Department of Electrical and Computer Engineering
University of Illinois at Chicago

ECE 452 | Homework 3 | Date: 2/25/2018

Due date: 3/6/2018

1. Compute the matrix exponential of the twist $\xi = \begin{bmatrix} -0.2 & 0.5 & 0.1 & -0.4 & 0.3 & -0.1 \end{bmatrix}^T$.

2. Consider the rigid body transformation

$$g = \begin{bmatrix} -0.573973 & 0.312409 & -0.756938 & 1.57652 \\ 0.520871 & -0.573973 & -0.631861 & -1.28446 \\ -0.631861 & -0.756938 & 0.16672 & 0.875234 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   (a) Compute the exponential coordinates of g.

   (b) Find the screw motion to which the above rigid body motion corresponds.

3. Compute the rigid-body transformation corresponding to the screw motion around the line $l = \{\begin{bmatrix} -2 & 3 & 1 \end{bmatrix}^T + \lambda \begin{bmatrix} -12 & 20 & 9 \end{bmatrix}^T, \lambda \in \mathbb{R}\}$ with the pitch $h = 1.4$ for $\theta = 140°$.

   Hint: Make sure that you appropriately normalize the corresponding twist.

4. A rigid body is moving along a screw. The coordinate transformation between the body-fixed frame B and the global reference frame A is given by:

$$g_{ab}(\vartheta) = \begin{bmatrix} \frac{1}{9}(-2\cos(\vartheta) + 6\sin(\vartheta) + 2) & \frac{1}{9}(-5\cos(\vartheta) - 4) & \frac{1}{9}(-4\cos(\vartheta) - 3\sin(\vartheta) + 4) & \frac{1}{9}(-12\vartheta + 23\cos(\vartheta) + 6\sin(\vartheta) + 13) \\ \frac{2}{9}(\cos(\vartheta) + 3\sin(\vartheta) - 1) & \frac{1}{9}(-4\cos(\vartheta) + 3\sin(\vartheta) + 4) & \frac{1}{9}(-5\cos(\vartheta) - 4) & \frac{1}{9}(12\vartheta + 22\cos(\vartheta) - 9\sin(\vartheta) - 31) \\ \frac{1}{9}(8\cos(\vartheta) + 1) & \frac{2}{9}(\cos(\vartheta) + 3\sin(\vartheta) - 1) & \frac{1}{9}(-2\cos(\vartheta) + 6\sin(\vartheta) + 2) & -\frac{2}{9}(3\vartheta + \cos(\vartheta) + 15\sin(\vartheta) - 10) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

   where ϑ is the magnitude of the screw motion. Find the screw along which the body is moving.

   Hint: Recall that if the body moves along a screw corresponding to a twist $\hat{\xi}$, the motion is described by a rigid body transformation $g_{ab}(\theta) = e^{\hat{\xi}\theta} g_{ab}(0)$.

5. Give the geometric interpretation of the rigid body motion represented by the matrix

$$g = \begin{bmatrix} 0.327697 & 0.229144 & 0.916574 & 0.632756 \\ -0.229144 & 0.960453 & -0.158189 & -1.26669 \\ -0.916574 & -0.158189 & 0.367244 & 1.23325 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   Hint: Think about the geometric interpretation of the exponential coordinates.