Quantum algorithms: Exercices 2

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1 Implementation of Grover's diffuser operator

Our goal is to design a quantum circuit for Grover's diffuser operator $U_\psi = 2 |\psi\rangle \langle\psi| - 1$, with $|\psi\rangle = 1/\sqrt{N} \sum_x |x\rangle$.

1. Write down a circuit U_1 that prepares $|\psi\rangle$ from $|0\rangle^{\otimes n}$

2. Evaluate U_1^2.

3. We aim at implementing U_ψ as $U_\psi = U_1 U_2 U_1$. Write down the circuit corresponding to U_2.

4. Prove that U_2 can be written as $U_2 = -X^{\otimes n} U_3 X^{\otimes n}$, with U_3 a n-qubit controlled Z gate

5. Write U_3 in terms of the Toffoli gate.

6. Write and represent graphically the full circuit for $-U_\psi$. Comment on the role of the minus sign.