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Trajectory Control of a Quadrotor Subject to 2D Wind Disturbances

Robust-Adaptive Approach

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Abstract The paper addresses the flight control of a quad-rotor subject to two dimensional unknown static/varying wind disturbances. A model separation is proposed to simplify the control of the six-degrees-of-freedom (6DOF) nonlinear dynamics of the flying robot. Such approach allows to deal with quad-rotor's 3D-motion via two subsystems: dynamic (altitude and MAV-relative forward velocity) and kinematic (nonholonomic-like navigation) subsystems. In terms of control, a hierarchical control is used as the overall control structure to stabilize the kinematic underactuaded subsystem. A control strategy based on sliding-mode and adaptive control techniques is proposed to deal with slow and fast time-varying wind conditions, respectively. This choice not only provides well tracking control but also improves the estimation of unknown disturbance. The backstepping technique is used to stabilize the inner-loop heading dynamics, such recursive design takes into account a constrained heading rate. Promising simulations results show the validity of the proposed control strategy while tracking a time-parameterized straight-line and sinusoidal trajectory.

Keywords Adaptive control · Sliding-mode control · Wind-tolerant flight · Trajectory tracking

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1 Introduction

Nowadays, the successful insertion of miniature aerial vehicles (MAVs) in the civilian and commercial fields is a consequence of the wide application range of these kind of flying robots. The variety of system configurations and flight capabilities of MAVs allows to cope with a diverse scenarios imposed by real-world missions. The application scope ranges from homeland security (customs and borders protection, coast guard), energy industry (surveillance of gas or oil pipelines, monitoring of nuclear facilities), damage assessment (earthquakes, floods) until weather forecasting (storms monitoring, air sampling). The majority of these tasks evolves outdoors exposing the vehicle to adverse atmospheric conditions that deteriorates the performance of navigation. For this reason, more reliable and robust control algorithms are required to achieve the motion control objective. Indeed, in air navigation the unknown wind patterns represent a major issue causing significant deviations in the path or trajectory of the MAV.

Previous research works related to the motion control of MAVs subject to wind disturbances, have particulary focused on fixed-wing configurations, whose flight conditions assumes unknown constant wind speeds up to 50 % of MAV airspeed. As in [1], where the authors use a linear quadratic regulator for optimal guidance of a fixed-wing MAV. The controller, evaluated at simulation level, aims to follow a straight-line and circular paths under different constant wind values. In [2], it is experimentally implemented a path-following controller using constant-velocity MAV under moderate wind conditions (20–50 % of MAV airspeed). McGee and Hedrick [3] presents presented a path planning and control algorithms meant to survey multiple waypoints while considering heading a constrained rate heading. The path planning is treated as an optimization problem and assumes the knowledge of the constant wind component, while the sliding surface controller is aimed at dealing with small time varying wind components. A nonlinear controller is presented in [4] regarding to the following of a straight-line reference considering various constant lateral wind components (crosswind) up to 40 % of the MAV speed.

Related works addressing the flight control of quad-rotor MAVs under wind conditions, have approached the problem at a dynamic level. That is to say, considering or including wind forces (unknown external force) in the overall dynamics, either translational or rotational. As in [5], where a robust control of the position of a quadrotor is presented. Robust behavior is achieved via a disturbance observer that encompasses not only wind gusts but also the nonlinear terms in a total disturbance force, this leads to obtain linear dynamics treated through standard PID controllers. Likewise, [6] focus on robust position control of a quad-rotor by modeling the wind influence, i.e. Dryden Wind Gust Model, on the quadrotor's dynamic behavior aiming at the estimation of the in-flight wind velocities. On the other hand, authors of [7] have dealt with the motion control problem through a hybrid backstepping controller that includes the desired acceleration obtained with the Frenet-Serret theory (Backstepping+FST). Simulations have shown that such control design provides a robust flight face to wind disturbance (0.8 and 0.5 m/s) at moderate rotorcraft veolocities (2 m/s) during a target-tracking task.

1.1 Paper Contributions

At modeling level, we aim to simplify the navigation control problem of quad-rotors by splitting the 6DOF dynamics into two parts: a dynamic subsystem, encompassing pitch and altitude dynamics (longitudinal dynamics), and a kinematic subsystem, similar to nonholonomic mobile robots. Indeed, the enhanced flight capabilities of quad-rotors as bidirectional velocity, not possible in fixed-wing MAVs [8], allows to introduce standard kinematics-based control approaches (e.g. [9, 10]). In terms of control, a standard two time-scale hierarchical control structure is used for the overall tracking control of the kinematic subsystem. The quad-rotor's translational motion subject to two-dimensional unknown wind disturbance is treated at the outer-loop layer. At this level, the control design approach aims towards the combination of the advantages provided by adaptive and sliding-mode control techniques. The adaptive control is intended to estimate slow time-varying wind components, while sliding-mode control deals with the unknown-but-bounded time-varying wind gusts. On the other hand, the inner-loop control regards to the stabilization of heading error dynamics generated during the tracking of the commanded orientation of the velocity control vector. In addition, the control design is based in the Backstepping technique and takes into account the heading rate constraint due to the limited response of rotors.

This work is organized as follows: The problem statement containing the modeling strategy is presented in Section 2. Section 3 presents the robust-adaptive control to mitigate the wind disturbance and thus accomplishing the tracking control objective. Numerical simulations results are presented in Section 4. Lastly, the conclusions and perspectives are given in Section 5.

2 Problem Statement

Consider the problem where a quad-rotor tracks a time-parameterized reference path $\zeta(t)$, that is to say, it is required that the vehicle reaches a time-varying spatial point according to a timing law. The two dimensional reference trajectory is assumed two-times differentiable and bounded.

The presence of wind causes an erroneous flight trajectories shifting away the vehicle from the pre-planned path as illustrated by Fig. 1. For this reason, such unknown disturbances must be included in the control design, either via estimation and/or robust techniques. The flight profile of the quad-rotor is not constrained to minimum-positive velocity, as is the case of fixed-wing MAVs (stall angle), in fact it might generate negative velocities. This property makes this air platform ideal for tracking purposes under two-dimensional wind conditions.

2.1 Classical Quadrotor Dynamic Model

Firstly, let us define the orthogonal right-handed reference frames where the overall motion of flying robot evolves:

- $\mathscr{F}^i = (e_x, e_y, e_z)^T$ defines the earth-fixed frame.

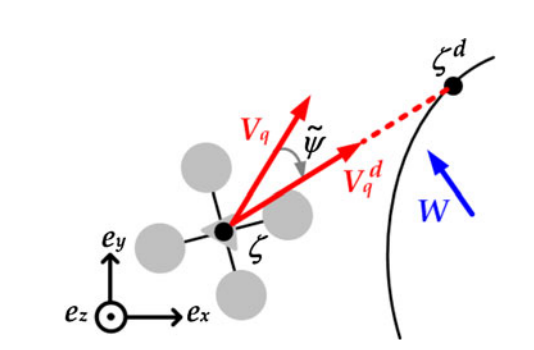


Fig. 1 Trajectory-tracking control problem

- $\mathscr{F}^b = (e_1, e_2, e_3)^T$, defines the body-fixed frame.
- $\mathscr{F}^{\eta_i} = (e_x^{\eta_i}, e_y^{\eta_i}, e_z^{\eta_i})^T$, defines the local frame resulting from a single rotation, where η_1, η_2 and η_3 correspond to roll (ϕ), pitch(θ) and yaw (ψ).

The majority of work related to the control design for quad-rotor MAVs have been based on the 6DOF dynamical model, written as

$$m\dot{\mathbf{V}}^i = \mathscr{R}(\eta)\mathbf{T}^b + m\mathbf{G}^i \tag{1}$$

$$I\dot{\Omega}^b = \tau^b - \Omega^b \times I\Omega^b \tag{2}$$

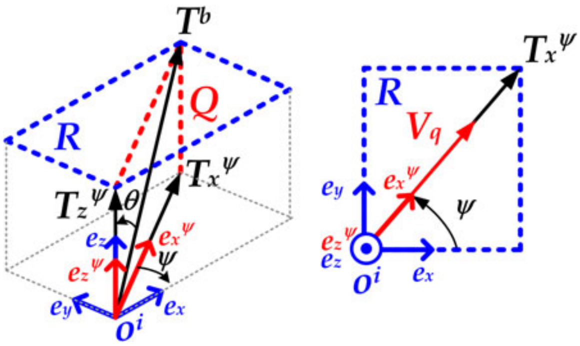
where, $\mathbf{G}^i = (0, 0, -g)$ is the gravitational acceleration vector, $\Omega^b = (P, Q, R)^T$ is the angular rate of the quad-rotor in the body frame, $\xi^i = (x, y, z)^T$, denotes the 3D-position of the UAV relative to the inertial frame, $\eta = (\phi, \theta, \psi)^T$ are the three Euler angles (yaw, pitch and roll), which represent aerial robot's orientation, and $\mathscr{R}^\eta$ is the righthanded orthonormal transformation from the body frame to the inertial frame.

2.2 Quadrotor Dymamic-Kinematic Model

We propose an alternative modeling approach regarding to simplify the navigation control task. Firstly, small angular deviations ($\phi \approx 0$) of rolling motion are assumed (i.e. rolling stable behavior). The approach focuses on the motion subsystems resulting from the two remaining rotations, i.e. pitching and yawing motion. The pitch rotation, $\mathscr{R}(\theta)$, applied to $\mathbf{T}^b$ defines the dynamic subsystem, which allows to control altitude and quad-rotor's relative forward velocity. Whereas, the following rotation, $\mathscr{R}(\psi)$, is applied to the velocity vector defines a kinematic subsystem that allows to address the quad-rotor navigation from a kinematic point of view, i.e. as nonholonomic-like mobile robot (see Fig. 2).

2.2.1 Dynamic Subsystem (Q-plane)

The pitch rotation $\mathscr{R}(\theta)$, about e_2, decomposes the main thrust vector $\mathbf{T}^b$ into two components $\mathbf{T}_x^\psi$ and $\mathbf{T}_z^\psi$, producing forward and vertical motion of the MAV with respect to (w.r.t.) frame $\mathscr{F}^\psi$. These dynamics, translational (Σ_{Q_T}) and



(a) Q-plane (b) R-plane

Fig. 2 3D motion based on two rotations (θ and ψ)

rotational (Σ_{Q_R}), evolve within the Q-plane $(e_x^{\psi} e_z^{\psi})$. Thus, the corresponding vectorial motion equations modeling such dynamics are

$$(\Sigma_{Q_T}) : m\dot{\mathbf{V}}^{\psi} = \mathscr{R}(\theta)\mathbf{T}^b + m\mathscr{R}\left(\psi^T\right)\mathbf{G}^i \tag{3}$$

$$(\Sigma_{Q_R}) : I_y\ddot{\theta} = \tau_\theta \tag{4}$$

The gravity vector $\mathbf{G}^i$ projected to $\mathscr{F}^{\psi}$ (Q-plane) is not modified due to $e_{z^{\psi}} = e_z$ (see Fig. 2a). This allows to account for gravity-based forces (vertical dynamics) within this plane or frame. Thus, Eqs. 3 and 4 are rewritten as

$$(\Sigma_{Q_T}) : m\dot{\mathbf{V}}^{\psi} = T\mathscr{R}(\theta)e_3 - mge_z \tag{5}$$

$$(\Sigma_{Q_R}) : I_y\ddot{\theta} = \tau_\theta \tag{6}$$

where $\mathbf{V}^{\psi} = (\mathbf{V}_q, 0, \mathbf{V}_z)^T$ contains relative forward and vertical quad-rotor velocities, with $\mathbf{V}_q = (v_q, 0, 0)^T$ and $V_z = (0, 0, \dot{z})$.

2.2.2 Kinematic Subsystem (R-plane)

The last rotation $\mathscr{R}(\psi)$, about e_z^{ψ}, heads the Q-plane, as well as the inner dynamics as well, in function of the yaw angle displacements. Thus, we apply the transformation $\mathscr{R}(\psi)$ to the 2D velocity vector $\mathbf{V}_q$ generating the inertial velocity vector $\dot{\zeta} = (\dot{x}, \dot{y})^T$, whose components lies in e_x and e_y (R-plane). Based on the latter, it is then possible to address the 2D motion evolving within R-planes with kinematic approach. The 4th-dimentional equation system representing the kinematic behavior within R-plane are written as

$$(\Sigma_{R_T}) : \dot{\zeta}^i = \mathscr{R}(\psi)\mathbf{V}_q \tag{7}$$

$$(\Sigma_{R_R}) : \ddot{\psi} = \frac{\tau_\psi}{I_z} \tag{8}$$

where $\dot{\zeta}^i = (\dot{x}, \dot{y})^T$ is the translational velocity within the R-plane.

2.3 Disturbed Kinematic Model

The vectorial kinematic model of the quad-rotor considering the nominal model 7 and 8 as well as the disturbance wind vector is written as

$$(\Sigma_{R_T}) : \dot{\zeta}^i = \mathscr{R}(\psi)\mathbf{V}_q + \mathbf{W}^i \tag{9}$$

$$(\Sigma_{R_R}) : \ddot{\psi} = \frac{\tau_\psi}{I_z} \tag{10}$$

The wind vector might be characterized by two vectors:

$$\mathbf{W}^i = \mathbf{W}_o + \Delta_W(t) \tag{11}$$

(a) The static wind vector evolves slowly along the time, it represents the mean value of the overall wind vector.

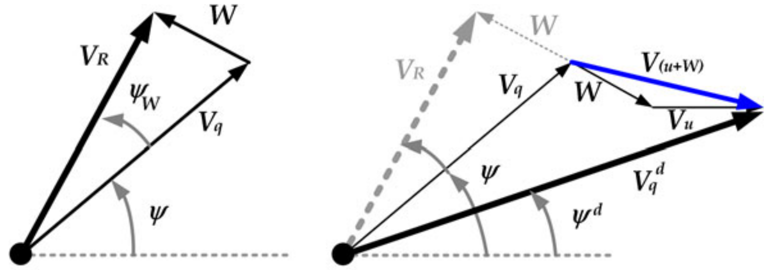
$$\mathbf{W}_o = \begin{pmatrix} w_{x_o} \\ w_{y_o} \end{pmatrix} \tag{12}$$

(b) The non-static vector $(\Delta_W(t))$, or fast time-varying, features higher frequencies and smaller amplitudes in comparison with the static wind vector.

$$\Delta_W(t) = \begin{pmatrix} \Delta_{w_x}(t) \\ \Delta_{w_y}(t) \end{pmatrix} = \begin{pmatrix} a_x \sin(\omega_x t + \varphi_x) \\ a_y \sin(\omega_y t + \varphi_y) \end{pmatrix} \tag{13}$$

where a_i, ω_i and φ_i denote the magnitude, frequency and phase of the non-static wind model [3, 5]. Likewise, it is assumed that that non-static wind and its time-derivative are both bounded, verifying

$$\|\Delta_w\| \leq \beta \tag{14}$$



(a) Erroneous velocity due to wind (b) Wind correction

Fig. 3 Vectorial scheme depicting the relationship between the $\mathbf{V}_q$, $\mathbf{W}$ and $\mathbf{V}_q^d$

3 Control Strategy

In this section is addressed the motion control of a rotorcraft MAV with the objective of reaching and remaining in the trajectory $\zeta^d(t)$, which is subject to unknown wind disturbances. To this end, we have exploited the structure of system 9 and 10 to use nonlinear hierarchical control scheme considering the time-scale separation between translational (slow time-scale) and rotational motion (fast time-scale) of the aerial robot.

The primary goal of the control design is to mitigate the wind so that the quad-rotor can achieve the tracking control objective. From a geometric perspective, the erroneous vector $\mathbf{V}_R$, due to rotorcraft's velocity $\mathbf{V}_q$ and wind $\mathbf{W}$, may be corrected by means of control vector $\mathbf{V}_{(u+W)}$, composed by the control of nominal model and the wind compensation, this lead the MAV to the desired velocity $\mathbf{V}_q^d$ (Fig. 3).

Let us recall the disturbed kinematic model[1] and the torque τ_ψ (Eqs. 9 and 10), where, for simplicity, we have considered the normalized value of I_z and replaced the expression corresponding to wind vector (Eq. 13).

$$\left(\Sigma_{R_T}\right) : \dot{\zeta}^i = \mathscr{R}(\psi)\mathbf{V}_q + \mathbf{W}_o + \Delta_W(t) \tag{15}$$

$$\left(\Sigma_{R_R}\right) : \ddot{\psi} = \tau_\psi \tag{16}$$

[1]For a more in-depth analysis (i.e. 3D motion) v_q is provided by the longitudinal dynamics (Q-plane). However, this goes beyond the goal of the present paper.

The overall control vector $\mathbf{V}_{(u+W)}$ for the translational kinematics (Eq. 15) acts through $\mathscr{R}(\psi)\mathbf{V}_q$, where its magnitude is directly introduced via V_q, which is an actual control input. This is not the case for vector's orientation, due to the dependence of translational motion on orientation (underactuation). For this reason, and following a backstepping-like procedure, we define $R(\psi)$ as virtual control input. This means, that control-vector's orientation becomes the reference trajectory for the heading dynamics (Fig. 4).

Our aim is to synthesize a controller capable of dealing with a close-to-reality wind profile encompassing both components:

– *Slow time-varying wind* ($\mathbf{W}_o$): Since the quasi-constant wind vector is not available, we replace it in the controller with its estimated $\hat{\mathbf{W}}_o = (\hat{w}_x, \hat{w}_y)^T$. Defining the parameter estimation error as

$$\tilde{\mathbf{W}}_o \triangleq \hat{\mathbf{W}}_o - \mathbf{W}_o \tag{17}$$

and the corresponding time derivative as

$$\dot{\tilde{\mathbf{W}}}_o = \dot{\hat{\mathbf{W}}}_o \tag{18}$$

assuming small variations of $\mathbf{W}_o$ w.r.t time, that is to say $\dot{\mathbf{W}}_o \approx 0$

– *Fast time-varying wind* (Δ_W): The sliding-mode control (SMC) technique provides the framework to deal time-varying, and nolinear, parametric uncertainties as is the case of

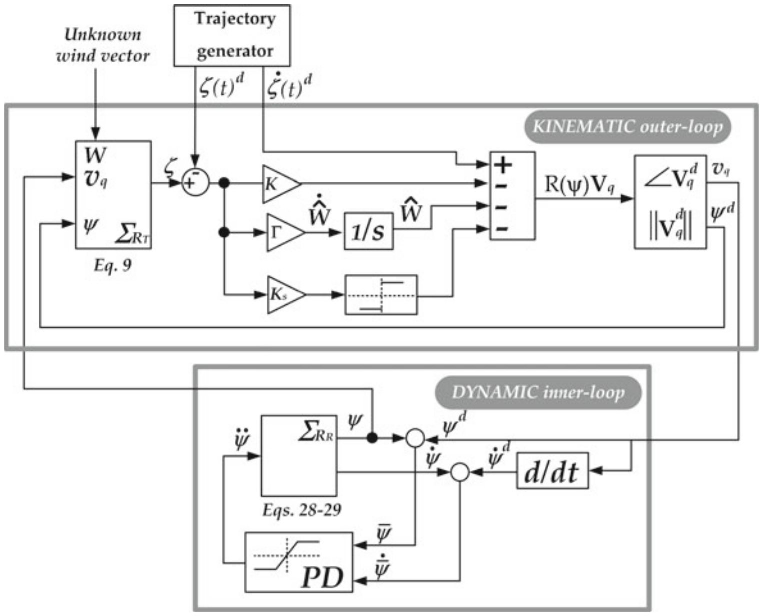


Fig. 4 Overall control scheme

the non-static wind disturbance. Let us, then, define the following sliding surface[2]

$$\mathbf{S} = \tilde{\zeta};\ \text{or} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \tag{19}$$

where $\tilde{\zeta} \triangleq \zeta - \zeta(t)^d$ stands for the position error.

3.1 Outer-Loop Adaptive-Robust Kinematic Controller

In order to provide a controller that guarantees the convergence of state vector to the equilibrium, consider the following candidate Lyapunov function (CLF)

$$\mathscr{V} = \frac{1}{2}\mathbf{S}^T\mathbf{S} + \frac{1}{2}\tilde{\mathbf{W}}_o^T\Lambda^{-1}\tilde{\mathbf{W}}_o \tag{20}$$

where Λ is positive-definite diagonal matrix representing the adaptation gain matrix. Differentiating w.r.t time leads to

$$\dot{\mathscr{V}} = \mathbf{S}^T(\dot{\zeta} - \dot{\zeta}^d) + \tilde{\mathbf{W}}_o^T\Lambda^{-1}\dot{\hat{\mathbf{W}}}_o \tag{21}$$

[2]Where we have used $\left(\frac{d}{dt} + \lambda\right)^{n-1}$.

Next, substituting Eqs. 15 and 18 lead to

$$\dot{\mathscr{V}} = \mathbf{S}^T(\mathscr{R}(\psi)\mathbf{V}_q + \mathbf{W}_o + \Delta_W - \dot{\zeta}^d) + \tilde{\mathbf{W}}_o^T\Lambda^{-1}\dot{\hat{\mathbf{W}}}_o \tag{22}$$

In order to provide an energy-decreasing behavior to $\dot{\mathbf{V}}$, we assign the controller to the translational kinematics as

$$\mathscr{R}(\psi)\mathbf{V}_q := -K_p\mathbf{S} - \hat{\mathbf{W}}_o - K_s\mathbf{Sign}(\mathbf{S}) + \dot{\zeta}^d \tag{23}$$

where K_p and K_s are positive-definite diagonal matrices and $\mathbf{Sign}(\mathbf{S}) = (\text{sign}(S_1), \text{sign}(S_2))^T$. The inclusion of the controller 23 in Eq. 22 results in

$$\begin{aligned}\dot{\mathscr{V}} &= -\mathbf{S}^T K_p\mathbf{S} + \mathbf{S}^T(\Delta_W - K_\alpha\mathbf{Sign}(\mathbf{S})) \\ &\quad - \mathbf{S}^T\tilde{\mathbf{W}}_o + \tilde{\mathbf{W}}_o^T\Lambda^{-1}\dot{\hat{\mathbf{W}}}_o\end{aligned} \tag{24}$$

which holds the inequality

$$\begin{aligned}\dot{\mathscr{V}} &\leq -\mathbf{S}^T K_p\mathbf{S} + \|\mathbf{S}\|\|\Delta_W\| \\ &\quad - \lambda_{\min}\{K_\alpha\}\|\mathbf{S}\|\|\mathbf{Sign}(\mathbf{S})\| \\ &\quad - \mathbf{S}^T\tilde{\mathbf{W}}_o + \tilde{\mathbf{W}}_o^T\Lambda^{-1}\dot{\hat{\mathbf{W}}}_o\end{aligned} \tag{25}$$

now, using the properties $\|\mathbf{x}\|\|\mathbf{y}\| \geq |\mathbf{x}^T\mathbf{y}|$ and the bound of time-varying wind (Eq. 14) yields

$$\dot{\mathscr{V}} \leq -\mathbf{S}^T K_p \mathbf{S} + \|\mathbf{S}\|\beta - \lambda_{\min}\{K_\alpha\}|\mathbf{S}^T \mathbf{Sign}(\mathbf{S})| - \mathbf{S}^T\tilde{\mathbf{W}}_o + \tilde{\mathbf{W}}_o^T \Lambda^{-1}\dot{\hat{\mathbf{W}}}_o \tag{26}$$

by means of properties $\|\mathbf{x}\| \leq \|\mathbf{x}\|_1$ and $\|\mathbf{x}\|_1 = \sum_{i=1}^n |x_i|$allow to rewrite previous equation as

$$\dot{\mathscr{V}} \leq -\mathbf{S}^T K_p \mathbf{S} + \|\mathbf{S}\|_1(\beta - \lambda_{\min}\{K_\alpha\}) - \mathbf{S}^T\tilde{\mathbf{W}}_o + \tilde{\mathbf{W}}_o^T \Lambda^{-1}\dot{\hat{\mathbf{W}}}_o \tag{27}$$

finally, defining $\lambda_{\min} \triangleq \alpha + \beta$ and considering the adaptation law $\dot{\hat{\mathbf{W}}} = \Lambda\mathbf{S}$ leads to

$$\dot{\mathscr{V}} = -\mathbf{S}^T K_p \mathbf{S} - \alpha\|\mathbf{S}\|_1 \tag{28}$$

it follows that $\dot{\mathscr{V}} \leq 0$. Hence, the fact that $\dot{\mathscr{V}}$ is negative semi-definite, allows to draw conclusions on the stability of the states for the motion control problem of the rotorcraft as follows:

(a) *Path-following*: If the reference path were time independent (ζ^d), the closed-loop system becomes autonomous. Thus, the negative semi-definiteness of $\dot{\mathscr{V}}$ complemented by the so-called La Salle's invariance principle, allows to conclude that the origin of the state vector $(\mathbf{S}, \tilde{\mathbf{W}})^T$ is stable and attractive, then $\mathbf{S}$ and $\tilde{\mathbf{W}}$ tend asymptotically to zero. In other words, the state trajectory reaches and remains the sliding surface in a finite time and the estimation vector converge to the actual value of static wind.

(b) *Trajectory-tracking*: In the present case the desired path is parameterized by time ($\zeta^d(t)$), which implies that the closed-loop system is non-autonomous. We might conclude stability of the origin, i.e. state vector $(\mathbf{S}, \tilde{\mathbf{W}})^T$ remains bounded. However, it might proven by means of the Barbalat's lemma [11] that $\mathbf{S} \to 0$, while $\tilde{\mathbf{W}}$ remains bounded (condition that is good enough in some cases).

Remark 1 In order to alleviate the inherent chattering problem, the hyperbolic function **Tanh** $(\mathbf{S})$ might be used to replace the $\mathbf{Sign}(\mathbf{S})$ in the kinematic tracking controller.

3.2 Inner-Loop Bounded Heading Controller

Let us recall the control vector obtained to drive the translational kinematic subsystem 23.

$$\mathscr{R}(\psi)\mathbf{V}_q := -K_p\mathbf{S} - \hat{\mathbf{W}}_o - K_s\mathbf{Sign}(\mathbf{S}) + \dot{\zeta}^d \tag{29}$$

where the norm and direction of Eq. 29 represent, respectively, the required speed and orientation so that the rotorcraft is capable of attaining and remaining on the reference path in presence of wind. The controller 29 assumes $\mathscr{R}(\psi) = \mathscr{R}(\psi^d)$, which is valid provided the time-scale separation between translational kinematics and the rotational dynamics. The latter implies the stabilization of the heading error dynamics,

$$\dot{\tilde{\psi}}_1 = \frac{1}{\varepsilon}\tilde{\psi}_2 \tag{30}$$

$$\dot{\tilde{\psi}}_2 = \frac{1}{\varepsilon}\left(\tau_\psi - \ddot{\psi}^d\right) \tag{31}$$

where, ε $(0 < \varepsilon << 1)$ is introduced to establish the time-scale separation, and the control input is considered bounded. Let us define the heading error as

$$\tilde{\psi} \triangleq \psi - \psi^d \tag{32}$$

where, the orientation of control vector (Eq. 29) stands for the heading reference

$$\psi^d = \arctan(u_x/u_y) \tag{33}$$

with u_x and u_y representing the components of Eq. 29.

The control design for the rotational inner-loop assumes the following:

A1 Limited response of the torque actuators (rotors differential thrust) is assumed, i.e. $|\tau_\psi| \leq \tau_{\max}$ with $\tau_{\max} > 0$.

A2 The heading rate, $\tilde{\psi}_2$, might be assumed bounded due to actuators saturation, i.e. $|\tilde{\psi}_2| \leq \gamma$ with $\gamma > 0$.

A3 Heading reference, ψ^d, is smooth time-varying function with the corresponding derivatives bounded.

A4 Due to the non-aggressive translational trajectories the heading reference acceleration might be disregarded.

To design the control law for the inner-loop heading dynamics, we proceed similarly as in [12], where a backstepping-based controller is obtained accounting a bounded velocity state, i.e. the heading rate error in our case (see **A2** and **A3**). Let us consider the CLF to obtain the controller that stabilizes the first-order scalar system 30

$$\mathscr{W}_1 = \ln\cosh(\tilde{\psi}) \tag{34}$$

whose time-derivative is

$$\dot{\mathscr{W}}_1 = \frac{1}{\varepsilon}\tilde{\psi}_2 \tanh(\tilde{\psi}_1) \tag{35}$$

From the latter, it is clear that via $\tilde{\psi}_2$ is possible to render $\mathscr{W}_1 < 0$ by using $\tilde{\psi}_2 = -\tanh(\tilde{\psi}_1)$. Thus, the next step considers $\tilde{\psi}_2^d = -\tanh(\tilde{\psi}_1)$ leading to the following error state

$$z = \tilde{\psi}_2 + \tanh(\tilde{\psi}_1) \tag{36}$$

whose time-derivative is written

$$\dot{z} = \frac{1}{\varepsilon}\left(\tau_\psi + \tilde{\psi}_2 \mathrm{sech}^2(\tilde{\psi}_1)\right) \tag{37}$$

Let us propose the overall CLF encompassing $\mathscr{W}_1$ as

$$\mathscr{W} = \ln\cosh(\tilde{\psi}) + \frac{1}{2}z^2 \tag{38}$$

whose the time-derivative is

$$\dot{\mathscr{W}} = \frac{1}{\varepsilon}\left[\tilde{\psi}_2 \tanh(\tilde{\psi}_1) + z\left(\tau_\psi + \tilde{\psi}_2 \mathrm{sech}^2(\tilde{\psi}_1)\right)\right] \tag{39}$$

now, using $\mathrm{sech}^2(\tilde{\psi}_1) \leq 1$, Eqs. 36 and 37 as well as the controller

$$\tau_\psi = -2z \tag{40}$$

yields

$$\dot{\mathscr{W}} = \frac{1}{\varepsilon}\left(-\tanh^2(\tilde{\psi}_1) - z^2\right) \tag{41}$$

which is negative definite and guarantees that the trajectories of the state vector $(\tilde{\psi}_1, z)^T$ converge asymptotically to the origin.

4 Numerical Simulations

The aim of this section is to provide a detailed description of the performance of the proposed control in presence of 2D wind disturbances. As mentioned before, the effect of wind disturbance causes erroneous trajectories as depicted by Fig. 5.

4.1 Straight-Line Tracking

In this case when the quad-rotor is tracking a straight-line trajectory ($v_q^d = 12$ m/s), we consider

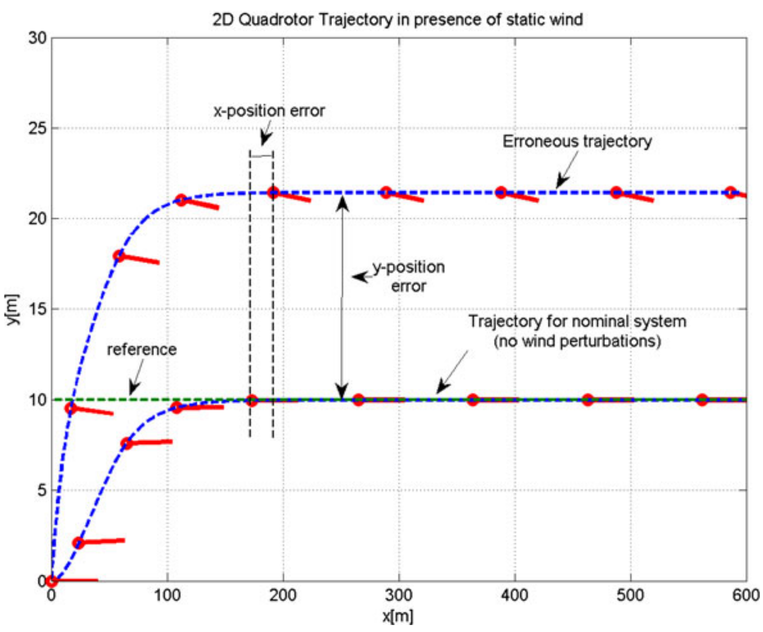


Fig. 5 Nominal response (no-wind conditions) vs Erroneous response (static-wind conditions)

Fig. 6 Straight-line tracking performance: (*top-left*) w_x vs $\hat{w}_x$, (*top-right*) w_y vs $\hat{w}_y$

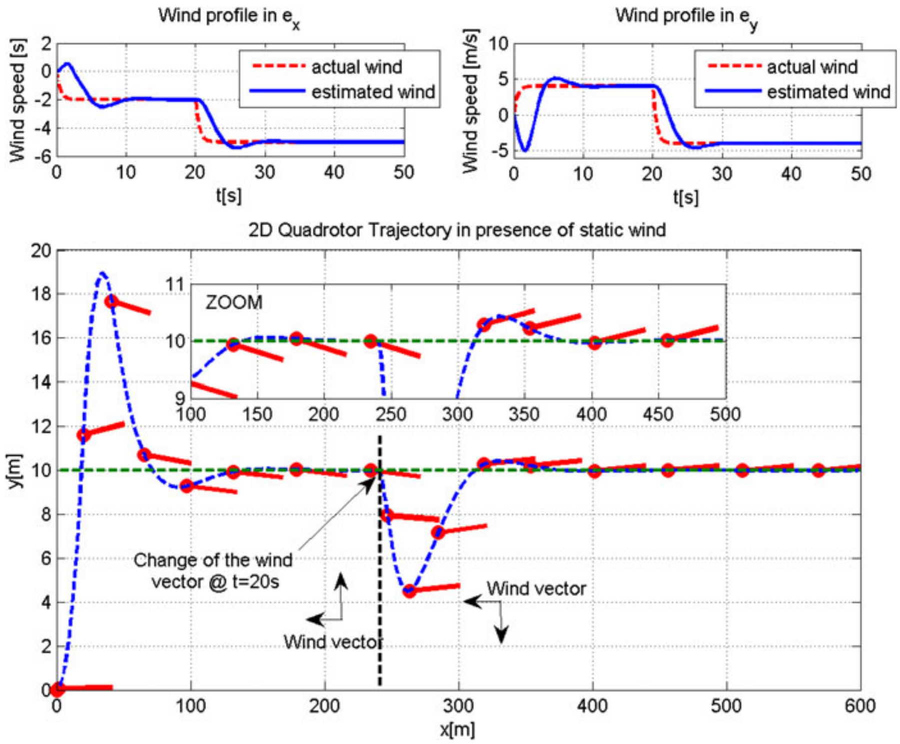


Fig. 7 Heading and control inputs performance

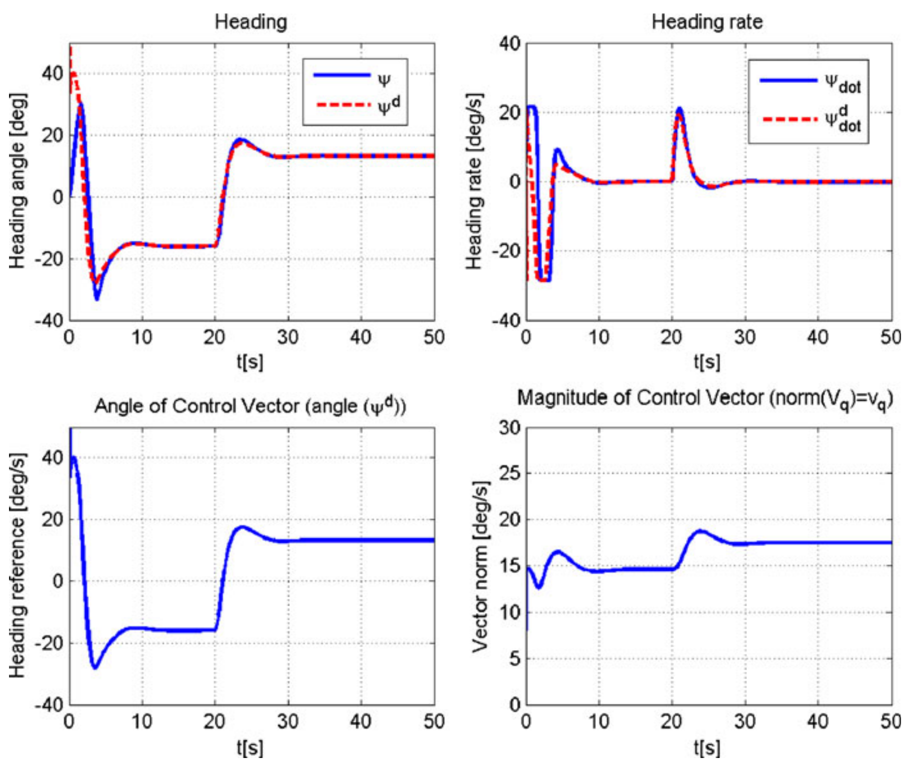


Fig. 8 2D translational performance

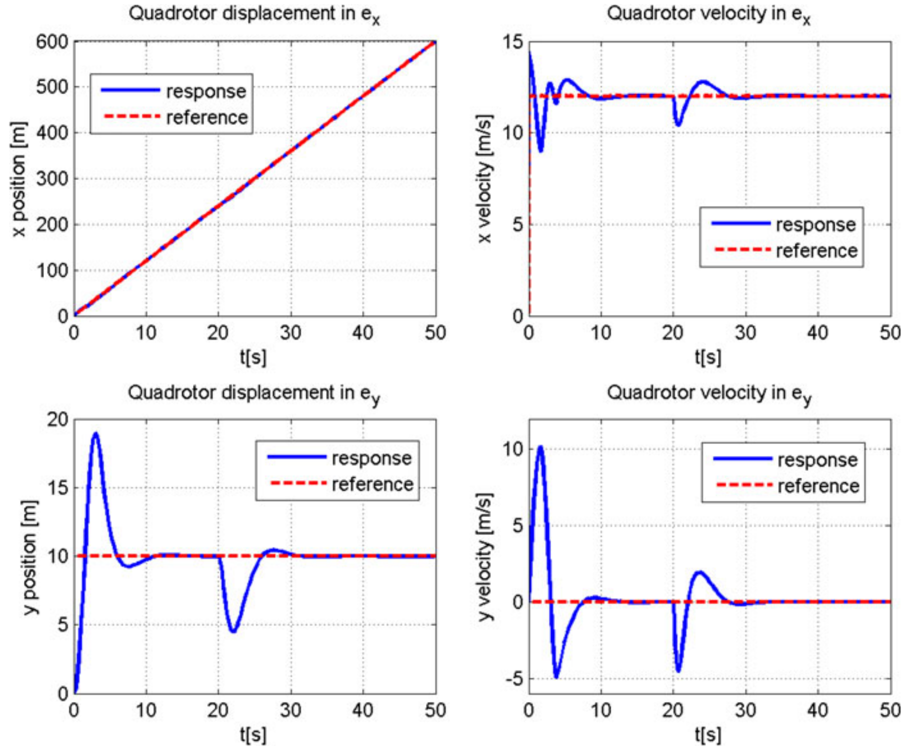
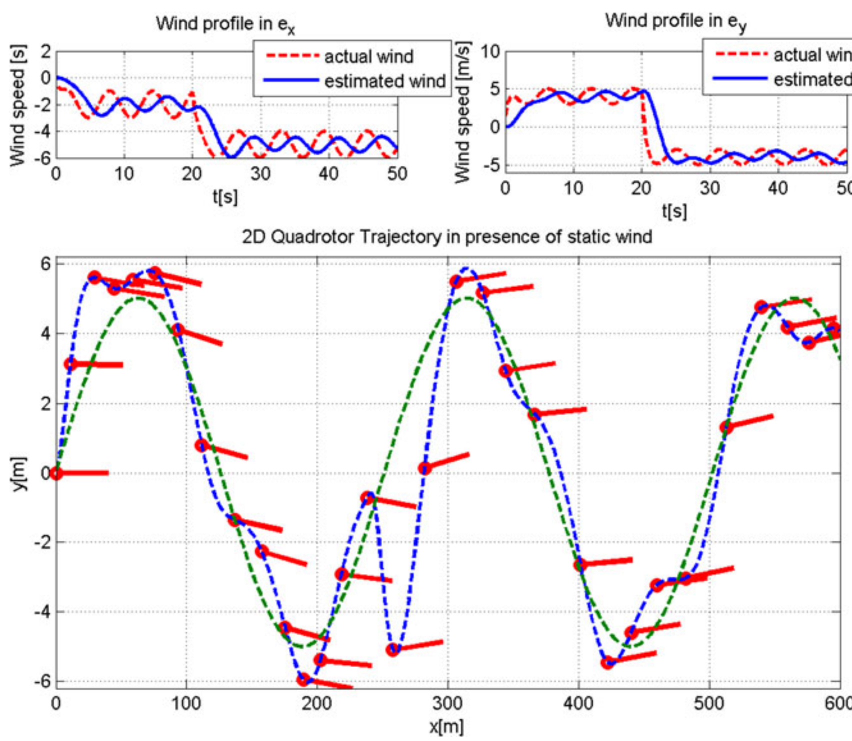


Fig. 9 Sinusoidal tracking performance (only adaptive controller): (*top-left*) w_x vs $\hat{w}_x$, (*top-right*) w_y vs $\hat{w}_y$



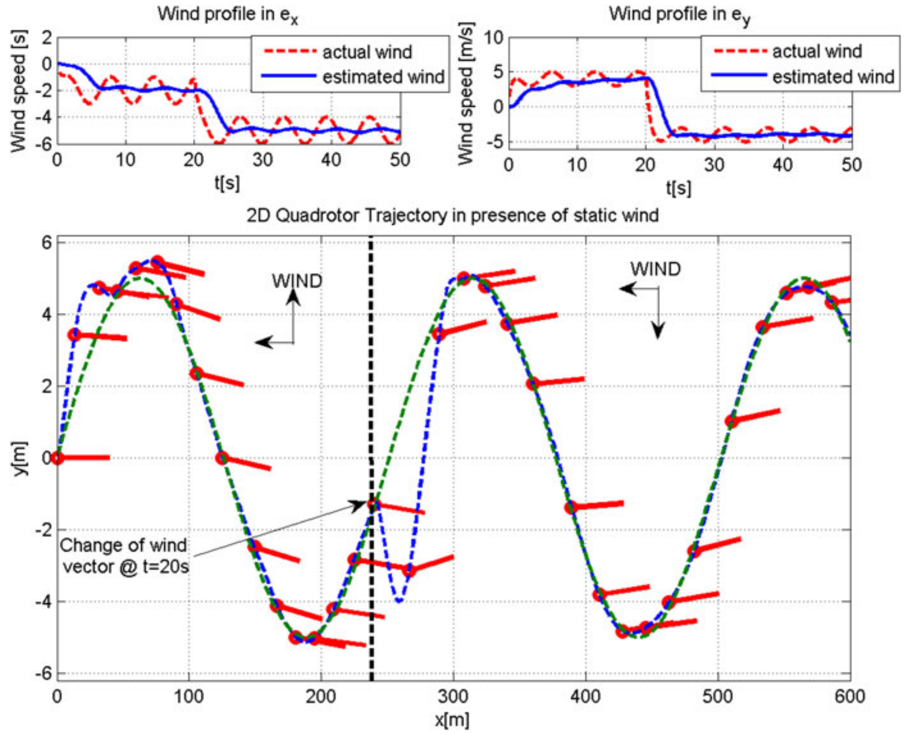


Fig. 10 Sinusoidal tracking performance (adaptive + sliding-mode controller): (*top-left*) w_x vs $\hat{w}_x$, (*top-right*) w_y vs $\hat{w}_y$

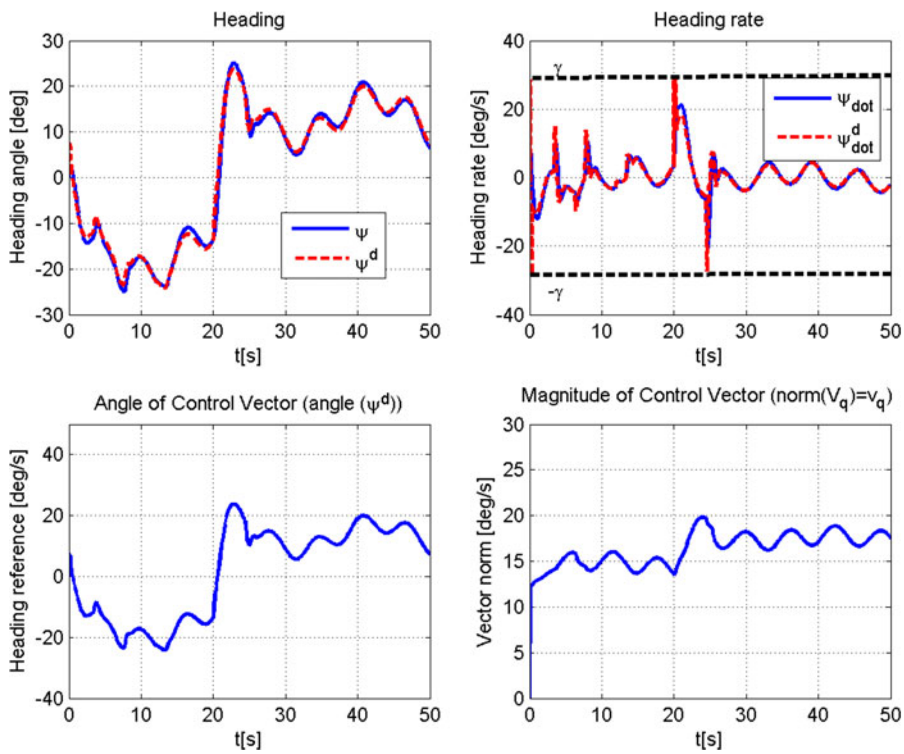


Fig. 11 Heading and control input performance

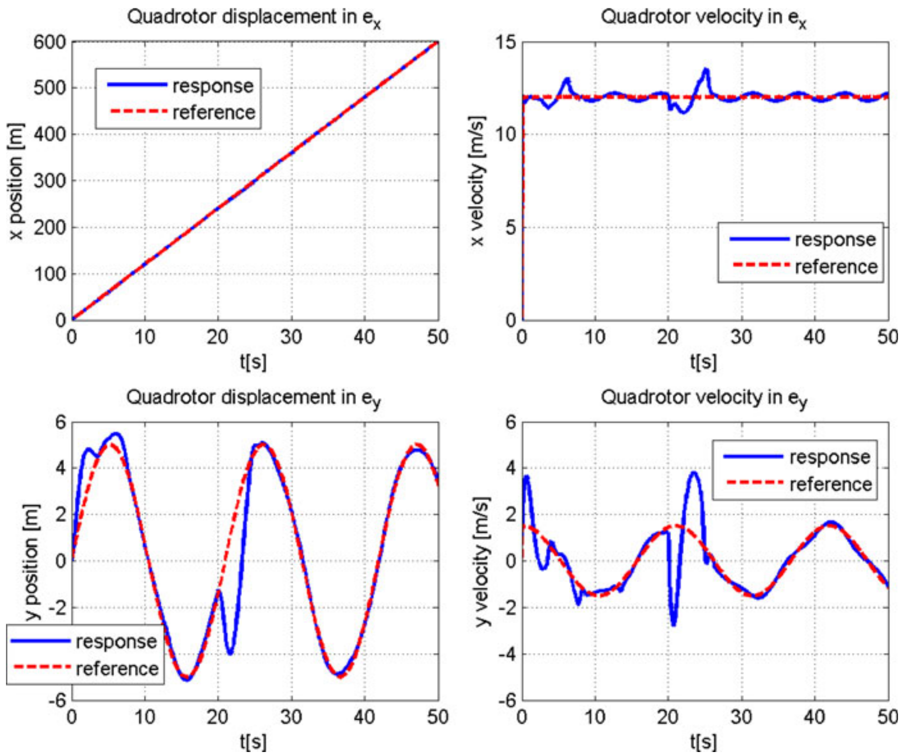


Fig. 12 2D translational motion performance

a static wind to evaluate the adaptive controller. In Fig. 6 it is observed that the quad-rotor is able to track the desired trajectory even if there is a drastic change in the static-wind values at $t = 20$ s. Also, notice that the estimation converge to the actual wind values. Likewise, it is observed that tracking the straight-line, the vehicle features a heading angle different than zero to compensate wind components. Thus, it is logical to expect, that velocity and orientation control inputs evolves in function of wind magnitude and orientation variations (Figs. 7 and 8).

4.2 Sinusoidal Trajectory

The simulation presented in this section retains the static wind component used in the latter simulation. However, now, the wind profile also includes the time-varying wind component. Firstly, it is evaluated the response of the controller having only the adaptive action. Figure 9 shows that the adaptive controller is not enough to track satisfactorily the desired trajectory since its performance is significantly deteriorated by the varying wind component, which is also the case of the wind estimation. Next, it is introduced both controllers (adaptive + sliding-mode) in order to mitigate the effect of non-static wind. Figure 10 and 12 show that the combinations improves considerably the tracking of the commanded trajectory. Indeed, the estimation is also improved converging to the actual static wind values. Also, it is important to underline that heading rate values remains within the saturation limits γ (see Fig. 11).

5 Concluding Remarks

The present paper proposes a model reduction regarding to simplify the navigation control task, consisting in partitioning the complete 6 DOF dynamic model into a dynamic subsystem (Q-plane), that focuses on controlling MAV's relative forward velocity and overall vertical motion, and a kinematic subsystem (R-plane), which allows to treat the quad-rotor motion problem in a nonholonomic-like navigation context. From this approach and considering motion capabilities of rotorcrafts MAVs arises the possibility of applying the wide variety of navigation schemes for ground mobile robots.

The paper addressed the tracking control problem of a quad-rotor MAV subject to unknown wind conditions within a kinematic framework. It is proposed a control strategy merging adaptive and sliding-mode control techniques to estimate static-wind and to cope with unknown time-varying wind, respectively. We distinguish two control complementarity provides the following benefits:

– Estimation of the slow-time varying uncertainty, i.e. static wind.
– The combination pure switching control (sliding-mode) which implies high control chattering.
– Modularity to tackle static/varying wind components.

Simulation study have exhibited the effectiveness of the control law for two different trajectory-tracking tasks under static/varying wind conditions up to 40 % of MAV velocity. The control design presented in this paper is based on a quad-rotor drone, however, such scheme might be applied to different miniature multi-rotor MAVs (three-rotors, hexa-rotors, etc.), since they share similar kinematic and dynamic models.

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