Quantum Algorithms 2021/2022: Exercices 5

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1 Density matrix and quantum state tomography

The density matrix ρ summarizes all the physical properties of a quantum S. For a system S embedded in an environment E, it is defined as

$$\rho = \mathrm{Tr}_E \left|\psi_{SE}\right\rangle \left\langle\psi_{SE}\right|, \tag{1}$$

where Tr_E is the trace over the environment, defined as $\mathrm{Tr}_E(.) = \sum_{i_E} \left\langle i_E\right| . \left|i_E\right\rangle$, and where $\left|\psi_{SE}\right\rangle$ is the combined state of the system and environnement.

1. Calculate ρ when the system is decoupled from the environnement, i.e., $\left|\psi_{SE}\right\rangle = \left|\psi_S\right\rangle \otimes \left|\psi_E\right\rangle$. Describe the physical meaning of this situation when S is a quantum computer.
2. Let us define an observable O acting on the system, i.e $O = O_S \otimes 1$. Write the expression of the expectation value $\langle O\rangle$ as a function of ρ.
3. Write the evolution of a density matrix via a unitary operation, i.e gate, U?
4. Quantum state tomography describes a protocol to measure the matrix ρ in a quantum computer. It is based on decomposing ρ is a basis of Pauli strings.

$$\rho = \sum_\sigma c_\sigma \sigma \tag{2}$$

   with $\sigma = \bigotimes_i \sigma_i$, $\sigma_i = 1_i, X_i, Y_i, Z_i$. Write the expression of c_σ as a function of ρ and σ.
5. Write a quantum circuit to measure c_σ. We recall the identities $X = HZH$, $Y = SXS^\dagger = SHZHS^\dagger$.