Department of Electrical and Computer Engineering
University of Illinois at Chicago

ECE 452 Homework 1 Date: 1/24/2018

Due date: 1/30/2018

1. Consider the following four vectors:

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -2 \\ 4 \\ -3 \\ 3 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 0 \\ -2 \\ 1 \\ 5 \end{bmatrix}.$$

   Compute:

   (a) $\vec{v}_1 + \vec{v}_2$;
   (b) $\vec{v}_1 + \vec{v}_3$;
   (c) $\vec{v}_2 \cdot \vec{v}_3$;
   (d) $\vec{v}_1 \cdot \vec{v}_3$;
   (e) $\vec{v}_1 \times \vec{v}_3$;
   (f) $\|\vec{v}_4\|$;
   (g) the angle θ_{24} between vectors $\vec{v}_2$ and $\vec{v}_4$.

   If necessary, comment on your answers.

2. Consider the matrices:

$$A = \begin{bmatrix} -4 & 5 & 4 \\ -2 & -4 & -2 \\ -3 & 4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & -5 & 3 \\ 5 & -5 & -4 \\ -3 & 1 & -1 \\ 3 & 2 & -4 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 1 & -2 \\ 1 & -2 & 3 \\ 0 & -2 & -3 \end{bmatrix}.$$

   Compute:

   (a) $A \cdot B$;
   (b) $A + B$;
   (c) $A + C$;
   (d) $B \cdot C$;
   (e) $C \cdot B$;

(f) A^2.

(g) B^2.

If necessary, comment on your answers.

3. Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & -1 & -4 \\ 6 & -2 & -6 \\ 2 & -1 & -3 \end{bmatrix}.$$

4. Solve the following system of linear ordinary differential equations for $x_1(t)$ and $x_2(t)$:

$$\begin{aligned} \dot{x}_1(t) &= -3\,x_1(t) - 2\,x_2(t), & x_1(0) &= -3 \\ \dot{x}_2(t) &= 4\,x_1(t) + 3\,x_2(t), & x_2(0) &= 1. \end{aligned}$$

Hint: Write the equations in the form $\dot{x}(t) = A \cdot x(t)$.

5. Let

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ -4 & -1 \end{bmatrix}.$$

Compute:

(a) $e^A \cdot e^B$;

(b) $e^B \cdot e^A$;

(c) e^{A+B}.

Comment on your answers.