# Department of Electrical and Computer Engineering
## University of Illinois at Chicago

ECE 452 $\qquad\qquad$ Homework 1 $\qquad\qquad$ Date: 2/9/2018

### Due date: 2/15/2018

1. Given a vector $v \in \mathbb{R}^3$, a skew-symmetric matrix $\hat{v} \in \mathbb{R}^{3\times3}$ can be defined so that $v \times w = \hat{v}\, w$ for any $w \in \mathbb{R}^3$.

   (a) Compute $\hat{v}$ for $v = [5, -1, -2]^T$.

   (b) Compute $R = e^{\hat{v}}$ for $\hat{v}$ computed above.

   (c) Recall that $R$ is a rotation matrix. Give the geometric interpretation (axis, angle) of the rotation described by $R$ computed in 1b.

2. What vector do you get if you rotate the vector $p = [5, 2, -4]^T$ by 75 degrees around the axis described by the vector $\omega = [4, 1, -3]^T$?

3. Consider a fixed frame $A$ and a rigid body that has a body-fixed frame $B$ attached to it. The two frames are initially aligned.

   Consider the following two sequences of rotations:

   (a) First rotate the object for 60 degrees around the $x$ axis of the frame $A$. Next, rotate the object for 45 degrees around the $z$ axis of the rotated frame $B$.

   (b) First rotate the object for 60 degrees around the $x$ axis of the frame $A$. Next, rotate the object for 45 degrees around the $z$ axis of the frame $A$.

   Compute the rotation matrix that describes the final rotation (configuration) of the frame $B$ in both cases.

4. Consider the following rotation matrix:
$$R = \begin{bmatrix} 0.4619 & -0.1189 & -0.8790 \\ -0.5615 & -0.8063 & -0.1860 \\ -0.6866 & 0.5794 & -0.4392 \end{bmatrix}.$$

   (a) Find the exponential coordinates of $R$.

   (b) Give the geometric interpretation (axis, angle) of the rotation described by $R$.

5. A rotation matrix between frames $A$ and $B$ is given by:
$$R_{ab} = \begin{bmatrix} 0.6325 & 0.2533 & -0.7319 \\ -0.7074 & 0.5737 & -0.4128 \\ 0.3154 & 0.7789 & 0.5421 \end{bmatrix}.$$

   A point $q$ is described in the frame $A$ by a vector $q_a = [-4, 3, 5]^T$. What is the description of $q$ in the frame $B$.