Midterm Exam CS-GY 6003 INET Spring 2021
March 24, 2021

Instructions:

Scheduling:

- The exam is available to download from Gradescope at 6:30pm EDT on March 24th 2021. It must be uploaded back to Gradescope by 10pm EDT on March 24th 2021. The time to complete the exam will depend on your preparation: a prepared student could finish it in less than two hours, an ill-prepared student might take up to three hours. It is your responsibility to allow time for scanning and uploading your exam.

Format:

- The exam consists of 6 questions, for a total of 80 points. You should plan for 30 minutes per question.
- You may write your solutions directly on the downloaded exam paper, *or* in your own format. You are responsible for providing clear and legible solutions to the problems. Your exam must be resubmitted into Gradescope electronically. Ensure that you know how to digitally scan any handwritten material. This is entirely the student's responsibility.

Questions during the exam:

- There is a ZOOM session for questions that will be open during the entire course of the exam. You may ask questions to the instructor with private chat during the exam. Any announcements made by the instructor during the exam will be made over ZOOM and also be email. **It is the student's responsibility to stay connected (either by ZOOM or email) during the exam.**

Rules:

- This exam is a **take-home exam**. You may use **only** the resources from the online class (any material on NYU classes for this course) and any type of calculator (although it is not needed).
- Your work must be entirely your own. It is **forbidden to discuss any work with *any* other person**. Furthermore, your work must be done without using internet searches (although this is completely unhelpful for this exam). Any breach of academic honesty will be handled in accordance with the *Student Code of Conduct*, (a copy of which is provided), and in this particular case, taken very seriously.
- You are asked to **read** the attached Student Code of Conduct Section III subsections A,B,C,D,E and **sign** below to acknowledge that you aware of the policy. Once signed, a copy of this page must be uploaded with your exam.

I acknowledge that my submitted Exam work is entirely my own. I have read and am in accordance with the Student Code of Conduct policy of NYU Tandon and fully accept the consequences of breaching the above instructions.

Name: ______________________________

Signature: ______________________________

Read questions carefully! Join the zoom meeting, microphones off. Private chat if you have a question

https://nyu.zoom.us/j/98287364690

Question 1

(15 points) Show your work

(a) Write the following statements in predicate logic using only the predicates $C(x)$ for x is cheating and $G(x)$ for x gets caught, where x is assumed in the domain of students. Using students a, b, c, give an example to show that the two statements below are *not equivalent*.

- If no student gets caught, then not everyone was cheating.
- Every student who is cheating gets caught.

(b) Prove or disprove each of the following statements:

- $A \cap B \subseteq B \cap (\overline{B} \cup A)$
- If A = the set of *even* integers, and $B = \{\frac{1}{n} | n \in \mathbb{N}\}$, then A and B have the same cardinality. (here $\mathbb{N}$ does not include 0).

Question 2

(10 points). Show your work!

(a) Let $f(n) = \sqrt{n}(\log n + 2^n)$ and $g(n) = 3^n + \log n$. Prove either $f(n)$ is $O(g(n))$ or $g(n)$ is $O(f(n))$. Only one is true.

(b) Let $A = \{0, 1, 2, 3 \ldots\}$. Define $f : A \to A$ where $f(n) = 2n$. Define $g : A \to A$ where $g(n) = \lfloor \frac{n}{2} \rfloor$. Explain whether or not $f \circ g$ is injective and/or surjective.

Question 3

(15 points) Show your work for all parts

(a) Evaluate the following sums, or provide a justification that the sum does not exist.

$$\sum_{k=0}^{\infty}\left(\frac{2\cdot 3^k+1}{6^{k-2}}\right) \qquad \sum_{k=1}^{\infty}(-1)^k(k+2)$$

(b) You have 3 friends, a, b, c. Over the next 7 days, you invite a *subset* of your friends to dinner. Each subset is called a "guest list". No guest list is repeated, and inviting *no* friends is no possible. You define a relation R on your set of possible guest lists as follows: two guest lists are *related* if there is *exactly* one person in common between the two guest lists. Draw the graph for this relation. Determine if this relation is reflexive, transitive, anti-symmetric, and/or symmetric. Justify your answer. Next, show the **minimum** number of arrows that must be added to the graph of R in order to make it an equivalence relation.

extra page if needed

Question 4

(10 points) Show your work

(a) A stadium has 35 rows of chairs, each row has exactly 5 seats labeled A, B, C, D, E. In each seat there is either a child or an adult. Prove that there are at least two rows with the same seating arrangement (ex. child, adult, child, child, adult).

(b) You have 3 buckets, the first of size 4 gallons, the second of size 10 gallons and the third of size 15 gallons. Show that using these 3 buckets you can measure any number of gallons over 22 gallons. For example, you can make 24 gallons using 6 refills of the first bucket.

Question 5

(15 points) Show your work

A pile of n bombs need to be safely separated into individual piles of 1 bomb each. A bomb squad has equipment that can separate a pile into *two* smaller piles. The **cost** of the separation depends on the size of the two smaller piles. The cost of separating an n-bomb pile into two piles of size k and m is exactly $2mk$. (For example, separating a 10-bomb pile into piles of size 3 and 7 costs $2 \cdot 3 \cdot 7 = 42$). Prove by induction that *regardless* of how the bomb squad choses to carry out the separation, that the total cost of placing all bombs into individual piles is $n(n-1)$.

Question 6

(15 points) Show your work

(a) I have \$100 and would like to give some to my children $C1, C2$, and $C3$. The first child is the oldest and should get **at least** \$20, the second child should get **at least** \$5, and the third child is a baby so she only gets **either nothing or** \$1. How many different ways are there of handing out the money (in dollars), assuming I don't need to give it *all* away.

(b) A password is made by selecting letters from $\{w, x, y, z\}$. How many ways can you make a password of length 5 if it must have no x's **or** exactly three y's ? (this is english inclusive or).

(c) There are 10 students waiting to order food from 4 different restaurants. Each student picks a restaurant and must line up in front of the restaurant. The restaurants are each different! How many different ways are there for the students to line up in front of the restaurants? Consider both the number of people in each line, and their order. A line may be empty.