Department of Electrical and Computer Engineering
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ECE 452 Homework 4 Date: 3/6/2018

Due date: 3/13/2018

1. A rigid body is moving along a screw around the line $l = \{\begin{bmatrix} 2 & -5 & 3 \end{bmatrix}^T + \lambda \begin{bmatrix} 20 & -9 & -12 \end{bmatrix}^T, \lambda \in \mathbb{R}\}$ with the pitch $h = 1.3$ and the magnitude $\theta(t) = 3t^2 + 2t$. At $t = 0$, the rigid body transformation between the global frame A and the body-fixed frame B is:

$$g_{ab}(0) = \begin{bmatrix} 0.510799 & 0.147375 & 0.846974 & 0.606065 \\ 0.767512 & -0.522024 & -0.372044 & 1.90451 \\ 0.38731 & 0.840102 & -0.379762 & 1.12323 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   (a) Compute the transformation matrix $g_{ab}(t)$.
   (b) Compute the spatial velocity $V_{ab}^s(t)$.
   (c) Compute the body velocity $V_{ab}^b(t)$.

   Note: *Problem 1b would be appropriate for the exam, you should be able to find the solution without the help of Matlab/Mathematica.*

2. Consider a rigid body with the body fixed frame B, whose configuration with respect to the frame A at time t_0 is given by:

$$g_{ab}(t_0) = \begin{bmatrix} 0.930114 & -0.209659 & -0.301549 & 1.35887 \\ -0.209659 & 0.371024 & -0.904646 & -2.7234 \\ 0.301549 & 0.904646 & 0.301137 & 0.738995 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   The spatial velocity of the rigid body at time t_0 (in vector form) is

$$V_{ab}^s(t_0) = \begin{bmatrix} -1.2 & 0.2 & 0.4 & -0.6 & -1.0 & 0.8 \end{bmatrix}^T.$$

   Find a screw motion $g(t) = e^{\hat{\xi}t} g(0)$ that passes through $g_{ab}(t_0)$ at time t_0 and would result in the same rigid body velocity at t_0. Note that $g(t)$ could be thought of as the screw motion along which the rigid body moves instantaneously at t_0.

3. Consider two frames A and B that are initially aligned (the axes are parallel, but the origins do not coincide). Assume that the frame B rotates with the rotational velocity described by a vector $\begin{bmatrix} -1 & -3 & 1 \end{bmatrix}^T$ in the frame A and that the origin of the frame B moves along the curve $\begin{bmatrix} -t^2 \cos t + t \sin t + 3 & t^2 \sin t + 2t \cos t - 3 \cos t & t^2 - 2 \sin t + 3t \cos t - 1 \end{bmatrix}^T$.

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(a) Find the spatial velocity V_{ab}^s as a function of time.

(b) Find the body velocity V_{ab}^b as a function of time.

(c) Find the transformation g_{ab} at time $t = 2s$.

4. The spatial velocity of the rigid body at $t = 1s$ is equal to

$$V_{ab}^s(1) = \begin{bmatrix} 0.5 & -0.2 & -0.3 & -0.7 & -0.4 & 0.6 \end{bmatrix}^T,$$

(in twist coordinates). What is the body velocity of the rigid body (in vector form) at $t = 1s$ if you know that

$$g_{ab}(1) = \begin{bmatrix} 0.544224 & 0.756286 & -0.363114 & 1.24676 \\ -0.821625 & 0.392997 & -0.412899 & -0.593889 \\ -0.169567 & 0.523052 & 0.835262 & -1.16116 \\ 0 & 0 & 0 & 1 \end{bmatrix}?$$

5. Consider three frames, A, B, and C. The **spatial** velocity of frame B relative to frame A is constant and equal to $V_{ab}^s = \begin{bmatrix} -2 & 7 & 5 & -1 & -4 & 8 \end{bmatrix}^T$. The **body** velocity of frame C relative to frame B is constant and equal to $V_{bc}^b = \begin{bmatrix} 6 & 9 & -5 & 4 & 7 & -1 \end{bmatrix}^T$. At $t = 0$, we have:

$$g_{ab}(0) = \begin{bmatrix} -0.43432 & -0.583051 & -0.686599 & 0.608676 \\ -0.876071 & 0.0961901 & 0.472491 & 1.37076 \\ -0.209442 & 0.806721 & -0.552571 & -1.28077 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and

$$g_{bc}(0) = \begin{bmatrix} 0.191896 & 0.929215 & 0.31581 & 1.15862 \\ 0.623914 & -0.363899 & 0.691599 & 0.385134 \\ 0.757567 & 0.0643232 & -0.649581 & 1.30157 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find the following quantities:

(a) Write the formulas (but do not compute) for $g_{ab}(t)$ and $g_{bc}(t)$.

(b) Compute the body velocity V_{ac}^b.

(c) Compute the spatial velocity V_{ac}^s.

Hint: *A motion with a constant velocity corresponds to a screw motion. See also page 58 in Murray, Li and Sastry.*

Note: *While this exercise is somewhat more computationally intensive than Problem 1b, it is reasonable enough that it would be appropriate for the exam (each part).*