Quantum algorithms

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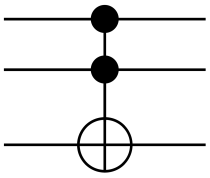
1 Universal reversible classical computing

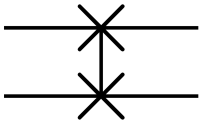
Adapted from Jones/Jaksche, Oxford. The Toffoli gate is universal for reversible classical computing. We will illustrate this result by expressing common gates in terms of the Toffli gate.

1. The Toffoli gate is universal for reversible classical computing.

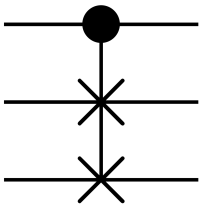


   Write the truth table . Show that this gate is reversible.

2. Write the truth table for a Swap gate.



3. Write the truth table for a Fredkin gate.



4. Write the Swap gate in terms of three CNOT gates.
5. Write a CNOT gate in terms of a Toffoli gate.
6. Write the Swap gate in terms of Toffoli gates.

Optional : Write the Fredkin gate in terms of the Toffoli

2 Universal quantum gates

The set $(H, P, T, CNOT)$ forms a set of universal quantum gates.

1. Recall the matrix and 'bra-ket' expressions of these gates.
2. Prove that H is unitary. Write how the state $|0\rangle$ is transformed.
3. How to implement a Z gate from the set written above?
4. An X gate? A Y gate? A CZ gate? A CY gate?
5. How to create the following Bell state?

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{1}$$

3 Measurements

1. How to measure the expectation value $\langle Z\rangle$ of a single qubit?
2. How to measure $\langle X\rangle$? How to measure $\langle ZZ\rangle$ for two qubits? $\langle ZX\rangle$, $\langle XX\rangle$? Interpret these quantities in terms of correlations between measurements.