# Lecture 6: Quantum oracles & Recent experiments with quantum computers

Quantum Algorithms

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# Outline

Information about the exam

Implementing a quantum oracle

Recent experimental breakthroughs with quantum computers

- Presentation of a superconducting qubit quantum computer
- The quantum supremacy experiment (Arute et al, 2019)
- The toric code experiment (Satzinger et al, 2021)

# Information about the exam

- Room D420 Jan 17th — 1:30 pm - 3:30 pm (2 hours).
- Two problems.
- Only the documents accessible on the Moodle (slides, exercices), and hand-written notes allowed.
- You can bring your laptop, but only to visualize the slides, not to use Internet.
- Use of smartphones/ipads, etc during the exam forbidden.

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# Grover's oracle: Reminder

- Problem given a $n$-bit Boolean function $f(x)$ . Find the solution $x = w$ such that $f(w) = 1$
- The quantum algorithm is given by

$$U = (U_{\mathrm{diffuser}} U_{\mathrm{oracle}})^t H^{\otimes n} \tag{1}$$

## Grover's oracle: Reminder

- Grover's algorithm converges to the solution for $t \propto \sqrt{N = 2^n}$.
- The diffuser $U_{\text{diffuser}} = 2 \left|\psi\right\rangle \left\langle\psi\right| - 1$ can be implemented with the Toffoli gate
- How can we implement an oracle $U_{\text{oracle}} \left|x\right\rangle = (-1)^{f(x)} \left|x\right\rangle$? (without knowing the solution $w \ldots$)?
- Let us show that this possible using elementary bitstring operations and the technique of 'uncomputation'.

## Case study: 3-SAT Problem

- 3-SAT: Given n bits, we are looking for the bit strings $x = (x_1, \ldots, x_n)$ that satisfy the Boolean function

$$f(x) = C_1(x) \wedge \cdots \wedge C_M(x), \tag{2}$$

  where $\wedge$ is a conjunction (AND).
- Each clause $C_m(x)$ is made of a disjunction (OR: $\vee$) of at most three litterals.
- Example with $M = 3, n = 2$

$$f(x) = (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2) \tag{3}$$

- 3-SAT is NP-Complete.

# Implementing a Grover's oracle for 3-SAT

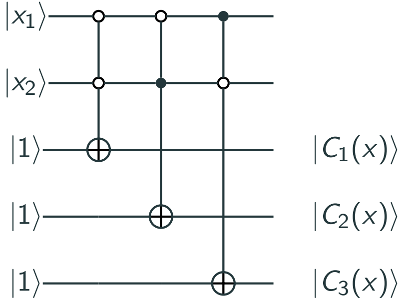
- Oracle $U_{\text{oracle}} |x\rangle = (-1)^{f(x)} |x\rangle$ with $f(x) = (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2)$
- Step 1: Convert each clause to conjunctions (de Morgan's law)

$$f(x) = \neg(\neg x_1 \wedge \neg x_2) \wedge \neg(\neg x_1 \wedge x_2) \wedge \neg(x_1 \wedge \neg x_2) \tag{4}$$

# Implementing a Grover's oracle for 3-SAT

- Step 2: Add one ancilla qubit and Tofolli gates to test each clause

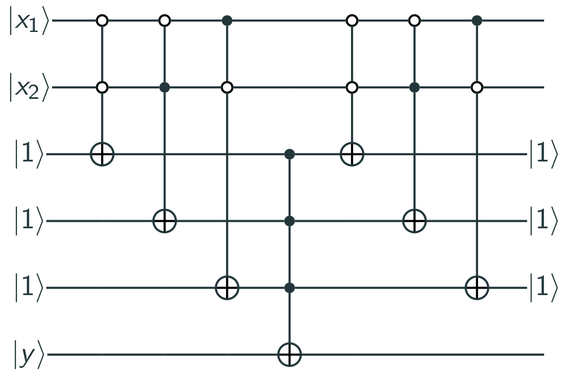
$$|x\rangle \otimes |00\rangle \to |x\rangle \otimes |C_1(x)\rangle\,|C_2(x)\rangle \tag{5}$$



# Implementing a Grover's oracle for 3-SAT

- Step 3: Add an extra ancilla bit for performing the conjunctions between clauses and 'uncompute' the first ancilla qubits

$$|x\rangle\,|1\rangle^{\otimes M}\,|y\rangle \rightarrow |x\rangle\,|1\rangle^{\otimes M}\,|y \oplus f(x)\rangle \tag{6}$$




- If the last ancilla is flipped, $f(x) = 1$, i.e., we can mark the solution!

# Implementing a Grover's oracle for 3-SAT

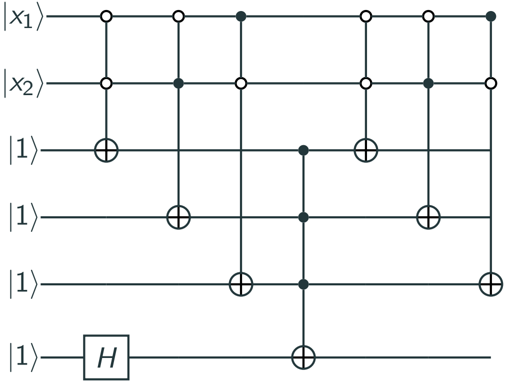
- All ancilla qubits except one have been uncomputed, we have effectively realized a *XOR* oracle

$$|x\rangle \otimes |y\rangle \rightarrow |x\rangle \otimes |y \oplus f(x)\rangle \tag{7}$$

- How to realize a *phase* oracle, as required for Grover's oracle, from a XOR oracle?

$$|x\rangle \rightarrow (-1)^{f(x)} |x\rangle \tag{8}$$

- XOR to phase oracle conversion: Simply initialize the last ancilla using the $H$ gate



# Implementing a Grover's oracle for 3-SAT

- We obtain

$$
\begin{aligned}
|x\rangle\,|1\rangle^{\otimes M}\,(H\,|1\rangle) \;&\rightarrow\; |x\rangle\,|1\rangle^{\otimes M}\,(|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle) \\
&\rightarrow\; |x\rangle\,|1\rangle^{\otimes M}\,(-1)^{f(x)}(|0\rangle - |1\rangle) \\
&\rightarrow\; (-1)^{f(x)}\,|x\rangle\,|1\rangle^{\otimes M}\,(H\,|1\rangle)
\end{aligned} \tag{9}
$$

- which effectively realizes our oracle (and uncomputes the last ancilla)!

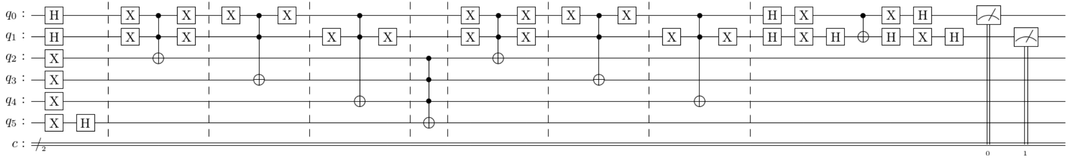
$$
|x\rangle \;\rightarrow\; (-1)^{f(x)}\,|x\rangle \tag{10}
$$

# The full circuit

- Problem function:

$$f(x) = (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2) \tag{11}$$

- Circuit




- Qiskit output: bistring 11 with probability 1.0: I coded the problem without knowing the solution, then the algorithm gives me the solution.

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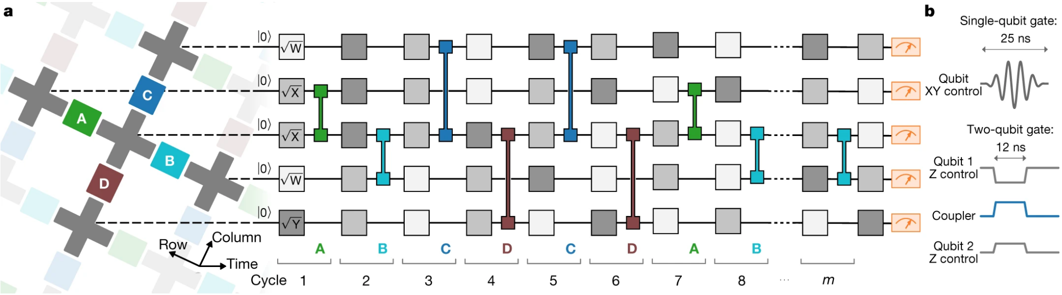
## How does it work?

- Superconducting material+Josephson junctions
  $\rightarrow$ 53 'anharmonic oscillators' $h_i = \omega_0 a_i^\dagger a_i + U a_i^\dagger a_i (a_i^\dagger a_i - 1) + \dots$
  $\rightarrow$ Reviews by Devoret, Girvin, etc
- Cool to 20 mK temperature via a dilution fridge
- We can control the qubit $|0\rangle_i, |1\rangle_i = a_i^\dagger |0\rangle_i$ with 'single qubit gates'.
- iSwap gates between qubits $U_{i,j} = e^{i(\pi/4)(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)}$
- Each qubit can be measured.
- We have a universal quantum computer: Every $N$-qubit state (dim $2^N$) can be created and measured (Deutsch 1989).

$$|\psi\rangle = \sum_{s_1,\dots,s_N} c_{s_1,\dots,s_N} |s_1\rangle \otimes \dots \otimes |s_N\rangle \tag{12}$$

# Programming a quantum computer



## The quantum computing roadmap

- Long-terms goals: Quantum algorithms to solve hard problems
  - Data search (Grover's 1996)
  - Factorization (Shor's 1995)
  - Large-scale optimization problems (on-going debate)
- Outstanding conceptual/technological challenges
  - Error propagation is typically exponential in problem size
  - Sophisticated algorithms for 'quantum error correction' required to achieve fault tolerance.
- Massive investments (USA, China, Europe, etc) are helpful, but should not hide conceptual challenges ('quantum hype').
- We need verification methods to validate technological progress

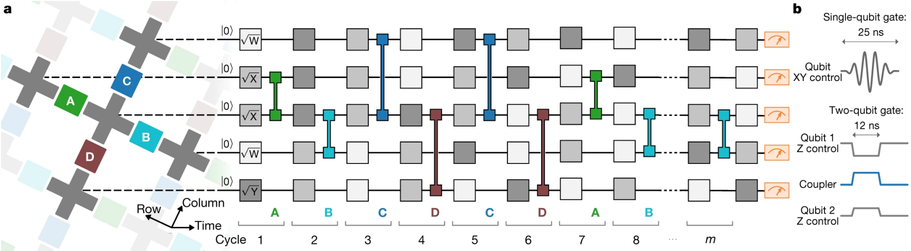
# What is this about?

- It will probably take years before we can run a quantum algorithm solving an open problem in computer science.
- An important milestone is to check that a quantum computer is able to perform a computation that is not achievable via a classical computer with reasonable resources (Preskill 2012).
- Two important conceptual questions
  - How can I check something that I cannot compute?
  - How can I explore an gigantic Hilbert space of dimension $2^{53} \sim 10^{15}$ with a finite number of measurements?

# The supremacy test

## Idea

- Use random circuits that are difficult to simulate classically for $N > 50$
- Make use of random matrix theory to create a faithful figure of merit, based on measuring the system in terms of 'bitstrings' (ex $s = 01001\ldots$).



# The supremacy test

## Algorithm

1. Consider a reduced or 'shallow' circuit $U$ that I can compute classically.
   1.1 Implement $|\psi\rangle = U|0\rangle^{\otimes N}$
   1.2 Measure $M \ll 2^N$ bitstrings $\{s_m\}$ (sampled ideally according to $P(s) = |\langle s|\psi||\rangle^2$).
   1.3 Evaluate the fidelity based on computing

$$\mathcal{F}_{\text{XEB}} = \frac{2^N}{M}\sum_{m=1}^{M} P(s_m) - 1 \tag{13}$$

2. Consider a non-simulatable 'supremacy' circuit $U$
   2.1 Measure the bitstrings $\{s_m\}$ as above
   2.2 Archive the results, waiting for the classical computer to 'catch up' and allow for the evaluation of the fidelity.

# The test is meaningful

- For sufficiently large M,

$$\mathcal{F}_{\mathrm{XEB}} = \frac{2^N}{M}\sum_{m=1}^{M} P(s_m) - 1 = 2^N \sum_{s=0}^{2^N-1}\sum_{m=1}^{M}\frac{\delta_{s,s_m}}{M} P(s) - 1 \approx 2^N \sum_{s=0}^{2^N-1} P(s)^2 - 1 \quad (14)$$

- For a random circuit this sum can be calculated analytically.

# The test is meaningful

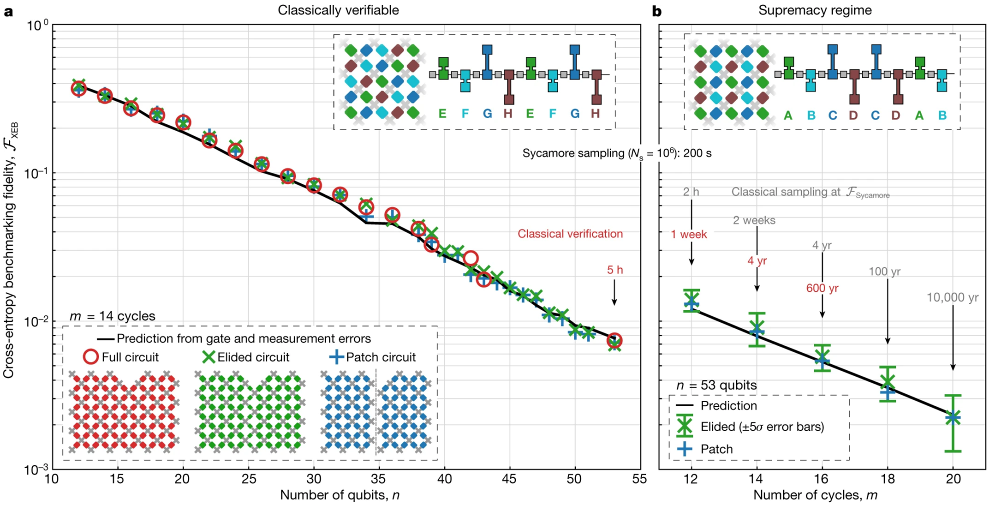
For sufficiently large $N$,

$$\mathcal{F}_{\mathrm{XEB}} \approx 2^N \sum_{s=0}^{2^N-1} P(s)^2 - 1 \approx 2^{2N} \langle P(s)^2 \rangle - 1 \tag{15}$$

$$\mathrm{Prob}(P(s) \in [p, p+dp]) = 2^N exp(-2^N p) dp \tag{16}$$

- In average, $\langle P(s) \rangle = 2^{-N}$, as for a uniform distribution $P_{\mathrm{uni}}(s) = 2^{-N}$.
- However $\langle P(s)^2 \rangle = 2 \times 2^{-2N}$, *twice* compared to $P_{\mathrm{uni}}(s)^2 = 2^{-2N}$!
- Therefore, $\mathcal{F}_{\mathrm{XEB}} = 1$. For uniform distribution, we get instead $\mathcal{F}_{\mathrm{XEB}} = 0$.
- Fast convergence of the estimation of $\mathcal{F}_{\mathrm{XEB}}$ with $\sim 10^6$ measurements.

# Results

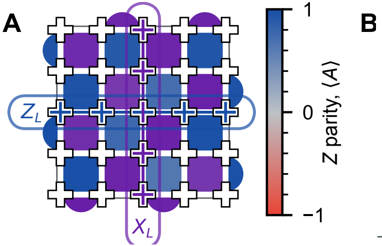

a
Classically verifiable
b
Supremacy regime
Cross-entropy benchmarking fidelity, $\mathcal{F}_{XEB}$
Number of qubits, $n$
Number of cycles, $m$
E F G H E F G H
A B C D C D A B
Sycamore sampling ($N_s = 10^6$): 200 s
Classical verification
5 h
$m$ = 14 cycles
Prediction from gate and measurement errors
Full circuit
Elided circuit
Patch circuit
2 h
Classical sampling at $\mathcal{F}_{Sycamore}$
1 week
2 weeks
4 yr
4 yr
600 yr
100 yr
10,000 yr
$n$ = 53 qubits
Prediction
Elided (±5σ error bars)
Patch

# Conclusion

- A remarkable technological achievement.
- Exponential propagation of errors. This was expected
- The power of classical simulations was underestimated (see eg X. Waintal et al, PRX 2020).

# Sycamore was made for quantum error correction

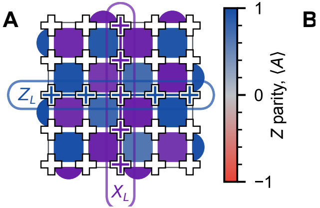
- Each qubit is subject to errors. This can be a spin flip error ($\sigma_x$ on the state), phase error ($\sigma_z$), or both.
- In quantum error correction, we fix errors by encoding the quantum information using several physical qubits.




$\rightarrow$ one *logical* qubit

# The toric code

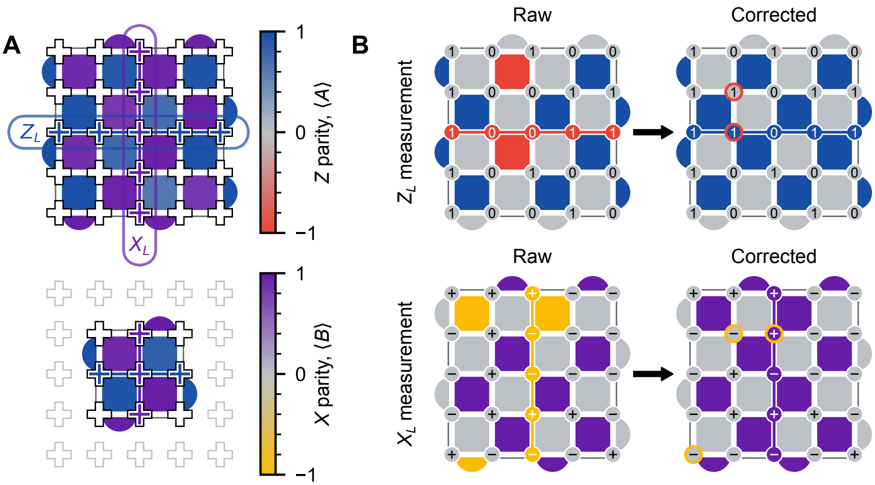
- The toric code $|G\rangle$ is the ground state of the Hamiltonian $H = -\sum_s A_s - \sum_p B_p$ (Kitaev 1997)



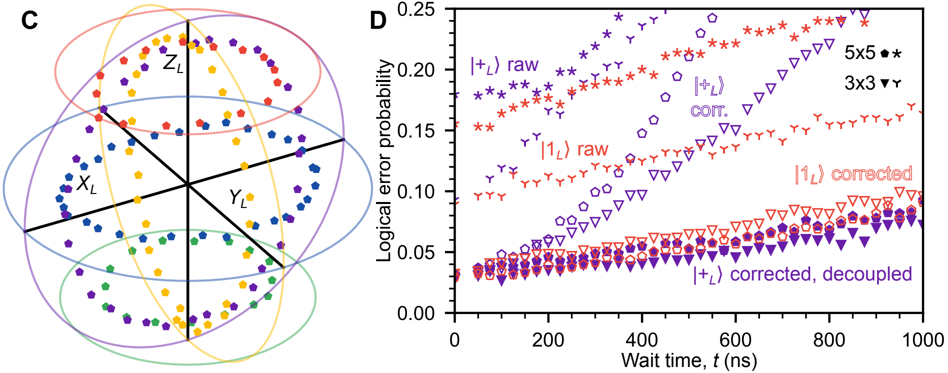

- Blue plaquettes $A_s = \sigma^z_{s_1}\sigma^z_{s_2}\sigma^z_{s_3}\sigma^z_{s_4}$
- Purple plaquettes $B_p = \sigma^x_{p_1}\sigma^x_{p_2}\sigma^x_{p_3}\sigma^x_{p_4}$

- $|0\rangle_L = |G\rangle$ can be created easily with Sycamore. $|1\rangle_L = X_L\,|G\rangle$.
- $A_s\,|G\rangle = |G\rangle$, $B_p\,|G\rangle = |G\rangle$. If it is not the case, an error occured!
- According to the quantum threshold theorem (Aharonov Ben-Or 1999), scaling up the toric code leads to fault-tolerant quantum computing

# Experimental results



# Experimental results



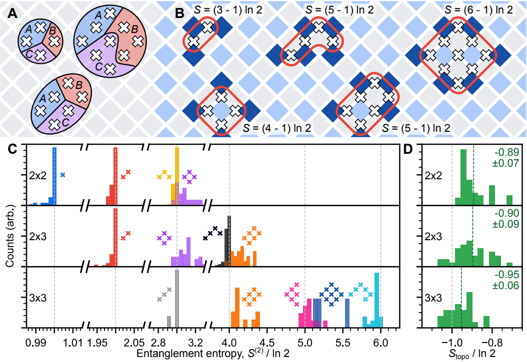
## The toric code

- The toric code is also the simplest example of a quantum phase with 'intrinsic' topological order.
    - Non-local order parameter
    - Stability versus local perturbations
    - The excitations (i.e the errors) on top of $|G\rangle$ have anyonic statistics.
- The non-local order parameter: the topological entanglement entropy (Kitaev, Wen, Levin, Preskill 2006)

$$S_{\mathrm{top}} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC} = -1 \tag{17}$$

with the entanglement entropy of subsystem $X$, $S_X = -\log(\mathrm{Tr}[\rho_X^2])$.

# Results




- First demonstration of intrinsic topological order
- Braiding of anyonic excitations (errors) have been also analyzed.
- Next step: non-abelian anyons in string nets models. . .