

# PPG motion artifact reduction using neural network and spline interpolation

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**Abstract**— A new method of removing motion artifact from the Photoplethysmogram signal using multilayer feed forward neural network is described in the paper. 3850 number of beats each containing 7 important clinical features collected from BIDMC dataset are used for training the neural network and 770 number of beats are used for testing. The proposed algorithm performs quite well yielding higher levels of accuracy in conserving the systolic peak to systolic peak distance (peak to peak distance) as compared to other existing algorithms. The error between the average peak to peak distance of the clean PPG signal and the processed PPG signal is 6.19%. Another salient contribution of this work is that it preserves other clinical features quite well. Thus the algorithm can be useful for clinical feature based classification of PPG signal in ambulatory monitoring.

**Key Words**— Motion artifact, Neural network, PPG, Spline interpolation

## I. INTRODUCTION

Photoplethysmography (PPG) is a non-invasive optical technique for measurement of blood volume change at the peripheral blood vessels with respect to cardiac cycle in the human body [1-2]. Owing to the low cost instrumentation and ease of collection, PPG has become a popular choice as diagnostic monitoring tool in many physiological measurements [3]. A PPG sensor, placed at peripheral body part like finger, consists of an infrared (IR) photosource and a photodetector. Various vital clinical information can be obtained from the PPG signal like heart rate, stroke volume, vascular health assessment, heart rate variability [4] etc. These physical information are derived from various features obtained from PPG signal e.g. pulse width, systolic amplitude, pulse interval [5] etc. Furthermore these clinical features are calculated from various signature points of a PPG waveform viz., Systolic Peak, Dicrotic Notch, Diastolic Peak and foot. Hence if the waveform is corrupted due to unnecessary movement of the sensor which is quite common in ambulatory monitoring, the acquired clinical information may vary to a great extent from its true value. Thus preprocessing is an essential prerequisite to reduce artefact in the PPG data for computerized analysis and classification applications. Many existing researches had focused on computerised PPG preprocessing to increase the efficiency of signal analysis algorithms. The evaluation methods reported

were mostly visual comparison between the preprocessed data and the (simultaneously acquired) clean PPG, and peak to peak distance comparison. Some popular methods implemented for preprocessing a PPG signals are wavelet analysis [6], DSP based cycle by cycle Fourier series analysis (CFSA) technique [7], Empirical Mode Decomposition (EMD) [8], Window's adaptive noise cancellation (ANC)[9], Principal Component Analysis (PCA) [10], Wiener filtering [11], Independent Component Analysis (ICA) [12] etc. Using a reference noise signal, constrained ICA (cICA) was combined with adaptive filters [13] to extract clean PPG from the MA corrupted signal with the amplitude information preserved. However, the authors commented that the technique may not perform well in real world applications, where the MA signal has overlapping spectra with PPG. In [14] adaptive step-size least mean squares (AS-LMS) filter is designed for reducing MA in corrupted PPG signals. A noise reference signal is also generated here for functioning of the adaptive filter using FFT, ICA and SVD. Another work [15] utilized the quasi periodicity of the PPG signal in designing a periodic Moving Average Filter (MAF) to remove the motion artifacts.

In this paper a new method combining the feed forward neural network and spline interpolation is proposed to remove the motion artifact from the signal. At first the raw PPG signal is normalized and the fiducial points were detected. Based on these fiducial points, individual beats were extracted. Various clinical features were extracted and these features are used to classify a beat into corrupt or clean based on a thresholding scheme. Finally the detected corrupted beats were replaced with a new synthetic beat generated by spline interpolation. The salient feature of this work is that we retained the fiducial points quite well. Also this algorithm doesn't require known noise to be recorded for preprocessing. This work aims to preserve the useful clinical features which can further be used for diagnosis of critical cardiovascular diseases such as Systemic Vascular Restrictions (SVR), Vasoconstriction and Vasodilation, Arterial stiffness, Stroke Volume etc.

The rest of the paper is organized as follows. In section II, the proposed algorithm is presented. Section III presents the experimental results and comparative analysis. Finally in section IV, the paper is concluded.

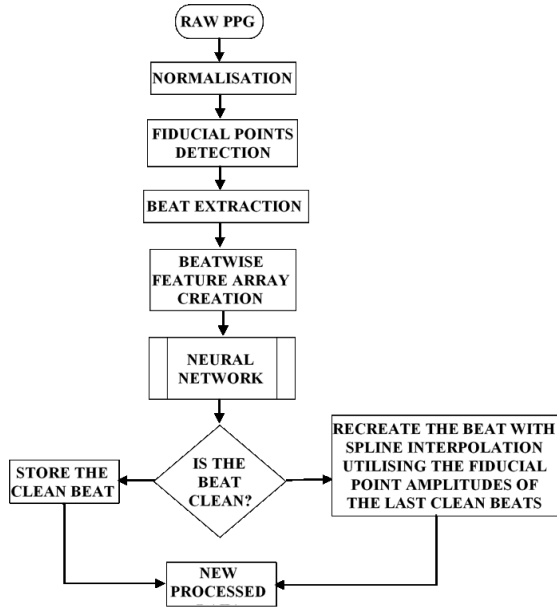


Fig. 1 Work flow diagram

## II. METHODOLOGY

### A. Database description

For implementing the algorithm, a publicly available dataset is being used. It is called BIDMC (The Beth Israel Deaconess Medical Center) dataset [16]. This dataset contains simultaneous recordings of PPG, electrocardiogram (ECG), and impedance pneumography respiratory signals which were acquired from intensive care patients. 30 volunteers were recruited purely on random basis for data collection. The age of the volunteers ranges from 19-90+. All the volunteers were admitted to intensive care units at the BIDMC, Boston, USA. The recordings are of 8-minutes duration with a sampling frequency of 125 Hz.

### B. Normalization of the dataset:

The PPG data was normalized within a range of 0 to 1 to increase the sensitivity of peak and foot detection algorithm. The formula employed for the normalization is:

$$\text{Normalized value} = \frac{\text{Actual value} - \text{Minimum Value}}{\text{Maximum value} - \text{Minimum value}} \quad (1)$$

### C. Detection of fiducial points:

The basic requirement of this work is to detect the quality of individual beat. So for detecting individual beats, the fiducial points are detected first. The fiducial points of a PPG signal are: peak, foot, diastolic peak and diastolic notch. A local maximum was provisionally selected as a peak by comparing the amplitudes of consecutive samples. The adjacent samples of the local maxima were compared using a threshold value. If that maximum amplitude exceeds that threshold, then the point was considered as a

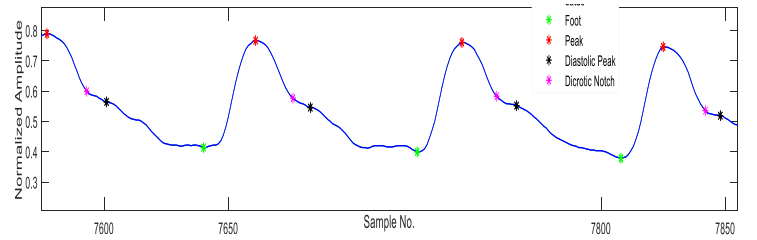


Fig. 2 Fiducial points of a PPG signal

peak. The foot was also detected using the same logic except here the minimum amplitude point was stored. The second maxima between peak and foot is detected as diastolic peak and the minima between diastolic peak and systolic peak is marked as diastolic notch. Fig. 2 shows the fiducial points of a PPG signal.

### D. Beatwise segmentation:

For beatwise feature calculation, a  $n \times m$  dimension matrix was constructed with each PPG beat, where  $n$  is the number of PPG beats in the signal and  $m$  is the length of the beats after necessary zero padding. All the beats are aligned to a common point of reference, the systolic peak where the signal-to-noise ratio (SNR) is maximum. The part of signal between two foots is considered as a beat. After extraction, the  $k^{\text{th}}$  beat vector may be represented as:

$$B_k = [b_{k1} \ b_{k2} \dots b_{km}]^T \quad (2)$$

Where,  $b_{kj}$  represents  $j^{\text{th}}$  sample of  $k^{\text{th}}$  beat of length  $m$ . Zero padding was done at both ends of the systolic peak, wherever necessary, to make the length of the beats equal. The final beat matrix  $B$  can be represented as:

$$B = \begin{bmatrix} b_{11} & b_{21} & \dots & b_{n1} \\ b_{12} & b_{22} & \dots & b_{n2} \\ . & . & . & . \\ b_{1m} & \dots & \dots & b_{nm} \end{bmatrix} \quad (3)$$

This  $B$  serves as the input to the next stage for calculation of beatwise clinical features, which will be the input to the neural networks in the further stage. Fig. 3 shows the pictorial representation of the matrix  $B$ .

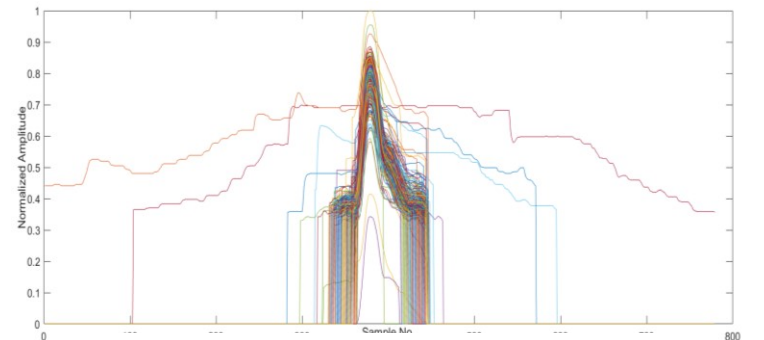


Fig. 3 Overlapped PPG beats aligned with respect to systolic peak

### E. Creation of feature matrix

Based on the fiducial points useful clinical features were extracted to form a beat wise feature set matrix which would be the input to the neural network. The features extracted are:

1. Feature 1: Systolic Amplitude (A)  
It is the normalized amplitude of the systolic peak which is used for finding the stroke volume, vasoconstriction and vasodilation in blood vessel;
2. Feature 2: Pulse Width (C)  
It is the width of the beat measured at half systolic peak height. Systolic peak height is the length from foot to the peak of a beat. Pulse Width is used for determining systemic vascular resistance.
3. Feature 3: Systolic Peak to Systolic Peak distance (B)  
It is the distance between two consecutive systolic peaks which correlates closely with the R-R interval of ECG and is used for finding heart rate.
4. Feature 4: Pulse Interval (D)  
It is the distance between the two foots of a beat. The ratio of PI with systolic amplitude gives a clear idea of the cardiovascular system of a person.
5. Feature 5: Number of slope reversal in the signal  
Ideally there should be three slope reversals in a PPG beat viz., at systolic peak, diastolic notch and diastolic peak. The more the beat is corrupted with motion artifact, the more slope reversals will be there in the signal.
6. Feature 6: Pulse Area  
The pulse area is measured as the total area under the PPG curve.
7. Feature 7: Augmentation Index (E/A)  
It is the ratio of the diastolic peak height to the systolic peak height. Arterial stiffness can be calculated from this clinical feature.

Fig. 4 shows the clinical features of the PPG that were extracted.

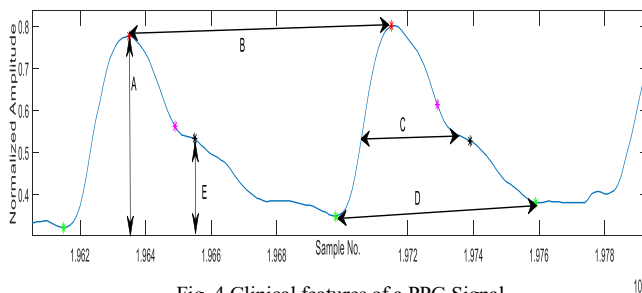


Fig. 4 Clinical features of a PPG Signal

### F. COF calculation from each beat:

Based on the multidimensional dataset created, the network is trained with equal number of clean beats and corrupted beats. The correlation coefficient (COF) of successive beats are calculated. Correlation coefficient (COF) establishes the statistical relationship between two or more

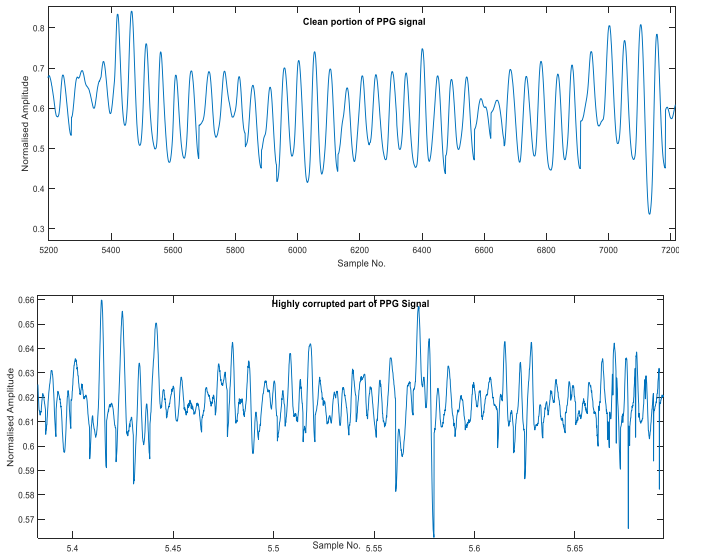


Fig. 5 (a) clean portion of PPG Signal (b) motion corrupted portion of PPG signal

datasets. It states the extent of similarity between the two arrays. So, the COF between the consecutive beats of the dataset (aligned to respective systolic peaks) is calculated by the formula:

$$COF(a, b) = \frac{1}{N-1} \sum_{i=1}^N \left( \frac{a - \mu_a}{\sigma_a} \right) \left( \frac{b - \mu_b}{\sigma_b} \right) \quad (4)$$

Where, N is the total number of samples in a beat a (b),  $\mu_a$  ( $\mu_b$ ) is the mean of data a (b),  $\sigma_a$  ( $\sigma_b$ ) are the standard deviation of a (b). The COF calculated for consecutive beats are stored in an array of dimension 1xM, where M is the number of beats-1. This array is further utilized in creation of target vector for training the network.

### G. Creation of target array based on COF:

The main aim of this work is to detect beats corrupted with motion artefact and clean beat without motion artefact. In a resting PPG signal without motion artefact, there is not much variation between consecutive beats. Hence in clean PPG signal the COF array formed will have value close to 1 since the beats are nearly identical with each other. However, the COF between consecutive beats of a motion artefact contaminated data will be very less since the beats are corrupted and vary widely with each other. This is evident in fig. 5 where both the clean part and the motion artifact corrupted part of the PPG signal is shown for comparison. The concept is utilized in creation of target vector for training the network. Manual creation of target vector is not realistic as the individual dataset length is very large (greater than 1000). A thresholding scheme is introduced to control the robustness of the algorithm. A threshold value ranging from 0 to 1 is introduced, where 0 defines no thresholding or accepting each beat as it is and higher value defines classifying a beat as bad even for slightest change in the general contour of the beat. For e.g., if the threshold is given

as 0.9, then for  $\text{COF}_j > 0.9$ , the beats are identified as clean and the target vector corresponding to these beats are marked as 1. Here  $j$  is the index of the COF array created in the previous stage. For  $\text{COF}_j < 0.9$ , the beats are identified as corrupted and the target vector corresponding to these beats are marked as 0.

#### H. Neural Network training:

The feature matrix described in section E is used as the input to the neural network. 2-layer Feed forward back propagation network having one hidden layer with architecture of 7-10-1 and with learning algorithm as Scaled conjugate gradient (SCG) is used to classify the beats as corrupt or good. SCG is a supervised learning algorithm which is faster than conjugate gradient method since the algorithm increases learning speed with each iteration. The detailed algorithm is discussed in [17]. The hidden layer uses tan sigmoid activation function, and the output layer uses log sigmoid activation function. Input feature matrix had a dimension of  $3850 \times 7$ , where 3850 is the total number of beats and 7 is the total number of features used. For training, 80% of the dataset i.e., 3080 beats were used. For testing, 770 (20%) beats were used. The network is trained for a total of 100 epoch. The training accuracy is found to be 98.15 % and the testing accuracy is 96.49 %. This trained network is used for evaluation of other normalized beats to classify them as corrupted or clean beat.

#### I. Spline interpolation:

The previous position and amplitude of a clean beat is stored and looked as the amplitude of a fiducial point for the next beat in case the beat is corrupted. Spline interpolation function returns a vector of interpolated values corresponding to the query points [18]. Cubic splines are most common. In this case the interval between each query points is represented by a cubic polynomial and has continuous first and second derivatives at the knots. Utilizing the last stored amplitude information of a clean beat, the synthetic beat is generated which is replaced whenever a corrupted beat is detected.

### III. RESULTS AND DISCUSSIONS

To show the improved performance in preserving the features of the PPG signal, two approaches were taken. The first one is objective evaluation targeted to show proximity of the processed data with the clean data. This approach also shows how the features of the clean signal widely varies with the degree of motion artefact. The feature, peak to peak interval, is used to compare with the literature. The second one is subjective evaluation where visual comparison are given. Accurate beat detection is imperative for the success of this algorithm. 100% accuracy in beat detection is achieved for all the PPG data used in this study. Using the first approach, the closeness of the preprocessed beat to the clean beat is evaluated by comparing seven important clinical features as

marked in fig 4 and explained in section E. The objective was to estimate how far a clinical feature in the clean signal could be restored in the process of denoising and is quantified by percentage change of the feature with reference to the original feature as given in this formula:

$$E_{\text{feature}} \% = \frac{|\text{Mean feature}_{\text{processed/corrupted}} - \text{Mean feature}_{\text{clean}}|}{\text{Mean feature}_{\text{clean}}} \times 100\% \quad (5)$$

The percentage error of the peak to peak interval is also calculated by the formula given in equation 6 to compare with the literature.

$E_{p,p} \%$  (Percentage error between the average Peak to Peak distance of clean PPG signal ( $\text{PPc}$ ) and the average Peak to Peak distance of processed PPG signal ( $\text{PPp}$ ))

$$= \frac{|\text{PPp} - \text{PPc}|}{\text{PPc}} \times 100 \% \quad (6)$$

Table I shows the  $E_{\text{feature}} \%$  (as calculated using equation 5) of seven clinical features obtained from 10 different data. As described in segment E, feature 1 is the systolic amplitude. The first sub column within feature 1 represents the average error between the movement signal and the clean signal and the second sub column shows the average error between the processed signal and the clean signal of that feature for different data. The same way of representation is adapted for all the other features. The objective of processing is to make the motion artifact data clean. Hence an essential factor for evaluating the quality of the preprocessing algorithm is to look into the difference between the features of the clean and processed signal. For a good motion artifact reduction algorithm, this difference should come very less. The average  $E_{\text{systolic amplitude}} \%$  between the movement and the clean is 14.50 % whereas the average  $E_{\text{systolic amplitude}} \%$  between the processed and the clean is 5.61%. This shows the variation of systolic amplitude between the clean and processed is much less than the difference of systolic amplitudes between the movements and clean. This is because the signal is distorted in such a way that the variation of features between clean and movement increases. The average  $E_{\text{pulse width}} \%$  between the movement and the clean is 44.26 % whereas the average  $E_{\text{pulse width}} \%$  between the processed and the clean is 4.64 %. The average  $E_{\text{pulse interval}} \%$  between the movement and the clean is 64.95 % whereas the average  $E_{\text{pulse interval}} \%$  between the processed and the clean is 2.98 %. The average  $E_{\text{no.of_slope_reversal}} \%$  between the movement and the clean is 65.23% whereas the average  $E_{\text{no.of_slope_reversal}} \%$  between the processed and the clean is 3.00%. This shows that the contour of the processed signal is much smoother like the clean signal and unlike the movement signal. The average  $E_{\text{pulse area}} \%$  between the movement and the clean is 24.91% whereas the average  $E_{\text{pulse area}} \%$  between the processed and the clean is 3.34%. The average  $E_{\text{augmantation_index}} \%$  between the movement and the clean is 11.38 % whereas the average  $E_{\text{augmantation_index}} \%$  between the processed and the clean is

TABLE I

COMPARISON OF AVERAGE PERCENTAGE ERROR: VARIOUS FEATURES OF CLEAN AND MOVEMENT VS CLEAN AND PROCESSED

Data ID	Feature 1		Feature 2		Feature 3		Feature 4		Feature 5		Feature 6		Feature 7	
	$E_{\text{feature1 mov \%}}$	$E_{\text{feature1 proc \%}}$	$E_{\text{feature2 mov \%}}$	$E_{\text{feature2 proc \%}}$	$E_{\text{feature3 mov \%}}$	$E_{\text{feature3 proc \%}}$	$E_{\text{feature4 mov \%}}$	$E_{\text{feature4 proc \%}}$	$E_{\text{feature5 mov \%}}$	$E_{\text{feature5 proc \%}}$	$E_{\text{feature6 mov \%}}$	$E_{\text{feature6 proc \%}}$	$E_{\text{feature7 mov \%}}$	$E_{\text{feature7 proc \%}}$
bidmc_01	17.67	1.75	39.48	0.27	66.05	1.06	65.52	0.60	66.03	0.56	6.48	1.64	0.09	0.05
bidmc_02	13.01	10.36	58.88	5.52	68.07	11.02	68.55	4.37	68.60	4.41	45.25	4.32	3.16	0.11
bidmc_04	7.16	2.97	46.10	1.32	67.33	2.96	69.45	1.05	70.37	1.06	37.88	1.80	10.95	0.04
bidmc_10	12.63	5.62	46.52	5.53	66.65	7.34	67.73	3.02	67.79	2.91	8.44	2.73	4.08	0.12
bidmc_12	15.85	12.24	49.42	6.98	64.85	12.61	62.21	6.68	62.22	6.69	13.72	12.69	2.57	0.51
bidmc_13	2.03	5.51	42.19	7.35	56.77	1.33	55.27	0.84	55.57	0.84	23.93	3.57	15.12	1.66
bidmc_14	5.89	7.67	43.51	13.22	59.00	17.90	57.59	9.84	57.85	10.12	19.60	0.04	1.16	0.31
bidmc_17	17.32	5.79	54.65	5.15	69.09	5.57	65.10	1.87	64.17	1.88	14.42	1.58	5.95	0.17
bidmc_22	19.58	1.91	51.54	0.21	64.94	0.53	70.52	0.52	71.94	0.52	61.29	2.86	49.49	0.87
bidmc_30	33.84	2.26	10.35	0.84	70.79	1.55	67.60	1.01	67.76	1.02	18.09	2.18	21.23	0.10

\*Feature 1-Feature 7 refer section E

0.39 %. The lowest variation between the clean and processed is obtained in this feature. The average  $E_{\text{peak-to-peak \%}}$  between the movement and the clean is 65.36 % whereas the average  $E_{\text{peak-to-peak \%}}$  between the processed and the clean is 6.19%. The  $E_{\text{P-P \%}}$  is same as  $E_{\text{peak-to-peak \%}}$ .  $E_{\text{P-P \%}}$  is used to compare with the literature. With the results published in [13], the  $E_{\text{P-P \%}}$  is calculated for various algorithms by taking average all kind of movements. Since the nature of movement is not given in BIDMC dataset, we considered it as random movement. Only those algorithms which are implemented for all kinds of movement or random movements, are used for comparing with this work. The comparison is given in Table II. Significant improvement can be seen with respect to other algorithms. Thus, with this method the clinical features can be restored without much distortion. For subjective measure, the graphical presentation of movement and preprocessed

PPG data were shown in fig 6. Close similarity between clean and processed PPG morphology can be seen from the fig 6.

#### IV. CONCLUSION

A new method of removing motion artifact from the Photoplethysmogram signal using multilayer feed forward neural network and spline interpolation is proposed in the paper. Experimental results show that this algorithm yields very small error between the clinical features of the clean and processed PPG signal. This signifies that the variation of clinical feature from clean PPG signal due to the preprocessing method is low. The results obtained and presented in this paper show that this algorithm performs better in conserving the clinical features than other literature. Another salient feature of this work is that it does not require

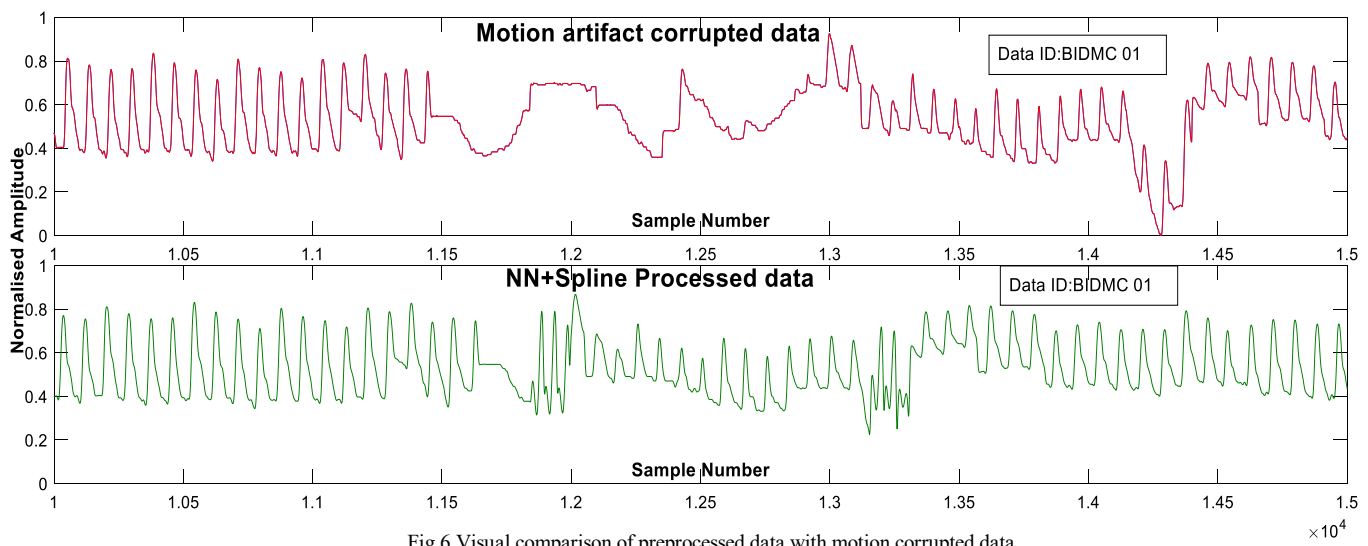


Fig.6 Visual comparison of preprocessed data with motion corrupted data



known noise to be recorded for preprocessing. The proposed method can be used for clinical feature based classification in ambulatory monitoring.

TABLE II  
EFFECTIVENESS OF THE PROPOSED METHOD IN RESTORING THE PEAK-TO-PEAK VALUES

Method	Average $E_{p-p}$ %
<b>cICA+LMS [13]</b>	9.406 %
<b>FFT+AS-LMS [14]</b>	17.75 %
<b>MAF [15]</b>	14.96 %
<b>NN+Spline</b>	6.19 %

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