Counting:

- n distinct objects
- Boxes of capacity $K_1 \ K_2 \ldots$
- Boxes are labelled!

1)
$$\frac{n!}{K_1! \ K_2! \ K_3!}$$

multinomial

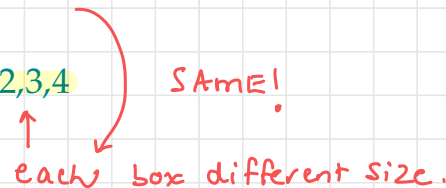Number of ways to rearrange 4 red blocks and 3 blue blocks and 2 yellow blocks

$$\frac{9!}{4! \ 3! \ 2!}$$

Number of ways to put 9 different colored blocks into 3 DISTINCT boxes of sizes 2,3,4

$$\frac{9!}{4! \ 3! \ 2!} \ .$$

Number of ways to put 9 different colored blocks into 3 boxes of sizes 2,3,4

$$\frac{9!}{4! \ 3! \ 2!}$$

SAME!

↑
each box different size.

Number of ways to put 11 different colored blocks into 4 unmarked boxes of sizes 2, (3, 3, 3)

$$\frac{11!}{2! \ 3! \ 3! \ 3! \cdot 3!}$$

boxes

orders of similar boxes

Put 9 books (distinct) into boxes of size 2, 3, 4 where the order of the books in each box matters

$$\frac{9!}{2! \; 3! \; 4!} \quad \times \quad 2! \cdot 3! \cdot 4! \quad = \quad 9!$$

$\underbrace{}$ Order in each box.

$\underbrace{}$ Place into boxes

2) $\dbinom{n+K-1}{K-1}$

- Put n <mark>identical objects</mark> into K "<mark>categories</mark>". ← no Capacity.
- Allowed <mark>empty</mark> categories

Put 9 books (<mark>identical looking</mark>) onto 3 shelves, where each shelf is unique.

$$\dbinom{9+3-1}{3-1} = \dbinom{11}{2}$$

Put 9 DISTINCT books onto 3 shelves, where each shelf is unique. The order on the books on the shelves matters because the books are distinct!

$$\binom{9+3-1}{3-1} \cdot 9!$$

order the books

Pick # of book per shelf

no Capacity.

3) $\binom{n-1}{k-1}$ • Put n identical objects into k "categories".

• No empty categories.
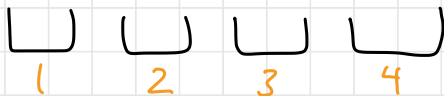
Put 9 coins into 3 distinct bags. . No empty bags!

$$\binom{9-1}{3-1} = \binom{8}{2}$$

4) 6 different colored balls into 4 labelled boxes. No empty box.

Given Capacities 3,1,1,1

$\lfloor\rfloor \lfloor\rfloor \lfloor\rfloor \lfloor\rfloor$
1　2　3　4

(balls) 1 2 3 4 5 6

ways:

Step 1) PICK "LARGE" box.

ways = 4

Step 2) Pick which balls go into LARGE box: $\binom{6}{3}$

Step 3) Assign remaining 3 balls into 3 boxes: 3!

Total:

$$4\binom{6}{3}\cdot 3\cdot 2$$

$$= 4!\binom{6}{3}$$

SAME!

OR!! Use multinomial:

$$\frac{6!}{3!\,1!\,1!\,1!}\times 4 = 4\cdot\frac{6!}{3!}$$

ways to order boxes 3,1,1,1 or 1,3,1,1 ...