Documentation Differential equation Algae population.

Simplified Situation.

Dimensions

The current used dimensions are

$Mixed\ layer\ dept$	$M(t)$	m
$Nutrients\ concentration$	$N(t)$	$mmol/m^2$
$Phytoplankton$	$P(t)$	$mmol/m^2$
$Herbivore$	$H(t)$	$mmol/m^2$

Constants

$Deep\ nutrients$	N_0	$mM\ m^{-3}$	10
$Uptake\ half\ saturation$	j	$mM\ m^{-3}$	0.5
$Plant\ metabolic\ loss$	r	d^{-1}	0.07
$Grazing\ threshold$	P_0	$mM\ m^{-3}$	0.1
$Grazing\ half\ saturation$	K	$mM\ m^{-3}$	1.0
$Maximum\ grazing\ rate$	c	d^{-1}	1.0
$Grazing\ efficiency$	f	$-$	0.5
$Loss\ to\ carnivores$	g	d^{-1}	0.07
$Low\ ligth\ photosynthetic\ slope$	α	$(ly)^{-1}$	0.04
$Diffusion\ rate$	m	md^{-1}	3

Differential equation

$$\frac{d}{dt}\begin{bmatrix} M \\ N \\ P \\ H \end{bmatrix} = \bar{f}(\bar{w}, t)$$

For simplification $\bar{f}$ is split up in understandable quantities in the code.

$$\bar{f}(\overline{w},t) = layerdeptchange + NPtransfere + Concequencesduetolayerdeptchange \\ + PHtransfere + losstoCarnifores$$

$\boldsymbol{layerdeptchange}$

the change of the layer dept is defined as $\xi(t)$.

$$layerdeptchange = \begin{bmatrix} \xi(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\xi(t)$ is created such that Figrue1b from A Model of Annual Plankton Cycles is recreated

$\boldsymbol{N \rightarrow P\ transfere}$

The Nutrients concentration is transferred to the python plankton due to eating it.

$$transfererate = \left(\frac{\alpha(t,M,P) * N}{j+N} - r\right) * P$$

$$NPtransfere = \begin{bmatrix} 0 \\ -transfererate \\ transfererate \\ 0 \end{bmatrix}$$

$growfactor\ \ \boldsymbol{\alpha(t)}$

The Phytoplankton grows at photosynthetic slope rate α. In this model α is varied by t.
In this model $\alpha(t)$ is defined as

$$\alpha(t) = 0.4 + 0.3 * \cos\left(\frac{2\pi}{365.25}t\right)$$

$\boldsymbol{Concequences\ due\ to\ layer\ dept\ change}$

The changing layer mixed layer dept can give the pythoplankton more space to spread but it can also throw living puthonplankton into the unlivable lower layer. If this happens the pythoplankton changes back into nutrients.

$$Concequences\ due\ to\ layer\ dept\ change = \begin{bmatrix} 0 \\ \frac{\max(\xi(t),0)}{M}(N_0 - N) \\ -\frac{\max(\xi(t),0)}{M} * P \\ -\frac{\xi(t)}{M} * H \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{m}{M}(N_0 - N) \\ -\frac{m}{M} * P \\ 0 \end{bmatrix}$$

$\boldsymbol{P \rightarrow H\ transfere}$

The transfer of nutrients from the concentration Phytoplankton to the concentration herbivores is given by.

$$transferrate = c \cdot \frac{P(t) - P_0}{k + P(t) - P_0}$$

$$PHtransfere = \begin{bmatrix} 0 \\ 0 \\ -transferrate \cdot H(t) \\ transferrate \cdot f \cdot H(t) \end{bmatrix}$$

$losstoCarnifores$

The Herbivores are lost to carnivores linearly by factor g.

$$losstoCarnifores = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g * H \end{bmatrix}$$

Thus the differential equation is given by:

$$\bar{f}(\bar{w}, t) = \begin{bmatrix} \xi(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \left(\frac{\alpha(t, M, P) * N}{j + N} - r \right) * P \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{m + \max(\xi(t), 0)}{M} (N_0 - N) \\ -\frac{m + \max(\xi(t), 0)}{M} * P \\ -\frac{\xi(t)}{M} * H \end{bmatrix}$$
$$+ \frac{P(t) - P_0}{k + P(t) - P_0} \begin{bmatrix} 0 \\ 0 \\ -c \cdot H(t) \\ c \cdot f \cdot H(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g * H \end{bmatrix}$$