In [1]:
```
'''
Author: Sameer
Date: 16/11/2018
RBF Classification = K-Means + Perceptron Learning Alogrithm
'''
```

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n Learning Alogrithm\n'
```

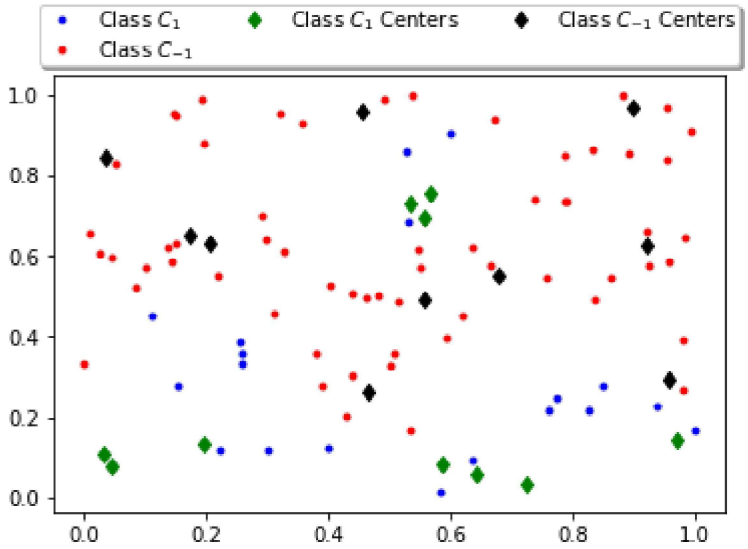
In [2]:
```
# Import Required Libraries
import numpy
import matplotlib.pyplot as plt
from copy import deepcopy
```

In [3]:
```
# Parameters
Input_Samples = 100
# C = 1
Centers = 20
grid_size = 1000
eta = 0.3 # Learning Rate
Epoch = numpy.array([]) # Storing Epoch for plotting
Misclassification = numpy.array([]) # No.of Wrong Classifications
```

In [4]:
```
# Check if data exists. If yes load it otherwise create new data and store it
 in text file.
# Data includes the points for the classification problem, Centers for K-Means
 Algo and Weights & Bias.
# Half of the centers will be for Positive Class and other half will be for  N
egative class.
# Weight includes bias.
if numpy.DataSource().exists('InputPattern.txt'):
    X = numpy.loadtxt('InputPattern.txt')
else:
    X = numpy.random.uniform(0, 1, (Input_Samples, 2))
    numpy.savetxt('InputPattern.txt', X)
if numpy.DataSource().exists('Weights.txt'):
    Weights = numpy.loadtxt('Weights.txt')
else:
#     Weights = numpy.random.uniform(-1, 1, (1 + int(Centers)))
    Weights = numpy.random.normal(0, 0.3086067, (1 + int(Centers)))
    numpy.savetxt('Weights.txt', Weights)
Gaussian_Std_Deviation = numpy.std(X)
```

In [5]:
```
# Create Desired Classification based on some conditions.
# Which will give a circle and sine wave boundary and points lying inside them
 will be positive class, negative otherwise.
D = numpy.array([])
Pos_Class = numpy.array([[0, 0]])
Neg_Class = numpy.array([[0, 0]])
for i in range(0, Input_Samples):
    if X[i, 1] < 0.2 * numpy.sin(10 * X[i, 0]) + 0.3 or numpy.square(X[i, 0] -
 0.5) + numpy.square(X[i, 1] - 0.8) < numpy.square(0.15):
        D = numpy.concatenate((D, [1]), axis=0)
        Pos_Class = numpy.concatenate((Pos_Class, [X[i]]), axis=0)
    else:
        D = numpy.concatenate((D, [-1]), axis=0)
        Neg_Class = numpy.concatenate((Neg_Class, [X[i]]), axis=0)
Pos_Centers = Pos_Class[numpy.random.choice(Pos_Class.shape[0], int(Centers/2
), replace=False), :]
Neg_Centers = Neg_Class[numpy.random.choice(Neg_Class.shape[0], int(Centers/2
), replace=False), :]
```

In [6]:
```
# Plot Results
plt.plot(Pos_Class[1:, 0], Pos_Class[1:, 1], 'b.', label=r'Class $C_1$')
plt.plot(Neg_Class[1:, 0], Neg_Class[1:, 1], 'r.', label=r'Class $C_{-1}$')
plt.plot(Pos_Centers[:, 0], Pos_Centers[:, 1], 'gd', label=r'Class $C_1$ Cente
rs')
plt.plot(Neg_Centers[:, 0], Neg_Centers[:, 1], 'kd', label=r'Class $C_{-1}$ Ce
nters')
plt.legend(loc='lower center', bbox_to_anchor=(0.5, 1), fancybox=True, shadow=
True, ncol=3, borderpad=0.1, labelspacing=0.1)
plt.savefig('Results1.pdf')
plt.show()
```



In [7]:
```
# Required Functions
def Gaussian_Kernel(X1, X2, sigma):
    return numpy.exp(-numpy.square(numpy.linalg.norm(X1 - X2))/(2 * numpy.squa
re(sigma)))

# def MulitQuadratic_Kernel(X1, X2, c):
#     return numpy.sqrt(numpy.square(numpy.linalg.norm(X1 - X2)) + numpy.squar
e(c))

# def Inverse_MulitQuadratic_Kernel(X1, X2, c):
#     return 1/(numpy.sqrt(numpy.square(numpy.linalg.norm(X1 - X2)) + numpy.sq
uare(c)))

def KMeans(points, centers, std_dev):
    labels = numpy.zeros(len(points))
    centers_old = numpy.zeros((len(centers), 2))
    centers_new = deepcopy(centers)
    Error = numpy.linalg.norm((centers_new - centers_old), axis=1)
    variance = numpy.array([])
    while Error.all() != 0:
        for i in range(0, len(points)):
            min_dist_label = numpy.argmin(numpy.linalg.norm((points[i] - cente
rs), axis=1))
            labels[i] = min_dist_label
        centers_old = deepcopy(centers_new)
        for i in range(0, len(centers)):
            temp = numpy.array([points[j] for j in range(0, len(points)) if la
bels[j] == i])
            if temp.size != 0:
                centers_new[i] = numpy.mean(temp, axis=0)
                variance = numpy.concatenate((variance, [numpy.std(temp)]), ax
is=0)
            else:
                centers_new[i] = centers_old[i]
                variance = numpy.concatenate((variance, [std_dev]), axis=0)
        Error = numpy.linalg.norm((centers_new - centers_old), axis=1)
    return centers_new, variance

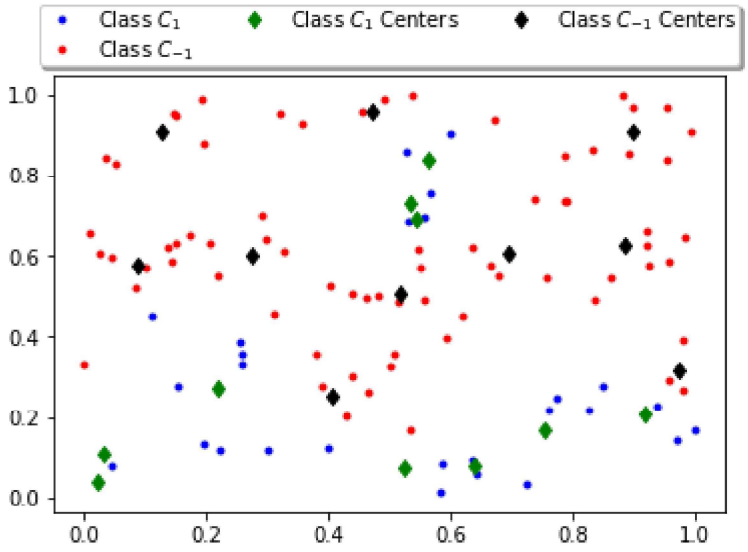
def eval_perceptron(points, labels, centers, variance, weights):
    m = 0
    for i in range(0, len(points)):
        input_vector = numpy.array([Gaussian_Kernel(points[i], centers[j], var
iance[j])
                                    for j in range(0, len(centers))])
        input_vector = numpy.concatenate(([1], input_vector))
        induced_local_field = numpy.dot(weights, input_vector)
        if induced_local_field * labels[i] < 0:
            m += 1
    return m

def update_weights(points, labels, centers, variance, weights, lr):
    for i in range(0, len(points)):
        input_vector = numpy.array([Gaussian_Kernel(points[i], centers[j], var
iance[j])
                                    for j in range(0, len(centers))])
        input_vector = numpy.concatenate(([1], input_vector))
```

```
        induced_local_field = numpy.dot(weights, input_vector)
        if induced_local_field >= 0:
            output = 1
        else:
            output = -1
        weights = weights + lr * (labels[i] - output) * input_vector
    return weights

def PTA(points, labels, final_centers, final_variance, weights, lr, epoch, mis
classification):
    e = 0
    m = eval_perceptron(points, labels, final_centers, final_variance, weights
)
    weights_current = update_weights(points, labels, final_centers, final_vari
ance, weights, lr)
    # Book Keeping
    misclassification = numpy.concatenate((misclassification, [m]), axis=0)
    epoch = numpy.concatenate((epoch, [e]), axis=0)
    # Print
    print('Number of errors in the Epoch ', e, ': \t', m)
    while m > 0:
        e += 1
        m = eval_perceptron(points, labels, final_centers, final_variance, wei
ghts_current)
        weights_current = update_weights(points, labels, final_centers, final_
variance, weights_current, lr)
        # Book Keeping
        misclassification = numpy.concatenate((misclassification, [m]), axis=0
)
        epoch = numpy.concatenate((epoch, [e]), axis=0)
        # Print
        print('Number of errors in the Epoch ', e, ': \t', m)
    print('Optimal Weight Reached!!!')
    return weights_current, epoch, misclassification
```

In [8]:
```
# K-Means Algo: Final Centers
Final_Pos_Centers, VP = KMeans(Pos_Class, Pos_Centers, Gaussian_Std_Deviation)
Final_Neg_Centers, VN = KMeans(Neg_Class, Neg_Centers, Gaussian_Std_Deviation)
Final_Centers = numpy.concatenate((Final_Pos_Centers, Final_Neg_Centers), axis
=0)
Final_Variance = numpy.concatenate((VP, VN), axis=0)
# Plot Results
plt.plot(Pos_Class[1:, 0], Pos_Class[1:, 1], 'b.', label=r'Class $C_1$')
plt.plot(Neg_Class[1:, 0], Neg_Class[1:, 1], 'r.', label=r'Class $C_{-1}$')
plt.plot(Final_Pos_Centers[:, 0], Final_Pos_Centers[:, 1], 'gd', label=r'Class
 $C_1$ Centers')
plt.plot(Final_Neg_Centers[:, 0], Final_Neg_Centers[:, 1], 'kd', label=r'Class
 $C_{-1}$ Centers')
plt.legend(loc='lower center', bbox_to_anchor=(0.5, 1), fancybox=True, shadow=
True, ncol=3, borderpad=0.1, labelspacing=0.1)
plt.savefig('Results2.pdf')
plt.show()
```



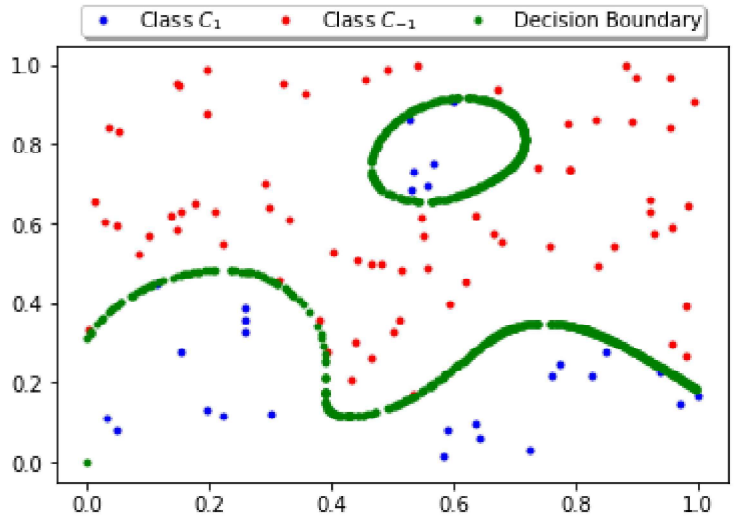
In [9]:
```
# PTA: Final Optimal Weights
Final_Weights, Epoch, Misclassification = PTA(X, D, Final_Centers, Final_Variance, Weights, eta, Epoch, Misclassification)
```

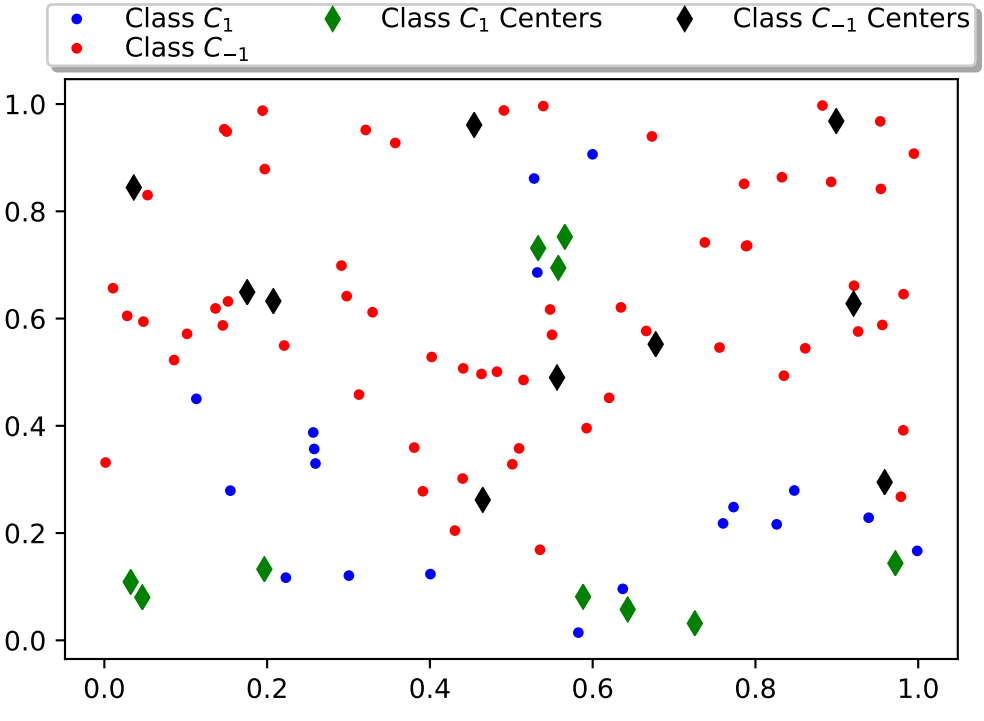
```
Number of errors in the Epoch  0 :       44
Number of errors in the Epoch  1 :       26
Number of errors in the Epoch  2 :       9
Number of errors in the Epoch  3 :       7
Number of errors in the Epoch  4 :       6
Number of errors in the Epoch  5 :       7
Number of errors in the Epoch  6 :       7
Number of errors in the Epoch  7 :       8
Number of errors in the Epoch  8 :       5
Number of errors in the Epoch  9 :       10
Number of errors in the Epoch  10 :      6
Number of errors in the Epoch  11 :      5
Number of errors in the Epoch  12 :      21
Number of errors in the Epoch  13 :      7
Number of errors in the Epoch  14 :      5
Number of errors in the Epoch  15 :      23
Number of errors in the Epoch  16 :      7
Number of errors in the Epoch  17 :      18
Number of errors in the Epoch  18 :      16
Number of errors in the Epoch  19 :      16
Number of errors in the Epoch  20 :      13
Number of errors in the Epoch  21 :      14
Number of errors in the Epoch  22 :      11
Number of errors in the Epoch  23 :      10
Number of errors in the Epoch  24 :      7
Number of errors in the Epoch  25 :      17
Number of errors in the Epoch  26 :      6
Number of errors in the Epoch  27 :      13
Number of errors in the Epoch  28 :      17
Number of errors in the Epoch  29 :      8
Number of errors in the Epoch  30 :      18
Number of errors in the Epoch  31 :      8
Number of errors in the Epoch  32 :      18
Number of errors in the Epoch  33 :      18
Number of errors in the Epoch  34 :      10
Number of errors in the Epoch  35 :      20
Number of errors in the Epoch  36 :      7
Number of errors in the Epoch  37 :      7
Number of errors in the Epoch  38 :      17
Number of errors in the Epoch  39 :      6
Number of errors in the Epoch  40 :      16
Number of errors in the Epoch  41 :      8
Number of errors in the Epoch  42 :      8
Number of errors in the Epoch  43 :      8
Number of errors in the Epoch  44 :      6
Number of errors in the Epoch  45 :      12
Number of errors in the Epoch  46 :      14
Number of errors in the Epoch  47 :      14
Number of errors in the Epoch  48 :      13
Number of errors in the Epoch  49 :      12
Number of errors in the Epoch  50 :      12
Number of errors in the Epoch  51 :      12
Number of errors in the Epoch  52 :      11
Number of errors in the Epoch  53 :      11
Number of errors in the Epoch  54 :      8
Number of errors in the Epoch  55 :      13
Number of errors in the Epoch  56 :      9
```

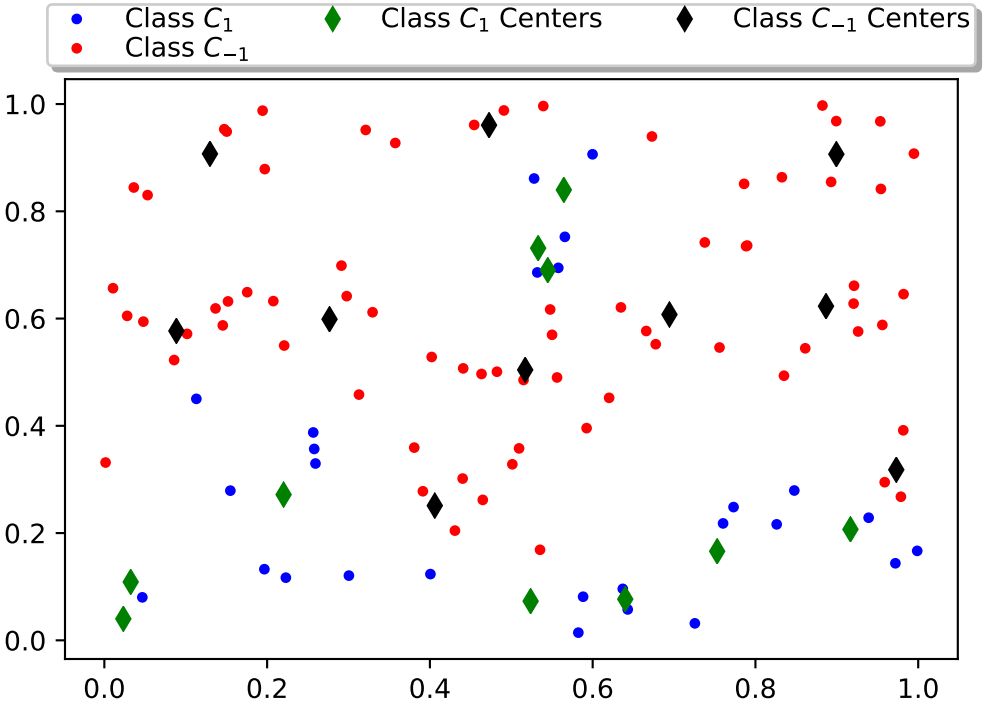
```
Number of errors in the Epoch  2850 :    7
Number of errors in the Epoch  2851 :    3
Number of errors in the Epoch  2852 :    4
Number of errors in the Epoch  2853 :    4
Number of errors in the Epoch  2854 :    4
Number of errors in the Epoch  2855 :    1
Number of errors in the Epoch  2856 :    3
Number of errors in the Epoch  2857 :    3
Number of errors in the Epoch  2858 :    2
Number of errors in the Epoch  2859 :    3
Number of errors in the Epoch  2860 :    2
Number of errors in the Epoch  2861 :    1
Number of errors in the Epoch  2862 :    3
Number of errors in the Epoch  2863 :    3
Number of errors in the Epoch  2864 :    3
Number of errors in the Epoch  2865 :    2
Number of errors in the Epoch  2866 :    7
Number of errors in the Epoch  2867 :    3
Number of errors in the Epoch  2868 :    4
Number of errors in the Epoch  2869 :    4
Number of errors in the Epoch  2870 :    1
Number of errors in the Epoch  2871 :    2
Number of errors in the Epoch  2872 :    3
Number of errors in the Epoch  2873 :    4
Number of errors in the Epoch  2874 :    5
Number of errors in the Epoch  2875 :    3
Number of errors in the Epoch  2876 :    3
Number of errors in the Epoch  2877 :    3
Number of errors in the Epoch  2878 :    2
Number of errors in the Epoch  2879 :    2
Number of errors in the Epoch  2880 :    1
Number of errors in the Epoch  2881 :    3
Number of errors in the Epoch  2882 :    1
Number of errors in the Epoch  2883 :    3
Number of errors in the Epoch  2884 :    1
Number of errors in the Epoch  2885 :    2
Number of errors in the Epoch  2886 :    3
Number of errors in the Epoch  2887 :    2
Number of errors in the Epoch  2888 :    0
Optimal Weight Reached!!!
```

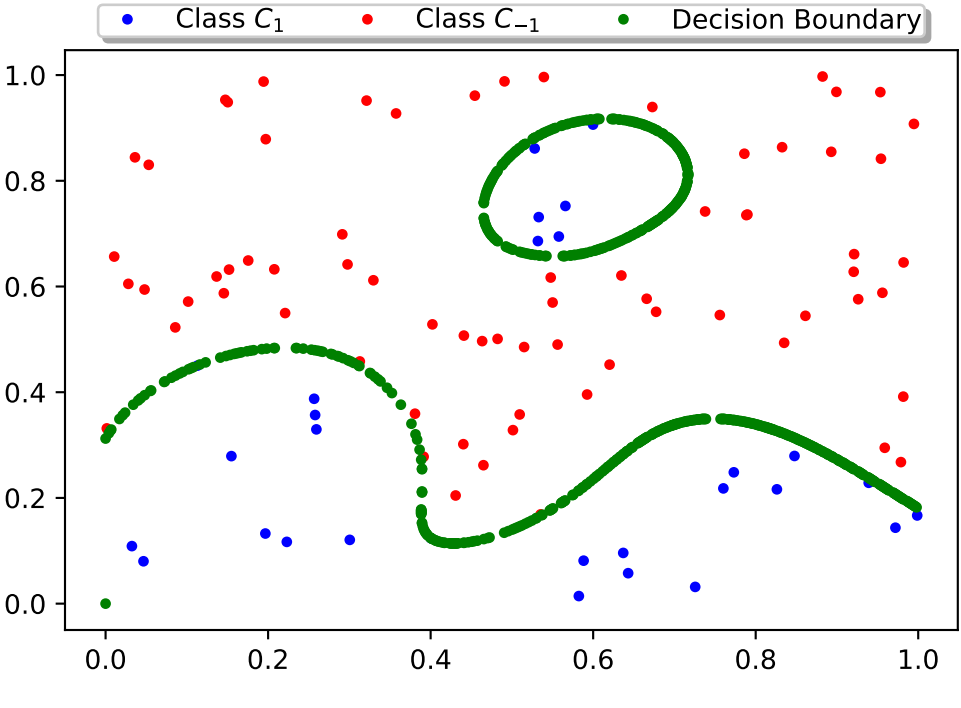
In [10]:
```
# Generate Boundary
x_points = numpy.linspace(0.0, 1.0, grid_size)
y_points = numpy.linspace(0.0, 1.0, grid_size)
H = numpy.array([[0, 0]]) # Decision Boundary
for i in range(0, grid_size):
    for j in range(0, grid_size):
        temp = numpy.array([x_points[i], y_points[j]])
        input_vector = numpy.array([Gaussian_Kernel(temp, Final_Centers[j], Fi
nal_Variance[j])
                                    for j in range(0, len(Final_Centers))])
        input_vector = numpy.concatenate(([1], input_vector))
        induced_local_field = numpy.dot(Final_Weights, input_vector)
        if -0.01 < induced_local_field < 0.01:
            H = numpy.concatenate((H, [temp]), axis=0)
```

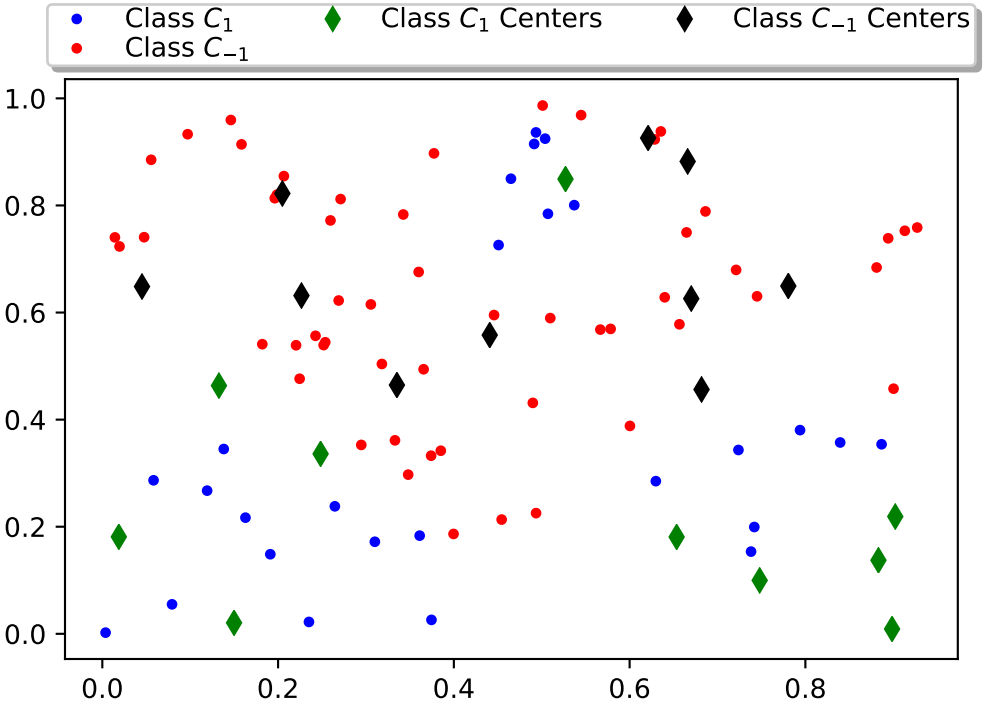
In [11]:
```
# Plot Results
plt.plot(Pos_Class[1:, 0], Pos_Class[1:, 1], 'b.', label=r'Class $C_1$')
plt.plot(Neg_Class[1:, 0], Neg_Class[1:, 1], 'r.', label=r'Class $C_{-1}$')
plt.plot(H[:, 0], H[:, 1], 'g.', label=r'Decision Boundary')
plt.legend(loc='lower center', bbox_to_anchor=(0.5, 1), fancybox=True, shadow=
True, ncol=3, borderpad=0.1, labelspacing=0.1)
plt.savefig('Results3.pdf')
plt.show()
```

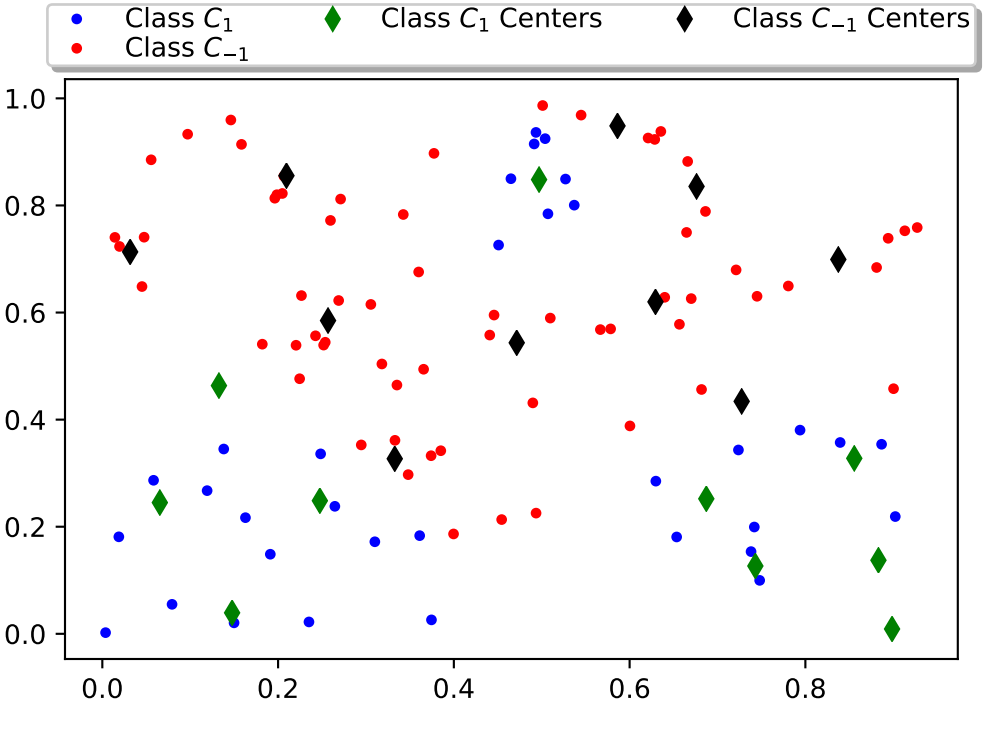


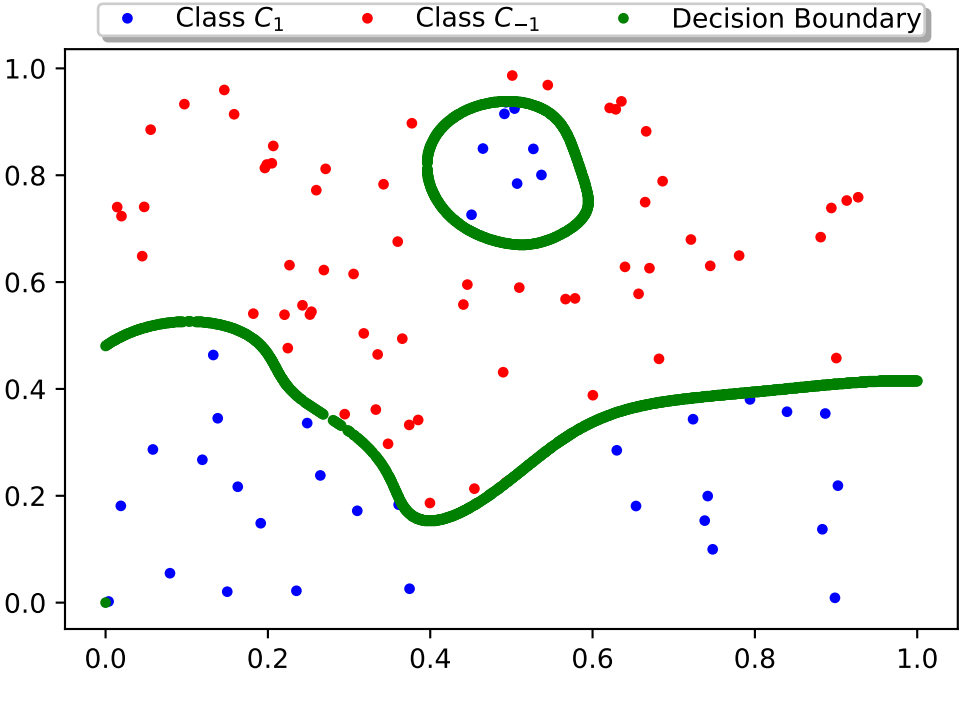

Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

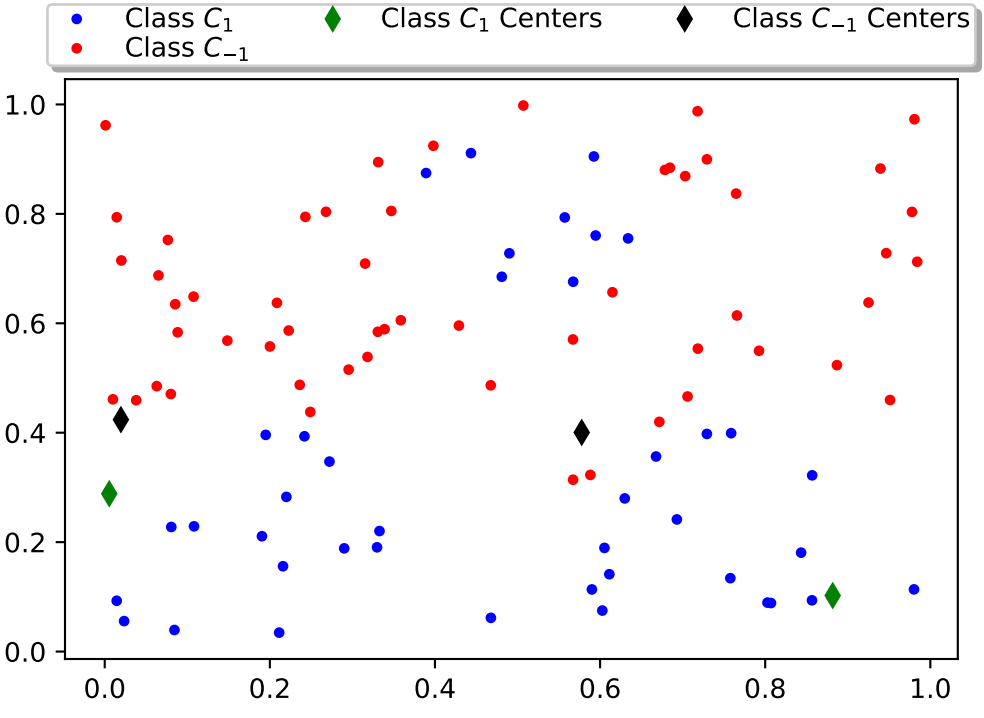

Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

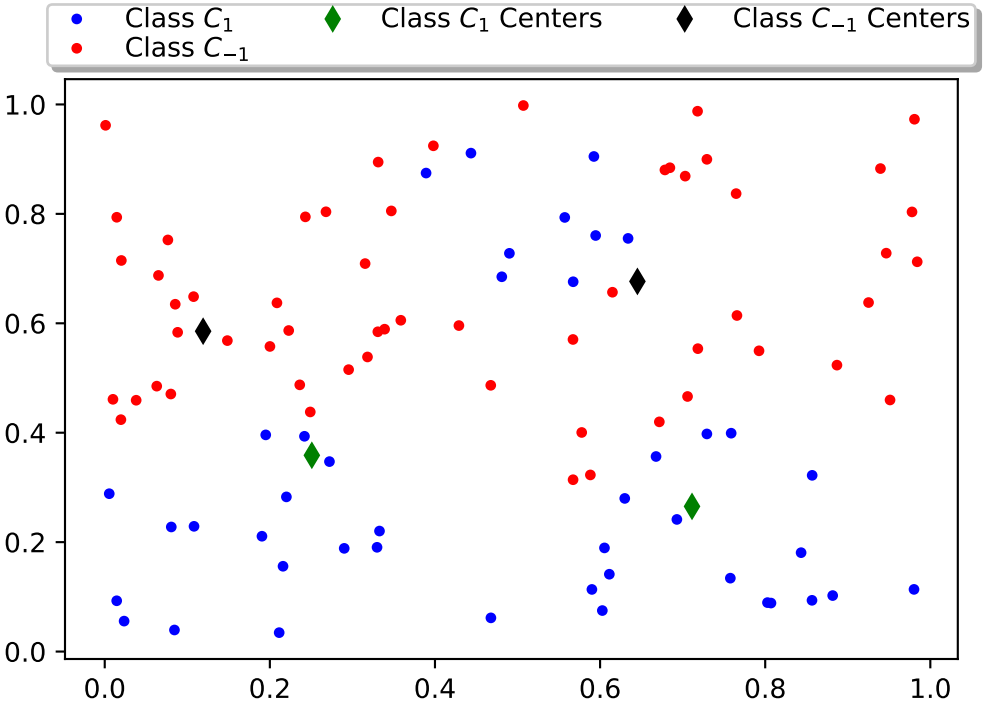

Class C_1
Class C_{-1}
Decision Boundary
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

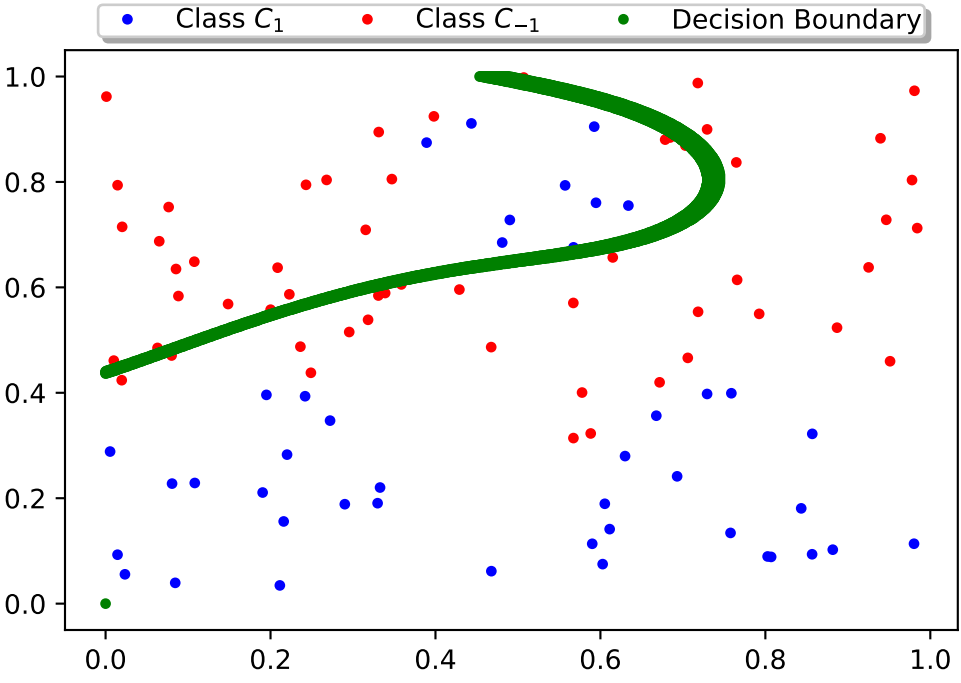

Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8


Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8


Class C_1
Class C_{-1}
Decision Boundary
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Class C_1
Class C_{-1}
Class C_1 Centers
Class C_{-1} Centers
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Class C_1
Class C_{-1}
Decision Boundary
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0