# Quantum Algorithms 2020/2021: Exercices 1

Benoît Vermersch (benoit.vermersch@lpmmc.cnrs.fr) -September 13, 2021

## 1   Universal reversible classical computing

Adapted from Jones/Jaksche, Oxford. The Toffoli gate is universal for resersible classical computing. We will illustrate this result by expressing common gates in terms of the Toffli gate.

1. The Toffoli gate is universal for reversible classical computing.
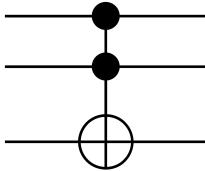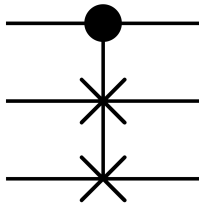
   

   Write the truth table and the corresponding matrix representation. Show that this gate is reversible.

2. Write the truth table for a Swap gate.
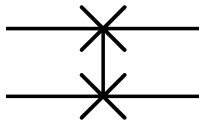
   

3. Write the truth table for a Fredkin gate.

   

4. Write the Swap gate in terms of three CNOT gates.

5. Write a CNOT gate in terms of a Toffoli gate.

6. Write the Swap gate in terms of Toffoli gates.

## 2   Universal quantum gates

The set of $(H, P, T, CNOT)$ form a set of universal quantum gates.

1. Recall the matrix and tensor-product expressions of these gates.

2. Write the transformation matrix associated with the Hadamard gate $H$. Prove that it's unitary. Write how the state $|0\rangle$ is transformed.

3. How to implement a $Z$ gate from the set written above?

4. An $X$ gate? A $Y$ gate?

5. A $CZ$ gate? A $CY$ gate?

6. How to create the following Bell state?

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{1}$$

# 3 Measurements

1. How to measure the expectation value $\langle Z \rangle$ of a single qubit?

2. How to measure $\langle X \rangle$?

3. How to measure $\langle ZZ \rangle$ for two qubits? $\langle ZX \rangle$, $\langle XX \rangle$? Interpret these quantities in terms of correlations between measurements.