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# HIND PHOTOSTAT AND HIND BOOK CENTER

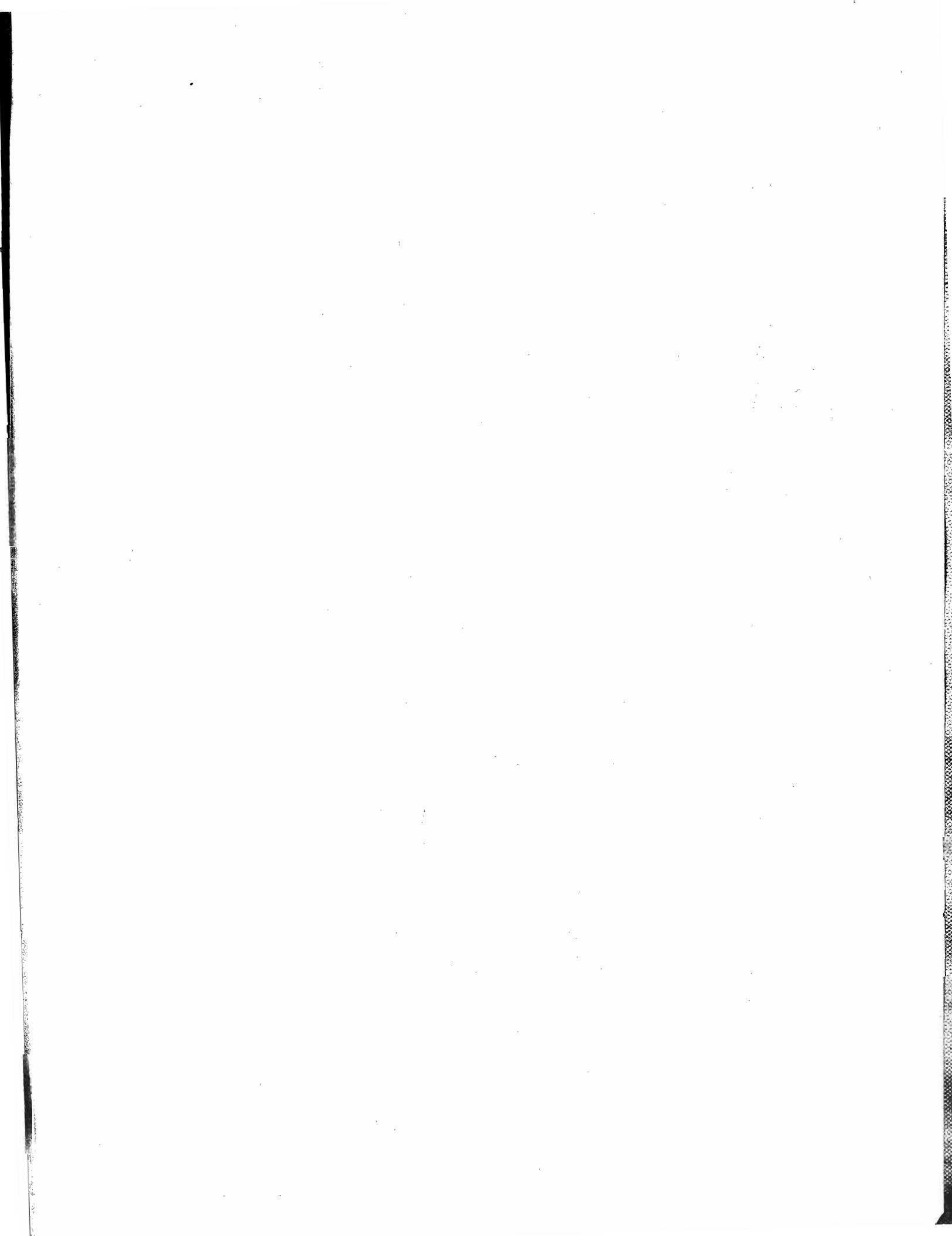
NAME:-.....

SUBJECT:- Control system (E.E.)

INSTITUTE:-.....

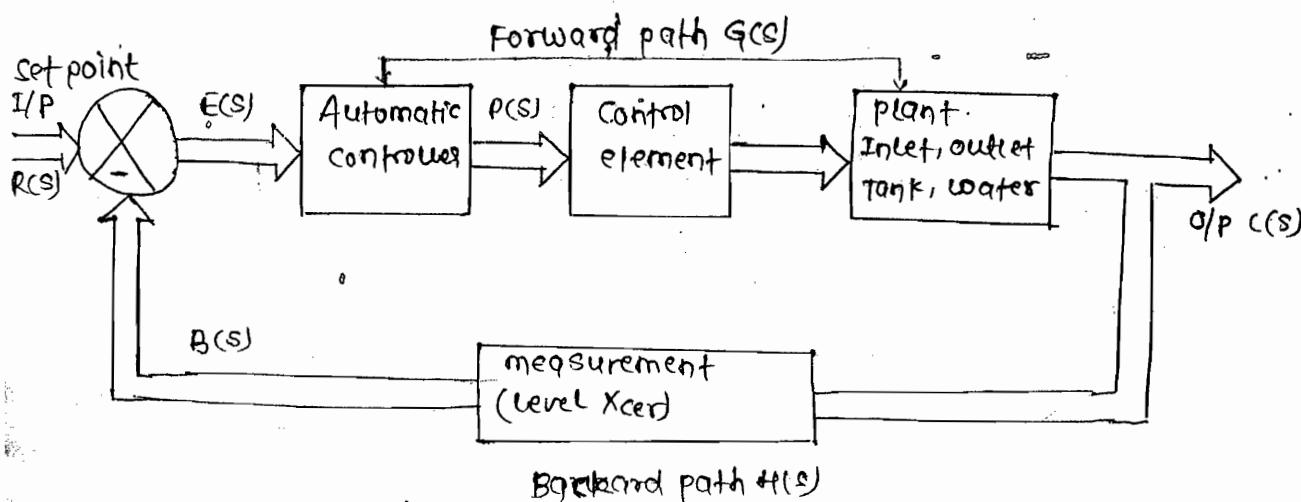
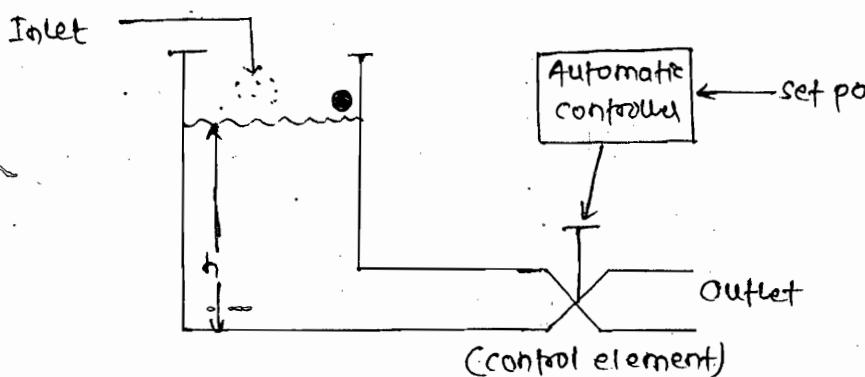
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Chapter-01  
Introduction to control sys.

- \* Consider liquid level control system whose control objective is to keep the water level into tank at a height 'h'.
- \* Controller is a automatic device with error signal  $E(s)$  as i/p & controller o/p  $P(s)$  affecting the dynamics of plant to achieve the control objective.
- Therefore controller o/p  $P = f(e)$   
where  $e = \text{steady state error}$ .
- \* The different modes of controller operation are proportional, proportional + integral & proportional + integral + derivative.
- \* There are 2 basic control loop configurations:-  
 (1.) Closed loop (or) feedback control system
  - In this configuration the changes in the o/p are measured through FB & compared with the i/p (or) set point to achieve control objective.
  - Feedback employs measurement (sensor or  $X_{cer}$ )



$$E(s) = R(s) - B(s)$$

$$\frac{C(s)}{G(s)} = R(s) - C(s) \cdot H(s)$$

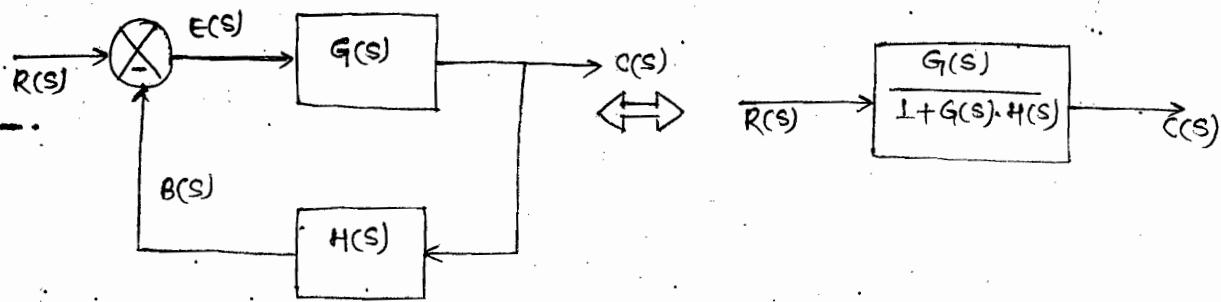
$$C(s) = G(s) R(s) - G(s) \cdot H(s) C(s)$$

$$C(s) [1 + G(s) \cdot H(s)] = G(s) R(s)$$

$$C(s) = \left[ \frac{G(s)}{1 + G(s) \cdot H(s)} \right] R(s)$$

Control canonical form →

Eq. mathematical model →



\* Sensitivity Analysis →

Let  $\alpha$  = A variable

that changes its value

$\beta$  = A parameter that changes the value of ' $\alpha$ '

$$S_{\beta}^{\alpha} = \frac{\% \text{ change in } \alpha}{\% \text{ change in } \beta}$$

$$S_{\beta}^{\alpha} = \frac{\frac{\partial \alpha}{\partial \beta}}{\frac{\alpha}{\beta}} = \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial \beta}$$

## Sensitivity of a closed loop control system (CLCS) $\Rightarrow$

Let  $\alpha = m(s) = \text{CLCS}$

$\beta = \text{disturbances in forward path elements}$

$$\frac{s^{m(s)}}{G(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

$$\text{since } m(s) = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\frac{G(s)}{m(s)} = 1 + G(s) \cdot H(s)$$

$$\frac{\partial m(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)} \left[ \frac{G(s)}{1 + G(s) \cdot H(s)} \right]$$

$$= \frac{1 + G(H) \cdot H(s) - G(s) \cdot H(s)}{(1 + G(s) \cdot H(s))^2}$$

$$= \frac{1}{(1 + G(s) \cdot H(s))^2}$$

$$\frac{s^{m(s)}}{G(s)} = [1 + G(s) \cdot H(s)] \times \frac{1}{[1 + G(s) \cdot H(s)]^2}$$

$$\boxed{\frac{s^{m(s)}}{G(s)} = \frac{1}{1 + G(s) \cdot H(s)}}$$

$1 + G(s) \cdot H(s) = \text{Noise reduction factor (OR) Return diff.}$

## \* Sensitivity of CLCS with respect to disturbances in $H(s)$ i.e. F/b elements $\Rightarrow$

$\alpha = m(s) = \text{CLCS}$

$\beta = \text{disturbances in F/b elements i.e. } H(s)$

$$\frac{s^{m(s)}}{H(s)} = \frac{H(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial H(s)}$$

$$\text{since } m(s) = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\frac{m(s)}{H(s)} = \frac{G(s)}{H(s)[1+G(s) \cdot H(s)]}$$

$$\frac{H(s)}{m(s)} = \frac{H(s)[1+G(s) \cdot H(s)]}{G(s)}$$

$$\frac{\partial m(s)}{\partial H(s)} = \frac{\partial}{\partial H(s)} \left[ \frac{G(s)}{1+G(s) \cdot H(s)} \right]$$

$$= \frac{[1+G(s) \cdot H(s)] \times 0 - G(s) \cdot G(s)}{[1+G(s) \cdot H(s)]^2}$$

$$= \frac{-[G(s)]^2}{[1+G(s) \cdot H(s)]^2}$$

$$\frac{s^m(s)}{H(s)} = \frac{H(s)[1+G(s) \cdot H(s)]}{G(s)} \times \frac{-[G(s)]^2}{[1+G(s) \cdot H(s)]^2}$$

$$\boxed{\frac{s^m(s)}{H(s)} = \frac{-G(s) \cdot H(s)}{1+G(s) \cdot H(s)}}$$

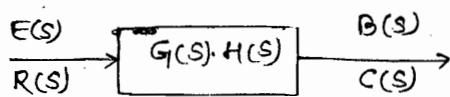
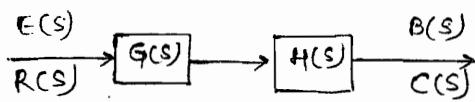
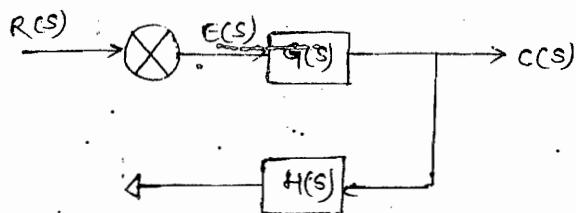
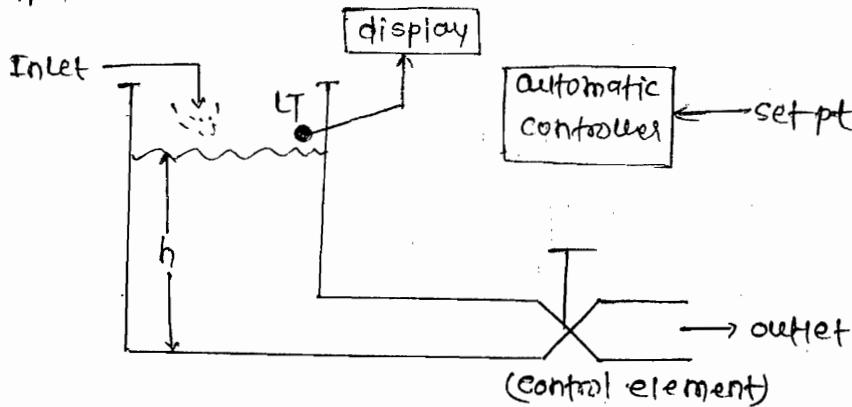
$$\boxed{\left| \frac{s^m(s)}{H(s)} \right| = \frac{|G(s) \cdot H(s)|}{1+|G(s) \cdot H(s)|}}$$

**Note →** The CLCS is more sensitive to disturbances in F/b elements i.e.  $H(s)$  than forward path element i.e.  $G(s)$ .

#### (Open loop Control system (OLCS) →

- (1) They are conditional control sys. Formulated under the basic condn that the sys. is not subjected to any type of disturbances.
- (2) In this configuration the F/b (or) measurement is not connected to Forward path (or) controller (open loop).
- (3) F/b in open loop sys. except for displaying the information about o/p do not have any major significance. This insignificance of F/b is known as elimination of F/b

Q.) Performance analysis is not applicable to these systems because they are not subjected to any type of disturbances. & give out a desired O/p for the desired I/p.



Sensitivity of OLCS w.r.t disturbances in  $G(s)$

Let  $\alpha = m(s) = \text{OLCS}$

$B = G(s)$

$$S_{G(s)}^{m(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

$$m(s) = G(s) \cdot H(s)$$

$$\frac{G(s)}{H(s)} = \frac{1}{G(s)}$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)} [G(s) \cdot H(s)] = H(s)$$

$$S \frac{M(s)}{G(s)} = \frac{1}{H(s)} \times H(s)$$

$$\boxed{S \frac{M(s)}{G(s)} = 1}$$

Ques. → The OL & CL sys. are shown in figs

For 10% change in the forward path, the responses of open loop sys. & CL sys. will be affected by:-

- (a) 10%, 1%
- (b) 10%, 0.1%
- (c) 1%, 10%
- (d) 1%, 0.1%

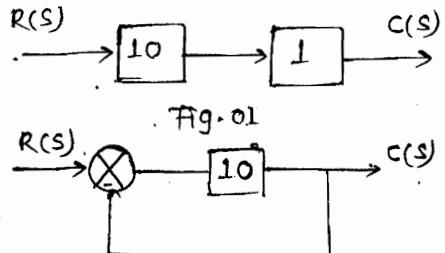


Fig:-02

Soln →

OPCS → 10%

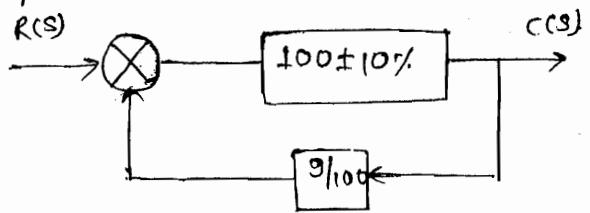
$$CLCS \rightarrow \frac{1}{1+G(s) \cdot H(s)} \times 10\%$$

$$= \frac{1}{1+10 \times 1} \times 10\%$$

$$= 0.1\%$$

Ans. (b)

Ques.(2) The closed loop gain when f/b of gain 9/100 is connected in the sys. as shown below will be:-



- (a)  $10 \pm 10\%$
- (b)  $10 \pm 1\%$
- (c)  $100 \pm 10\%$
- (d)  $100 \pm 1\%$

$$\text{Soln} \rightarrow \text{C.L. gain} = \frac{G}{1+GH}$$

$$= \frac{100}{1+100 \times \frac{9}{100}}$$

$$\text{C.L. gain} = 10$$

$$= \frac{1}{1+GH} \times 10 \times$$

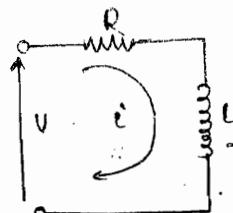
$$= \frac{1}{1+100 \times \frac{9}{100}} \times 10 \times$$

$$= \frac{1}{10} \times 10 \times = 1.$$

### \* Concept of TF $\rightarrow$

A TF is a mathematical model representing the CS relating I/P & O/P in the form of a ratio i.e. O/p divided by I/p.

Ex:-



$$V = iR + \frac{Ldi}{dt}$$

Applying LT

$$V(s) = I(s)R + sLI(s)$$

$$V(s) = I(s)[R + LS]$$

$$\frac{I(s)}{V(s)} = \frac{1}{R + LS}$$

$$\frac{I(s)}{V(s)} = \frac{1/L}{s + R/L}$$

$$\text{TF} = F(s) = \frac{C(s)}{R(s)} = \frac{N(s)}{D(s)} = \frac{k(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \dots \quad (i)$$

where  $k = \text{sys. gain}$

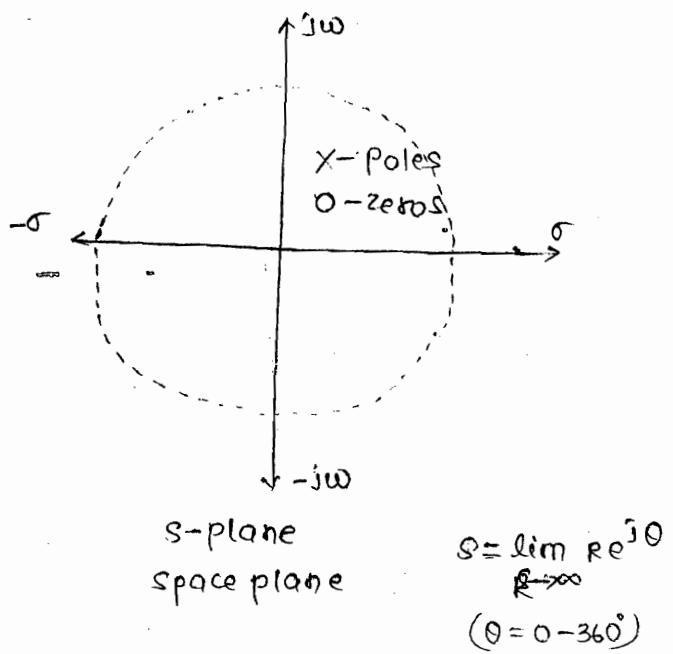
Time constant form  $(1+TS)$

$$\text{TF} = F(s) = \frac{C(s)}{R(s)} = \frac{N(s)}{D(s)} = \frac{k'(1+T_a s)(1+T_b s)}{(1+T_1 s)(1+T_2 s)} \quad (ii)$$

where;  $k' = \frac{k \cdot z_1 z_2}{p_1 p_2} = \text{sys. dc gain}$

$$T_a = \frac{1}{z_1}, T_b = \frac{1}{z_2}$$

$$T_1 = \frac{1}{p_1}, T_2 = \frac{1}{p_2}$$



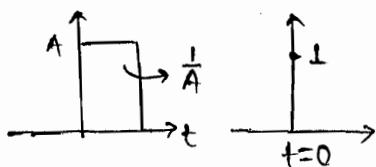
From eqn (i) the TF of LTI sys. may be defined as the ratio of Laplace Xform of o/p to Laplace Xform of i/p under the assumption that the sys. initial cond'n are zero.

Poles & zeros are those critical freq. which make the TF  $\infty$  (OR) zero.

#### \* Impulse Response & TF →

##### \* Impulse signal

$$\begin{aligned} r(t) &= 1, t=0 \\ &= 0, t \neq 0 \end{aligned}$$



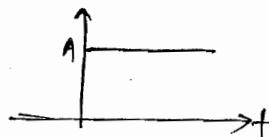
$$LT \rightarrow R(s) = \frac{1}{s}$$

##### \* Step signal

$$r(t) = A u(t)$$

$$u(t) = 1 \quad t > 0$$

$$= 0 \quad t < 0$$

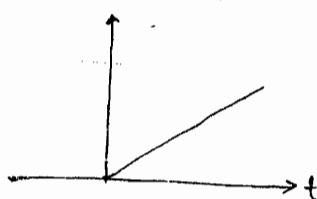


$$LT \rightarrow R(s) = \frac{A}{s}$$

\* Ramp signal

$$r(t) = At, t > 0$$

$$= 0, t < 0$$



$$\mathcal{L}[r] \rightarrow R(s) = \frac{A}{s^2}$$

$$T.F. = F(s) = \frac{C(s)}{R(s)}$$

$$C(s) = F(s) R(s)$$

Let  $R(s) = \text{Impulse Response} = 1$

$$C(s) = \text{Impulse Response} = F(s) \times 1$$

$$= T.F. \quad (3.)$$

$\mathcal{L}(\text{Impulse Response}) = T.F.$  "Weighting function"

$$\frac{d}{dt} (\text{Parabolic Response}) = \text{Ramp Response}$$

$$\frac{d}{dt} (\text{Ramp Response}) = \text{Step Response}$$

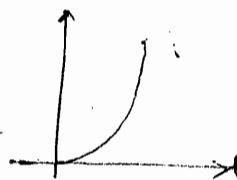
$$\frac{d}{dt} (\text{Step Response}) = \text{Impulse Response}$$

$$\boxed{\mathcal{L}(\text{Impulse Response}) = T.F.}$$

\* Parabolic signal

$$r(t) = A \cdot \frac{t^2}{2}, t > 0$$

$$= 0, t < 0$$



$$\mathcal{L}[r] \rightarrow R(s) = \frac{A}{s^3}$$

Ques. (1)

Ques. (1) The IR of a system is

$$c(t) = -t\bar{e}^t + 2\bar{e}^t \quad (t > 0)$$

Its open loop TF will be :-

- (a.)  $\frac{2s+1}{(s+1)^2}$     (b.)  $\frac{2s+1}{s^2}$     (c.)  $\frac{2s+1}{s}$     (d.)  $\frac{2s+1}{(s+1)}$

Soln  $\rightarrow$   $L(IR) = TF$

$$TF = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$TF = \frac{-1 + 2(s+1)}{(s+1)^2}$$

$$\frac{G(s)}{1+G(s)H(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s) \cdot H(s) = 1$$

$$\text{put } H(s) = 1$$

$$\frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s) [(s+1)^2 - (2s+1)] = 2s+1$$

$$G(s) = \frac{2s+1}{s^2}$$

shortcut method

(for unity f/b sys.)

$$G(s)$$

$$\text{open loop TF} = \frac{\text{Num.}}{\text{Den - Num.}}$$

$$TF = \frac{2s+1}{(s+1)^2 - (2s+1)}$$

$$G(s) = \frac{2s+1}{s^2}$$

Ques. (2)

Ques. (2) What is the open loop de-gain of unity -ve f/b sys. having closed loop TF.

$$\frac{s+4}{s^2+7s+13}$$

- (a.)  $\frac{4}{13}$     (b.)  $\frac{4}{9}$     (c.) 4    (d.) 13

Soln  $\rightarrow$

$$\text{Open loop TF. } G(s) = \frac{s+4}{s^2+7s+13 - s - 4}$$

$$= \frac{s+4}{s^2+6s+9}$$

$$= \frac{4(s+\frac{5}{4})}{9(1+\frac{6s}{9}+\frac{s^2}{9})}$$

$$K = \frac{4}{9}$$

Que. → Find the dc gain of TF.

$$\frac{C(s)}{R(s)} = \frac{10(s+5)}{s(s+1)(s+2)}$$

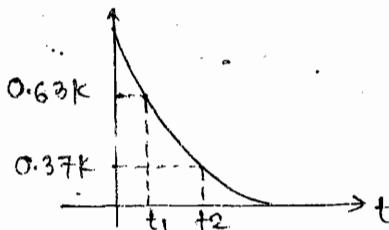
Soln →

$$\begin{aligned} \frac{C(s)}{R(s)} &= \frac{10 \times 5 (1+0.2s)}{(s) 1(1+s) 2(1+0.5s)} \\ &= \frac{25(1+0.2s)}{s(1+s)(1+0.5s)} \end{aligned}$$

$$[k=25]$$

Que. → The IR of the sys. is

$$\frac{C(s)}{R(s)} = \frac{k}{s+\alpha} \text{ is shown in fig.}$$



The value of  $\alpha$  will be

- (a)  $t_1$  (b)  $1/t_1$  (c)  $t_2$  (d)  $1/t_2$

Soln → Impulse Response =  $L^{-1}$ (T.F.)

$$\begin{aligned} &= L^{-1}\left(\frac{k}{s+\alpha}\right) \\ &= k e^{-\alpha t} \end{aligned}$$

At  $t=t_2$

$$k e^{-\alpha t_2} = k(0.37)$$

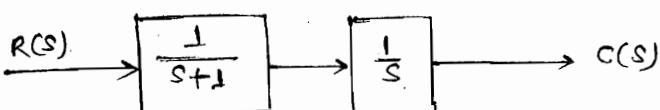
Since  $e^{-\alpha t_2} = 0.37$

$$k e^{-\alpha t_2} = k e^{-1}$$

$$\alpha t_2 = 1$$

$$\boxed{\alpha = \frac{1}{t_2}}$$

Chap(01)  
Que.(2)



Impulse Response = ?

Ans.  $\rightarrow$  Impulse Response =  $\mathcal{L}^{-1}(T.F)$

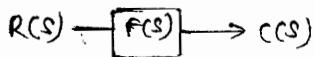
$$= \mathcal{L}^{-1} \frac{1}{s(s+1)} = \mathcal{L}^{-1} \left[ \frac{1}{s} - \frac{1}{s+1} \right]$$

$$= e^t - e^{-t}$$

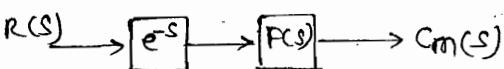
Q.  $\rightarrow$  A certain sys has i/p  $r(t)$  & o/p  $c(t)$ . If the i/p is 1st passed through a block whose TF is  $e^{-s}$  & then applied to the sys. the modified o/p will be.

- (a)  $c(t) u(t-1)$       (c)  $c(t-1) u(t-1)$   
 (b)  $c(t-1) u(t)$       (d) None.

Soln  $\rightarrow$



$$C(s) = R(s) F(s)$$



$$C_m(s) = R(s) e^{-s} F(s)$$

$$C_m(s) = C(s) e^{-s}$$

$$\mathcal{L}^{-1} F(s) e^{-as} = f(t-a) u(t-a)$$

$$C_m(t) = c(t-1) u(t-1)$$

Chap(2)(1)  
(S9)

Ans.(b)

(2.) CL gain =  $\frac{G}{1+GH}$

$$100 = \frac{10^3}{1 + 10^3 \times B}$$

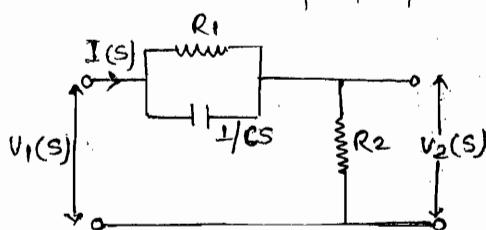
$$B = 9 \times 10^3$$

\* TF For Compensators →

\* Compensators in CS are used for improving the performances specifications i.e. transient & steady state response C/S.

\* There are 3 type:-

(i) Lead compensator → It is used for improving the transient state or speed of response of sys.



$$V_1(s) = I(s) \left[ \frac{R_1}{R_1 Cs + 1} \right] + I(s) \cdot R_2$$

$$= I(s) \left[ \frac{R_1 + R_2 + R_1 R_2 C s}{R_1 C s + 1} \right]$$

$$V_0(s) = I(s) R_2$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{R_2(R_1 C s + 1)}{R_1 + R_2 + R_1 R_2 C s}}$$

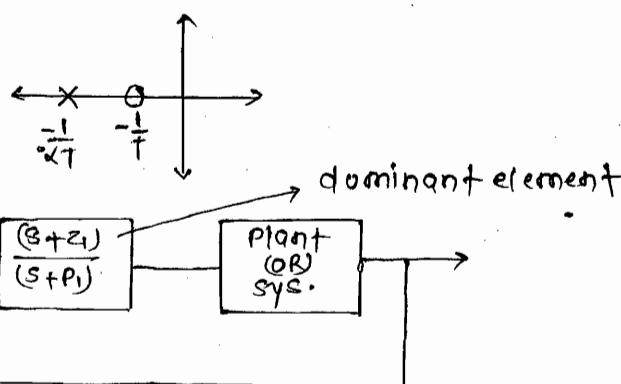
$$T = R_1 C ; \quad \alpha = \frac{R_2}{R_1 + R_2} \quad (\alpha < 1)$$

$$R_2(R_1 C s + 1)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{\alpha(1+Ts)}{1+\alpha Ts}$$

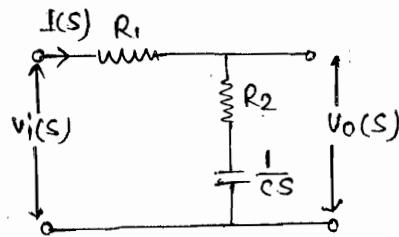
$$\text{Zero at } s = -\frac{1}{T}$$

$$\text{Poles at } s = -\frac{1}{\alpha T}$$



Note → Adding a zero to a sys. TF in terms of compensators represents a lead compensator.

(2) Lag compensator → It is used for improving steady state response of the sys. i.e. elimination of steady state error b/w i/p & o/p.



$$V_i(s) = I(s) \left[ R_1 + R_2 + \frac{1}{Cs} \right]$$

$$= I(s) \left[ \frac{R_1 Cs + R_2 Cs + 1}{Cs} \right]$$

$$V_o(s) = I(s) \left[ R_2 + \frac{1}{Cs} \right]$$

$$V_o(s) = I(s) \left[ \frac{R_2 Cs + 1}{Cs} \right]$$

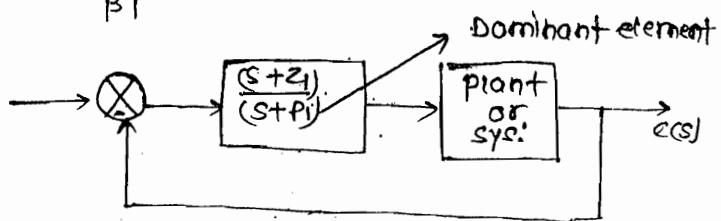
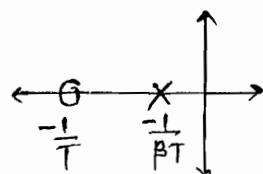
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2 Cs + 1}{R_1 Cs + R_2 Cs + 1}}$$

$$T = R_2 C ; \quad B = \frac{R_1 + R_2}{R_2} = \frac{1}{\alpha} \quad (\alpha > 1)$$

$$\frac{R_2 Cs + 1}{R_2 Cs \left( \frac{R_1 + R_2}{R_2} + 1 \right)}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = - \frac{1 + TS}{1 + \beta TS}}$$

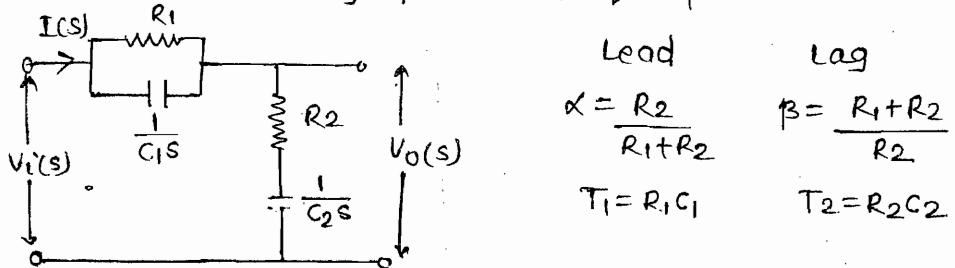
$$\text{zero at } s = -\frac{1}{T} ; \text{ pole at } s = -\frac{1}{\beta T}$$



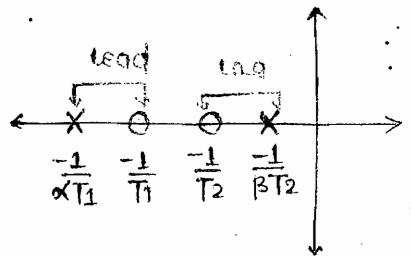
Note → Adding the pole to a sys. TF in terms of compensators represent lag compensator.

- (3.) LAG-LEAD compensator → It improves both transient & steady state response c/s.

\* It exhibits both Lead & Lag c/s in its freq. response.



$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1s)(1+T_2s)}{(1+\alpha T_1s)(1+\beta T_2s)}$$



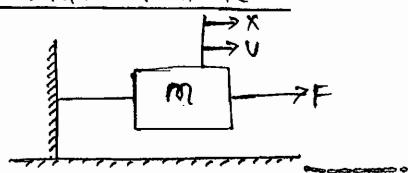
\* mechanical systems → All mech. sys. are classified into 2 types :-

(1.) Mechanical translational sys. →

I/P = Force (F) ; O/P = linear disp (x) (or) linear velocity (v)

\* The 3 ideal elements are :-

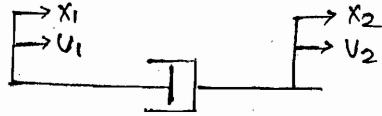
(1.) Mass element →



(a)  $F = m \frac{dv}{dt}$

(b.)  $F = m^2 \frac{d^2x}{dt^2}$

(2.) Damper element →

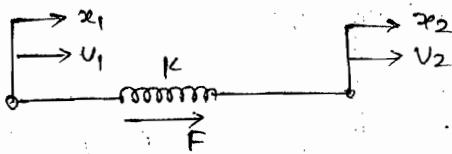


$F \propto (v_1 - v_2)$

(a)  $F = f(v_1 - v_2) = f v \quad [v = v_1 - v_2]$

(b.)  $F = f \frac{d}{dt} (x_1 - x_2) = f \frac{dx}{dt} \quad (x = x_1 - x_2)$

### (3.) Spring element →



$$(a) F = k \int (v_1 - v_2) dt = k \int v dt \quad (v = v_1 - v_2)$$

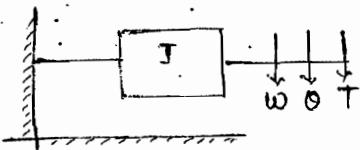
$$(b) F = k(x_1 - x_2) = kx \quad (x = x_1 - x_2)$$

### (2.) Mech. Rotational sys. →

I/p = torque ( $T$ ) ; o/p = Angular disp ( $\theta$ ) or Angular velocity ( $\omega$ )

3 ideal elements are →

#### (1.) Inertia element

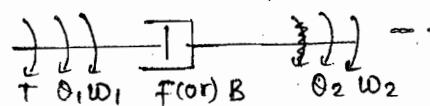


$$(a) T = J \frac{d\omega}{dt}$$

$$(b) T = J \frac{d^2\theta}{dt^2}$$

#### Torsional

#### (2.) Damper element (friction)

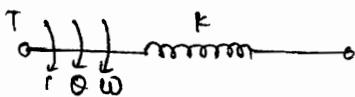


$$(a) T = f(\omega_1 - \omega_2) = f\omega (\omega_1 - \omega_2 = \omega)$$

$$(b) T = f \frac{d}{dt} (\theta_1 - \theta_2) = f \frac{d\theta}{dt} \quad (\theta = \theta_1 - \theta_2)$$

#### Torsional

#### (3.) Spring element (stiffness)



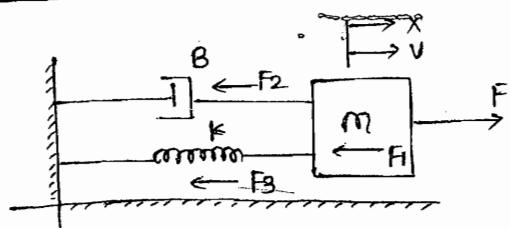
$$(a) T = k \int \omega dt$$

$$(b) T = k\theta$$

### \* Analogous system →

The electrical eq's of mech. elements are known as analogous sys.

(1.) mech. translational sys →

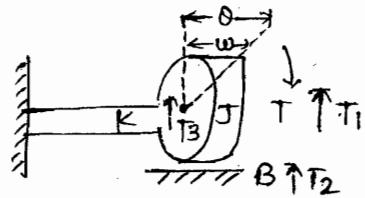


$$F = F_1 + F_2 + F_3$$

$$F = m \frac{dv}{dt} + Bv + k \int v dt$$

$$F = m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx -$$

(2.) mech. rotational sys →

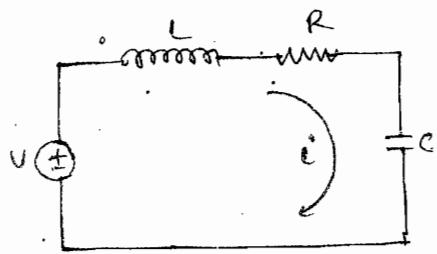


$$T = T_1 + T_2 + T_3$$

$$T = J \frac{d\omega}{dt} + B\omega + k \int \omega dt$$

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + K\theta \quad \text{--- (ii)}$$

(3.) Electrical system →



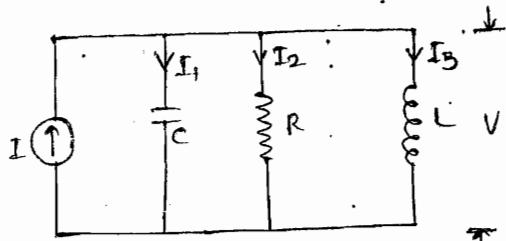
$$V = V_1 + V_2 + V_3$$

$$V = L \frac{di}{dt} + iR + \frac{1}{C} \int i dt$$

$$\left[ i = \frac{dq}{dt}, q = \text{charge} \right]$$

$$V = L \frac{dq}{dt} + R \frac{dq}{dt} + \frac{q}{C} \quad \text{--- (iii)}$$

(4.) Electrical System →



$$I = I_1 + I_2 + I_3$$

$$I = C \frac{dv}{dt} + \frac{V}{R} + \frac{1}{L} \int v dt$$

$$\left[ V = \frac{d\phi}{dt}; \phi = \text{flux} \right]$$

$$I = C \frac{d\phi}{dt} + \frac{1}{R} \frac{d\phi}{dt} + \frac{\phi}{L} \quad \text{--- (iv)}$$

Comparing eqn (i) - (iv)

(1) F-T-V Analogy    (2) F-T-I Analogy

$$F - T - V - I$$

$$m - J - L - C$$

$$B - R - \frac{1}{R}$$

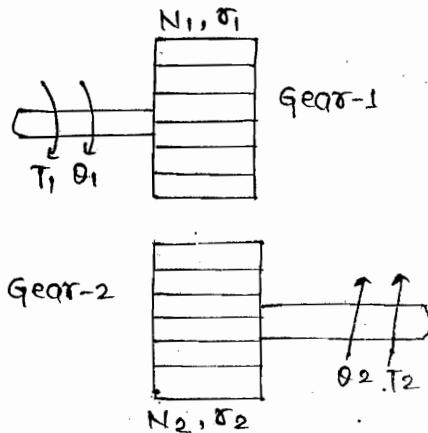
$$k - \frac{1}{C} - \frac{1}{L}$$

$$V - \omega - i - v$$

$$X - \theta - q - \phi$$

## GEARs →

- \* These are mech. devices which are used as intermediate elements b/n electrical motor & load.
- \* They are used for stepping up (or) stepping down either torque (or) speed.
- \* They are analogous to electrical TF.



$N$  = No. of teeths

$r$  = Radius of gear wheel (m)

$T$  = Torque on gear wheel (N-m)

$\theta$  = Angular disp (radians)

## Dynamics of gear train

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{\tau_1}{\tau_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{\tau_x}{\tau_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}}$$

Ex:-



(i) If  $T_1 = 10 \text{ N-m}$ , find  $T_2$  &  $T_3$ ?

(ii)  $\omega_1 = 20 \text{ r/s (ccw)}$ , find  $\omega_2$  &  $\omega_3$ ?

(i)  $\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{100}{50} = \frac{10}{T_2} \Rightarrow T_2 = 5 \text{ N-m}$

$\frac{N_1}{N_3} = \frac{T_1}{T_3} = \frac{100}{200} = \frac{10}{T_3} \Rightarrow T_3 = 20 \text{ N-m}$

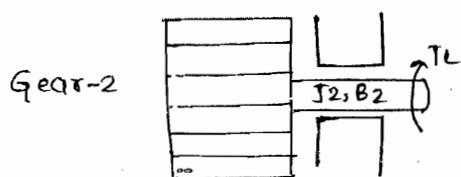
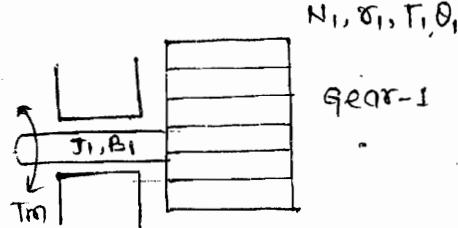
(ii)  $\frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{100}{50} = \frac{\omega_2}{20} \Rightarrow \omega_2 = 40 \text{ r/s (ccw)}$

$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1} = \frac{100}{200} = \frac{\omega_3}{20} \Rightarrow \omega_3 = 10 \text{ r/s (cw)}$

### Observations →

(1)  $N_1 > N_2 \Rightarrow T \downarrow, \omega \uparrow$

(2)  $N_1 = N_2 \Rightarrow$  There will be no change on  $T$  &  $\omega$  (speed & torque)



$T_2, \omega_2, N_2, \theta_2$

$T_m = \text{motor torque (N-m)}$

$T_1 = \text{Torque on gear-1 due to } T_m (\text{N-m})$

$T_2 = \text{Torque on gear-2 due to } T_1 (\text{N-m})$

$T_L = \text{Torque on load due to } T_2 (\text{N-m})$

$J, B = \text{Inertia \& Friction of gears.}$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} ; \quad T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_1 = \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) \cdot T_L$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

\* (I) = eq; Inertia \& friction of motor side gear (gear-1)

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} ; \quad \dot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \dot{\theta}_1 ; \quad \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \ddot{\theta}_1$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = \left[ J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 \right] \frac{d^2\theta_1}{dt^2} + \left[ B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 \right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

\* (ii) Eq. inertia & friction of load side gear (gear-2) →

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\theta_2^o}{\theta_1^o} = \frac{\theta_2^o}{\theta_1^{oo}} ; \quad \theta_1^o = \left(\frac{N_2}{N_1}\right) \theta_2^o, \quad \theta_1^{oo} = \left(\frac{N_2}{N_1}\right) \theta_2^{oo}$$

$$T_m = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right) B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) \cdot \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

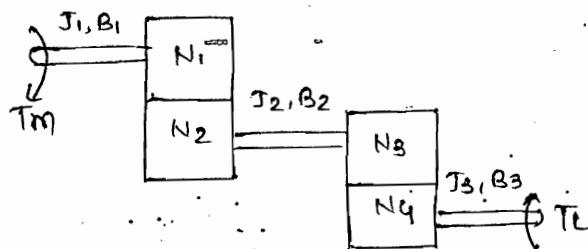
$$\left(\frac{N_2}{N_1}\right) T_m = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left[\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2\right] \frac{d^2\theta_2}{dt^2} + \left[\left(\frac{N_2}{N_1}\right)^2 B_1 + B_2\right] \frac{d\theta_2}{dt} + T_L$$

$$J_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 J_1 + J_2$$

$$B_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 B_1 + B_2$$

Q. →



Soln → (i) motor side gear →

$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 J_3$$

$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 B_3$$

(ii) Load side gear →

$$J_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 J_1 + \left(\frac{N_4}{N_3}\right)^2 J_2 + J_3$$

$$B_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 B_1 + \left(\frac{N_4}{N_3}\right)^2 B_2 + B_3$$

For NODAL METHOD →

(i) No. of nodes = No. of displacements.

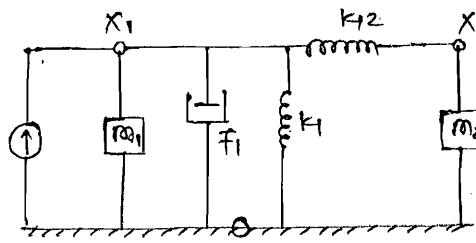
(ii) Take an additional node which is a ref node.

(iii) Connect the mass or inertia elements b/w the principal node & ref node only.

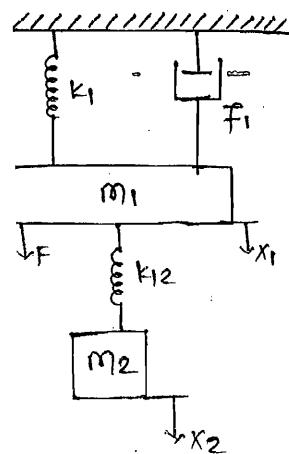
(iv) Connect an spring & damping elements either b/n the principal node  
(or) b/n the principal node & ref. depending on there position.

(v) Obtain the nodal dia. & write the describing diffn eqn at each node.

\* Nodal diagram  $\rightarrow$   
(mech. eq. diagram)



\* mech. system  $\rightarrow$



At Node X<sub>1</sub>

$$F = m_1 \frac{d^2x_1}{dt^2} + f_1 \frac{dx_1}{dt} + k_1 x_1 + k_{12}(x_1 - x_2)$$

at node X<sub>2</sub>

$$0 = m_2 \frac{d^2x_2}{dt^2} + k_{12}(x_2 - x_1)$$

\* Transfer function  $\rightarrow$

$$F(s) = [m_1 s^2 + f_1 s + k_1 + k_{12}] X_1(s) - k_{12} X_2(s)$$

$$0 = [m_2 s^2 + k_{12}] X_2(s) - k_{12} X_1(s)$$

$$X_2(s) = \begin{bmatrix} k_{12} \\ m_2 s^2 + k_{12} \end{bmatrix} X_1(s)$$

$$\boxed{\frac{X_1(s)}{X_2(s)} = \frac{m_2 s^2 + k_{12}}{(m_1 s^2 + f_1 s + k_1 + k_{12})(m_2 s^2 + k_{12}) - k_{12}^2}}$$

\* Order  $\rightarrow$

(1) mass element  $\rightarrow$  Order (2)

3 mass element  $\rightarrow$  order - 6

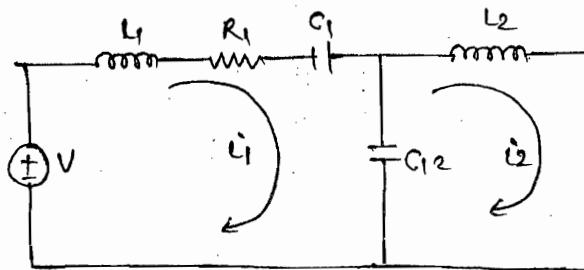
(2) mass element  $\rightarrow$  order - 4

n mass element  $\rightarrow$  Order - 2n

F-V Analogy →

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_{12}} \int (i_1 - i_2) dt$$

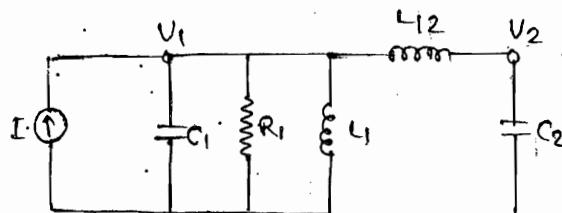
$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}} \int (i_2 - i_1) dt$$



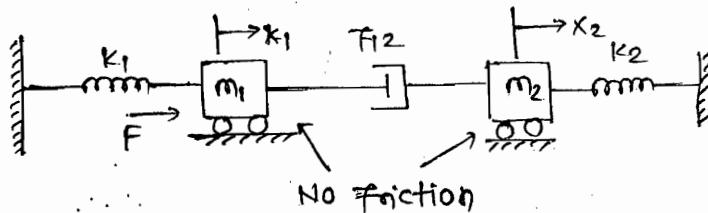
F-I Analogy →

$$I = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1} \int V_1 dt + \frac{1}{L_{12}} \int (V_1 - V_2) dt$$

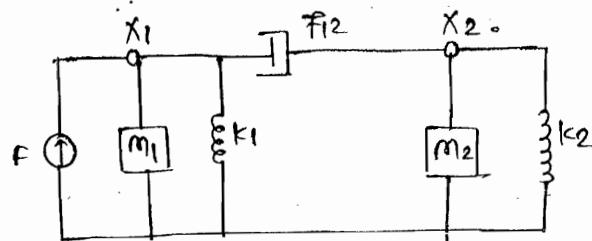
$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}} \int (V_2 - V_1) dt$$



Conv.(2)  
Q.(2)



801<sup>n</sup> → Nodal diagram → (mech. eq. dia.)



At node  $x_1 \rightarrow$

$$F = m_1 \frac{d^2x_1}{dt^2} + f_{12} \frac{d}{dt}(x_1 - x_2) + k_1 x_1$$

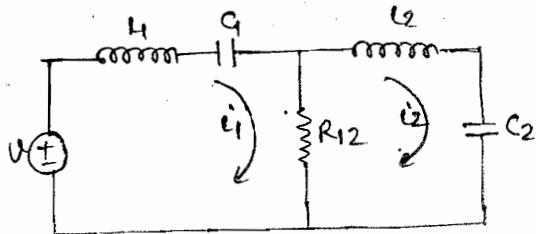
at node  $x_2 \rightarrow$

$$0 = m_2 \frac{d^2x_2}{dt^2} + f_{12} \frac{d}{dt}(x_2 - x_1) + k_2 x_2$$

F-V analogy  $\rightarrow$

$$V = L_1 \frac{di_1}{dt} + R_{12}(i_1 - i_2) + \frac{1}{C_1} \int i_1 dt$$

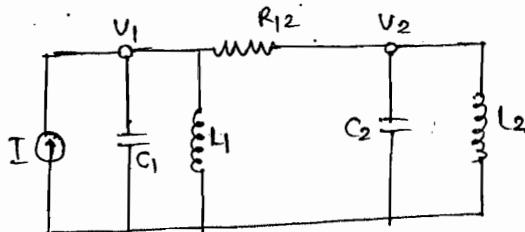
$$0 = L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2} \int i_2 dt$$



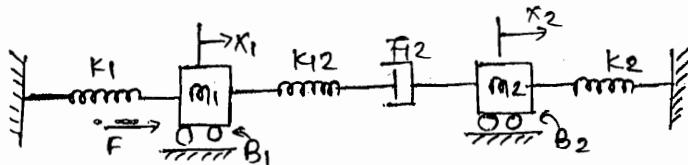
F-I analogy  $\rightarrow$

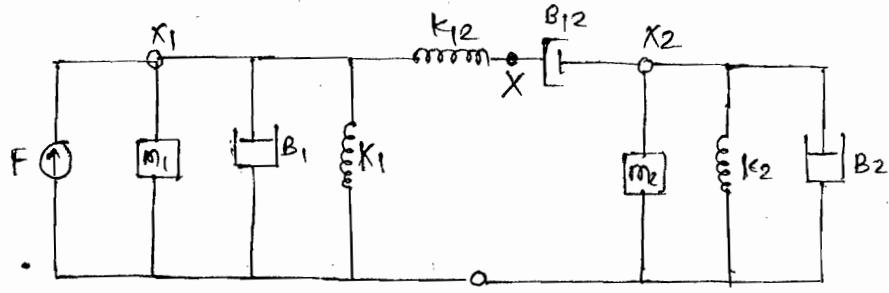
$$I = C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_1} \int V_1 dt$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_2} \int V_2 dt$$



Conv.  
Que.





at node \$X\_1 \rightarrow\$

$$F = m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1 + k_{12}(x_1 - x_2)$$

at dummy node \$X \rightarrow\$

$$0 = k_{12}(x - x_1) + B_{12} \frac{d}{dt}(x - x_2)$$

at Node \$X\_2 \rightarrow\$

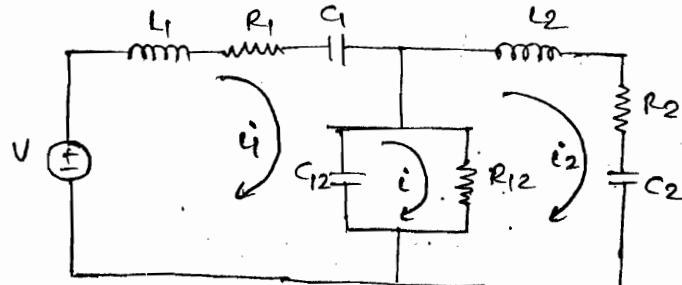
$$0 = m_2 \frac{d^2 x_2}{dt^2} + B_{12} \frac{dx_2}{dt} + k_2 x_2 + B_{12} \frac{d}{dt}(x_2 - x)$$

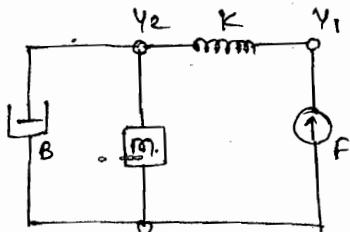
F-v analogy \$\rightarrow\$

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_{12}} \int (i_1 - i_2) dt$$

$$0 = \frac{1}{C_{12}} \int (i_1 - i_2) dt + R_{12} (i_1 - i_2)$$

$$0 = L_2 \frac{di_2}{dt} + R_{12} i_2 + \frac{1}{C_2} \int i_2 dt + R_{12} (i_2 - i_1)$$



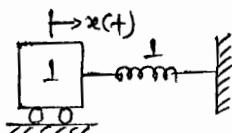
(Q.5  
55)at dummy node ( $y_1$ )

$$F = k(y_1 - y_2)$$

at node ( $y_2$ )

$$0 = \frac{md^2y_2}{dt^2} + B \frac{dy_2}{dt} + k(y_2 - y_1)$$

$$k(y_1 - y_2) = m \frac{d^2y_2}{dt^2} + B \frac{dy_2}{dt}$$

(4  
55)

$$F = m \frac{d^2x}{dt^2} + kx$$

Given  $m = k = 1$ 

$$F = \frac{d^2x}{dt^2} + x$$

$$F(s) = (s^2 + 1)x(s)$$

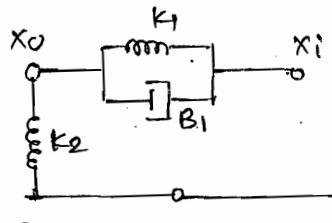
$$x(s) = \frac{1}{s^2 + 1} \cdot F(s)$$

Given

$$f(s) = \text{unit impulse force} = 1$$

$$x(s) = \frac{1}{s^2 + 1} \times 1$$

$$x(t) = \sin t$$

(3  
55)at node  $x_0 \rightarrow$ 

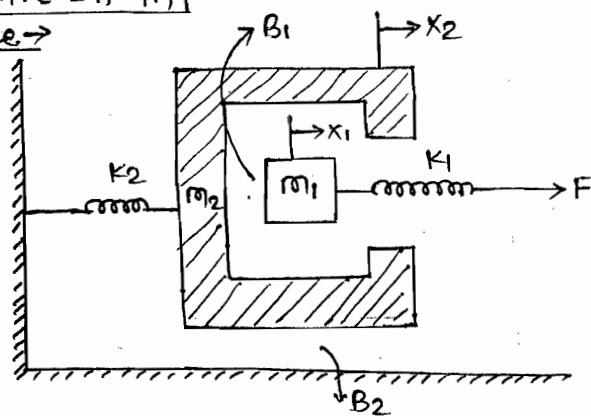
$$0 = k_2 x_0 + k_1(x_0 - x_1) + B_1 \frac{dx_0}{dt} (x_0 - x_1)$$

$$[k_2 + k_1 + B_1 s] x_0(s) = x_1(s) [B_1(s)] + 1$$

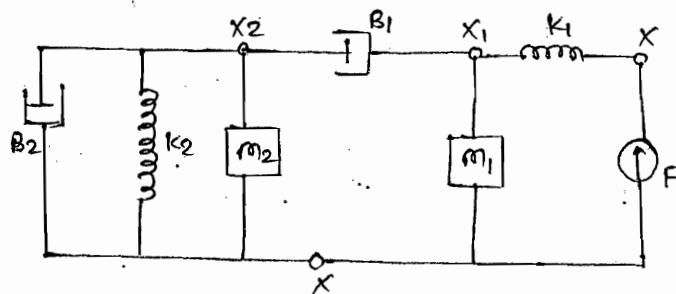
$$\frac{x_0(s)}{x_1(s)} = \frac{k_1 + B_1(s)}{(k_1 + k_2 + B_1 s)}$$

**DATE-14/11/14**

**Que →**



**SOL → Nodal diagram →**



At dummy node 'x' →

$$F = k_1(x - x_1) \quad \text{--- (1)}$$

at node 'x<sub>2</sub>' →

$$0 = m_2 \frac{d^2 x_2}{dt^2} + b_2 \frac{dx_2}{dt} + k_2 x_2 + b_1 \frac{dx_1}{dt} + k_1 (x_1 - x_2) \quad \text{--- (ii)}$$

$$k_1 (x - x_1) = m_1 \frac{d^2 x_1}{dt^2} + b_1 \frac{dx_1}{dt} (x_1 - x_2) \quad \text{--- (iii)}$$

$$F = m_1 \frac{d^2 x_1}{dt^2} + b_1 \frac{dx_1}{dt} (x_1 - x_2) \quad \text{--- (iv)}$$

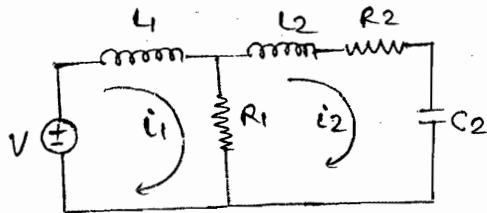
At node 'x<sub>2</sub>' →

$$0 = m_2 \frac{d^2 x_2}{dt^2} + b_2 \frac{dx_2}{dt} + k_2 x_2 + b_1 \frac{dx_1}{dt} (x_2 - x_1) \quad \text{--- (v)}$$

F-V analogy →

$$V = L_1 \frac{di_1}{dt} + R_1 (i_1 - i_2)$$

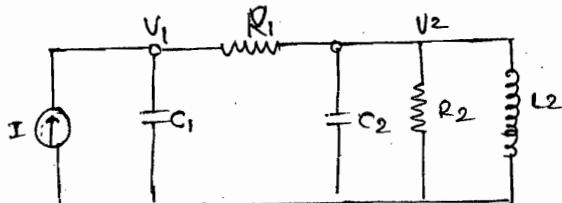
$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 dt + R_1 (i_2 - i_1)$$



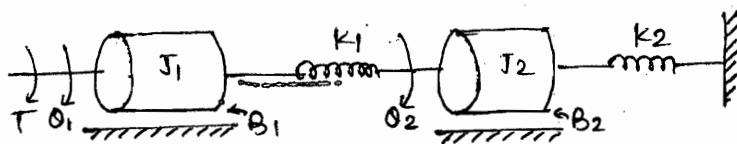
F-I Analogy →

$$I = C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_1}$$

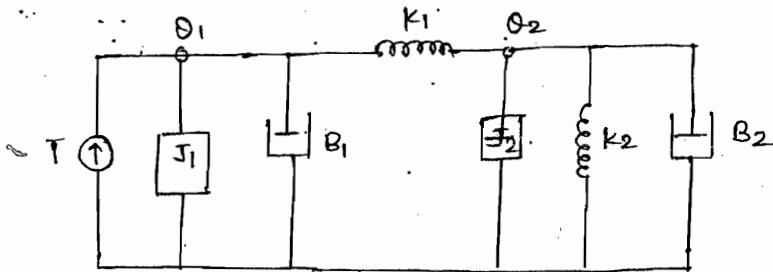
$$\Theta = C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2} \int V_2 dt + \frac{V_2 - V_1}{R_1}$$



Que. →



SOLN →



At node O1 →

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

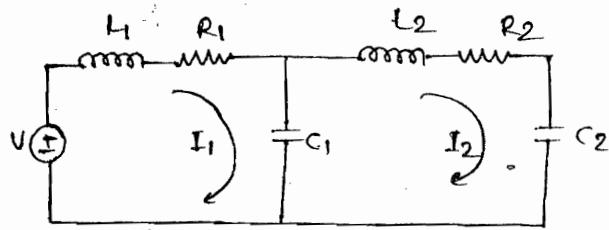
At node O2 →

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2 \theta_2 + K_1(\theta_2 - \theta_1)$$

T-U Analogy →

$$U = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2) dt$$

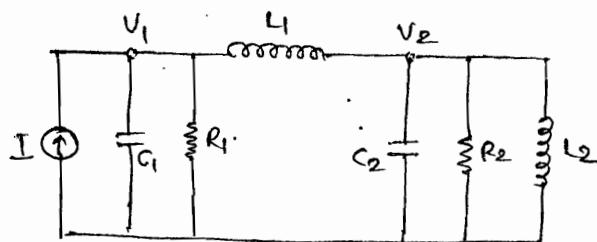
$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 dt + \frac{1}{C_1} \int (i_2 - i_1) dt$$



T-I analogy →

$$I = C_1 \left( \frac{dV_1}{dt} \right) + \left( \frac{V_1}{R_1} \right) + \frac{1}{L_1} \int (V_1 - V_2) dt$$

$$0 = C_2 \frac{dV_2}{dt} + \left( \frac{V_2}{R_2} \right) + \frac{1}{L_2} \int (V_2 - V_1) dt$$



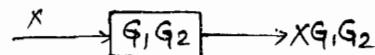
### Block diagram →

#### Rules

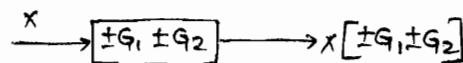
i) Combining blocks in series.



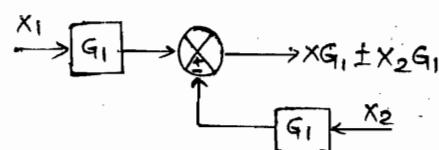
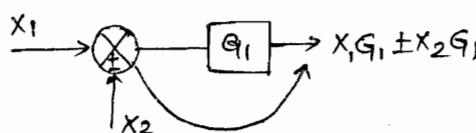
#### Eq. diagram



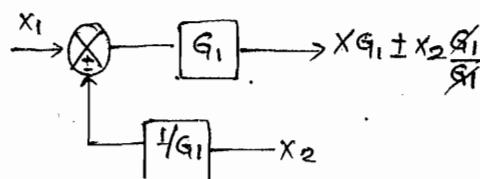
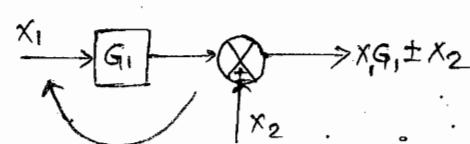
ii) Combining blocks in parallel.



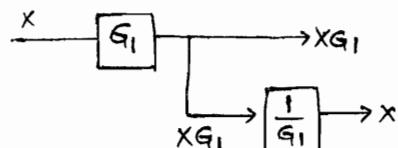
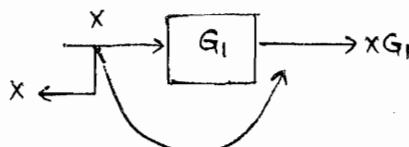
iii) Shifting the summing elements after the blocks.



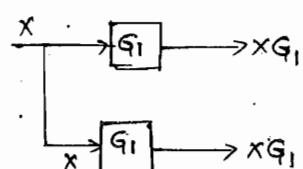
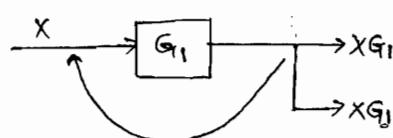
iv) Shifting the summing elements before blocks.



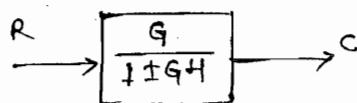
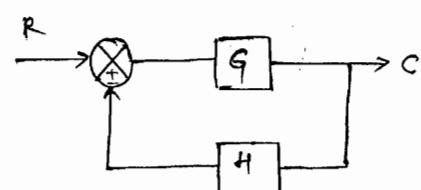
v) Shifting the take off point after the block.



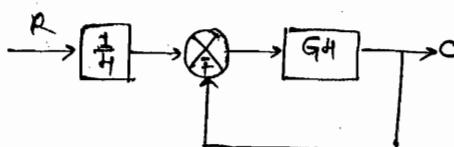
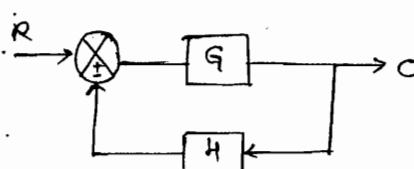
vi) —II— before the block.



vii) TF of CLCS.

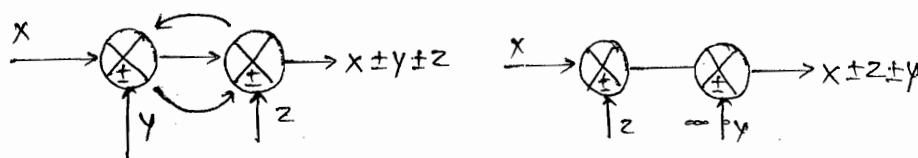


viii) Block dia. transformation



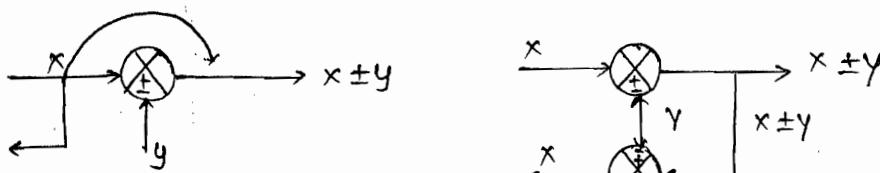
$$\frac{C}{R} = \frac{1}{H} \times \frac{GH}{1 \pm GH}$$

9.) Interchanging  
the summing ele-  
ments.

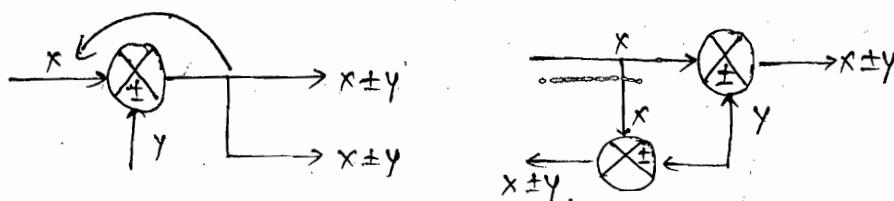


CRITICAL RULE →

10.) Shifting the  
take-off point after  
the summing element



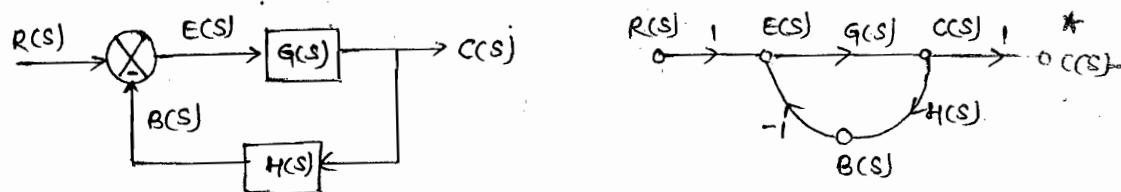
III — II — Before



- (7) (d)      (8) (b)      (9) (c)

\* SIGNAL FLOW GRAPH → It is the graphical representation of CS in which nodes representing each of the system variable & are connected by direct branches.

SFG for CLCS →



Terminology of SFG's →

(i) Node— It represents sys. variable & is equal to the sum of all incoming signal at it.

(ii) I/p Node (OR) Source node→ It is a node having only outgoing branches

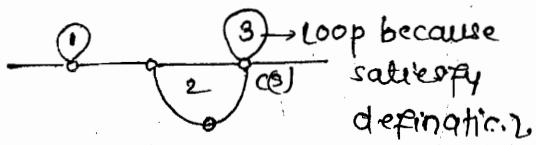
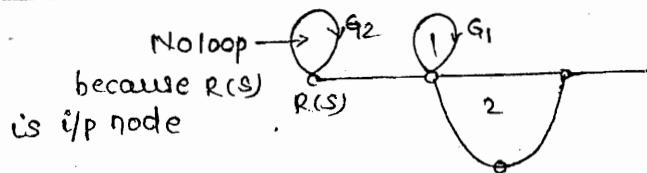
(iii) O/p Node (OR) Sink node→ It is a node having only incoming branches.

(4) Mix (or) chain node → It is a node having both incoming & outgoing branches.

(5) Path → It is the traversal of the connected branches in the direction of branch arrow such that no node is traversed more than one.

(6) forward path → It is a path from i/p node to o/p node.

(7) Loop → It is a path which starts & ends at the same node.



Note: Self loops on the defined i/p nodes are not valid loops.

Loops (or) self-loops on the defined o/p nodes are valid loops.

(8) Non-touching loops → 2 (or) more loops are said to be non-touching if they do not have a common node.

\* MASON'S GAIN Formula →

$$\text{The overall Gain} = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + \dots + P_R \Delta_R}{\Delta}$$

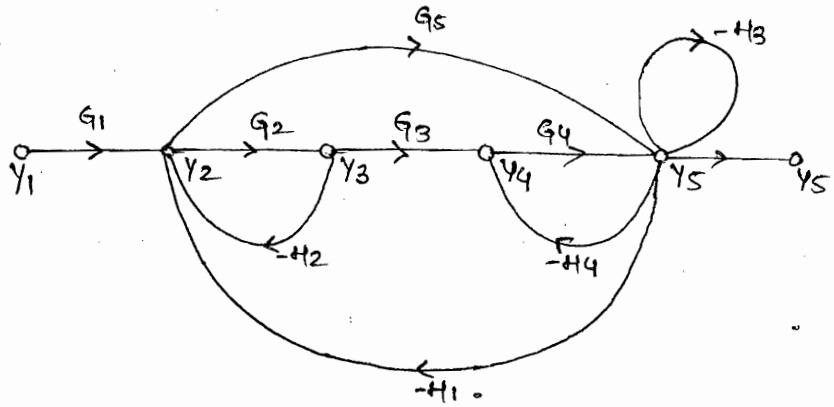
(OR)  
transfer function

where;  $P_k$  = Path gain of  $k^{\text{th}}$  forward path,

$$\Delta = 1 - \left[ \begin{array}{l} \text{sum of all gains} \\ \text{of all individual} \\ \text{loops} \end{array} \right] + \left[ \begin{array}{l} \text{sum of gain products} \\ \text{of 2 non-touching} \\ \text{loops} \end{array} \right]$$

$$- \left[ \begin{array}{l} \text{sum of gain products} \\ \text{of 3 non-touching loops} \end{array} \right] + \dots$$

$\Delta_k$  = It is that value of  $\Delta$  obtained by removing all the loops touching  $k^{\text{th}}$  forward path.



$$\underline{\text{Case-(i)}} \quad \frac{Y_5}{Y_1}$$

(i) Forward paths

$$P_1 = G_1 G_2 G_3 G_4$$

$$P_2 = G_1 G_5$$

(ii) To find  $\Delta \rightarrow$

(a) Individual loops

$$I_1 = -G_2 H_2$$

$$I_3 = -H_3$$

$$I_5 = -G_5 H_1$$

$$I_2 = -G_4 H_4$$

$$I_4 = -G_2 G_3 G_4 H_1$$

(b) Two NTL's  $\rightarrow$

$$L_1 = I_1 I_2 = G_2 H_2 = G_2 H_2 H_4 G_4$$

$$L_2 = I_1 I_3 = G_2 H_2 H_3$$

(iii) To find  $\Delta_1$  &  $\Delta_2$

$$\Delta_1 = \Delta_2 = 1$$

$$\frac{Y_5}{Y_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \quad \frac{Y_5}{Y_1} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{\Delta} \quad (\text{i})$$

$$\Delta = 1 - \left[ -G_2 H_2 - G_4 H_4 - H_3 \right] - \left[ -G_2 G_3 G_4 H_1 - G_5 H_1 \right] + \left[ G_2 H_2 G_4 H_4 \right] + \left[ G_2 H_2 H_3 \right]$$

$$\underline{\text{Case(ii)}} \rightarrow \text{To find } \frac{Y_5}{Y_3} = \frac{\left( \frac{Y_5}{Y_1} \right)}{\left( \frac{Y_3}{Y_1} \right)} \quad (\text{ii})$$

To find  $\frac{Y_3}{Y_1}$

(i) forward path

$$P_1 = G_1 G_2$$

(ii) To find  $\Delta$

$\Delta$  is independant of forward path

(iii) To find  $\Delta_1$

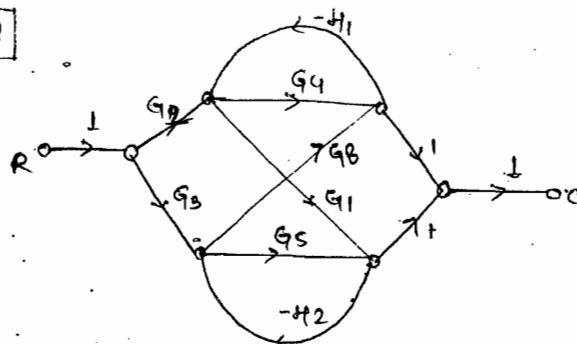
$$\Delta_1 = 1 - (G_4 H_4 + H_3)$$

$$\frac{Y_3}{Y_1} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 (1 + G_4 H_4 + H_3)}{\Delta} \quad \text{--- (2)}$$

$$\frac{Y_C}{Y_3} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{G_1 G_2 (1 + G_4 H_4 + H_3)} \quad \text{--- (3)}$$

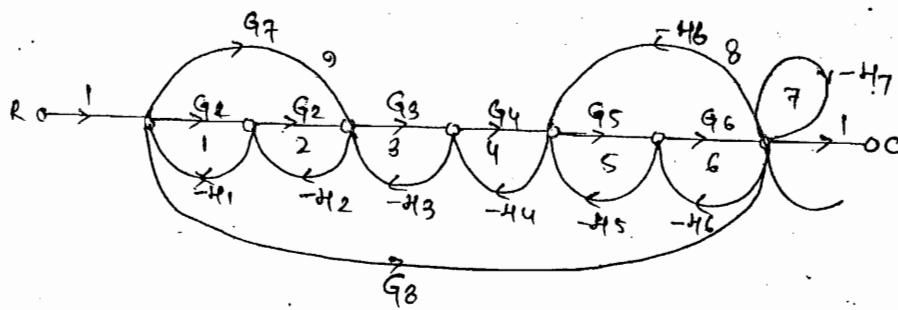
TE-15/01/14

UV  
(4.1)  
8



$$\frac{C}{R} = \frac{G_2 G_4 (1 + G_5 H_2) + G_3 G_5 (1 + G_4 H_1) + G_2 G_1 (1) + G_3 G_8 (1) + G_2 G_1 (-H_2) G_8 (1) + G_3 G_8 (-H_1) G_1 (1)}{1 - [-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2] + [G_4 H_1 G_5 H_2]}$$

Q.



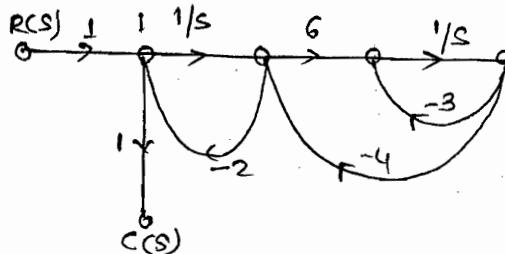
$$\text{Sol} \rightarrow L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(H_1)$$

$$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$$

No. of forward paths = 3

No. of loops = 11

Q(11)  
56



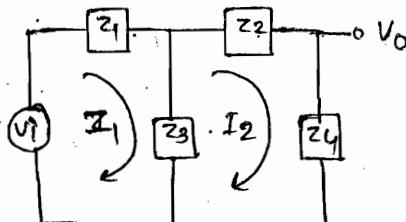
$$\text{calc. } \frac{C(s)}{R(s)} = ?$$

Sol →

$$\frac{C(s)}{R(s)} = \frac{1 \left\{ 1 - \left( \frac{-3}{s} - \frac{24}{s^2} \right) \right\}}{1 - \left[ \frac{-2}{s} - \frac{3}{s} - \frac{24}{s^2} \right] + \frac{6}{s^2}}$$

$$= \frac{\frac{s+27}{s}}{\frac{s^2+29s+6}{s^2}} = \frac{s(s+27)}{s^2+29s+6}$$

Q(12)  
56



$$\text{calc. } G_2 \& H = ?$$

Sol → Loop-(1) →

$$V_i = I_1 z_1 + (I_1 - I_2) z_3$$

$$V_i = I_1(z_1 + z_3) - I_2 z_3$$

$$I_1 = \frac{V_i}{z_1 + z_3} + \frac{I_2 z_3}{z_1 + z_3}$$

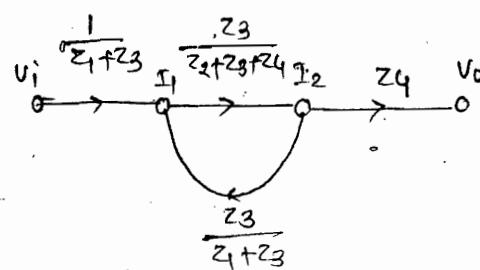
Consider the variables as node in the given que.  
ie,

loop(2) →

$$0 = I_2 z_2 + I_2 z_4 + (I_2 - I_1) z_3$$

$$I_2(z_2 + z_3 + z_4) = I_1 z_3$$

$$I_2 = I_1 \cdot \frac{z_3}{z_2 + z_3 + z_4}$$



$$\boxed{G_2 = \frac{z_3}{z_2 + z_3 + z_4}}$$

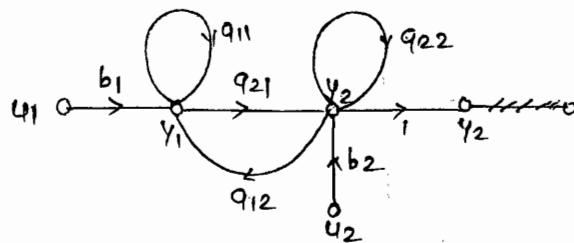
$$\boxed{H = \frac{z_3}{z_1 + z_3}}$$

Q. → Find  $\frac{Y_2}{U_1}$  &  $\frac{Y_2}{U_2}$  using SFG

$$Y_1 = q_{11}Y_1 + q_{12}Y_2 + b_1 U_1$$

$$Y_2 = q_{21}Y_1 + q_{22}Y_2 + b_2 U_2$$

Soln →



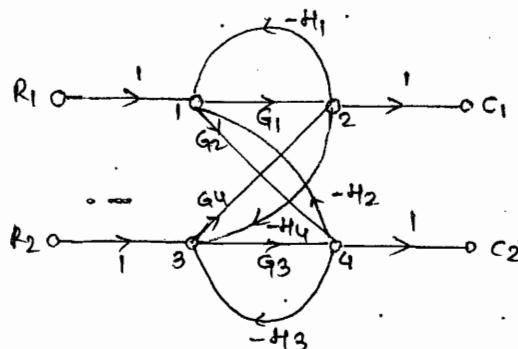
Case(1) →  $\frac{Y_2}{U_1} \Big|_{U_2=0}$

$$\frac{Y_2}{U_1} = \frac{b_1 q_{21}(1)}{1 - (q_{11} + q_{22} + q_{12} q_{21}) + (q_{11} q_{22})}$$

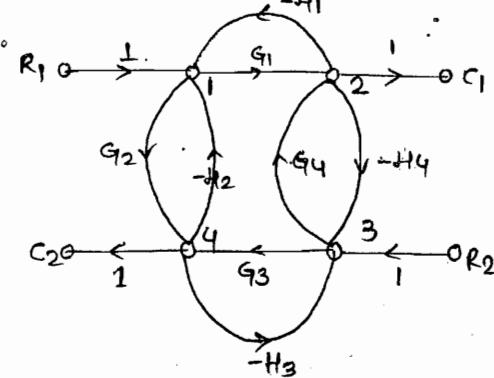
Case(2)  $\frac{Y_2}{U_2} \Big|_{U_1=0}$

$$\frac{Y_2}{U_2} = \frac{b_2(1-q_{11})}{1 - (q_{11} + q_{22} + q_{12} q_{21}) + (q_{11} q_{22})}$$

Q. →



Soln →



Case(1)  $\frac{C_1}{R_1} \Big|_{R_2=C_2=0}$

$$\frac{C_1}{R_1} = \frac{G_1(1+G_3H_3) + G_2(-H_3)G_4(1)}{1 + [-G_1H_1 - G_4H_4 - G_2H_2 - G_3H_3 + G_2H_3G_4H_1 + G_1H_2G_3H_4] + (G_1H_1G_3H_3 + G_2H_2G_4H_4)}$$

$$\underline{\text{Case(2)}} \quad \left| \frac{C_2}{R_2} \right|_{C_1=R_1=0} ; \quad \frac{C_2}{R_2} = \frac{G_3(1+G_4H_1) + G_4(-H_1)G_2(1)}{\Delta}$$

$$\underline{\text{Case(3)}} \quad \left| \frac{C_2}{R_1} \right|_{C_1=R_2=0} ; \quad \frac{C_2}{R_1} = \frac{G_3(1+G_4H_4) + G_4(-H_4)G_3(1)}{\Delta}$$

$$\underline{\text{Case(4)}} \quad \left| \frac{C_1}{R_2} \right|_{C_2=R_1=0} ; \quad \frac{C_1}{R_2} = \frac{G_4(1+G_2H_2) + G_3(-H_2)G_1(1)}{\Delta}$$

(contd.)  
58

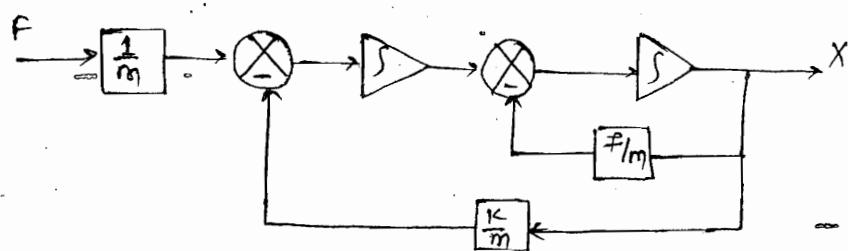
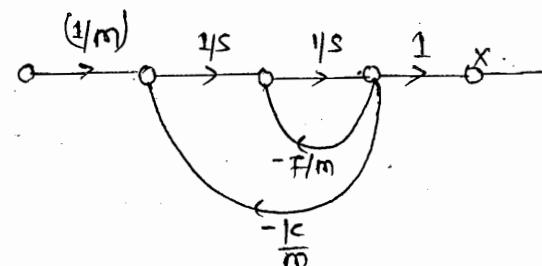
$$F = m \frac{d^2x}{dt^2} + f \frac{dx}{dt} + kx$$

$$F(s) = (ms^2 + fs + k) X(s)$$

$$\begin{aligned} \frac{X(s)}{F(s)} &= \frac{1}{ms^2 + fs + k} \\ &= \frac{(1/m)}{s^2 + (\frac{f}{m})s + (\frac{k}{m})} \end{aligned}$$

State diagram →

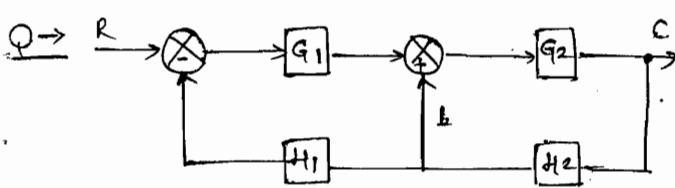
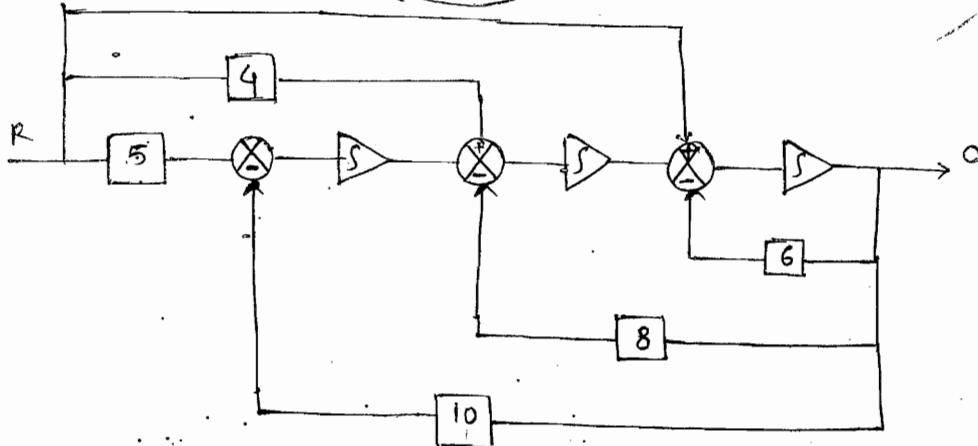
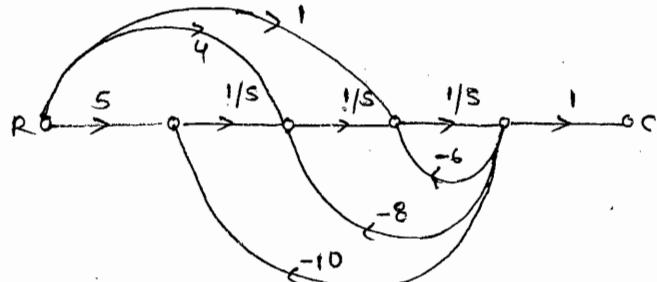
$$\begin{aligned} &\frac{(1/m)}{s^2} \\ \frac{1}{&1 + \frac{(f/m)}{s} + \frac{(k/m)}{s^2}} \\ &\frac{(1/m)}{s^2} \\ \frac{1 - \left[ \frac{-f/m}{s} - \frac{k/m}{s^2} \right]}{&} \end{aligned}$$



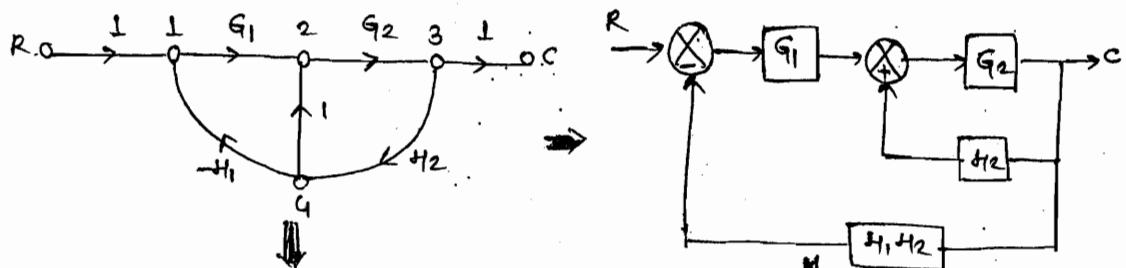
$$Q \rightarrow \frac{C(s)}{R(s)} = \frac{s^2 + 4s + 5}{s^3 + 6s^2 + 8s + 10}$$

SOL<sup>n</sup>

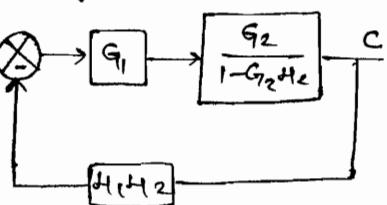
$$\frac{C(s)}{R(s)} = \frac{\frac{s^2}{s^3} + \frac{4s}{s^3} + \frac{5}{s^3}}{1 + \frac{6s^2}{s^3} + \frac{8s}{s^3} + \frac{10}{s^3}} = \frac{\frac{1}{s} + \frac{4}{s^2} + \frac{5}{s^3}}{1 - \left( \frac{-6}{s} - \frac{8}{s^2} - \frac{10}{s^3} \right)}$$



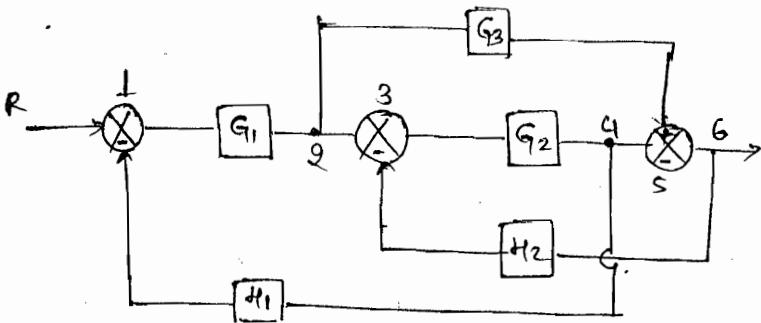
SOL<sup>n</sup>



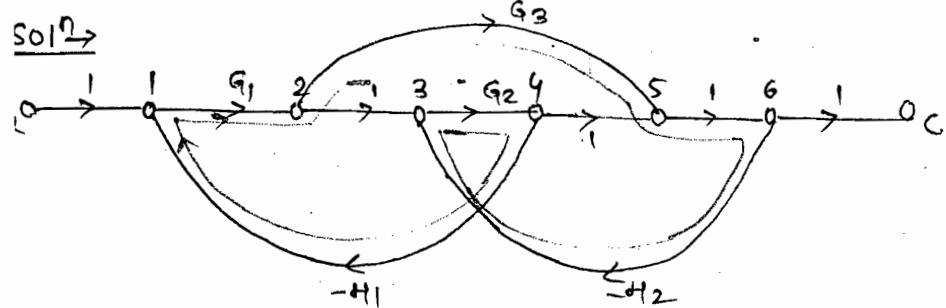
$$\frac{C}{R} = \frac{G_1 G_2 (1)}{1 - (G_2 H_2 - G_1 G_2 H_1 H_2)} = \frac{G_1 G_2}{1 - G_2 H_2 + G_1 G_2 H_1 H_2}$$



1(conv)  
56

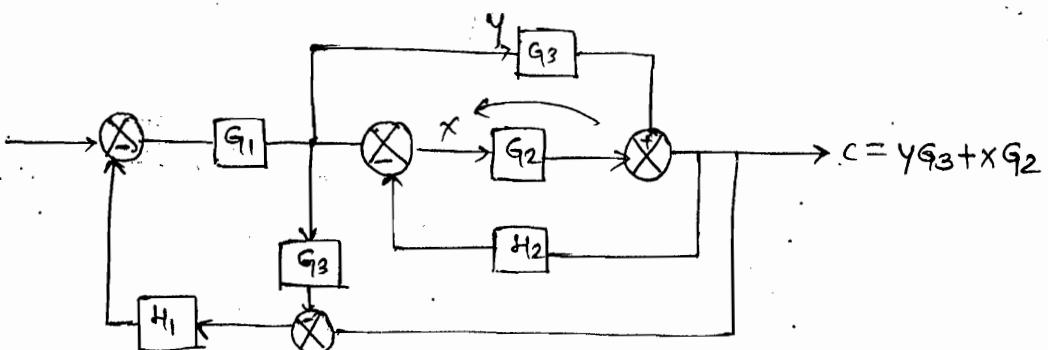
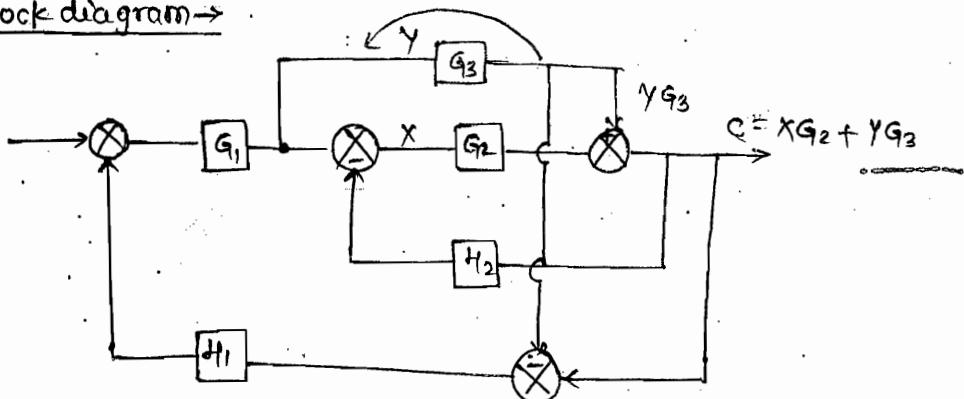


SOLN

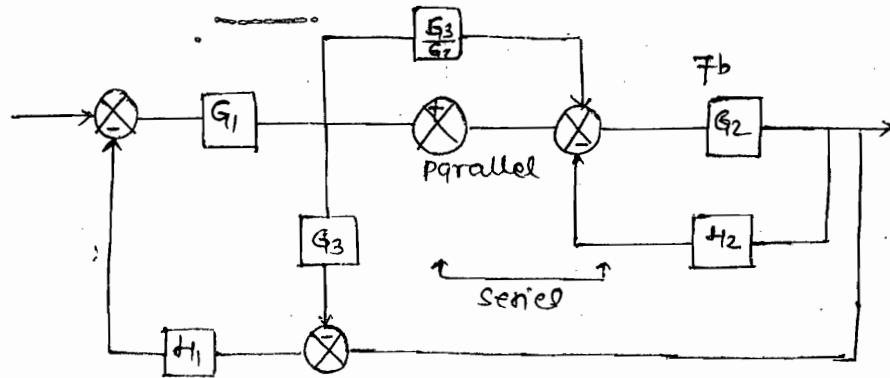
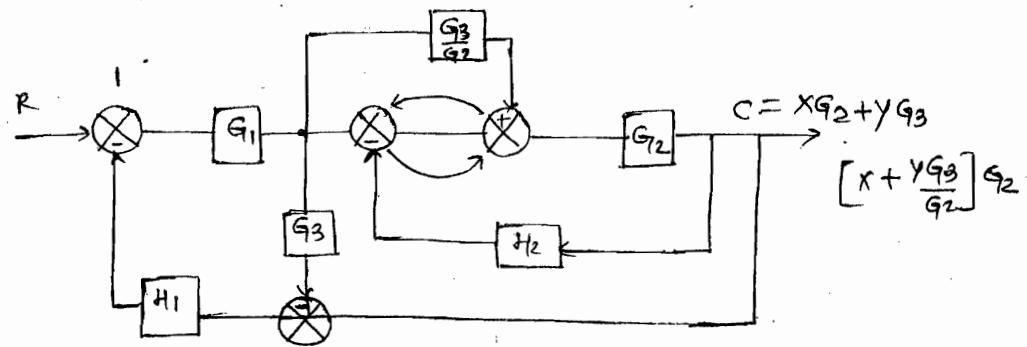


$$\frac{C}{R} = \frac{G_1 G_2 (1) + G_1 G_3 (1)}{1 - (-G_1 G_2 H_1 - G_2 H_2 + G_1 G_2 G_3 H_1 + H_2)}$$

Block diagram →



Sagor Nagpal  
8871453536

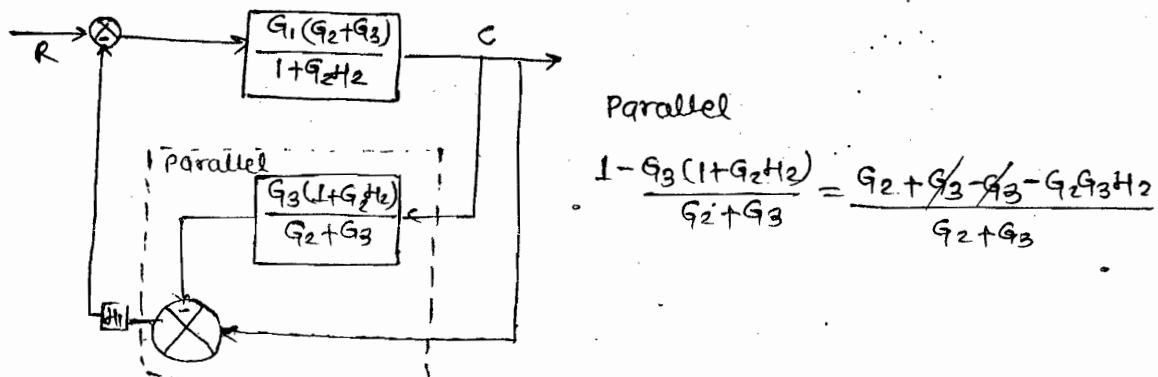
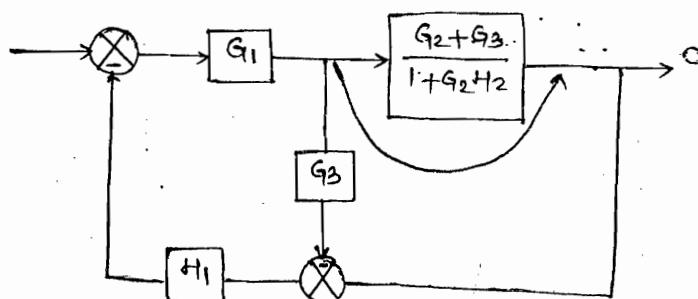


$$\text{Parallel} = 1 + \frac{G_3}{G_2}$$

$$= \frac{G_2 + G_3}{G_2}$$

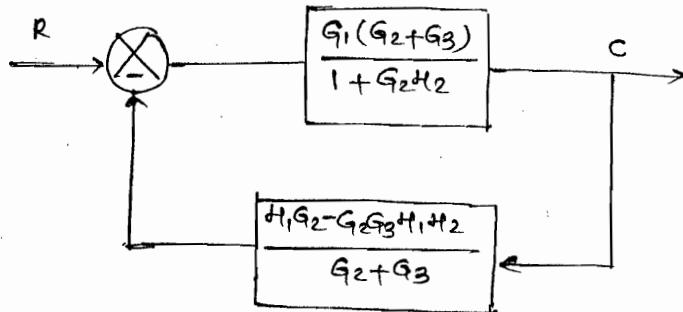
$$\text{feedback} = \frac{G_2}{1 + G_2 H_2}$$

$$\text{series} = \frac{G_2 + G_3}{1 + G_2 H_2}$$



Parallel

$$\frac{1 - G_3(1 + G_2 H_2)}{G_2 + G_3} = \frac{G_2 + G_3 - G_2 G_3 - G_2 G_3 H_2}{G_2 + G_3}$$



$$\frac{C}{R} = \frac{G_1(G_2+G_3)}{1+G_2H_2+G_1G_2H_1-G_1G_2G_3H_1H_2}$$

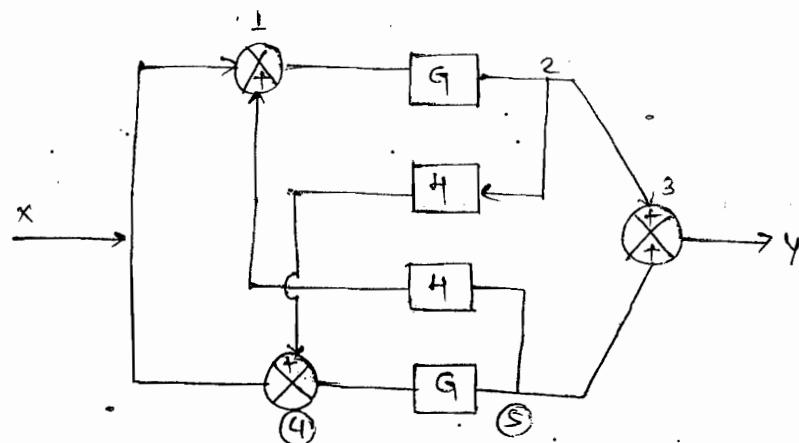
Q.  $\frac{Y}{X}$  equals

(a.)  $\frac{2G}{1-2GH}$

(b.)  $\frac{2G}{1-GH}$

(c.)  $\frac{G}{1-2GH}$

(d.)  $\frac{G}{1-GH}$

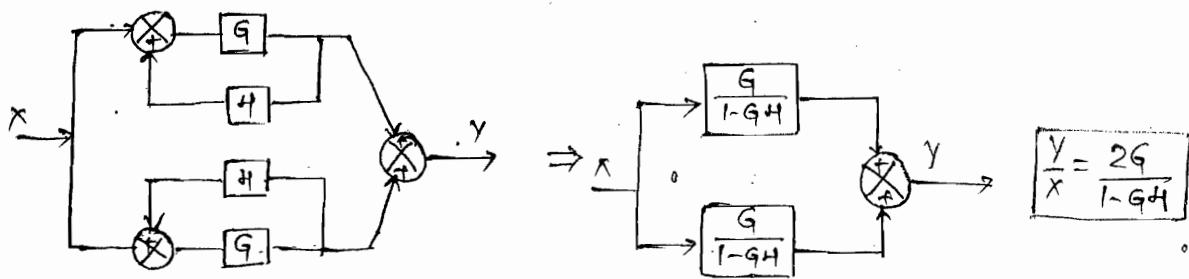


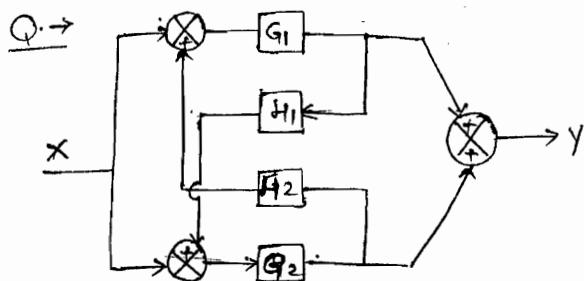
SOL

$$\frac{Y}{X} = \frac{G(1) + G(1) + G^2H(1) + G^2H(1)}{1 - G^2H^2} = \frac{2G + 2G^2H}{1 - G^2H^2} = \frac{2G(1 + GH)}{(1 + GH)(1 - GH)}$$

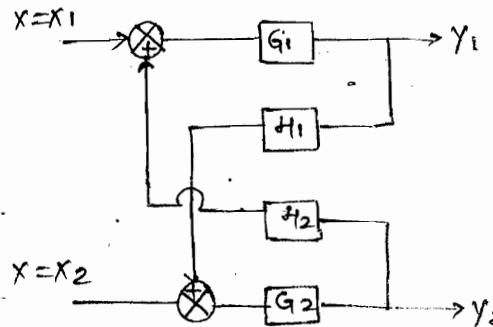
$$\boxed{\frac{Y}{X} = \frac{2G}{1 - GH}}$$

By Block dia. reduction  $\rightarrow$

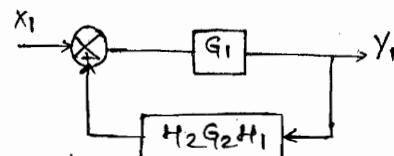


SOLN

$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

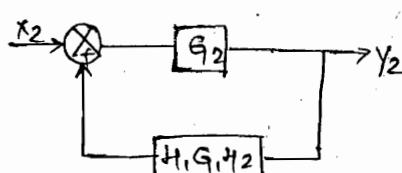
By Block diagram →

$$\text{case(i)} \rightarrow \frac{Y_1}{X_1} \Big|_{Y_2=0} \quad X_2 = 0$$



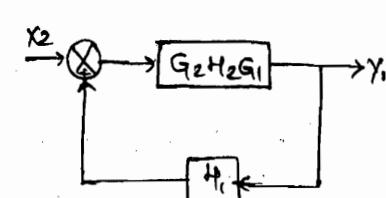
$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1 H_1 G_2 H_2}$$

$$\text{case(2)} \rightarrow \frac{Y_2}{X_2} \Big|_{Y_1=0} \quad X_1 = 0$$



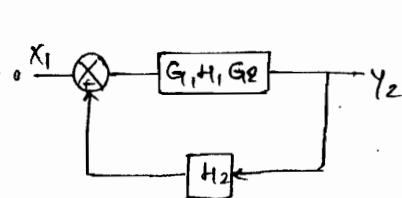
$$\frac{Y_2}{X_2} = \frac{G_2}{1 - G_1 H_1 G_2 H_2}$$

$$\text{Case(3)} \rightarrow \frac{Y_1}{X_2} \Big|_{Y_2=0} \quad X_1 = 0$$



$$\frac{Y_1}{X_2} = \frac{G_1 H_1 G_2}{1 - G_1 H_1 G_2 H_2}$$

$$\text{Case(4)} \rightarrow \frac{Y_2}{X_1} \Big|_{X_2=Y_1=0} \quad X_1 = 0$$



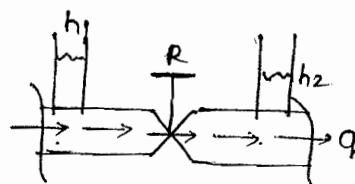
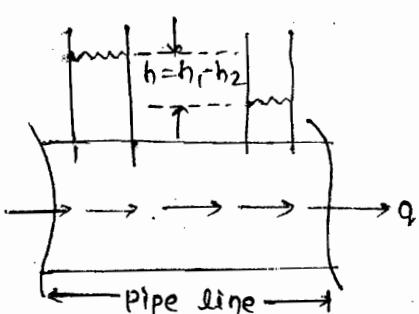
$$\frac{Y_2}{X_1} = \frac{G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

## TF for physical systems →

\* A general physical sys. is said to be constituted of 5 elements :-

- 1) Resistance type element.
- 2) Capacitance type elemet.
- 3) Time-constant element.
- 4) Oscillatory element.
- 5) Dead time element.

### (1) Resistance type element →

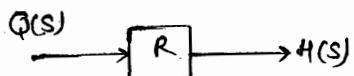


$$h \propto q$$

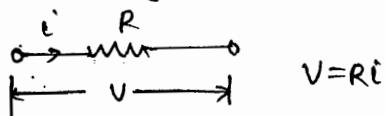
$$h = Rq$$

$R$  = Hydraulic Resistance

$$H(s) = RQ(s)$$

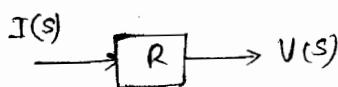


It is analogous to electrical resistance

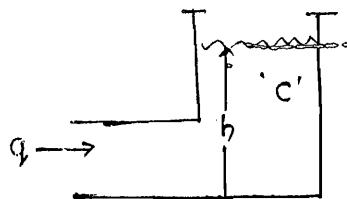


$$V = RI$$

$$V(s) = RI(s)$$



### (2.) Capacitance type element →



$C = \text{Hydraulic capacitance}$   
 $= (\text{Area} / \text{volume})$

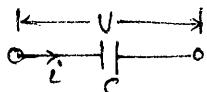
$$q \propto \frac{dh}{dt}$$

$$q = C \frac{dh}{dt}$$

$$Q(s) = CS H(s)$$

$$Q(s) \xrightarrow{\frac{1}{CS}} H(s)$$

It is analogous to "electrical capacitance"

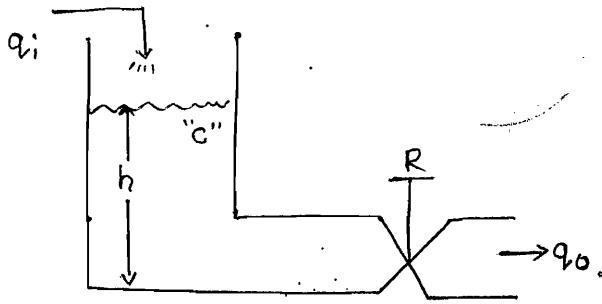


$$i = C \frac{dv}{dt}$$

$$I(s) = CS V(s)$$

$$I(s) \xrightarrow{\frac{1}{CS}} V(s)$$

### 3.) Time-constant element →



$$q_i - q_o = \frac{cdh}{dt}$$

$$q_i = \frac{cdh}{dt} + q_o$$

since  $h = Rq_o$

$$q_o = \frac{h}{R}$$

$$q_i = C \frac{dh}{dt} + \frac{h}{R}$$

$$Rq_i = R C \frac{dh}{dt} + h$$

$$RQ_i(s) = (RCs + 1) H(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1}$$

It is analogous to RC  $\eta/\omega$

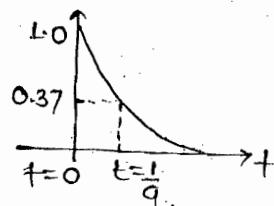
$$V_i(s) = I(s) \left[ R + \frac{1}{Cs} \right]$$

$$= I(s) \left[ \frac{Rcs + 1}{Cs} \right]$$

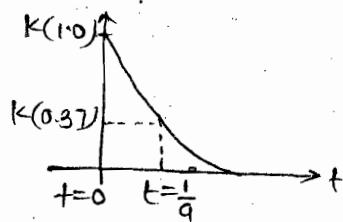
$$V_o(s) = I(s) \frac{1}{Cs}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{Rcs + 1}}$$

Eq:-  $\frac{1}{(s+q)} \Rightarrow e^{-qt} \Rightarrow$

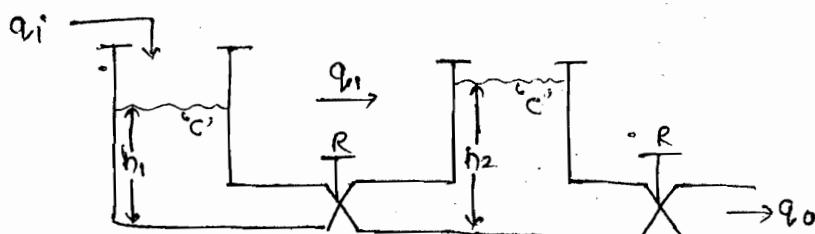


$$\frac{k}{s+q} \Rightarrow ke^{-qt} \Rightarrow$$



#### 4) Oscillatory element $\rightarrow$

#### \* Interacting & non-interacting system $\rightarrow$



$$q_i - q_1 = \frac{cdh_1}{dt}$$

$$Q_1(s) - Q_i(s) = CsH_1(s) \quad \text{--- (i)}$$

$$h_1 - h_2 = Rq_1$$

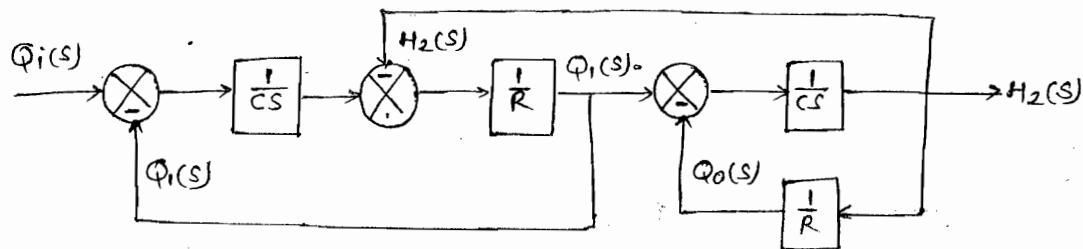
$$H_1(s) - H_2(s) = RQ_1(s) \quad \text{--- (ii)}$$

$$q_1 - q_0 = \frac{cdh_2}{dt}$$

$$Q_1(s) - Q_0(s) = CsH_2(s) \quad \text{--- (iii)}$$

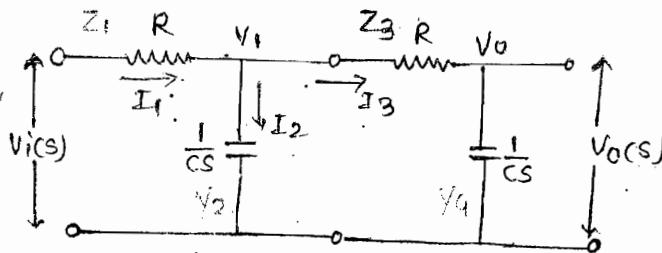
$$h_2 = Rq_0$$

$$H_2(s) = RQ_0(s) \quad \text{--- (iv)}$$



$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{R^2 C^2 s^2} (1)}{1 - \left[ \frac{-1}{R C S} - \frac{1}{R C S} - \frac{1}{R C S} \right] + \frac{1}{R^2 C^2 s^2}} = \frac{\frac{1}{R^2 C^2 s^2}}{\frac{R^2 C^2 s^2 + 3 R C S + 1}{R^2 C^2 s^2}}$$

$$\boxed{\frac{H_2(s)}{Q_i(s)} = \frac{R}{R^2 C^2 s^2 + 3 R C S + 1}}$$



(1.) Current through  $y_4$  ( $I_3$ ) =  $V_o \cdot y_4 = V_o C S$ .

(2.) To find  $V_1$

$$I_3 = \frac{V_1 - V_0}{Z_3} \Rightarrow V_1 = I_3 Z_3 + V_0 = V_1 = V_0 C S \cdot R + V_0$$

$$V_1 = V_0 (1 + R C S)$$

(3.) To find  $I_2$

$$I_2 = V_1 \cdot y_2 = V_0 (1 + R C S) \cdot C S \Rightarrow V_0 (C S + R C^2 S^2)$$

(4.) To find  $I_1$

$$I_1 = I_2 + I_3 = V_0 (C S + R C^2 S^2) + V_0 C S = V_0 (2 C S + R C^2 S^2)$$

(5.) To find  $V_i$

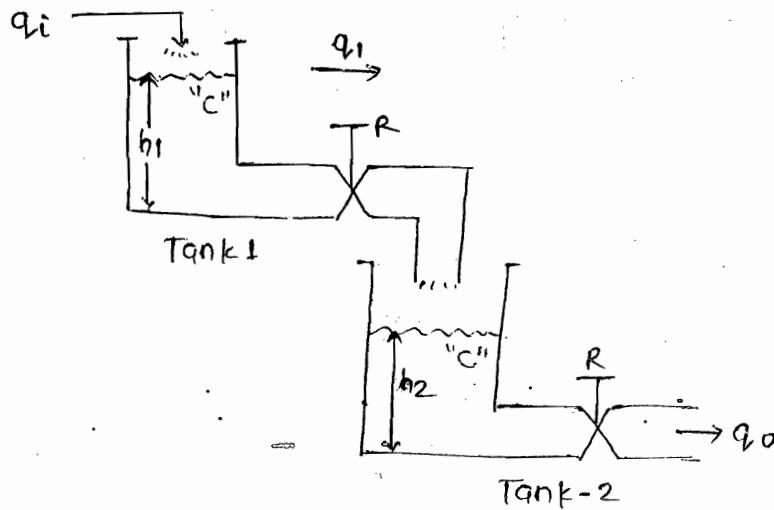
$$I_1 = \frac{V_i - V_1}{Z_1} \Rightarrow V_i = I_1 Z_1 + V_1 \Rightarrow V_i = V_0 (2 C S + R C^2 S^2) R + V_0 (1 + R C S)$$

$$V_i = V_0 (2 R C S + R^2 C^2 S^2 + 1 + R C S)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{R^2 C^2 S^2 + 3 R C S + 1}}$$

Note → When 2 time constant elements are cascaded interactively the overall TF of such an arrangement is not the product of 2 individual TF.

\* Non-interacting system →



Tank (01)

$$q_i - q_1 = \frac{cdh_1}{dt}$$

$$q_i = \frac{cdh_1}{dt} + q_1$$

$$\text{Since } h_1 = Rq_1$$

$$q_1 = h_1$$

$$q_i = \frac{cdh_1}{dt} + \frac{h_1}{R}$$

$$Rq_i = \frac{cdh_1}{dt} + h_1$$

$$Rq_i(s) = (Rcs+1) H_1(s) \quad \dots \text{(i)}$$

Tank (02)

$$q_1 - q_o = \frac{cdh_2}{dt}$$

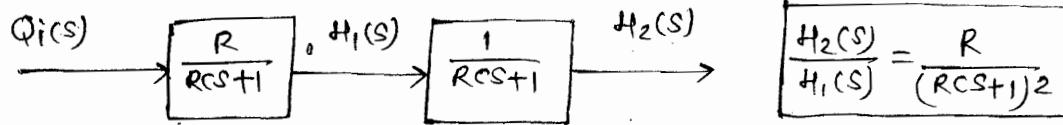
$$q_1 = \frac{cdh_2}{dt} + q_o$$

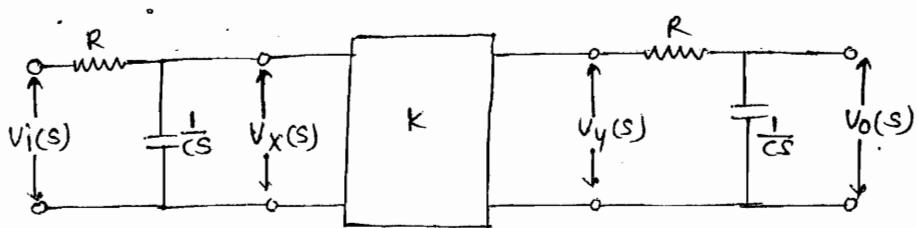
$$\frac{h_1}{R} = \frac{cdh_2}{dt} + \frac{h_2}{R}$$

$$(\because h_2 = Rq_o)$$

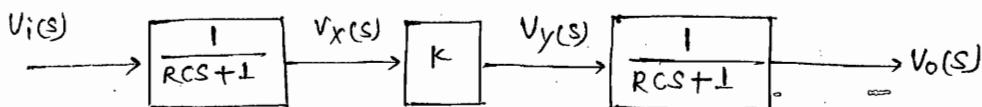
$$h_1 = RC \frac{dh_2}{dt} + h_2$$

$$H_1(s) = (Rcs+1) H_2(s) \quad \dots \text{(ii)}$$





$$\frac{V_x(s)}{V_i(s)} = \frac{1}{RCS+1} \quad | \quad K = \frac{V_y(s)}{V_x(s)} \quad | \quad \frac{V_o(s)}{V_y(s)} = \frac{1}{RCS+1}$$



$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{k}{(RCS+1)^2}}$$

Note → When 2 time constant elements are cascaded non-interactively the overall TF of such an arrangement is the product of 2 individual TF.

\* Control sys. components → CS, there are 2 types:-

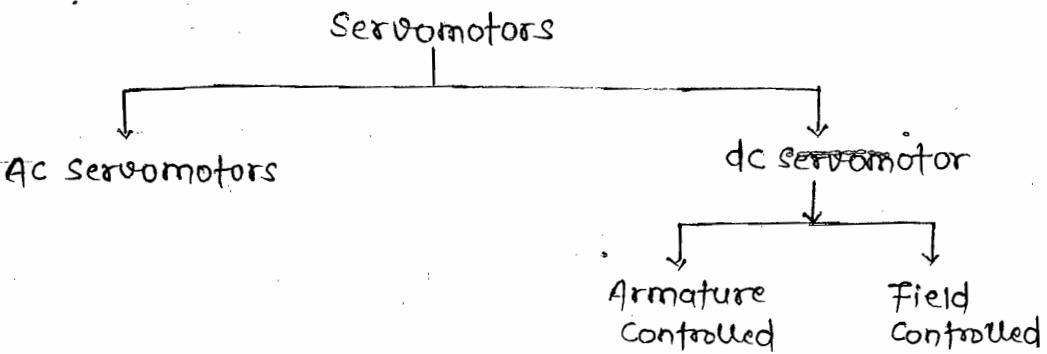
(1) Transducers → A Xcer is a device which when actuated with one form of energy is capable of converting it into any other related form; the conversion is usually from non-electrical to electrical.

(2) Servo mechanisms → They are electromech. sys. whose i/p is elec. voltage & o/p is mech. position (or) speed.

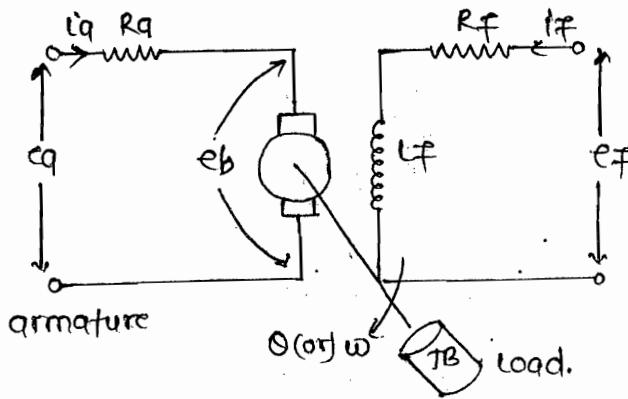
\* They are also known as inverse Xcers.

Servo - low power low freq. application

\* The term servo indicates low power & low freq. appli? It also indicates that the i/p & o/p c/s must be approximately linear.



### \* DC Servomotor →



### - Armature Controlled dc servomotors →

$$\text{Input} = e_q ; \text{output} = \Theta (\text{or}) \omega$$

### Principle of operation →

$$(i) \text{ Air gap flux } \propto \text{field current } (i_f)$$

$$\phi = k_f i_f$$

$$(ii) T_m \propto \phi \cdot i_q$$

$$T_m \propto k_f \cdot i_f \cdot i_q$$

$$T_m = k_f k_f \cdot i_f \cdot i_q$$

$k_f$  = motor torque const.

$$T_m = k_f i_q$$

$$T_m(s) = k_f I_q(s) \quad (i)$$

$$(3.) \text{ Back emf } (e_b) \propto \text{speed } (w)$$

$$e_b = k_b w$$

$$e_b(s) = k_b w(s) \quad (2)$$

$$(4.) \text{ Analyze of arm. current}$$

$$e_q = i_q R_q + e_b$$

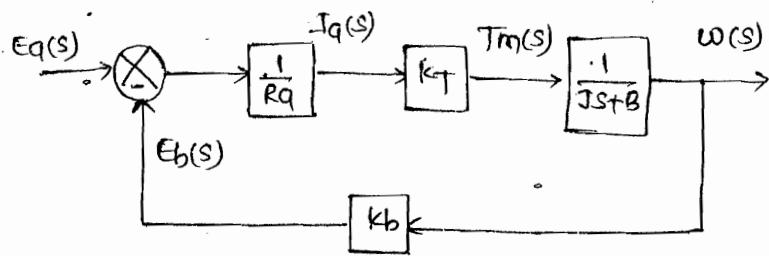
$$e_q - e_b = i_q R_q$$

$$E_q(s) - E_b(s) = I_q(s) R_q \quad (3)$$

$$(5.) \text{ At load}$$

$$T_m = \frac{J d\omega}{dt} + B\omega$$

$$T_m(s) = (Js + B) \omega(s) \quad (4)$$



$$\begin{aligned} \frac{w(s)}{E_q(s)} &= \frac{\frac{k_T}{Rq(Js+B)}}{1 + \frac{k_T k_b}{Rq(Js+B)}} = \frac{k_T}{Rq(Js+B) + k_T k_b} \\ &= \frac{k_T / Rq}{Js + B + \frac{k_T k_b}{Rq}} = \frac{k_T / Rq}{Js + F} \\ \boxed{\frac{w(s)}{E_q(s)} = \frac{k_T / Rq}{Js + F}} \end{aligned}$$

Note:- The TF of arm. control dc servomotor has a single time constant element & it represents the F/b control mechanism as seen from the block dia.

### \* Field control servomotor →

$$I/p = e_f \rightarrow \theta/p = 0 \text{ (or) } w$$

### Principle of operation →

$$(1.) \text{ Air gap flux } \propto \text{Field current } (i_f)$$

$$\phi = k_f i_f$$

$$(2.) \quad T_m \propto \phi \cdot i_q$$

$$T_m \propto k_f i_f \cdot i_q$$

$$T_m = k_f k_f i_f \cdot i_q$$

↓  
k\_f = motor torque const.

$$T_m = k_f i_f$$

$$T_m(s) = k_f I_f(s) \quad (1)$$

$$(3.) \quad \text{Back emf } (e_b) \propto \text{speed } (w)$$

$$e_b = k_b w$$

$$E_b(s) = k_b w(s) \quad (2)$$

### (4) Analysis of field circuit

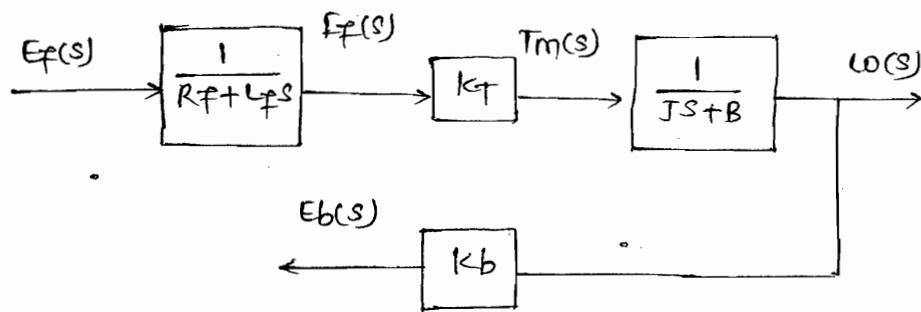
$$e_f = i_f R_f + L_f \frac{di_f}{dt}$$

$$E_f(s) = I_f(s) [R_f + L_f s] \quad (3.)$$

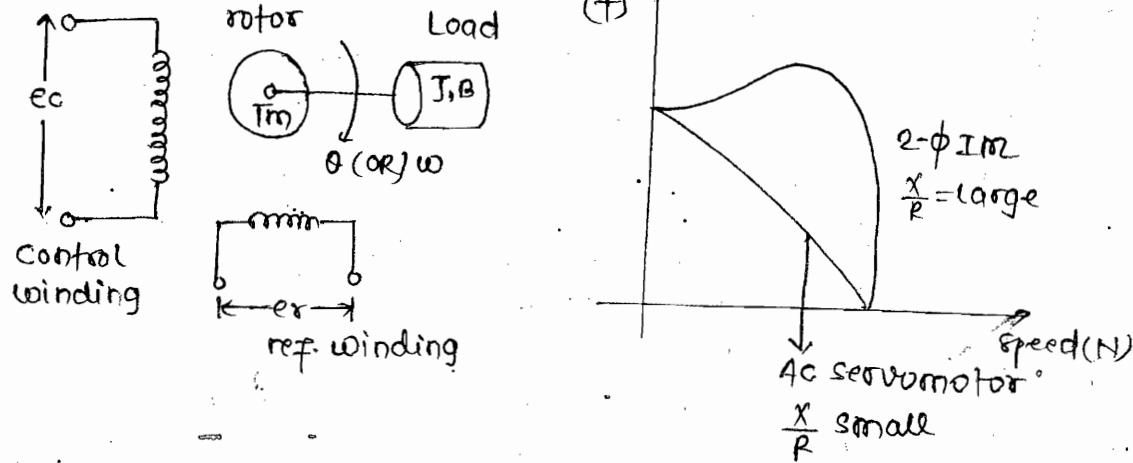
(5.) At load.

$$T_m = J \frac{d\omega}{dt} + B \omega$$

$$T_m(s) = (Js + B) \omega(s) \quad (4)$$



### AC servomotor →



- \* It is constructionally similar to a 2-φ ind'n motor.
- \* Out of the 2 wdg's placed in quadrature. One of the wdg is excited by a constant vol. & is known as ref wdg.
- \* The torque developed by the rotor is proportional to control winding vol.
- \* The rotor of Ac. servomotor is built with high resistance so that its  $\frac{X}{R}$  ratio is small & torque-speed c/s are approximately linearised.
- \* In the TF modelling since the torque developed by the rotor is proportional to control winding vol.; the resistance & inductance of the control wdg is assumed to be negligible.

$$T_m \propto E_c$$

$$T_m = K_m E_c$$

$$K_m = \frac{T_0}{E_c}$$

\* where;  $T_0$  = stall torque

$$T_m(s) = K_m E_c(s) \quad \text{--- (1)}$$

at load →

$$T_m = J \frac{d\omega}{dt} + B\omega$$

$$T_m(s) = (Js + B) \cdot \omega(s) \quad \text{--- (2)}$$

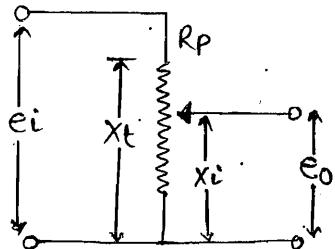
From eqn (1) & (2)

$$K_m E_c(s) = (Js + B) \omega(s)$$

$$\frac{\omega(s)}{E_c(s)} = \frac{K_m}{Js + (B - m)}$$

where;  $m$  = correction factor. slope of N-T charts.

\* Potentiometer →



\* It is a variable resistive displacement  $x_{cer}$  used as error detector in CS app?

\* A pair of potentiometer act as error detector.

Input = wiper disp.  $x_i$ , o/p =  $e_0$

Total resistance of POT. =  $R_p$

Resistance / per unit length =  $\frac{R_p}{x_t}$

Resistance for wiper disp of  $x_i$  units =  $\frac{R_p}{x_t} \cdot x_i$

Applying Voltage divider;

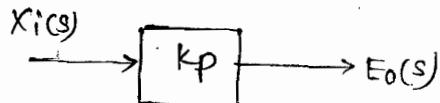
$$e_0 = \frac{R_p \cdot x_i}{x_t} \cdot e_i$$

$$e_0 = \frac{x_i}{x_t} \cdot e_i$$

Let  $K_p = \text{POT. gain} = \frac{e_i}{x_t} \left( \frac{\text{V}}{\text{mm}} \right)$

$$e_0 = K_p x_i$$

$$E_0(s) = K_p X_i(s)$$

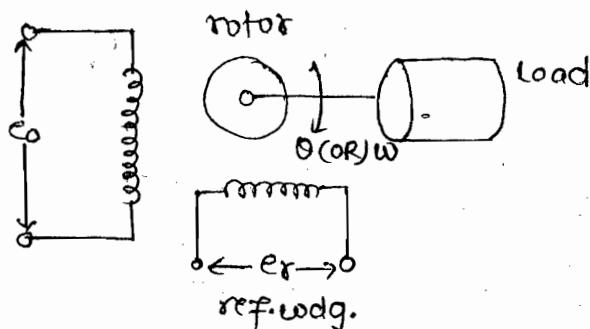


\* Tachometer → \* They are speed sensors used as f/b elements in cs appn.

↳ There are 2 types :-

(i) Dc tachometer → These are small dc gen's whose i/p is mech. speed & o/p is ele. voltage proportional to the speed.

(ii) Ac. tachometer →



↳ Out of the 2 wdg's placed in quadrature only one of the wdg is excited by a constant vol. If is known as ref. wdg.

↳ When the rotor is stationary the peripheral flux links the ref. wdg only. As the rotor rotates the rate of change of flux induces an emf

which is directly proportional to speed of rotor.  
The ac tachometer is also known as drag cup because the rotor Transfer fl. is of drag cup shape.

### Transfer fn.

$$e_0 \propto \text{speed}$$

$$(a.) \text{ Input} = 0, o/p = e_0 \quad (b.) I/p = \omega$$

$$e_0 \propto \frac{d\theta}{dt}$$

$$e_0 = k_T \cdot \frac{d\theta}{dt}$$

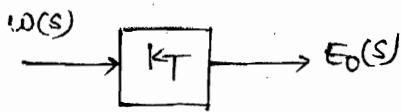
$$E_0(s) = k_T \cdot s \cdot \theta(s)$$

$$\theta(s) \xrightarrow{k_T s} E_0(s)$$

$$o/p = e_0$$

$$e_0 \propto \omega$$

$$E_0(s) = k_T \omega(s)$$

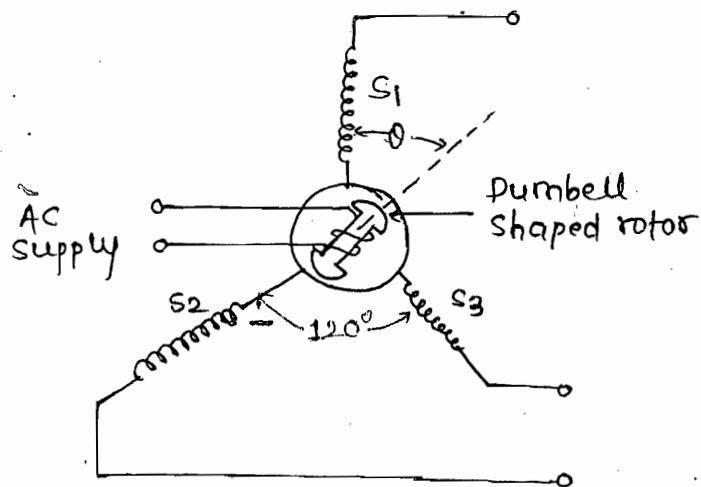


where;  $k_T$  = Tachometer gain

Ex. → The TF of a tachometer  $\frac{E(s)}{\theta(s)}$  is

- (a)  $ks$  (b)  $ks^2$  (c)  $\frac{k}{s}$  (d)  $k$

### \* SYNCHRO →

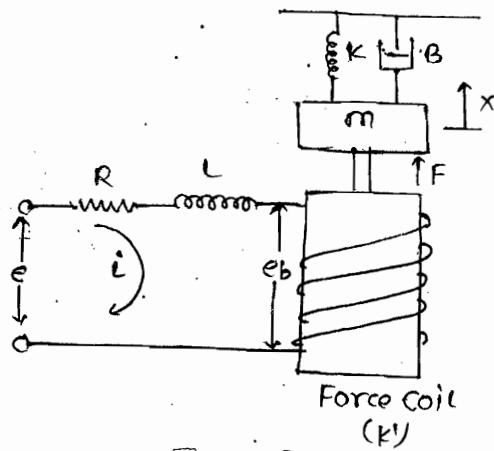


It is commercially known as SELSYN (OR) autosyn.

It is an electromagnetic device which converts angular position of the rotor into proportional voltages.

- \* It is constructionally similar to a 3-φ alternator but operationally the principle is based on Xmer action.
- \* When the rotor is inclined along any one of the stator access completely then maxm Vol. will be induced in that wedg. & negligible Vol. is induced in the other 2 wedges.
- \* Such a position of rotor is known as ele. zero position.
- ↳ A pair of synchro known as synchro Xmitter & synchro control Xmer (receivers) act as error detector.
- ↳ The Xmitter rotor is dumbbell shape & th

Due. →



$$\text{Soln} \rightarrow I/p = e ; 0/p = x$$

$$e = iR + \frac{Ldi}{dt} + eb$$

$$e - eb = iR + \frac{Ldi}{dt}$$

$$E(s) - Eb(s) = I(s)[R + Ls] \quad \text{---(i)}$$

eb (transduced vol.)

$$eb \propto k_T \frac{dx}{dt}$$

$$Eb(s) = k_T s X(s) \quad \text{---(ii)}$$

where;  $k_T$  = Tachometer gain.

At Force coil

$$F \propto i$$

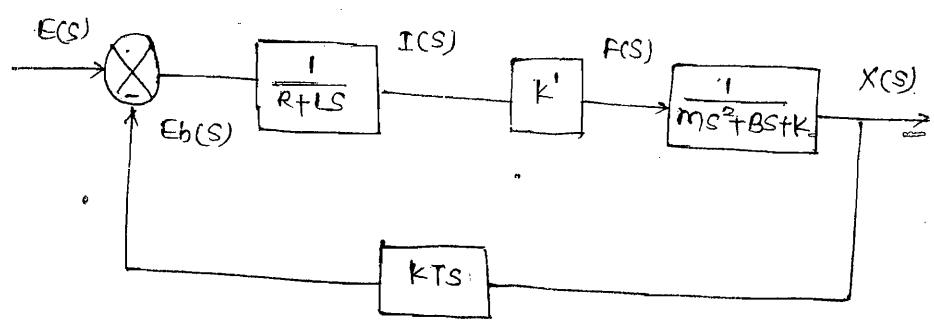
$$F = k'i$$

$$F(s) = k'I(s) \quad \text{---(iii)}$$

At mech. sys.

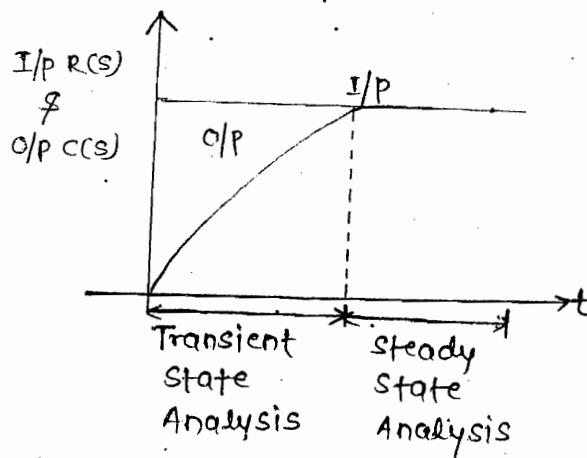
$$F = \frac{md^2x}{dt^2} + B \frac{dx}{dt} + ks$$

$$F(s) = (ms^2 + Bs + k) X(s) \quad \text{---(iv)}$$



chapter-02  
 Time domain analysis

- \* The time response analysis is the analysis on time taken by the response of the sys. when subjected to an i/p.
- \* It is divided into 2 parts :-
- (i) Transient state analysis → It deals with the nature of response of sys. when subjected to an i/p.
- (ii) Steady state response analysis → It deals with the estimation of magnitude of steady state error b/w i/p & o/p.



Standard test signals →

- (1) Sudden i/p → Step signal
- (2) Velocity type i/p → Ramp signal
- (3) Acceleration type i/p → Parabolic signal
- (4) Sudden shocks → Impulse signal → stability

↑  
Time domain  
analysis

Note:- The transient state analysis & the transient state specification are defined for step signal only because the magnitude of the i/p sig. should not change with time.

### \* Type & Order →

- (1.) Every TF representing the CS has certain type & order.
- (2.) Steady-state response analysis depends on type of the CS.
- (3.) The type of the sys. obtained from open loop TF  $G(s)H(s)$ , by observing the no. of open loop poles occurring at origin.

$$\text{Let } G(s) \cdot H(s) = \frac{K(1+T_1 s)}{s^p(1+T_1 s)}$$

$p=0$ , type-0 system

$p=1$ , type-1 system

$\vdots$

$p=n$ , type-n system

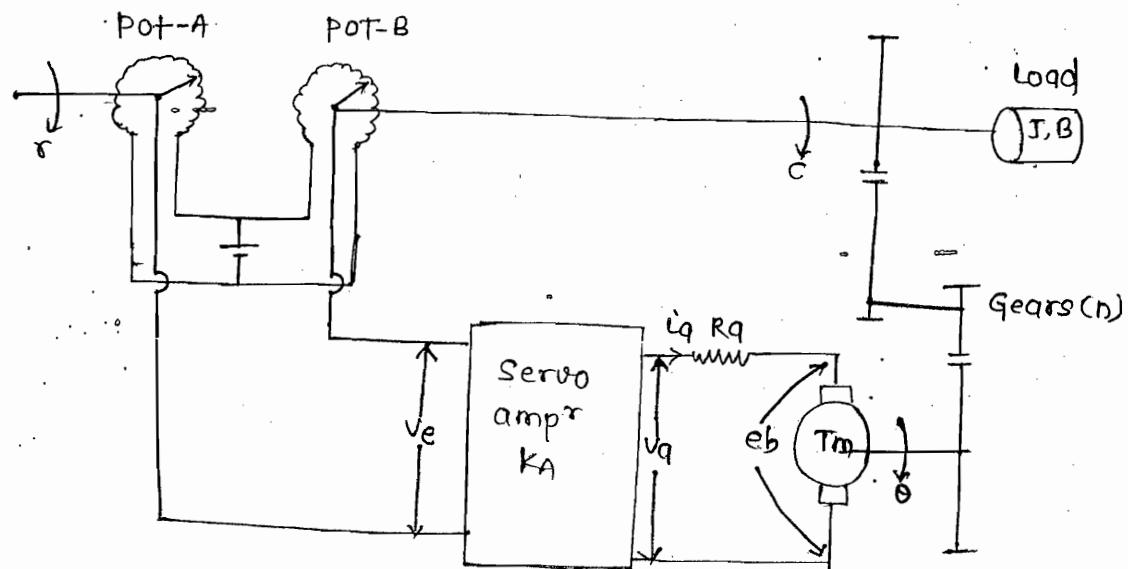
- (4.) The transient state analysis depends upon the order of CS.

- (5.) The order of sys is obtained from closed loop TF  $\frac{G(s)}{1+G(s)H(s)}$  by

Observing the highest power of a C/S eqn.

$$1+G(s)H(s)=0$$

Eg:- Position Control system



Arm. controlled  
dc servomotor.

$$(1) I/P = r [R(s)], O/P = c; [C(s)]$$

### (2) Principle of operation

#### (i) At potentiometer

$$V_e \propto (r - c)$$

$$V_e = k_p (r - c)$$

$$V_e(s) = k_p [R(s) - C(s)] \quad \text{--- (i)}$$

#### (ii) At Amp

$$V_q \propto V_e$$

$$V_q = k_A V_e$$

$$V_q(s) = k_A V_e(s) \quad \text{--- (ii)}$$

#### (iii) Analysis of arm. controlled dc servomotor

$$T_m = k_T I_q(s) \quad \text{--- (iii)}$$

$$E_b = k_b \frac{d\theta}{dt}$$

$$E_b(s) = k_b(s) \theta(s) \quad \text{--- (iv)}$$

$$V_q = i_q R_q + e_b$$

$$V_q(s) = E_b(s) = I_q(s) \cdot R_q \quad \text{--- (v)}$$

$$T_m = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$$

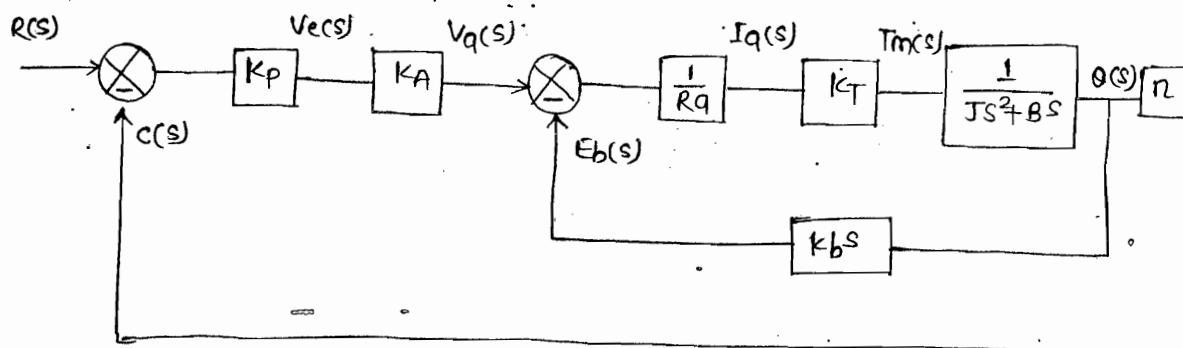
$$T_m(s) = (J s^2 + B s) \theta(s) \quad \text{--- (vi)}$$

#### (iv) At gears:-

$$c \propto \theta$$

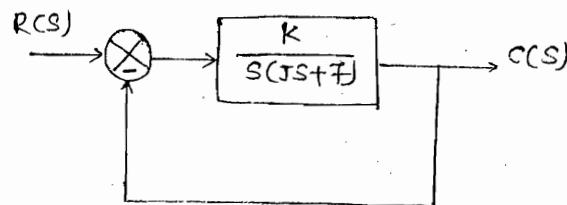
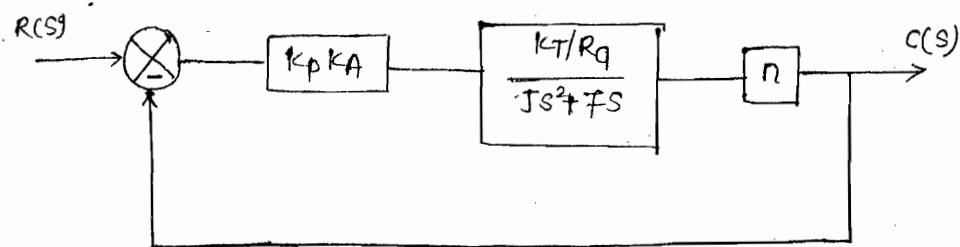
$$c = n \theta$$

$$c(s) = n \theta(s) \quad \text{--- (vii)}$$



Inner f/b loop →

$$\frac{\frac{k_T}{R_q(J s^2 + B s)}}{1 + \frac{k_T k_b s}{R_q(J s^2 + B s)}} = \frac{k_T}{R_q(J s^2 + B s) + k_T k_b s} = \frac{\frac{k_T / R_q}{J s^2 + B s + \frac{k_T k_b s}{R_q}}}{\frac{1}{R_q}} = \frac{k_T / R_q}{J s^2 + \frac{1}{R_q} s}$$



$$\text{where; } K = \frac{K_p K_A K_T n}{R_q}$$

For type of sys →

$$G(s) = \frac{K}{s(Js + f)}, H(s) = 1$$

$$G(s) \cdot H(s) = \frac{K}{s(Js + f)}$$

Type = 1

Order of the sys. →

$$1 + G(s) H(s) = 0$$

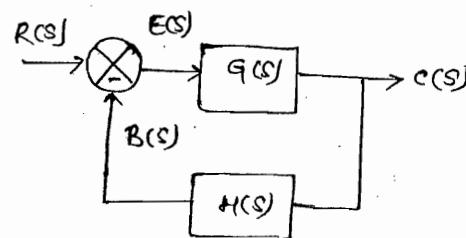
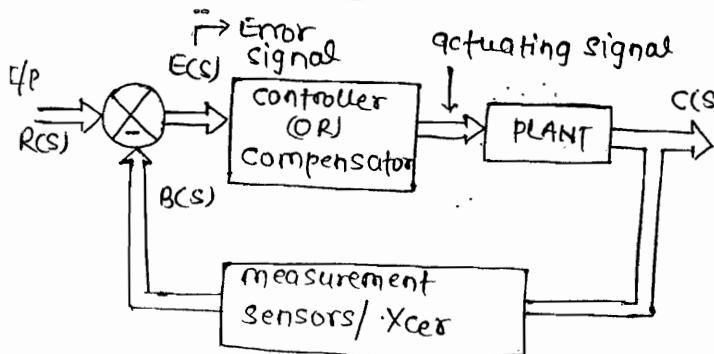
$$1 + \frac{K}{s(Js + f)} = 0$$

$$Js^2 + fs + K = 0$$

Order = 2

\* Steady state Response analysis → It deals with estimation of magnitude of steady state error b/n i/p & o/p & depends on type of cs.

Error Compensation →



To obtain an expression for error

$$E(s) = R(s) - B(s)$$

↓      ↓  
I/P    O/P

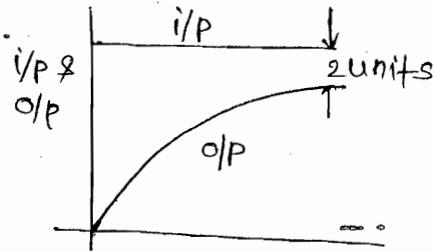
$$E(s) = R(s) - C(s) \cdot H(s)$$

$$E(s) = R(s) - E(s) \cdot G(s) \cdot H(s)$$

$$E(s) [1 + G(s) \cdot H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}$$

error Ratio



$$\lim_{t \rightarrow \infty} e(t) = 2 \text{ units}$$

(ess)

$$ess = \lim_{t \rightarrow \infty} e(t)$$

By FUT

$$ess = \lim_{s \rightarrow 0} s \cdot E(s)$$

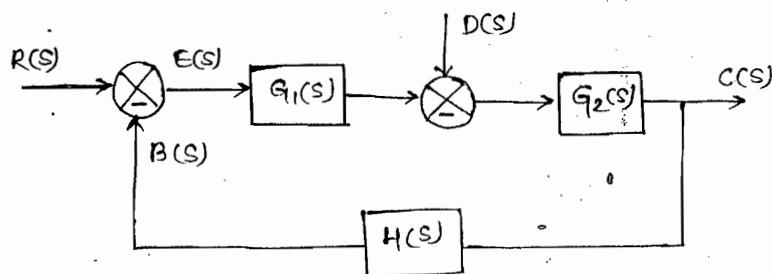
$$ess = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s) \cdot H(s)}$$

$$ess = \frac{\lim_{s \rightarrow 0} s \cdot R(s)}{1 + \lim_{s \rightarrow 0} G(s) \cdot H(s)}$$

Note:- \* Since the steady state error is defined as the diff b/w I/P & O/P,  
to find the type of the sys. & hence steady state error the F/b  
gain should be unity [ $H(s) = 1$ ]

\* For non-unity F/b elements they should be specified as measuring element  
i.e sensors (OR) Xcer element.

\* To obtain an exp^n for error with disturbance  $\rightarrow$



$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s) \cdot H(s)$$

$$\therefore C(s) = [E(s) G_1(s) + D(s)] G_2(s)$$

$$C(s) = E(s) \cdot G_1(s) + G_2(s) + D(s) \cdot G_2(s)$$

$$E(s) = R(s) - E(s) \cdot G_1(s) \cdot G_2(s) \cdot H(s) - D(s) \cdot G_2(s) \cdot H(s)$$

$$E(s) [1 + G_1(s) \cdot G_2(s) \cdot H(s)] = R(s) - D(s) \cdot G_2(s) \cdot H(s)$$

$$E(s) = \frac{R(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)} - \frac{D(s) \cdot G_2(s) \cdot H(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)}$$

$$ess = \lim_{s \rightarrow 0} s \cdot E(s)$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)} - \lim_{s \rightarrow 0} \frac{s \cdot D(s) \cdot G_2(s) \cdot H(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)}$$

(4)  
59

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot D(s) \cdot G(s)}{1 + G_1(s) \cdot G_2(s)}$$

$$= \lim_{s \rightarrow 0} \frac{s \times \frac{1}{s} \cdot G_2(s)}{1 + G_1(s) \cdot G_2(s)}$$

ans (c)

$$ess = \frac{-G_2}{G_1 G_2 + 1} \Rightarrow |ess| = \frac{G_2}{1 + G_1 G_2}$$

ess ↓ by  $G_1 \uparrow$

\* Steady state error for different types of i/p →

(1.) Step i/p

$$R(s) = \frac{A}{s}$$

$k_p$  = position error constant

$$ess = \lim_{s \rightarrow 0} \frac{\frac{A}{s}}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \rightarrow 0} G(s) \cdot H(s)$$

$$= \frac{A}{1 + \lim_{s \rightarrow 0} G(s) \cdot H(s)} = \frac{A}{1 + k_p}$$

$$ess = \frac{A}{1 + k_p}$$

### (2) Ramp signal

$$R(s) = \frac{A}{s^2}$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{A}{s^2}}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{A}{s + SG(s) \cdot H(s)}$$

$$= \frac{A}{\lim_{s \rightarrow 0} s + \lim_{s \rightarrow 0} s \cdot H(s) \cdot G(s)}$$

$$= \frac{A}{Kv}$$

$Kv$  = velocity error const.

$$= \lim_{s \rightarrow 0} s \cdot G(s) \cdot H(s)$$

### (3) Parabolic signal

$$R(s) = \frac{A}{s^3}$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{A}{s^3}}{1 + G(s) \cdot H(s)}$$

$$\frac{A}{\lim_{s \rightarrow 0} s^2 + \lim_{s \rightarrow 0} s^2 G(s) \cdot H(s)} = \frac{A}{KA}$$

$KA$  = acceleration error const.

$$= \lim_{s \rightarrow 0} s^2 G(s) \cdot H(s)$$

- Steady state error for different types of system →

Type-0 →

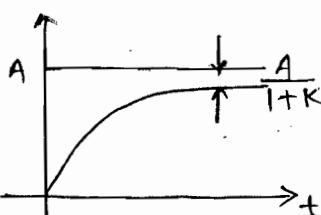
$$G(s) \cdot H(s) = \frac{K(1+T_0 s)}{(1+T_1 s)}$$

1) step i/p

$$R(s) = \frac{A}{s}$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{A}{s}}{1 + K(1+T_0 s)} = \frac{A}{1+K}$$

$$= \frac{A}{1+K}$$



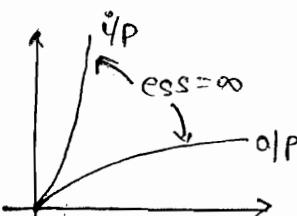
2) Ramp i/p

$$R(s) = \frac{A}{s^2}$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{A}{s^2}}{1 + K(1+T_0 s)} = \frac{A}{(1+T_0 s)(1+T_1 s)}$$

$$ess = \lim_{s \rightarrow 0} \frac{A}{s + ks(1+T_0 s)} = \frac{A}{(1+T_1 s)}$$

= ∞



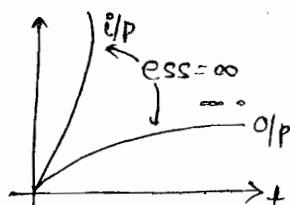
3) Parabolic i/p

$$R(s) = \frac{A}{s^3}$$

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{A}{s^3}}{1 + K(1+T_0 s)} = \frac{A}{(1+T_0 s)^2}$$

$$= \lim_{s \rightarrow 0} \frac{A}{s^2 + ks^2(1+T_0 s)} = \frac{A}{(1+T_1 s)}$$

= ∞



System	Step Input	Ramp Input	Parabolic Input
TYPE-0	$\frac{A}{1+K}$ $K_p = K$	$\infty$ $K_v = 0$	$\infty$ $K_a = 0$
TYPE-1	0 $K_p = \infty$	$\frac{A}{K}$ $K_v = K$	$\infty$ $K_a = 0$
TYPE-2	0 $K_p = \infty$	0 $K_v = \infty$	$A/K$ $K_a = K$

$$\begin{array}{cccc}
 1 & \infty & \infty \\
 0 & 1 & \infty \\
 0 & 0 & 1 \\
 \\ 
 K & 0 & 0 & K_p = K_v = K_a \\
 \infty & K & 0 \\
 \infty & \infty & K
 \end{array}$$

### Observations →

(1)  $e_{ss} \propto \frac{1}{K}$   
As  $K \uparrow e_{ss} \downarrow$

(2) The maximum type no. for a linear CS is 2. Beyond type-2 the sys. tends to become unstable & also exhibits non-linear c/s more dominantly.

(3)  
61.

$$G(s) = \frac{10}{s^2(4+s)} ; r(t) = 2 + 3t + 4t^2$$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot (2s^2 + 3s + 8)}{s^2(4+s)}$$

$$= \frac{1}{1 + \frac{10}{s^2(4+s)}}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{2s^2 + 3s + 8}{s^2 + s^2 \cdot \frac{10}{s^2(4+s)}}$$

$$= \frac{8 \times 4}{10}$$

$$\boxed{e_{ss} = 3.2}$$

### shortcut method

type-2.

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$e_{ss} = 0 \quad 0 \quad \frac{A}{K}$$

$$A = 8$$

$$K = K_a = \lim_{s \rightarrow 0} s^2 \cdot \frac{10}{s^2(4+s)} = \frac{10}{4}$$

$$e_{ss} = \frac{8}{\left(\frac{10}{4}\right)} = 3.2$$

(4)  
61

$$r(t) = (1-t^2) 3u(t)$$

$$R(s) = \frac{3}{s} - \frac{6}{s^3}$$

$$ess = \lim_{s \rightarrow 0} s \left[ \frac{3}{s} - \frac{6}{s^3} \right] / 1 + G(s)$$

$$ess = \lim_{s \rightarrow 0} \frac{s \left( \frac{3}{s} \right)}{1 + G(s)} - \lim_{s \rightarrow 0} \frac{s \cdot \frac{6}{s^3}}{1 + G(s)}$$

$$ess = \frac{3}{1 + \lim_{s \rightarrow 0} G(s)} - \frac{6}{\lim_{s \rightarrow 0} s^2 G(s)}$$

$$ess = \frac{3}{1 + K_p} - \frac{6}{K_A}$$

1)

Type-0

 $\xrightarrow{(1/s)}$ 

Type-1

$$ess = \frac{1}{1+K}$$

$$0.2 = \frac{1}{1+K}$$

$$K=4$$

$$ess = \frac{1}{K}$$

$$ess = \frac{1}{4} = 0.25 \text{ units}$$

$$\boxed{ess = 0.25}$$

}) Type-1 system

$$ess = \frac{1}{K}$$

$$ess = 5\% \Rightarrow \frac{5}{100} = \frac{1}{20}$$

$$\frac{1}{20} = \frac{1}{K}$$

$$\boxed{K=20}$$
  

$$\boxed{\text{Type-1}}$$

(1)  
61

$$G(s) = \frac{10}{s^2 + 14s + 50}$$

Type-0

$$ess = \frac{1}{1+K}$$

$$K = K_p = \lim_{s \rightarrow 0} \frac{10}{s^2 + 14s + 50} = \frac{10}{50}$$

$$ess = \frac{1}{1 + \frac{10}{50}} = \frac{50}{60} = 0.83 \text{ units}$$

$$\boxed{ess = 0.83}$$

shortcut method

$$R(s) = \frac{3}{s} - \frac{6}{s^3}$$

$$= \frac{A}{1+K} - \frac{A}{K}$$

$$= \frac{3}{1+K_p} - \frac{6}{K_p}$$

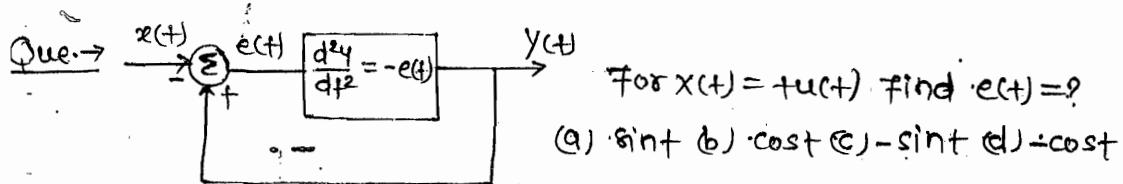
If I/p is not specified then  
finite error is taken.

(3)  
59)  $G(s) = \frac{K}{s(s+q)}$   
Type-1 system  
 $e_{ss} = \frac{A}{K}$   
 $A=1, K=kq = \lim_{s \rightarrow 0} s \cdot \frac{K}{s(s+q)}$   
 $= \frac{K}{q}$   
 $e_{ss} = \frac{q}{K}$

(i)  $s_K^{ess} = \frac{K}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial K}$   
 $e_{ss} = \frac{q}{K}$   
 $\frac{e_{ss}}{K} = \frac{q}{K^2} \Rightarrow \frac{K}{e_{ss}} = \frac{K^2}{q}$   
 $\frac{\partial e_{ss}}{\partial K} = \frac{\partial}{\partial K} \left( \frac{q}{K} \right) = -\frac{q}{K^2}$   
 $s_K^{ess} = \frac{K^2}{q} \times -\frac{q}{K^2} = -1$   
 $s_K^{ess} = -1$

(ii)  $s_q^{ess} = \frac{q}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial q}$   
 $e_{ss} = \frac{q}{K} \Rightarrow \frac{q}{e_{ss}} = K$   
 $\frac{\partial e_{ss}}{\partial q} = \frac{\partial}{\partial q} \left( \frac{q}{K} \right) = \frac{1}{K}$   
 $s_q^{ess} = K \times \frac{1}{K} = 1$   
 $s_q^{ess} = 1$

Note:- Sensitivity of  $e_{ss}$  w.r.t  $K$  &  $q$  is same for the above system.



Soln  $\rightarrow$   
 $e(t) = -x(t) + y(t)$   
 $E(s) = -X(s) + Y(s)$   
 $\frac{d^2y}{dt^2} = -e(t); s^2Y(s) = -E(s)$   
 $y(s) = \frac{-E(s)}{s^2}$   
 $E(s) = -X(s) - \frac{E(s)}{s^2}$   
 $E(s) + \frac{E(s)}{s^2} = -X(s)$

$$\left(\frac{s^2+1}{s^2}\right) E(s) = -X(s)$$

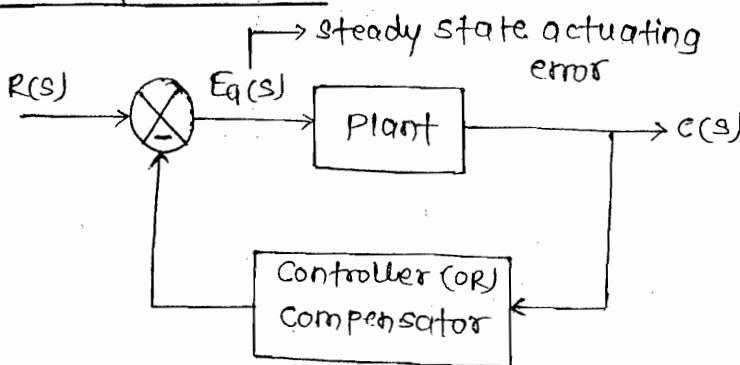
$$E(s) = \frac{-X(s) \cdot s^2}{s^2+1}$$

Given  $x(t) = tu(t)$

$$\begin{aligned} X(s) &= \frac{1}{s^2} \\ E(s) &= \frac{\frac{-1}{s^2} \cdot s^2}{(s^2+1)} \\ &= \frac{-1}{s^2+1} \end{aligned}$$

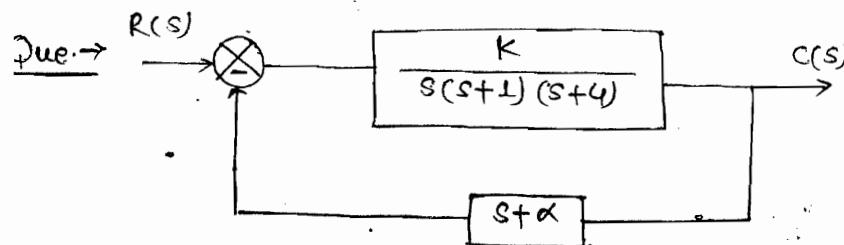
$$e(t) = -\sin t$$

\* Output Compensation →



→ A controller/compensator placed in the f/b path compensates for changes in o/p & a steady state actuating error signal effects the dynamics of the plant to achieve the control objective.

∴ In such cases to find steady state error which is the diff. b/w i/p & o/p convert the cs into unity f/b system.



(i) Which i/p will yield constant error?

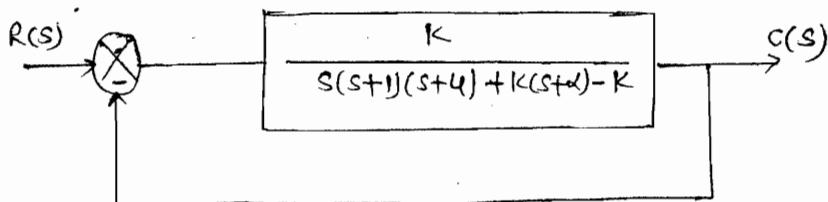
- (a) Step i/p. (b) Ramp i/p (c) parabolic i/p. (d) Impulse i/p

(ii) Find steady state error for the above i/p?

- (a)  $\frac{K}{\alpha-1}$  (b)  $\frac{K-1}{\alpha}$  (c)  $\alpha$  (d)  $\alpha-1$

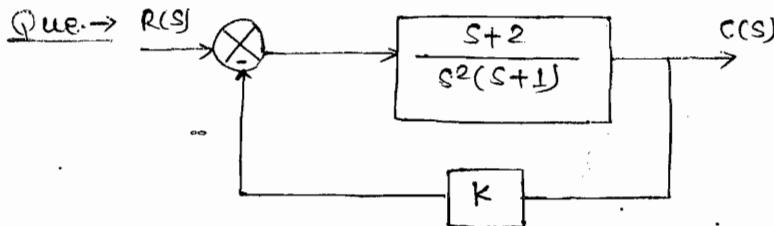
∴  $\text{Soln} \rightarrow$

$$\begin{aligned}\frac{C(s)}{R(s)} &= \frac{K}{s(s+1)(s+4)} \\ &\quad \times \frac{1 + \frac{K(s+\alpha)}{s(s+1)(s+4)}}{1 + \frac{K(s+\alpha)}{s(s+1)(s+4)}} \\ &= \frac{K}{s(s+1)(s+4) + K(s+\alpha)} \\ G(s) &= \frac{K}{s^3[(s+1)(s+4)s + K(s+\alpha)] - K^2}\end{aligned}$$



$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{K}{s(s+1)(s+4)+K(s+\alpha)-K}} \\ &= \frac{1}{1 + \frac{K}{K\alpha - K}} = \frac{K\alpha - K}{K\alpha - K + K} \end{aligned}$$

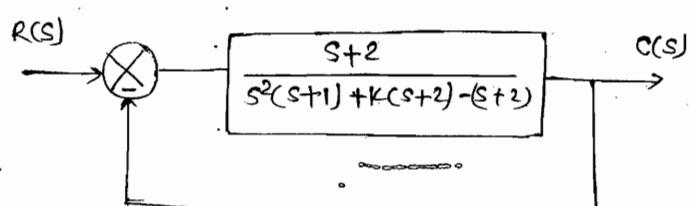
$$\begin{aligned} e_{ss} &= \frac{k(\alpha-1)}{k\alpha} && \text{qns.(a) \& (b)} \\ e_{ss} &= \frac{\alpha-1}{\alpha} \end{aligned}$$



Soln.  $\rightarrow$

$$\frac{C(s)}{R(s)} = \frac{\frac{s+2}{s^2(s+1)}}{1 + \frac{(s+2)K}{s^2(s+1)}} = \frac{s+2}{s^2(s+1)+(s+2)K}$$

$$G(s) = \frac{(s+2)}{s[s^2(s+2)+K(s+2)-(s+2)]}$$



$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{(s+2)}{s^2(s+1)+K(s+2)-(s+2)}} = \frac{1}{1 + \frac{2}{9K-2}}$$

$$e_{ss} = \frac{K-1}{K}$$

### \* Error Series →

$$E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$$\text{Let; } F(s) = \frac{1}{1 + G(s) \cdot H(s)}$$

$$\mathcal{L}^{-1} E(s) = R(s) \cdot F(s)$$

$$\mathcal{L}^{-1} E(s) = \mathcal{L}^{-1} R(s) \cdot F(s)$$

$$e(t) = \int_0^\infty f(\tau) \cdot r(t-\tau) d\tau$$

Expanding  $r(t-\tau)$  using Taylor series

$$r(t-\tau) = r(t) - \tau \dot{r}(t) + \frac{\tau^2}{2!} \ddot{r}(t) - \frac{\tau^3}{3!} \dddot{r}(t) + \dots$$

$$e(t) = r(t) \int_0^\infty f(\tau) d\tau - \dot{r}(t) \int_0^\infty \tau f(\tau) d\tau + \frac{\ddot{r}(t)}{2!} \int_0^\infty \tau^2 f(\tau) d\tau - \frac{\dddot{r}(t)}{3!} \int_0^\infty \tau^3 f(\tau) d\tau \dots$$

Defining "Dynamic error const's"

$$k_0 = \int_0^\infty f(\tau) d\tau, k_1 = - \int_0^\infty \tau f(\tau) d\tau, k_2 = \int_0^\infty \tau^2 f(\tau) d\tau, k_3 = - \int_0^\infty \tau^3 f(\tau) d\tau$$

$$e(t) = k_0 r(t) + k_1 \dot{r}(t) + \frac{k_2}{2!} \ddot{r}(t) + \frac{k_3}{3!} \dddot{r}(t) + \dots$$

$$ess = \lim_{t \rightarrow \infty} e(t)$$

### To find dynamic error constants →

$$\mathcal{L} f(t) = F(s) = \int_0^\infty f(\tau) e^{-st} d\tau$$

$$\lim_{s \rightarrow 0} F(s) = \lim_{s \rightarrow 0} \int_0^\infty f(\tau) e^{-st} d\tau = \int_0^\infty f(\tau) d\tau \Rightarrow k_0$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \int_0^\infty f(\tau) e^{-st} d\tau = - \int_0^\infty \tau f(\tau) e^{-st} d\tau$$

$$\lim_{s \rightarrow 0} \frac{d}{ds} F(s) = \lim_{s \rightarrow 0} - \int_0^\infty \tau f(\tau) e^{-st} d\tau \Rightarrow - \int_0^\infty \tau f(\tau) d\tau \Rightarrow k_1$$

$$K_0 = \lim_{s \rightarrow 0} F(s)$$

$$K_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(s)$$

$$K_2 = \lim_{s \rightarrow 0} \frac{d^2 F(s)}{ds^2}$$

$$\text{where; } F(s) = \frac{1}{1+G(s)H(s)}$$

### \* Relationship b/n static & dynamic error constants $\rightarrow$

$$G(s)H(s) = \frac{100}{s(s+2)}$$

#### (I) Static error constants ..

$$K_p = \lim_{s \rightarrow 0} \frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s \rightarrow 0} \frac{s \cdot 100}{s(s+2)} = 50$$

$$K_A = \lim_{s \rightarrow 0} \frac{s^2 \cdot 100}{s(s+2)} = 0$$

#### (II) Dynamic error constants.

$$F(s) = \frac{1}{1+G(s)H(s)} = \frac{1}{1+\frac{100}{s(s+2)}}$$

$$K_0 = \lim_{s \rightarrow 0} F(s)$$

$$K_0 = \lim_{s \rightarrow 0} \frac{1}{1 + \frac{100}{s(s+2)}}$$

$$K_0 = \frac{1}{1 + \lim_{s \rightarrow 0} \frac{100}{s(s+2)}} = \frac{1}{1+\infty} = 0$$

$$K_0 = \boxed{\frac{1}{1+K_p}}$$

$$K_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \frac{1}{1 + \frac{100}{s(s+2)}} = \frac{d}{ds} \frac{s(s+2)}{s^2 + 2s + 100}$$

$$= \frac{(s^2 + 2s + 100)(s+2) - s(s+2)(2s+2)}{(s^2 + 2s + 100)^2}$$

$$K_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(s) = \frac{(0+0+100)(0+2)-0}{(0+0+100)^2} = \frac{100 \times 2}{(100)^2} = \frac{1}{50}$$

$$K_1 = \frac{1}{K_V}$$

$$K_2 = \frac{1}{K_A}$$

Note:- Static & dynamic error constant are inversely related to each other however they need not be direct reciprocal value because the dynamic error constant are defined for error series.

$$\text{Given } G(s) \cdot H(s) = \frac{100}{s(s+2)}$$

Find ess for  $r(t) = 5+2t$

Soln i) Error ratio

$$R(s) = \frac{5}{s} + \frac{2}{s^2} = \frac{5s+2}{s^2}$$

$$\text{ess} = \lim_{s \rightarrow 0} \frac{s \cdot \left( \frac{5s+2}{s^2} \right)}{1 + \frac{100}{s(s+2)}}$$

$$\text{ess} = \frac{2 \times 2}{100} = \frac{2}{50} \text{ units}$$

ii) Error series

$$\text{ess} = \lim_{t \rightarrow 0} [k_0 r(t) + k_1 \dot{r}(t) + k_2 \ddot{r}(t) + \dots]$$

$$r(t) = 5+2t \Rightarrow k_0 = 0$$

$$\dot{r}(t) = 0+2 = 2 \Rightarrow k_1 = \frac{1}{50}$$

$$\ddot{r}(t) = 0$$

$$\text{ess} = \lim_{t \rightarrow \infty} [0 \times (5+2t) + \frac{1}{50} \times 2] = \frac{2}{50}$$

iii) Short cut methods

Type-i)

$$R(s) = \frac{5}{s} + \frac{2}{s^2}$$

$$\text{ess} = 0 + \frac{A}{K}$$

$$A=2, K=K_V=50$$

$$\text{ess} = \frac{2}{50}$$

**[DATE-17/11/14].**

\* Transient state analysis → \* It deals with the nature of response of sys. when subjected to an i/p & depends

on order of CS.

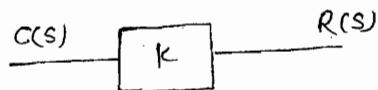
$$\frac{C(s)}{R(s)} = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

(1.) zero order system →

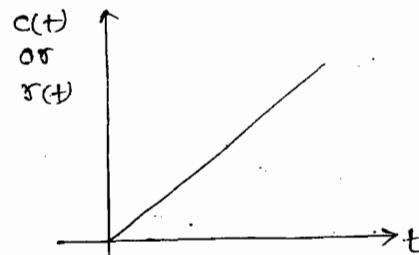
$$\frac{C(s)}{R(s)} = \frac{b_0}{a_0}$$

$$\text{let } k = \text{gain} = \frac{b_0}{a_0}$$

$$\frac{C(s)}{R(s)} = k$$



EX:- Sensors / Xcers



(2.) 1st order sys →

$$\frac{C(s)}{R(s)} = \frac{b_0}{a_1 s + b_0}$$

$$= \frac{b_0/a_0}{a_1 s + 1/a_0}$$

$$\text{let } k = \text{gain} = \frac{b_0}{a_0}$$

$$T = \text{time constant} = \frac{a_1}{a_0}$$

$$\frac{C(s)}{R(s)} = \frac{k}{1+TS}$$

EX:- RC n/w

$$\frac{U_o(s)}{U_i(s)} = \frac{1}{R_s s + 1} \quad T = RC$$

Transient analysis →

$$\text{let } R(s) = 1/s$$

$$C(s) = \frac{k}{s(1+Ts)}$$

$$= \frac{k}{s(1+Ts)}$$

$$= k \left[ \frac{1}{s} - \frac{T}{s+T} \right]$$

$$= k \left[ \frac{1}{s} - \frac{1}{s+\frac{1}{T}} \right]$$

$$c(t) = k(1 - e^{-t/T})$$

$$\lim_{t \rightarrow \infty} c(t) = k$$

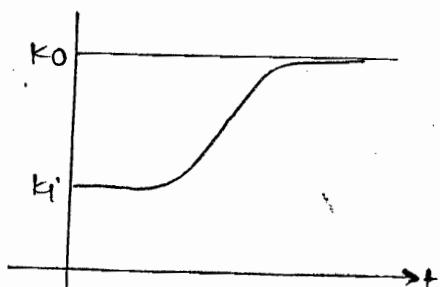
$$\text{at } t=0, c(t) = k(1 - e^0) = 0$$

$$t=T, c(t) = k(1 - e^1) = (0.63)k$$

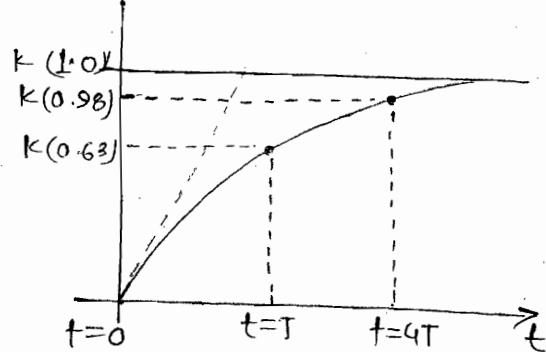
$$t=4T, c(t) = k(1 - e^4) = (0.98)k$$

The time const is defined as time taken by the response of the sys. to reach 63% of the final value.

Liquid level sys., pneumatic sys., thermometers, RC or RL n/w are eg. of 1<sup>st</sup> order system.

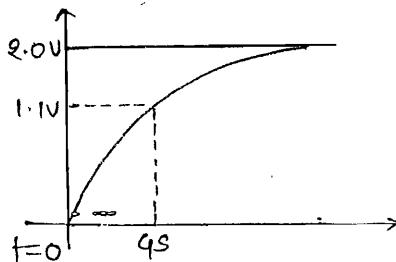


$$c(t) = k_0 + (k_i - k_0)e^{-t/\tau}$$



Que. → Certain 1st order sys. is initially at rest & subjected to sudden i/p at  $t=0$ , its response reaches 1.1V in 4s & eventually reaches a steady state of 2V. Find the time const.

Soln →



$$C(t) = k(1 - e^{-t/T})$$

$$\lim_{t \rightarrow \infty} C(t) = k = 2$$

$$\text{at } t=4s$$

$$1.1 = 2(1 - e^{-4/T})$$

$$2e^{-4/T} = 0.9$$

$$\boxed{T = 5s}$$

(17)  
63

$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

$$r(t) = 10u(t)$$

$$R(s) = \frac{10}{s}$$

$$C(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2}$$

$$c(t) = 5(1 - e^{-2t})$$

$$\lim_{t \rightarrow \infty} c(t) = 5$$

$$\frac{99}{100} \times 5 = 4.95$$

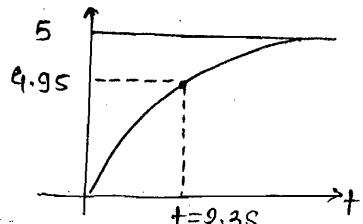
$$4.95 = 5(1 - e^{-2t})$$

$$5e^{-2t} = 0.05$$

$$e^{-2t} = 0.01$$

$$-2t = \ln(0.01)$$

$$t = 2.3s$$



Que. → A thermometer having 1st order dynamics is subjected to sudden temp. change of  $30^\circ\text{C} - 150^\circ\text{C}$ . If it has a time constant of 4s. what temp. it will indicate after 4s.

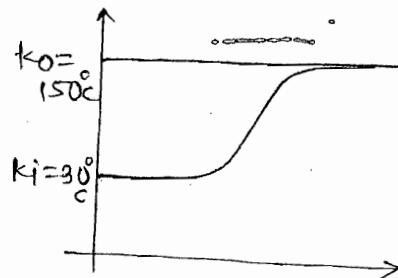
Soln

$$C(t) = k_0 + (k_i - k_0)e^{-t/T}$$

$$9t \Rightarrow t = 4s.$$

$$C(t) = 150 + (30 - 150)e^{-4/4}$$

$$C(t) = 105.6^\circ C$$



Que. → The TF of the 1<sup>st</sup> order sys. is

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

Its type & ess/unit ramp i/p are

- (a) 0, ∞ (b) 0, T (c) 1, ∞ (d) 1, T

Soln

$$G(s) = \frac{1}{1+Ts-1} = \frac{1}{Ts}$$

Type-1 system

$$ess = \frac{1}{K} \quad \text{where } K = K_U = \lim_{s \rightarrow 0} s \cdot \frac{1}{Ts} = \frac{1}{T}$$

ess = T

2nd order system →

The response of 2<sup>nd</sup> order (or) higher order sys. exhibits continuous (or) sustained oscillation about the steady state value of i/p with a freq. known as undamped natural freq.  $\omega_n$ .

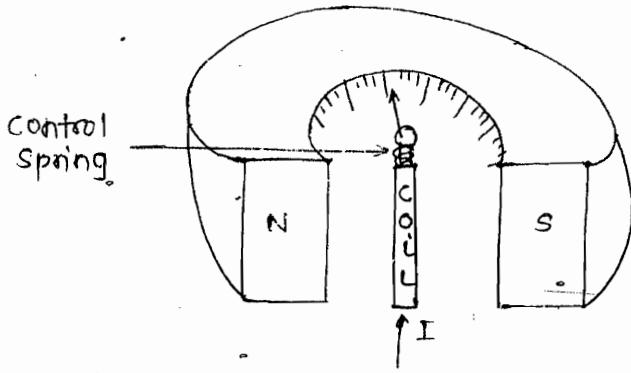
This oscillation in the response are damped to the steady state value of i/p using appropriate damping methods, the damping is mathematically expressed as damping ratio  $\xi$  (eta).

Ques - PMMC.

Undamped Natural freq.  $\omega_m$   $\omega_n$  r/s

- damping Ratio "ξ (Getta)"

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



$\Gamma/p$  = deflecting torque ( $T_d$ )

$\theta/p$  = angular deflection of pointer ( $\theta$ )

$J$  = Inertia of moving system,  $B$  = Inherent friction

$K$  = Spring constant.

$$T_d = \frac{J d^2\theta}{dt^2} + \frac{B d\theta}{dt} + K\theta ; T_d(s) = (Js^2 + Bs + K) \theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2 + Bs + K} = \frac{1/J}{s^2 + \frac{B}{J}s + \frac{K}{J}}$$

$$s^2 + \frac{B}{J}s + \frac{K}{J} = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{\frac{K}{J}} \text{ rad/s} ; 2\zeta\sqrt{\frac{K}{J}} = \frac{B}{J} , \zeta = \frac{B}{2\sqrt{KJ}} \quad [\zeta \propto B]$$

" $K$  TYPE" of std. 2nd order system →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 - \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \quad \text{Type-1 sys}$$

Effect of damping on the nature of response →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + 1 = 0$$

$$= \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

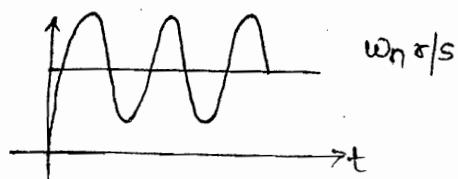
$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$D = \zeta^2 - 1 = 0 \Rightarrow \zeta = 1$$

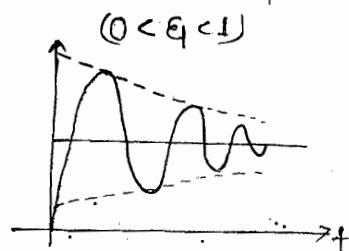
$$D = \zeta^2 - 1 < 0 \Rightarrow \zeta < 1$$

$$D = \zeta^2 - 1 > 0 \Rightarrow \zeta > 1$$

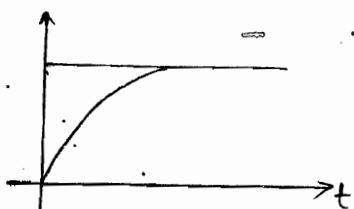
Case(1) → Un-damped case ( $\zeta=0$ )



Case(2) → Under-damped case ( $0 < \zeta < 1$ )



Case(3) → Critically damped case ( $\zeta=1$ )



Case(4) → Overdamped case ( $\zeta > 1$ )

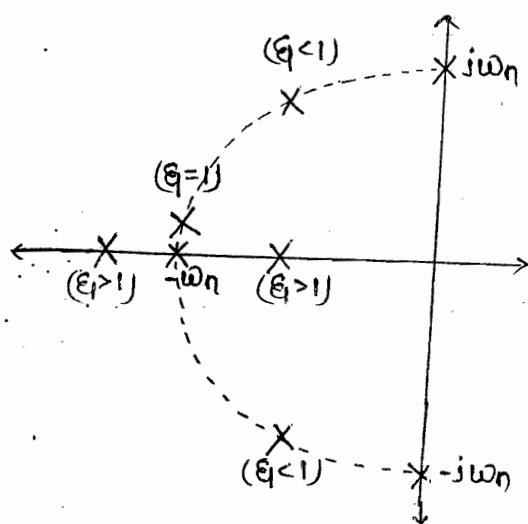
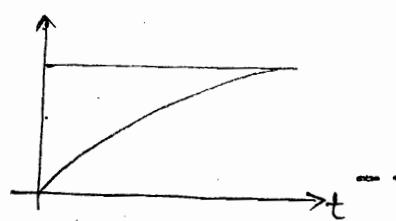


Fig :- Root locus

- \* Most of the CS are designed  $\xi < 1$  because the response can be analysed using more no. of performance specification (optimum values of  $\xi$  in CS design are b/w 0.3-0.8).
- \* The root locus of 2nd order sys. obtained by varying the damping ratio  $\xi$  is a semicircular path with a radius of  $\omega_n$  & breakaway point at  $-\omega_n$  on the -ve real axis.

C/S of underdamped sys.  $\rightarrow$

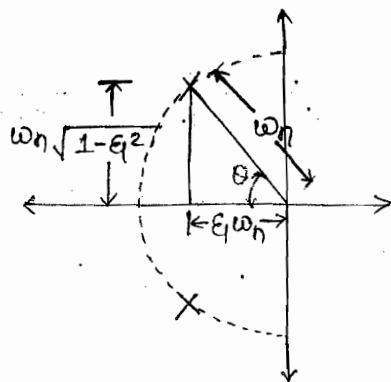
$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s = -\xi\omega_n \pm \sqrt{(\xi^2 - 1)} \omega_n$$

For  $\xi < 1$

$$(1) \cos \theta = \frac{\xi \omega_n}{\omega_n} = \xi$$

$\theta = \cos^{-1} \xi$
$\sin \theta = \sqrt{1 - \xi^2}$
$\tan \theta = \frac{\sqrt{1 - \xi^2}}{\xi}$



2) Damping coefficient (or) actual damping (or) damping factor

$$\zeta = \xi \omega_n$$

3) Timeconstant of underdamped response

$$T = \frac{1}{\zeta} = \frac{1}{\xi \omega_n}$$

4) Damped natural freq:  $\omega_d = \omega_n \sqrt{1 - \xi^2}$  rad/s

5) For  $\xi < 1$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = (s + \xi\omega_n)^2 + \omega_d^2$$

$$1) \text{Damping ratio} = \frac{\text{Actual damping}}{\text{critical damping}} = \frac{\xi \omega_n}{\omega_n} = \xi$$

$$\text{actual damping} = \xi \omega_n, \text{ at } \Rightarrow \xi = 1$$

Actual damping becomes critical damping  
critical damping = 1

### \* Transient analysis $\rightarrow$ (Underdamped Response)

$$\text{Let } R(s) = \frac{1}{s}, C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

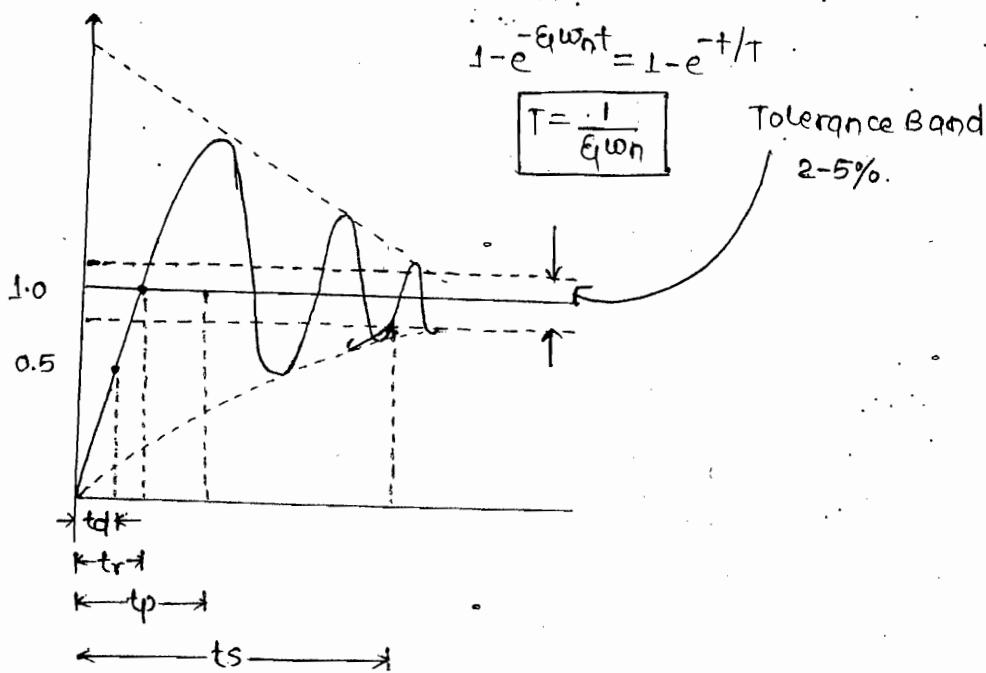
$$\begin{aligned} C(s) &= \frac{1 - (s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ &= \frac{1}{s} - \frac{(s + \zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \times \frac{\zeta\omega_n}{\omega_n \sqrt{1 - \zeta^2}} \end{aligned}$$

$$C(t) = 1 - e^{-\zeta\omega_n t} \cdot \cos \omega_d t - e^{-\zeta\omega_n t} \sin \omega_d t \cdot \frac{\zeta\omega_n}{\sqrt{1 - \zeta^2}}$$

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \left[ \sqrt{1 - \zeta^2} \cos \omega_d t + \zeta \sin \omega_d t \right]$$

$$\therefore A \sin \omega t + B \cos \omega t = \sqrt{A^2 + B^2} \sin \left[ \omega t + \tan^{-1} \left( \frac{B}{A} \right) \right]$$

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin \left[ \omega_d t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right]$$



(1) Delay time ( $t_d$ )  $\rightarrow$

$$t_d = \frac{1+0.7\zeta}{\omega_n} \text{ secs.}$$

(2) Rise time ( $t_r$ )  $\rightarrow$

$$\text{At } t=t_r; c(t)=1$$

$$\therefore c(t) \Big|_{t=t_r} = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r + \theta) = 1$$

$$\frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r + \theta) = 0$$

$$\text{since } \sin(\omega_d t_r + \theta) = 0$$

$$\omega_d t_r + \theta = \pi$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \text{ rad.}$$

(3) Settling time ( $t_s$ )  $\rightarrow$

$$\text{For } 2\% \text{ of Tolerance band; } t_s = 4T \rightarrow \frac{4}{\zeta\omega_n} \text{ secs.}$$

$$\text{For } 5\% \text{ of tolerance band; } t_s = 3T \rightarrow \frac{3}{\zeta\omega_n} \text{ secs.}$$

(4) No. of cycles  $\rightarrow$

$$\omega_d = 2\pi f_d \Rightarrow f_d = \frac{\omega_d}{2\pi} \left( \frac{\text{cycles}}{\text{sec}} \right)$$

$$2\% \text{ of TB} \rightarrow t_s \times f_d \Rightarrow \frac{4f_d}{\zeta\omega_n} \text{ (cycles)}$$

$$5\% \text{ of TB.} \rightarrow t_s \times f_d \Rightarrow \frac{3f_d}{\zeta\omega_n} \text{ (secs)(cycles)}$$

(5) Time period / Time interval of damped sinusoid  $\rightarrow$

$$T = \frac{1}{f_d} \text{ secs.}$$

(6) Peak time ( $t_p$ )  $\rightarrow$

$$\text{At } t=t_p; c(t) = \text{max}^m \text{ value}$$

$$\frac{d}{dt} c(t) = 0$$

$$\Rightarrow \frac{d}{dt} \left[ 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \theta) \right] = 0$$

$$\Rightarrow 0 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_d t + \theta) \omega_d + \sin(\omega_d t + \theta) \frac{-\xi \omega_n}{\sqrt{1-\xi^2}} (-\xi \omega_n) = 0$$

$$\Rightarrow \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_d t + \theta) \cdot \omega_d = \sin(\omega_d t + \theta) \cdot \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \xi \omega_n$$

$$\Rightarrow \frac{\sin(\omega_d t + \theta)}{\cos(\omega_d t + \theta)} = \frac{\omega_d}{\xi \omega_n} = \frac{\omega_n \sqrt{1-\xi^2}}{\xi \omega_n}$$

$$\tan(\omega_d t + \theta) = \tan \theta$$

$$\tan(n\pi + \theta) = \tan \theta$$

$$\omega_d t = n\pi$$

$$\therefore t = t_p$$

$$\omega_d t_p = n\pi$$

$$t_p = \frac{n\pi}{\omega_d} \text{ sec.}$$

maximum peak overshoot ( $m_p$ )  $\rightarrow$

$$c(t) \Big|_{t=t_p} = \frac{\pi}{\omega_d}$$

$$c(t) = 1 - \frac{e^{-\xi \omega_n t} \cdot \frac{\pi}{\omega_d}}{\sqrt{1-\xi^2}} \cdot \sin\left(\frac{\omega_d \pi}{\omega_d} + \theta\right)$$

$$c(t) = 1 + e^{-\xi \omega_n t} \cdot \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

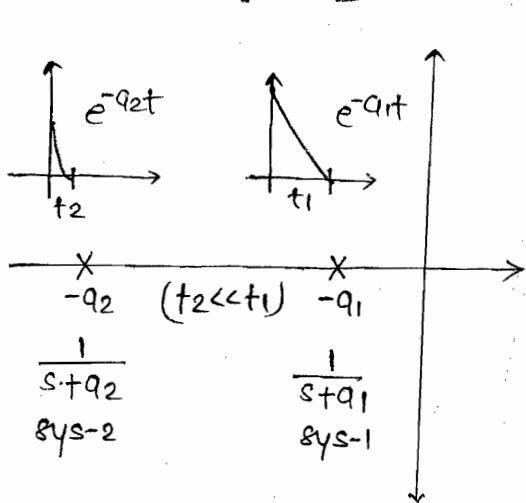
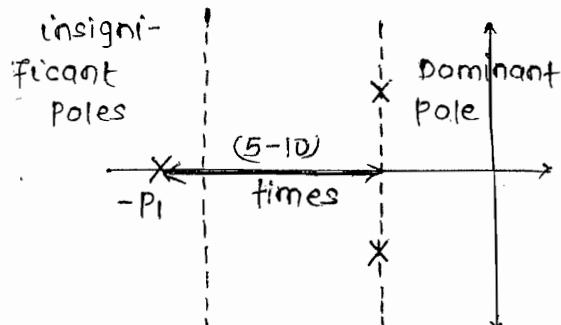
$$c(t) = 1 + e^{-\xi \pi / \sqrt{1-\xi^2}}$$

$$m_p = c(t) \Big|_{t=t_p} - 1 = e^{-\xi \pi / \sqrt{1-\xi^2}} \cong e^{-\xi \pi / \sqrt{1-\xi^2}}$$

- \* Time Response analysis for higher order sys. → Consider a 3rd order c/s eqn;

$$s^3 + ps^2 + qs + k = 0$$

$$(s+p_1)(s^2+q_1s+k_1) = 0$$



\* The time response analysis of higher order sys. is obtained by approximating to 2nd order system. w.r.t dominant poles.

\* The time domain specification obtained for approximated lower order sys. are valid for original higher order sys. also because poles lie in the insignificant region have insignificant effect on the time response. c/s.

Que. → The 2nd order approximation using dominant pole concept is

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

(a)  $\frac{10}{s^2+s+1}$    (b)  $\frac{2}{s^2+s+1}$    (c)  $\frac{10}{(s+5)(s+1)}$    (d)  $\frac{2}{(s+5)(s+1)}$

Soln → Note:- When approximating higher order TF to a lower order TF convert in time constant form before eliminating the insignificant pole.

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

$$= \frac{10 \cdot 2}{5(s+1)(s^2+s+1)}$$

$$T(s) = \frac{2}{s^2+s+1}$$

ATE-18/11/14

ue. → The c/s eqn of the sys. is

$$s(s^2 + 6s + 13) + k = 0$$

Find the value of  $k$  such that the c/s eqn has a pair of complex roots with real part -1.

- (a) 10 (b) 20 (c) 30 (d) 40

⇒  $s^3 + 6s^2 + 13s + k = 0 \dots\dots\dots (i)$

$$(s+a)(s^2 + bs + c) = 0$$

$$\text{Roots are } s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

$$\text{Given } \frac{-b}{2} = -1, b = 2$$

$$(s+a)(s^2 + 2s + c) = 0$$

$$s^3 + as^2 + 2s^2 + 2as + as + ac = 0 \dots\dots\dots (ii)$$

from eqn (i) & (ii)

$$s^3 + s^2(2+a) + s(2a+c) + ac = 0 \dots\dots\dots (iii)$$

$$\begin{aligned} (2+a) &= 6, a = 4, 2a+c = 13 & ac = k \\ &c = 5 & 6 \times 5 = k \\ &k = 30 \end{aligned}$$

ue. → The open loop TF of a unity f/b sys. is  $G(s) = \frac{k(s+b)}{s^2(s+20)}$

for what value of  $b$  does all the 3 roots of the c/s eqn converge at the same point on the real axis.

⇒  $1 + \frac{k(s+b)}{s^2(s+20)} = 0$

$$s^2(s+20) + k(s+b) = 0$$

$$s^3 + 20s^2 + ks + kb = 0 \dots\dots\dots (i)$$

$$\& (s+a)(s+a)(s+a) = 0$$

$$s^3 + s^2(3a) + s(3a^2) + a^3 = 0 \dots\dots\dots (ii)$$

$$3a = 20$$

$$a^3 = 3a^2 b$$

$$3a^2 = k$$

$$a^3 = kb$$

$$b = \frac{a}{3}$$

$$\boxed{b = \frac{20}{9}}$$

5  
64  
\* (a)

$$15\% < \eta_p < 30\%, t_s < 0.75s.$$

$$\eta_p = 15\%$$

$$\eta_p = 0.15$$

$$e^{-\theta \pi / \sqrt{1-\epsilon^2}} = 0.15$$

$$\epsilon = 0.55$$

$$\theta = \cos^{-1} \epsilon = 57^\circ$$

$$t_s = \frac{4}{\epsilon \omega_n} = 0.75$$

$$\epsilon \omega_n = \frac{4}{0.75}$$

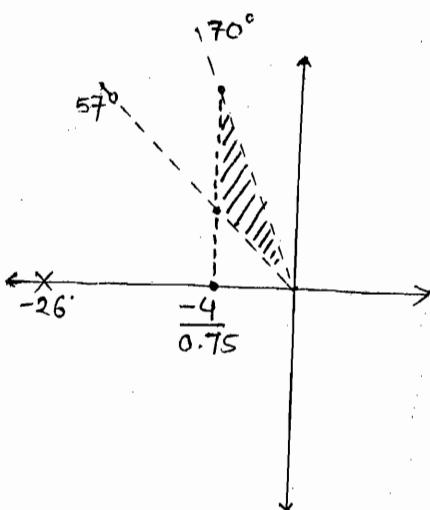
$$\eta_p = 30\%$$

$$\eta_p = 0.3$$

$$e^{-\theta \pi / \sqrt{1-\epsilon^2}} = 0.3$$

$$\epsilon = 0.35$$

$$\theta = \cos^{-1} (0.35) = 70^\circ$$



$$* (b.) 3rd roots; s = 5 \times \frac{-4}{0.75} = -26$$

$$* (c.) \eta_p = 30\%, \epsilon = 0.35$$

$$t_s = 0.75s = \frac{4}{0.35 \times \omega_n} = 0.75$$

$$\omega_n = 15 \text{ rad/s.}$$

$$(s+26)(s^2 + 2 \times 0.35 \times 15s + 225) = 0$$

$$s^3 + 36.5s^2 + 498s + 5850 = 0$$

$$1 + \frac{5850}{s^3 + 36.5s^2 + 498s} = 0$$

$$1 + G(s) = 0$$

$$G(s) = \frac{5850}{s(s^2 + 36.5s + 498)}$$

3  
63

$$1 + \frac{k(s+2)}{s^3 + ks^2 + 4s + 1} = 0$$

$$s^3 + ks^2 + 4s + 1 + ks + 2k = 0$$

$$s^3 + ks^2 + s(4+k) + 2k + 1 = 0 \quad \dots \text{(i)}$$

$$\therefore (s+a)(s^2 + bs + c) = 0$$

$$\text{Given } \epsilon = 0.2, \omega_n^2 = 9$$

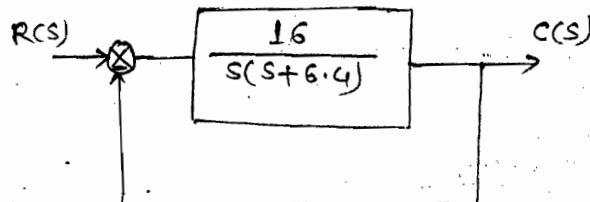
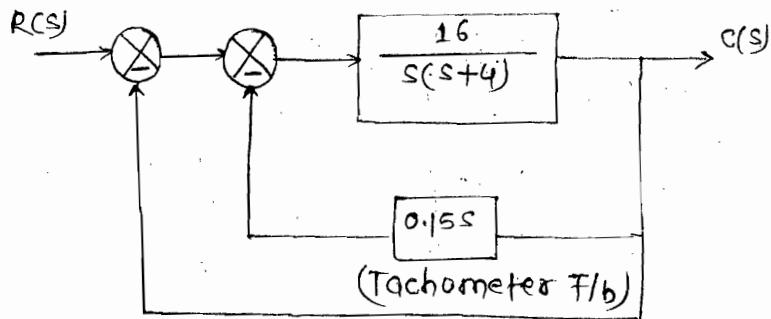
$$c = \omega_n^2 = 9, b = 2\epsilon \omega_n = 2 \times 0.2 \times 3 = 1.2$$

$$(s+a)(s^2 + 1.2s + 9) = 0$$

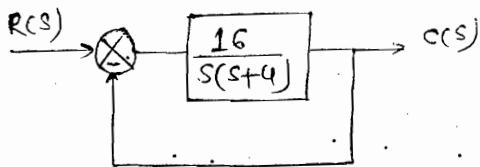
$$s^3 + 1.2s^2 + 9s + 9s^2 + 1.2s + 9 = 0$$

If the pre

\* Effect of tachometer F/b on the performance c/s →



Case(1) → without tachometer F/b.



$$1 + \frac{16}{s(s+4)} = 0$$

$$s^2 + 4s + 16 = 0$$

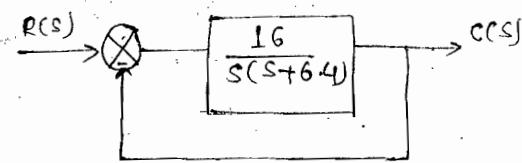
$$\omega_n = 4, \zeta = 0.5 \times 4 = 4$$

$$\omega_n = 4; \zeta = 0.5$$

$$T = \frac{1}{\zeta \omega_n} = \frac{1}{0.5 \times 4} = 0.5s$$

$\zeta = 0.5; T = 0.5s$
$\omega_n = 4$

Case(2) → with tachometer F/b



$$1 + \frac{16}{s(s+6.4)} = 0$$

$$\omega_n = 4, \zeta = 0.8$$

$$T = \frac{1}{\zeta \omega_n} = \frac{1}{0.8 \times 4} = 0.32s$$

$\zeta = 0.8, \omega_n = 4$
$T = 0.32s$

$\omega_n$  = Remains fixed

$$\zeta = \zeta \uparrow$$

$$T = 0.32s, T \downarrow$$

Response is faster

(19)  
63

(d)

$$G(s) = \frac{10K}{s(1+0.1s)}$$

$$I/P = \frac{1}{2} \text{ rps}$$

$$\cdot \text{ess} < 0.2^\circ$$

$$(a.) I/P = \frac{1}{2} \text{ rps} \Rightarrow \pi \text{ rad/s}$$

$$\text{ess} = \frac{0.2\pi}{180} \text{ radians}$$

$$\frac{0.2\pi}{180} = \lim_{s \rightarrow 0} \frac{s \cdot \frac{\pi}{180}}{1 + \frac{10K}{s(1+0.1s)}}$$

$$\frac{0.2\pi}{180} = \frac{\pi}{10K}$$

$$K = 90$$

$$\frac{9d^2c(t)}{dt^2} + \frac{8dc(t)}{dt} + 16c(t) = 16u(t)$$

$$(9s^2 + 8s + 16)c(s) = 16u(s)$$

$$\begin{aligned} \frac{c(s)}{u(s)} &= \frac{16}{9s^2 + 8s + 16} \\ &= \frac{4}{s^2 + 2s + 4} \end{aligned}$$

$$\omega_n = 2 \text{ rad/s}$$

$$2\zeta \times 2 = 2$$

$$\zeta = 0.5$$

(10)  
62

$$\omega_n = 4 \text{ rad/s}$$

$$2\zeta \times 4 = 4$$

$$\zeta = 0.5$$

$$t_p = \frac{3\pi}{4\sqrt{1-(0.5)^2}}$$

$$t_p = \frac{1.5\pi}{\sqrt{3}}$$

(9)  
62

$$\frac{d}{dt}(1 - e^{-st} - 5te^{-st})$$

$$\text{Imp. res.} = 25 + e^{-5t}$$

$$TF = \frac{25}{(s+5)^2} = \frac{25}{s^2 + 10s + 25}$$

$$[\omega_n = 5 \text{ rad/s}; \zeta = 1] \quad \text{ans(d)}$$

$$(b.) 1 + \frac{1000}{s(1+0.1s)} = 0$$

$$0.1s^2 + s + 900 = 0$$

$$s^2 + 10s + 900 = 0$$

$$\omega_n = \sqrt{900} = 30 \text{ rad/s}$$

$$2\zeta \times 30 = 10$$

$$\zeta = 0.05$$

$$\frac{dy}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$$

$$at + f = 0$$

$$y(t) = 0$$

ans(a)

(8)  
62

$$\frac{H(s+c)}{(s+a)(s+b)}$$

$$(1) \quad u(t) = 2 + D\bar{e}^t + E\bar{e}^{-3t}$$

$$(2) \quad \bar{e}^{2t}u(t) = F\bar{e}^t + G\bar{e}^{-3t}$$

$$\begin{aligned} (i) \quad \frac{H(s+c)}{s(s+a)(s+b)} &= \frac{k_1}{s} + \frac{k_2}{(s+a)} + \frac{k_3}{(s+b)} \\ &= 2 + D\bar{e}^t + E\bar{e}^{-3t} \\ q = 1, b = 3 \end{aligned}$$

$$(ii) \quad \frac{H(s+c)}{(s+2)(s+a)(s+b)}$$

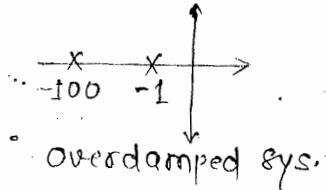
$$\boxed{\begin{array}{l} C=2 \\ H=6 \end{array}}$$

$$\lim_{s \rightarrow 0} \frac{s \cdot H(s+c)}{s(s+a)(s+b)} = \lim_{t \rightarrow \infty} [2 + D\bar{e}^t + E\bar{e}^{-3t}]$$

$$\frac{4C}{ab} = 2; \quad \frac{4C}{3} = 2, \quad 4E = 6$$

(7)  
61

$$TF = G(s) = \frac{100}{(s+1)(s+100)}$$



$$\frac{1}{(1+s)(1+\cancel{s})} = \frac{1}{(1+Ts)} \quad T = 1 \text{ sec.}$$

$$2\% TB = 4T = 4 \text{ sec.}$$

2nd pole is dominant insignificant effect.

5) (A) 6  
65 (C)

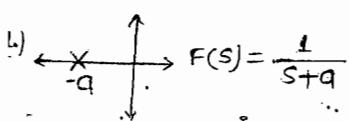
Chapter-03  
Stability

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- \* The stability of LTI system may be defined as when the sys. is subjected to bounded i/p the o/p should be bounded.
  - \* BIBO implies the IR of the sys. should tend to zero as time  $t$  approaches  $\infty$ .
  - \* The stability of a sys. depends on roots of the cl/s eqn
- $$1 + G(s)H(s) = 0$$
- i.e. closed loop poles.

\* IR & stability →

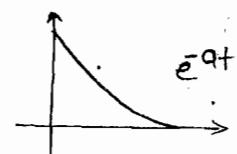
Closed loop pole loc

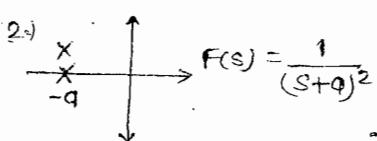
1)   $F(s) = \frac{1}{s+q}$

Stability Criteria

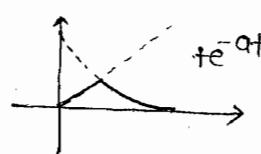
absolutely stable

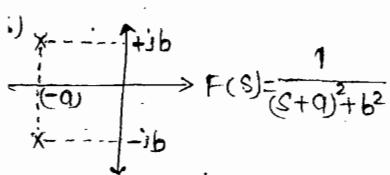
Impulse Response



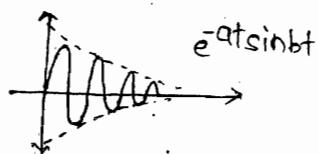
2)   $F(s) = \frac{1}{(s+q)^2}$

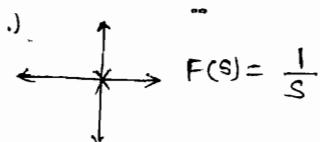
absolutely stable



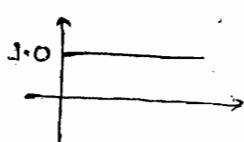
3)   $F(s) = \frac{1}{(s+q)^2+b^2}$

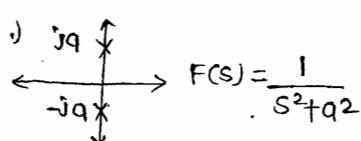
absolutely stable



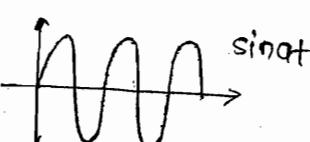
4)   $F(s) = \frac{1}{s}$

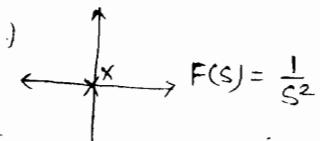
Marginally stable/  
critically stable



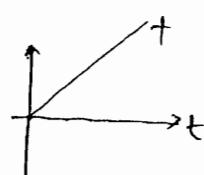
5)   $F(s) = \frac{1}{s^2+q^2}$

Marginally/critically

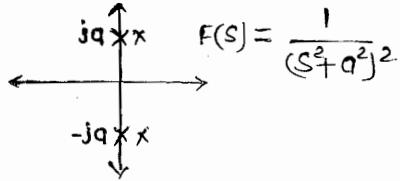


6)   $F(s) = \frac{1}{s^2}$

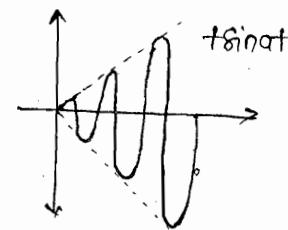
Unstable



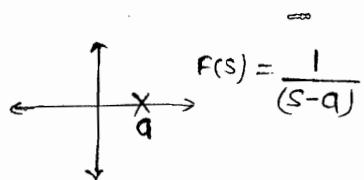
(7)



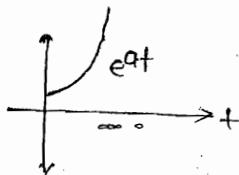
Unstable



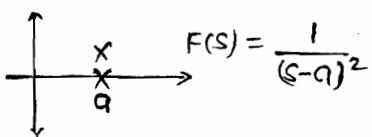
(8)



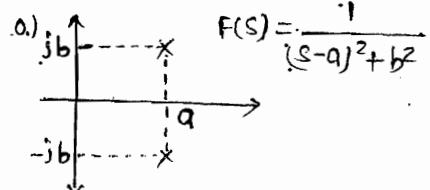
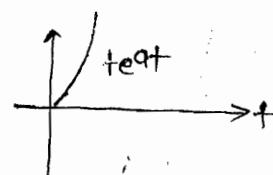
Unstable



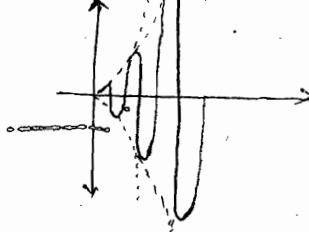
9)



Unstable



Unstable



### Routh-Hurwitz Criteria →

$$\textcircled{2} \quad P(s) = s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

ROUTH =  
ARRAY

$s^4$	1	18	5
$s^3$	8	16	0
$s^2$	$b_1 = 16$	$b_2 = 5$	0
$s$	$c_1 = 135$	0	0
$s^0$	$d_1 = 5$	0	0

$$b_1 = \frac{8 \times 18 - 1 \times 16}{3} = 16$$

$$b_2 = \frac{8 \times 5 - 1 \times 0}{3} = 5$$

$$c_1 = \frac{16 \times 16 - 8 \times 5}{3} = 13.5$$

$$d_1 = \frac{13.5 \times 5 - 16 \times 0}{3} = 5$$

$$\textcircled{3} \quad P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$$

$s^5$	1	2	3
$s^4$	1	2	15
$s^3$	0	-12	0
$s^2$	$\frac{2\epsilon+12}{\epsilon}$	15	0
$s^1$	$\frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12}$		
$s^0$	15		

To check for sign changes

$$(i) \lim_{\epsilon \rightarrow 0} \frac{2\epsilon+12}{\epsilon} = \frac{2(0)+12}{0} = +\infty$$

$$(ii) \lim_{\epsilon \rightarrow 0} \frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12} = \frac{-144}{12} = -12$$

Sign changes:  $+\infty \rightarrow -12$   
 $-12 \rightarrow 15$

2 poles are in RHS

Difficulty-1 → When the 1st element of any row is 0 while the rest of the row has atleast one non-zero term then in such case substitute  $+\epsilon$  (small +ve no) in place of zero & evaluate the rest of RA (Routh array) in terms of  $\epsilon$ .

Check for sign changes by taking  $\lim_{\epsilon \rightarrow 0}$  for the 1st column elements to comment on stability.

(4)  
65

$$P(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

$s^6$	1	8	20	16
$s^5$	2	12	16	20
$s^4$	2	12	16	0
$s^3$	0	8	24	0
$s^2$	6	16	0	0
$s^1$	2.6	0	0	0
$s^0$	16	0	0	0

(i) Construct an auxiliary eq'n  $A(s)$

$$A(s) = 2s^4 + 12s^2 + 16s$$

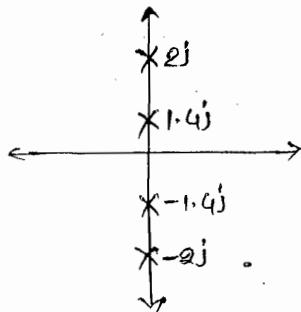
$$(ii) \frac{d}{ds} A(s) = 8s^3 + 24s^2 + 16s$$

The roots of  $A(s) =$  poles symmetric about origin

$$\frac{-12 \pm \sqrt{144-8 \times 16}}{4} = -2, -4$$

$$(s^2+2)(s^2+4)=0$$

$$s = \pm j1.4, \pm j2$$



(5/65)

$s^5$	2	4	2
$s^4$	1	2	1
$s^3$	0	4	0
$s^2$	1	1	0
$s^1$	0	0	0
$s^0$	1	0	0

$$A_1(s) = s^4 + 2s^2 + 1$$

$$\frac{d}{ds} A_1(s) = 4s^3 + 4s$$

$$A_2(s) = s^2 + 1$$

$$\frac{d}{ds} A_2(s) = 2s$$

### The roots of $A_1(s)$

$$\frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$$

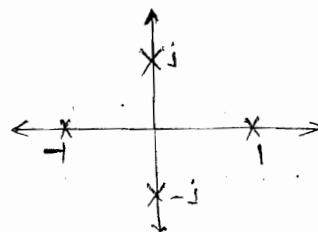
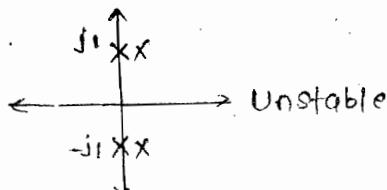
Poles symmetric about origin

$$(s^2+1)(s^2+1) = 0$$

$$s = \pm j, s = \pm i$$

$$(s^2+1)(s^2+1) = 0$$

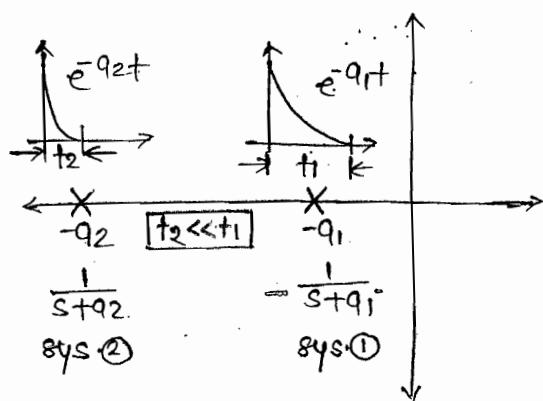
$$s = \pm j, \pm i$$



difficulty (02) → when one complete row of Routh array is 0, then in such cases construct an AE;  $A(s)$  differentiated to get new coefficient & evaluate rest of the RA.

\* Check the roots of AE which are poles symmetric about origin to comment on stability.

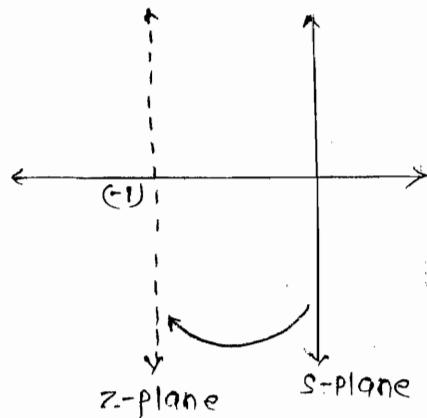
### Relative stability Analysis →



Both sys(1) & sys(2) are said to be absolutely stable  
sys(2) is said to be relatively more stable than sys(1) bcoz  $t_2 \ll t_1$

$$P(s) = s^3 + 7s^2 + 25s + 39 = 0$$

check whether the roots are lying more -ve ly or +ve = 1?



$$s+1=z$$

$$s=z-1$$

$$P(z) = (z-1)^3 + (z-1)^2 + 25(z-1) + 39 = 0$$

$$P(z) = z^3 + 4z^2 + 14z + 20 = 0$$

$z^3$	1	14
$z^2$	4	20
$z$	9	0
$z^0$	20	0

Shortcut → \* Put  $s=-1$  in given  $P(s)$  & if +ve value is coming means all the roots are lying in LHS

If  $P(s)=0$ ; then only 4 values are lying on RHS.

Conditionally stable → A sys. is said to be conditionally stable if its stability depends on one or more parameters.

(7/6)

$$P(s) = s^4 + 2s^3 + 3s^2 + 2s + k = 0$$

$s^4$	1	2	k
$s^3$	2	2	0
$s^2$	2	k	0
$s^1$	$\frac{4-2k}{2}$	0	0
$s^0$	k	0	0

$$(i) \frac{4-2k}{2} > 0 ; k < 2$$

$$(ii) k > 0$$

$$0 < k < 2$$

$$\text{at } k = k_{\max} = 2$$

$$s_{\text{res}} = 0$$

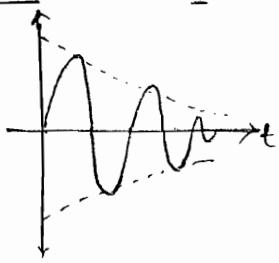
$$A(s) = 2s^2 + k = 0$$

$$2s^2 + 2 = 0$$

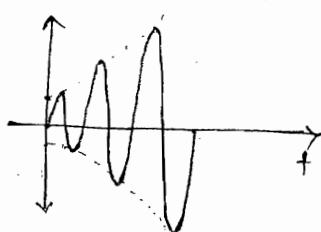
$$s = \pm j \rightarrow j\omega$$

$$\omega = \omega_{\max} = 1 \text{ rad/s}$$

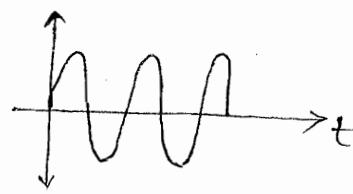
case(1)  $\rightarrow s = -\alpha \pm j\omega$



case(2)  $\rightarrow s = \alpha \pm j\omega$



case(3)  $\rightarrow s = +j\omega$



Q6)

$$1 + \frac{k(s-2)^2}{(s+2)^2} = 0$$

$$(s+2)^2 + k(s-2)^2 = 0$$

$$s^2(1+k) + s(4-4k) + (4+4k) = 0$$

$s^2$	1+k	4+4k
$s^1$	4-4k	0
$s^0$	4+4k	0

$$(i) 1+k > 0 \quad (ii) 4-4k > 0$$

$$k > -1$$

$$k < 1$$

$$-1 < k < 1$$

$$0 \leq k < 1$$

Q7)

$$1 + \frac{10(k_p s + k_I)}{s(s^2 + s + 20)} = 0$$

$$s^3 + s^2 + s(20 + 10k_p) + 10k_I = 0$$

$s^3$	1	$20 + 10k_p$
$s^2$	1	$10k_I$
$s^1$	$20 + 10k_p - 10k_I$	0
$s^0$	$10k_I$	0

$$(i) 10k_I > 0, \boxed{k_I > 0}$$

$$(ii) 20 + 10k_p = 10k_I > 0$$

$$\boxed{k_p > k_I - 2}$$

Q8)

$$1 + \frac{k(s+2)^2}{s(s^2 + 1)(s+4)} = 0$$

$$s^4 + 4s^3 + s^2(1+k) + s(4+4k) + 4k = 0$$

$s^4$	1	$1+k$	$4k$
$s^3$	4	$(4+4k)$	0
$s^2$	$\phi$	$4k$	0
$s^1$	$\frac{(4+4k)e-16k}{\leftarrow}$	0	0
$s^0$	$9k^2$	0	0

the sys. is unstable  
for all  $k > 0$

DATE-19/11/14

(8)  
66

$$1 + \frac{k}{(s^2+2s+2)(s+2)} = 0$$

SOLN

$$(s^2+2s+2)(s+2) + k = 0$$

$$s^3 + 4s^2 + 6s + (4+k) = 0$$

$s^3$	1	6	0
$s^2$	4	$(4+k)$	0
$s^1$	$\frac{24-(4+k)}{4}$	0	0
$s^0$	4		

$$\text{i) } \frac{24-(4+k)}{4} > 0$$

$$k < 20$$

$$\text{ii) } 4+k > 0$$

$$k > -4$$

$$-4 < k < 20$$

$$q+k = k_{max} = 20$$

$$A(s) = qs^2 + (q+k)s = 0$$

$$qs^2 + (q+20)s = 0$$

$$s^2 = -6$$

$$s = \pm j\sqrt{6} = j1.0$$

$$\omega = \sqrt{6} \text{ rad/s}$$

Shortcut  $\rightarrow$  (3rd order system) (only when all coefficients of eqn are +ve)

$$s^3 + qs^2 + 6s + (q+k) = 0$$

(i) Product of external coefficients < product of internal = Stable

(ii) Product of external coefficients > product of internal = Unstable

(iii)  $=$  marginally stable

$$q+k=24$$

$$km\omega = 20$$

$$4+20=24$$

$$A(s) = 4s^2 + (q+k) = 0$$

$$4s^2 + (q+20) = 0$$

$$s^2 = -6$$

$$s = \pm j\sqrt{6} \approx j\omega$$

$$\boxed{\omega = \sqrt{6} \pi / s}$$

Q6

$$1 + \frac{k(s+1)}{s^3 + q s^2 + 2s + 1} = 0$$

Soln  $\rightarrow$

$$s^3 + qs^2 + s(2+k) + (k+1) = 0$$

$$k+1 = q(k+2) \text{ (Given)}$$

$$q = \frac{k+1}{k+2}$$

$$A(s) = qs^2 + (k+1) = 0$$

$$s^2 = \frac{-(k+1)}{q}$$

$$s^2 = \frac{-(k+1)(k+2)}{(k+1)}$$

$$s = \pm j\sqrt{k+2} \approx j\omega$$

$$\omega = \sqrt{k+2}$$

$$\omega = \sqrt{k+2}$$

$$k = 2$$

$$q = \frac{2+1}{2+2} = \frac{3}{4}$$

$$\boxed{k=2, q=\frac{3}{4}}$$

CON 03)	SOLN (a)
66	$1 + \frac{k(s+\alpha)}{s(s+2)(s+4)^2} = 0$

$$s^4 + 10s^3 + 32s^2 + (k+32)s + k\alpha = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 32 & k\alpha \\ s^3 & 10 & 32+k & 0 \\ s^2 & 288-k & k\alpha & 0 \\ \hline & 10 & & \\ s^1 & A & 0 & 0 \\ \hline s^0 & k\alpha & 0 & 0 \end{array}$$

where  $A = \frac{(288-k)(k+32)}{10} - 10k\alpha$

\* (i)  $\frac{288-k}{10} > 0$

$k < 288$

\* (ii)  $\frac{(288-k)(k+32)}{10} - 10k\alpha > 0$

$(\frac{288-k}{10})$

$(288-k)(k+32) - 100k\alpha > 0$

$k\alpha < \frac{(288-k)(k+32)}{100}$

$0 < k\alpha < \frac{(288-k)(k+32)}{100}$

k(b.)  $e_{ss} = \lim_{t \rightarrow 0} s \cdot \frac{1}{s^2}$

$$\frac{1 + \frac{k(s+\alpha)}{s(s+2)(s+4)^2}}{s^2}$$

$e_{ss} = \frac{32}{k\alpha} \quad \therefore \text{let } e_{ss} = 0.16 \text{ (16%)}$

$0.16 = \frac{32}{200 \times \alpha} \quad \therefore \alpha = 1 \quad k = 200$

$k\alpha = 200 \times 1 = 200$

$200 < \frac{(288-200)(200+32)}{100}$

$200 < 204$

THE ROOT LOCUS  
Technique

- The Root locus is defined as the Locus of closed loop poles obtained when sys. gain  $k$  is varied from 0 to  $\infty$ .
- The RL determines relative stability of the sys.

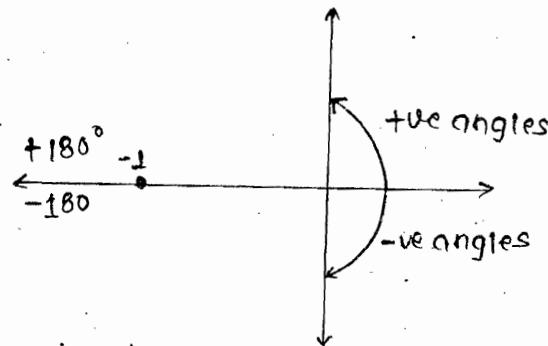
### Angle & magnitude cond<sup>n</sup>

- The angle cond<sup>n</sup> is used for checking whether certain points lie on RL (or) not & also the validity of RL for closed loop poles.

$$1 + G(s) \cdot H(s) = 0$$

$$G(s) \cdot H(s) = -1 + j0$$

$$\underline{G(s) \cdot H(s)} = +180^\circ - \tan^{-1}\left(\frac{0}{1}\right) \left(\frac{\text{Imag.}}{\text{Real}}\right) = 180^\circ \pm 180^\circ \text{ or } \pm (2q+1) 180^\circ$$



- The angle cond<sup>n</sup> may be stated as for a point to lie on RL the angle evaluated at that point must be odd multiple of  $\pm 180^\circ$ .
- The magnitude cond<sup>n</sup> is used for finding the sys. gain  $k$  at any point on RL.

$$1 + G(s) \cdot H(s) = 0$$

$$G(s) \cdot H(s) = -1 + j0$$

$$|G(s) \cdot H(s)| = \sqrt{(-1)^2 + 0^2} = 1$$

$|G(s) \cdot H(s)| = 1$

(4/68)

$$s_1 = -3+4j, s_2 = -3-2j$$

$$G(s) \cdot H(s) = \frac{k}{(s+1)^4}$$

$$G(s) \cdot H(s) \Big|_{(s=s_1=-3+4j)} = \frac{k+j0}{(-3+4j+1)^4} = \frac{0^\circ}{(-2+4j)^4} = \frac{0^\circ}{116.5 \times 4} = -466^\circ$$

$$G(s) \cdot H(s) \Big|_{(s=s_2=-3-2j)} = \frac{k+j0}{(-3-2j+1)^4} = \frac{0^\circ}{-135 \times 4} = +540^\circ$$

Because of the odd multiple is present in the  $s_2$  then  $180 \times 3 = 540^\circ$  it is lying on RL.

(5/68)

$$G(s) = \frac{k}{s(s^2+7s+12)}$$

$$s = -1+j1$$

$$G(s) \Big|_{s=-1+j1} = \frac{k}{(-1+j1)[(-1+j1)^2 + 7(-1+j1)+12]} = \frac{k+j0}{(-1+j1)(s+5j)}$$

$$\angle G(s) \Big|_{s=-1+j1} = \frac{0^\circ}{(135^\circ)(45^\circ)} = -180^\circ \text{ (lying on the R-L)}$$

$$|G(s)|_{s=-1+j1} = 1 \Rightarrow \frac{\sqrt{k^2+0^2}}{\sqrt{(-1)^2+(1)^2}\sqrt{5^2+5^2}} \Rightarrow \frac{k}{\sqrt{2} \times \sqrt{50}} = 1 \quad \therefore k=10$$

Ques. → The OLT of U-FB sys. is  $G(s) = \frac{k}{s(s+1)(s+3)}$

A zero is added to the sys. so that the locus passes through  $-1+j1$ .

The locn of zero would be?

- (a) -1.33 (b) -2.33 (c) -1.66 (d) -2.66.

Soln. → Locus passes through  $-1+j1$  means it is lying on R-L.

$$G(s) = \frac{k}{s(s+1)(s+3)}$$

$$\begin{aligned} G(s) \Big|_{(s=-1+j1)} &= \frac{k(-1+j1+2)}{(-1+j1)(-1+j1+1)(-1+j1+3)} \\ &= \frac{(k+j0)(z-1+j1)}{(-1+j1)(0+j1)(2+j1)} \end{aligned}$$

$$\begin{aligned} \angle G(j\omega) &= (0^\circ) + \arg\left(\frac{1}{z-1}\right) \\ &= (135^\circ)(90^\circ)(26.5^\circ) \end{aligned}$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) - 251.5^\circ = 180^\circ$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) = 180 + 251.5^\circ$$

$$\left(\frac{1}{z-1}\right) = \tan(431.5^\circ)$$

$$\left(\frac{1}{z-1}\right) = 3$$

$$z = 1.33$$

$$(s+z) = 0$$

$$(s+1.33) = 0$$

$$\boxed{s = -1.33}$$

### \* Construction Rules of R-L →

(1.) The R-L is symmetrical about real axis.

(2.) Let  $p$  = no. of open loop poles;  $z$  =

$z$  = no. of open loop zeros

&  $p > z$ ; then the no. of branches of R-L =  $p$

(3.) The no. of branches terminating at zeros =  $z$

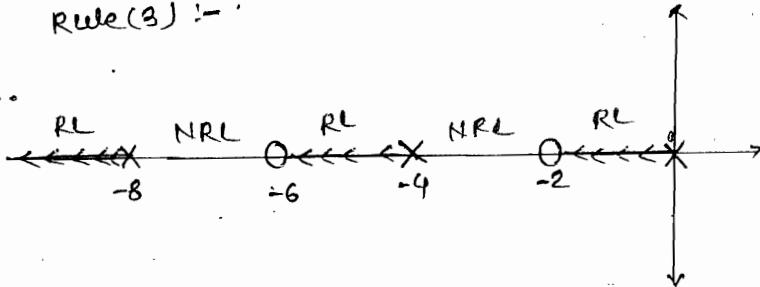
The no. of branches terminating at  $\infty$  =  $p-z$

(4.) A point on Real axis is said to be on RL if to the right side of this point the sum of open loop poles & zeros is odd.

$$\text{eg:- } G(s) = \frac{k(s+2)(s+6)}{s(s+4)(s+8)}$$

Rule (2) :-  $p=3, z=2, p-z=1$

Rule (3) :-



$$\begin{aligned}\frac{G(s)}{1+G(s)} &= \frac{k(s+2)(s+6)}{s(s+4)(s+8)} \\ &\quad - \frac{k(s+2)(s+6)}{s(s+4)(s+8)} \\ &= \frac{k(s+2)(s+6)}{s(s+4)(s+8) + k(s+2)(s+6)}\end{aligned}$$

$$\text{Closed-loop poles} = s(s+4)(s+8) + k(s+2)(s+6)$$

when  $k=0$

$$\text{closed loop poles} = 0, -4, -8$$

(4) Angle of Asymptotes → The p-z branches terminate at  $\infty$  along certain straight line known as asymptotes of RL.

Therefore no. of asymptotes = p-z.

$$\theta = \frac{(2q+1)180^\circ}{(p-z)} \quad q = 0, 1, 2, 3, \dots$$

e.g.: p-z=2

$$\theta_1 = \frac{[2(0)+1] \times 180^\circ}{2} = 90^\circ \quad ; \quad \theta_2 = \frac{[2(1)+1] \times 180^\circ}{2} = 270^\circ$$

6  
68

$$s(s+4)(s^2+2s+5) + k(s+1) = 0$$

$$1 + \frac{k(s+1)}{s(s+4)(s^2+2s+5)} = 0$$

$$1 + \frac{G(s) \cdot H(s)}{s(s+4)(s^2+2s+5)} = 0$$

$$G(s) \cdot H(s) = \frac{k(s+1)}{s(s+4)(s^2+2s+5)} = \frac{k(s+1)}{s^2(s+4)(s+3)}$$

(Q.) p=4, z=1, p-z=3

$$\theta_1 = \frac{[2(0)+1] \times 180^\circ}{3} = 60^\circ \quad \theta_2 = \frac{[2(2)+1] \times 180^\circ}{3} = 300^\circ$$

$$\theta_3 = \frac{[2(1)+1] \times 180^\circ}{3} = 180^\circ$$

\* \* \*

Angle b/w asymptotes	$= \frac{2\pi}{p-z}$
-------------------------	----------------------

5.) Centroid → It is the intersection point of asymptotes on the real axis. It may or may not be a part of RL.

$$\text{Centroid} = \frac{\sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeros}}{P-Z}$$

$$\frac{7}{8} \quad s^3 + 5s^2 + (k+6)s + k = 0$$

$$s^3 + 5s^2 + 6s + ks + k = 0$$

$$s^3 + 5s^2 + 6s + k(s+1) = 0$$

$$1 + \frac{k(s+1)}{s^3 + 5s^2 + 6s} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = \frac{k(s+1)}{s(s^2 + 5s + 6)} = \frac{k(s+1)}{s(s+3)(s+2)}$$

$$(2.) P=3, Z=1, P-Z=2$$

5.) Centroid →

$$\text{Zero at } s = -1+j0 = -1$$

$$\text{Centroid} = \frac{-5 - (-1)}{2} = -2$$

$$\text{Poles at } s = 0+j0$$

$$\begin{array}{r} -2+j0 \\ -3+j0 \\ \hline -5 \end{array}$$

$$\text{Centroid} = -2, 0$$

6.) Break away points → They are those points where multiple roots of the c/s eqn occur.

Procedure → (1.) Construct  $1+G(s)H(s) = 0$   
 (2.) Write 'k' in terms of 's'

(3.) find  $\frac{dk}{ds} = 0$ .

(4.) The roots of  $\frac{dk}{ds} = 0$  will give breakaway points.

(5.) To test valid BA points substitute in step (2.)

If  $k$  is +ve  $\Rightarrow$  valid BA points.

General predictions about BA points  $\rightarrow$

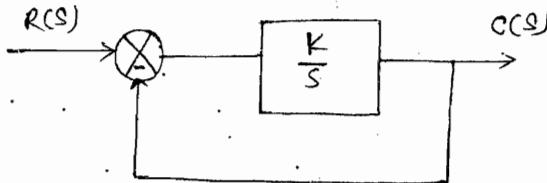
(1) The branches of RL either approach (or) <sup>leave</sup> the BA points at an angle of  $\pm 180 \frac{\pi}{n}$ , where  $n = \text{no of branches approaching (or) leaving}$  the BA point.

(2) The complex conjugate path for the branches of RL approaching (or) leaving the BA point is a circle.

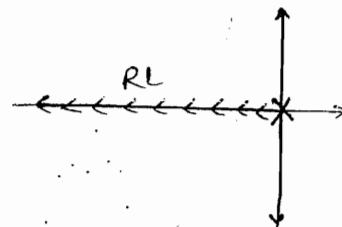
(3.) whenever there are 2 adjacently placed poles on the real axis with the section of real axis b/w them as a part of RL then there exist some BA point b/w the adjacently placed poles.

Conc(4)  
69

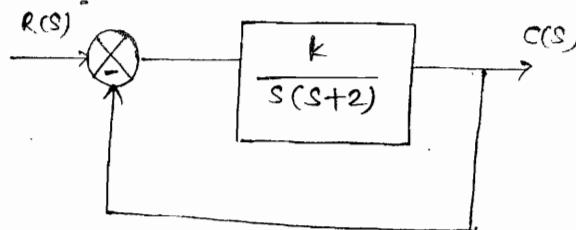
First order system  $\rightarrow$



$$\frac{O(s)}{R(s)} = \frac{k}{s+k} \quad G(s) = \frac{k}{s}$$



\* 2nd order system  $\rightarrow$

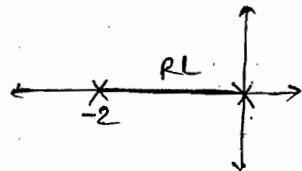


$$\frac{C(s)}{R(s)} = \frac{k}{s^2 + 2s + k}$$

$$G(s) = \frac{k}{s(s+2)}$$

(2.)  $P=2$ ;  $Z=0$ ;  $P-Z=2$

(3.)



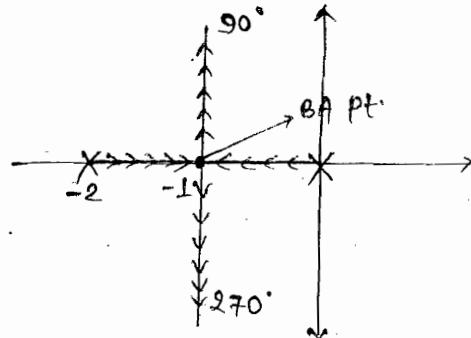
(4.)  $\theta_1=90^\circ$ ,  $\theta_2=270^\circ$

(5.) Centroid  $= \frac{0+(-2)}{2} = -1$

(6.) BA point  $\rightarrow$   
 $s^2 + 2s + k = 0$

$$k = -s^2 - 2s$$

$$\frac{dk}{ds} = 0, -2s - 2 = 0; s = -1$$

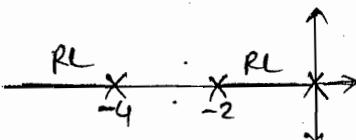


\* 3rd order system  $\rightarrow$

$$G(s) = \frac{k}{s(s+2)(s+4)}$$

Effect of adding poles to a TF:-

(7.)  $P=3$ ,  $Z=0$ ,  $P-Z=3$  (3)



(8.)  $\theta_1=60^\circ$ ,  $\theta_2=180^\circ$ ,  $\theta_3=300^\circ$

(5.)  $\frac{0+(-2)+(-4)-0}{3} = -2$

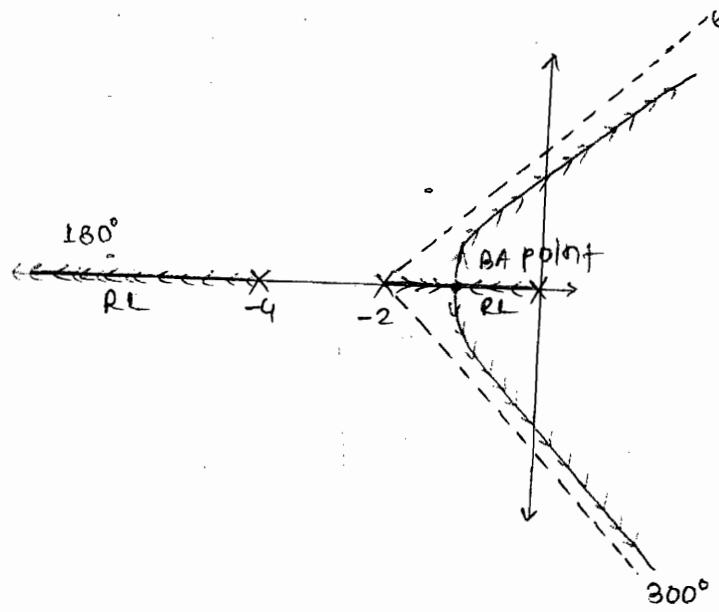
(6.) BA point

$$s^3 + 6s^2 + 8s + k = 0$$

$$k = -s^3 - 6s^2 - 8s$$

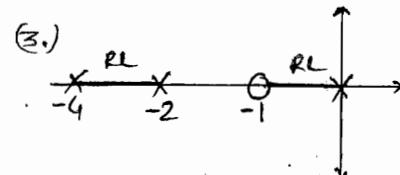
$$\frac{dk}{ds} = 0, 3s^2 + 12s + 8 = 0$$

$$s = -0.8, -3.15X$$



\*  $G(s) = \frac{k(s+1)}{s(s+2)(s+4)}$  Effect of adding zero to a TF.

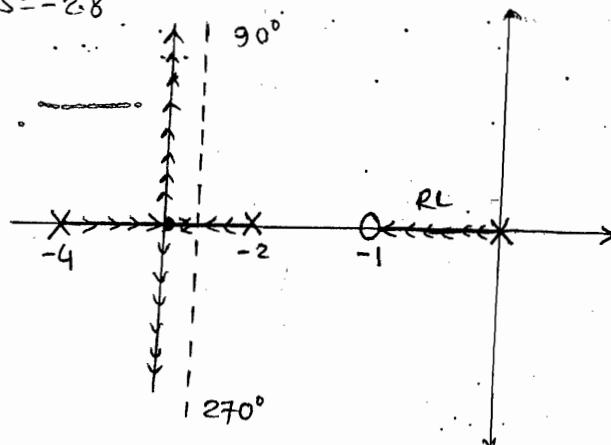
(2.)  $P=3$ ,  $Z=1$ ,  $P-Z=2$



(4.)  $\theta_1 = 90^\circ$ ,  $\theta_2 = 270^\circ$

(5.)  $\frac{0 + (-2) + (-4) - (-1)}{2} = -2.5$

(6.) BA point  $s = -2.8$



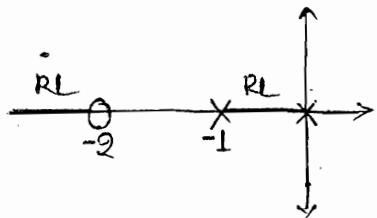
\*\*  
Stability ↑  
RL shifts to  
LHS

Prediction (4) whenever there is a zero on real axis & to the left side of that zero there are no poles (or) zeros on the real axis with the entire section of real axis to the left side of zero as a part of RL then there exists a BA points to the left side of that zero.

$$\text{Eq: } G(s) = \frac{k(s+2)}{s(s+1)}$$

$$(2.) P=2, Z=1, P-Z=1$$

(3.)



(6.) BA points:-

$$s(s+1) + k(s+2) = 0$$

$$k = \frac{-s^2 - s}{s+2}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(s+2)(-2s-1) - [(-s^2 - s)]}{(s+2)^2} = 0$$

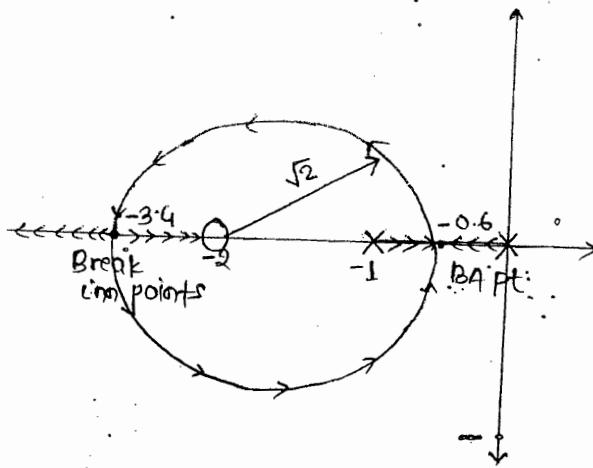
$$-2s^2 + s - 4s - 2 + s^2 + s = 0$$

$$-s^2 - 4s - 2 = 0$$

$$s^2 + 4s + 2 = 0$$

$$s = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$s = -2 \pm \sqrt{2} = -0.6, -3.4$$



To evaluate the center & radius  $\rightarrow$

$$G(s) = \frac{k(s+b)}{s(s+a)}$$

$$\text{Let } s = x + iy$$

$$G(s) = \frac{k[x + iy + b]}{(x + iy)(x + iy + a)}$$

$$G(s) = \frac{k[(x+b) + iy]}{x^2 + xiy + iyx - y^2 + ax + aiy}$$

$$g(s) = \frac{k[(x+b)+jy]}{s^2+ax-y^2+j(2xy+ay)}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{A-B}{1+AB}\right) = 180^\circ$$

$$\Rightarrow \frac{A-B}{1+AB} = \tan(180^\circ)$$

$$\Rightarrow \frac{A-B}{1+AB} = 0$$

$$\Rightarrow A-B = 0$$

$$\Rightarrow \left(\frac{y}{x+b}\right) - \left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 0$$

$$\Rightarrow x^2+ax-y^2-(2x^2+exb+ax+ab)=0$$

$$\Rightarrow (x+b)^2+y^2=b(b-a)$$

$$\Rightarrow \text{center} = -b, 0 = -2, 0$$

$$\Rightarrow \text{Radius} = \sqrt{b(b-a)} = \sqrt{2(2-1)} = \sqrt{2}$$

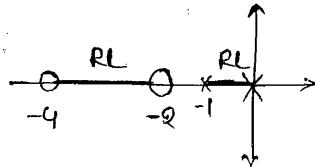
BA points shortcuts = Center ± Radius.

Prediction (5.) whenever there are 2 adjacently placed zeros on the real axis with the section of real axis b/w them as a part of RL then there exists BA point b/w the adjacently placed zeros.

Eg:-  $g(s) = \frac{k(s+2)(s+4)}{s(s+1)}$

(2.)  $P=2, Z=2; P-Z=0$

(3.)



(G) BA points  $\rightarrow$

$$s(s+1) + k(s^2 + 6s + 8) = 0$$

$$k = \frac{-s^2 - s}{s^2 + 6s + 8}$$

$$\frac{dk}{ds} = 0$$

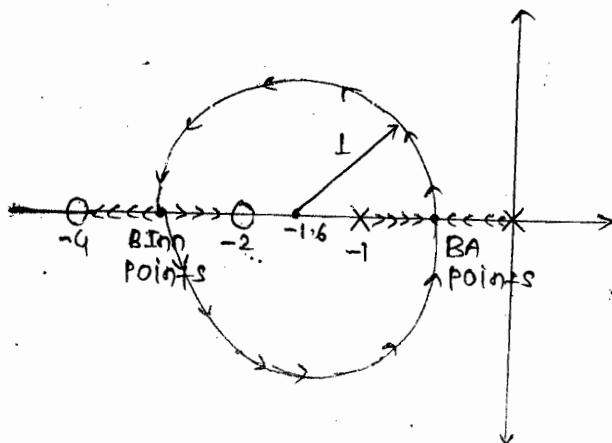
$$\frac{(s^2 + 6s + 8)(-2s - 1) - [(-s^2 - s)(2s + 6)]}{(s^2 + 6s + 8)^2} = 0$$

$$\Rightarrow 5s^2 + 16s + 8 = 0$$

$$s = \frac{-16 \pm \sqrt{256 - 160}}{10}$$

$$s = -1.6 \pm 1 = -2.6, -0.6$$

(Center Radius)



Rule(7)  $\rightarrow$  Intersection of RL with imaginary axis :- The roots of the AE A(s) at  $k = k_{max}$  from Routh Array give the intersection of RL with imaginary axis.

e.g:-

$$G(s) = \frac{k}{s(s+2)(s+4)}$$

$$s^3 + 6s^2 + 8s + k = 0$$

$$(i) \frac{48-k}{6} > 0 \quad (ii) k > 0$$

$$k < 48$$

$$0 < k < 48$$

$$k = k_{max} = 48$$

$$A(s) = 6s^2 + k = 0$$

$$6s^2 + 48 = 0$$

$$s = \pm i\sqrt{8}$$

$$s = \pm i2\sqrt{2}$$

$s^3$	1	8
$s^2$	6	$k$
$s^1$	$\frac{48-k}{6}$	0
$s^0$	$k$	0

### Intersection of asymptotes with jw axis

$$\tan \theta = \frac{y}{x}$$

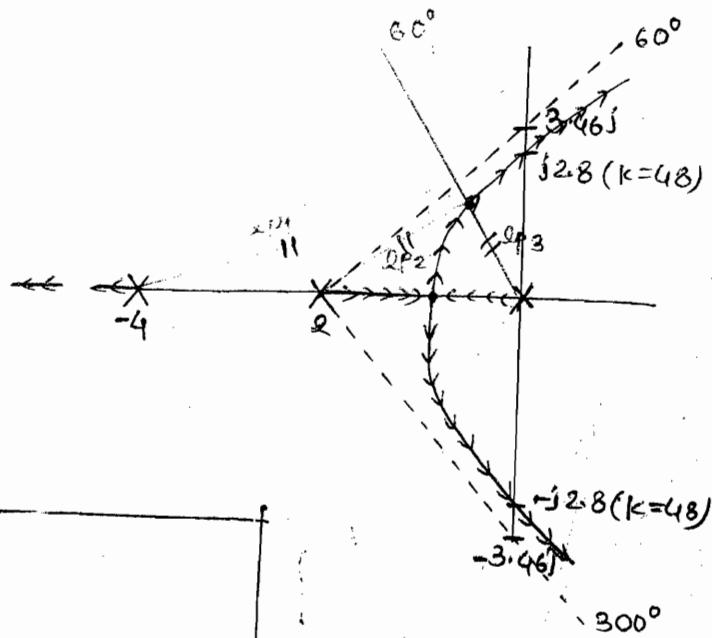
$$\tan 60^\circ = \frac{y}{x}$$

$$y = \tan 60^\circ \times 2$$

$$y = \sqrt{3} \times 2$$

$$= 3.46$$

$$= \pm 3.46j$$



Short cut method →

$$G(s) = \frac{k}{s(s+a)(s+b)}$$

$$\text{Intersection of RL with jw axis} = \pm \sqrt{ab}$$

Q. → Find k when  $\theta = 0.5$  from RL?

$$\text{Soln} \rightarrow \theta = \cos^{-1}(0.5)$$

$$\theta = 60^\circ$$

$$k = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeros}}$$

$$= \frac{l_{P_1} \times l_{P_2} \times l_{P_3}}{1}$$

Linearity	$q_1x_1(t) + q_2x_2(t) \rightleftharpoons$ $q_1c_1(\eta) + q_2c_2(\eta)$	$q_1\mathcal{F}_1(t) + q_2\mathcal{F}_2(t) \rightleftharpoons$ $q_1F_1(s) + q_2F_2(s)$	$q_1x_1(\eta) + q_2x_2(\eta) \rightleftharpoons$ $q_1c_1(z) + q_2c_2(z)$
Time-Reversal	$x(-t) \rightleftharpoons c_{\eta}(t)$	$x(-t) \rightleftharpoons x(-\omega)$	$x(-\eta) \rightleftharpoons x(z^{-1})$
Conjugation	$x^*(t) \rightleftharpoons c_{-\eta}^*$	$x^*(t) \rightleftharpoons x^*(-\omega)$	$x^*(\eta) \rightleftharpoons x^*(z^*)$
Time-shifting	$x(t-t_0) \rightleftharpoons c_\eta e^{-j\eta\omega_0 t_0}$	$x(t-t_0) \rightleftharpoons x(\omega) e^{-j\omega_0 t_0}$	$x(\eta-\eta_0) \rightleftharpoons x(z) z^{-\eta_0}$
freq: shifting	$x(t)e^{j\eta\omega_0 t} \rightleftharpoons c_{\eta-m}$	$x(t) e^{j\omega_0 t} \rightleftharpoons x(\omega-\omega_0)$	$x(\eta-1) \rightleftharpoons x(\eta-1) z^{-1}$
Convolution in time	$x_1(t) * x_2(t) \rightleftharpoons T_0(c_1\eta, c_2\eta)$ where $T_0 = \text{lcm}(T_1, T_2)$	$x_1(t) * x_2(\omega) = x_1(\omega) x_2(\omega)$	$x_1(\eta) * x_2(\eta) \rightleftharpoons F_1(s) \cdot F_2(s)$ $x_1(\eta) * x_2(\eta) \rightleftharpoons -x_1(z) x_2(z)$
multiplication in time	$x_1(t) \cdot x_2(t) \rightleftharpoons (c_1\eta * c_2\eta)$	$x_1(t) \cdot x_2(t) = \frac{1}{2\pi} [x_1(\omega) * x_2(\omega)]$	$x_1(\eta) \cdot x_2(\eta) \rightleftharpoons \frac{1}{2\pi} j [F_1(s) * F_2(s)]$
Integration in time	$\int x(k) dk = \frac{c_n}{j\eta\omega_0}$	$\int x(t) dt = \frac{x(\omega)}{j\omega} + \pi T x(0) \delta(\omega)$ $x(0) = x(\omega) \Big _{\omega=0}$	$\int x(k) dk = \int_{-\infty}^t x(\tau) d\tau = \begin{cases} \frac{F(s)}{s} & \text{Bilateral} \\ \frac{F(s)}{s} + \int_0^\infty f(\tau) d\tau & \text{Unilateral} \end{cases}$ $\frac{d^n x(t)}{dt^n} = (j\omega)^n X(\omega)$
DIFF in time	$\frac{d^n x(t)}{dt^n} = (j\eta\omega_0)^n c_n$		$\frac{d^n F(s)}{ds^n} = \begin{cases} s^n F(s) & \text{Bilateral} \\ s^n F(s) - s^{n-1} f(0) & \text{Unilateral} \end{cases}$ $-s^{n-3} f'(0)$

$x(\eta) - x(\eta-1) z \rightleftharpoons (1-z^{-1}) X(z)^{\frac{1}{z}}$

Parseval's Power theorem	$P = \frac{1}{T_0} \int_0^{T_0}  x(t) ^2 dt$ $P = \sum_{n=-\infty}^{\infty}  c_n ^2$	<u>Initial value theorem</u> $x(0) = \lim_{s \rightarrow \infty} s \cdot F(s)$ applicable only causal sys. $f(t) = 0, t < 0$	<u>Final value theorem</u> $x(\infty) = \lim_{s \rightarrow 0} s x(s)$ condn: applicable only for causal type of sys. i.e. $x(\eta) = 0, \eta < 0$
	$E = \int_{-\infty}^{\infty}  x(t) ^2 dt$ $= \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$	$x(at), a \neq 0 \Rightarrow x\left(\frac{\omega}{a}\right)$ $x(t) \cdot \cos \omega_0 t = \frac{1}{2} [x(\omega + \omega_0) + x(\omega - \omega_0)]$ $x(t) \cdot \sin \omega_0 t = \frac{1}{2} [x(\omega + \omega_0) - x(\omega - \omega_0)]$	<u>Scaling of z:</u> $q^n x(n) \Leftrightarrow X(q^{-1}z)$
Time scaling	$x(at), a \neq 0 \Rightarrow x\left(\frac{\omega}{a}\right)$ $x(t) \cdot \cos \omega_0 t = \frac{1}{2} [x(\omega + \omega_0) + x(\omega - \omega_0)]$ $x(t) \cdot \sin \omega_0 t = \frac{1}{2} [x(\omega + \omega_0) - x(\omega - \omega_0)]$	$t^n x(t) \Leftrightarrow \frac{d^n x(\omega)}{d\omega^n}$	$\eta \cdot x(\eta) \Leftrightarrow -z \frac{d x(z)}{dz}$
modulation	$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$	$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$ $x(0) = \int_{-\infty}^{\infty} x(t) dt$ $x(t) = x(\omega) \Big _{\omega=0}$	<u>Final value theorem</u> $F(\infty) = \lim_{s \rightarrow 0} s \cdot F(s)$ (i). causal type $F(H) = 0, t < 0$ (ii). $s F(s)$ should have only its poles in s-plane, $x(\omega) = 2\pi \text{rect}(H) \Big _{t=0}$
Differentiation in freq.	$dx(t)/dt = j\omega x(\omega)$	$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$ $x(0) = \int_{-\infty}^{\infty} x(t) dt$ $x(t) = x(\omega) \Big _{\omega=0}$	<u>Final value theorem</u> $x(\infty) = \lim_{z \rightarrow 1} (1-z)^{-1} x(z)$ $x(\infty) = \lim_{s \rightarrow 0} s x(s)$ condn: (i) applicable for causal signals i.e. $x(t)=0$ $t < 0$ (ii) $(1-z)^{-1} x(z)$ should have poles inside unit circle in z-plane.
Area under time domain	$A = \int_{-\infty}^{\infty} x(t) dt$	$A = \int_{-\infty}^{\infty} x(t) dt$	$\frac{F(t)}{t} \Leftrightarrow \int_s^{\infty} F(s) ds$
Area under freq. domain	$A = \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$	$A = \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$	(i) $(1-z)^{-1} x(z)$ should have poles inside unit circle in z-plane.
Integration in freq.	$A = \int_{-\infty}^{\infty} x(t) dt$	$A = \int_{-\infty}^{\infty} x(t) dt$	



DATE-20/11/14

\* Rule no.(B)  $\rightarrow$  Angle of departure & arrival  $\rightarrow$  The angle of departure is obtained when complex poles terminate at  $\infty$

\* The angle of arrival is obtained at complex zeros.

$$\phi_D = 180^\circ + \phi ; \phi_A = 180^\circ - \phi$$

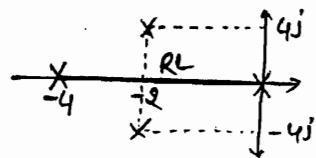
where;  $\phi = \sum \phi_z - \sum \phi_p$

(8)  
68

$$G(s) \cdot H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

(Q.)  $P=4$ ;  $Z=0$ ;  $P-Z=4$

(3.)



(4.)  $\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$

(5.)  $\frac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6.) BA points  $\rightarrow$ (Q.) Shortcut methods  $\rightarrow$ 

Avg. value of real poles  $= \frac{0+(-4)}{2} = -2$

\* If the avg. value of real poles = Real part of complex pole

There will be 3 BA points.

\* The avg. value of real poles  $\neq$  Real part of complex poles.

There will be 1 BA points

(b.) nature of BA points  $\rightarrow$ 

Absolute value of avg. value = 2  
of real poles.

$$2 \times 2 = 20$$

$$x = 10$$

$x \geq 5 \Rightarrow$  There will be 1 real & 2 complex BA points

$x < 5 \Rightarrow$  There will be 3 real BA points.

\* Original method →

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

$$K = -s^4 - 8s^3 - 36s^2 - 80s$$

$$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80 = 0$$

$$4s^3 + 24s^2 + 72s + 80 = 0$$

$$s = -2; -2 \pm 2.45j$$

Note:- To check the validity of complex BA points we angle condn.

$$G(s)H(s) \Big|_{s=-2 \pm 2.45j} = \frac{K}{(-2+2.45j)(2+2.45j)(0+6.45j)(0-1.55j)}$$

$$\left| G(s) \cdot H(s) \right|_{s=-2+j2.45} = \frac{0^\circ}{(130^\circ)(50^\circ)(90^\circ)(-90^\circ)} = -180^\circ$$

(7.)

$$\begin{array}{c|ccc} s^4 & 1 & 36 & K \\ s^3 & 8 & 80 & 0 \\ s^2 & 26 & K & 0 \\ s^1 & \frac{2080-8K}{26} & 0 & 0 \\ s^0 & K & 0 & 0 \end{array}$$

$\frac{2080-8K}{26} > 0, K > 0$   
 $K < 260$   
 $0 < K < 260$

$$K_{\max} = 260$$

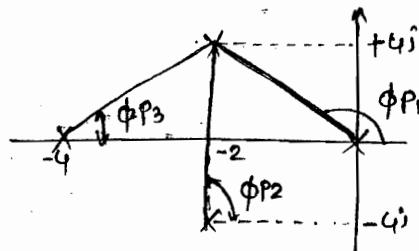
$$f(s) = 26s^2 + K = 0$$

$$26s^2 + 260 = 0$$

$$s = \pm 3.16j$$

$$\begin{aligned} Y &= \tan 45^\circ \times 2 = 2 \\ &= \pm j2 \end{aligned}$$

(8.) Angle of departure →



$$\phi_P = 180^\circ - \tan^{-1} \left( \frac{4-0}{0-(-2)} \right)$$

$$= 116.6^\circ$$

$$\phi_{P2} = 90^\circ$$

$$\phi_{P3} = \tan^{-1} \left[ \frac{4-0}{-2-(-4)} \right] = 63.4^\circ$$

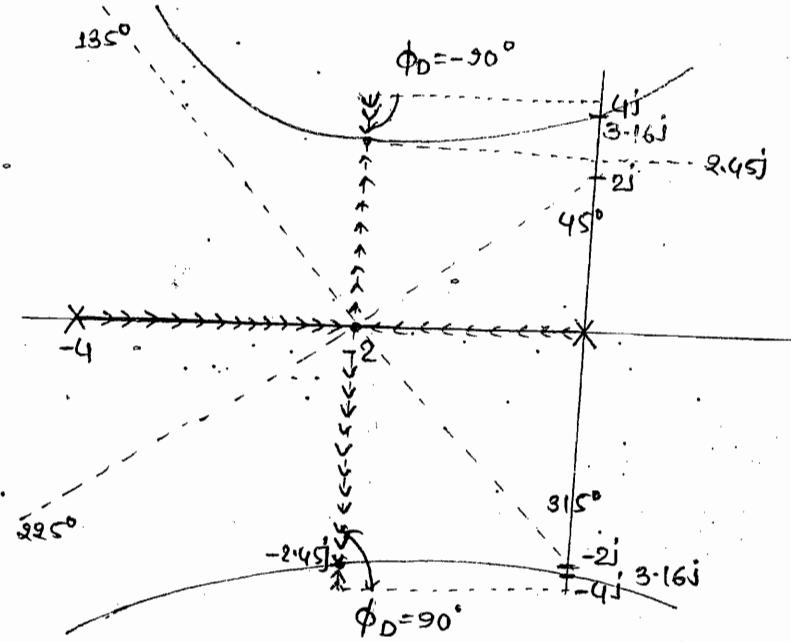
$$\phi = \sum \phi_Z - \sum \phi_P$$

$$= 0 - [116.6 + 90 + 63.4]$$

$$\phi = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180^\circ - 270^\circ$$

$$\boxed{\phi_D = -90^\circ}$$

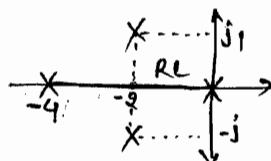


(9)  
68

$$G(s) \cdot H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$$

(i) P=4; Z=0, P-Z=4

(ii)



(iii) θ<sub>1</sub>=45°, θ<sub>2</sub>=135°, θ<sub>3</sub>=225°, θ<sub>4</sub>=315°

$$(5.) \frac{0+(-2)+(-2)+(-4)-0}{4} = -2$$

(6.) BA points  $\rightarrow$

$$s^4 + 8s^3 + 21s^2 + 20s + k = 0$$

$$k = -s^4 - 8s^3 - 21s^2 - 20s$$

$$\frac{dk}{ds} = 0.$$

$$-4s^3 - 24s^2 - 42s - 20 = 0$$

$$4s^3 + 24s^2 + 42s + 20 = 0$$

$$s = -0.78, -2, -3.22$$

(7.)

$s_4$	1	21	$k$
$s_3$	8	20	0
$s_2$	18s	$k$	0
$s_1$	$\frac{370-8k}{18s}$	0	0
$s^0$	$k$	0	0

$$\frac{370-8k}{18s} > 0, k < 46.25$$

$$0 < k < 46.25$$

$$k = k_{\text{max}} = 46.25$$

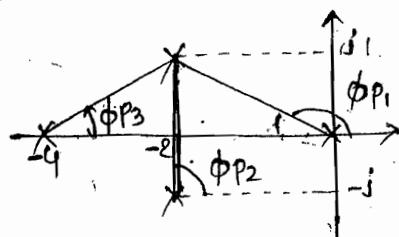
$$A(s) = 18.5s^2 + k = 0$$

$$18.5s^2 + 46.25 = 0$$

$$s = \pm j1.58$$

$$\gamma = \tan^{-1} 45^\circ \times 2 = \pm 2j$$

(8.) Angle of departure  $\rightarrow$



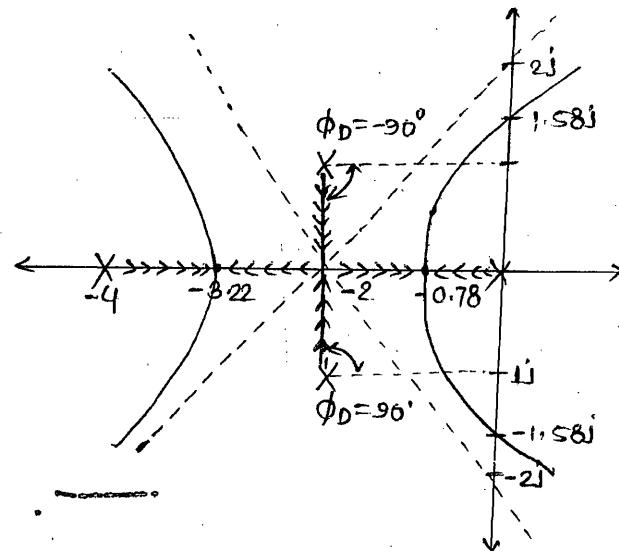
$$\phi_{P1} = 180^\circ + \tan^{-1} \left( \frac{1-0}{0-(-2)} \right) = 153.5^\circ$$

$$\phi_{P2} = 90^\circ; \quad \phi_{P3} = \tan^{-1} \left( \frac{1-0}{-2-(-4)} \right) = 26.5^\circ$$

$$\phi = 0^\circ - (153.5^\circ + 90^\circ + 26.5^\circ) = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180 - 270^\circ = -90^\circ$$

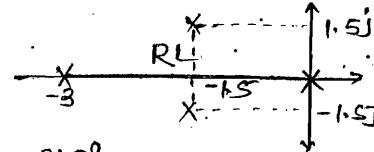
$$\phi_n = -90^\circ$$



Conv(L)

$$G(s) = \frac{K}{s(s+3)(s^2 + 3s + 4s)}$$

$$(2.) P=4, Z=0, P-Z=4 \quad (3.)$$



$$(4.) \theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$$

$$(5.) \frac{0 + (-6s) + (-1.5) + (-3) - 0}{4} = -1.5$$

(6.) BA points :-

$$s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$$

$$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$$

$$\frac{dK}{ds} = -4s^3 - 18s^2 - 27s - 13.5 = 0$$

$$4s^3 + 18s^2 + 27s + 13.5 = 0$$

$$\boxed{s = -1.5, -1.5, -1.5}$$

(7.)

$s^4$	+	13.5	K
$s^3$	6	13.5	0
$s^2$	11.25	K	0
$s^1$	$\frac{151.87 - 6K}{11.25}$	0	0
$s^0$	K	0	0

$$\frac{151.87 - 6K}{11.25} > 0$$

$$K < 25.3$$

$$0 < K < 25.3$$

$$\text{at } k = k_{\text{min}} = 25.3$$

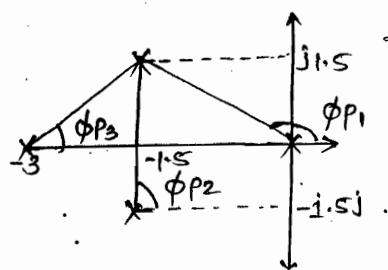
$$A(s) = 11.25s^2 + k = 0$$

$$11.25s^2 + 25.3 = 0$$

$$[s = \pm 1.5]$$

$$Y = \tan^{-1} 45^\circ \times 1.5 = 1.5 \\ = \pm 1.5j$$

(8.) Angle of departure  $\rightarrow$



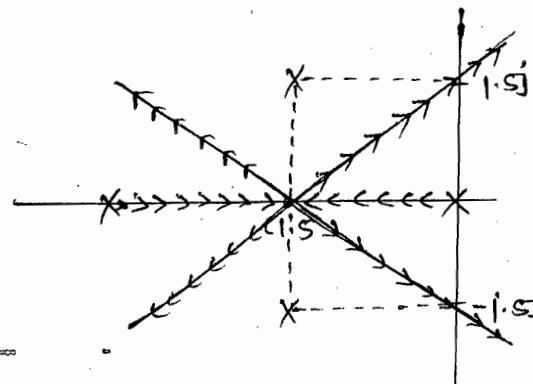
$$\phi_{P_1} = 180^\circ - \tan^{-1} \left( \frac{1.5}{0 - (-1.5)} \right) = 135^\circ$$

$$\phi_{P_2} = 90^\circ, \quad \phi_{P_3} = \tan^{-1} \left( \frac{1.5 - 0}{1.5 - (-3)} \right) = 45^\circ$$

$$\phi = 0^\circ - (135^\circ + 90^\circ + 45^\circ) = -270^\circ$$

$$\phi_D = 180^\circ - 270^\circ = -90^\circ$$

$$[\phi_D = -90^\circ]$$



con 0(2)  
69

$$s(s+4)(s^2+2s+2) + K(s+1) = 0$$

119

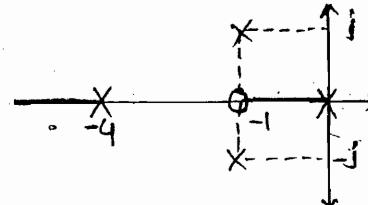
$$1 + \frac{K(s+1)}{s(s+4)(s^2+2s+2)} = 0$$

$$1 + G(s) \cdot H(s) = 0$$

$$G(s) \cdot H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

(2.)  $P=4$ ;  $Z=1$ ,  $P-Z=3$

(3.)



(4.)  $\theta_1 = 60^\circ$ ,  $\theta_2 = 180^\circ$ ,  $\theta_3 = 360^\circ$

(5.)  $\frac{0 + (-1) + (-1) + (-4) - (-1)}{3} = -1.6$

(6.) B4 points:- Nill.

(7.)  $s^4 + 6s^3 + 10s^2 + (K+B)s + K = 0$

$s^4$	1	10	K
$s^3$	6	$(K+B)$	0
$s^2$	$\frac{52-K}{6}$	K	0
$s^1$	$\frac{(52-K)(K+B)-6K}{6}$	0	0
$s^0$	$K$	$\frac{(52-K)}{6}$	0

$$(52-K)(B+K) - 36K = 0$$

$$K^2 - BK - 416 = 0$$

$$K = 24.78, -16.75$$

$$K_{max} = 24.78$$

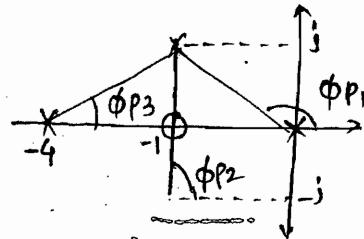
$$A(s) = \frac{(52-24.78)}{6} s^2 + 24.78 = 0$$

$$s = \pm 52.84$$

$$Y = \tan 60^\circ \times 1.6 = \sqrt{3} \times 1.6 = 2.77$$

$$= \pm 2.77j$$

(8.)



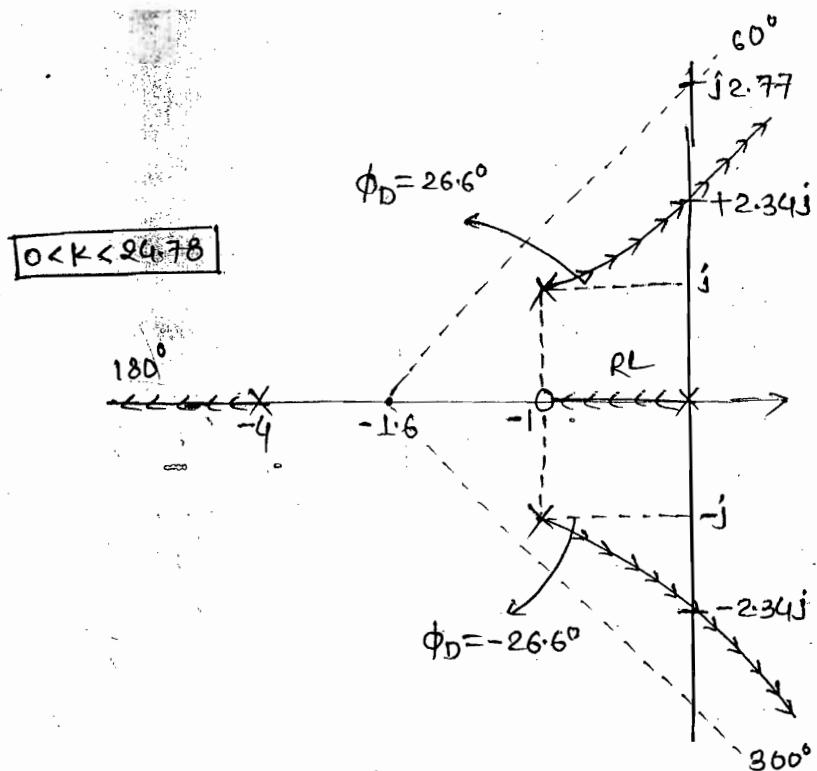
$$\phi_{P1} = 180^\circ + \tan^{-1}\left(\frac{1-0}{0-(-1)}\right) = 135^\circ$$

$$\phi_{P2} = 90^\circ; \quad \phi_{P3} = 90^\circ$$

$$\phi_P = \tan^{-1}\left(\frac{1-0}{-1-(-4)}\right) = 18.4^\circ$$

$$\phi = 90^\circ - (135^\circ + 90^\circ + 18.4^\circ) = -153.4^\circ$$

$$\phi_D = 180 - 153.4^\circ = 26.6^\circ$$

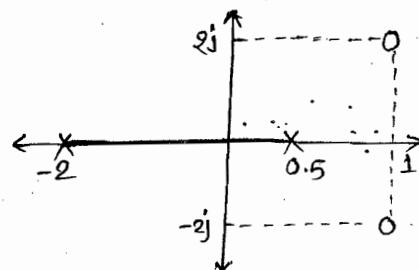


(Q.3)  
69

$$G(s) = \frac{k(s^2 - 2s + 5)}{(s+2)(s-0.5)}$$

$$(2.) P=2, Z=2, P-Z=0$$

(3.)



(4.) BA. point  $\rightarrow$

$$(s+2)(s-0.5) + k(s^2 - 2s + 5) = 0$$

$$s^2 + 1.5s - 1 + k(s^2 - 2s + 5) = 0$$

$$k = \frac{-s^2 - 1.5s + 1}{s^2 - 2s + 5}$$

$$\frac{dk}{ds} = 0$$

$$\Rightarrow 3.5s^2 - 12s - 5.5 = 0$$

$$s = -0.4, 3.8$$

$$(5.) s^2(1+k) + s(1.5-2k) + (5k-1) = 0$$

$$\begin{matrix} S^2 & (1+k) & 5k-1 \\ S^1 & (1.5-2k) & 0 \\ S^0 & 5k-1 & 0 \end{matrix}$$

$$1.5 - 2k = 0$$

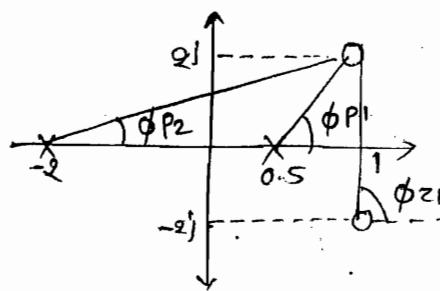
$$k_{max} = 0.75$$

$$A(s) = (1+k)s^2 + (5k-1) = 0$$

$$(1+0.75)s^2 + (5 \times 0.75 - 1) = 0$$

$$s = \pm j1.25$$

(8.) Angle of arrival  $\rightarrow$



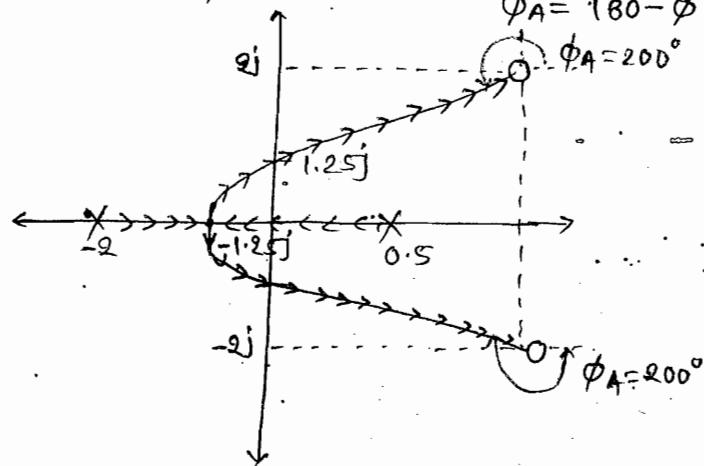
$$\phi z_1 = 90^\circ$$

$$\phi P_1 = \tan^{-1}\left(\frac{2-0}{1-0.5}\right) = 76^\circ$$

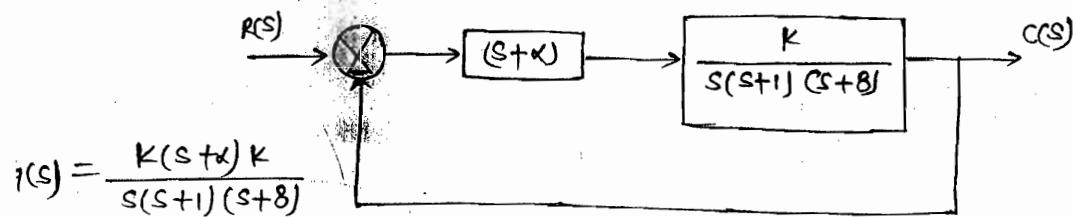
$$\phi P_2 = \tan^{-1}\left(\frac{2-0}{1-(-0.5)}\right) = 34^\circ$$

$$\phi = 90^\circ - (76 + 34) = -20^\circ$$

$$\phi_A = 180^\circ - \phi = 180 - (-20) = 200^\circ$$



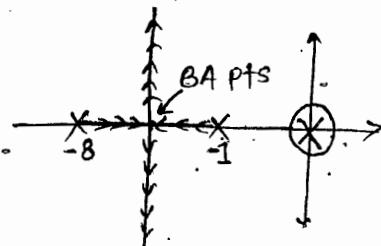
### \* Root Contour



\* These are multiple RL diagrams obtained by varying multiple parameter in a TF drawn on same s-plane.

case(1)  $\rightarrow \alpha = 0$

$$G(s) = \frac{Ks}{s(s+1)(s+8)}$$



Case(2)  $\rightarrow$

$$1 + \frac{K(s+\alpha)}{s(s+1)(s+8)} = 0$$

$$s(s+1)(s+8) + K(s+\alpha) = 0$$

$$s(s+1)(s+8) + ks + k\alpha = 0$$

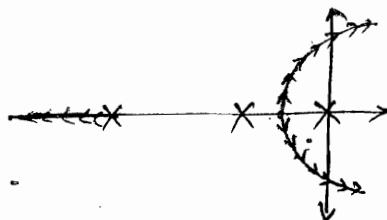
$$1 + \frac{k\alpha}{s(s+1)(s+8) + ks} = 0$$

$$1 + G(s) \cdot H(s) = 0$$

$$G(s) \cdot H(s) = \frac{k\alpha}{s(s+1)(s+8) + ks}$$

if value of K is not given then  $K=1$

$$G(s) \cdot H(s) = \frac{\alpha}{s(s+1)(s+8) + s} \leftarrow \frac{\alpha}{s(s^2 + 9s + 9)}$$



Q Find BA points for  $k=10$ ?

$$\text{Soln} \rightarrow G(s) \cdot H(s) = \frac{10s}{s(s+1)(s+8)+10s}$$

$$\text{let } 10s = k'$$

$$= \frac{k'}{s(s+1)(s+8)+10s}$$

$$= \frac{k'}{s(s^2+9s+18)}$$

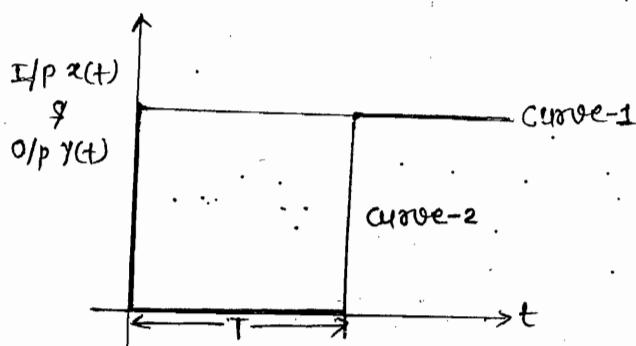
$$1 + \frac{k'}{s(s^2+9s+18)} = 0$$

$$s^3 + 9s^2 + 18s + k' = 0$$

$$k' = -s^3 - 9s^2 - 18s$$

$$\frac{dk'}{ds} = -3s^2 - 18s - 18 = 0$$

\* Analysis of sys. having DEAD TIME (or) TRANSPORTATION LAG  $\rightarrow$



For curve-(1)

$$O/p Y(t) = I/p X(t)$$

$$Y(s) = e^{-Ts} X(s)$$

for curve-(2)

$$O/p Y(t) = X(t-T)$$

$$\boxed{\frac{Y(s)}{X(s)} = e^{-Ts}}$$

Time domain approximation  $\rightarrow$  (time domain analysis, R.H, RL plots)

$$Y(t) = X(t-T) = X(t) - T\dot{X}(t) + \frac{T^2}{2!} \ddot{X}(t) - \frac{T^3}{3!} \dddot{X}(t) + \dots$$

$$Y(t) = X(t) - TX(t)$$

$$Y(s) = X(s)(1-Ts) \quad \& \quad Y(s) = X(s) \cdot e^{-Ts}$$

$$e^{-Ts} \approx (1-Ts)$$

$$\text{Ex:- } G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$$

\* Dead time is one of the forms of non-linearity & is approximated as zero in RHS of s-plane.

\* TF having poles (or) zeros in RHS of s-plane are known as non-min<sup>m</sup> phase fn.

Non min<sup>m</sup> phase fn  $\rightarrow$

$$\boxed{|F(s)| \Big|_{\omega \rightarrow \infty} \neq -(P-Z)90^\circ}$$

$$\text{Eq:- } G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(s-3)}{s(s+3)} \frac{K(1-s)}{s(s+3)} = \frac{K(1-s)}{(s+3)(1+\frac{s}{3})}$$

$$G(j\omega) = \frac{\left(\frac{K}{3} + j0\right)\left(1-j\omega\right)}{\left(0+j\omega\right)\left(1+\frac{j\omega}{3}\right)}$$

$$|G(j\omega)| = \frac{(0^\circ)e^{j\tan^{-1}\omega}}{(90^\circ)(j\tan^{-1}\frac{\omega}{3})} = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$|G(j\omega)| \Big|_{\omega=\infty} = -90^\circ - 90^\circ - 90^\circ = -270^\circ \neq -(P-Z)90^\circ$$

\* TF having poles & zeros in the LHS of s-plane are known as min<sup>m</sup> phase fn. LTI TF should be min<sup>m</sup> phase fn.

min<sup>m</sup> phase fn  $\rightarrow$

$$\boxed{|F(s)| \Big|_{\omega \rightarrow \infty} = -(P-Z)90^\circ}$$

$$\text{Eq:- } G(s) = \frac{k(1+s)}{s(s+3)} = \frac{(k/3)(1+s)}{s(s+\frac{s}{3})}$$

$$G(j\omega) = \frac{\left(\frac{k}{3} + j0\right)(1+j\omega)}{(0+j\omega)\left(1+\frac{j\omega}{3}\right)}$$

$$|G(j\omega)| = \frac{(0^\circ)[\tan'(j\omega)]}{(90^\circ)(\tan'\frac{\omega}{3})} = -90^\circ + \tan'\omega - \tan'\frac{\omega}{3}$$

$$|G(j\omega)|_{\omega=\infty} = -90 + 90^\circ - 90^\circ = -90^\circ$$

$$-90^\circ = -(P-Z)90^\circ$$

\* Since  $s$  indicates time & can't be -ve ( $t-s$ ) factor should be expressed as  $s-(s-1)$  in time domain methods.

$$G(s) = \frac{ke^{-s}}{s(s+3)} = \frac{k(1-s)}{s(s+3)} = \frac{-k(s-1)}{s(s+3)}$$

\* Complementary R-L (CRL) or Inverse RL (IRL)  $\rightarrow$

$$G(s)H(s) = 1+j0$$

(1) Angle cond<sup>n</sup>

$$|G(s) \cdot H(s)| = 0^\circ = \pm(2q)180^\circ$$

(2) Mag. Cond<sup>n</sup>

$$|G(s) \cdot H(s)| = 1$$

\* construction Rule of CRL  $\rightarrow$

Rule(1): The CRL is symmetrical about real axis.

Rule(2): Same as RL.

Rule(3): A point on real axis is said to be on CRL if to the right side of this point the sum of open loop poles & zeros is even.

Rule(4): Angle of asymptotes  $\rightarrow$

$$\theta = (eq)180^\circ$$

where,  $q = 0, 1, 2, \dots$

Rule(5) Centroid  $\rightarrow$  same as RL

Rule(6) BA points  $\rightarrow$  same as RL

Rule(7) Intersection of CRL with 3w axis  $\rightarrow$  same as RL

Rule(8) Angle of departure & arrival  $\rightarrow$

$$\phi_D = 0^\circ + \phi$$

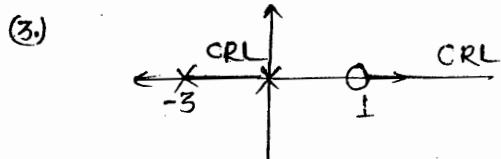
$$\phi_A = 0^\circ - \phi$$

where;

$$\phi = \sum \phi_z - \sum \phi_p$$

Que.  $\rightarrow$   $g(s) = \frac{Ke^s}{s(s+3)} = \frac{k(1-s)}{s(s+3)}$

SOLN  $\rightarrow$  (1)  $P=2$ ,  $Z=1$ ,  $P-Z=1$



(6.) BA points:-

$$1 + \frac{k(1-s)}{s(s+3)} = 0$$

$$s(s+3) + k(1-s) = 0$$

$$k = \frac{-s^2 - 3s}{1-s}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2 - 3s)(-1)]}{(1-s)^2} = 0$$

$$s^2 - 2s - 3 = 0$$

$$s = \frac{2 \pm \sqrt{4+12}}{2}$$

$$s = 1 \pm 2 = -1, 3$$

(7.)  $s^2 + s(3-k) + k = 0$

$$\begin{matrix} s^2 & 1 & k \\ s & 3-k & 0 \\ s^0 & k & 0 \end{matrix}$$

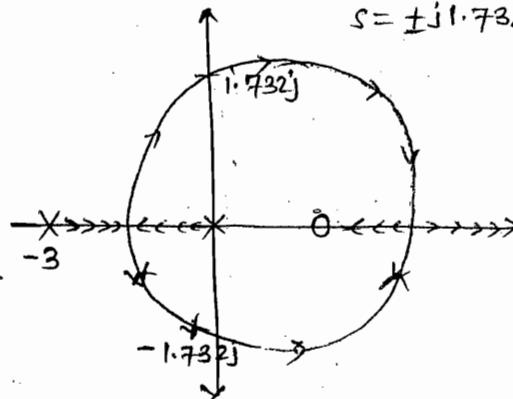
$$s-k=0$$

$$k_{max}=3$$

$$A(s) = s^2 + k = 0$$

$$s^2 + 3 = 0$$

$$s = \pm j 1.732$$



(1)  
67

$$G(s) = \frac{k(s+a)}{s^2(s+b)}$$

ans(c)

$$1 + G(s) = 0$$

$$1 + \frac{k(s+a)}{s^2(s+b)} = 0$$

$$s^3 + bs^2 + ks + ak = 0$$

$$(i) \quad ak > 0 \quad (ii) \quad \frac{bk-ak}{b} > 0 \\ k > 0 \quad k(b-a) > 0$$

$k > 0$

$$\begin{matrix} s^3 & 1 & k \\ s^2 & b & ak \\ s^1 & \frac{bk-ak}{b} & 0 \\ s^0 & ak & 0 \end{matrix}$$

$$\frac{bk-ak}{b} = 0$$

$$k(b-a) = 0$$

$$k = k_{max} = 0$$

$$A(s) = bs^2 + ak = 0$$

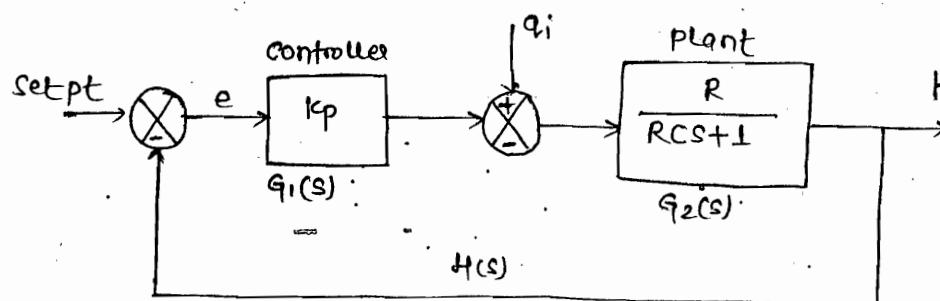
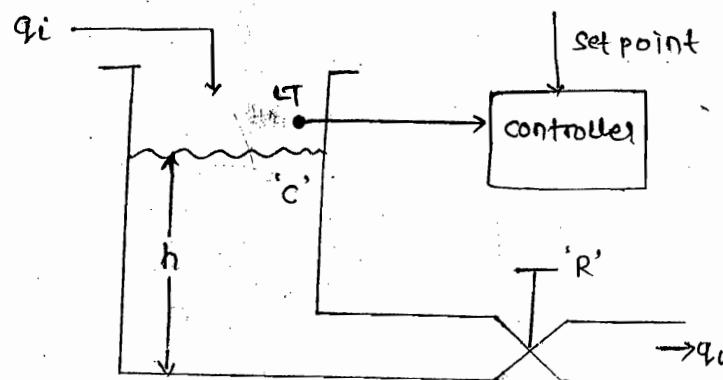
$$bs^2 + 0 = 0$$

$$s = 0$$

(10)  
69

$0 \leq k < 1 \rightarrow$  over damped (1, 3) ans(cc)

$k > 5$  over damped

**INDUSTRIAL  
CONTROLLER**


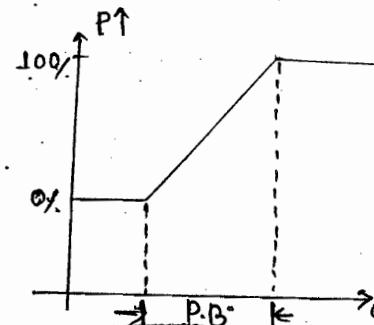
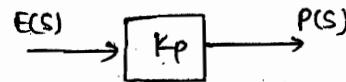
$$|ess| = \lim_{s \rightarrow 0} \frac{s \cdot Q_1(s) \cdot Q_2(s)}{1 + G_1(s)G_2(s)}$$

Proportional mode →

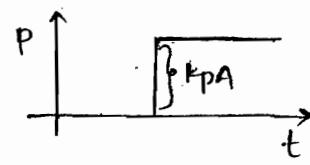
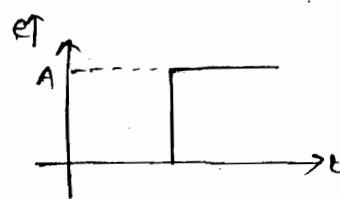
$$P \propto e$$

$$P = k_p e \quad ; \quad k_p = \text{proportional gain}$$

$$P(s) = k_p E(s)$$



$$\text{Proportional Band} = \frac{100}{k_p}$$



$$P = k_p e \quad (e=A)$$

$$P = k_p A$$

$$|ess| = \lim_{s \rightarrow 0} \frac{s \cdot A \cdot \frac{R}{s \cdot RCs + 1}}{1 + \frac{Rk_p}{s \cdot RCs + 1}}$$

$$|ess| = \frac{AR}{1 + Rk_p}$$

Offset

$$\text{Offset} \propto \frac{1}{k_p}$$

- \* It is a natural extension of ON/OFF controller.
- \* The band of error where every value of error has unique value of controller o/p is known as proportional band.
- \* The disadvantage of this controller is it exhibits a permanent residual error known as offset.

(20)  
63

$$\begin{aligned} P &= K_p e \\ 100 &= K_p \times 1 \\ K_p &= 100 \\ PB &= \frac{100}{K_p} = \frac{100}{100} = 1 \end{aligned}$$

$$100\% PB = 1 \times 100\% = 100\%$$

$$100\% PB = 100\%$$

$$20\% PB = ?$$

10, 20

$$\frac{20}{100} \times 100 = 20\%$$

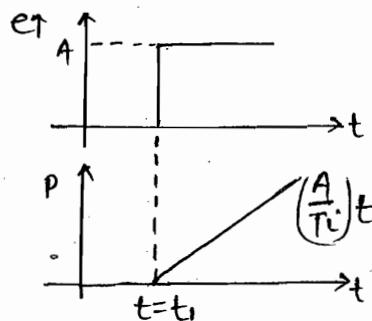
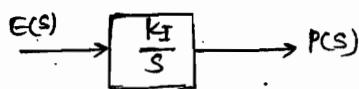
## (2) Integral mode →

$$\frac{dp}{dt} \propto e$$

$$\frac{dp}{dt} = K_I e \quad (K_I = \text{Integral scaling})$$

$$P = K_I \int e dt$$

$$P(s) = \frac{K_I}{s} E(s)$$



$$P = \frac{1}{T_i} \int A dt \quad (e=A)$$

$$P = \left( \frac{A}{T_i} \right) t$$

Let  $e = \sin \omega t$

Defining "RESET TIME!"

$$T_i = \frac{1}{K_I}$$

$$P = \frac{1}{T_i} \int e dt$$

$$P(s) = \frac{1}{T_i s} E(s)$$



$$P = \frac{1}{T_i} \int \sin \omega t dt$$

$$P = \frac{1}{\omega T_i} (-\cos \omega t)$$

$$P = \frac{1}{\omega T_i} \sin(\omega t + \frac{\pi}{2})$$

$$|P_{ess}| = \lim_{s \rightarrow 0} \frac{\frac{1}{s} \cdot \frac{A}{T_i} \times \frac{R}{RCs+1}}{1 + \frac{R}{T_i s (1+s)}}$$

$$|P_{ess}| = \frac{AR}{1+\infty} = 0$$

\* The disadvantage of integral controller is its response to errors is slow. However it is capable of eliminating the error completely in the sys.

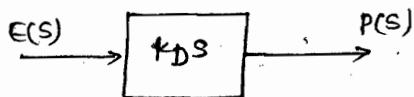
### 3) Derivative (OR) Rate mode $\rightarrow$

$$P \propto \frac{de}{dt}$$

$$P = k_D \frac{de}{dt}$$

$k_D$  = Rate constant

$$P(s) = k_D s \cdot E(s)$$

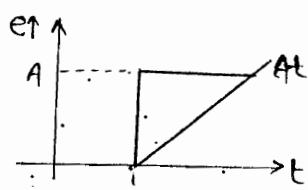
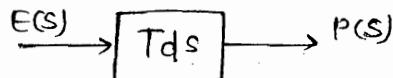


Defining "RATE TIME"

$$Td = k_D$$

$$P = Td \frac{de}{dt}$$

$$P(s) = Td s \cdot E(s)$$

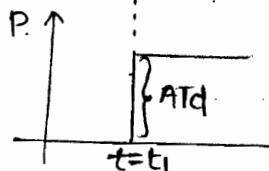


$$P = Td \frac{d(A)}{dt} \quad (e=A)$$

$$P=0$$

$$P = Td \frac{d(At)}{dt} \quad (e=At)$$

$$P = Td A$$



$$|ess| = \lim_{s \rightarrow 0} \frac{s \cdot A \cdot \frac{R}{s^2 RCs + 1}}{\frac{1 + Tds \cdot R}{RCs + 1}}$$

$$|ess| = \lim_{s \rightarrow 0} \frac{\frac{AR}{s^2 RCs + 1}}{\frac{s + s^2 Td R}{RCs + 1}}$$

$$|ess| = \frac{AR}{0} = \infty$$

$$\boxed{|ess| = \infty}$$

\* The disadvantage of this controller is it can't respond to sudden error.

\* It is also called as anticipatory controller because it sends a control signal in anticipation of error.

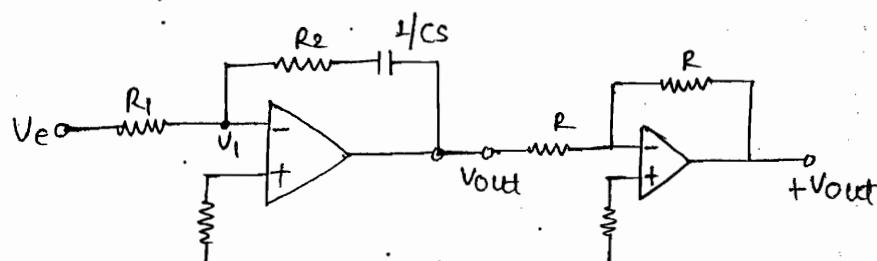
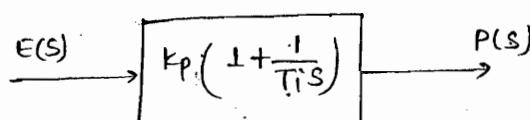
\* This anticipatory nature may result in large instability in the system.

\* Composite controller mode →

(I) P+I mode →

$$P = k_p e + \frac{k_p}{T_i} \int e dt$$

$$P(s) = \left[ k_p \left( 1 + \frac{1}{T_i s} \right) \right] E(s)$$



$$\frac{U_e - V_1}{R_1} = \frac{V_1 - V_{out}}{R_2 C S + 1}$$

$$\therefore V_1 = 0$$

$$\frac{U_e (R_2 C S + 1)}{R_1 C S} = -V_{out}$$

$$-V_{out} = \frac{U_e R_2 C S}{R_1 C S} + \frac{U_e}{R_1 C S}$$

$$V_{out} = \frac{R_2}{R_1} U_e + \frac{R_2}{R_1} \frac{1}{R_2 C} \int U_e dt$$

$$k_p = \frac{R_2}{R_1}, T_i = R_2 C$$

Effect on transient state

$$\text{let } e = \sin \omega t$$

$$P = k_p \sin \omega t + \frac{k_p}{T_i} \int \sin \omega t dt$$

$$P = k_p \sin \omega t + \left( \frac{-k_p}{\omega T_i} \right) \cos \omega t$$

$$P = \sqrt{k_p^2 + \left( \frac{k_p}{\omega T_i} \right)^2} \sin \left( \omega t + \tan^{-1} \frac{1}{\omega T_i} \right)$$

- \* It is capable of improving steady state response c/s of the sys. i.e. elimination of steady state error b/w i/p & o/p.
- \* The integral controller eliminates offset of proportional controller.
- \* It is also known as proportional + Reset controller because the rate of change of controller o/p can be reset by changing the value of reset time  $T_i$ .
- \* For sinusoidal i/p the phase of the controller o/p lags by  $\tan^{-1}(\frac{1}{\omega T_i})$ . Hence it is similar to phase lag compensator.
- \* In terms of filtering property it acts as low pass filter.
- \* The P+I controller increases the type & order of system by one.

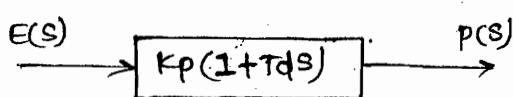
#### Effect on performance specification →

- 1) It increases rise time.
- 2) It reduces BW.
- 3) It reduces the stability of the system.
- 4) It increases the damping ratio & hence reduces the peak overshoot.
- 5) It eliminates steady state error.

#### 2) P + D mode →

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = [K_p(1+T_d s)] E(s)$$



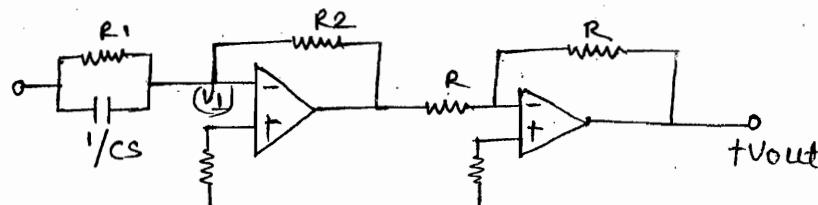
#### Effect on transient state

$$\text{let } e = \sin \omega t$$

$$P = K_p \sin \omega t + K_p T_d \frac{d}{dt} \sin \omega t$$

$$P = K_p \sin \omega t + \omega K_p T_d \cos \omega t$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \cdot \sin(\omega t + \tan^{-1}(\omega K_p T_d))$$



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{R_2}$$

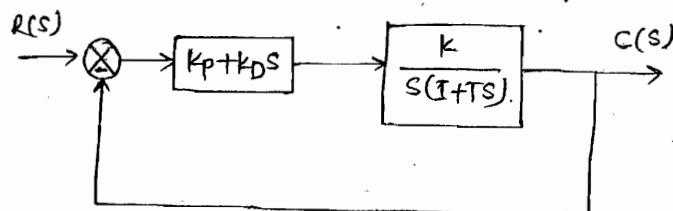
$$V_{out} = \frac{V_e (R_1 + R_2)}{R_1}$$

$$K_p = \frac{R_2}{R_1}; T_d = R_1 C$$

$$-V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot R_1 C V_e$$

$$+V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot R_1 C \cdot \frac{dV_e}{dt}$$

Chapter (3.)  
Conv. (4.)



(1) without P+D controller  $\rightarrow$

$$G(s) = \frac{K}{s(1+Ts)}$$

Type-1/Order-2

with P+D controller

$$G(s) = \frac{K(K_p + K_d s)}{s(1+Ts)}$$

Type-1/Order-2

(2) with p-controller  $\rightarrow$

$$G(s) = \frac{K K_p}{s(1+Ts)}$$

$$1 + \frac{K K_p}{s(1+Ts)} = 0$$

$$s(1+Ts) + K K_p = 0$$

$$Ts^2 + s + K K_p = 0$$

$$s^2 + \frac{s}{T} + \frac{K K_p}{T} = 0$$

$$\omega_n = \sqrt{\frac{K K_p}{T}} \text{ rad/s}$$

$$2 \zeta \sqrt{\frac{K K_p}{T}} = \frac{1}{T}$$

$$\zeta = \frac{1}{2\sqrt{KK_pT}}$$

(III) With P+D controller  $\rightarrow$

$$G(s) = \frac{K(K_p + K_D s)}{s(1+Ts)}$$

$$1 + \frac{K(K_p + K_D s)}{s(1+Ts)} = 0$$

$$Ts^2 + s + K_D s + K_K p = 0$$

$$s^2 + \frac{s(1+K_D)}{T} + \frac{KK_p}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KK_p}{T}} \text{ rad/s}}$$

$$2\zeta\sqrt{\frac{KK_p}{T}} = \frac{1+K_D}{T}$$

$$\boxed{\zeta = \frac{1+K_D}{2\sqrt{KK_p T}}}$$

(IV) ess/ unit ramp i/p  $\rightarrow$

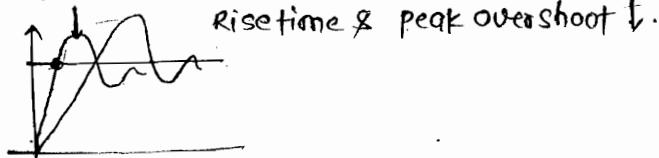
$$ess = \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K(K_p + K_D s)}{s(1+Ts)}} = \frac{1}{1 + \frac{K(K_p + K_D \cdot 0)}{1+Ts}} = \frac{1}{1 + \frac{KK_p}{1+Ts}}$$

$$\boxed{ess = \frac{1}{KK_p}}$$

- \* It is capable of improving the transient state c/s of the system only.  
i.e. it improves the speed of response of sys.
- \* for sinusoidal i/p the phase of controller o/p leads by  $\tan^{-1}\omega_n T$ . Hence it is similar to phase lead compensator.
- \* In terms of filtering property it acts as HPF.
- \* The P+D controller does not affect the type & order of the system.

\* effect on performance specification  $\rightarrow$

- (1) Reduces the rise time.
- (2) Increases the BW.
- (3) It amplifies noise & hence reduces  $\frac{S}{N}$  ratio.
- (4) It increases the stability of the sys.
- (5) It increases the damping ratio  $\zeta$  & hence reduces peak overshoot.



(14) (d)

(15) (c.)

(16) (c.)

(17) (c.)

(18) (c.)

(c.)

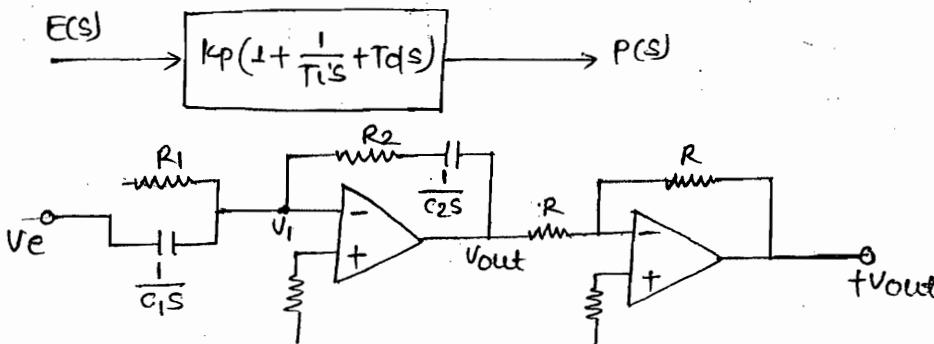
(19) (b.)

(20) (a.)

(3.) PID mode →

$$P = k_p e + \frac{k_p}{T_i} \int e dt + k_p T_d \frac{de}{dt}$$

$$P(s) = \left[ k_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \right] E(s)$$



$$\Rightarrow \frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2 C_2 s + 1}{C_2 s}}$$

$$\Rightarrow -V_{out} = \frac{V_e(R_1 C_1 R_2 C_2 s + 1)}{R_1 C_2 s}$$

$$\Rightarrow -V_{out} = \frac{V_e(R_1 C_1 R_2 C_2 s)}{R_1 C_2 s} + \frac{V_e s (R_1 C_1 + R_2 C_2)}{R_1 C_2 s} + \frac{V_e}{R_1 C_2 s}$$

$$\Rightarrow -V_{out} = V_e \left[ \frac{R_1 C_1}{R_1 C_2} + \frac{R_2 C_2}{R_1 C_2} \right] + \frac{V_e}{R_1 C_2 s} + R_2 C_2 V_e$$

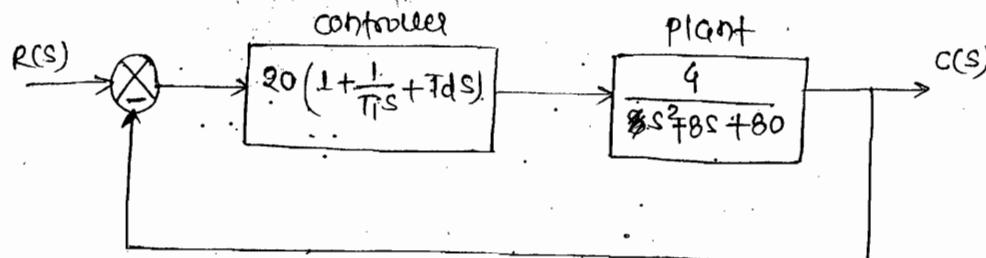
$$\Rightarrow tV_{out} = \frac{R_2}{R_1} V_e t + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C_2} \int V_e dt + \frac{R_2}{R_1} \cdot R_2 C_2 \frac{dV_e}{dt}$$

$k_p = \frac{R_2}{R_1}$ , $T_i = R_2 C_2$ $T_d = R_1 C_1$
--

- \* It improves both transient state & steady state response c/s.
- \* It is similar to lag lead compensator.
- \* In terms of filtering property it acts as band reject filter.
- \* Effect on performance specification →

- (1) It reduces rise time.
- (2) It increases BW.
- (3) It amplifies noise & Hence reduces S/N ratio.
- (4) It increases stability of the sys.
- (5) It increases damping ratio & Hence reduces peak overshoot.
- (6) It eliminates steady state error b/n i/p & o/p.
- (7) The PID controller increases type & order of sys. by 1.

(1/78)  $G_c(s) = \left\{ 20 \left[ 1 + \frac{1}{T_i s} + T_d s \right] \right\} E(s)$



(a)  $G(s) = \frac{4 \times 20 (1 + T_d s)}{s^2 + 8s + 80}$

( $T_i s \neq 0$  because given)

$1 + G(s) = 0$

$$1 + \frac{80(1+T_d s)}{s^2 + 8s + 80} = 0$$

$$s^2 + 8s + 80 + 80(1+T_d s) = 0$$

$$s^2 + s(8 + 80T_d) + 160 = 0$$

$$\omega_n = \sqrt{160} = 12.64 \text{ rad/s}$$

$$24 \times 12.64 = 8 + 80T_d$$

$$T_d = 1 \text{ (given)}$$

$$2 \times 1 \times 12.64 = 8 + 80T_d$$

$$T_d = 0.2s$$

(b)  $G(s) = \frac{20(T_i s + 20 + 4T_i s^2) 4}{T_i s (s^2 + 8s + 80)}$

$$1 + G(s) = 0$$

$$T_i s^3 + 8T_i s^2 + 80T_i s + 80T_i s^2 + 80 + 16T_i s^2 = 0$$

$$T_i s^3 + 24T_i s^2 + 160T_i s + 80 = 0$$

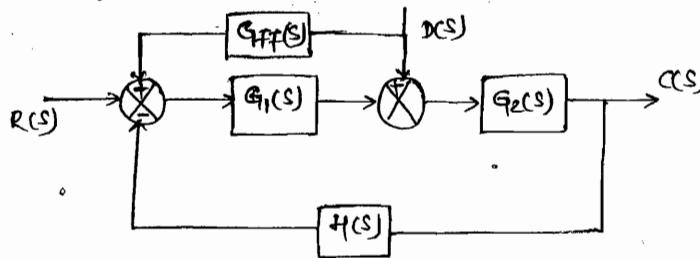
$$s^3 + \frac{24}{T_i} s^2 + \frac{160}{T_i} s + \frac{80}{T_i} = 0$$

$$\frac{80}{T_i} = 24 \times 160$$

$$T_i = 0.02s$$

## \* Feed Forward Compensation →

Sagar sen<sub>37</sub>  
8871453536



$$\left. \frac{C(s)}{R(s)} \right|_{D(s)=0} = \frac{G_1(s) \cdot G_2(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)}$$

$$\left. \frac{C(s)}{R(s)} \right|_{R(s)=0} = \frac{G_2(s) + G_{FF}(s) \cdot G_1(s) \cdot G_2(s)}{1 + G_1(s) \cdot G_2(s) \cdot H(s)}$$

$$C(s) = \frac{R(s) [G_1(s) G_2(s)] + D(s) [G_2(s) + G_{FF}(s) G_1(s) G_2(s)]}{1 + G_1(s) G_2(s) H(s)}$$

To eliminate the effect of the disturbance in the sys., the cond'n for feed forward controller is

$$G_{FF}(s) = \frac{-1}{G_1(s)}$$

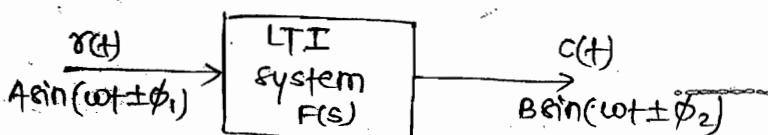
$$\left. \frac{Q(s)}{D(s)} \right|_{G_{FF}(s)} = \frac{k(s+a)(s+c)}{(s+b)(s+d)} ; \quad G_2(s) = 1$$

$$G_C(s) = \frac{1}{G_1(s)} = \frac{(s+b)(s+d)}{k(s+a)(s+c)}$$

$$G_C(s) = \frac{(s+b)(s+d)}{k(s+a)(s+c)}$$

Chapter-(04)  
Freq. domain analysis

- \* When any sys. is subjected to sinusoidal i/p the o/p is also sinusoidal having diff. magnitude & phase angle but same i/p freq,  $\omega$  & s.
- \* freq. response analysis implies varying  $\omega$  from  $(0 \rightarrow \infty)$  & observing variation in magnitude & phase angle of the response.



$$\boxed{B = A |F(s)|}$$

$$\boxed{\phi_2 = \pm \phi_1 + \arg F(s)}$$

$$T.F. = F(s) = \frac{C(s)}{R(s)}$$

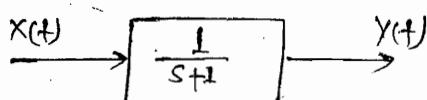
put  $s = j\omega$

$F(j\omega)$  = sinusoidal T.F.  
 $\cong$  sinusoidal response

$$F(j\omega) = |F(j\omega)| \angle F(j\omega) \longrightarrow \tan^{-1} \left( \frac{\text{I.P.}}{\text{R.P.}} \right)$$

$$\sqrt{(\text{R.P.})^2 + (\text{I.P.})^2}$$

Ex:-  
Chap.(06)  
Q.(1)



$$x(t) = 8 \sin t ; \text{ find } y(t) = ?$$

$$F(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1}{j\omega + 1} = \frac{1+j\omega}{1+\omega^2}$$

$$|F(j\omega)| = \frac{\sqrt{1^2 + \omega^2}}{\sqrt{1 + \omega^2}} = \frac{1}{\sqrt{1 + \omega^2}}$$

$$F(j\omega) = \frac{\tan'(0)}{\tan'(\omega)} = -\tan(\omega)$$

$$F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \boxed{-\tan'(\omega)}$$

Given;  $\sin t$

$$x \sin \omega t ; \omega = 1 \text{ rad/s}$$

$$F(j\omega) = \frac{1}{\sqrt{2}} \sin(-45^\circ)$$

$$\boxed{y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)}$$

Ex:- Find  $y(t)$  when  $x(t) = 10 \sin(t + 30^\circ)$

$$\boxed{y(t) = \frac{10}{\sqrt{2}} \sin(t - 15^\circ)}$$

Q.(2)  $G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 3)(s + 4)(s + 5)}$

Soln  $\rightarrow$   $G(j\omega) = \frac{(-\omega^2 + 9)(j\omega + 2)}{(j\omega + 3)(j\omega + 4)(j\omega + 5)} = 0$

$$\omega^2 = 9, \omega = 3 \text{ rad/s}$$

\* Freq. Response plot  $\rightarrow$

(1) Polar plots  $\rightarrow$

Absolute value of  $|F(j\omega)|$  vs  $\omega$

&  $\angle F(j\omega)$  (degree)

(2) Bode plot  $\rightarrow$

decimal value (db) of  $|F(j\omega)|$  vs  $\log \omega$

$[20 \log |F(j\omega)|]$

&  $\angle F(j\omega)$  (degree)

Freq. response analysis of 2<sup>nd</sup> order sys. →

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$F(s) = \frac{1}{s^2 + \frac{2\zeta s}{\omega_n} + 1}$$

Put ( $s = j\omega$ )

$$F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{2\zeta(j\omega)}{\omega_n} + 1}$$

$$F(j\omega) = \frac{1}{\frac{-\omega^2}{\omega_n^2} + 1 + \frac{j2\zeta\omega}{\omega_n}}$$

$$|F(j\omega)| = \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

Its dB value,

$$-20 \log \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

Asymptotic approximation →

$$-20 \log \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

Calc'd Low freq:

$$1 \gg \left(\frac{\omega}{\omega_n}\right)^2$$

$$-20 \log 1 = 0 \text{ db}$$

calc'd High freq:

$$\left(\frac{\omega}{\omega_n}\right)^2 \gg 1$$

$$-20 \log \sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

$$-40 \log \left(\frac{\omega}{\omega_n}\right) \quad \text{①}$$

$$-40 \log \omega + 40 \log \omega_n \\ m X + C$$

$$\boxed{\text{Slope}(m) = -40 \frac{\text{db}}{\text{rad/s}}}$$

Corner freq. ( $\omega_{cf}$ )

$$\Omega = -40 \log \left( \frac{\omega}{\omega_n} \right)$$

$$\log \left( \frac{\omega}{\omega_n} \right) = 0$$

$$\left( \frac{\omega}{\omega_n} \right) = \log(\Omega) = 1$$

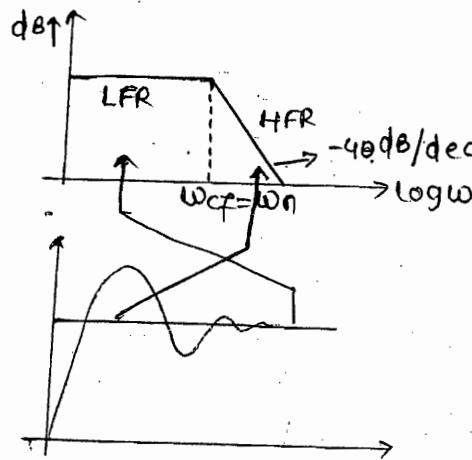
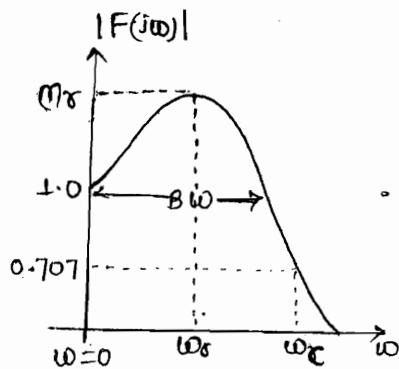
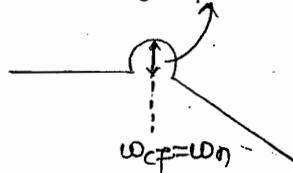
$$\omega = \omega_{cf} = \omega_n$$

Error at  $\omega_{cf} \rightarrow$ 

$$\text{at } \omega = \omega_{cf} = \omega_n$$

$$-20 \log \sqrt{(-1)^2 + (2\zeta)^2}$$

$$-20 \log 2\zeta$$

\* Freq. domain specification  $\rightarrow$ 

- (1) Resonant freq. ( $\omega_0$ )  $\rightarrow$  It is defined as the freq. at which the magnitude has max<sup>m</sup> value.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$$

where;  $u = \frac{\omega}{\omega_n}$

$\omega + \omega_r = \omega_r$  = resonant freq. ;  $\omega = \omega_r = \frac{\omega_r}{\omega_n}$

$$\frac{d}{d\omega_r} \left[ (1 - \omega_r^2)^2 + (2\zeta \omega_r)^2 \right]^{-1/2} = 0$$

$$-\frac{1}{2} \left[ (1 - \omega_r^2)^2 + (2\zeta \omega_r)^2 \right]^{-3/2} \cdot \frac{d}{d\omega_r} \left[ (1 - \omega_r^2)^2 + (2\zeta \omega_r)^2 \right] = 0.$$

$$2(1 - \omega_r^2)(-2\omega_r) + 4\zeta^2 \cdot 2\omega_r = 0$$

$$(2 - 2\omega_r^2)(-2\omega_r) + 8\zeta^2 \omega_r = 0$$

$$-4\omega_r + 4\omega_r^3 + 8\zeta^2 \omega_r = 0$$

$$-1 + \omega_r^2 + 2\zeta^2 = 0$$

$$\omega_r^2 = 1 - 2\zeta^2$$

$$\omega_r = \sqrt{1 - 2\zeta^2}$$

$$\boxed{\omega_r = \omega_n \sqrt{1 - 2\zeta^2} \text{ rad/s}}$$

for ' $\omega_r$ ' to be real & +ve

$$2\zeta^2 < 1 \Rightarrow \boxed{\zeta < \frac{1}{\sqrt{2}}}$$

It is correlated with  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$  rad/s

for ' $\omega_d$ ' to be real & +ve

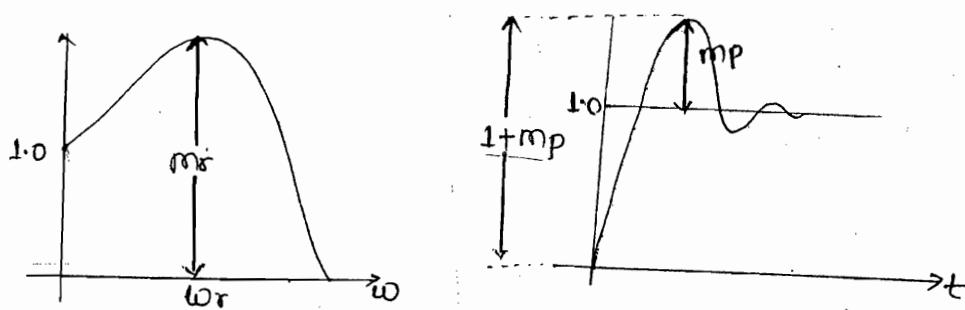
$$\zeta^2 < 1 \Rightarrow \boxed{\zeta < 1}$$

Q.) Resonant peak  $\rightarrow$  It is the max<sup>m</sup> value of magnitude occurring at resonant  
peak magnitude (or) (mr) freq.  $\omega_r$ .

$$|F(j\omega)|_{\omega=\omega_r} = Mr$$

$$Mr = \frac{1}{\sqrt{\left[ 1 - \left( \frac{\omega_r \sqrt{1 - 2\zeta^2}}{\omega_n} \right)^2 \right]^2 + \left( \frac{2\zeta \omega_n \sqrt{1 - 2\zeta^2}}{\omega_n} \right)^2}}$$

$$M_r = \frac{1}{2\sqrt{1-\xi^2}}$$



$$\begin{aligned} \xi < \frac{1}{\sqrt{2}} &\Rightarrow M_r > 1 \\ \xi = \frac{1}{\sqrt{2}} &\Rightarrow M_r = 1 \\ \xi > \frac{1}{\sqrt{2}} &\Rightarrow \text{No } m_r \end{aligned}$$

(Q.10  
71)

$$M(j\omega) = \frac{10}{100 - \omega^2 + j10\sqrt{2}\omega} = \frac{1}{1 - \frac{\omega^2}{100} + j\frac{10\sqrt{2}\omega}{10}}$$

$$\frac{1}{1 - \left(\frac{\omega}{10}\right)^2 + j\frac{\sqrt{2}\omega}{10}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\frac{\sqrt{2}\omega}{\omega_n}}$$

$$\frac{2\sqrt{10}}{\omega_n} = \frac{\sqrt{2}\omega}{10} ; M_r = 1$$

(3.) Band width (BW)  $\rightarrow$  It is the range of freq. over which the magnitude has a value of  $1/\sqrt{2}$ .

\* It indicates the speed of response of the sys.

\* wider BW implies faster response.

$$BW \propto \frac{1}{tr}$$

where; tr = rise time

(4) Cut off freq.  $\rightarrow$  It is the freq. at which the magnitude has a value of  $\frac{1}{\sqrt{2}}$

\* It indicates the ability of a sys. to distinguish signal from noise.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-\omega_c^2)^2 + (2\zeta\omega_c)^2}}$$

$$\text{where; } \omega = \frac{\omega}{\omega_n}$$

at  $\omega = \omega_c = \text{cut off freq.}$

$$\omega = \omega_c = \frac{\omega_c}{\omega_n}$$

$$\frac{1}{\sqrt{(1-\omega_c^2)^2 + (2\zeta\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$(1-\omega_c^2)^2 + (2\zeta\omega_c)^2 = 2$$

$$\omega_c^4 + 1 - 2\omega_c^2 + 4\zeta^2\omega_c^2 - 2 = 0$$

$$\Rightarrow \omega_c^4 + \omega_c^2(4\zeta^2 - 2) - 1 = 0$$

$$\Rightarrow \frac{-(4\zeta^2 - 2) \pm \sqrt{(4\zeta^2 - 2)^2 + 4}}{2}$$

$$\Rightarrow 1 - 2\zeta^2 \pm \frac{\sqrt{16\zeta^4 - 16\zeta^2 + 8}}{2}$$

$$= 1 - 2\zeta^2 \pm \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$\omega_c^2 = 1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$\omega_c = \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\omega_c(\text{or}) \text{ BW} = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \text{ r/s}$$

linear approximation

$$\omega_c(\text{or}) \text{ BW} = \omega_n (-1.19\zeta + 1.85)$$

## \* Freq. Response analysis of dead time (or) transportation lag $\rightarrow$

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$$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$$

$$F(j\omega) = e^{-j\omega T} = \cos\omega T - j\sin\omega T$$

$$|F(j\omega)| = \sqrt{\cos^2\omega T + \sin^2\omega T} = 1$$

$$\angle F(j\omega) = \tan^{-1} \left( \frac{-\sin\omega T}{\cos\omega T} \right)$$

$$= \tan^{-1}(-\tan\omega T)$$

$$= -\omega T \text{ (radians)}$$

$$e^{-j\omega T} = 1 \angle -\omega T \text{ (radians)}$$

$$\pi \text{ rad} = 180^\circ$$

$$-\omega T \text{ rad} = ?$$

$$-\frac{\omega T}{\pi} \times 180^\circ = -57.3 \omega T \text{ (degree)}$$

$$e^{-j\omega T} = 1 \angle -57.3 \omega T \text{ (degree)}$$

## \* Stability from freq. response plot $\rightarrow$

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

$$\text{put } (s=j\omega)$$

$$G(j\omega)H(j\omega) = -1 + j0 \text{ (critical point)}$$

### Stability criteria $\rightarrow$

(1.) Gain cross over freq. ( $\omega_{gc}$ )

$$|G(j\omega)H(j\omega)| \Big|_{\omega=\omega_{gc}} = 1 \text{ (or) } 0 \text{ db}$$

(2.) Phase cross over freq. ( $\omega_{pc}$ )

$$|G(j\omega)H(j\omega)| \Big|_{\omega=\omega_{pc}} = -180^\circ$$

(3.) Gain margin ( $G_m$ ):- It is the "allowable Gain"

$$|G(j\omega)H(j\omega)| \Big|_{\omega=\omega_{pc}} = X ; G_m(\text{db}) = 20 \log \left( \frac{1}{X} \right)$$

$$G_m = \frac{1}{X}$$

(4) Phase margin (PM)  $\rightarrow$  It is the "allowable phase lag"

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \phi^{\circ}; \quad PM \Rightarrow 180 + \phi^{\circ}$$

Stable  $\Rightarrow GM & PM = +ve \Rightarrow \omega_{gc} < \omega_{pc}$

Unstable  $\Rightarrow GM & PM = -ve \Rightarrow \omega_{gc} > \omega_{pc}$

Marginaly stable  $\Rightarrow GM = PM = 0; \omega_{gc} = \omega_{pc}$

KGM & PM for 2nd order sys.  $\rightarrow$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\zeta\omega_n)} = \frac{\omega_n^2 + j0}{(0 + j\omega)(j\omega + 2\zeta\omega_n)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega \sqrt{\omega^2 + 4\zeta^2\omega_n^2}}$$

$$|G(j\omega)| = \frac{0^{\circ}}{(90^{\circ}) \left( \tan^{-1} \frac{\omega}{2\zeta\omega_n} \right)}$$

$$= -90^{\circ} - \tan^{-1} \left( \frac{\omega}{2\zeta\omega_n} \right)$$

$$G(j\omega) = \frac{\omega_n^2 + j0}{-\omega^2 + 2\zeta\omega \cdot \omega_n j}$$

$$|G(j\omega)| = \frac{0^{\circ}}{180 - \tan^{-1} \left( \frac{2\zeta\omega_n}{\omega} \right)}$$

$$= -180^{\circ} + \tan^{-1} \left( \frac{2\zeta\omega_n}{\omega} \right)$$

at  $\omega = \omega_{pc} = \infty \text{ rad/s.}$

$$|G(j\omega)| = -180^{\circ}$$

$$|G(j\omega)|_{\omega=\omega_{pc}=\infty} = X$$

$$X = 0 \Rightarrow GM = \frac{1}{0} = \infty$$

$$\boxed{\begin{aligned} \omega_{pc} &= \infty \\ GM &= \infty \end{aligned}}$$

at  $\omega = \omega_{gc}$

$$\frac{\omega_n^2}{\omega \sqrt{\omega^2 + 4\epsilon^2 \omega_n^2}} = 1$$

$$\omega_n^4 = \omega^2(\omega^2 + 4\epsilon^2 \omega_n^2)$$

$$\omega_n^4 + \omega^2 - 4\epsilon^2 \omega_n^2 - \omega^4 = 0$$

$$\omega^2 = \frac{-4\epsilon^2 \omega_n^2 \pm \sqrt{16\epsilon^4 \omega_n^4 + 4\omega_n^4}}{2}$$

$$= -2\epsilon^2 \omega_n^2 \pm \omega_n^2 \sqrt{4\epsilon^4 + 1}$$

$$\omega^2 = -2\epsilon^2 \omega_n^2 + \omega_n^2 \sqrt{4\epsilon^4 + 1}$$

$$\boxed{\omega = \omega_{gc} = \omega_n \sqrt{-2\epsilon^2 + \sqrt{4\epsilon^4 + 1}}} \text{ r/s}$$

$$\underline{G(\omega)}|_{\omega=\omega_{gc}} = \phi = -90^\circ + \tan^{-1} \left( \frac{\omega_n \sqrt{-2\epsilon^2 + \sqrt{4\epsilon^4 + 1}}}{2\epsilon \omega_n} \right)$$

$$P_m = 180^\circ + \phi$$

$$\boxed{P_m = 90^\circ - \tan^{-1} \left[ \frac{\sqrt{-2\epsilon^2 + \sqrt{4\epsilon^4 + 1}}}{2\epsilon} \right]}$$

$$\underline{G(\omega)}|_{\omega=\omega_{gc}} = \phi = -180^\circ + \tan^{-1} \left[ \frac{2\epsilon \omega_n}{\omega_n \sqrt{-2\epsilon^2 + \sqrt{4\epsilon^4 + 1}}} \right]$$

$$P_m = 180^\circ + \phi$$

$$\boxed{P_m = \tan^{-1} \left[ \frac{2\epsilon}{\sqrt{-2\epsilon^2 + \sqrt{4\epsilon^4 + 1}}} \right]}$$

linear approximation

$$\boxed{P_m = 100\epsilon}$$

(3)  
70

$$\%mp = 50\%$$

$$mp = 0.5$$

$$e^{-8\pi}/\sqrt{1-e^2} = 0.5$$

$$\epsilon = 0.215$$

Period of oscillations = 0.2s.

$$T = \frac{1}{f_d} = 0.2s$$

$$f_d = 5\text{Hz}$$

$$\omega_d = 2\pi f_d$$

$$\omega_d = 2\pi \times 5 = 31.41 \text{ rad/s}$$

$$\omega_n \sqrt{1-\epsilon^2} = 31.41$$

$$\omega_n \sqrt{1-(0.215)^2} = 31.41$$

$$\omega_n = 32.16 \text{ rad/s}$$

$$\omega_r = \omega_n \sqrt{1-2\epsilon^2}$$

$$\omega_r = 32.16 \sqrt{1-2(0.215)^2}$$

$$\boxed{\omega_r = 30.63 \text{ rad/s}}$$

Conv(2)  
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$$1 + \frac{k}{s(c+a)} = 0$$

$$s^2 + qst + k = 0$$

$$k = \omega_r^2$$

$$q = 2\zeta\omega_n$$

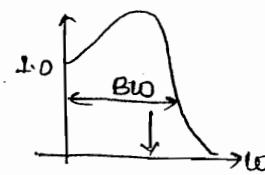
Given;

$$M\ddot{x} = \frac{1}{2\zeta\sqrt{1-\epsilon^2}} = 1.04$$

$$\epsilon = 0.6 ; \cancel{0.8}$$

$$\omega_r = \omega_n \sqrt{1-2(0.6)^2} = 11.55$$

$$\omega_n = 22.2 \text{ rad/s}$$

(4)  
70 (c) (11)  
71 (a)(4)  
70

Sharp cut off c/s  $\Rightarrow$  less BW  $\Rightarrow$  more tr  
less stable

BW  $\downarrow$ , Mr  $\uparrow$ , stability  $\downarrow$ , tr  $\uparrow$

(5)  
70

$$1 + \frac{100}{s(s+10)} = 0$$

$$s^2 + 10s + 100 = 0$$

$$\omega_n = 10 \text{ rad/s}$$

$$2\zeta \times 10 = 10$$

$$\epsilon = 0.5$$

$$\omega_r = 10 \sqrt{10 - 2 \times 0.5^2} = 7.07 \text{ rad/s}$$

$$BW = \omega_n (-1.19\epsilon + 1.85)$$

$$= 10 (-1.19 \times 0.5 + 1.85)$$

$$\boxed{BW = 12.7 \text{ rad/s}}$$

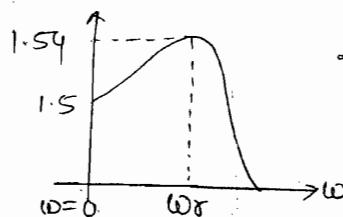
$$K = (22.2)^2 = 492.84$$

$$q = 2 \times 0.6 \times 22.2 = 26.4$$

Ques. → The TF of 2nd order sys. is

$$\frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Its freq. response is shown in fig.



The value of 'K' is (a) 1 (b) 1.54 (c) 1.5 (d) 1.04

Soln →

$$\begin{aligned} \text{TF} &= \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ &= \frac{k}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} \end{aligned}$$

(put  $s = j\omega$ );

$$\begin{aligned} \text{TF} &= \frac{k}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}} \\ &= \frac{k}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} \end{aligned}$$

$$at \omega = 0;$$

$$|F(j\omega)| = k = 1.5$$

(6)  
70

$$G(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$1 + \frac{2\sqrt{3}}{s(s+1)} = 0$$

$$s^2 + s + 3.46 = 0$$

$$\omega_n = \sqrt{3.46} = 1.86 \text{ rad/s}$$

$$2\zeta \times 1.86 = 1$$

$$\zeta = 0.27$$

$$P_m = 100\zeta = 100 \times 0.27 = 27^\circ \approx 30^\circ$$

(7)  
70

$$G(s) = \frac{qs+1}{s^2}$$

$$1 + \frac{qs+1}{s^2} = 0$$

$$s^2 + qs + 1 = 0$$

$$\omega_n = 1 \text{ rad/s}$$

$$2\zeta \times 1 = q$$

$$\text{Given; } P_m = 45^\circ, \quad \zeta = \frac{P_m}{180} = \frac{45}{180} = 0.25$$

$$\therefore q = 2 \times 0.25 = 0.9$$

Ans(c)

(8)

$$G(s) = \frac{1}{s(s^2+s+1)}$$

$$G(j\omega) = \frac{1}{j\omega(j\omega^2+j\omega+1)}$$

$$G(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\underline{|G(j\omega)|} = -90^\circ + \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

$$\text{at } \omega = \omega_{pc}$$

$$-90^\circ + \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -180^\circ$$

$$\therefore -\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -90^\circ$$

$$1-\omega^2=0$$

(9)

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3 \times 1}{\omega\sqrt{\omega^2+4}}$$

$$\begin{aligned} \underline{|G(j\omega)|} &= \frac{0^\circ (-57.3^\circ \times 2\omega)}{(90^\circ)(+\tan^{-1} 0.5\omega)} \\ &= (-90^\circ) - 114.6\omega - \tan^{-1}(0.5\omega) \end{aligned}$$

$$\text{at } \omega = \omega_{gc}$$

$$\frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$9 = \omega^2(\omega^2+4)$$

$$\omega^4 + 4\omega^2 - 9 = 0$$

$$\omega^2 = \frac{-4 \pm \sqrt{16+36}}{2}$$

$$\omega = \omega_{pc} = 1 \text{ rad/s}$$

$$|G(j\omega)| \Big|_{\omega=\omega_{pc}=1} = X$$

$$X = \frac{1}{\sqrt{(1-1)^2+1^2}} = 1$$

$$G_m = \frac{1}{X} = \frac{1}{1} = 1$$

$$G_m(\text{db}) = 20 \log 1 = 0 \text{ db}$$

$$\boxed{G_m(\text{db}) = 0 \text{ db}}$$

$$\omega^2 = -2 \pm 3.6$$

$$\omega^2 = 1.6, -5.6$$

$$\omega = \sqrt{1.6} = 1.26 \text{ rad/s}$$

$$|G(j\omega)| \Big|_{\omega=\omega_{gc}} = 1.26 \text{ rad/s} = \phi$$

$$\phi = -90^\circ - 114.6 \times 1.26 - \tan^{-1} 0.5 \times 1.26$$

$$\phi = -267.5^\circ$$

$$P_m = 180^\circ - 267.5^\circ$$

$$P_m = -87.5^\circ$$

By observation

$$\omega_{pc} < \omega_{gc}$$

$$G_m = P_m = -87.5^\circ$$

(12)  
71

$$G(s) = \frac{s+5}{s^2 + 4s + 9}$$

$$G(j\omega) = \frac{j\omega + 5}{9 - \omega^2 + 4j\omega}$$

$$\boxed{|G(j\omega)|}_{\omega < 3} = \frac{\tan^{-1}(\omega/5)}{\tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) = \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\boxed{|G(j\omega)|}_{\omega > 3} = \frac{\tan^{-1}\omega/5}{180^\circ - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) - 180^\circ + \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\omega = 0, \boxed{|G(j\omega)| = 0^\circ}$$

$$\omega = \infty; \boxed{|G(j\omega)| = 90^\circ - 180^\circ + 0^\circ = -90^\circ}$$

$$0^\circ \rightarrow 90^\circ$$

correction

~~$\times \epsilon \rightarrow \epsilon$~~  qns.(q)

(16)  
72

$$G(s) = \frac{e^{-Ts}}{s(s+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$\begin{aligned} |G(j\omega)| &= \frac{-\omega T}{\left(\frac{\pi}{2}\right)\left(\tan^{-1}\omega\right)} \\ &= -\frac{\pi}{2} - \omega T - \tan^{-1}\omega \end{aligned}$$

$$At \omega = \omega_1$$

$$\boxed{|G(j\omega)| = 0}$$

$$-\frac{\pi}{2} - \omega_1 T - \tan^{-1}\omega_1 = 0$$

$$-\tan^{-1}\omega_1 = \frac{\pi}{2} + \omega_1 T$$

$$-\omega_1 = \tan\left(\frac{\pi}{2} + \omega_1 T\right)$$

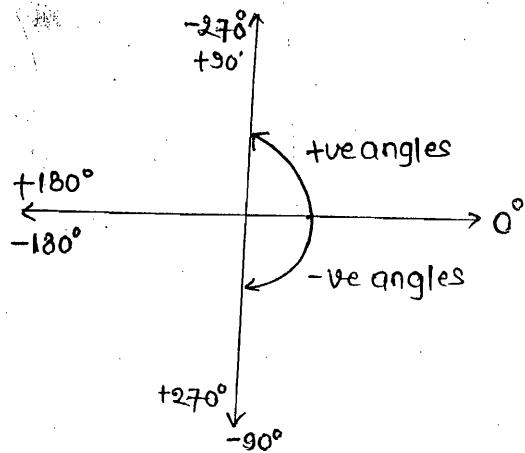
$$-\omega_1 = -\cot(\omega_1 T)$$

$$\boxed{\omega_1 = \cot(\omega_1 T)}$$

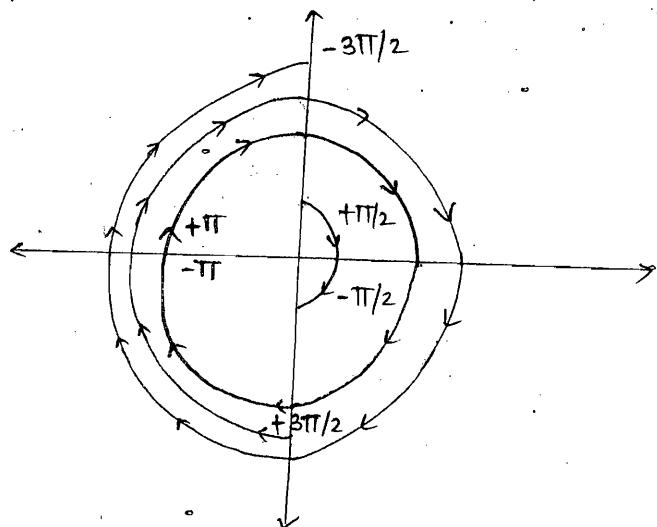
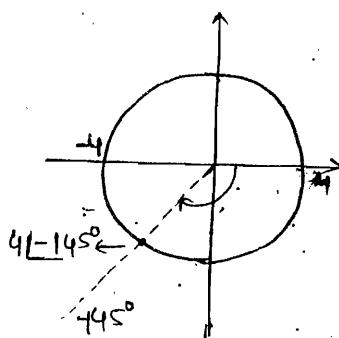
POLAR PLOT

\* It is a plot of absolute value of magnitude & phase angle in degrees of open loop TF  $G(j\omega) \cdot H(j\omega)$  vs  $\omega$  drawn on polar co-ordinates.

Polar co-ordinates :-



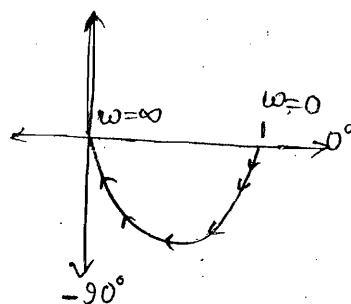
Eg:-  $41-45^\circ$

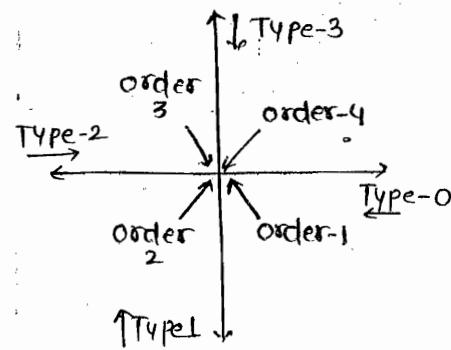


$$G(s) = \frac{1}{(s+1)} = \frac{1}{(1+s)} = \frac{1}{(1+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} ; |G(j\omega)| = -\tan^{-1}\omega$$

$\omega$	0	$\infty$
$ G(j\omega) $	1	0
$\angle G(j\omega)$	$0^\circ$	$-90^\circ$





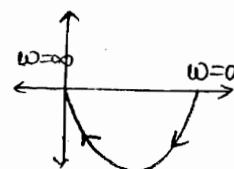
### \* General shapes of polar plot →

#### Type/order

(1) Type-0/ order-1

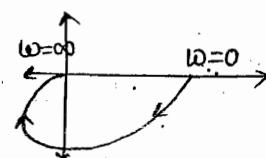
$$G(s) = \frac{1}{(1+Ts)}$$

#### Polar plot



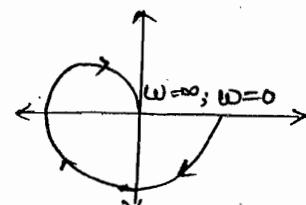
(2) Type-0/ order-2

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)}$$



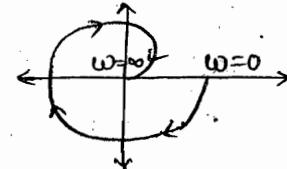
(3) Type-0/ order-3

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)}$$



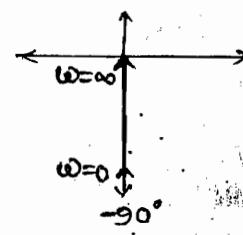
(4) Type-0/ order-4

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)(1+T_4s)}$$



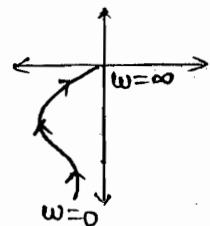
(5) Type-1/ order-1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega} e^{-j90^\circ}$$



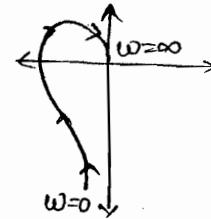
(6) Type-1/ order-2

$$G(s) = \frac{1}{s(1+Ts)}$$



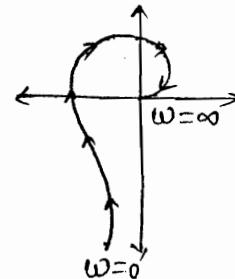
(7.) Type-1 / order-3

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



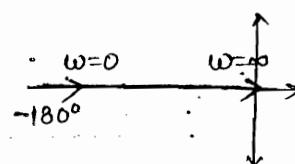
(8.) Type-1 / order-4

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)(1+T_3s)}$$



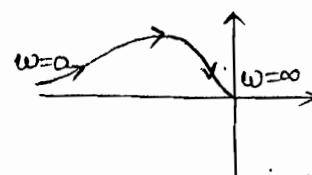
(9.) Type-2 / order-2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2} e^{-180^\circ}$$



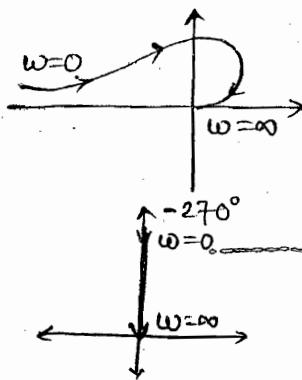
(10.) Type-2 / order-3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



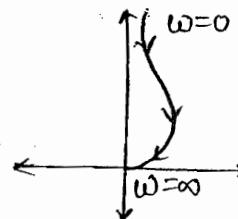
(11.) Type-2 / order-4

$$G(s) = \frac{1}{s^2(1+T_1s)(1+T_2s)}$$



(12.) Type-3 / order-3

$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} e^{-270^\circ}$$



(13.) Type-3 / order-4

$$G(s) = \frac{1}{s^3(1+Ts)}$$

DATE-23/11/14

- \* Effect of adding zeros on the of the polar plot → for minm phase or Non-minm phase  
↑ when zeros are added it is necessary to check the intersection of polar plot with -ve real axis b/m  $\omega=0 \& \omega=\infty$ .

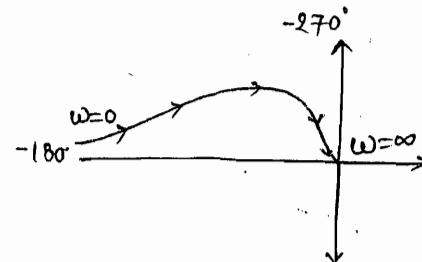
4  
75

$$G(s) = \frac{1}{s^2(1+s)}$$

$$\angle G(j\omega) = -180^\circ + \tan^{-1}(\omega)$$

$$\omega=0, |G(j\omega)|=\infty \ ; \ \underline{\angle G(j\omega)} = -180^\circ$$

$$\omega=\infty, |G(j\omega)|=0 \ ; \ \underline{\angle G(j\omega)} = -270^\circ$$

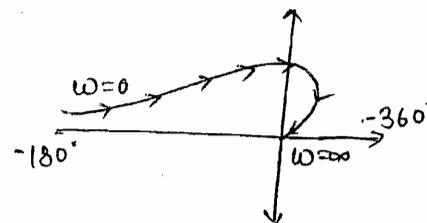


$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\omega=0, |G(j\omega)|=\infty \ ; \ \underline{\angle G(j\omega)} = -180^\circ$$

$$\omega=\infty, |G(j\omega)|=0 \ ; \ \underline{\angle G(j\omega)} = -360^\circ$$



$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$$

$$\omega=0, |G(j\omega)|=\infty \ ; \ \underline{\angle G(j\omega)} = -180^\circ$$

$$\omega=\infty, |G(j\omega)|=0 \ ; \ \underline{\angle G(j\omega)} = -270^\circ$$

Because of present of one zero, the polar plot will cut the -ve real axis at some value.  
so for finding that value,

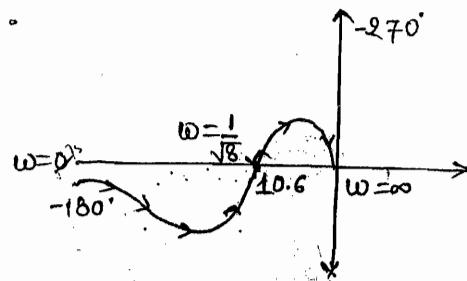
$$-180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega = -180^\circ$$

$$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$$

$$4\omega = \frac{\omega + 2\omega}{1 - 2\omega^2}$$

$$4 - 8\omega^2 = 3;$$

$$\omega = \omega_{pc} = \frac{1}{\sqrt{8}} \text{ rad/s}$$



$$|G(j\omega)| (\omega = \omega_{pc} = \frac{1}{\sqrt{8}}) = X$$

$$X = \frac{\sqrt{1 + \left(\frac{4}{\sqrt{8}}\right)^2}}{\sqrt{1 + \left(\frac{1}{\sqrt{8}}\right)^2} \sqrt{1 + \left(\frac{2}{\sqrt{8}}\right)^2}} = 10.6$$

Conv(4)  
75

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

$$= \frac{2(1+0.5s)}{(1+s)(1-s)}$$

$$G(s) = \frac{-2(1+0.5s)}{(1+s)(1-s)}$$

$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

$$|G(j\omega)| = \frac{2}{\sqrt{1+(0.5\omega)^2}}$$

$$|G(j\omega)| = \frac{(-180^\circ) + \tan^{-1} 0.5\omega}{(\tan^{-1}\omega)(-\tan^{-1}\omega)}$$

$$= -180^\circ + \tan^{-1} 0.5\omega$$

Note:- -ve gain  $(-k+50)$  contributes  $-180^\circ$  for all  $\omega$

$$G(s) = \frac{(s+2)}{(s+1)(s-4)} = \frac{-0.5(1+0.5s)}{(1+s)(1-0.25s)}$$

$$G(j\omega) = \frac{-0.5(1+0.5j\omega)}{(1+j\omega)(1-0.25j\omega)}$$

$$|G(j\omega)| = \frac{0.5}{\sqrt{1+\omega^2}} \cdot \frac{\sqrt{1+(0.5\omega)^2}}{\sqrt{1+(0.25\omega)^2}}$$

$$|G(j\omega)| = \frac{(-180^\circ) + \tan^{-1} 0.5\omega}{(\tan^{-1}\omega)(-\tan^{-1} 0.25\omega)}$$

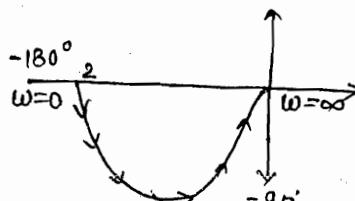
$$-180^\circ + \tan^{-1} 0.5\omega + \tan^{-1} 0.25\omega - \tan^{-1}\omega$$

$\omega$	0	$\sqrt{2}$	$\infty$
$ G(j\omega) $	0.5	0.33	0
$ G(j\omega) $	$-180^\circ$	$-180^\circ$	$-90^\circ$

$$-180^\circ + \tan^{-1} 0.5\omega + \tan^{-1} 0.25\omega - \tan^{-1}\omega = -180^\circ$$

$$\tan^{-1}\omega = \tan^{-1} 0.5\omega + \tan^{-1} 0.25\omega$$

$\omega$	0	$\infty$
$ G(j\omega) $	2	0
$ G(j\omega) $	$-180^\circ$	$-90^\circ$

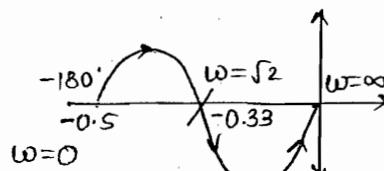


$$\omega = \frac{0.5\omega + 0.25\omega}{1 - 0.125\omega}$$

$$1 - 0.125\omega^2 = 0.75$$

$$\omega = \omega_{pc} = \sqrt{2} \text{ rad/s}$$

$$X = \frac{0.5 \sqrt{1 + (0.5 \times \sqrt{2})^2}}{\sqrt{1 + (\sqrt{2})^2} \sqrt{1 + (0.25 \times \sqrt{2})^2}} = 0.33$$



\* Find  $qm$  &  $pm$  from polar plots →

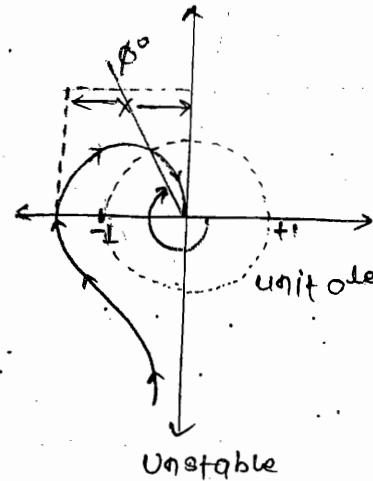
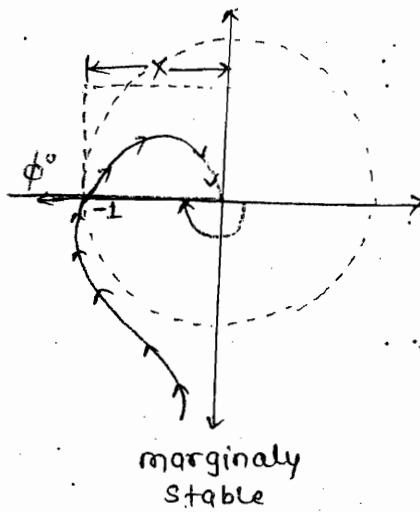
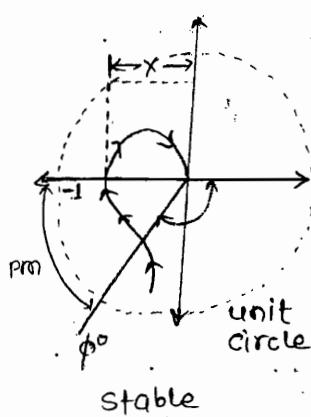
$$qm = \frac{1}{x} : qm(\text{db}) = 20 \log\left(\frac{1}{x}\right) \quad x = 1 : qm = \frac{1}{x} = \frac{1}{1} = 1, qm(\text{db}) = 20 \log 1 = 0 \text{ db}$$

$= +ve$

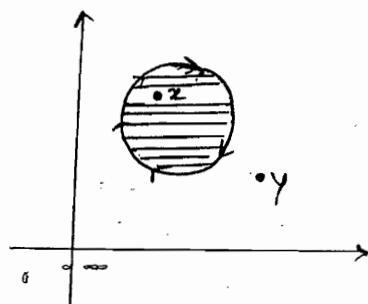
$$pm = \phi - (-180^\circ) = 180^\circ + \phi = +ve \quad \phi = -180^\circ; pm = 180^\circ - 180^\circ = 0$$

$$qm = \frac{1}{x} = qm(\text{db}) = 20 \log\left(\frac{1}{x}\right) = -v$$

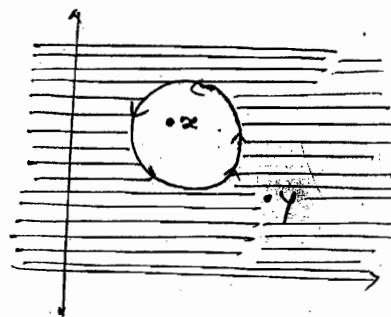
$$pm = 180^\circ + \phi = -ve$$



Concept of enclosure & encirclement →



Fig(1)



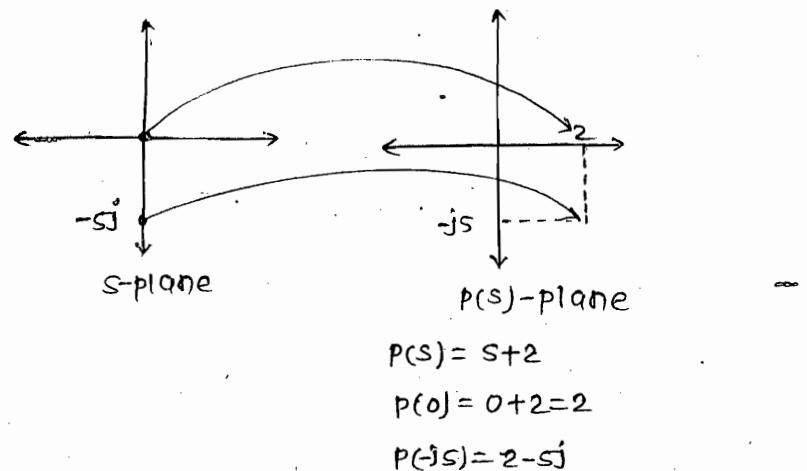
Fig(2)

- \* A point is said to be enclosed by a contour if it lies to the right side of the dirn of contour.
- \* A point is said to be encircled if the contour is a closed path.
- \* In fig (2) point y is said to be enclosed whereas point x is said to be only encircled in ACW dirn.
- \* In polar plots if the critical point  $-1+j0$  is not enclosed than the sys. is said to be stable.

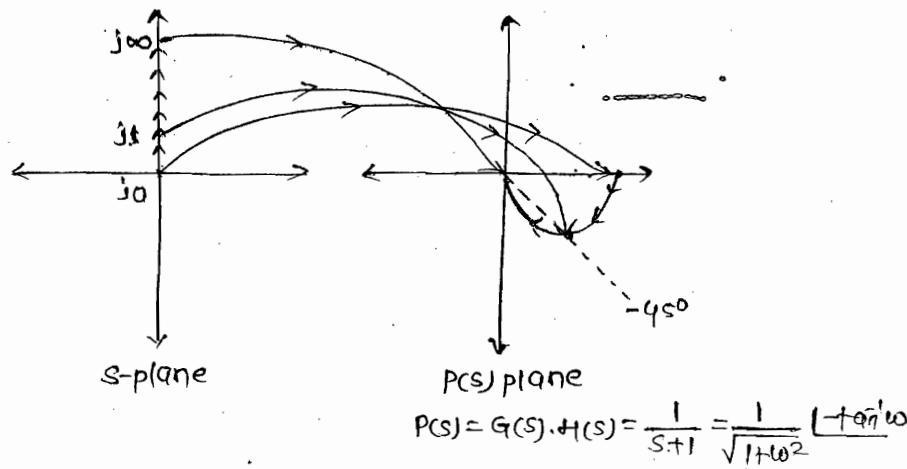
### Theory of Nyquist plots $\rightarrow$

\* The mapping theorem states that every point of s-plane will get mapped onto a corresponding point in  $p(s)$  plane where  $p(s)$  is any fn of  $s$ .

### Principle of mapping $\rightarrow$



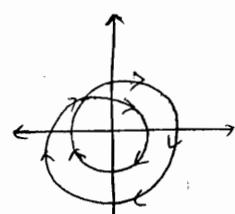
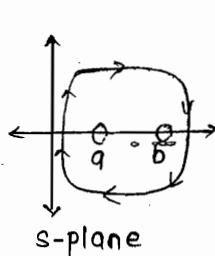
### Eg:- Polar plot.



## Principle of argument $\rightarrow$

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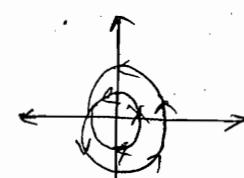
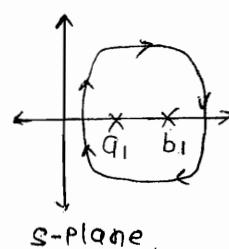
Case(i)  $\rightarrow$



$$P(s) = (s-a)(s-b)$$

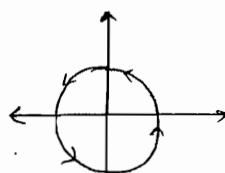
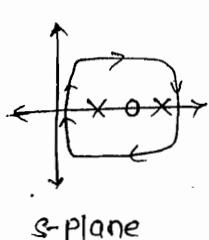
PA  $\rightarrow$  Pole ACW

case(ii)  $\rightarrow$



$$P(s) = \frac{1}{(s-a_1)(s-b_1)}$$

Case(iii)  $\rightarrow$



$$P(s) = \frac{s-c}{(s-a)(s-b)}$$

$$\boxed{N = P - Z}$$

where;  $N =$  no. of encirclement

= +ve (ACW dirn)

= -ve (CCW dirn)

$P =$  No. of poles in RHS of s-plane

$Z =$  No. of zeros in RHS of s-plane.

\* The principle of argument may be stated as if the s-plane closed contour encloses  $p$  poles &  $z$  zeros ( $p > z$ ) in RHS of s-plane then the origin of  $P(s)$  plane is encircled  $(p-z)$  times in ACW dirn.

\* Nyquist stability criteria  $\rightarrow$

$$G(s)H(s) = \frac{K(s+z_1)}{s(s+p_1)}$$

$$1 + G(s)H(s)$$

$$1 + \frac{K(s+z_1)}{s(s+p_1)}$$

$$\frac{s(s+p_1) + K(s+z_1)}{s(s+p_1)} \rightarrow CL \text{ poles}$$

$$\frac{s(s+p_1)}{s(s+p_1)} \rightarrow OL \text{ poles}$$

UC  $\rightarrow$  Upper closed loop pole

LO  $\rightarrow$  Lower open loop zeros

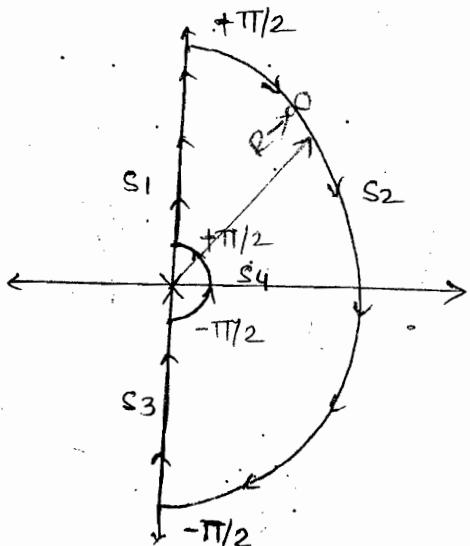
Applying  $N = P - Z$  to eqn (i)

$P$  = No. of OL poles in RHS of s-plane

$Z$  = No. of CL poles in RHS of s-plane

(i)  $Z=0, N=P$

Nyquist Path  $\rightarrow$



To map  $s_1 \Rightarrow$  polar plot

To map  $s_2 \Rightarrow$  put  $s = e^{j\theta} \lim_{R \rightarrow \infty} Re^{j\theta}$

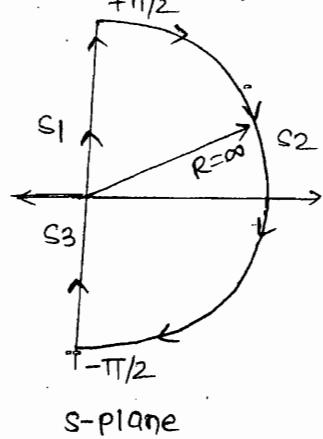
$$\left( \theta = +\frac{\pi}{2} \rightarrow -\frac{\pi}{2} \right)$$

To map  $s_3 \Rightarrow$  inverse polar plot

To map  $s_4 \Rightarrow$  put  $s = \lim_{r \rightarrow 0} re^{j\theta}$

$$\left( \theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2} \right)$$

Eq:-  $G(s) = \frac{10}{(s+2)(s+4)}$



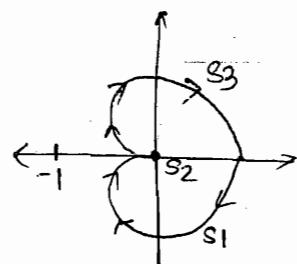
P = RHS pole  
N = Encirclement

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$N = P - Z$

$O = 0 - Z$

$Z=0$  Stable



$$G(s) = \frac{10}{(s+2)(s+4)}$$

To map  $s_2 \rightarrow$

$$G(s) = \frac{10}{s^2} = \frac{10}{\lim_{R \rightarrow \infty} (Re^{j\theta})^2} = \frac{10}{\lim_{R \rightarrow \infty} R^2 e^{j2\theta}} = 0e^{j2\theta}$$

Que:- w/o constructing Nyquist plot find the no. of encirclement about  $-1+j0$ ?

Soln →

$$1 + \frac{10}{(s+2)(s+4)} = 0$$

$$s^2 + 6s + 18 = 0$$

$s^2$	-1	18
$s$	6	0
$s^0$	18	0

$N = P - Z$

$\downarrow \quad \downarrow$

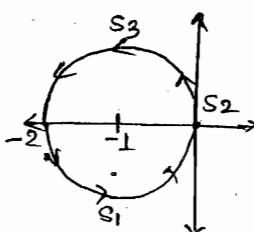
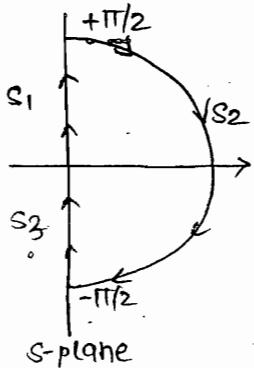
$N=0$

$Z = \text{sign change}$

$P = \text{open loop pole}$

Que:-  $G(s) = \frac{(s+2)}{(s+1)(s-1)}$

Soln → for also this que section will 3, because of No. poles in origin.



$N = P - Z$

$1 = 1 - Z$

$Z=0$

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

$$1 + \frac{s+2}{s^2-1} = 0$$

$$s^2 - 1 + s + 2 = 0$$

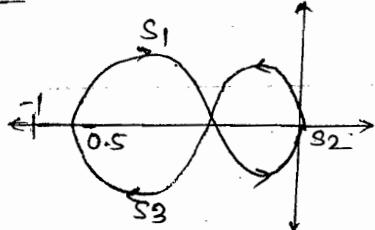
$$s^2 + s + 1 = 0$$

$s^2$	1	1
$s^1$	1	0
$s^0$	1	0

$$\begin{array}{c} N = P - Z \\ \downarrow \\ 1 \\ \boxed{N=1} \end{array}$$

Q.  $\rightarrow G(s) = \frac{(s+2)}{(s+1)(s+4)}$

SOL<sup>n</sup>  $\rightarrow$



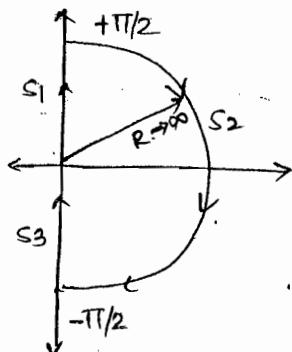
$$N = P - Z$$

$$O = 1 - Z$$

$\boxed{Z=1}$  Unstable

Q.  $\rightarrow G(s) = \frac{100}{(s+2)(s+4)(s+8)}$

ID<sup>n</sup>  $\rightarrow$



$$\begin{array}{c} N = P - Z \\ \downarrow \\ 0 \\ \downarrow \\ 0 - Z \end{array}$$

$\boxed{Z=0}$  Stable

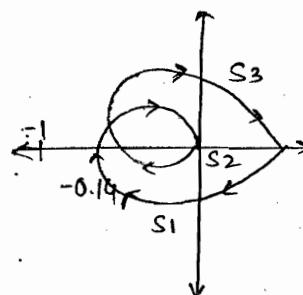
$$1 + \frac{s+2}{(s+1)(s+4)} = 0$$

$$s^2 + 4s + s - 4 + s + 2 = 0$$

$$s^2 + 2s - 2 = 0$$

$s^2$	1	-2
$s^1$	-2	0
$s^0$	-2	0

$$\begin{array}{c} N = P - Z \\ \downarrow \\ 1 - 1 \\ \downarrow \\ \boxed{N=0} \end{array}$$



$$G(s) = \frac{100}{(s+2)(s+4)(s+8)}$$

shortcut method →

$$\frac{100}{(-\omega^2 + 6j\omega + 8)} \cdot \frac{100}{(\omega^2 + 6j\omega + 8)(j\omega + 8)}$$

$$\frac{100}{-\omega^3 - 8\omega^2 - 6\omega^2 + 48j\omega + 8j\omega + 64}$$

$$\frac{100}{64 - 14\omega^2 + j(56\omega - \omega^3)}$$

$$56\omega - \omega^3 = 0$$

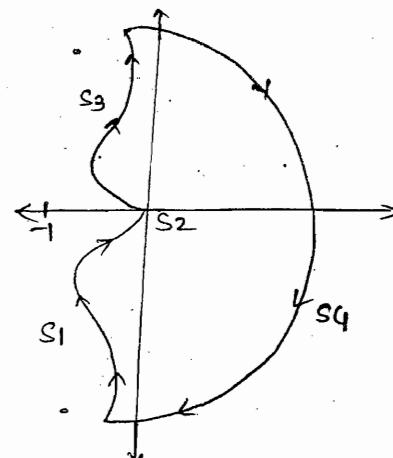
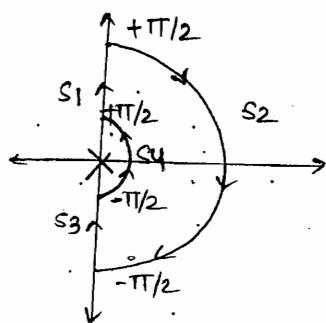
$$\omega^2 = 56$$

$$\omega = \omega_{pc} = \sqrt{56} \text{ rad/s}$$

$$X = \frac{100}{64 - 14(7.4)^2} = -0.14$$

Ques. →  $G(s) = \frac{10}{s(1+5s)}$

For 1^n →



$N = P - Z$   
 $O = O - Z$   
 $[Z=0]$  Stable

$$G(s) = \frac{10}{s(1+5s)}$$

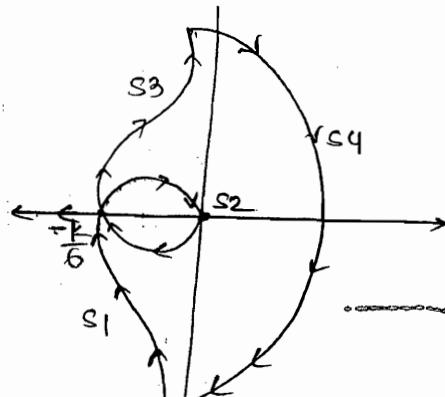
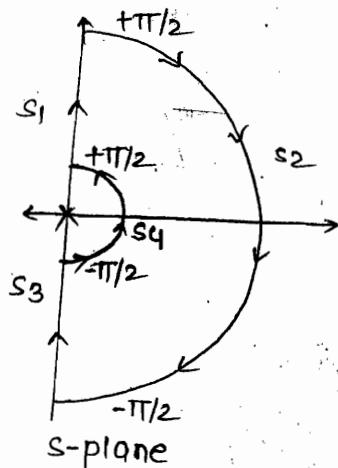
To map  $s_4 \rightarrow$

$$G(s) = \frac{10}{s(1)} = \frac{10}{\lim_{t \rightarrow 0} re^{j\theta}} = \infty e^{j\theta} \quad (\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2})$$

$$= \infty e^{j\pi/2} \rightarrow \infty e^{-j\pi/2}$$

Ques:-  $G(s) = \frac{K}{s(s+1)(s+2)}$

Soln →



$$G(s) = \frac{K}{s(s+1)(s+2)}$$

To map  $s_4 \rightarrow$

$$G(s) = \frac{K}{s(s+1)(s+2)} = \frac{0.5K}{\lim_{s \rightarrow \infty} se^{j\theta}} = \infty e^{j\theta} \quad (\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2})$$

$$= \infty e^{j\pi/2} \rightarrow \infty e^{-j\pi/2}$$

$$\frac{K}{j\omega(-\omega^2 + j\omega + 2)}$$

$$\frac{K}{-3\omega^2 + j(2\omega - \omega^2)}$$

$$2\omega - \omega^2 = 0$$

$$\omega = \omega_{pe} = \sqrt{2} \text{ rad/s.}$$

$$\frac{K}{-3(\sqrt{2})^2} = -\frac{K}{6}$$

$$-\frac{K}{6} = -1 \quad ; \quad K_{max} = 6$$

case (1)  $K > 6$

$$-2 = 0 - z$$

$$z = 2$$

unstable

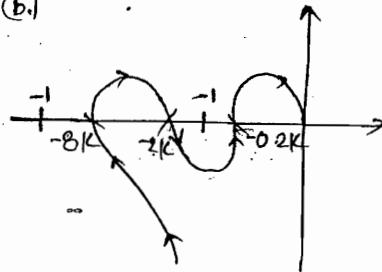
case (2)  $K < 6$

$$0 = 0 - z$$

$$z = 0$$

stable

$X = \frac{K}{6}$
$GM = \frac{1}{X} = \frac{6}{K}$
$GM \propto \frac{1}{K}$

(22)  
74 (b)

(i)  $-0.2K = -1$   
 $K_{max} = 5$   
 $K < 5 \rightarrow \text{stable}$

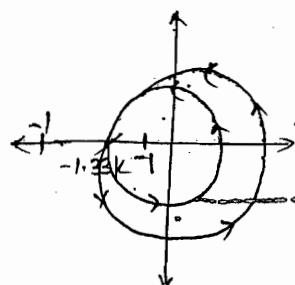
(ii)  $-2K = -1$   
 $K_{max} = \frac{1}{2}$   
 $K > \frac{1}{2} \rightarrow \text{stable}$

(iii)  $+8K = -1$   
 $K_{max} = \frac{1}{8}$   
 $K < \frac{1}{8} \rightarrow \text{stable}$

$$\boxed{\begin{array}{l} \frac{1}{2} < K < 5 \\ K < \frac{1}{8} \end{array}}$$

(19)  
73

$$G(s) = \frac{k(s+3)(s+5)}{(s-2)(s-4)}$$



$$-1.33k = -1$$

$$K_{max} = \frac{1}{1.33} = 0.75$$

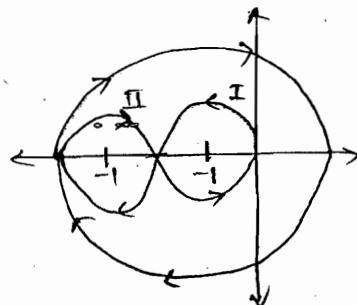
(i)  $k < \frac{1}{1.33}$

$$0 = 2 - z$$

$$z = 2 \quad (\text{Unstable})$$

(ii)  $k > \frac{1}{1.33}$

$$2 = 2 - z ; z = 0 \quad (\text{stable})$$

(18)  
72Given  $P=0$ .

Region (I)

$$1 - 1 = 0 - z$$

$$z = 0$$

$$\text{stable}$$

Region (II)

$$-2 = 0 - z$$

$$z = 2$$

$$\text{unstable}$$

(15)  
71

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{-\omega^2(1 + \frac{1}{\omega^2 T_1^2} + \frac{2}{\omega^2 T_1 T_2} + \frac{1}{\omega^2 T_2^2})}$$

$$= \frac{1}{j\omega [1+j\omega(T_1-T_2) - \omega^2 T_1 T_2]}$$

$$= \frac{1}{-\omega^2(T_1+T_2) + j(\omega - \omega^2 T_1 T_2)}$$

$$\omega - \omega^2 T_1 T_2 = 0$$

$$1 - \omega^2 T_1 T_2 = 0$$

$$\omega = \omega_{PC} = \frac{1}{\sqrt{T_1 T_2}} \text{ rad/s}$$

$$= \left( \frac{1}{\sqrt{T_1 T_2}} \right)^2 (T_1 + T_2)$$

$$= -\frac{T_1 T_2}{T_1 + T_2}$$

$$X = \frac{T_1 T_2}{T_1 + T_2}$$

$$Gm = \frac{T_1 + T_2}{T_1 T_2}$$

Date: - 24/11/14

(14)  
71 (q)

- If the sys. is unstable then its pole will be R.H.S & its magnitude is not affected but the phase will be affected.

(A)  $K = K_1$        $K \uparrow \Rightarrow \theta \uparrow, m_r \downarrow \rightarrow 4$

(B)  $K = K_2$        $K \uparrow \Rightarrow \theta \downarrow, m_r \uparrow \rightarrow 3$   
 $K_2 > K_1$

ans(q)

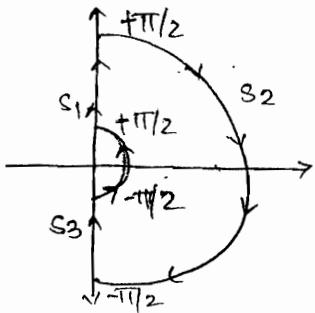
(C) marginally stable

(Response is oscillatory  $\Rightarrow \theta = 0, m_r = \infty \rightarrow 2$ )

(D) Unstable sys. ~~not~~  
 Phase c/s will tend to tend to  $-270^\circ \rightarrow 1$

Que.  $\rightarrow \frac{10}{s(s-5)}$

Soln.  $\rightarrow$



To map  $S_1$   
 Polar plot  $\rightarrow$

$$G(s) = \frac{-2}{s(1-0.2s)}$$

$$G(j\omega) = \frac{-2}{j\omega(1-0.2j\omega)}$$

$$|G(j\omega)| = \frac{2}{\omega \sqrt{1+(0.2\omega)^2}}$$

$$|G(j\omega)| = \frac{-180^\circ}{90(-\tan' 0.2\omega)} = -180^\circ - 90 + \tan' 0.2\omega = -270^\circ + \tan' 0.2\omega$$

$\omega^\circ$	0	$\infty$
$ G(j\omega) $	$\infty$	0
$G(j\omega)$	$-270^\circ$	$-180^\circ$

To map  $s_4 \rightarrow$

$$G(s) = \frac{10}{s(-5)} = \frac{2}{s(-1)}$$

$$= \frac{2}{\lim_{r \rightarrow 0} r e^{j\theta} e^{j\pi}} \quad (\because -1 = e^{j\pi})$$

$$= \infty e^{-j(\theta + \pi)}$$

$$\theta = -\frac{\pi}{2} + 0 + \frac{\pi}{2}$$

$$\infty e^{-j(\frac{\pi}{2} + \pi)} = \infty e^{-j\pi/2}$$

$$\infty e^{-j(\pi/2 + \pi)} = \infty e^{-j3\pi/2}$$

$$\infty e^{-j\pi/2} \longrightarrow \infty e^{-j3\pi/2}$$

$$N = P - Z$$

$$-1 = 1 - Z$$

$$Z = 2 \text{ (Unstable)}$$

Ques.  $\rightarrow G(s) = \frac{s+2}{s^2(s-1)}$

Soln.  $\rightarrow G(s) = \frac{s+2}{s^2(s-1)}$

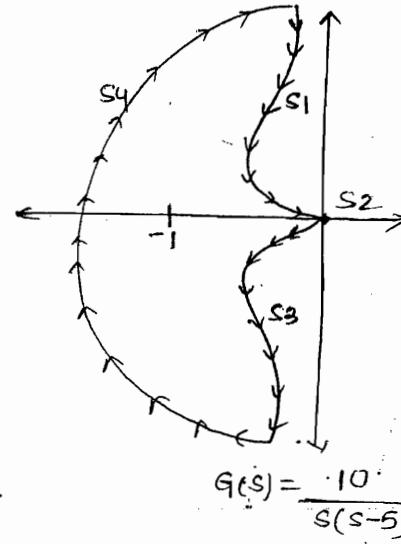
$$G(s) = \frac{2(1+0.5s)}{s^2(1)(1-s)}$$

$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(j\omega)^2(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\omega^2\sqrt{1+\omega^2}}$$

$$|G(j\omega)| = \frac{-180(\tan' 0.5\omega)}{(180)(-\tan' \omega)}$$

$$= -180 - 180 + \tan' 0.5\omega + \tan' \omega$$



$$G(s) = \frac{10}{s(s-5)}$$

$\omega^\circ$	0	$\infty$
$ G(j\omega) $	$\infty$	0
$G(j\omega)$	$-360^\circ$	$-180^\circ$

Because of the presence of zero

$$-180 - 180 + \tan' 0.5\omega + \tan' \omega = 180$$

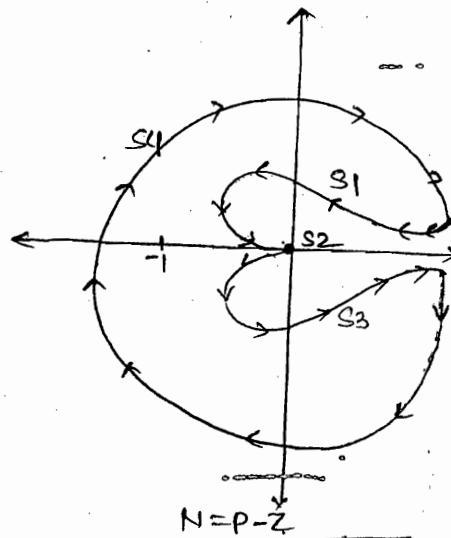
$$\tan' (0.2\omega) + \tan' \omega = 180$$

$$\tan' \left( \frac{0.2\omega + \omega}{1 - 0.2\omega^2} \right) = 180$$

$$0.2\omega + \omega = 0$$

To map  $s_4 \rightarrow$

$$\begin{aligned} g(s) &= \frac{2}{s^2(-1)} \\ &= \frac{2}{\lim_{s \rightarrow 0} (se^{j\theta})^2 e^{j\pi}} \\ &= \frac{2}{\lim_{s \rightarrow 0} s^2 e^{j2\theta} e^{j\pi}} \\ &\propto e^{j(2\theta + \pi)} \\ \theta &= -\frac{\pi}{2} \rightarrow +\frac{\pi}{2} \\ \infty e^{j0} &\rightarrow \infty e^{-j(\pi)} \end{aligned}$$



$$N = P - Z \quad -1 = 1 - 2 \quad [Z = 2] \text{ (Unstable)}$$

$N = -1$  (Because of two dirn encirclement)

Check  $\rightarrow$

$$1 + \frac{s+2}{s^2(s-1)} = 0$$

$$s^3 - s^2 + s + 2 = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 0 \\ s^2 & -1 & 2 \\ s^1 & 3 & 0 \\ s^0 & 2 & 1 \end{array}$$

$$\begin{array}{c} N = P - Z \\ \downarrow \quad \downarrow \\ 1 \quad -2 \\ \boxed{N = -1} \end{array}$$

Ques  $\rightarrow$

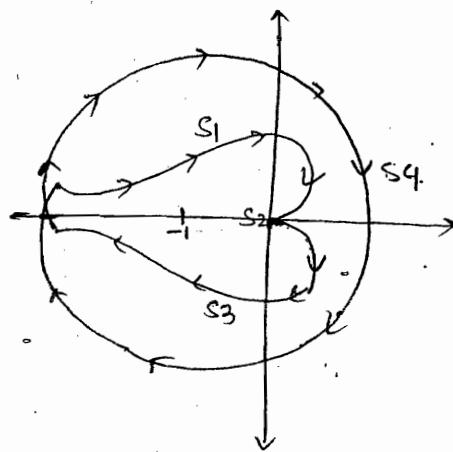
$$g(s) = \frac{1}{s^2(1+s)(1+2s)}$$

Sol  $\rightarrow$

$$\begin{aligned} g(s) &= \frac{1}{s^2} \\ &= \frac{1}{\lim_{s \rightarrow 0} (se^{j\theta})^2} \\ &= \frac{1}{\lim_{s \rightarrow 0} s^2 e^{j2\theta}} \\ &\propto e^{-j2\theta} \end{aligned}$$

$$\theta = -\pi/2 \rightarrow +\pi/2$$

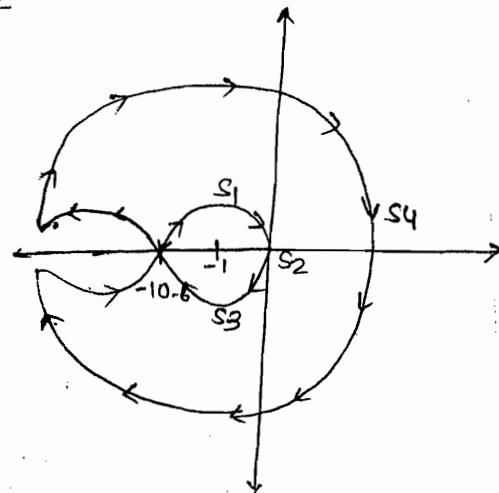
$$= \infty e^{j\pi} \rightarrow \infty e^{-j\pi}$$



$$N = P - Z$$

$$\text{Q. } \rightarrow G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

SOL



$$N = P - Z$$

$$-2 = 0 - Z$$

$$Z = 2$$

stable

$N = -2$  (No. of encirclement of -1 are 2 times in CW dirn)

(Contd.)  
75

$$G(s) = \frac{s}{1-0.2s}$$

To map of  $s_1 \rightarrow$

$$G(j\omega) = \frac{(j\omega)}{1-0.2(j\omega)}$$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+(0.2\omega)^2}}$$

$$|G(j\omega)| = 90 + \tan^{-1}(0.2\omega)$$

To map  $s_2 \rightarrow$

$$G(s) = \frac{s}{-0.2s} = -5$$

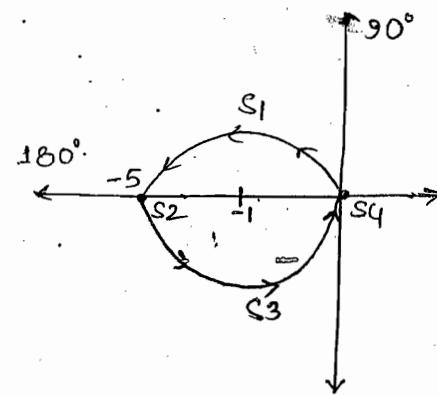
To map  $s_4 \rightarrow$

$$G(s) = s$$

$$= \lim_{T \rightarrow 0} (re^{j\theta})$$

$$= oe^{j0}$$

$\omega$	0	$\infty$
$ G(j\omega) $	0	5
$\angle G(j\omega)$	$90^\circ$	$180^\circ$



$$N = P - Z$$

$$1 = 1 - Z$$

$$Z = 0$$

(stable)

$$\text{Ques} \rightarrow G(s) = \frac{k(1+s)^2}{s^3}$$

Soln  $\rightarrow$  To map  $s_1 \rightarrow$

$$G(j\omega) = \frac{k(1+j\omega)^2}{(j\omega)^3}$$

$$|G(j\omega)| = \frac{k(1+\omega^2)}{\omega^3}$$

$$\angle G(j\omega) = -270^\circ + 2\pi n \omega$$

$$-270^\circ + 2\pi n \omega = -180^\circ$$

$$2\pi n \omega = 90^\circ$$

$$2\pi n \omega = 45^\circ$$

$$\omega = \omega_{pc} = \pm \pi/s$$

$$X = \frac{k(1+1^2)}{j^3} = 2k$$

To map  $s_4 \rightarrow$

$$G(s) = \frac{k}{s^3}$$

$$= \frac{k}{\lim_{\tau \rightarrow 0} \gamma^3 e^{j3\theta}}$$

$$\infty e^{-j3\theta} (\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2})$$

$$\infty e^{j3\pi/2} \rightarrow \infty e^{j3\pi/2}$$

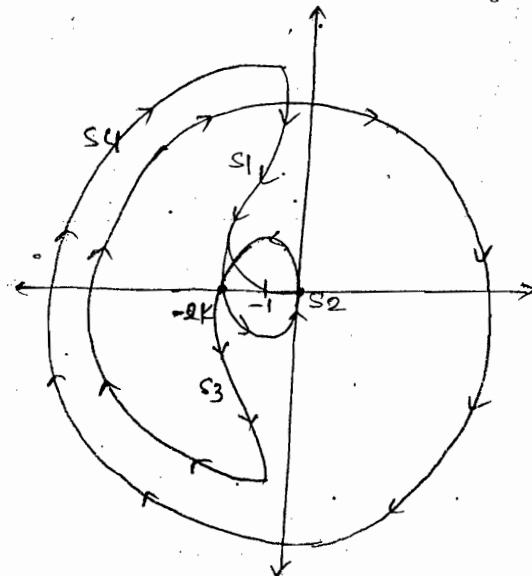
$$-2k = -1, k_{max} = 0.5$$

$$k > 0.5$$

$$1-1=0-2$$

$\boxed{z=0}$  stable

$\omega$	0	1	$\infty$
$ G(j\omega) $	$\infty$	$2k$	0
$\angle G(j\omega)$	$-270^\circ$	$-180^\circ$	$-90^\circ$



$$\text{Que.} \rightarrow G(s) = \frac{10e^{-s}}{s(s+5)}$$

Soln → To map  $s_1 \rightarrow$

polar plot

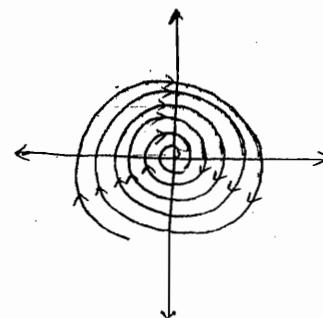
$$G(s) = \frac{2e^{-s}}{s(1+0.2s)}$$

$$G(j\omega) = \frac{2e^{-j\omega}}{(j\omega)(1+0.2j\omega)}$$

$$|G(j\omega)| = \frac{2}{\sqrt{1+0.2\omega^2}}$$

$$|G(j\omega)| = -90^\circ - 57.3\omega - j0^\circ 0.2\omega$$

$\omega$	0	$\infty$
$ G(j\omega) $	$\infty$	0
$\angle G(j\omega)$	-90°	-90°



Sagar Sen  
8871453536

BODE PLOT

\* It is a plot of the dB value of magnitude & phase angle in degree vs  $\log \omega$ .

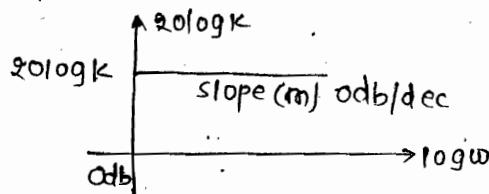
(1) System Gain ( $\pm k$ ) →

$$F(s) = \pm k$$

$$F(j\omega) = \pm k + j0$$

$$|F(j\omega)| = \sqrt{(\pm k)^2 + 0^2} = k$$

Its dB value is



$$F(j\omega) = k + j0 \rightarrow 0^\circ \text{ for all } \omega$$

$$= -k + j0 \rightarrow -180^\circ \text{ for all } \omega$$

(2) Integral & derivative factors ( $s^{\pm n}$ ) →  
(pole/zeros at origin)

$$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$$

$$|F(j\omega)| = \left[ \sqrt{0 + \omega^2} \right]^{\pm n} = (\omega)^{\pm n}$$

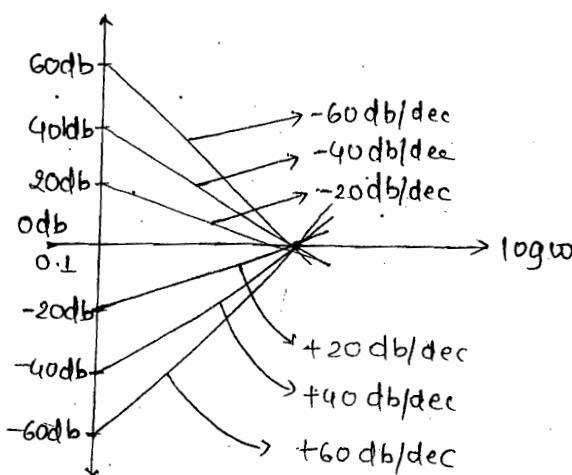
Its dB value is

$$20\log(\omega)^{\pm n} \Rightarrow \pm 20 \log \times n \log \omega \quad (1)$$

$$\boxed{\text{slope (m)} = \pm 20 \times n \text{ db/dec}}$$

$$\pm 20 \log \pm 20 \times n \log \omega = 0$$

$$\log \omega = 0, \omega = \log(0) = 18/s$$



\* First Order factors →  $(1 + TS)^{\pm 1}$

$$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$$

$$|F(j\omega)| = \left[ \sqrt{1 + (\omega T)^2} \right]^{\pm 1}$$

Its dB value

$$\pm 20 \log \left[ \sqrt{1 + (\omega T)^2} \right]^{\pm 1}$$

$$\pm 20 \log \sqrt{1 + (\omega T)^2} \longrightarrow \text{U}$$

Asymptotic approximations →

case(i) → Low freq.

$$1 \gg (\omega T)^2$$

$$\pm 20 \log \sqrt{1} = \pm 0 \text{db}$$

case(ii) High freq:

$$(\omega T)^2 \gg 1$$

$$\pm 20 \log \sqrt{(\omega T)^2}$$

$$\pm 20 \log (\omega T) \text{ --- (2)}$$

$$\pm 20 \log \omega \pm 20 \log T$$

$$(m x + c)$$

$$\boxed{\text{Slope}(m) = \pm 20 \text{ db/dec}}$$

Corner freq: ( $\omega_{cf}$ ) →

$$\pm 20 \log \omega T = 0$$

$$\log \omega T = 0$$

$$\omega T = \log'(0) = 1$$

$$\boxed{\omega = \omega_{cf} = \frac{1}{T} \text{ rad/s}}$$

$$\text{Ex: } (s \pm j)^{\pm 1}$$

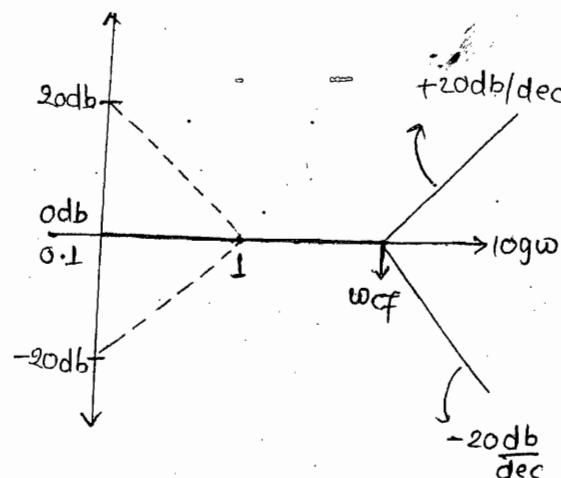
$$\left(1 \pm \frac{j}{2}\right)^{\pm 1}$$

$$T = \frac{1}{2} \Rightarrow \omega_{cf} = 2 \text{ rad/s}$$

Error at  $\omega_{cf}$  →

$$\text{At } \omega = \omega_{cf} = \frac{1}{T}$$

$$\pm 20 \log \sqrt{1 + \left(\frac{1}{T} \times T\right)^2} = \pm 20 \log \sqrt{2} = \pm 3 \text{ db}$$



$\leftarrow$  Quadratic factor  $\rightarrow (s^2 + 2\zeta\omega_n s + \omega_n^2)^{\pm 1}$

$$\left( \frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1 \right)^{\pm 1}$$

(put  $s=j\omega$ )

$$\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + j2\zeta \left( \frac{\omega}{\omega_n} \right) \right]^{\pm 1}$$

$$\pm 20 \log \sqrt{\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right]^2 + 2\zeta \left( \frac{\omega}{\omega_n} \right)^2}$$

LFR  
0db/dec

HFR  
 $\pm 40 \log \left( \frac{\omega}{\omega_n} \right)$

$$\boxed{\text{slope } (m) = \pm 40 \text{ db/dec}}$$

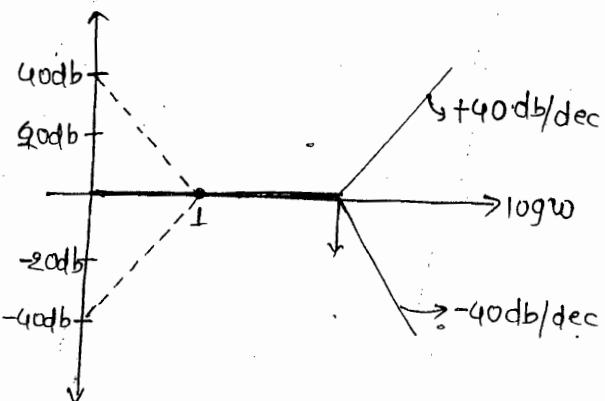
corner freq. ( $\omega_{cf} = \omega_n \sqrt{s}/s$ )

$$(s^2 + 4s + 25)^{\pm 1}$$

$$\omega_n^2 = 25, \omega_n = \omega_{cf} = 5 \text{ rad/s.}$$

error at  $\omega_{cf} \rightarrow$

$$\pm 20 \log 2\zeta$$



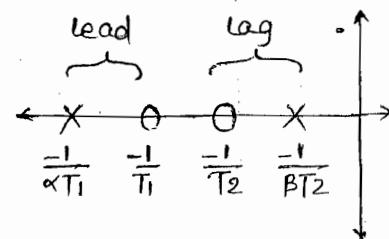
factors	corner freq.	magnitude	slope
$k$	-	$20 \log k$	0db/dec
$j(\omega)^{\pm n}$	-	$\pm 20n \log \omega$	$\pm 20n \text{ db/dec}$
$(1+j\omega T)^{\pm 1}$	$\frac{1}{T}$	$\pm 20 \log \omega T$	$\pm 20 \text{ db/dec}$
$\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2j\zeta \left( \frac{\omega}{\omega_n} \right) \right]^{\pm 1}$	$\omega_n$	$\pm 40 \log \left( \frac{\omega}{\omega_n} \right)$	$\pm 40 \text{ db/dec}$

\* Bode plot for lag-lead compensator →

175

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+\tau_1 s)(1+\tau_2 s)}{(1+\alpha\tau_1 s)(1+\beta\tau_2 s)}$$

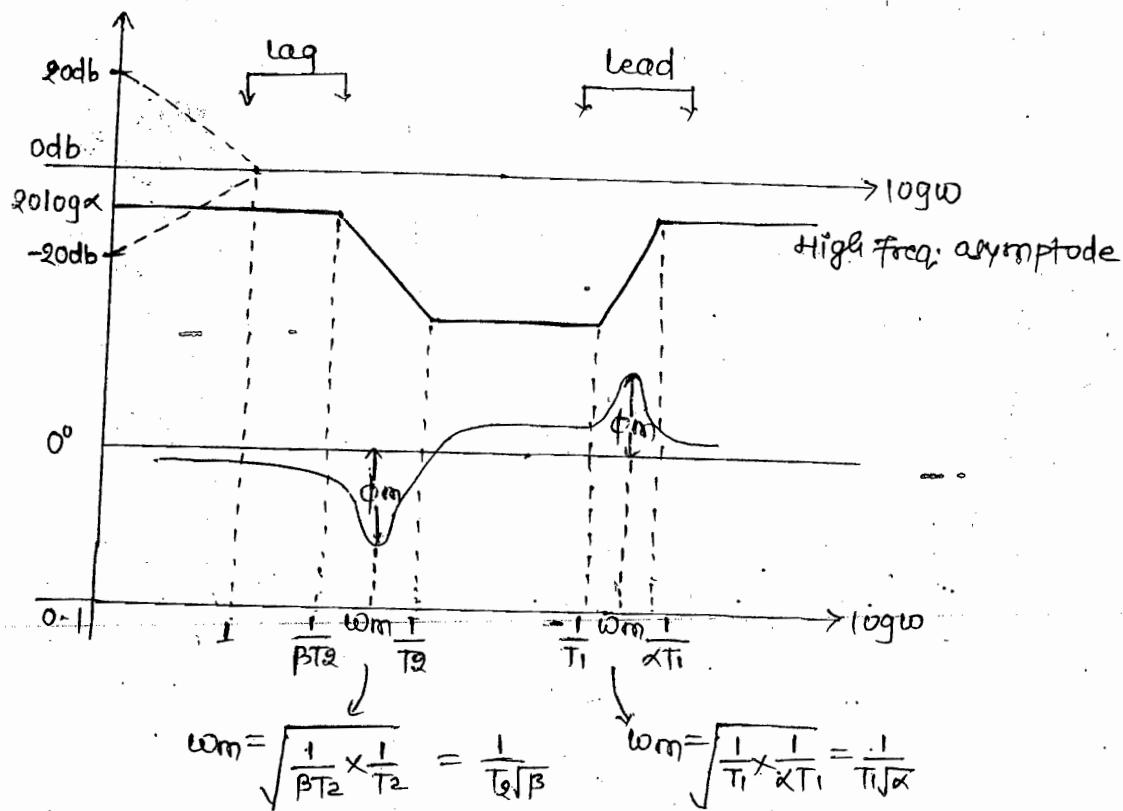
$$F(j\omega) = \frac{\alpha(1+j\omega\tau_1)(1+j\omega\tau_2)}{(1+j\omega\tau_1)(1+j\omega\beta\tau_2)}$$



$$|F(j\omega)| = +\alpha\eta'\omega\tau_1 - +\alpha\eta'\omega\alpha\tau_1 + +\alpha\eta'\omega\tau_2 - +\alpha\eta'\omega\beta\tau_2$$

magnitude table →

Factor	corner freq.	magnitude
(1) $\alpha$	-	+log $\alpha$
(2) $(j\omega)^{\pm\eta}$	-	Nil
(3) $\frac{1}{1+j\omega\beta\tau_2}$	$\frac{1}{\beta\tau_2}$	-20db/dec (-20)
(4) $1+j\omega\tau_2$	$\frac{1}{\tau_2}$	+20db/dec
(5) $1+j\omega\tau_1$	$\frac{1}{\tau_1}$	+20db/dec +20
(6) $\frac{1}{1+j\omega\alpha\tau_1}$	$\frac{1}{\alpha\tau_1}$	-20db/dec (-20)



for calculate High freq. asymptote

\* C/s of phase lead compensator  $\rightarrow$  The phase lead compensator shifts the gain cross over freq; to higher values where the desired phase margin is acceptable.  
Hence it is effective when the slope of uncompensated sys. near the gain cross over freq; is low.

\* The max<sup>m</sup> phase lead occurs at the geometric mean of 2 corner freq;

$$|F(j\omega)| = \phi = \tan^{-1}\omega T_1 - \tan^{-1}\omega x T_1$$

$$\tan \phi = \tan(\tan^{-1}\omega T_1 - \tan^{-1}\omega x T_1)$$

$$\tan \phi = \frac{\omega T_1 - \omega x T_1}{1 + (\omega T_1)^2 \alpha} = \frac{\omega T_1(1-\alpha)}{1 + (\omega T_1)^2 \alpha}$$

$$\text{At } \omega = \omega_m = \frac{1}{T_1\sqrt{\alpha}} ; \phi = \phi_m$$

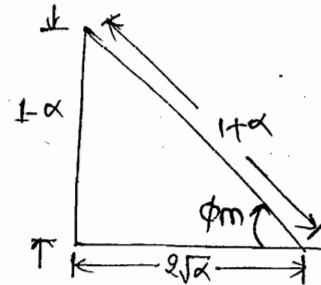
$$\tan \phi_m = \frac{\frac{1}{T_1\sqrt{\alpha}} \cdot T_1(1-\alpha)}{1 + \left(\frac{1}{T_1\sqrt{\alpha}}\right)^2 \cdot \alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\bar{\tan \phi_m} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)}$$



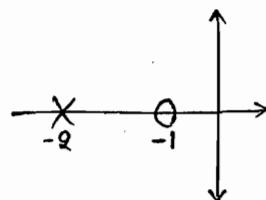
$$\sin \phi_m (1+\alpha) = 1-\alpha$$

$$\alpha \sin \phi_m + \alpha = 1 - \sin \phi_m$$

$$\alpha (1 + \sin \phi_m) = 1 - \sin \phi_m$$

$$\boxed{\alpha = \frac{1 - \sin \phi_m}{1 + \sin \phi_m}}$$

### (3.) Polar plot →



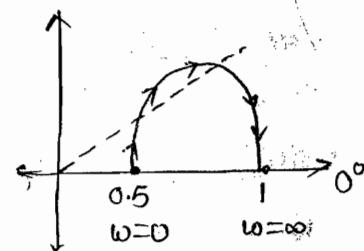
$$|F(j\omega)| = \tan^{-1}\omega - \tan^{-1}0.5\omega$$

$\omega$	0	1	$\infty$
$ F(j\omega) $	0.5	1	
$\angle F(j\omega)$	0°	18.4°	0°

$$F(s) = \frac{s+1}{s+2} = \frac{0.5(1+s)}{(1+0.5s)}$$

$$F(j\omega) = \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$



\* c/s of phase lag compensator  $\rightarrow$  \* phase lag compensator shifts the gain cross over freq. to lower values

where the desired  $\phi$  margin is acceptable.

Hence it is effective when the slope of the uncompensated sys. near gain cross over freq. is high.

\* The max<sup>m</sup> phase lag occurs at the geometric mean of the 2 corner freq.

$$\underline{f(\omega)} = \phi = \tan^{-1}\omega T_2 - \tan^{-1}\omega \beta T_2$$

$$\tan\phi = \tan(\tan^{-1}\omega T_2 - \tan^{-1}\omega \beta T_2)$$

$$\tan\phi = \frac{\omega T_2 - \omega T_2 \beta}{1 + (\omega T_2)^2 \cdot \beta} = \frac{\omega T_2(1-\beta)}{1 + (\omega T_2)^2 \beta}$$

$$\text{At } \omega = \omega_m = \frac{1}{T_2 \sqrt{\beta}} ; \phi = \phi_m$$

$$\tan\phi_m = \frac{\frac{1}{T_2 \sqrt{\beta}} \times T_2(1-\beta)}{1 + \left[ \frac{1}{T_2 \sqrt{\beta}} \cdot T_2 \right]^2 \beta} = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\tan\phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\boxed{\phi_m = \tan^{-1} \left( \frac{1-\beta}{2\sqrt{\beta}} \right)}$$

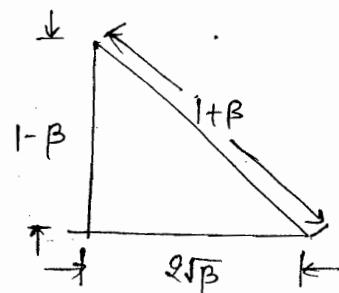
$$\sin\phi_m = \frac{1-\beta}{1+\beta}$$

$$\boxed{\phi_m = \sin^{-1} \left( \frac{1-\beta}{1+\beta} \right)}$$

$$\sin\phi_m(1+\beta) = 1-\beta$$

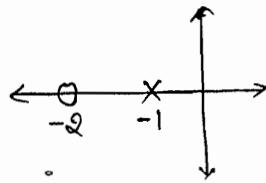
$$\beta \sin\phi_m + \beta = 1 - \sin\phi_m$$

$$\beta(1 + \sin\phi_m) = 1 - \sin\phi_m$$



$$B = \frac{1 - \sin \phi_m}{1 + \sin \phi_m}$$

(3.) Polar plot  $\rightarrow$



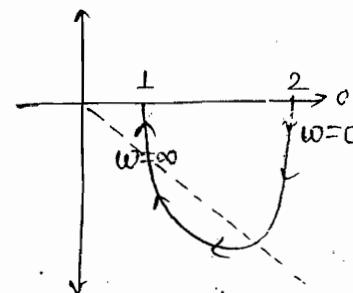
$$F(s) = \frac{s+2}{s+1} = \frac{2(1+0.5s)}{(1+s)}$$

$$F(j\omega) = \frac{2(1+0.5j\omega)}{(1+j\omega)}$$

$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \tan^{-1} 0.5\omega - \tan^{-1} \omega$$

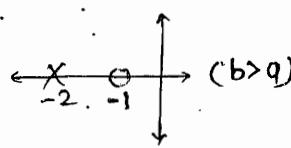
$\omega$	0	1	$\infty$
$ F(j\omega) $	2	1	1
$\angle F(j\omega)$	$0^\circ$	$-18.4^\circ$	$0^\circ$



(1)  
76

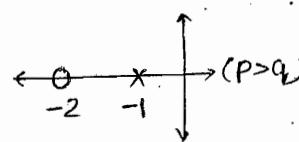
lead :-

$$\frac{s+a}{s+b} = \frac{s+1}{s+2}$$



lag :-

$$\frac{s+p}{s+q} = \frac{s+2}{s+1}$$



(3)  
76

$$\alpha = \frac{R_2}{R_1 + R_2}$$

$$\phi_m = \sin^{-1} \left( \frac{1 - \frac{R_2}{R_1 + R_2}}{1 + \frac{R_2}{R_1 + R_2}} \right)$$

$$\phi_m = \sin^{-1} \left( \frac{R_1}{R_1 + 2R_2} \right)$$

(2)  
76

$$G_C(s) = \frac{0.5(1+0.3s)}{(1+0.1s)}$$

(4)  
76

$$\frac{1+0.3s}{1+0.1s} = \frac{1+Ts}{1+kTs}$$

$$T_1 = 0.3, kT = 0.1$$

$$\alpha \times 0.3 = T; \alpha = \frac{1}{3}$$

$$\phi_m = \sin^{-1} \left( \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \right) = \sin^{-1} \left( \frac{2}{3} \right)$$

lead compensator

$$\phi_m = \sin^{-1} \left( \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \right)$$

$$\phi_m = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ$$

ans(c)

(6)  
76

$$\frac{K(1+0.3s)}{(1+0.17s)} = \frac{1+Ts}{1+\alpha Ts}$$

$$T=0.3; \quad \alpha T=0.17$$

$$\alpha \times 0.3 = 0.17$$

$$\alpha = 0.56$$

$$T=R_C$$

$$R_C \times 10^{-6} = 0.3$$

$$R_C = 300\text{ k}\Omega$$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

$$0.56 = \frac{R_2}{300 + R_2}$$

$$R_2 = 400\text{ k}\Omega$$

(10)  
77

$$\frac{1+2s}{1+0.2s} = \frac{1+Ts}{1+Kts}$$

$$T=2; \quad Kt=0.2$$

$$Kx2 = 0.2, \quad K=0.1$$

(Lead Controller)

(11)  
74

$$(b) \frac{s+9.9}{s+3} \Rightarrow \text{lag.}$$

$$(d) \frac{s+6}{s} = 1 + \frac{6}{s} \rightarrow \text{PI}$$

$$\frac{K_p + K_I}{3}$$

(c) Can't be ans.

(e) To improved transient state lead compensator must be used.

(5)  
76

(d)

(7)  
77

(d)

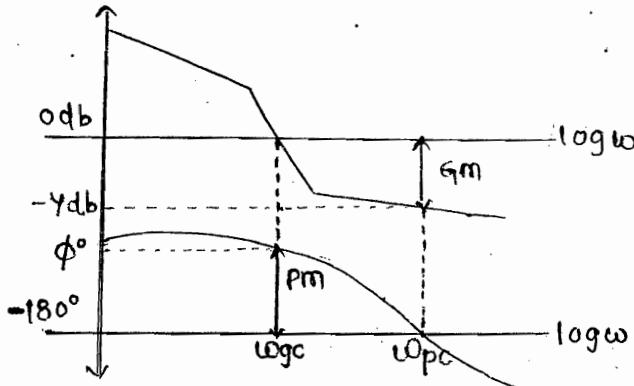
(10)  
77

(c)

(12)  
77

(g)

DATE-25/11/14

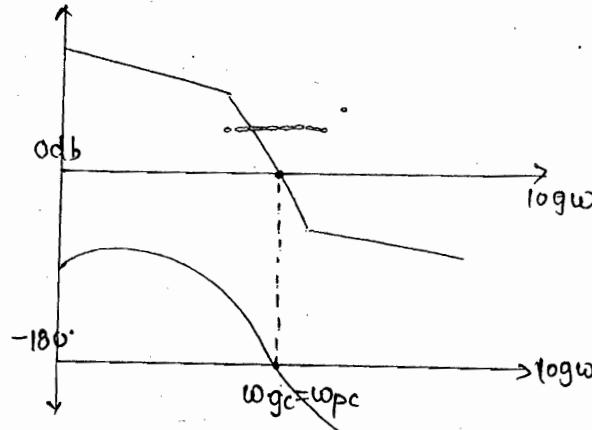
\* To find  $G_m$  &  $P_m$  from BODE plots  $\rightarrow$ 

stable

$$PM = \phi - (-180^\circ) = 180^\circ + \phi = +ve$$

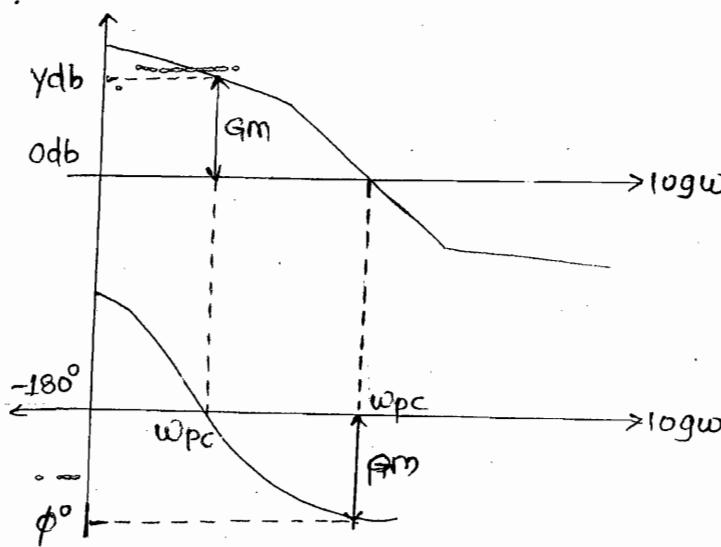
$$GM = 0 - (-4) = +4\text{ db} = +ve$$

$$\boxed{\omega_{gc} < \omega_{pc}}$$



$$GM = PM = 0$$
  
 marginally stable

$$\boxed{\omega_{gc} = \omega_{pc}}$$



$$\rho_m = 180^\circ + \phi = -\nu e$$

$$G_m = 0 - y = -y_{db} = -\nu e$$

$$\boxed{\omega_{qc} > \omega_{pc}}$$

Unstable

Que.  $\rightarrow G(s) = \frac{10^3(s+20)}{(s+10)(s+200)}$

Soln.  $\rightarrow G(s) = \frac{\frac{10^3}{20}(20)\left(\frac{s}{20}+1\right)}{10 \times 200 \left(\frac{s}{10}+1\right)\left(\frac{s}{200}+1\right)}$

$$= \frac{10 \left(1 + \frac{j\omega}{20}\right)}{\left(1 + \frac{j\omega}{10}\right)\left(1 + \frac{j\omega}{200}\right)}$$

Factors CF magnitude

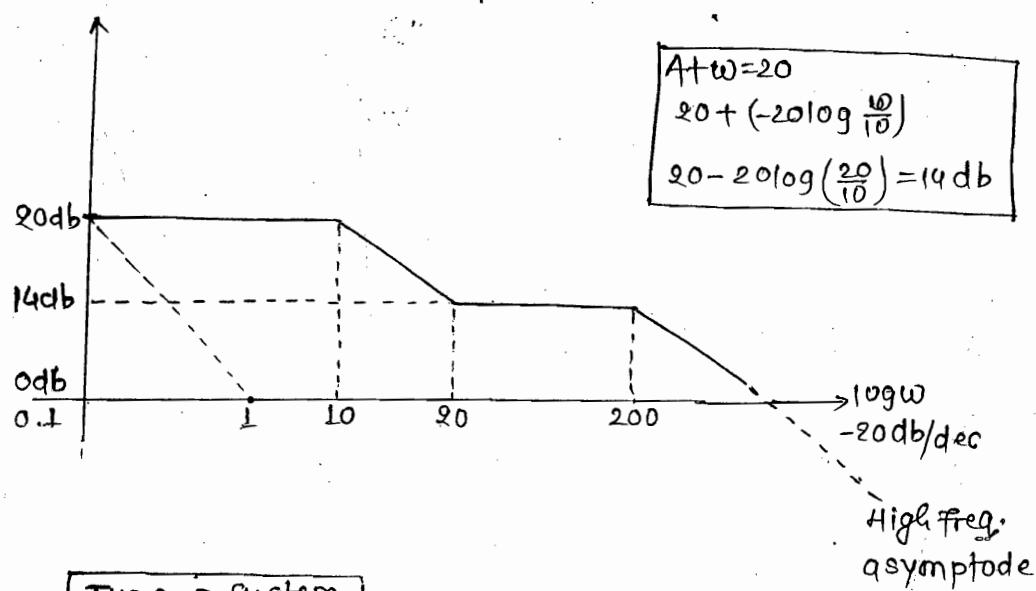
(1.)  $k=10$  -  $20 \log 10 = 20 \text{ db}$

(2.)  $(j\omega)^{\pm n}$  -  $\text{Nil}$

(3.)  $\frac{1}{1 + \frac{j\omega}{10}}$   $10 \rightarrow -20 \log \left(\frac{\omega}{10}\right)$   
 $-20 \rightarrow (-20 \text{ db/dec})$

(4.)  $\frac{1 + \frac{j\omega}{20}}{1 + \frac{j\omega}{10}}$   $20 \rightarrow +20 \log \left(\frac{\omega}{20}\right)$   
 $0 \rightarrow (+20 \text{ db/dec})$

(5.)  $\frac{1}{1 + \frac{j\omega}{200}}$   $200 \rightarrow -20 \log \left(\frac{\omega}{200}\right)$   
 $-20 \rightarrow (-20 \text{ db/dec})$



Type-0 system

$$K_U = K_A = 0$$

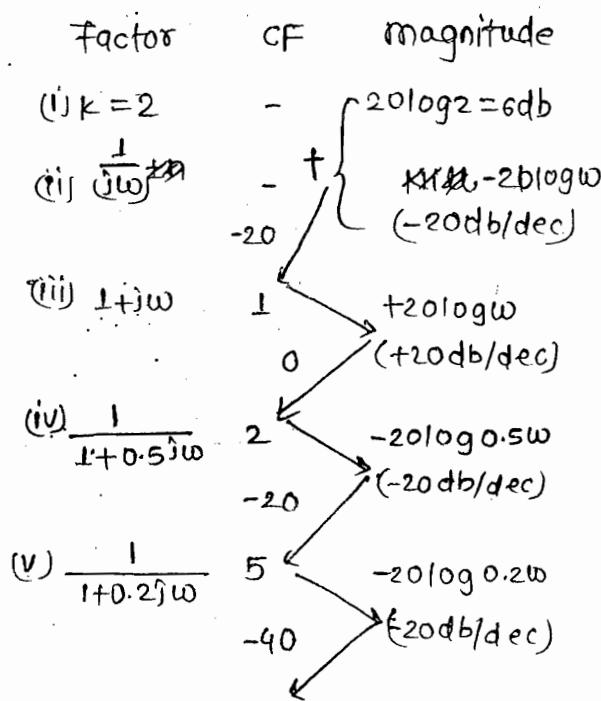
$$20 \log K_P = 20$$

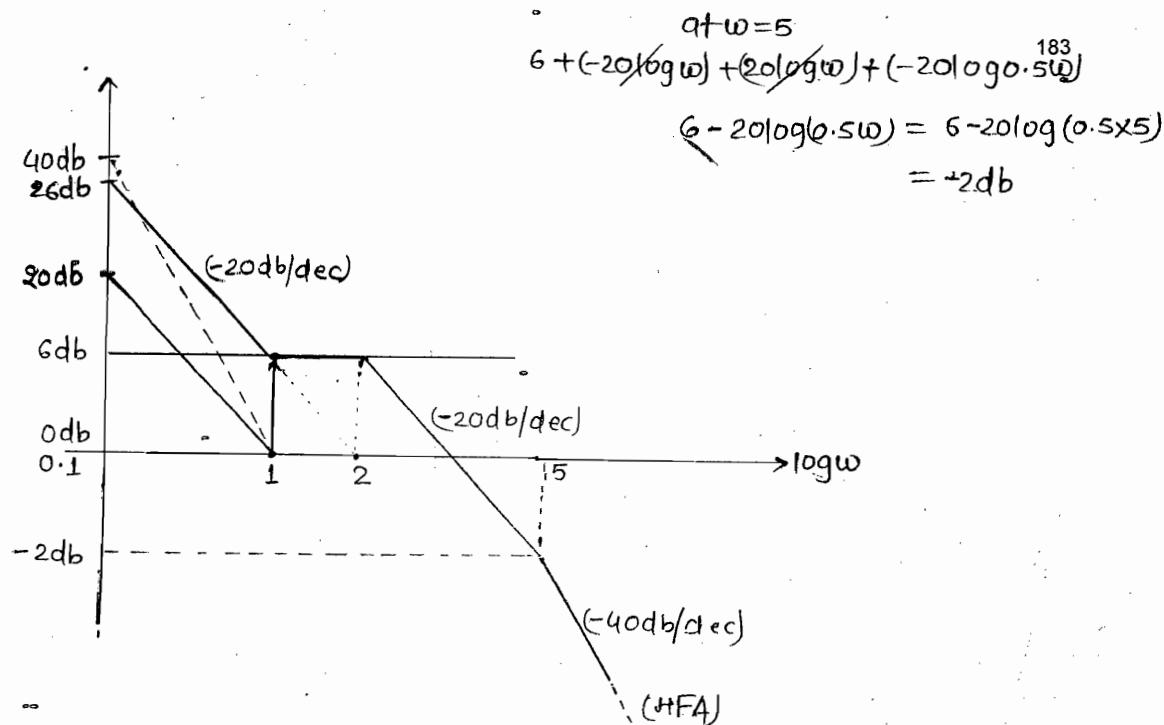
$$K_P = \log g'(1) = 10$$

Que.  $\rightarrow G(s) = \frac{20(s+1)}{s(s+2)(s+5)}$

Soln.  $\rightarrow G(s) = \frac{20(1+s)}{10 \cdot s \left(1+\frac{s}{2}\right) \left(1+\frac{s}{5}\right)}$

$$G(j\omega) = \frac{2(1+j\omega)}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)}$$





$$Y \quad M \quad X \quad C \quad C = 20 \log K$$

$$Y = -20 \log \omega + G$$

$$\text{At } \omega = 0.1$$

$$Y = -20 \log 0.1 + 6 = 20 + 6 = 26 \text{ db}$$

Type-1 system

$$K_p = \infty, K_A = 0$$

$$K_V = \omega = 2$$

$$20 \log K - 20 \log \omega = 0$$

$$K = K_V = \omega$$

Consequence(1) →

(74)

$$G(s) = \frac{3(s+1)(s+700)}{s^2(s^2 + 18s + 400)}$$

SOL<sup>n</sup> → quadratic factors must be considered as quadratic factors.

$$G(s) = \frac{3(1+s)(1+\frac{s}{700})700}{s^2 \times 400 \left[ 1 + \frac{18s}{400} + \frac{s^2}{400} \right]}$$

$$G(j\omega) = \frac{3(1+j\omega)(1+\frac{j\omega}{700})700}{-\omega^2 \times 400 \left[ 1 - \frac{\omega^2}{400} + \frac{j18\omega}{400} \right]}$$

$$\boxed{G(j\omega)} \Big|_{\omega < 20} = \frac{(0^\circ) (+\tan^{-1}\omega) (+\tan^{-1}\frac{\omega}{700})}{(180^\circ) \left[ +\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right) \right]}$$

$$\boxed{G(j\omega)} \Big|_{\omega > 20} = \frac{(0^\circ) (+\tan^{-1}\omega) (+\tan^{-1}\frac{\omega}{700})}{(180^\circ) \left[ 180^\circ - +\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right) \right]}$$

Expt. calc<sup>n</sup> of phase angle  
for quadratic term

$$\omega =$$

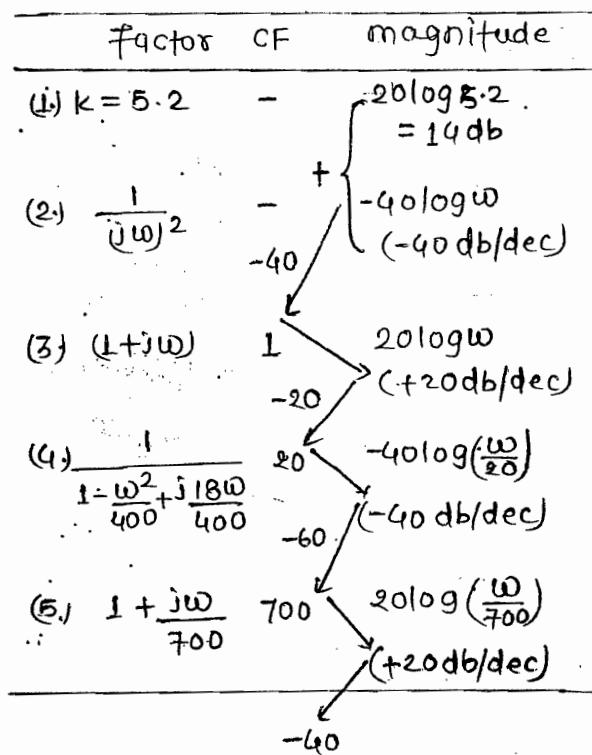
$$-180 + \left[ -\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right) \right]$$

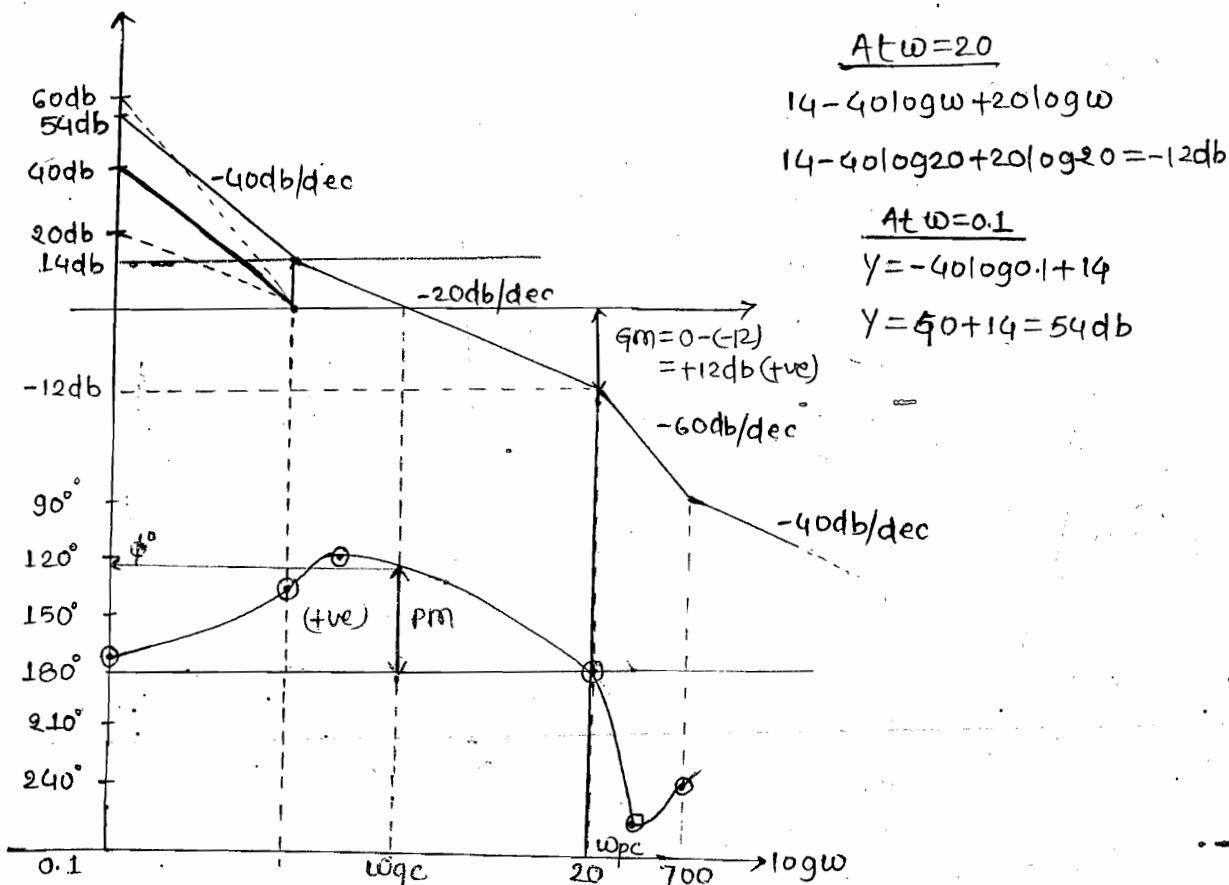
$$(180^\circ + 10.6^\circ) = -169.4^\circ$$

Phase angle table

$\omega$	$-180^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{700} + \tan^{-1}\frac{18\omega}{400-\omega^2}$	$\phi_R$
0.1	$-180^\circ + 5.7^\circ + 0.008^\circ = -174^\circ$	
1	$-180^\circ + 45^\circ + 0.08^\circ = -137^\circ$	
5	$-180^\circ + 78.6^\circ + 0.4^\circ = -114^\circ$	
20	$-180^\circ + 87^\circ + 1.6^\circ = -181^\circ$	
100	$-180^\circ + 89.4^\circ + 8^\circ = -163.4^\circ$	$-251^\circ$
700	$-180^\circ + 89.9^\circ + 45^\circ = -178.5^\circ$	$-223^\circ$

magnitude table





Type-2 system

$$K_p = K_v = \infty$$

$$K_A = \omega^2$$

$$20 \log K - 20 \log \omega^2 = 0$$

$$K = K_A = \omega^2$$

$$\omega_{pc} > \omega_{gc}$$

stable system

$$G_m = P_m = +ve$$

(13)  
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$$G(s) = \frac{k(s+10)(s+20)}{s^3(s+100)(s+200)}$$

$$G(s) = \frac{0.01k(1+0.1s)(1+0.05s)}{s^3(1+\frac{s}{100})(1+\frac{s}{200})}$$

$$G(j\omega) = \frac{k}{100} \left(1 + \frac{j\omega}{10}\right) \left(1 + \frac{j\omega}{20}\right) / \left(\frac{j\omega}{100}\right)^3 \left(1 + \frac{j\omega}{100}\right) \left(1 + \frac{j\omega}{200}\right)$$

$$G(j\omega) = \frac{(0^\circ) \left(\tan^{-1} \frac{\omega}{10}\right) \left(\tan^{-1} \frac{\omega}{20}\right)}{90^\circ \left(\tan^{-1} \frac{\omega}{100}\right) \left(\tan^{-1} \frac{\omega}{200}\right)}$$

$$\phi_R = -270^\circ + \tan^{-1} \frac{\omega}{10} + \tan^{-1} \frac{\omega}{20} - \tan^{-1} \frac{\omega}{100} - \tan^{-1} \frac{\omega}{200}$$

$\omega$	0.1	1	10	20	50	100	200
$\phi_R$	$-269^\circ$	$-262^\circ$	$-207^\circ$	$-179^\circ$	$-163^\circ$	$-179^\circ$	$-207^\circ$

mag. table →

Factor	CF	magnitude
(1.) $k' = \frac{k}{100}$	-	$20 \log k'$

(2.)  $\frac{1}{(j\omega)^3}$  -  $-60 \log \omega$   
 $(-60 \text{db/dec})$

(3.)  $1 + \frac{j\omega}{10}$  10  $20 \log \frac{\omega}{10}$   
 $-40 \rightarrow (+20 \text{db/dec})$

(4.)  $1 + \frac{j\omega}{20}$  20  $20 \log \frac{\omega}{20}$   
 $-20 \rightarrow (+20 \text{db/dec})$

(5.)  $\frac{1}{1 + \frac{j\omega}{100}}$  100  $-20 \log \frac{\omega}{100}$   
 $-40 \rightarrow (-20 \text{db/dec})$

(6.)  $\frac{1}{1 + \frac{j\omega}{200}}$  200  $-20 \log \frac{\omega}{200}$   
 $-60 \rightarrow (-20 \text{db/dec})$

$A + \omega = 20$ 

$$-60\log 20 + 20\log 2 = -72 \text{ db}$$

 $A + \omega = 100$ 

$$60\log 100 + 20\log 10 + 20\log 5 = -86 \text{ db}$$

$$Gm(1) = 0 - (-72) = 72 \text{ db}$$

$$20\log K' = 72 \Rightarrow K' = 3981$$

$$K' = \frac{K}{100} = 3981 ; K = 398100$$

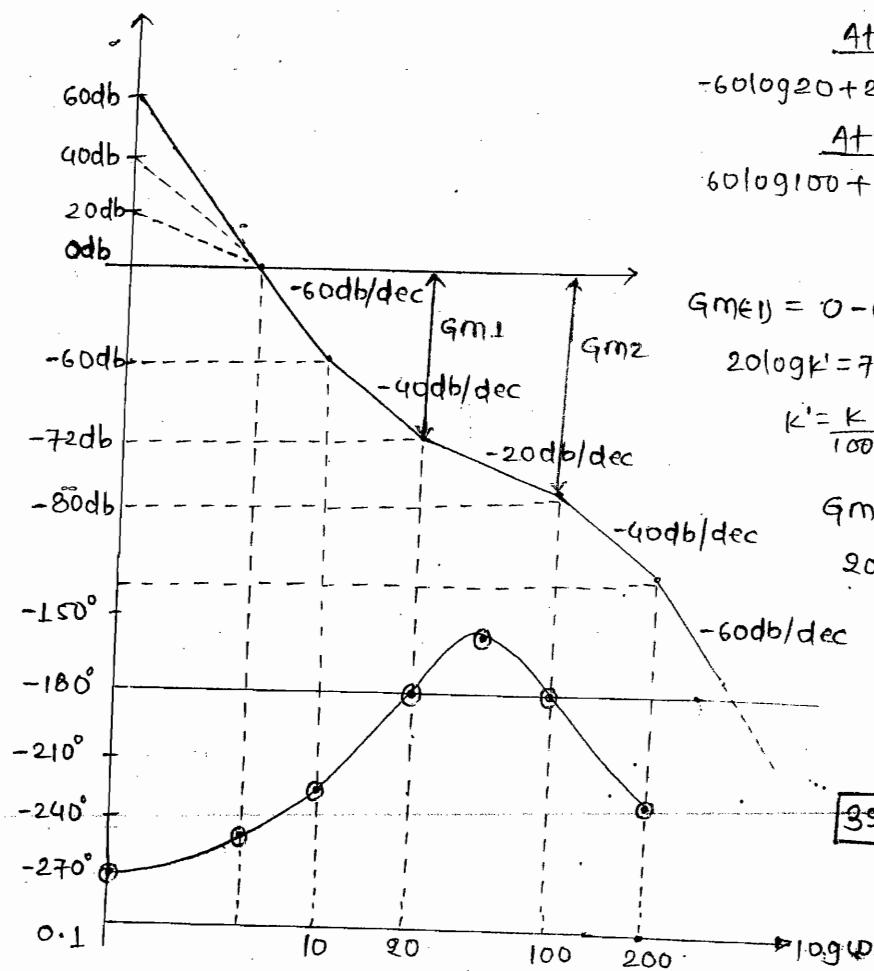
$$Gm(2) = 0 - (-86) = 86 \text{ db}$$

$$20\log K' = 86 \Rightarrow K' = 19952$$

$$K' = \frac{K}{100} = 19952 ; K = 1995200$$

$$K = 1995200$$

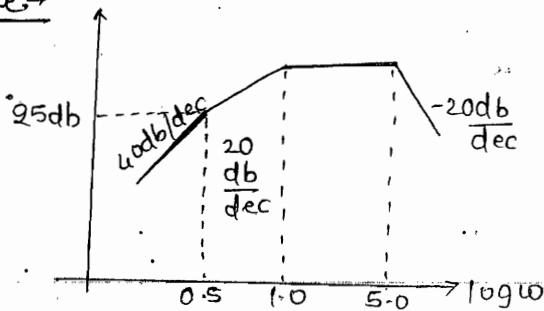
$$\boxed{398100 < K < 1995200}$$



### Inverse Bode Plot

- \* Observe the starting slope. This will give the information of poles (or) zeros at origin.
- \* For the starting slope write the eqn  $y = mx + c$ .  
 $c = 20 \log k$ .
- \* At every corner freq: observe the change in slopes. This will give the information of 1st (or) higher order factors.

Que →



Soln

$$G(s) = \frac{70 s^2}{(1+2s)(1+s)(1+\frac{s}{5})}$$

$$\text{At } \omega = 0.5$$

$$y = mx + c$$

$$25 = 20 \log 0.5 + c$$

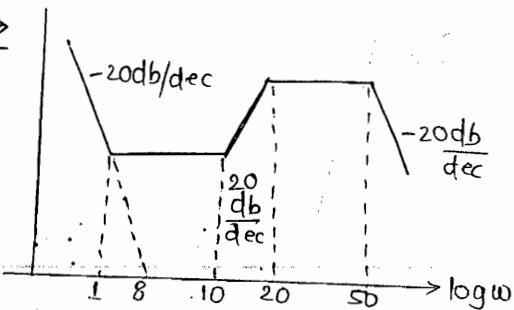
$$c = 37$$

$$20 \log k = 37$$

$$k = 10^{3.7} = 50$$

$$k = 70$$

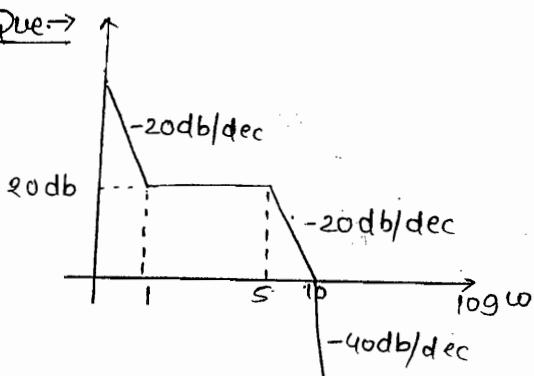
Que →



Soln

$$G(s) = \frac{8(1+s)(1+\frac{s}{10})}{s(1+\frac{s}{20})(1+\frac{s}{50})}$$

Que →



Soln

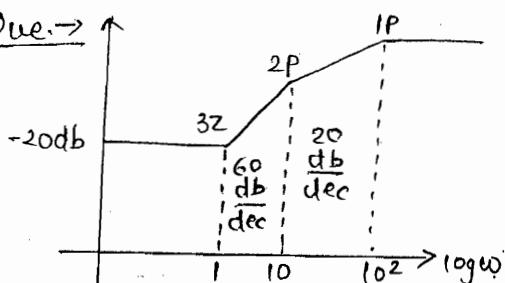
$$G(s) = \frac{10(1+s)}{s(1+\frac{s}{5})(1+\frac{s}{10})}$$

$$\text{At } \omega = 1$$

$$y = mx + c \quad 20 \log k = 20$$

$$20 = 20 \log 1 + c \quad k = 10^{10} = 100$$

Que →



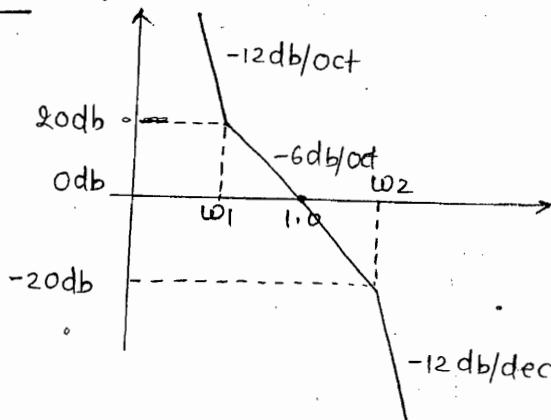
Soln

$$G(s) = \frac{0.1(1+s)^3}{(1+\frac{s}{10})^2(1+\frac{s}{100})}$$

$$20 \log k = 10$$

$$k = 10^{10} = 100$$

$$k = 0.1$$

Que. →Soln →decade scale

$$\omega_2 = 10\omega_1$$

$$\text{db value magnitude} = \pm 20 \times n \log \omega$$

$$\text{slope}(m) = \pm 20 \times n \log 10$$

$$= \pm 20 \times n \text{ db/dec}$$

$$\boxed{\pm 6 \times n \frac{\text{db}}{\text{oct}} \cong \pm 20 \times n \frac{\text{db}}{\text{dec}}}$$

Octave scale

$$\omega_2 = 2\omega_1$$

$$\text{db value magnitude} = \pm 20 \times n \log 2$$

$$\text{slope}(m) = \pm 20 \times n \log 2$$

$$= \pm 6 \times n \text{ db/oct}$$

2nd line →

$$\text{At } \omega = 1$$

$$y = mx + c$$

$$0 = -20 \log 1 + c$$

$$c = 0$$

$$\text{At } \omega = \omega_1$$

$$y = mx + c$$

$$20 = -20 \log \omega_1 + 0$$

$$\omega_1 = 10^{\frac{1}{2}}(1) = 0.1\pi/\text{s}$$

$$\text{At } \omega = \omega_2$$

$$y = mx + c$$

$$-20 = -20 \log \omega_2 + 0$$

$$\omega_2 = 10^{\frac{1}{2}}(1) = 10\pi/\text{s}$$

To Find k →1st line

$$\text{At } \omega = \omega_1 = 0.1$$

$$y = mx + c$$

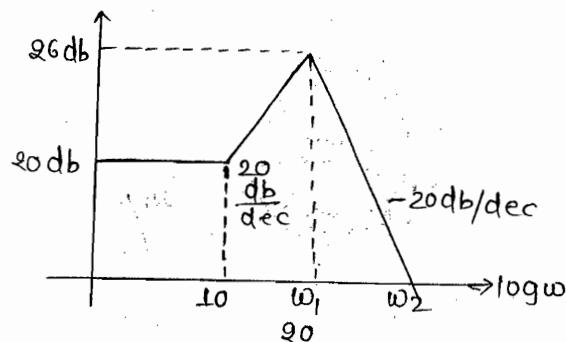
$$20 = -20 \log 0.1 + c$$

$$c = 20$$

$$20 \log k = -20$$

$$\boxed{k = 0.1}$$

$$\boxed{G(s) = \frac{0.1(1+10s)}{s^2(1+0.1s)}}$$

Que.→Soln→To find  $\omega_1$ 

2nd line

At  $\omega = 10$

$y = mx + c$

$20 = 20 \log 10 + c$

$c = 0$

At  $\omega = \omega_1$

$y = mx + c$

$26 = 20 \log \omega_1 + 0$

$\omega_1 = \log\left(\frac{26}{20}\right) = 20 \text{ rad/s}$

To find  $\omega_2$ 

3rd line

At  $\omega = \omega_1 = 20$

$y = mx + c$

$26 = -20 \log 20 + c$

$c = 52$

At  $\omega = \omega_2$

$y = mx + c$

$0 = -20 \log \omega_2 + 52$

$-52 = -20 \log \omega_2$

$\omega_2 = \log\left(\frac{52}{20}\right) = 400 \text{ rad/s}$

To find K

$20 \log K = 20$

$K = \log^{-1}(1)$

$= 10$

$$G(s) = \frac{10 \left( 1 + \frac{s}{10} \right)}{\left( 1 + \frac{s}{20} \right)^2}$$

M & N circles

\* Nicol charts →

\* The Nicol chart consists of magnitudes & phase angles of a closed loop system represented as a family of circles known as M & N circles.

\* This chart gives information about closed loop freq; response.

\*

\* M-circles →

Let the CLTF

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\text{Let } G(s) = X + jY$$

$$\frac{C(s)}{R(s)} = \frac{X + jY}{1 + X + jY}$$

The magnitude ( $m$ )

$$m = \sqrt{x^2 + y^2}$$

$$m^2 [ (1+x)^2 + y^2 ] = x^2 + y^2$$

$$m^2 x^2 - x^2 + m^2 y^2 - y^2 + 2x m^2 + m^2 = 0$$

$$(m^2 - 1)x^2 + y^2(m^2 - 1) + 2x m^2 + m^2 = 0 \quad \dots \dots \quad (i)$$

In eqn (i)  $m = 1$

$2x + 1 = 0$ , It represents a straight line passing through  $-\frac{1}{2}, 0$

If  $m \neq 1$  eqn (i) represents a family of circles.

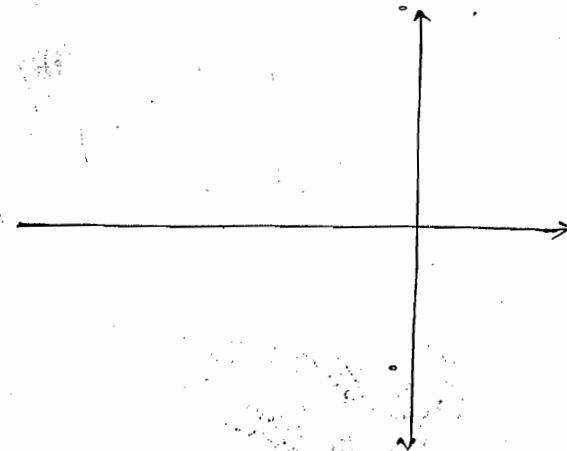
$$x^2 + y^2 + 2x \frac{m^2}{m^2 - 1} + \frac{m^2}{m^2 - 1} = 0 \quad \dots \dots \quad (ii)$$

$$x^2 + y^2 + 2x \frac{m^2}{m^2 - 1} + \frac{m^2}{m^2 - 1} + \frac{m^2}{(m^2 - 1)^2} = \frac{m^2}{(m^2 - 1)^2}$$

$$x^2 + 2x \frac{m^2}{m^2 - 1} + \frac{m^4}{(m^2 - 1)^2} + y^2 = \frac{m^2}{(m^2 - 1)^2}$$

$$\left[ x + \frac{m^2}{m^2 - 1} \right]^2 + y^2 = \frac{m^2}{(m^2 - 1)^2}$$

$$\text{Center } z = \frac{-m^2}{m^2-1}, 0 \quad \text{Radius } = \frac{m}{m^2-1}$$



#### (4.) N-circles $\rightarrow$

Let  $\alpha$  = phase angle of CL system

$N = \tan \alpha$  represents a family of circles

$$\alpha = \tan^{-1} \frac{y}{x} - \tan^{-1} \left( \frac{y}{1-x} \right)$$

$$N = \tan \alpha = \tan \left[ \tan^{-1} \frac{y}{x} - \tan^{-1} \frac{y}{1+x} \right]$$

$$\frac{y}{x} - \frac{y}{1+x} \over 1 + \frac{y^2}{x(1+x)}$$

$$x^2 + x + y^2 - \frac{y}{N} = 0 \quad \dots \dots \dots (i)$$

adding term  $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$  on both side.

$$x^2 + x + \frac{1}{4} + y^2 - \frac{y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\text{Center} = -\frac{1}{2}, \frac{1}{2N} \quad \text{Radius} = \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$$

All N-circles intersects the real axis b/w -1 & origin only.

(b) 3 is wrong because geometric rules are not applicable for log scale.

$$\frac{23}{74} \quad x^2 + y^2 + 2x \frac{m^2}{m^2-1} + \frac{m^2}{m^2-1} = 0 \quad \dots \dots \dots \text{(i)}$$

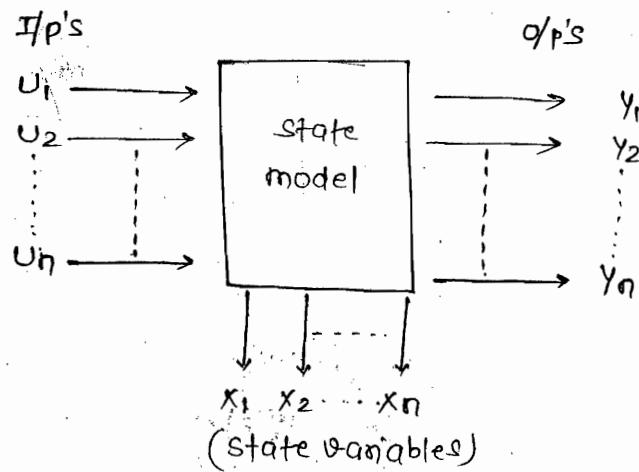
$$x^2 + y^2 + 2.25x + 1.125 = 0$$

$$\frac{m^2}{m^2-1} = 1.125$$

$$\boxed{m=3}$$

$\frac{25}{74}$  (q.)

Chapter-(05)  
State Space Variables



(1) State eqn.  $\rightarrow$

$$\dot{X}(t) = AX(t) + BU(t)$$

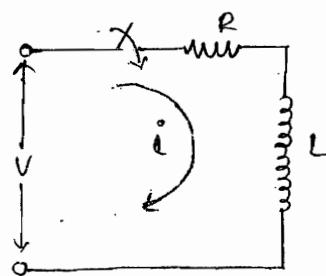
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

(2) O/p eqn.  $\rightarrow$

$$Y(t) = CX(t) + DU(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

Ex:-



$$\text{at } t=0^-$$

$$i(0)^- = i_L(0)^- = 0 \text{ Amps}$$

$$\text{At } t=0^+$$

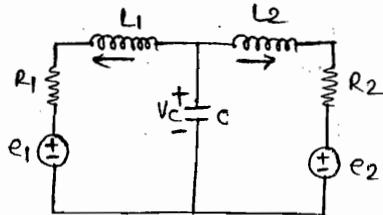
$$i_L(0)^+ = \frac{1}{L} \int_{t=0}^t V dt = 0 \text{ Amps}$$

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{V}{L} \quad (\text{A/S})$$

state eqn of N/w.

Conv(L)  
80



Loop-(1)

$$L_1 \frac{di_1}{dt} + i_1 R_1 + e_1 - V_C = 0$$

$$\frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 + \frac{V_C}{L_1} - \frac{e_1}{L_1} \quad \dots \text{(i)}$$

Loop(2)

$$L_2 \frac{di_2}{dt} + i_2 R_2 + e_2 - V_C = 0$$

$$\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 + \frac{V_C}{L_2} - \frac{e_2}{L_2} \quad \dots \text{(ii)}$$

KCL at Node Vc  $\rightarrow$

$$i_1 + i_2 + C \frac{dV_C}{dt} = 0$$

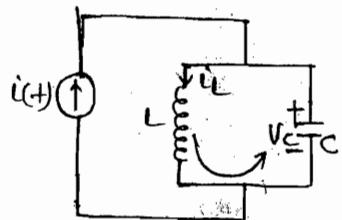
$$\frac{dV_C}{dt} = -\frac{i_1}{C} - \frac{i_2}{C} \quad \dots \text{(iii)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ 0 \end{bmatrix}$$

Let O/p  $y = i_1$

$$y = [1 \ 0 \ 0] \begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix} + 0$$

(b.)



$$i(t) = i_L + i_C$$

$$\dot{i}(t) = \dot{i}_L + \frac{C dV_C}{dt}$$

$$\frac{dV_C}{dt} = -\frac{\dot{i}_L}{C} + \frac{\dot{i}(t)}{C} \quad \text{--- (i)}$$

$$L \frac{di}{dt} - V_C = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_C}{L} \quad \text{--- (ii)}$$

### \* Type-(I) Problem →

To obtain state model from differential eqn

$$\frac{d^3y}{dt^3} + 4 \frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 10y = 20u$$

$$[s^3 + 4s^2 + 6s + 10] Y(s) = 20 U(s)$$

Let  $y = x_1$

$$\frac{dy}{dt} = \overset{\circ}{x}_1 = x_2 ; \quad \frac{d^2y}{dt^2} = \overset{\circ}{x}_2 = x_3 ; \quad \frac{d^3y}{dt^3} = \overset{\circ}{x}_3$$

$$\overset{\circ}{x}_1 = x_2$$

$$\overset{\circ}{x}_2 = x_3$$

$$\overset{\circ}{x}_3 = 20u - 4x_3 - 6x_2 - 10x_1$$

$$\begin{bmatrix} \overset{\circ}{x}_1 \\ \overset{\circ}{x}_2 \\ \overset{\circ}{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + 0$$

**BUSH / COMPANION FORM**

\* Shortcut method for BUSH Form →

(7/80)  $\frac{Y(s)}{U(s)} = \frac{1}{s^4 + 5s^3 + 8s^2 + 6s + 3}$  Reverse order with rev. sign

$$\begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \ddot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

constant numerator.

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Que. →  $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = \frac{2du}{dt} + u$

Sol'n →  $(s^2 + 7s + 9) Y(s) = (2s + 1) U(s)$

$$\frac{Y(s)}{U(s)} = \frac{(2s+1)}{s^2 + 7s + 9}$$

Que.(8) phase variable method →

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 7s + 9} \quad (2s+1) \quad \leftarrow \text{only rev order *}$$

$\leftarrow$  Rev order with rev sign

$$\begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} * & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Ex:-

$$\frac{Y(s)}{U(s)} = \frac{10(s^2 + 2s + 4)}{s^3 + 6s^2 + 8s + 12}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = [4 \quad 2 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

\* Type(2) problem →

- To obtain TF from state model.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (i)$$

$$y(t) = cx(t) + du(t)$$

Applying LT on eqn (i)

$$sx(s) = ax(s) + bu(s)$$

$$-x(0)$$

$$y(s) = cx(s) + du(s)$$

for TF:  $x(0) = 0$

$$sx(s) - ax(s) = bu(s)$$

$$(sI - A)x(s) = Bu(s)$$

$$x(s) = (sI - A)^{-1}Bu(s)$$

$$y(s) = [c(sI - A)^{-1}B + D]u(s)$$

$$\frac{y(s)}{u(s)} = c(sI - A)^{-1}B + D$$

(Transfer matrix)

$$Q(1) \rightarrow \overset{\circ}{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

Sagar 1986  
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$$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T \cdot X$$

$$\begin{aligned} \text{Soln} \rightarrow SI - A &= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} (s+1) & 0 \\ 0 & (s+2) \end{bmatrix} \end{aligned}$$

$$\text{adj}(SI - A) = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$|(SI - A)| = (s+1)(s+2)$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$(SI - A)^{-1} \cdot B = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C(SI - A)^{-1} \cdot B \Rightarrow [1 \quad 1] \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} Y(s) \\ U(s) \end{bmatrix} = \frac{1}{s+1}}$$

$$Q(2) \quad \overset{\circ}{X}(t) = -2X(t) + 2U(t)$$

$$Y(t) = 0.5X(t)$$

$$SX(s) = -2X(s) + 2U(s)$$

$$(s+2)X(s) = 2U(s)$$

$$X(s) = \frac{2}{s+2}U(s)$$

$$Y(s) = 0.5X(s)$$

$$Y(s) = 0.5 \times \frac{2}{s+2} U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s+2}}$$

\* Type(3.) Problem  $\rightarrow$  stability for sm  $\rightarrow$

$$\begin{aligned}\frac{Y(s)}{U(s)} &= C[S\mathbf{I} - A]^{-1}B + D \\ &= C \cdot \frac{\text{Adj}(S\mathbf{I} - A) \cdot B}{|S\mathbf{I} - A|} + D\end{aligned}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{C \cdot \text{Adj}(S\mathbf{I} - A) \cdot B + |S\mathbf{I} - A| \cdot D}{|S\mathbf{I} - A|}}$$

$$\text{Zeros} \rightarrow C \cdot \text{Adj}(S\mathbf{I} - A) \cdot B + (S\mathbf{I} - A) \cdot D$$

$$\text{Poles} \rightarrow (1 + G(s) \cdot H(s) = 0) = |S\mathbf{I} - A| = 0$$

Eigen values of  $= CL$  poles  
sys. matrix [A]

Cond(2)  
80

$$\overset{o}{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \gamma$$

$$C = \begin{bmatrix} -17 & -5 \end{bmatrix} X + [1] \gamma$$

8010  $\rightarrow$

$$\begin{aligned}(S\mathbf{I} - A) &= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \\ &= \begin{bmatrix} S & -1 \\ 20 & S+9 \end{bmatrix}\end{aligned}$$

$$\text{Adj}(S\mathbf{I} - A) = \begin{bmatrix} S+9 & 1 \\ -20 & S \end{bmatrix}$$

$$\text{Adj}(sI - A) \cdot B \Rightarrow \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C \cdot \text{Adj}(sI - A) \cdot B \Rightarrow \begin{bmatrix} -17 & -5 \\ 0 & s \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} = -17 - 5s$$

$$|sI - A| = s(s+9) + 20$$

$$|sI - A| = (s^2 + 9s + 20)(1)$$

$$\text{Zeros} \Rightarrow -17 - 5s + s^2 + 9s + 20 = 0$$

$$s^2 + 4s + 3 = 0$$

$$s = -1, -3$$

$$\text{Poles} \Rightarrow |sI - A| = 0$$

$$s^2 + 9s + 20 = 0$$

$s = -4, -5$  (sys. is stable because of -ve eigen values)

Que.(6)  
80

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad u = [-0.5 \ -3 \ -5] \vec{x} + v$$

Ans 10

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \underbrace{[-0.5 \ -3 \ -5] \vec{x}}_{3 \times 3} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

The sum of principle diagonal = sum of eigen elements  
values

ans (a)

\* Type (04) Diagonalization  $\rightarrow$

$$\ddot{X} = AX + BU$$

$$Y = CX + DU$$

Let  $X = PZ \Rightarrow$  transformation matrix

$$\ddot{Z} = APZ + BU$$

$$Y = CPZ + DU$$

$$\boxed{\begin{aligned}\ddot{Z} &= [P^T A P] Z + [P^T B] U \\ Y &= CPZ + DU\end{aligned}}$$

$$P = \text{Vandermonde matrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_n \\ z_1^2 & z_2^2 & \dots & z_n^2 \end{bmatrix}$$

$P^T A P \Rightarrow$  diagonal matrix with diagonal elements as eigen values.

Ex:  $\rightarrow$  Diagonalize

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$P^T = \frac{\text{adj}(P)}{|P|} = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}$$

$$P^T A P = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$|SI - A| = s(s+5) + 6 = 0$$

$$s^2 + 5s + 6 = 0$$

$$s = -2, -3$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

## \* TYPE (05) Controllability & observability →

203

Controllability → To control the state variables

Observability → To measure the state variables.

### KALMAN'S Test →

$$Q_C = [B \ AB \ A^2B \ \dots \ A^{n-1}B] = |Q_C| \neq 0$$

$$Q_0 = [C^T \ A^T C^T \ (A^T)^2 C^T \ \dots \ (A^T)^{n-1} C^T] = |Q_0| \neq 0$$

Q.1 →  $\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

$$y = [1 \ 1]x$$

Soln →  $Q_C = [B \ AB]$

$$Q_0 = [C^T \ A^T C^T]$$

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = -1$$

$$Q_0 = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4$$

Q.10)  $\dot{x} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \quad y = [b \ 0]x$

Soln →  $A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$$Q_0 = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix} = 2b^2 \neq 0$$

ans.(c)

\* Type (c) Solution of state eqn  $\rightarrow$

$$\dot{X}(t) = AX(t) + BU(t)$$

(a) Free Response  $\rightarrow$   $U(t) = 0$

$$\dot{X}(t) = AX(t)$$

$$\dot{X}(t) - AX(t) = 0 \quad \dots \dots \dots \text{(i)}$$

$$X(t) = Ke^{At} \quad \dots \dots \dots \text{(ii)}$$

Applying LT to eqn (ii)

$$SX(s) - X(0) - AX(s) = 0$$

$$SX(s) - AX(s) = X(0)$$

$$(SI - A) X(s) = X(0)$$

$$X(s) = (SI - A)^{-1} \cdot X(0)$$

$$\phi(s) = (SI - A)^{-1} = \text{Resolvent matrix}$$

$$X(t) = [e^{-At}] (SI - A)^{-1} X(0)$$

$$X(t) = e^{At} \quad K$$

$$\boxed{\phi(t) = e^{At} = [e^{-At}] (SI - A)^{-1}}$$

state transition matrix

(b) forced Response  $\rightarrow$

$$SX(s) - X(0) = AX(s) + BU(s)$$

$$SX(s) - AX(s) = X(0) + BU(s)$$

$$(SI - A) X(s) = X(0) + BU(s)$$

$$X(s) = (SI - A)^{-1} X(0) + (SI - A)^{-1} BU(s)$$

$$X(t) = [e^{-At}] X(0) + [e^{-At}] [(SI - A)^{-1} BU(s)]$$

Ques. (3) →  $\overset{\circ}{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Soln →

ZIR (zero impulse response)  
short cut method  
At  $t=0$   
 $X(t) = X(0)$

$$SI - A \Rightarrow \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (s-1) & 0 \\ 0 & (s-1) \end{bmatrix}$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ 0 & \frac{1}{(s-1)} \end{bmatrix}$$

$$e^{At} = \phi(t) = e^{-t}(SI - A)^{-1}$$

$$S.T.M. \Rightarrow \begin{bmatrix} e^t & 0 \\ t e^t & e^t \end{bmatrix}$$

at  $t=0$

$$X(t) = X(0), e^{A(0)} = I$$

Property of STM  
 $A \cdot t = 0, e^{A \cdot 0} = I$

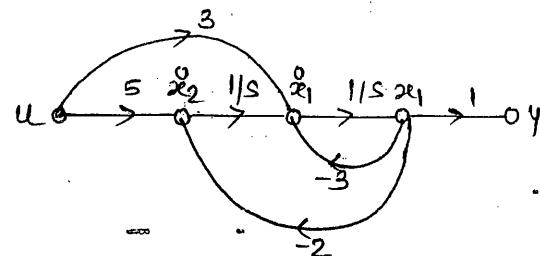
$$X(t) = \begin{bmatrix} e^t & 0 \\ t e^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ t e^t \end{bmatrix}$$

Ans. (c)

\* TYPE(7) state diagrams →

(1). physical variable form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2} = \frac{\frac{3}{s} + \frac{5}{s^2}}{1 - \left[ \frac{-3}{s} - \frac{2}{s^2} \right]}$$



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = -2x_1 + 5u$$

$$y = x_2$$

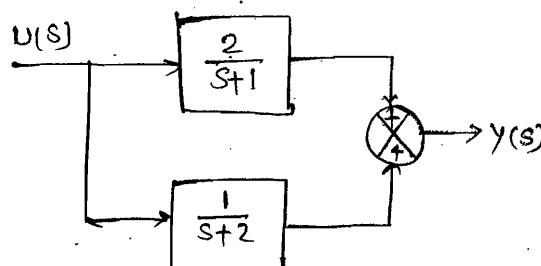
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$$

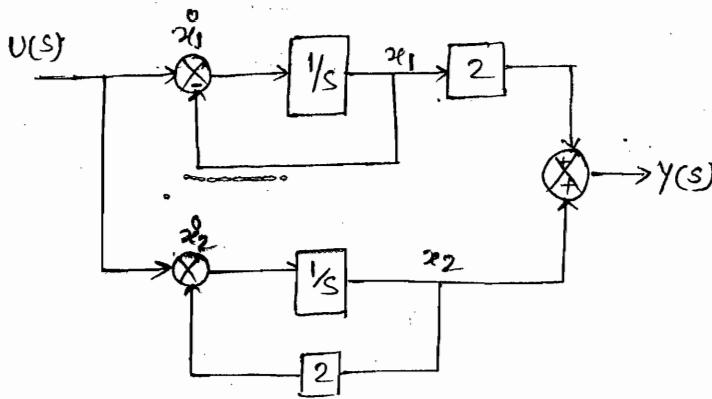
$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(2). Canonical variable form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{(s+1)(s+2)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$Y(s) = \frac{2U(s)}{s+1} + \frac{U(s)}{s+2}$$





$$\dot{x}_1 = -x_1 + u$$

$$\dot{x}_2 = 2x_2 + u$$

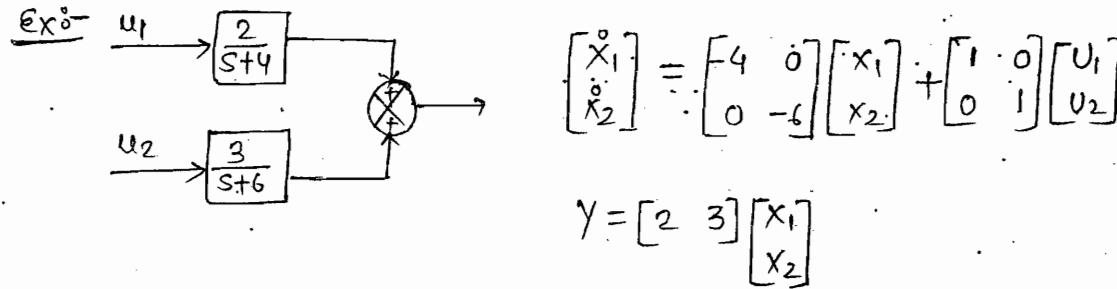
$$\dot{y} = 2x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

Unit matrix

$$y = [2 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Residues  
OF PF's



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