Quantum Algorithms 2021/2022: Exercices 3

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1 Measurement of ZZ stabilizers

The error syndromes in the three qubit bit flip code correspond to the measurement of the operators Z_1Z_2 and Z_2Z_3.

1. Explain the physical process underlying the measurement of an operator Z_1Z_2. Why cannot we simply measure the state of each qubit to perform an error syndrome?
2. How can we measure Z_1Z_2 using an ancilla qubit?

2 The three qubit phase flip code

The three qubit phase flip code can correct against qubit phase errors Z_1, Z_2, Z_3. This is achieved via the error syndromes associated with the measurements of the operators X_1X_2 and X_2X_3.

1. Prove that such code is a [3,1] stabilizer code.
2. Show that the code can correct against phase errors.
3. Define the two logical states $|0_L\rangle$, $|1_L\rangle$
4. Write a circuit to encode a logical qubit from a physical qubit $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$.
5. Write a circuit to perform an error syndrome X_1X_2 using an ancilla qubit.

3 Fault tolerance with the surface code

We consider the surface code. We will illustrate the concept of fault tolerance by studying the scaling of false detection of X errors with increasing sizes of the code.

1. Consider a single row of the code of length $d = 5$ (number of white physical qubits)

   

   Describe the state of the system after initialization, and after one X error (bit flip).
2. Suppose the error syndrome step gives $-1, 1, -1, 1$. Gives a possible error assignment with two errors. Show that the complementary error assignement with three errors also explain the error syndrome. Comment.
3. For an arbitrary value of odd d, the most likely undetected errors corresponds to an error of $d_e = (d+1)/2$ qubits (wrongly attributed to the complementary error assignement with $d-(d+1)/2 = (d-1)/2$). Assume a physical qubit error occurs with probability p after one circuit operation. What is the probability p_L for such a logical error as a function of p and d, after one logical cycle? We consider here that a logical cycle is assumed to be of duration 8 circuit operations.
4. Adapt the expression for a logical error for a $2D$ surface code. Plot p_L as a function of p and d. Comment w.r.t the notion of fault tolerance.