Thermal History of the Universe and the Cosmic Microwave Background.
I. Thermodynamics of the Hot Big Bang

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Part 1: Thermodynamics of the Hot Big Bang

1. assumptions
2. properties of ideal quantum gases
3. adiabatic expansion of ideal gases
4. particle freeze-out
5. neutrino background
6. photons and baryons
7. physics of recombination
8. nucleosynthesis
9. the isotropic microwave background

1. Assumptions

1. adiabatic expansion
2. thermal equilibrium
3. ideal gases

1.1. Adiabatic Expansion

The Universe expands adiabatically.

- isotropy requires it to expand adiathermally: no heat can flow because any flow would define a preferred direction

- an adiathermal expansion is adiabatic if it is reversible, but irreversible processes may occur

- the entropy of the Universe is dominated by far by the cosmic microwave background, so entropy generation by irreversible processes is negligible

1.2. Thermal Equilibrium

Thermal equilibrium can be maintained despite the expansion.

- thermal equilibrium can only be maintained if the interaction rate of particles is higher than the expansion rate of the Universe
- the expansion rate of the Universe is highest at early times, so thermal equilibrium may be difficult to maintain as $t \to 0$
- nonetheless, for $t \to 0$, particle densities grow so fast that interaction rates are indeed higher than the expansion rate
- as the Universe expands, particle species drop out of equilibrium

1.3. Ideal Gases

Cosmic “fluids” can be treated as ideal gases.

- ideal gas: no long-range interactions between particles, interact only by direct collisions
- obviously good approximation for weakly interacting particles like neutrinos
- even valid for charged particles because oppositely charged particles shield each other
- consequence: internal energy of ideal gas does not depend on volume occupied
- cosmic “fluids” can be treated as possibly relativistic quantum gases

2. Properties of Ideal Quantum Gases

1. occupation number
2. number density
3. energy density
4. grand canonical potential
5. pressure
6. entropy density
7. results for bosons and fermions
8. numbers

2.1. Occupation Number

- in thermal equilibrium with a heat bath of temperature T, the phase-space density for a quantum state of energy ε is

$$\langle f \rangle = \left\{ \exp\left[-\frac{\varepsilon - \mu}{kT} \right] \pm 1 \right\}^{-1}$$

(“$+$” for fermions, “$-$” for bosons)

- in thermal equilibrium with a radiation background, the chemical potential $\mu = 0$: Helmholtz free energy $F(T,V,N) = E - TS$ is minimised in equilibrium for a system at constant T and V, so from $\mathrm{d}F = -S\mathrm{d}T - P\mathrm{d}V + \mu\mathrm{d}N$:

$$\frac{\partial F}{\partial N} = 0 = \mu$$

2.2. Number Density

- particles in volume V, number of states in k-space element:

$$\mathrm{d}N = g\frac{V}{(2\pi)^3}\mathrm{d}^3k$$

- g is statistical weight, e.g. spin degeneracy factor
- momentum $\vec{p} = \hbar\vec{k}$, related to energy by

$$\varepsilon(p) = \sqrt{c^2p^2 + m^2c^4}$$

- spatial number density in thermal equilibrium:

$$n = \frac{g}{(2\pi\hbar)^3}\int_0^\infty \frac{4\pi p^2\mathrm{d}p}{\exp[\varepsilon(p)/kT] \pm 1}$$

2.3. Energy Density

- mean energy density: number of states per phase-space cell, times occupation number, times energy per state, integrated over momentum space:

$$u = \frac{g}{(2\pi\hbar)^3} \int_0^\infty \frac{4\pi p^2 \varepsilon(p)\, \mathrm{d}p}{\exp[\varepsilon(p)/kT] \pm 1}$$

- integrals can most easily be carried out by substituting a geometrical series:

$$\int_0^\infty \frac{x^m \mathrm{d}x}{e^x - 1} = \int_0^\infty \frac{x^m e^{-x} \mathrm{d}x}{1 - e^{-x}} = \int_0^\infty \mathrm{d}x x^m e^{-x} \sum_{n=0}^\infty e^{-nx} = \sum_{n=1}^\infty \int_0^\infty \mathrm{d}x x^m e^{-nx} = m!\zeta(m+1)\;;$$

for fermions, use $\dfrac{1}{e^x + 1} = \dfrac{1}{e^x - 1} - \dfrac{2}{e^{2x} - 1}$

2.4. Grand Canonical Potential

- extremal principles yield: (thermodynamical potential) $= -kT\ln($partition sum$)$
- adequate potential here: grand canonical potential $\Phi(T,V,\mu)$ (particle numbers can change)
- grand-canonical partition sum:

$$Z = \sum_{N=0}^{\infty} e^{\mu N/kT} \sum_{\{N_\alpha|N\}} e^{-\sum \varepsilon_\alpha N_\alpha/kT}$$

- grand-canonical potential:

$$\Phi(T,V,\mu) = \mp kT\frac{gV}{(2\pi\hbar)^3}\int_0^\infty \mathrm{d}p 4\pi p^2 \ln\left[1 \pm e^{\mu/kT} e^{-\varepsilon(p)/kT}\right] \quad \left(\begin{array}{c}\text{Fermions}\\ \text{Bosons}\end{array}\right)$$

2.5. Pressure

- Helmholtz free energy:

$$F(T,V,N) = U - TS$$

- grand-canonical potential:

$$\Phi(T,V,\mu) = F - \mu N = U - TS - \mu N$$

- thermodynamical Euler relation:

$$U = TS - PV + \mu N$$

- it follows:

$$\Phi = -PV \;\Rightarrow\; P = -\frac{\Phi}{V}$$

- example: relativistic boson gas in thermal equilibrium; $\varepsilon = cp$, $\mu = 0$,

$$P_{\mathrm{B}} = g\,\frac{\pi^2}{90}\,\frac{(kT)^4}{(\hbar c)^3}$$

2.6. Entropy Density

- Helmholtz free energy:

$$\mathrm{d}F(T,V,N) = -S\mathrm{d}T - P\mathrm{d}V + \mu\mathrm{d}N$$

- grand-canonical potential:

$$\mathrm{d}\Phi(T,V,\mu) = -S\mathrm{d}T - P\mathrm{d}V - N\mathrm{d}\mu$$

- thus, the entropy is:

$$S = -\frac{\partial\Phi}{\partial T}$$

- example: entropy density for relativistic boson gas in thermal equilibrium

$$s = \frac{S}{V} = gk\frac{2\pi^2}{45}\left(\frac{kT}{\hbar c}\right)^3$$

2.7. Results for Bosons and Fermions

	relativistic Bosons	relativistic Fermions	non-relativistic
n	$g_\mathrm{B}\dfrac{\zeta(3)}{\pi^2}\left(\dfrac{kT}{\hbar c}\right)^3$	$\dfrac{3}{4}\dfrac{g_\mathrm{F}}{g_\mathrm{B}}n_\mathrm{B}$	$g\left(\dfrac{kT}{2\pi\hbar}\right)^{3/2} e^{-kT/mc^2}$
u	$g_\mathrm{B}\dfrac{\pi^2}{30}\dfrac{(kT)^4}{(\hbar c)^3}$	$\dfrac{7}{8}\dfrac{g_\mathrm{F}}{g_\mathrm{B}}u_\mathrm{B}$	$\dfrac{3}{2}nkT$
P	$g_\mathrm{B}\dfrac{\pi^2}{90}\dfrac{(kT)^4}{(\hbar c)^3}=\dfrac{u_\mathrm{B}}{3}$	$\dfrac{7}{8}\dfrac{g_\mathrm{F}}{g_\mathrm{B}}P_\mathrm{B}$	nkT
s	$g_\mathrm{B}k\dfrac{2\pi^2}{45}\left(\dfrac{kT}{\hbar c}\right)^3$	$\dfrac{7}{8}\dfrac{g_\mathrm{F}}{g_\mathrm{B}}s_\mathrm{B}$	

2.8. Numbers

note: $1\,\mathrm{eV} = 1.6 \times 10^{-12}\,\mathrm{erg}$ correspond to $kT = 1.16 \times 10^{4}\,\mathrm{K}$

- $$n_\mathrm{B} = 10 g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3} = 1.6 \times 10^{13} g_\mathrm{B} \left(\frac{kT}{\mathrm{eV}}\right)^3 \mathrm{cm}^{-3}$$

- $$u_\mathrm{B} = 3.8 \times 10^{-15} g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^4 \frac{\mathrm{erg}}{\mathrm{cm}^3} = 2.35 \times 10^{-3} g_\mathrm{B} \left(\frac{kT}{\mathrm{eV}}\right)^4 \frac{\mathrm{erg}}{\mathrm{cm}^3}$$

- $$\frac{s_\mathrm{B}}{k} = 36 g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3} = 5.7 \times 10^{13} g_\mathrm{B} \left(\frac{kT}{\mathrm{eV}}\right)^3 \mathrm{cm}^{-3}$$

3. Adiabatic Expansion of Ideal Gases

1. temperature
2. density
3. matter-radiation equality

3.1. Temperature

- for relativistic boson or fermion gases:

$$P = \frac{u}{3} = \frac{E}{3V}$$

- first law of thermodynamics, $\mathrm{d}E + P\mathrm{d}V = 0$, then implies:

$$\mathrm{d}E = -P\mathrm{d}V = 3\mathrm{d}(PV) \Rightarrow P \propto V^{-4/3}$$

(*adiabatic index* is $\gamma = 4/3$)

- for non-relativistic ideal gas: $\gamma = 5/3$

- since $P \propto T^4$ for relativistic gases:

$$T \propto V^{-1/3} \propto a^{-1}$$

(a is cosmological scale factor)

- for non-relativistic ideal gas:

$$T \propto PV \propto V^{-5/3+1} \propto a^{-2}$$

3.2. Density

- non-relativistic gases:
  rest mass energy $\gg$ kinetic energy
  mass density:

$$\Rightarrow \rho \propto V^{-1} \propto a^{-3}$$

- relativistic gases: mass density

$$\rho = \frac{u}{c^2} \propto T^4 \propto a^{-4}$$

- density of relativistic particles drops faster than that of non-relativistic particles:

- dilution due to volume expansion, plus energy loss due to redshift

3.3. Matter-Radiation Equality

- today's matter density in the Universe:

$$\rho_0 = \Omega_0 \frac{3H_0^2}{8\pi G} = 1.9 \times 10^{-29}\,\Omega_0 h^2\,\mathrm{g\,cm^{-3}}$$

- today's radiation density in the Universe:

$$T = 2.73\,\mathrm{K} \;\Rightarrow\; \rho_{\mathrm{R},0} = \frac{u}{c^2} = 4.7 \times 10^{-34}\,\mathrm{g\,cm^{-3}}$$

- radiation domination:

$$\rho_{\mathrm{R}}(a_{\mathrm{eq}}) = \rho_{\mathrm{R},0} a_{\mathrm{eq}}^{-4} = \rho_0 a_{\mathrm{eq}}^{-3} = \rho(a_{\mathrm{eq}}) \;\Rightarrow\; a_{\mathrm{eq}} = \frac{\rho_{\mathrm{R},0}}{\rho_0} = 2.5 \times 10^{-5}\,(\Omega_0 h^2)^{-1}$$

4. Particle Freeze-Out

1. interaction rates

2. freeze-out conditions

3. particle distributions after freeze-out

4.1. Interaction Rates

- expansion timescale:

$$t_{\mathrm{exp}} \sim (G\rho)^{-1/2}$$

ρ is density of dominant fluid

- in early Universe, radiation dominates, thus

$$\rho \propto a^{-4} \;\Rightarrow\; t_{\mathrm{exp}} \propto a^2$$

expansion timescale is shortest at early times and increases as a^2

- thermal equilibrium maintained by two-body interactions, collision rate for one particle species is

$$\Gamma \propto n \propto T^3 \propto a^{-3}$$

- thus, the collision time scale is

$$t_{\mathrm{coll}} \propto \Gamma^{-1} \propto a^3$$

- for $a \to 0$, t_{coll} decreases faster than t_{exp}, so equilibrium can be maintained

4.2. Freeze-Out Conditions

- continuity equation in absence of collisions:

$$\dot{n} + 3Hn = 0$$

- collision rate:

$$\Gamma = \langle \sigma v \rangle \, n$$

- source term from thermal particle creation:

$$S = \langle \sigma v \rangle \, n_{\mathrm{T}}^2$$

- continuity equation now:

$$\dot{n} + 3Hn = -\Gamma n + S$$

- introduce comoving number density $N = a^3 n$:

$$\frac{\mathrm{d} \ln N}{\mathrm{d} \ln a} = -\frac{\Gamma}{H} \left(1 - \frac{N_{\mathrm{T}}}{N} \right)$$

- freeze-out can occur if $\Gamma \ll H$: particle number N cannot adapt

4.3. Particle Distributions After Freeze-Out

- relativistic particles:

$$n \propto T^3 \;\Rightarrow\; N = a^3 n = \text{const.}$$

- from freeze-out equation:

$$\frac{d\ln N}{d\ln a} = 0 \;\Rightarrow\; N = N_\mathrm{T}$$

independent of Γ!

- relativistic particles maintain thermal distribution even after freeze-out

- non-relativistic particles, comoving number density:

$$N_\mathrm{T} \propto T^{-3/2} \exp\left(-\frac{mc^2}{kT}\right)$$

- once $T \ll mc^2$, N_T drops rapidly, hence $N_\mathrm{T} \ll N$, and

$$\frac{d\ln N}{d\ln a} = -\frac{\Gamma}{H} \rightarrow 0$$

particle production stops: freeze-out

5. Neutrino Background

1. neutrino decoupling

2. electron-positron annihilation

3. photon heating

5.1. Neutrino Decoupling

- neutrinos are kept in thermal equilibrium by the weak interaction

$$\nu + \bar{\nu} \leftrightarrow e^+ + e^-$$

- this interaction freezes out when the temperature drops to

$$T_\nu \sim 10^{10.5}\,\mathrm{K} \sim 2.7\,\mathrm{MeV}$$

- neutrinos are ultrarelativistic at that "time", i.e. their comoving number density is that of an ideal, relativistic fermion gas

5.2. Electron-Positron Annihilation

- the reaction

$$e^+ + e^- \leftrightarrow 2\gamma$$

  is suppressed once temperature drops below

$$T \sim 2m_e c^2 \approx 1\,\mathrm{MeV} \approx 10^{10}\,\mathrm{K}$$

- thus, electrons and positrons annihilate shortly after neutrinos freeze out

- the entropy of the annihilating electron-positron gas heats the photons, but not the neutrinos

- the microwave background is heated above the neutrino background by the annihilation

5.3. Photon Heating

- entropy before annihilation equals entropy after annihilation:

$$s'_{e^+} + s'_{e^-} + s'_\gamma = s_\gamma$$

- before annihilation,

$$T_{e^+} = T_{e^-} = T_\gamma \equiv T'$$

thus

$$s'_{e^+} = s'_{e^-} = \frac{7}{8} s'_\gamma$$

$(g_{e^+} = g_{e^-} = g_\gamma = 2)$

- since $s \propto T^3$,

$$\left(2 \cdot \frac{7}{8} + 1\right) (T')^3 = T^3$$

- temperature after annihilation:

$$T = \left(\frac{11}{4}\right)^{1/3} T' \approx 1.4\, T'$$

- temperature of ν background today:

$$T_{\nu,0} = 1.95\,\mathrm{K}$$

6. Photons and Baryons

1. baryon number

2. photon-baryon ratio

6.1. Baryon Number

- number density of baryons today:

$$n_\mathrm{B} \approx \frac{\rho_\mathrm{B}}{m_\mathrm{p}} = \frac{\Omega_\mathrm{B}}{m_\mathrm{p}} \frac{3H_0^2}{8\pi G} = 1.1 \times 10^{-5}\,\Omega_\mathrm{B} h^2\,\mathrm{cm}^{-3}$$

- measurements: abundance ratios of light elements, microwave background (see later), gas in galaxy clusters:

$$\Omega_\mathrm{B} h^2 \approx 0.025$$

- only $\sim 10\% - 20\%$ of the matter in the Universe is contributed by baryons
- baryon number density $\propto a^{-3} \propto T^3$

6.2. Photon-Baryon Ratio

- number density of photons today:

$$n_\gamma = 407\,\mathrm{cm}^{-3}$$

- scales with temperature like T^3 like baryon number density, hence their ratio is independent of time

- constant baryon-photon-ratio:

$$\eta = \frac{n_\mathrm{B}}{n_\gamma} = 2.7\times 10^{-8}\,\Omega_\mathrm{B} h^2$$

- there is one baryon for about a billion photons in the Universe

- photon entropy completely dominates entropy of the Universe

- very important for process of recombination

7. Physics of Recombination

1. recombination process
2. Saha equation
3. recombination temperature
4. two-photon recombination
5. thickness of the recombination shell
6. expectation on radiation spectrum

7.1. Recombination Process

- as the temperature drops, electrons and protons can combine to form atoms:

$$e^- + p^+ \leftrightarrow H + \gamma$$

  freezes out

- How does recombination proceed, i.e. how does the electron density change with T?

- solution: minimisation of Helmholtz free energy $F(T,V,N)$

- canonical partition function Z,

$$F = -kT \ln Z$$

  with

$$Z = \frac{Z_\mathrm{e}^{N_\mathrm{e}}}{N_\mathrm{e}!} \frac{Z_\mathrm{p}^{N_\mathrm{p}}}{N_\mathrm{p}!} \frac{Z_\mathrm{H}^{N_\mathrm{H}}}{N_\mathrm{H}!}$$

- baryon number $N_\mathrm{B} = N_\mathrm{p} + N_\mathrm{H}$, electron number $N_\mathrm{e} = N_\mathrm{p}$, thus $N_\mathrm{H} = N_\mathrm{B} - N_\mathrm{e}$

- Stirling: $\ln N! \approx N \ln N - N$

7.2. Saha Equation

- from $\partial F/\partial N_\mathrm{e} = 0$:

$$\frac{N_\mathrm{e}^2}{N_\mathrm{B} - N_\mathrm{e}} = \frac{Z_\mathrm{e} Z_\mathrm{p}}{Z_\mathrm{H}}$$

- partition function for single species:

$$Z = \frac{4\pi g V}{(2\pi\hbar)^3} \int_0^\infty \mathrm{d}p\, p^2 e^{(\mu-\varepsilon)/kT}$$

- nonrelativistic limit:

$$\varepsilon = mc^2 + \frac{p^2}{2m}$$

- chemical potential:

$$\mu = 0 \;\Rightarrow\; \mu_\mathrm{e} + \mu_\mathrm{p} = \mu_\mathrm{H}$$

- ionisation potential:

$$(m_\mathrm{e} + m_\mathrm{p} - m_\mathrm{H})c^2 = \chi = 13.6\,\mathrm{eV}$$

- $x = N_\mathrm{e}/N_\mathrm{B}$, $n_\mathrm{B} = N_\mathrm{B}/V$; Saha equation:

$$\frac{x^2}{1-x} = \frac{(2\pi m_\mathrm{e} kT)^{3/2}}{(2\pi\hbar)^3 n_\mathrm{B}} e^{-\chi/kT}$$

7.3. Recombination Temperature

- n_B is the baryon number density; we had

$$n_\mathrm{B} = \eta n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3$$

- insert this into Saha equation:

$$\frac{x^2}{1-x} \approx \frac{0.26}{\eta} \left(\frac{m_\mathrm{e} c^2}{kT}\right)^{3/2} e^{-\chi/kT}$$

- want $x \ll 1$, hence $x^2/(1-x) \approx x^2$

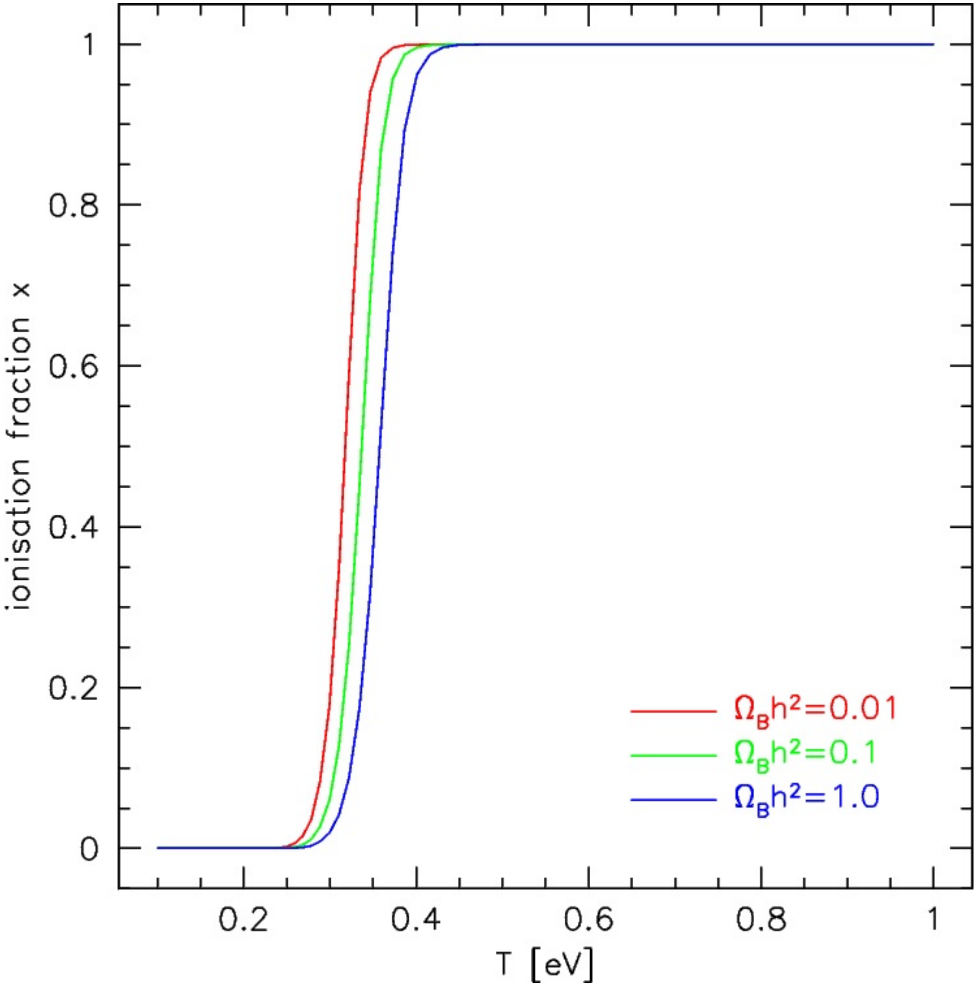
- $1/\eta$ is *huge* number, so $kT \ll \chi$ required for x to be small

- putting $x = 0.1$ yields $kT_\mathrm{rec} \approx 0.3\,\mathrm{eV}$, or

$$T_\mathrm{rec} \approx 3500\,\mathrm{K}$$

- high photon number density delays recombination considerably

7.3. Recombination Temperature



7.4. Two-Photon Recombination

- direct hydrogen recombination produces energetic photons; last step to ground state is Lyman-α $(2P \rightarrow 1S)$;

$$h\nu \geq E_{\mathrm{Ly}\alpha} = \frac{3}{4}\chi = 10.2\,\mathrm{eV}$$

- abundant Ly-α photons can reionise the cosmic gas
- photons cannot be lost as in clouds; energy loss due to cosmic expansion is slow
- How can recombination proceed at all?
- production of lower-energy photons: forbidden transition $2S \rightarrow 1S$, requires emission of two photons; this process is slow
- recombination proceeds at a somewhat slower rate than predicted by the Saha equation

7.5. Thickness of Recombination Shell

- recombination is not instantaneous; “time” interval between beginning and end?

- time: measured in terms of scale factor a, or redshift $1+z=a^{-1}$

- optical depth through recombination shell:

$$\tau = \int n_\mathrm{e} x \sigma_\mathrm{T} \mathrm{d}r$$

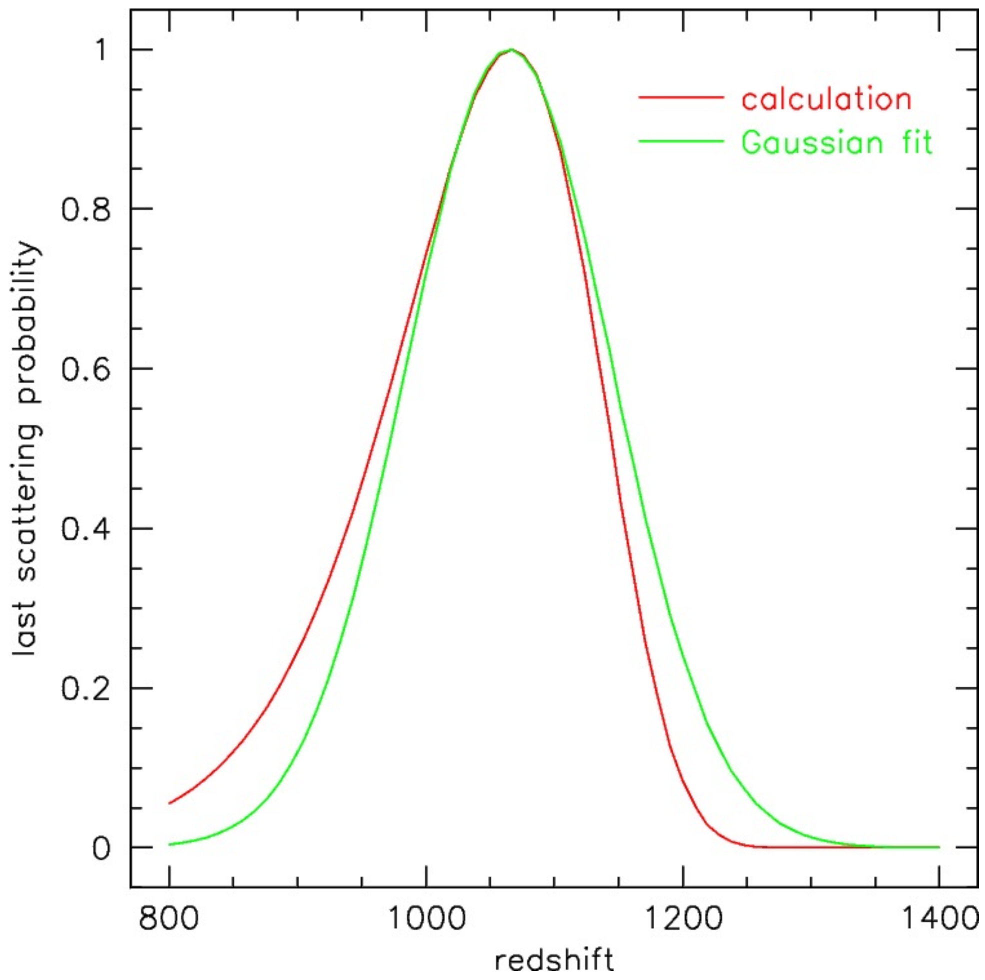
(σ_T: Thomson cross section)

- scattering probability for photons when travelling from z to $z-\mathrm{d}z$:

$$p(z)\,\mathrm{d}z = e^{-\tau}\frac{\mathrm{d}\tau}{\mathrm{d}z}\mathrm{d}z$$

- probability distribution $e^{-\tau}\mathrm{d}\tau/\mathrm{d}z$ is well described by Gaussian with mean $\bar{z}\sim 1100$ and standard deviation $\delta z\sim 80$

7.5. Thickness of Recombination Shell



7.6. Expectation on Radiation Spectrum

- last-scattering shell has finite width: CMB photons received today were released at different redshifts

- cosmic plasma cooled during recombination, photons were released at different mean temperatures: $T = T_0(1+z)$, thus

$$\delta T = T_0 \delta z \approx 200\,\mathrm{K}$$

- photons were redshifted after release: those released earlier, i.e. from hotter plasma, were redshifted by larger amount: $E = E_0(1+z)$

- these effects cancel *if* plasma temperature depends on scale factor like $T \propto a^{-1}$; *then*: Planck spectrum of single temperature expected

8. Nucleosynthesis

1. formation of the light elements

2. Gamow criterion

3. prediction of the microwave background temperature

8.1. Formation of the Light Elements

- high temperature in the early Universe allows nuclear reactions like in stars; density is much lower, so higher temperatures are required:

$$T_{\text{nuc}} \sim 10^9\,\text{K}$$

- before: neutrons and protons formed; equilibrium through

$$p + e^- \leftrightarrow n + \nu\,,\; p + \bar{\nu} \leftrightarrow n + e^+$$

- relative abundance freezes out once weak interaction becomes too slow;

$$T_{\text{freeze-out}} \sim 1.4 \times 10^{10}\,\text{K}$$

afterwards, free neutrons decay

- nucleosynthesis proceeds through strong interactions until these freeze out, e.g. deuterium formation $n + p \leftrightarrow \text{D} + \gamma$ stops at

$$T_{\text{D}} \sim 7.9 \times 10^8\,\text{K}$$

8.2. Gamow Criterion

- deuterium fusion is the most important step towards the fusion of higher elements; e.g. Helium:

$$\mathrm{D} + p \longrightarrow {}^3\mathrm{He} + \gamma\,,\ {}^3\mathrm{He} + n \longrightarrow {}^4\mathrm{He}$$

- deuterium needs to be produced in sufficient abundance for higher elements to form, but if all neutrons are immediately locked up into deuterium, no higher elements can form either

- George Gamow noticed in 1948 that deuterium formation has to proceed at “just right” rate:

$$n_\mathrm{B}\,\langle\sigma v\rangle\,t \sim 1\,,$$

  i.e. collision rates for baryons should not be too small or too large

8.3. Prediction of CMB Temperature

- for deuterium formation, $t \sim 3\,\text{min}$

- from a theoretical estimate for $\langle \sigma v \rangle$, Gamow estimated the baryon density at deuterium formation:

$$n_{\rm B} \sim 10^{18}\,\text{cm}^{-3}$$

- today's baryon density is

$$n_{\rm B,0} \sim 1.1 \times 10^{-5}\,\Omega_{\rm B} h^2\,\text{cm}^{-3}$$

- baryon density drops like T^3; thus:

$$\frac{n_{\rm B}}{n_{\rm B,0}} = \left(\frac{T_{\rm D}}{T_0}\right)^3$$

- using $T_{\rm D} = 7.9 \times 10^8\,\text{K}$:

$$T_0 = \left(\frac{n_{\rm B,0}}{n_{\rm B}}\right)^{1/3} T_{\rm D} \sim 4\,\text{K}$$

CMB temperature is predicted by the Big-Bang model!

9. The Isotropic Microwave Background

1. temperature and spectrum

2. limits on chemical potential and Compton parameter

3. the dipole

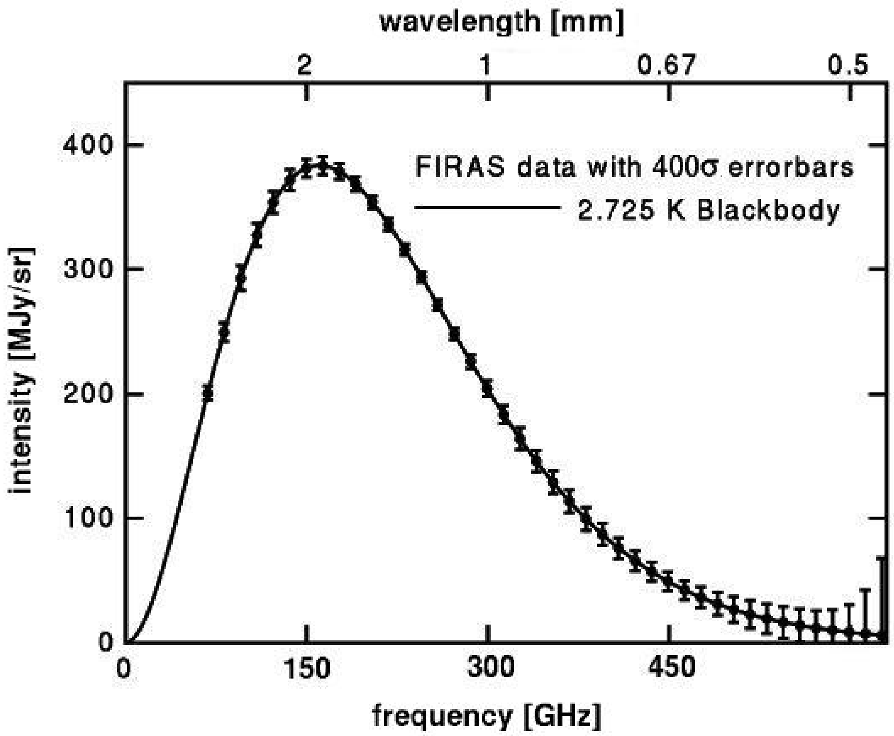
9.1. Temperature and Spectrum

- microwave background was detected, but not recognised, by Penzias and Wilson in 1965

- NASA's Cosmic Background Explorer (COBE) precisely measured its spectrum (and other things, see below)

- its temperature is $T_0 = (2.726 \pm 0.002)\,\mathrm{K}$

- its spectrum is the best black-body spectrum ever measured



9.2. Limits on μ and y

- finite width of the last-scattering surface: it is important that the CMB has a black-body spectrum

- shape of the spectrum allows constraints on the chemical potential:

$$|\mu| \leq 9 \times 10^{-5}$$

- hot gas between the last-scattering surface and us can distort the spectrum through Compton scattering

- Compton-y parameter:

$$y = \frac{kT}{m_\mathrm{e}c^2} \int n_\mathrm{e}\sigma_\mathrm{T}\mathrm{d}l$$

(typical energy change times scattering probability)

- constraint from COBE-FIRAS spectrum strictly contrains heat input in young Universe:

$$y \leq 1.5 \times 10^{-5}$$

9.3. Dipole

- "freely falling" observers in the Universe define comoving coordinates
- the CMB is at rest with respect to this coordinate frame
- Earth moves around the Sun, Sun around the centre of the Galaxy, Galaxy within the Local Group etc.
- motion causes temperature dipole:

$$T(\theta) = T_0\left(1 + \frac{v}{c}\cos\theta\right) + O(v^2/c^2)$$

- COBE has measured the dipole:

$$v = (371 \pm 1)\,\mathrm{km\,s}^{-1}$$

towards

$$l = (264.3 \pm 0.2)^\circ\ ,\ b = (48.1 \pm 0.1)^\circ$$