frac{1}{\infty} = 0.$

PM:

$\xrightarrow{\omega_{gc}}$ $M = 1$; $\frac{1}{\omega^3} = 1$

$\Rightarrow \omega_{gc} = 1.$

$PM = 180 - 270 = -90.$

Q. In G(s)H(s) plane the Nyquist plot of loop T/F $G(s)H(s) = \frac{\pi \cdot e^{-0.25s}}{s}$ passes through −ve real axis at the point is ——.

(a). $(-0.25, j0)$ (b). $(-0.5, j0)$

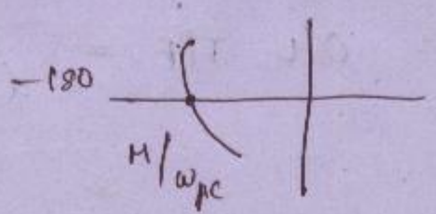
(c). $(-1, j0)$ (d). $(-2, j0)$.

Simply find magnitude at $\omega = \omega_{pc}$.

$e^{j\theta} = \cos\theta + j\sin\theta$

$\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta)$.



$$-180 = \angle GH \Big/_{\omega = \omega_{pc}}$$

$$\Rightarrow -180 = -90 + 0.25\omega \times \frac{180}{\pi}$$

$$\Rightarrow \omega_{pc} = 2\pi \text{ rad/sec.}$$

$$M = \frac{\pi}{\omega}\Big/_{\omega_{pc} = 2\pi}$$

$$= \frac{\pi}{2\pi} = 0.5 \qquad (-0.5, j0)$$

$$GM = \frac{1}{0.5} = 2.$$

PM:

$$\xrightarrow{\omega_{gc}} \left|\frac{\pi}{\omega}\right| = 1$$

$$\Rightarrow \omega_{gc} = \pi \text{ rad/sec.}$$

$$PM = 180 - 90 - 0.25\cancel{\omega}^{\pi} \times \frac{180}{\pi}$$

$$= 45^\circ$$

find e_{ss} for above T/F (which is worked on old notes) for unit step - ?

$$\frac{Y(s)}{U(s)} = \frac{3s+14}{(s+2)(s+4)}$$

↑ c/L T/F.

$$\therefore \text{O/L T/F} = \frac{3s+14}{(s+2)(s+4)-(3s+14)}$$

$$= \frac{3s+14}{(s^2+2s-8)}$$

$$e_{ss} = \frac{A}{1+k_p}$$

$$= \frac{1}{1+\frac{14}{-8}} = -\frac{8}{6} = -1.33.$$

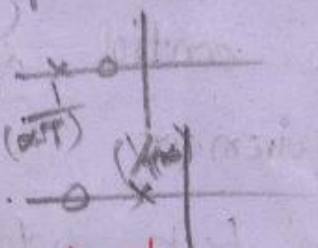
Compensator:-

A compensator is a n/w which add 1 finite pole, 1 finite zero, such that system performance is improved.

Lag } T/F: $\dfrac{1+Ts}{1+\alpha Ts}$ pole
lead }

$\alpha < 1 \longrightarrow$ lead

$\alpha > 1 \longrightarrow$ lag.



* Lead - lag } T/F : $\underbrace{\left[\dfrac{1+T_1 s}{1+\alpha T_1 s}\right]}_{\text{lead}} \cdot \underbrace{\left[\dfrac{1+T_2 s}{1+\alpha T_2 s}\right]}_{\text{lag}}$
Lag - lead }

$\rightarrow$ $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$ [freq. when max. ph. lead or ph. lag occurs].

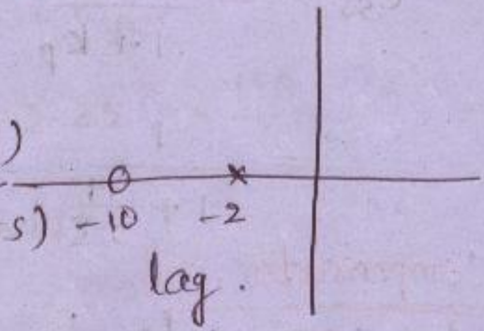
$\rightarrow$ $\phi_m = \sin^{-1}\left[\dfrac{1-\alpha}{1+\alpha}\right] \longrightarrow$ lead. $(\alpha < 1)$

$\rightarrow$ $\phi_m = \sin^{-1}\left[\dfrac{\alpha-1}{\alpha+1}\right] \longrightarrow$ lag $(\alpha > 1)$

$\rightarrow$ $M/_{\omega_m} = 10 \log\left(\frac{1}{\alpha}\right)$.

Q. $GH = \dfrac{s+10}{s+2}$

$$\frac{1+\tau s}{1+\alpha\tau s} = \frac{10(1+0.1s)}{2(1+0.5s)}$$



$\tau = 0.1$

$\alpha\tau = 0.5$

$\Rightarrow \alpha = 5.$

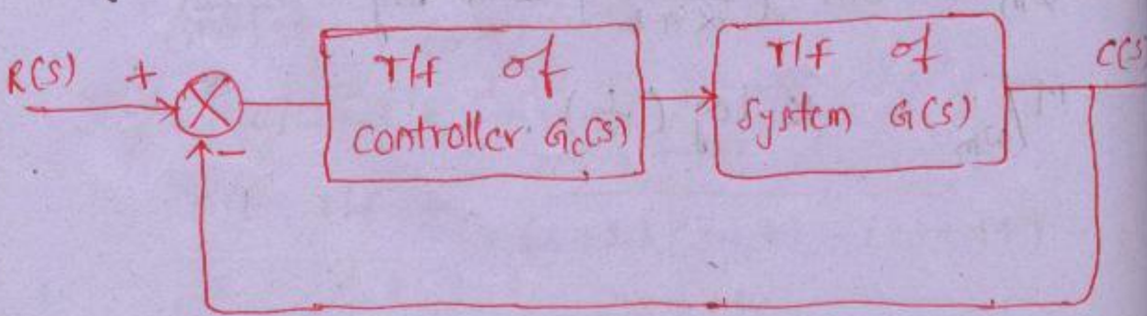
$$\omega_m = \frac{1}{0.1\sqrt{5}}$$

PID controllers :

A controller is a device which is used to control ss & tr. response as per requirement.

The best system demands smallest t_r, smallest t_s, smallest e_{ss} and smallest M_p, which is not possible without PID controllers.

Block diagram with controller shown in fig.



P- CONTROLLER:

To change tr. response as per requirement. T/F of p- controller is $= k_p$

Let consider the system $G(s) = \frac{1}{s(s+10)}$

$$\Longrightarrow \frac{C(s)}{R(s)} = \frac{1}{s^2+10s+1}$$

$\omega_n = 1$

$\xi = 5 \longrightarrow$ over damped.

with controller,

$$G(s) = \frac{k_p}{s(s+10)}$$

$$\frac{C(s)}{R(s)} = \frac{k_p}{s^2+10s+k_p}$$

If $k_p = 100$, $\omega_n = 10$

$\xi = 0.5 \longrightarrow$ under damped

$k_p = 25$, $\omega_n = 5$

$\xi = 1 \longrightarrow$ Critical damped.

→ The main dis. adv. in p- controller is as k_p value increases, ξ decreases hence $\% \, M_p$ increases.

As $\% \, M_p \uparrow\uparrow$, the system becomes less stable.

By using P-controller we get required nature by changing k_p value.

I - CONTROLLER:

purpose: To decrease ss error.

The T/F of Integral controller is $\frac{k_i}{s}$

I- controller add one ~~zero~~ pole at origin hence type is increased. As type increases the ss error decreases but system stability effected.

Eg: $G(s) = \frac{1}{s(s+10)}$ (without controller)

type = 1

CE: $s^2 + 10s + 1 \longrightarrow$ S.

with controller,

$$G(s) = \frac{k_i}{s^2(s+10)}$$

type = 2

CE: $s^3 + 10s^2 + k_i = 0 \longrightarrow$ (U.S).

D – CONTROLLER :

purpose : To improve the stability.

T/F of d- controller is $k_D . s$

D- controller add one zero at origin hence type is decreased. As type decreases stability improved But ss error increased.

Eg : $G(s) = \dfrac{1}{s^2(s+10)}$

type = 2

CE : $s^3 + 10s^2 + 1 = 0 \longrightarrow$ (U.S).

with controller.

$G(s) = \dfrac{k_D . \cancel{s}}{s^{\cancel{2}}(s+10)}$

Type = 1

CE : $s^2 + 10s + k_D = 0$

PI – CONTROLLER :

purpose :

To decrease ss error without effecting stability

T/f of PI - controller is $k_p + \dfrac{k_i}{s}$.

ie $\frac{sk_p + k_i}{s}$.

PI - controller add one pole at origin hence type is increased.

As Type increases e_{ss} decreases.

PI - controller add one finite zero in left of s-plane, which avoid effect on stability.

PD - CONTROLLER

PURPOSE:

To improve stability without effecting e_{ss}.

T/f of PD - controller is $k_p + k_D s$

PD - controller add only one finite zero in the left of s-plane hence system stability improved.

No change in type with PD - controller, hence no effect on ss error.

ξ value with PD - controller is

$$\xi_{PD} = \xi + \frac{\omega_n k_D}{2}$$

As $\xi_{PD} \uparrow\uparrow$, $\% M_p \downarrow\downarrow$ → more stable.

PID - CONTROLLER:

Purpose:

To improve stability as well as to decrease e_{ss}.

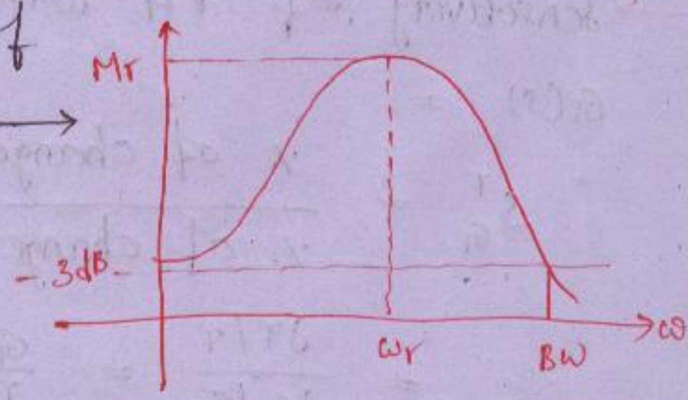
$$T/F \text{ of PID} = k_p + \frac{k_I}{s} + k_D s$$

PID, adds one pole at origin which increases type hence ss error decreases.

PID, adds two finite zeros in the left hand side. one finite zero avoid effect on system stability and the other zero improve stability of the system.

FREQUENCY DOMAIN SPECIFICATIONS:

freq. response of any RLC n/w →



→ Resonant freq.

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

→ Resonant peak (or) max. peak

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

→ BW of 1st order system $= \frac{1}{\tau}$

→ BW of 2nd order system = ω_p & ω_c

$$= \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi+4}}$$

→ $\omega_n = \frac{1}{\sqrt{LC}}$

→ $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

→ $Q = \frac{1}{2\xi} = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}$

→ Sensitivity describes the relative variations in the parameters.

→ Sensitivity of T/F w.r.t. variations in G(s) =

$$S_G^T = \frac{\%\text{ of change in T/F}}{\%\text{ of change in } G(s)}$$

$$= \frac{\partial T/T}{\partial G/G} = \frac{G}{T}\left(\frac{\partial T}{\partial G}\right)$$

$$= \frac{1}{1+GH}$$

$$S_H^T = \frac{H}{T}\frac{\partial T}{\partial H} = \frac{-GH}{1+GH}.$$

o/l Sensitivity $= 1 = S_T^{o/l} > S_T^{c/l\ loop}$

→ The o/l system is more sensitive compare to c/l system. In a c/l system f/b n/w is more sensitive compare to forward path.

FRI.
15/08/08

Subjects to be Targetted:

1. Signals & Systems — 20 M.
2. Digital & MP — 15 to 20 M
3. Mathematics — 20 M
4. EDC — 15 M
5. Networks — 15 to 20 M.
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7. Measurements — 10 to 15 M
8. Machines — 20 to 25 M
9. Power Systems — 20 to 25 M
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Previous papers:

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