

Third Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024
Mathematics for Computer Science

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. VTU Formula Hand Book is permitted.
 3. M : Marks, L: Bloom's level, C: Course outcomes.
 4. Mathematics hand book is permitted.

Module - 1				M	L	C																	
Q.1	a.	A Random variable X has the following probability function for variable values of x. <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k²</td><td>2k²</td><td>7k²+k</td></tr></table> (i) Find the value of k. (ii) Evaluate P(x ≥ 6) and P(3 < x ≤ 6).	x	0	1	2	3	4	5	6	7	P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k	6	L2	CO1
x	0	1	2	3	4	5	6	7															
P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k															
	b.	Find the mean and variance of Binomial distribution.	7	L2	CO2																		
	c.	In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for, (i) 10 minutes or more. (ii) Less than 10 minutes. (iii) Between 10 and 12 minutes.	7	L3	CO2																		
OR																							
Q.2	a.	A random variable x has the following density function $P(x) = \begin{cases} Kx^2 & -3 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$ Find the value of K. Evaluate (i) P(1 ≤ x ≤ 2) (ii) P(x ≤ 2)	6	L2	CO1																		
	b.	In a factory producing blades, the probability of any blade being defective is 0.002. If blades are supplied in packets of 10, using Poisson distribution determine the number of packets containing. (i) No defective. (ii) One defective (iii) Two defective blades respectively in a consignment of 10,000 packets.	7	L2	CO2																		
	c.	In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for. (i) More than 2100 hours. (ii) Between 1900 to 2100 hours. (iii) Less than 1950 hours. (Given φ(1.67) = 0.4525, φ(0.83) = 0.2967)	7	L3	CO2																		

Module - 2

Q.3	a.	The joint probability distribution table for two random variable x and y is as follows:		6	L2	CO2																		
		<table border="1"> <tr> <td></td><td>Y</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr> <tr> <td>X</td><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr> <tr> <td></td><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr> </table>		Y	-2	-1	4	5	X	1	0.1	0.2	0	0.3		2	0.2	0.1	0.1	0				
	Y	-2	-1	4	5																			
X	1	0.1	0.2	0	0.3																			
	2	0.2	0.1	0.1	0																			
		Determine the marginal probability distribution of x and y. Obtain the correlation coefficient between x and y.																						
	b.	Find the unique fixed probability vector for the regular stochastic matrix		7	L2	CO3																		
		$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 6 & 2 & 3 \\ 0 & 2 & 1 \\ 3 & 3 & 3 \end{bmatrix}$																						
	c.	Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws : (i) A has the ball. (ii) B has the ball. (iii) C has the ball.		7	L3	CO3																		

OR

Q.4	a.	The joint probability distribution of two discrete random variables x and y is given by $f(x, y) = k(2x+y)$ where x and y are integers. Such that $0 \leq x \leq 2$, $0 \leq y \leq 3$. (i) Find the value of the constant K. (ii) Find the marginal probability distribution of X and Y. (iii) Show that the random variables X and Y are dependent.		6	L2	CO2
	b.	Find the unique fixed probability vector for the matrix, $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$		7	L2	CO3
	c.	Each year a man trades his car for a new car in 3 brands of the popular company. If he has a 'swift' he trades it for 'Dzire'. If he has a 'Dzire' he trades it for a 'Wagor'. If he has a 'Wagor' he is just as likely to trade it for a new 'Wagor' or for a 'Dzire' or a 'Swift' one. In 2020 he bought his first car which was 'Wagor'. Find the probability that he has (i) 2022 Wagor. (ii) 2022 Swift. (iii) 2023 Dzire. (iv) 2023 Wagor.		7	L3	CO3

Module - 3

Q.5	a.	Explain the following terms: (i) Statistical Hypothesis. (ii) Critical region of statistical test. (iii) Test for significance.		6	L1	CO5
-----	----	--	--	---	----	-----

b.	In 324 throws of a six faced die an odd number turned up 181 times. Is it reasonable to think that the die is an unbiased one at 5% level of significance?	7	L3	CO4
c.	One type of aircraft is found to develop engine trouble in 5 flights out of a total of 100 and another type in 7 flights out of a total 200 flights. Is there a significant difference in the two types of aircrafts so far as engine defects are concerned? Test at 5% significance level.	7	L3	CO4

OR

Q.6	a. Define : (i) Null Hypothesis. (ii) Significance level. (iii) Type I and II error.	6	L1	CO5
b.	A coin was tossed 1000 times and head turns up 540 times. Test the hypothesis that the coin is unbiased at 1% level of significance.	7	L3	CO4
c.	In an exit poll enquiry it was revealed that 600 voters in one locality and 400 voters from an other locality favoured 55% and 48% respectively a particular party to come to power. Test the hypothesis that there is a difference in the locality in respect of the opinion at 1% level of significance.	7	L3	CO4

Module - 4

Q.7	a.	A random sample of size 64 is taken from an infinite population having mean 112 and variance 144. Using central limit theorem, find the probability of getting the sample mean \bar{X} greater than 114.5	6	L2	CO5																	
	b.	<p>The following data shows the runs scored by two batsman: Can it be said that the performance of batsman A is more consistent than the performance of batsman B? Use 1% level of significance ($F_{0.01, 4, 7} = 7.85$)</p> <table border="1"> <tr> <td>Batsman A</td><td>40</td><td>50</td><td>35</td><td>25</td><td>60</td><td>70</td><td>65</td><td>55</td></tr> <tr> <td>Batsman B</td><td>60</td><td>70</td><td>40</td><td>30</td><td>50</td><td>-</td><td>-</td><td>-</td></tr> </table>	Batsman A	40	50	35	25	60	70	65	55	Batsman B	60	70	40	30	50	-	-	-	7	L2
Batsman A	40	50	35	25	60	70	65	55														
Batsman B	60	70	40	30	50	-	-	-														
	c.	<p>A coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and calculate the theoretical frequencies.</p> <table border="1"> <tr> <td>Number of heads</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr> <td>Frequency</td><td>5</td><td>29</td><td>36</td><td>25</td><td>5</td></tr> </table> <p>(Given $\chi^2_{0.05} = 9.49$ for 4 degree of freedom)</p>	Number of heads	0	1	2	3	4	Frequency	5	29	36	25	5	7	L3	CO4					
Number of heads	0	1	2	3	4																	
Frequency	5	29	36	25	5																	

OR

Q.8	a. Suppose that 10, 12, 16, 19 is a sample taken from a normal population with variance 6.25. Find at 95% confidence interval for the population mean.	6	L2	CO4
b.	The individuals are choosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given $t_{0.05} = 2.262$ for 9 degree of freedom).	7	L3	CO5
c.	A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories (Given $\chi^2_{0.05} = 7.81$ for 3 degree of freedom).	7	L3	CO4

Module - 5

Q.9	a. Three different kinds of food are tested on three groups of rats for 5 weeks. The objective is to check the difference in mean weight (in grams) of the rats per week. Apply one-way ANOVA using a 0.05 significance level to the following data :	10	L3	CO6																																
	<table border="1"><tr><td>Food 1</td><td>8</td><td>12</td><td>19</td><td>8</td><td>6</td><td>11</td></tr><tr><td>Food 2</td><td>4</td><td>5</td><td>4</td><td>6</td><td>9</td><td>7</td></tr><tr><td>Food 3</td><td>11</td><td>8</td><td>7</td><td>13</td><td>7</td><td>9</td></tr></table>	Food 1	8	12	19	8	6	11	Food 2	4	5	4	6	9	7	Food 3	11	8	7	13	7	9														
Food 1	8	12	19	8	6	11																														
Food 2	4	5	4	6	9	7																														
Food 3	11	8	7	13	7	9																														
	b. Analyze and interpret the following statistics concerning output of wheat per field obtained as a result of experiment conducted to test four varieties of wheat viz. A, B, C, D under a Latin-square design.	10	L4	CO6																																
	<table border="1"><tr><td>C</td><td>B</td><td>A</td><td>D</td></tr><tr><td>25</td><td>23</td><td>20</td><td>20</td></tr><tr><td>A</td><td>D</td><td>C</td><td>B</td></tr><tr><td>19</td><td>19</td><td>21</td><td>18</td></tr><tr><td>B</td><td>A</td><td>D</td><td>C</td></tr><tr><td>19</td><td>14</td><td>17</td><td>20</td></tr><tr><td>D</td><td>C</td><td>B</td><td>A</td></tr><tr><td>17</td><td>20</td><td>21</td><td>15</td></tr></table>	C	B	A	D	25	23	20	20	A	D	C	B	19	19	21	18	B	A	D	C	19	14	17	20	D	C	B	A	17	20	21	15			
C	B	A	D																																	
25	23	20	20																																	
A	D	C	B																																	
19	19	21	18																																	
B	A	D	C																																	
19	14	17	20																																	
D	C	B	A																																	
17	20	21	15																																	
OR																																				
Q.10	a. Set up an analysis of variance table for the following per acre production data for three varieties of wheat, each grown on four plots and state if the variety differences are significant at 5% significant level (Two way ANOVA).	10	L3	CO6																																
	<table border="1"><tr><th rowspan="2">Plot of land</th><th colspan="3">Per acre production data</th></tr><tr><th colspan="3">Variety of wheat</th></tr><tr><td></td><td>A</td><td>B</td><td>C</td></tr><tr><td>1</td><td>6</td><td>5</td><td>5</td></tr><tr><td>2</td><td>7</td><td>5</td><td>4</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td></tr><tr><td>4</td><td>8</td><td>7</td><td>4</td></tr></table>	Plot of land	Per acre production data			Variety of wheat				A	B	C	1	6	5	5	2	7	5	4	3	3	3	3	4	8	7	4								
Plot of land	Per acre production data																																			
	Variety of wheat																																			
	A	B	C																																	
1	6	5	5																																	
2	7	5	4																																	
3	3	3	3																																	
4	8	7	4																																	
	b. Set up ANOVA table for the following information relating to three drugs testing to judge the effectiveness in reducing blood pressure for three different groups of people.	10	L4	CO6																																
	<table border="1"><tr><th rowspan="2">Group of people</th><th colspan="3">Drug</th></tr><tr><th>X</th><th>Y</th><th>Z</th></tr><tr><td rowspan="2">A</td><td>14</td><td>10</td><td>11</td></tr><tr><td>15</td><td>9</td><td>11</td></tr><tr><td rowspan="2">B</td><td>12</td><td>7</td><td>10</td></tr><tr><td>11</td><td>8</td><td>11</td></tr><tr><td rowspan="2">C</td><td>10</td><td>11</td><td>8</td></tr><tr><td>11</td><td>11</td><td>7</td></tr></table> <p>Do the drugs act differently? Are the different groups of people affected differently? Is the interaction term significant? Answer the above questions taking a significant level of 5%?</p>	Group of people	Drug			X	Y	Z	A	14	10	11	15	9	11	B	12	7	10	11	8	11	C	10	11	8	11	11	7							
Group of people	Drug																																			
	X	Y	Z																																	
A	14	10	11																																	
	15	9	11																																	
B	12	7	10																																	
	11	8	11																																	
C	10	11	8																																	
	11	11	7																																	