||JAI SRI GURUDEV||

S.J.C. INSTITUTE OF TECHNOLOGY, CHICKBALLAPUR

Department of Mathematics

LECTURE NOTES

MATHEMATICS-3 FOR COMPUTER SCIENCE STREAM (BCS301)

MODULE - 5

DESIGN OF EXPERIMENTS AND ANOVA

Prepared by:

Purushotham P
Assistant Professor
SJC Institute of Technology
Email id: purushotham@sjcit.ac.in
Phone: 7338005244

Experimental unit:

For conducting an experiment, the experimental material is divided into smaller parts and each part is referred to as an experimental unit. The experimental unit is randomly assigned to treatment is the experimental unit. The phrase “randomly assigned” is very important in this definition.

Experiment:

A way of getting an answer to a question which the experimenter wants to know.

Treatment

Different objects or procedures which are to be compared in an experiment are called treatments.

Sampling unit:

The object that is measured in an experiment is called the sampling unit. This may be different from the experimental unit.

Factor:

A factor is a variable defining a categorization. A factor can be fixed or random in nature. A factor is termed as a fixed factor if all the levels of interest are included in the experiment. A factor is termed as a random factor if all the levels of interest are not included in the experiment and those that are can be considered to be randomly chosen from all the levels of interest.

Replication:

It is the repetition of the experimental situation by replicating the experimental unit.

Experimental error:

The unexplained random part of the variation in any experiment is termed as experimental error. An estimate of experimental error can be obtained by replication.

Treatment design:

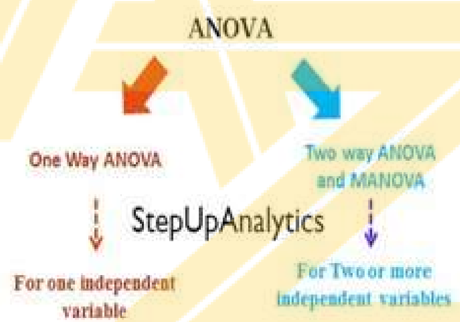
A treatment design is the manner in which the levels of treatments are arranged in an experiment.

ANOVA:

Analysis of variance (ANOVA) is an analysis tool used in statistics that splits an observed aggregate variability found inside a data set into two parts: systematic factors and random factors. The systematic factors have a statistical influence on the given data set, while the random factors do not.

ANOVA stands for Analysis of Variance. It is a statistical method used to analyze the differences between the means of two or more groups or treatments. It is often used to determine whether there are any statistically significant differences between the means of different groups

There are two main types of ANOVA: one-way (or unidirectional) and two-way. There also variations of ANOVA.



Real Life Applications of ANOVA:

- In social sciences, ANOVA tests can be used to study the statistical significance of various study environments on test scores. Medical research. In medical research, the ANOVA test can be used to identify the relationship between various types or brands of medications on individuals with migraines or depression.
- We can use the ANOVA test to compare different suppliers and select the best available. ANOVA (Analysis of Variance) is used when we have more than two sample groups and determine whether there are any statistically significant differences between the means of two or more independent sample groups.

CRD: A completely randomized design (CRD) is one where the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment.

RBD: A randomized block design is a restricted randomized design, in which experimental units are first organized into homogeneous blocks and then the treatments are assigned at random to these units within these blocks. The main advantage of this design is, if done properly, it provides more precise results.

LSD: The Latin Square Design gets its name from the fact that we can write it as a square with Latin letters to correspond to the treatments. The treatment factor levels are the Latin letters in the Latin square design. The number of rows and columns has to correspond to the number of treatment levels.

ONE WAY CLASSIFICATION:

- Define the problem for different varieties and different treatments.

Verities					Sum	Squares
x_{11}	x_{12}	x_{13}	$\dots$	x_{1n_1}	T_1	${T_1}^2$
x_{21}	x_{22}	x_{23}	$\dots$	x_{2n_2}	T_2	${T_2}^2$
x_{31}	x_{32}	x_{33}	$\dots$	x_{3n_3}	T_3	${T_3}^2$
---	---	----	$\dots$	---		---
x_{k1}	x_{k2}	x_{k3}	$\dots$	x_{kn_k}	T_k	${T_k}^2$

- Define the null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$ for the level of significance.
- Find the sum of all the verities (Row wise) Find the sum of all the contents of N varieties, say T.
- Find the correction factor $CF = \frac{T^2}{N}$
- Find the sum of squares of individual items $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$
- Find the sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$
- Find the sum of squares with in the class or sum of squares due to error by subtraction SEE=TSS-SST.
- Here k represents total number of verities, N represents the total number of observations.
- Plot the ANOVA table

Sources variation	d.f	SS	MSS	F Ratio
Between treatments	k-1	SST	$MST = \frac{SST}{k-1}$	$F = \frac{MST}{MSE}$
Error	N-k	SSE	$MSE = \frac{SSE}{N-k}$	
Total	N-1	-	-	

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11

PROBLEMS:

1. Three processes A, B and C are tested to see whether their outputs are equivalent. The following observations of outputs are made:

A	10	12	13	11	10	14	15	13
B	9	11	10	12	13	-	-	-
C	11	10	15	14	12	13	-	-

Carry out the analysis of variance and state your conclusion.

Sol. To carry out the analysis of variance, we form the following tables

									Total	Squares
A	10	12	13	11	10	14	15	13	T_1=98	T^2_1=9604
B	9	11	10	12	13				T_2=55	T^2_2=3025
C	11	10	15	14	12	13			T_3=75	T^2_3=5625
Total T									228	-

The squares are as follows

									Sum of Squares
A	100	144	169	121	100	196	225	169	1224
B	81	121	100	144	169				615
C	121	100	225	196	144	169			955
Grand Total - $\sum_i \sum_j {x_{ij}}^2$									2794

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(228)^2}{19} = \frac{51984}{19} = 2736$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 2794 - 2736$$
$$\Rightarrow TSS = 58$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{9604}{8} + \frac{3025}{5} + \frac{5625}{6} - 2736$$
$$\Rightarrow SST = 1200.5 + 605 + 937.5 - 2736$$
$$\Rightarrow SST = 2743 - 2736$$
$$\Rightarrow SST = 7$$

Therefore, sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 58 - 7$$
$$\Rightarrow SSE = 51$$

Sources variation	**d.f.**	**SS**	**MSS**	**F Ratio**
Between treatments	3-1=2	SST=7	$MST = \frac{7}{2} = 3.5$	$F = \frac{3.5}{3.1875} = 1.0980$
Error	19-3=16	SSE=51	$MSE = \frac{51}{16} = 3.1875$	
Total	19-1=18	-	-	

Since evaluated value 1.0980<3.63 for F(2,16) at 5% level of significance
Hence the null hypothesis is accepted, there is no significance between the three process.

2. A test was given to five students taken at random from the fifth class of three schools of a town. The individual scores are

School I	9	7	6	5	8
School II	7	4	5	4	5
School III	6	5	6	7	6

Carry out the analysis of variance.

Sol.

To carry out the analysis of variance, we form the following tables

						Total	Squares
S1	9	7	6	5	8	T_1=35	T^2_1=1225
S2	7	4	5	4	5	T_2=25	T^2_2=625
S3	6	5	6	7	6	T_3=30	T^2_3=900
			Total T=			90	-

The squares are as follows

						Sum of Squares
S1	81	49	36	25	64	255
S2	49	16	25	16	25	131
S3	36	25	36	49	36	182
		Grand Total - $\sum_i \sum_j {x_{ij}}^2$				568

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(90)^2}{15} = \frac{8100}{15} = 540$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 568 - 540$$
$$\Rightarrow TSS = 28$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{1225}{5} + \frac{625}{5} + \frac{900}{5} - 540$$
$$\Rightarrow SST = 245 + 125 + 180 - 540$$
$$\Rightarrow SST = 550 - 540$$
$$\Rightarrow SST = 10$$

Therefore sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 28 - 10 \Rightarrow SSE = 18$$

Sources variation	**d.f.**	**SS**	**MSS**	**F Ratio**
Between treatments	3-1=2	SST=10	$MST = \frac{10}{2} = 5$	$F = \frac{5}{1.5} = 3.33$
Error	15-3=12	SSE=18	$MSE = \frac{18}{12} = 1.5$	
Total	15-1=14	-	-	

Since evaluated value 3.33<3.63 for F(2,12) at 5% level of significance

Hence the null hypothesis is accepted, there is no significance between the three process.

3. Three different kinds of food are tested on three groups of rats for 5 weeks. The objective is to check the difference in mean weight (in grams) of the rats per week. Apply one-way ANOVA using a 0.05 significance level to the following data:

Food 1	8	12	19	8	6	11
Food 2	4	5	4	6	9	7
Food 3	11	8	7	13	7	9

Sol. To carry out the analysis of variance, we form the following tables

							Total	Squares
F1	8	12	19	8	6	11	T_1=64	T^2_1=4096
F2	4	5	4	6	9	7	T_2=35	T^2_2=1225
F3	11	8	7	13	7	9	T_3=55	T^2_3=3025
			Total T				154	-

The squares are as follows

							Sum of Squares
F1	64	144	361	64	36	121	790
F2	16	25	16	36	81	49	223
F3	121	64	49	169	49	81	533
			Grand Total - $\sum_i \sum_j {x_{ij}}^2$				1546

Set the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(154)^2}{18} = \frac{23716}{18} = 1317.55$

Therefore Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 1546 - 1317.55$$
$$\Rightarrow TSS = 228.45$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{4096}{6} + \frac{1225}{6} + \frac{3025}{6} - 1317.55$$
$$\Rightarrow SST = 682.66 + 204.166 + 504.166 - 1317.55$$
$$\Rightarrow SST = 1391 - 1317.55$$
$$\Rightarrow SST = 73.45$$

Therefore, sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 228.45 - 73.45 \Rightarrow SSE = 155$$

Sources variation	d.f.	SS	MSS	F Ratio
Between treatments	3-1=2	SST=73.45	$MST = \frac{73.45}{2} = 36.725$	$F = \frac{36.725}{10.33} = 3.55$
Error	18-3=15	SSE=155	$MSE = \frac{155}{15} = 10.33$	
Total	18-1=17	-	-	

Since evaluated value 3.55 <3.68 for F(2,15) at 5% level of significance

Hence the null hypothesis is accepted , there is no significance between the three process.

4. Three types of fertilizers are used on three groups of plants for 5 weeks. We want to check if there is a difference in the mean growth of each group. Using the data given below apply a one-way ANOVA test at 0.05 significant level

Fertilizer 1	6	8	4	5	3	4
Fertilizer 2	8	12	9	11	6	8
Fertilizer 3	13	9	11	8	7	12

Sol.

To carry out the analysis of variance, we form the following tables

							Total	Squares
F1	6	8	4	5	3	4	T_1=30	T^2_1=900
F2	8	12	9	11	6	8	T_2=54	T^2_2=2916
F3	13	9	11	8	7	12	T_3=60	T^2_3=3600
Total T							144	-

The squares are as follows

							Sum of Squares
F1	36	64	16	25	9	16	166
F2	64	144	81	121	36	64	510
F3	169	81	121	64	49	144	628
Grand Total - $\sum_i \sum_j {x_{ij}}^2$							1304

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(144)^2}{18} = \frac{20736}{18} = 1152$

Therefore Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 1304 - 1152$$
$$\Rightarrow TSS = 152$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{900}{6} + \frac{2916}{6} + \frac{3600}{6} - 1152$$
$$\Rightarrow SST = 150 + 486 + 600 - 1152$$
$$\Rightarrow SST = 1236 - 1152$$
$$\Rightarrow SST = 84$$

Therefore sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 152 - 84 \Rightarrow SSE = 68$$

Sources variation	d.f.	SS	MSS	F Ratio
Between treatments	3-1=2	SST=84	$MST = \frac{84}{2} = 42$	$F = \frac{42}{4.533} = 9.2653$
Error	18-3=15	SSE=68	$MSE = \frac{68}{15} = 4.533$	
Total	18-1=17	-	-	

Since evaluated value 9.2653 >3.68 for F(2,15) at 5% level of significance
Hence the null hypothesis is rejected, there is significance between the tree process.

5. Set an analysis of variance table for the following data.

A	6	7	3	8
B	5	5	3	7
C	5	4	3	4

Sol.

To carry out the analysis of variance, we form the following tables

					Total	Squares
A	6	7	3	8	$T_1=24$	$T_1^2=576$
B	5	5	3	7	$T_2=20$	$T_2^2=400$
C	5	4	3	4	$T_3=16$	$T_3^2=256$
Total T					60	-

The squares are as follows

					Total Squares
A	36	49	9	64	158
B	25	25	9	49	108
C	25	16	9	16	66
Grand Total - $\sum_i \sum_j {x_{ij}}^2$					332

Set the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(60)^2}{12} = \frac{3600}{12} = 300$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 332 - 300$$
$$\Rightarrow TSS = 32$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{576}{4} + \frac{400}{4} + \frac{256}{4} - 300$$
$$\Rightarrow SST = 144 + 100 + 64 - 300$$
$$\Rightarrow SST = 308 - 300$$
$$\Rightarrow SST = 8$$

Therefore sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 32 - 8$$
$$\Rightarrow SSE = 24$$

Sources variation	d.f.	SS	MSS	F Ratio
Between treatments	3-1=2	SST=8	$MST = \frac{8}{2} = 4$	$F = \frac{4}{2.66} = 1.5037$
Error	12-3=9	SSE=24	$MSE = \frac{24}{9} = 2.66$	
Total	12-1=11	-	-	

Since evaluated value 1.5037<4.26 for F(2,9) at 5% level of significance
Hence the null hypothesis is accepted, there is no significance between the three process.

6. A trial was run to check the effects of different diets. Positive numbers indicate weight loss and negative numbers indicate weight gain. Check if there is an average difference in the weight of people following different diets using an ANOVA Table.

Low Fat	Low Calorie	Low protein	Low carbohydrate
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3

Sol.

To carry out the analysis of variance, we form the following tables

	Low Fat	Low Calorie	Low protein	Low carbohydrate	
	8	2	3	2	
	9	4	5	2	
	6	3	4	-1	
	7	5	2	0	
	3	1	3	3	
T	33	15	17	6	71
T^2	1089	225	289	36	-

The squares are as follows

	Low Fat	Low Calorie	Low protein	Low carbohydrate	
	64	4	9	4	
	81	16	25	4	
	36	9	16	1	
	49	25	4	0	
	9	1	9	9	
$\sum_i \sum_j x_{ij}^2$	239	55	63	18	375

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(71)^2}{20} = \frac{5041}{20} = 252$

Therefore Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 375 - 252$$
$$\Rightarrow TSS = 123$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{1089}{5} + \frac{225}{5} + \frac{289}{5} + \frac{36}{5} - 252$$
$$\Rightarrow SST = 217.8 + 45 + 57.8 + 7.2 - 252$$
$$\Rightarrow SST = 327.8 - 252$$
$$\Rightarrow SST = 75.80$$

Therefore sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 123 - 75.80$$
$$\Rightarrow SSE = 47.2$$

Sources variation	**d.f.**	**SS**	**MSS**	**F Ratio**
Between treatments	4-1=3	SST=75.80	$MST = \frac{75.80}{3} = 25.26$	$F = \frac{25.26}{2.95} = 8.56$
Error	20-4=16	SSE=47.20	$MSE = \frac{47.20}{16} = 2.95$	
Total	20-1=19	-	-	

Since evaluated value 8.56 >3.24 for $F_{(3,16)}$ at 5% level of significance

Hence the null hypothesis is rejected, there is significance between the four process.

7. The following data show the number of worms quarantined from the GI areas offour groups of muskrats in a carbon tetrachloride anthelmintic study. Conduct a two-way ANOVA test.

I	II	III	IV
33	41	12	38
32	38	35	43
26	40	46	25
14	23	22	13
30	21	11	26

Sol.

Given

I	II	III	IV
33	41	12	38
32	38	35	43
26	40	46	25
14	23	22	13
30	21	11	26

Subtract 30 from all the observations, we get

I	II	III	IV
3	11	-18	8
2	8	5	13
-4	10	16	-5
-16	-7	-8	-17
0	-9	-19	-4

	I	II	III	IV	
	3	11	-18	8	
	2	8	5	13	
	-4	10	16	-5	
	-16	-7	-8	-17	
	0	-9	-19	-4	
T	-15	13	-24	-5	-31
T^2	225	169	576	25	

The squares are as follows

	I	II	III	IV	
	9	121	324	64	
	4	64	25	169	
	16	100	256	25	
	256	49	64	289	
	0	81	361	16	
$\sum_i \sum_j {x_{ij}}^2$	285	415	1030	563	2293

Set the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

Correction Factor $CF = \frac{T^2}{N} = \frac{(-31)^2}{20} = \frac{961}{20} = 48$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 2293 - 48$$
$$\Rightarrow TSS = 2245$$

Sum of the squares of between the treatments $SST = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SST = \frac{225}{5} + \frac{169}{5} + \frac{576}{5} + \frac{25}{5} - 48$$

$$\Rightarrow SST = 45 + 33.8 + 115.2 + 5 - 48$$
$$\Rightarrow SST = 199 - 48$$
$$\Rightarrow SST = 151$$

Therefore sum of squares due to error SEE=TSS-SST

$$\Rightarrow SSE = 2245 - 151$$
$$\Rightarrow SSE = 2094$$

Sources variation	d.f.	SS	MSS	F Ratio
Between treatments	4-1=3	SST=151	$MST = \frac{151}{3} = 50.33$	$F = \frac{130.87}{50.33} = 2.6$
Error	20-4=16	SSE=2094	$MSE = \frac{2094}{16} = 130.87$	
Total	20-1=19	-	-	

Since evaluated value 2.6<3.24 for F(3,16) at 5% level of significance
Hence the null hypothesis is accepted, there is no significance between the four process.

TWO WAY CLASSIFICATION:

- Define the problem for different varieties and different treatments.

	Verities					Sum	Squares
	x_{11}	x_{12}	x_{13}	…	x_{1n_1}	T_1	$T_1{}^2$
	x_{21}	x_{22}	x_{23}	…	x_{2n_2}	T_2	$T_2{}^2$
	x_{31}	x_{32}	x_{33}	…	x_{3n_3}	T_3	$T_3{}^2$
	---	---	----	…	---	---	---
	x_{k1}	x_{k2}	x_{k3}	…	x_{kn_k}	T_k	$T_k{}^2$
Sum	P_1	P_2	P_3	---	P_K	=G	
Squares	P_1^2	P_2^2	P_3^2	---	P_K^2		

- Define the null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_n$ for the level of significance.
- Find the sum of all the verities (Row wise) Find the sum of all the observations of N varieties, say T.
- Find the correction factor $CF = \frac{T^2}{N}$
- Find the sum of squares of individual items $TSS = \sum_i \sum_j x_{ij}{}^2 - CF$
- Find the sum of the squares of rows $SSR = \sum_i \frac{T_i{}^2}{n_i} - CF$
- Find the sum of the squares of columns $SCC = \sum_i \frac{P_i{}^2}{n_i} - CF$
- Find the sum of squares with in the class or sum of squares due to error by subtraction SEE=TSS-SSR-SSC.
- Plot the ANOVA table

Sources variation	d.f.	SS	MSS	F Ratio
Rows	r-1	SSR	$MSR = \frac{SSR}{r-1}$	$F_r = \frac{MSR}{MSE}$
Columns	c-1	SSC	$MSC = \frac{SSC}{c-1}$	
Error	(r-1)(c-1)	SSE	$MSE = \frac{SSE}{(r-1)(c-1)}$	$F_c = \frac{MSC}{MSE}$
Total	N-1	-	-	

1. Set up an analysis of variance table for the following per acre production data for three varieties of wheat, each grown on 4 plots and state it the variety differences are significant at 5% significant level.

Per acre production data			
Plot of land	Variety of wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Sol.

To carry out the analysis of variance, we form the following tables

Per acre production data				T	T^2
Plot of land	Variety				
	A	B	C		
1	6	5	5	16	256
2	7	5	4	16	256
3	3	3	3	9	81
4	8	7	4	19	361
P	24	20	16	=60	-
P^2	576	400	256		

The squares are as follows

Variety			
A	B	C	
36	25	25	
49	25	16	
9	9	9	
64	49	16	Grand Total - $\sum_i \sum_j {x_{ij}}^2$=332

Set the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3$, N=12

Correction Factor $CF = \frac{T^2}{N} = \frac{(60)^2}{12} = \frac{3600}{12} = 300$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 332 - 300$$
$$\Rightarrow TSS = 32$$

Sum of the row squares $SSR = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SSR = \frac{256}{3} + \frac{256}{3} + \frac{81}{3} + \frac{361}{3} - 300$$
$$\Rightarrow SSR = 85.33 + 85.33 + 27 + 120.33 - 300$$
$$\Rightarrow SSR = 318 - 300$$
$$\Rightarrow SSR = 18$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = \frac{576}{4} + \frac{400}{4} + \frac{256}{4} - 300$$
$$\Rightarrow SSC = 144 + 100 + 64 - 300$$
$$\Rightarrow SSC = 308 - 300$$
$$\Rightarrow SSC = 8$$

Therefore SSE=TSS-SSR-SSC

SSE=32-18-8=6

Sources variation	d.f.	SS	MSS	F Ratio
Rows	4-1=3	SSR=18	$MSR = \frac{18}{3} = 6$	$F_r = \frac{6}{1} = 6$
Columns	3-1=2	SSC=8	$MSC = \frac{8}{2} = 4$	
Error	3X2=6	SSE=6	$MSE = \frac{6}{6}=1$	$F_c = \frac{4}{1} = 4$
Total	12-1=11	-	-	

$F_r = 6>$ F(3,6)=4.76 &

$F_c = 4<$ F(6,2)=19.33

2. Three varieties of coal were analysed by four chemists and the ash-content in the varieties was found to be as under.

Varieties	Chemists			
	1	2	3	4
A	8	5	5	7
B	7	6	4	4
C	3	6	5	4

Carry out the analysis of variance.

Sol. To carry out the analysis of variance, we form the following tables

Chemists					T	T^2
Variety	1	2	3	4		
A	8	5	5	7	25	625
B	7	6	4	4	21	441
C	3	6	5	4	18	324
P	18	17	14	15	=64	-
P^2	324	289	196	225		

The squares are as follows

Chemists			
1	2	3	4
64	25	25	49
49	36	16	16
9	36	25	16

Grand Total - $\sum_i \sum_j {x_{ij}}^2$=366

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$, N=12

Correction Factor $CF = \frac{T^2}{N} = \frac{(64)^2}{12} = \frac{4096}{12} = 341.33$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 366 - 341.33$$
$$\Rightarrow TSS = 24.67$$

Sum of the row squares $SSR = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SSR = \frac{625}{4} + \frac{441}{4} + \frac{324}{4} - 341.33$$
$$\Rightarrow SSR = 156.25 + 110.25 + 81 - 341.33$$
$$\Rightarrow SSR = 347.50 - 341.33$$
$$\Rightarrow SSR = 6.17$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = \frac{324}{3} + \frac{289}{3} + \frac{196}{3} + \frac{225}{3} - 341.33$$
$$\Rightarrow SSC = 108 + 96.33 + 65.33 + 75 - 341.33$$
$$\Rightarrow SSC = 344.66 - 341.33$$
$$\Rightarrow SSC = 3.33$$

Therefore SSE=TSS-SSR-SSC

SSE=24.67-6.17-3.33=15.17

Sources variation	d.f.	SS	MSS	F Ratio
Rows	3-1=2	SSR=6.17	$MSR = \frac{6.17}{2} = 3.085$	$F_r = \frac{3.085}{2.53} = 1.22$
Columns	4-1=3	SSC=3.33	$MSC = \frac{3.33}{3} = 1.11$	
Error	3X2=6	SSE=15.17	$MSE = \frac{15.17}{6}$=2.53	$F_c = \frac{2.53}{1.11} = 2.28$
Total	12-1=11	-	-	

$F_r = 1.22 <$ F$_{(2,6)}$ &

$F_c = 2.28<$ F$_{(6,3)}$

3. Perform ANOVA and test at 0.05 level of significant whether these are differences in the detergent or in the engines for the following data:

Detergent	Engine		
	I	II	III
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Sol.

Given the data

Detergent	Engine		
	I	II	III
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Subtract 45 from all the observations, we get

Detergent	Engine			T	T^2
	I	II	III		
A	0	-2	6	**4**	**16**
B	2	1	7	**10**	**100**
C	3	5	10	**18**	**324**
D	-3	-8	4	**-7**	**49**
P	**2**	**-4**	**27**	**2**	**=25**
P^2	**4**	**16**	**729**	**4**	**-**

The squares are

Detergent	Engine			Sum
	I	II	III	
A	0	4	36	**40**
B	4	1	49	**54**
C	9	25	100	**134**
D	9	64	16	**89**
Grand Total - $\sum_i \sum_j {x_{ij}}^2=$				**317**

Set the null hypotheses $H_0: \mu_1 = \mu_{2=}\mu_3$, N=12

Correction Factor $CF = \frac{T^2}{N} = \frac{(25)^2}{12} = \frac{625}{12} = 52.08$

Therefore Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 317 - 52.08$$
$$\Rightarrow TSS = 264.92$$

Sum of the row squares $SSR = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SSR = \frac{16}{3} + \frac{100}{3} + \frac{324}{3} + \frac{49}{3} - 52.08$$
$$\Rightarrow SSR = 5.33 + 33.33 + 108 + 16.33 - 52.08$$
$$\Rightarrow SSR = 163 - 52.08$$
$$\Rightarrow SSR = 110.92$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = \frac{4}{4} + \frac{16}{4} + \frac{729}{4} - 52.08$$
$$\Rightarrow SSC = 1 + 4 + 182.25 - 52.08$$
$$\Rightarrow SSC = 187.25 - 52.08$$
$$\Rightarrow SSC = 135.17$$

Therefore SSE=TSS-SSR-SSC

SSE=264.92-110.92-135.17=18.83

Sources variation	**d.f.**	**SS**	**MSS**	**F Ratio**
Rows	4-1=3	SSR=110.92	$MSR = \frac{110.92}{3} = 36.97$	$F_r = \frac{36.97}{3.14} = 11.77$
Columns	3-1=2	SSC=135.17	$MSC = \frac{135.17}{2} = 67.58$	
Error	3X2=6	SSE=18.83	$MSE = \frac{18.83}{6}$=3.14	$F_c = \frac{67.58}{3.14} = 21.52$
Total	12-1=11	-	-	

$F_r = 11.77$ > F(3,6) &
$F_c = 21.52$> F(6,2)

Since the null hypothesis is rejected and there is a significance between Detergent and Engine.

4. Analyze and interpret the following statistics concerning output of wheat for field obtained as result of experiment conducted to test for Four varieties of wheat viz. A,B,C and D under Laton square design.

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Sol.

Given observations are

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Null hypothesis H_o : There is no significant difference between rows, columns and treatment

Code the data by subtracting 20 from each value, we get

					T	T^2
	C 5	B 3	A 0	D 0	8	64
	A -1	D -1	C 1	B -2	-3	9
	B -1	A -6	D -3	C 0	-10	100
	D -3	C 0	B 1	A -5	-7	49
P	0	-4	-1	-7	=- 12	
P^2	0	16	1	49	-	-

The squares are as follows

C 25	B 9	A 0	D 0	
A 1	D 1	C 1	B 4	
B 1	A 36	D 9	C 0	
D 9	C 0	B 1	A 25	
36	46	11	29	$\sum_i \sum_j {x_{ij}}^2$ =122

Correction Factor $CF = \frac{T^2}{N} = \frac{(-12)^2}{16} = \frac{144}{16} = 9$

Therefore, Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 122 - 9$$
$$\Rightarrow TSS = 113$$

Sum of the row squares $SSR = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SSR = \frac{64}{4} + \frac{9}{4} + \frac{100}{4} + \frac{49}{4} - 9$$
$$\Rightarrow SSR = 16 + 2.25 + 25 + 12.25 - 9$$
$$\Rightarrow SSR = 55.5 - 9$$
$$\Rightarrow SSR = 4$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = 0 + \frac{16}{4} + \frac{1}{4} + \frac{49}{4} - 9$$
$$\Rightarrow SSC = 4 + 0.25 + 12.25 - 9$$
$$\Rightarrow SSC = 16.5 - 9$$
$$\Rightarrow SSC = 7.5$$

To find the sum of the treatments

Observations					$Q = \sum(Observations)$	Q^2
A	0	-1	-6	-5	-12	144
B	3	-2	-1	1	1	1
C	5	1	0	0	6	36
D	0	-1	-3	-3	-7	49

Sum of the squares of treatments $SST = \sum_i \frac{{Q_i}^2}{n_i} - CF$

$$SST = \frac{144}{4} + \frac{1}{4} + \frac{36}{4} + \frac{49}{4} - 9$$
$$\Rightarrow SST = 36 + 0.25 + 9 + 12.25 - 9$$

$$\Rightarrow SST = 57.50 - 9$$
$$\Rightarrow SST = 48.50$$

$\therefore SSE = TSS - SSR - SSC - SST \Rightarrow SSE = 113 - 46.5 - 7.5 - 48.50 = 10.5$, We know that $F(3,6) = 4.76$

Sources variation	d.f	SS	MSS	F Ratio	Conclusion
Rows	4-1=3	SSR=46.5	$MSR = \frac{46.5}{3} = 15.5$	$F_r = \frac{15.5}{1.75} = 8.85$	$F_r > F(3,6)$ H_0-Rejected
Columns	4-1=3	SSC=7.5	$MSC = \frac{7.5}{3} = 2.5$	$F_c = \frac{2.5}{1.75} = 1.428$	$F_c < F(3,6)$ H_0-Accepted
Treatments	4-1=3	SST=48.5	$MST = \frac{48.5}{3} = 16.16$	$F_T = \frac{16.16}{1.75} = 9.23$	$F_T > F(3,6)$ H_0-Rejected
Error	3x2=6	SSE=10.5	$MSE = \frac{10.5}{6} = 1.75$	-	-
Total	25-1=24	-	-	-	-

5. Five varieties of paddy A, B, C, D, and E are tried. The plan, the varieties shown in each plot and yields obtained in Kg are given in the following table (LSD)

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93

Test whether there is a significant difference between rows and columns at 5% LOS.

Sol. Given observations are

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93

Null hypothesis H_o: There is no significant difference between rows, columns and treatment,
Code the data by subtracting 100 from each value, we get

						T	T^2
	B -5	E -15	C 39	A 17	D -3	33	1089
	E -10	D -11	B -25	C 46	A -13	-13	169
	C 16	A -5	D -8	B -11	E -26	-34	1156
	A -15	C 30	E -10	D -19	B -23	-37	1369
	D -13	B -35	A -1	E -11	C -7	-67	4489
P	-27	-36	-5	22	-72	= - 118	
P^2	729	1296	25	484	5184	-	-

The squares are as follows:

B 25	E 225	C 1521	A 289	D 9	
E 100	D 121	B 625	C 2116	A 169	
C 256	A 25	D 64	B 121	E 676	
A 225	C 900	E 100	D 361	B 529	
D 169	B 1225	A 1	E 121	C 49	
775	2496	2311	3008	1432	$\sum_i \sum_j x_{ij}{}^2 = 10022$

Correction Factor $CF = \frac{T^2}{N} = \frac{(-118)^2}{25} = \frac{13924}{25} = 557$

Therefore Total sum of squares $TSS = \sum_i \sum_j x_{ij}{}^2 - CF$

$$\Rightarrow TSS = 10022 - 557$$
$$\Rightarrow TSS = 9465$$

Sum of the row squares $SSR = \sum_i \frac{T_i{}^2}{n_i} - CF$

$$SSR = \frac{1089}{5} + \frac{169}{5} + \frac{1156}{5} + \frac{1369}{5} + \frac{4489}{5} - 557$$
$$\Rightarrow SSR = 217.8 + 33.8 + 231.2 + 273.8 + 897.8 - 557$$
$$\Rightarrow SSR = 1654.4 - 557$$
$$\Rightarrow SSR = 1097.4$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = \frac{729}{5} + \frac{1296}{5} + \frac{25}{5} + \frac{484}{5} + \frac{5184}{5} - 557$$
$$\Rightarrow SSC = 145.8 + 259.2 + 5 + 96.8 + 1036.8 - 557$$
$$\Rightarrow SSC = 1543.6 - 557$$
$$\Rightarrow SSC = 986.6$$

To find the sum of the treatments,

	Observations					$Q = \sum(Observations)$	Q^2
A	17	-13	-5	-15	-1	-17	289
B	-5	-25	-11	-23	-35	-99	9801
C	39	46	16	30	-7	124	15376
D	-3	-11	-8	-19	-13	-54	2916
E	-15	-10	-26	-10	-11	-72	5184

Sum of the squares of treatments $SST = \sum_i \frac{{Q_i}^2}{n_i} - CF$

$$SST = \frac{289}{5} + \frac{9801}{5} + \frac{15376}{5} + \frac{2916}{5} + \frac{5184}{5} - 557$$
$$\Rightarrow SST = 57.8 + 1960.2 + 3075.2 + 583.2 + 1036.8 - 557$$
$$\Rightarrow SST = 6713.2 - 557$$
$$\Rightarrow SST = 6156.2$$

$$\therefore SSE = TSS - SSR - SSC - SST$$
$$\Rightarrow SSE = 9465 - 1097.4 - 986.6 - 6156.2$$
$$\Rightarrow SSE = 1224.8$$

Sources variation	d.f.	SS	MSS	F Ratio	Conclusion
Rows	5-1=4	SSR=1097.4	$MSR = \frac{1097.4}{4} = 274.3$	$F_r = \frac{274.3}{102.66} = 2.672$	$F_r < F(4,12)$ H_0-Accepted
Columns	5-1=4	SSC=986.6	$MSC = \frac{986.6}{4} = 246.65$	$F_c = \frac{246.65}{102.66} = 2.4026$	$F_c < F(4,12)$ H_0-Accepted
Treatments	5-1=4	SST=6156.2	$MST = \frac{6156.2}{4} = 1539.05$	$F_T = \frac{1539.05}{102.66} = 15$	$F_T > F(4,12)$ H_0-Rejected
Error	4x3=12	SSE=1224.8	$MSE = \frac{1224.8}{12}$=102.66		
Total	25-1=24	-	-	-	

6. Present your conclusions after doing analysis of variance to the following results of the Latin-square design experiment conducted in respect of five fertilizers which were used on plots of different fertility.

A 16	B 10	C 11	D 9	E 9
E 10	C 9	A 14	B 12	D 11
B 15	D 8	E 8	C 10	A 18
D 12	E 6	B 13	A 13	C 12
C 13	A 11	D 10	E 7	B 14

Sol. Given observations are

A 16	B 10	C 11	D 9	E 9
E 10	C 9	A 14	B 12	D 11
B 15	D 8	E 8	C 10	A 18
D 12	E 6	B 13	A 13	C 12
C 13	A 11	D 10	E 7	B 14

Null hypothesis H_o: There is no significant difference between rows, columns and treatment,
Code the data by subtracting 10 from each value. We get,

						T	T^2
	A 6	B 0	C 1	D -1	E -1	5	25
	E 0	C -1	A 4	B 2	D 1	6	36
	B 5	D -2	E -2	C 0	A 8	9	81
	D 2	E -4	B 3	A 3	C 2	6	36
	C 3	A 1	D 0	E -3	B 4	5	25
P	16	-6	6	1	14	= 31	
P^2	256	36	36	1	196	-	-

The squares are as follows:

A 36	B 0	C 1	D 1	E 1	
E 0	C 1	A 16	B 4	D 1	
B 25	D 4	E 4	C 0	A 64	
D 4	E 16	B 9	A 9	C 4	
C 9	A 1	D 0	E 9	B 16	
74	22	30	23	86	$\sum_i \sum_j {x_{ij}}^2 = 235$

Correction Factor $CF = \frac{T^2}{N} = \frac{(31)^2}{25} = \frac{961}{25} = 38.44$

Therefore Total sum of squares $TSS = \sum_i \sum_j {x_{ij}}^2 - CF$

$$\Rightarrow TSS = 235 - 38.44$$
$$\Rightarrow TSS = 196.56$$

Sum of the row squares $SSR = \sum_i \frac{{T_i}^2}{n_i} - CF$

$$SSR = \frac{25}{5} + \frac{36}{5} + \frac{81}{5} + \frac{36}{5} + \frac{25}{5} - 38.44$$
$$\Rightarrow SSR = 5 + 7.2 + 16.2 + 7.2 + 5 - 38.44$$
$$\Rightarrow SSR = 40.60 - 38.44$$
$$\Rightarrow SSR = 2.16$$

Sum of the column squares $SSC = \sum_i \frac{{P_i}^2}{n_i} - CF$

$$SSC = \frac{256}{5} + \frac{36}{5} + \frac{36}{5} + \frac{1}{5} + \frac{196}{5} - 38.44$$

$$\Rightarrow SSC = 51.2 + 7.2 + 7.2 + 0.2 + 39.2 - 38.44$$

$$\Rightarrow SSC = 105 - 38.44 \Rightarrow SSC = 66.56$$

To find the sum of the treatments

	Observations					$Q = \sum(Observations)$	Q^2
A	6	4	8	3	1	22	484
B	0	2	5	3	4	14	196
C	1	-1	0	2	3	5	25
D	-1	1	-2	2	0	0	0
E	-1	0	-2	-4	-3	-10	100

Sum of the squares of treatments $SST = \sum_i \frac{{Q_i}^2}{n_i} - CF$

$$SST = \frac{484}{5} + \frac{196}{5} + \frac{25}{5} + \frac{0}{5} + \frac{100}{5} - 38.44$$

$$\Rightarrow SST = 96.8 + 39.2 + 5 + 0 + 20 - 38.44$$

$$\Rightarrow SST = 161 - 38.44$$

$$\Rightarrow SST = 122.56$$

$$\therefore SSE = TSS - SSR - SSC - SST$$

$$\Rightarrow SSE = 196.56 - 2.16 - 66.56 - 122.56$$

$$\Rightarrow SSE = 5.28$$

Sources variation	d.f.	SS	MSS	F Ratio	Conclusion
Rows	5-1=4	SSR=2.16	$MSR = \frac{2.16}{4} = 0.54$	$F_r = \frac{0.54}{0.44} = 1.227$	$F_r < F(4,12)$ H_0-Accepted
Columns	5-1=4	SSC=66.56	$MSC = \frac{66.56}{4} = 16.64$	$F_c = \frac{16.64}{0.44} = 37.81$	$F_c > F(4,12)$ H_0-Rejected
Treatments	5-1=4	SST=122.56	$MST = \frac{122.56}{4} = 30.64$	$F_T = \frac{30.64}{0.44} = 69.63$	$F_T > F(4,12)$ H_0-Rejected
Error	4x3=12	SSE=5.28	$MSE = \frac{5.28}{12}=0.44$	-	-
Total	25-1=24	-	-	-	-

7. Set up ANOVA table for the following information relating to three drugs testing to judge the effectiveness in reducing blood pressure for three different groups of people:

Group of people	Drug		
	X	Y	Z
A	14 15	10 9	11 11
B	12 11	7 8	10 11
C	10 11	11 11	8 7

Do the drugs act differently? Are the different groups of people affected differently? Is the interaction term significant? Answer the above questions taking a significant level of 5%.

Sol.

Given observations from different people (A, B, C) to the different drugs (X, Y, Z) are as

Group of people	Drug			T	T^2
	X	Y	Z		
A	14 15	10 9	11 11	70	4900
B	12 11	7 8	10 11	59	3481
C	10 11	11 11	8 7	58	3364
P	73	56	58	=187	-
P^2	5329	3136	3364	-	-

Where N=6+6+6=18

Correction Factor $CF = \frac{T^2}{N} = \frac{(187)^2}{18} = \frac{34969}{18} = 1942.722$

The squares are as follows

Group of people	Drug			Sum of Squares	
	X	Y	Z		
A	196 225	100 81	121 121	844	
B	144 121	49 64	100 121	599	
C	100 121	121 121	64 49	576	$\sum_i \sum_j x_{ij}{}^2 = 2019$

Therefore, Total sum of squares $TSS = \sum_i \sum_j x_{ij}{}^2 - CF$

$$\Rightarrow TSS = 2019 - 1942.722$$
$$\Rightarrow TSS = 76.28$$

Sum of the row squares $SSR = \sum_i \frac{T_i{}^2}{n_i} - CF$

$$SSR = \frac{4900}{6} + \frac{3481}{6} + \frac{3364}{6} - 1942.722$$
$$\Rightarrow SSR = 816.67 + 580.16 + 560.67 - 1942.722$$
$$\Rightarrow SSR = 14.78$$

Sum of the column squares $SSC = \sum_i \frac{P_i{}^2}{n_i} - CF$

$$SSC = \frac{5329}{6} + \frac{3136}{6} + \frac{3364}{6} - 1942.722$$
$$\Rightarrow SSC = 888.16 + 522.66 + 560.67 - 1942.722$$
$$\Rightarrow SSC = 28.77$$

SS within samples (SST)= $(14 - 14.5)^2 + (15 - 14.5)^2 + (10 - 9.5)^2 + (9 - 9.5)^2 + (11 - 11)^2 + (11 - 11)^2 + (12 - 11.5)^2 + (11 - 11.5)^2 + (7 - 7.5)^2 + (8 - 7.5)^2 + (10 - 10.5)^2 + (11 - 10.5)^2 + (10 - 10.5)^2 + (11 - 10.5)^2 + (11 - 11)^2 + (11 - 11)^2 + (8 - 7.5)^2 + (7 - 7.5)^2$

SST=3.50

Therefore,

SSE=TSS-SSR-SSC-SST

$$\Rightarrow \text{SSE} = 76.28 - 14.78 - 28.77 - 3.5$$
$$\Rightarrow \text{SSE} = 29.23$$

We have $F_{(2,9)}$=4.26 , F(4,9)=3.63

Sources variation	**d.f.**	**SS**	**MSS**	**F Ratio**	**Conclusion**
Rows	3-1=2	SSR=14.78	$MSR = \frac{14.78}{2} = 7.39$	$F_r = \frac{7.39}{0.389} = 19$	$F_r > F(2,9)$ H_0-Rejected
Columns	3-1=2	SSC=28.77	$MSC = \frac{28.77}{2} = 14.385$	$F_c = \frac{14.385}{0.389} = 37$	$F_r > F(2,9)$ H_0-Rejected
Treatments	9	SST=3.5	$MST = \frac{3.5}{9} = 0.389$	$F_T = \frac{7.33}{0.389} = 18.84$	$F_T > F(4,9)$ H_0-Rejected
Error	4	SSE=29.33	$MSE = \frac{29.33}{4}$=7.33	-	-

All the very best…..

Purushotham